FINANCIAL RISK CAPACITY

Saki Bigio

Financial crises seem particularly lengthy when banks fail to recapitalize after large losses. I explain this through a model where banks provide intermediation in markets with informational asymmetries. Large equity losses reduce a bank’s capacity to bear further losses. Losing this capacity leads to reductions in intermediation and exacerbate adverse selection. Adverse selection, in turn, lowers profits from intermediation which explains the failure to attract equity injections or retain earnings quickly. Financial crises are infrequent events characterized by low economic growth which is overcome only as banks slowly recover by retaining earnings. The model is contrasted against data on financial crises and several policy interventions are explored.

Keywords: Financial Crisis, Adverse Selection, Capacity Constraints.

1. INTRODUCTION

Oftentimes financial crises originate from episodes of extreme bank losses. Prolonged declines in economic activity follow when banks persistently retract lending after their equity is lost. This suggests that the impact and duration of crises could be mitigated if equity were reallocated into the financial industry. Not surprisingly, the slow recovery of bank equity was a major concern for policy makers, academics, and practitioners during the financial crisis of 2008-2009.\(^1\) In fact, during his only television interview, the Chairman of the Federal Reserve, Ben Bernanke, was asked when he would consider the crisis to be over. He answered, “When banks start raising capital on their own.”\(^2\) Why is it then that banks cannot attract capital during crises and prolong their duration?

This paper approaches this question from a novel angle. I view banks as intermediaries in financial markets that feature asymmetric information.\(^3\) Indeed, by dealing with a large number of parties, banks play a role in diluting transaction risks when asymmetric information is considerable. However, in practice, banks cannot dilute risk altogether so intermediation is risky. I also follow the literature on financial frictions arguing that the capacity to tolerate financial intermediation risk, \textit{i.e.} the \textit{financial risk capacity}, is tied to bank net worth. This features imply that large intermediation losses reduce bank net worth and magnify financial crises by exacerbating adverse selection. The intuition is that when banks lose net worth, they must cut back on lending per unit of collateral to decrease their exposure to future potential losses. In turn, borrowers respond using lower-quality assets as collateral. This response leads to an adverse selection cycle. Ultimately, adverse selection reduces bank profitability. Without profitable opportunities, the financial system can neither attract new equity nor accumulate profits quickly. This paper shows that this transmission mechanism operates even when the information structure or the production possibility of the economy are invariant.

Asymmetric information is key for this result. Absent other frictions, competition arguments suggest that banks should attract new equity in times when the economy most needs to rebuild its intermediation capacity. After

\(^1\)For example, the slow recovery of intermediary capital is the subject of Darrell Duffie’s 2010 Presidential Address to the American Finance Association (see Duffie, 2010).


\(^3\)This is a common view in banking theory (see Freixas and Rochet, 2008). A concrete example is Gorton (2010) who argues that “The essential function of banking is to create a special kind of debt, debt that is immune to adverse selection.”
all, like with any other service, marginal profits from intermediation should be high when the supply is low. High marginal profits should naturally attract the reallocation of equity to the financial system. A theory that links financial intermediation to bank net worth must explain why banks are not quickly recapitalized after large losses. This paper shows that asymmetric information breaks this equilibrium force. Asymmetric information can lead to reductions in returns on equity (ROE) that preclude banks from being recapitalized despite that financial resources are readily available. This feature distinguishes this from other macroeconomic models that study financial intermediation. In other models, banks cannot raise equity because bankers are fully-invested specialists and/or face other agency costs. However, these models deliver increases in bank ROE after bank losses, an opposite testable implication. Moreover, the economy studied in this paper responds very differently to bank losses of different magnitude. Equity injections and retained earnings are effective stabilizers of financial crises for moderate intermediation losses but fail to occur after large losses. This feature has important consequences for policy interventions as I argue later.

The model has four ingredients. (1) There is a demand for capital reallocation. (2) Financial intermediation facilitates this trade because there is asymmetric information in the market for capital. (3) Net worth evolves through time because this intermediation is risky. (4) Net worth is essential to provide intermediation because banks face limited-liability. Together, these four ingredients deliver recurrent and persistent financial crises. These crises are characterized by contractions in volumes of intermediation and reductions in collateral quality that lead to economic declines. In parallel, banks expect low equity returns, which is why they fail to build their equity.

I model the banking system as a competitive sector that provides intermediation in the reallocation of capital. As in Kiyotaki and Moore (2008), the first sector comprises producers of capital goods in need of funds. The second sector comprises agents that lack investment projects but, in contrast, have resources. The fundamental economic problem is that funds must flow from the latter to the former group, and capital must flow in the opposite way. The friction that prevents the direct flow of resources is private information about the quality of capital sold. Banks ameliorate this problem. They offer risk-free deposits to obtain funds from consumption producers. They transfer these funds to capital producers and take positions over a part their capital units (under asymmetric information). By facilitating trade, financial intermediation is fundamental for economic growth.

When parties share common information, the environment collapses to a classic stochastic-growth economy. These economies fluctuate only in response to shocks that affect the production possibility. Moreover, there is no need for financial intermediation. With asymmetric information about the quality of capital, banks dilute the idiosyncratic risk of transacting under disadvantageous information. However, if the distribution of capital quality is known, despite asymmetric information, banking is not risky. In equilibrium, banks can infer which qualities of capital are traded and, therefore, the value of their asset positions is risk-free. Without this additional source of risk, business cycles can originate from shocks that disperse the quality capital as in, for example, Bigio (2011) or Kurlat (2011). These shocks do not affect the production possibility but have real effects by aggravating adverse selection. However, fluctuations in these models are unrelated to the condition of banks. Furthermore, in these economies, severe adverse selection persists only if shocks are persistent.

To associate financial crisis to low bank net worth and adverse selection, this theory needs two additional ingredients. First, net worth must fluctuate. Second, net worth must matter. In the model, profits from intermediation are risky because, in addition to asymmetric information, the capital-quality distribution is, a priori, uncertain. Since this distribution evolves through time, the value of the banks’ asset positions is risky. This induces fluctuations in bank net worth.

To induce a role for net worth, I assume banks face limited-liability constraints. This constraint makes bank net worth a key state variable. Lower net worth banks must downsize their assets and liabilities to guarantee solvency upon future contingencies. Eventually, this reaction is responsible for the feedback-loop between asymmetric informa-

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4 In these models, banks cannot raise equity because bankers are fully-invested specialists and/or face other agency costs. However, these models deliver increases in bank return on equity after bank losses, an opposite but testable implication.

5 Formally, this presumes banks can exploit the law of large numbers to wipe out financial risk. Without risk, competition drives profits to 0 and bank equity plays no role.
tion and the evolution of net worth. Furthermore, this aspect also induces an externality: banks can fail to internalize that larger positions today may lead to greater losses tomorrow. This inefficiency is particularly damaging if adverse selection prevents banks from raising equity, and consequently, prolongs downturns. This calls for the policy interventions I study towards the end of the paper.

To summarize the mechanics of the model, consider the following hypothetical simulation. Suppose there is a sequence of shocks to the distribution of capital quality that lead to systematic financial losses. Losses are financed by liquidating a substantial portion of a bank’s net worth. Thus, net worth is depleted making it impossible to sustain the same magnitude of losses in the future. Banks respond by scaling down their operations and thereby exacerbating adverse selection. Adverse selection decreases expected bank profits and precludes external recapitalization. Instead, the financial system is recapitalized only through retained earnings. The problem is that this process must be very slow since volumes of intermediation and marginal profits are already low.

Figure 1 suggests how similar mechanics could have operated during the 2008-2009 financial crisis. The top-left panel plots the evolution of tangible net worth for a group of selected US bank holding companies (with and without TARP injections) during the last decade. The figure shows that, excluding TARP, this variable deviated from trend in the quarters prior to the Great Recession (second gray shaded area). The bottom-left panel describes the decline in the nominal stock of capital, a symptom of the deceleration in economic activity. The top-right panel shows that returns on equity (ROE) were stable during the period prior to the crisis. However, bank ROE falls during the recession and remains persistently below its historical average since. The bottom-right panel presents bank dividends and equity injections. Issuances were high in comparison to historical records but far from replenishing private bank equity.6

I tackle several technical challenges to solve the model. The model features asymmetric information, limited-liability constraints, and aggregate shocks. Despite this, it is highly tractable. I develop an algorithm that allows studying

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6The figure reports reconstructed time series of banking indicators based on the empirical strategy described in Section 6.
the global solution to the model. I show how to calibrate the model and provide rough measures of the effects of dividend taxes and capital requirements. These policy exercises show that the frequency and duration of financial crises can be reduced at the expense of economic growth.

The rest of the paper is organized in the following manner. The next section relates the paper to the literature. Section 3 introduces the model. Section 4 characterizes equilibria. Section 5 presents two analytic examples that underscore the role of asymmetric information in this economy. Section 6 presents the results of a quantitative example. Section 7 describes the effects of several policy experiments in this environment. Section 8 concludes. The computational algorithm and proofs to the appendix.

2. RELATIONSHIP WITH LITERATURE


This paper is tightly connected with two literatures. The first studies financial intermediation with a focus on bank net worth and the second one studies asymmetric information in financial markets. As portrayed in Figure 1, losses of financial net worth seem as a potential driving factor during the Great Recession. The literature explains this connection through several angles and a detailed description of all of these theories would require a more lengthy summary. However, they share a main theme, agency frictions on the side of intermediaries. Agency frictions were first incorporated into business cycle models by Bernanke and Gertler (1989) and Bernanke et al. (1996) placing them first on non-financial firms. Holmstrom and Tirole (1997) were the first to incorporate a similar frictions into financial firms but that model abstracted from any business cycle dynamics. Since the great recession, several papers have introduced these friction to state-of-the art business cycle models. Notably, Gertler and Karadi (2011) or Gertler and Kiyotaki (2010) use these models to analyze the benefits of government interventions.

This paper is closer to Brunnermeier and Sannikov (2011) and He and Krishnamurthy (2009) because intermediary losses here have an additional propagation mechanism and stress the importance of global methods. The distinction is that the propagation mechanism in those papers occurs through fire-sale spirals. These environments are typically constrained inefficient and call for government intervention.\(^7\) Another very related paper is Martinez-Miera and Suarez (2011) who incorporate a closer mechanism whereby contractions in lending also affect the riskiness of loans. Like that paper, I also analyze capital requirements.\(^8\)

The novelty in this paper relative to those is that financial risk is exacerbated by asymmetric information. Besides this propagation mechanism, a main distinction is in the incentives to recapitalize banks during a crises. In the literature, agency costs increase as net worth is lost. This means that the value of an additional unit of bank equity is greater in times of crises because the shadow value of relaxing those constraints is higher. If in those models, if outside equity injections were This distinction is relevant because the return to bank equity fell and remained low after the crisis. Here, in contrast, adverse selection reduces the profitability of equity injections. As a consequence, bankers choose not to inject equity despite free entry.

Another common explanation to the slow recapitalization of banks is the debt overhang problem stressed by Myers (1977). Philippon (2010) and Philippon and Schnabl (2011) apply this corporate finance insight to the banking industry and study government policies. I believe the mechanism here strengthens the debt overhang problem stress in those papers.

Early work by Stiglitz and Weiss (1981) and Myers and Majluf (1984) stressed that asymmetric information in financial markets can cause credit rationing. My paper also follows others in incorporating these ideas into general-equilibrium. For example, Carlstrom and Fuerst (1997) use private information in the return to investment to explain credit rationing during the cycle. Eisfeldt (2004) studies a model where assets are sold under asymmetric information

\(^7\)Similar feedback between losses in intermediary capital and reductions in the value of entrepreneurial capital occur in models such as Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009). Vayanos and Wang (2011) introduce asymmetric information into this framework. Fire sales were first analyzed by Shleifer and Vishny (1992).

\(^8\)In a similar environment, Rampini and Viswanathan (2011) study the effects of bank losses on investment and financing premia.
for self-insurance. The model is closer to Bigio (2011) or Kurlat (2011) who study models in which assets are sold under asymmetric information to relaxing financial constraints. These papers explain how shocks that exacerbate adverse selection can generate recessions. However, these models have no persistence in adverse selection. In contrast, here low bank equity leads to the persistent deterioration of credit markets. Other models that study lemons markets, such as Hendel and Lizzeri (1999), Plantin (2009), Kurlat (2011) or Daley and Green (2011), obtain persistence through learning dynamics.

Thus, the main contribution of this paper is to present a formal model of the interaction between financial intermediation and asymmetric information. Although the setup is new the idea is not. A similar mechanism has been described in the context of liability and catastrophe insurance that have featured recurrent large swings insurance premia and volumes after losses of equity. Winter (1991a) and Gron (1994a,b) survey the liability insurance market. The connection between this mechanism in insurance markets and credit markets is pointed by Duffie (2010).

Several modeling choices are taken from different papers. Banks here resemble those in the seminal work of O’Hara (1983) or Winter (1991b). The real side follows directly from Kiyotaki and Moore (2008) and asymmetric information is introduced similarly to Bigio (2011).

3. MODEL

3.1. Environment

The model is formulated in discrete time with an infinite horizon. Every period is divided into two stages, \( s \in \{1, 2\} \).

There are two goods: consumption goods (the numeraire) and capital goods. Consumption is perishable by the end of the period. There are two aggregate shocks: (1) a TFP shock \( A_t \in \{A_1, A_2, ..., A_M\} \) realized during the first stage and (2) a shock \( \phi_t \in \Phi \equiv \{\phi_1, \phi_2, ..., \phi_N\} \) that affects the depreciation of capital realized during the second stage. \((A_t, \phi_t)\) form a joint Markov process that evolves according to a transition probability \( \chi: (A \times \Phi) \times (A \times \Phi) \rightarrow [0, 1] \) with the standard assumptions.

It is worth noting now that in the description to follow, assets are traded during the first stage before the realization of \( \phi_t \). Claims are settled after the realization \( \phi_t \), and only then will producers decide on consumption and investment.

**Notation.** I use \( y_{t,s} \) to refer to the value of a variable \( y \) in period \( t \) stage \( s \) when the variable changes value between stages. Otherwise, if the variable remains constant through the period, I use the time subscript only.

**Demography.** There are two populations of agents, producers and bankers, and each population has a unit mass. Producers are in charge of productive activities, whereas bankers are intermediaries that facilitate the exchange of capital for consumption goods.

**Producers.** Producers are identified by a number \( z \in [0, 1] \) and carry their capital stock \( k_t(z) \) as an idiosyncratic state variable. Producers have log preferences over consumption streams and evaluate these according to an expected utility criterion:

\[
E \left[ \sum_{t \geq 0} \beta^t \log (c_t) \right],
\]

where \( c_t \) is consumption at time \( t \) and \( \beta \) their discount factor.

**Production Activities and Technologies.** At the beginning of the first stage, producers are randomly segmented into two groups, capital-goods producers (k-producers) and consumption-goods producers (c-producers). In particular, producers become k-producers with a probability \( \pi \) independent of time and \( z \).\(^9\)

A c-producer operates a linear technology during the first stage that produces \( A_t k_t(z) \) units of consumption. Although they have consumption goods, c-producers lack the possibility of augmenting their capital stock by building capital. In contrast, k-producers have access to an investment technology that allows them to produce one unit of

\(^9\)Randomization across activities is convenient to reduce the dimension of the state space of the model.
new capital with one unit of consumption. New capital is ready for production the following period. The investment technology is operated during the second stage. The capital held by k-producers remains idle, but it can be sold in exchange for consumption.

**Economic Problem.** The segmentation of activities induces a need for trade. On one hand, k-producers need consumption goods to consume and operate the investment technology. C-producers, on the other hand, have consumption goods but lack access to the investment technology. Thus, the economic problem here is that consumption goods must be transferred from c-producers to k-producers, and capital must be reallocated in the opposite direction.

**Capital Units.** At the beginning of the period, capital held by producers is homogeneous. During the first stage, the producer’s capital stock is divided into a continuum of units. Each unit is identified by some \( \omega \in [0, 1] \). The capital of every producer is divided in the same way: they all have the same composition across \( \omega \)'s and only differ in that the scale is proportional to the total capital stock.

These units can be individually reallocated across entrepreneurs. The rate at which units depreciate is given by their \( \omega \) and by the realization of \( \phi \). In particular, there is a function \( \lambda: [0, 1] \times \Phi \to \mathbb{R}_+ \), such that \( 1 - \lambda(\omega, \phi) \) is the depreciation rate of an \( \omega \)-unit when \( \phi \) is the realization of the shock. Once units are scaled by their corresponding \( \lambda(\omega, \phi) \), they become homogeneous units of \( t+1 \) capital. The following period, the capital stock held by producers is divided once again into different \( \omega \)'s, depreciated depending on \( \phi_{t+1} \), and the process is repeated indefinitely.

The maintained assumption is that \( \lambda(\omega, \phi) \) is increasing in \( \omega \). This implies that lower \( \omega \)'s depreciate faster. Note that since \( \phi_{t} \) is only realized during the second stage, the effective depreciation of a given quality is unknown at the first stage. However, producers know that for any realization of \( \phi_{t} \), higher \( \omega \)'s depreciate less and, therefore, are of better quality.

By the end of the second stage, the capital that remains for \( t+1 \) production from an original \( t \)-period stock \( k \) is \( \tilde{k} = k \int_{0}^{1} \lambda(\omega, \phi_{t}) \, d\omega \). However, producers may choose to sell particular units before the realization of \( \phi_{t} \). These decisions are summarized by an indicator function \( \mathbb{I}(\omega): [0, 1] \to \{0, 1\} \). \( \mathbb{I}(\omega) \) takes a value of 1 when an \( \omega \)-unit is sold. When a producer chooses \( \mathbb{I}(\omega) \), he is transferring \( k(z) \int_{0}^{1} \mathbb{I}(\omega) \, d\omega \) units. These units evolve into \( k(z) \int_{0}^{1} \mathbb{I}(\omega) \lambda(\omega, \phi_{t}) \, d\omega \) of \( t+1 \) capital after the realization of \( \phi_{t} \). Accounting shows that the \( t+1 \) that remains with the producer is \( k(z) \int_{0}^{1} [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi_{t}) \, d\omega \). Taking into consideration investments and possible purchases, the producer’s capital stock evolves according to:

\[
(1) \quad k_{t+1}(z) = i + k^b + k_t(z) \int_{0}^{1} [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi_{t}) \, d\omega.
\]

In this expression, \( i \) is \( t+1 \) capital created by investing (when possible) and \( k^b \) are purchases of \( t+1 \) capital.

Given \( \phi \), the average quality under a certain quality \( \omega^* \) is given by,

\[
\mathbb{E}[\lambda(\omega, \phi) | \omega \leq \omega^*] \equiv \frac{\int_{0}^{\omega^*} \lambda(\omega, \phi) \, d\omega}{\omega^*}.
\]

Let \( \bar{\lambda}(\phi) \equiv \mathbb{E}[\lambda(\omega, \phi) | \omega \leq 1] \) denote the unconditional quality average given \( \phi \). I impose some structure on \( \lambda(\omega, \phi) \) to provide an interpretation to the \( \phi \) shocks:

**Assumption 1** \( \lambda(\omega, \phi) \) satisfies that \( \mathbb{E}[\lambda(\omega, \phi) | \omega < \omega^*] \) is weakly decreasing in \( \phi \) for any \( \omega^* \).

The condition above states that the average quality under some cutoff \( \omega^* \) gets worse with larger \( \phi \). The key friction of the model is that \( \omega \) is private information. For this reason, in equilibrium, producers will sell qualities under an endogenously determined threshold. Assumption 1 implies that the buyer of these units, in this case the banker, is worse off with larger \( \phi \).

**Private Information.** The \( \omega \)-quality behind a given unit of capital is known only to its owner. This means that a buyer can only observe the size of a pool of units, \( \int_{0}^{1} \mathbb{I}(\omega) \, d\omega \), but cannot discern which \( \omega \)'s are in that pool.

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\(^{10}\) The assumption applies the definition of first-order-dominance to the conditional expectation function.
After $\phi$ is realized, the $t+1$ capital that remains from that pool is $\int_0^1 \mathbb{I}(\omega) \lambda(\omega, \phi_t) \, d\omega$. Therefore, any agent buying capital faces two sources of uncertainty: $\phi$ is unknown at the time of the purchase as is the $\omega$—composition of a given pool. In contrast, producers selling capital only face uncertainty about $\phi_t$. At the beginning of the second stage, the quality of a given pool becomes common knowledge and all uncertainty is gone.

**Bankers.** The reallocation of capital units in this economy is intermediated through bankers. Bankers are identified by some $j \in [0, 1]$. They have linear preferences over consumption streams and evaluate these according to an expected utility criterion:

$$E \left[ \sum_{t \geq 0} (\beta^f)^t c_t \right],$$

where $c_t$ is their consumption and $\beta^f$ the banker’s time discount factor. Bankers own legal institutions called banks. In addition, banks transform one unit of consumption into $R^k$ where $\mathbb{P}$ is interpreted as the bank’s net worth. In addition, at the beginning of every period, bankers receive a large exogenous endowment of consumption goods $\bar{e}_t(j)$. The variable $\bar{e}_t(j)$ is the banker’s personal wealth. Although $\bar{e}$ and $n$ are both consumption goods, there is an important legal distinction between the two. The banker’s personal endowment is protected by limited liability. Although these assets differ in that respect, during the first stage, bankers can alter the composition of their wealth by injecting equity from their personal wealth to their banks’ net worth, or do the opposite by paying dividends. After equity injections and dividends, their banks’ net worth evolves from $n_{t,1}(j)$ to $n_{t,2}(j)$. Bankers consume what remains of their endowment after dividends and equity injections.

Bankers intermediate in the market for capital by purchasing capital units from k-producers during the first stage and reselling them to c-producers during the second stage. Banks fund their capital purchases borrowing consumption goods from c-producers in the first stage and paying back during the second. Formally, they do so issuing tradeable riskless IOUs that entitle the holder to a unit of consumption in the second stage.\(^{11}\) These IOUs are redeemed by the second period and therefore bear no interest. Bankers buy capital units from k-producers under asymmetric information with the consumption goods obtained from this issuance.

After the shock is realized and depreciation is accounted, the pool of capital bought is resold as a homogeneous $t+1$ capital. In the process, the law of large numbers holds; given the realization of $\phi_t$, the actual average quality of capital bought coincides with the expected quality among the pool of sold units. However, since $\phi_t$ is realized between stages, depreciation is random and this is the source intermediation risk. Notice that bankers fund themselves with risk-less liabilities so they bear all the financial risk. Bankers experience losses if their issued IOUs exceed the market value of the pool of capital they sell.

When a banker experiences losses, he is forced to draw funds from his bank’s net worth to honor his liabilities. In principle, financial losses could be financed with the banker’s personal wealth. Instead, this does not happen because of limited liability. If IOUs are risk free, this imposes a limited-liability constraint (LLC): losses from financial intermediation cannot exceed his bank’s net worth under any contingency.\(^{12}\) Consequently, the bank’s net worth will restrain the amount of capital that banker’s can buy. The greater the amount of capital bought, the greater the risk and the greater the need for net worth as a cushion to support losses. Thus, the distinction between a banker’s personal endowment and his bank’s equity arises only because his net worth relaxes the LLC constraint. Since bankers can choose to inject equity from their personal endowment, they can relax that constraint at will. In equilibrium, they must have the incentives to do so.

Bankers face an exogenous constant probability of exit, $\rho$. When, $\rho = 0$, bankers are characterized by an infinite-horizon problem. When $\rho = 1$, bankers live one period, and one can solve the model analytically. Upon exit, bankers

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\(^{11}\)I explain how this institutional environment can be reinterpreted to resemble commercial bank practices in an online appendix.

\(^{12}\)Equivalently, this a *solvency* or *non-default* constraint.
sell their bank to an entrant banker. Most of the paper works with long-lived bankers but I set $\rho = 1$ to obtain closed form solutions by making the banker’s problem static.

**Public Information.** There are two endogenous aggregate states: $K_t = \int_0^1 k_t(z) \, dz$ and $N_{t,s} = \int_0^1 n_{t,s}(j) \, dj$. These states are the aggregate capital stock and the net worth of the financial sector respectively. It is only necessary to keep track of their ratio $\kappa_{t,s} \equiv N_{t,s}/K_t$, the relative size of the financial sector, as an aggregate endogenous state variable. Since $\kappa$ determines the capacity to bear losses relative to the wealth of producers, I refer to it as the financial risk capacity. Thus, the aggregate state of this economy is summarized by $X_{t,1} = \{A_t, \phi_{t-1}, \kappa_{t,1}\} \in \mathbb{X} \equiv \Lambda \times \Phi \times \mathbb{K}$ and $X_{t,2} = \{A_t, \phi_t, \kappa_{t,2}\} \in \mathbb{X} \equiv \Lambda \times \Phi \times \mathbb{K}$. At every point, $X_{t,s}$ is common knowledge. For tractability, I assume that the producer’s type is known. This assumption ensures that, in equilibrium, $c$-producers are excluded from selling capital. Bankers are informed about their exit at the beginning of the second stage.

**Markets.** The only possible transactions are purchases and sales of capital intermediated by bankers and the issuance of IOUs. There are two capital markets. The first market is where capital units are sold by $k$-producers and bought by banks under asymmetric information. This market opens during the first stage and satisfies the following assumption:

**Assumption 2** *Capital markets are anonymous and non-exclusive. Banks are competitive.*

Without anonymity (hidden capital), bankers could pay different prices depending on the volume of capital sold. With exclusivity, bankers could use dynamic incentives to screen. Assumption 2 therefore implies that the market in the first stage is a pooling market. Hence, there is a unique pooling price $p_t$ in this market.\(^\text{13}\) I refer to this market as the pooling market. The assumption that banks are competitive means that they take prices and traded qualities as given.

The second market opens during the second stage. This is the market in which bankers sell back all the units purchased during the first stage. This market clears at a price $q_t$. I refer to it as the resale market.

**Timing.** The summary of the timing is the following. At the beginning of the period, $A_t$ is realized and observed by all. Then, consumption goods are produced by $c$-producers. Bankers decided over equity injections and dividend payouts. $\kappa_{t,1}$ is updated and observed by all. Then, bankers obtain consumption goods issuing IOUs and use these funds to buy capital under asymmetric information. $k$-producers, in turn, choose the set of qualities to sell in the pooling market. During the second stage, $\phi_t$ is realized and $X_{t,2}$ is updated. Bankers learn the average quality of the capital they bought and resell the pool as homogeneous $t+1$ capital. By the end of the period, bankers settle claims against all issued IOUs realizing profits or losses. All producers simultaneously choose over consumption and purchases of capital and $k$-producers the scale of their investment.

The timing of the model is summarized by Figure 2. The following section describes the problem faced by the agents in this economy and the corresponding market-clearing conditions that define equilibria. This economy has a recursive representation, so from now on, I drop time subscripts. I use $x'$ to denote the value of a variable $x$ in the subsequent stage. I denote by $n$ bank equity brought from the previous period. $n'$ is bank equity after equity injections and dividends brought from stage 1 to stage 2. $n''$ is the bank equity after realizing profits in the second stage and taken to the subsequent period. The same notation is used for $\kappa$.

### 3.2. First-Stage Problems

**K-producer’s First Stage.** During the first stage, a $k$-producer enters the period with a capital stock $k$. At this stage, his only decision is about which qualities to sell:

**Problem 1 (k-producer’s $s=1$ problem)** *The k-producer’s first stage problem is:*

$$ V^k_1(k, X) = \max_{\{l(\omega)\in\{0,1\}} E \left[V^k_2(k' (\phi'), x, X') | X \right] $$

\(^\text{13}\)In a similar problem, Guerrieri and Shimer (2011) allow agents to trade in multiple markets for which capital is exchanged with some probability. Differences in probability and prices allow for separation. Bigio (2011) allows for repurchase contracts. Repurchase contracts improve over allocations but do not alter the essence of the results.
subject to $x = pk \int_0^1 \mathbb{I}(\omega) \, d\omega$ and $k'(\phi') = k \int_0^1 [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi') \, d\omega$.

The first equation is the k-producer’s budget constraint. $x$ is the quantity of consumption goods available during the second stage. This is obtained by selling $k \int_0^1 \mathbb{I}(\omega) \, d\omega$ units of capital at a pooling price $p$. The second equation accounts for the following period’s capital stock that remains with him. The solution to this problem determines a supply schedule for capital units in the pooling market.

**C-producer’s First Stage.** Since c-producers are excluded from the pooling market, they take no actions in this stage. Their value function is the expected value of their second stage’s value function:

**Problem 2 (c-producer’s $s=1$ problem)** The c-producer’s first stage value function:

$$V^c_1(k, X) = \mathbb{E}[V^c_2(k'(\phi'), x, X') | X]$$

where $x = Ak$ and $k'(\phi') = k \int_0^1 \mathbb{I}(\omega) \, d\omega$.

**Banker’s First Stage.** A banker enters the period with $n$ consumption goods stored in his bank and $\bar{e}$ as a personal endowment. A banker chooses an amount from his endowment, call this amount $e$, to be his equity injections to his bank. These injections come at the expense of decreasing consumption. He can reduce his bank’s net worth by transferring $d$ consumption units as dividends to be consumed after dividend taxes $\tau$. Equity injections and dividends are limited by their sources so $e \in [0, \bar{e}]$ and $d \in [0, n]$. In this paper I focus on equilibria in which $e \leq \bar{e}$ never binds with the interpretation that there are always resources available to recapitalize banks. The banker’s consumption is $c = (\bar{e} - e) + (1 - \tau) d$. His bank’s net worth is transformed instantaneously according to $n' = n + e - d$. The presence the dividend tax is introduced to obtain inaction regions for the banker’s financial policy. Without this tax, would feature very dramatic cycles. One can replace this convexity by introducing smooth-convex equity injection costs.

Let $Q$ be the volume of capital units purchased by a banker in the pooling market. He funds this purchases by issuing IOUs at face value $pQ$. These IOUs bear no interest and are redeemed by the second stage. During the second stage, the value of the capital pool purchased is $q \mathbb{E}[\lambda(\omega, \phi') | X] Q$. The resale market trades following-period capital at a resale price $q$. The $Q$ units are purchased in the current period, so these units are scaled by their average depreciation, $\mathbb{E}[\lambda(\omega, \phi') | X]$ to count them as following period capital. $\mathbb{E}[\lambda(\omega, \phi') | X]$ results from $\phi$ and

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14 A distinction between costly equity injections and dividend taxes is common in the dynamic corporate finance literature. See examples, Hennessy and Whited (2005) and Palazzo (2010), among others. In this environment, only the ratio of the cost of equity and dividend taxes matters, so I normalize the tax rate to account for this differences.
the \( (\omega) \)-policies that depend on \( X \).

The LLC states that the amount of issued IOUs cannot exceed the bank’s net worth plus the value of this capital under any realization of \( \phi' \):

\[
pQ \leq qE[\lambda(\omega, \phi') | X] Q + n' \text{ for any } (X, X') \in X \times X.
\]

Let \( \Pi(X, X') = qE[\lambda(\omega, \phi') | X] - p \) be the banker’s marginal profit from intermediation. \( \Pi(X, X') \) is a function of \( X \) and \( X' \) since qualities sold depend on \( X \), but their depreciation depends on \( X' \) through \( \phi' \). The banker’s problem is,

**Problem 3** The banker’s first stage problem is

\[
V_1^f(n, X) = \max_{Q, c \in [0, e], \bar{e}, d \in [0, n]} \left( c + E[V_2^f(n' + \Pi(X, X') Q, X') | X]\right)
\]

subject to

\[
\begin{align*}
-\Pi(X, X') Q & \leq n', \forall X' \\
\bar{c} & = (\bar{\varepsilon} - c) + (1 - \tau) d \\
n' & = n + e - d.
\end{align*}
\]

The first constraint is the LLC. The second and third constraints are the banker’s budget constraints and the law of motion of his net worth.

### 3.3. Second-Stage Problems

**K-producer’s Second Stage.** During the first stage, k-producers have sold part of their capital stocks in exchange for \( x \) consumption goods brought into the current stage. They also bring their remaining capital. Given their individual and the aggregate state, they solve,

**Problem 4** (i-entrepreneur’s s=2 problem) The k-producer’s problem in the second stage is:

\[
V_2^k(k, x, X) = \max_{c \geq 0, i, k^b \geq 0} \log(c) + \beta E[V_1^k(k', X') | X], \ j \in \{i, p\}
\]

subject to

\[
\begin{align*}
c + i + qk^b & = x \text{ and } k' = k^b + i + k
\end{align*}
\]

This budget constraint says that the k-producer uses \( x \) to consume \( c \), invest \( i \), or purchase \( k^b \) capital at price \( q \). The capital accumulation equation is consistent with equation (1) since \( k \) already incorporates sales and depreciation (accounted in the previous stage).

**C-producer’s Second Stage.** The c-producer’s problem is identical to the k-producer’s, except when the c-producer is restricted to set \( i \leq 0 \) because he lacks the investment technology:

**Problem 5** (p-entrepreneurs s=2 problem) The c-producer’s problem at the second stage is:

\[
V_2^c(k, x, X) = \max_{c \geq 0, i, k^b \geq 0} \log(c) + \beta E[V_1^c(k', X') | X], \ j \in \{i, p\}
\]

subject to

\[
\begin{align*}
c + i + qk^b & = x \text{ and } k' = k^b + i + k
\end{align*}
\]

**Banker’s Second Stage.** A banker’s only action during the second stage consists on reselling all the capital units purchased during the first stage after accounting for depreciation. Thus, their value is \( V_2^f(n, X) = \beta F E[V_1^f(R^b n, X') | X] \) if they remain in the industry or \( V_2^f = (1 - \tau) \beta F R^b n \) if they exit.

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15Investment reversibility is introduced for tractability.
3.4. Market-Clearing Conditions and Equilibrium

**Notation.** I append terms like \( j(k, X) \) to variables that indicate the policy function of a producer of type \( j \) in state \((k, X)\). I use \( I(\omega, k, X) \) to refer to a k-producer’s decision to sell an \( \omega \)-quality when his state is \((k, X)\).

**Aggregation.** In every period and stage, there are measures over capital holdings across the population of k and c-producers. I denote these measures by \( \Gamma^k \) and \( \Gamma^c \) respectively. By independence of the producer’s activities, these satisfy:

\[
\int_0^\infty \Gamma^k (dk) = \pi K \quad \text{and} \quad \int_0^\infty \Gamma^c (dk) = (1 - \pi) K.
\]

Their evolution is consistent with individual decisions and the activity segmentation process. In addition, there is also a measure \( \Lambda \) for the bankers net worth.

**First Stage.** Market clearing during the first stage requires that the demand for capital by bankers be equal to the supply of capital by k-producers. This condition is given by:

\[
\int_0^\infty Q(n, X) \Lambda (dn) = \int_0^\infty k \int_0^1 \| (\omega, k, X) \| d\omega \Gamma^k (dk)
\]

**Second Stage.** The demands for following-period capital by c- and k-producers are respectively:

\[
D^c (X, X') = \int_0^\infty k^{b,c} \left( x(k, X), k \int_0^1 \lambda (\omega, \phi') d\omega, X' \right) \Gamma^c (dk)
\]

and

\[
D^k (X, X') = \int_0^\infty k^{b,k} \left( x(k, X), k \int_0^1 [1 - \| (\omega, k, X) \|] \lambda (\omega, \phi') d\omega, X' \right) \Gamma^k (dk).
\]

Bankers sell all the units bought, so the supply of capital in the second stage is:

\[
S (X, X') = \mathbb{E} [\lambda (\omega, \phi') X] \int_0^\infty Q(n, X) \Lambda (dn).
\]

The market clearing condition in the second stage is \( S (X, X') = D^c (X, X') + D^k (X, X') \). A recursive competitive equilibrium is:

**Definition 1** (Recursive Competitive Equilibrium) A recursive competitive equilibrium (RCE) is \((1)\), a set of price functions, \( \{q(X, X'), p(X)\} \), \((2)\) a set of policy functions for c-producers \( c^c (x, k, X) \), \( k^{b,c} (x, k, X) \), \( \phi^c (x, k, X) \), a set of policy functions for k-producers \( c^k (x, k, X) \), \( k^{b,k} (x, k, X) \), \( \phi^k (x, k, X) \), a set of policy functions for bankers \( Q(n, X) \), \( e(n, X) \), \( d(n, X) \), \((3)\) sets of value functions, \( \{V^l_1 (k, X), V^l_2 (x, k, X)\}_{l=c,k} \), \( \{V^s_j (n, X)\}_{s=1,2} \), and \((4)\), a law of motion for the aggregate state \( X \), such that for any measures \( \Gamma^c \), \( \Gamma^k \) and \( \Lambda \) satisfying the consistency condition \((3)\) the following hold: \((I)\) The producers’ policy functions are solutions to their problems taking \( q(X, X') \), \( p(X) \) and the law of motion for \( X \) as given. \((II)\) \( Q(n, X) \), \( e(n, X) \), \( d(n, X) \) are solutions to the banker’s problem taking \( q(X, X') \), \( p(X) \) and the law of motion for \( X \) as given. \((III)\) Capital markets clear during the first and second stages. \((IV)\) The law of motion \( X \) is consistent with policy functions and the transition function \( \chi \). All expectations are consistent with the law of motion for \( X \) and agent policies.

The definition of equilibrium does not depend on the measures of asset holdings because this economy admits aggregation. This is shown once I characterize the solution.
4. CHARACTERIZATION

4.1. Policy Functions

Producers’ Second-Stage Policies. As a result of log-preferences, the c-producer’s policy functions are linear functions of his virtual wealth \( W^c(k, x, X) \equiv (A + q\lambda(\phi')) k \). His virtual wealth is the sum of produced consumption goods and the market value of his capital.

Proposition 1 In any RCE, the c-producer’s policy functions are \( k^c, l(k, x, X) = \beta \frac{W^c(k, x, X)}{q} \) and \( c^c(k, x, X) = (1 - \beta) W^c(k, x, X) \) and his value function is of the form \( V^c_2(k, x, X) = \psi^c(X) + \log(W^c(k, x, X)) \) where \( \psi^c(X) \) is a function of the aggregate state.

Policy functions for k-producers are also linear in their wealth. Their wealth is \( W^k(k, x, X) \equiv (x + q_i \mathbb{E}[\lambda(\omega, \phi')]|\omega < \omega^*(X)] k \). The k-producer’s wealth is the sum of the consumption goods obtained by selling capital units, \( x \), and the replacement value of the unsold units. The average depreciation of the unsold units is \( \mathbb{E}[\lambda(\omega, \phi')]|\omega < \omega^*(X)] \). This is the average depreciation under a cutoff quality \( \omega^*(X) \). The following section shows that, indeed, his capital sales decision is characterized by a threshold quality. The replacement cost of capital for k-producers is \( q_i = \min(1, q) \). The replacement cost is the minimum between the market value of capital and the production costs. Their policy functions take a similar form.

Proposition 2 In any RCE, the k-producer’s policy functions are \( k^{k, l}(k, x, X) = \beta \frac{W^k(k, x, X)}{q} \) and \( c^k(k, x, X) = (1 - \beta) W^k(k, x, X) \) and his value function is of the form \( V^k_2(k, x, X) = \psi^k(X) + \log(W^k(k, x, X)) \) where \( \psi^k(X) \) is a function of the aggregate state.

Producers’ First-Stage Policies. One can use Proposition 2 to obtain a closed-form expression for the k-producer’s value function during the first stage. Replacing the definitions of \( x \) and \( \mathbb{E}[\lambda(\omega, \phi')|\omega < \omega^*(X)] \), their value function is:

\[
V^k_1(k, X) = \max_{\mathbb{I}(\omega) \in \{0, 1\}} \mathbb{E} \left[ \log \left( p(X) \int_0^1 \mathbb{I}(\omega) d\omega + q_i(X, X') \int_0^1 \lambda(\omega, \phi') [1 - \mathbb{I}(\omega)] d\omega \right) | X \right] + \psi^k(X) + \log(k).
\]

Through this expression, it is clear that quality-sales decisions have to be the same across entrepreneurs regardless of their wealth. These decisions are characterized by a portfolio problem:

Proposition 3 In any RCE, the k-producer’s policy function in the first stage is given by \( \mathbb{I}^*(\omega, k, X) = 1 \) if \( \omega < \omega^* \) and 0 otherwise. The cutoff quality is given by:

\[
\omega^* = \arg \max_{\omega} \mathbb{E} \left[ \log \left( p\omega + q_i(X, X') \int_0^1 \lambda(\omega, \phi') d\omega \right) | X \right].
\]

Moreover, \( \omega^* \) is increasing in \( p \).

Proposition 3 shows that the solution to the producer’s problem during the first stage is given by a unique cutoff quality. This outcome resembles the solution to lemons problem of Akerlof (1970) but there is a distinction. Here, \( \omega^* \) is chosen by solving a portfolio problem because the producer does not know the outcome of \( \phi \) when selling capital. This portfolio problem has an intuitive interpretation: \( \omega^* \) is the fraction of the producer’s capital stock sold to banks. Once he exchanges these units, the producer loads the depreciation risk to the bank. In doing so, \( \omega^* \) becomes the risk-less portion of his portfolio. The remaining fraction, \( (1 - \omega^*) \), is risky because \( \phi' \) is realized after \( \omega^* \) is chosen. Since, \( \omega^* \) is increasing in \( p(X) \), the supply of capital has a typical upward sloping form. Consequently, the cutoff quality, \( \omega^*(X) \), indicates both the highest quality of capital traded and the volume of intermediation. From now on, I use threshold quality and volume of intermediation to refer to \( \omega^* \) interchangeably.

Bankers’ policies/ At the beginning of every period, bankers choose \( c, d \) and \( Q \) to maximize expected profits. The following Proposition shows that their value function and policies are linear in \( n \):

Proposition 4 The banker’s value function is a linear function of \( n \): \( V^f_1(n, X) = v^f_1(X) n \) and \( V^f_2(n, X) = v^f_2(X) n \) where \( v^f_1(X) \) and \( v^f_2(X) \) are the marginal value of financial equity in stages 1 and 2 respectively. \( v^f_1(X) \) solves the
The following Bellman equation:

\[ v_1^f (X) = \max_{Q \geq 0, e \in [0, e], d \in [0, 1]} (1 - \tau) d - e + E \left[ v_2^f (X') \left( \Pi (X, X') Q + n' \right) | X \right] \]

subject to,

\[-\Pi (X, X') Q \leq n', \forall X' \]

\[ n' = 1 + e - d. \]

The Bellman equation (5) is solved by linear optimal policies, \( e (X) \), \( d (X) \) and \( Q (X) \) given by:

\[ e (n, X) = e^* (X) n, \ d (n, X) = d^* (X) n, \ Q (n, X) = Q^* (X) (1 + e^* (X) - d^* (X)) n. \]

In addition, \( v_2^f (X) = \beta F R^b \) if the banker exits and \( v_2^f (X) = \beta F E \left[ v_1^f (X) R^b \right] \) otherwise.

The value function of the banker is linear in his net worth because of risk-neutrality and the linearity of the LLC. In equilibrium, there may be multiple solutions to \( d (n, X) \) and \( e (n, X) \) but, without loss of generality, I restrict the attention to linear policies. **Proposition 5** \( Q^* (X) \) is given by,

\[ Q^* (X) = \arg \max_{Q} E \left[ v_2^f (X') \Pi (X, X') | X \right] \hat{Q} \text{ subject to } \Pi (X, X') \hat{Q} \leq 1, \forall X'. \]

In equilibrium, \( \min_{X} \Pi (X, \hat{X}) < 0 \), and \( (e^* (X), d^* (X)) \), satisfy:

\[ e^* (X) > 0 \text{ only if } \beta F \left[ E \left[ v_2^f (X') \right] + \max \left\{ \frac{E \left[ v_2^f (X') \Pi (X, X') \right]}{-\min_{X} \Pi (X, \hat{X})}, 0 \right\} \right] \geq 1 \]

\[ d^* (X) > 0 \text{ only if } \beta F \left[ E \left[ v_2^f (X') \right] + \max \left\{ \frac{E \left[ v_2^f (X') \Pi (X, X') \right]}{-\min_{X} \Pi (X, \hat{X})}, 0 \right\} \right] \leq (1 - \tau). \]

When the inequalities are strict, \( e = \hat{e} \) and \( d = 1 \). \( e^* (X) \) and \( d^* (X) \) are indeterminate at the individual level when the relations hold with equality. \( e^* (X) \) and \( d^* (X) \) equal 0 when the inequalities are violated.

This proposition states that the LLC is binding whenever the expected discounted value of additional intermediation is positive, i.e., when \( E \left[ v_2^f (X') \Pi (X, X') | X \right] > 0 \). This term is the product of marginal profits, \( \Pi \), and the marginal value of bank equity \( v_2^f \). When, \( E \left[ v_2^f (X') \Pi (X, X') | X \right] = 0 \), \( Q \) is indeterminate and is 0 when this term is negative.

The proposition also states conditions for capital injections and dividend payoffs. These policies depend on the marginal value of keeping equity within the bank:

\[ \hat{v} (X) = \beta F \left[ E \left[ v_2^f (X') \right] + \max \left\{ \frac{E \left[ v_2^f (X') \Pi (X, X') \right]}{-\min_{X} \Pi (X, \hat{X})}, 0 \right\} \right]. \]

\( \hat{v} (X) \) has an intuitive interpretation. \( \beta F \) is the discount factor of future utility. The first term inside the bracket, \( E [v_2^f (X')] \), is the future marginal value of an additional unit of equity. This term is the bank’s stochastic discount factor. The second term represents the shadow value of equity obtained by relaxing the LLC. Recall that the inverse of the worst-case scenario losses, \( -\min_{X} \Pi (X, \hat{X}) \), is the bank’s marginal leverage. Increasing a unit of equity allows the bank to issue this amount in IOUs to purchase capital. This additional units of capital have an expected marginal value of \( E [v_2^f (X') \Pi (X, X')] \), which is the marginal value of an additional unit of intermediation. The max operator sets this marginal value to 0 when \( E [v_2^f (X') \Pi (X, X')] < 0 \).

When \( \hat{v} (X) < (1 - \tau) \), the banker prefers to pay out dividends: the marginal value of equity is \( \hat{v} (X) \), but the
after-tax marginal benefit of dividends is higher. In contrast, the banker injects equity to his bank when the value of holding equity exceeds one, the opportunity cost of equity in terms of foregone consumption. This result implies that banks have (S,s)-bands for their dividend policies.

The following section shows that in absence of asymmetric information, \( \hat{v}(X) \) is, in fact, monotone decreasing in \( \kappa \). However, if asymmetric information is sufficiently severe, \( \hat{v}(X) \) there are multiple inaction regions. This feature explains why the financial sector is not recapitalized or grows slowly when adverse selection is strong.

4.2. Market Prices and Bank Profits

**Resale-market Price Function.** The linearity of policy functions allows for aggregation. Integrating across the measure of c-producer’s capital stock yields their aggregate demand for following-period capital:

\[
D^c(X, X') = \left[ \frac{\beta (A + q(X, X') \lambda (\phi'))}{q(X, X')} - \tilde{\lambda} (\phi') \right] (1 - \pi) K.
\]

The demand for capital units by k-producers is obtained similarly. In this case, their aggregate demand function is broken up into three tranches because \( q \) determines whether they buy capital or produce it:\(^{15}\)

\[
D^k(X, X') = \begin{cases} 
\beta \frac{p(X) \omega^*(X)}{q(X, X')} & \text{if } q(X, X') < 1 \\
[0, \beta p(X) \omega^*(X) \pi K] & \text{if } q(X, X') = 1 \\
0 & \text{if } q(X, X') > 1.
\end{cases}
\]

Aggregate investment is the difference between the k-producer’s desired capital holdings and their demand for units sold by banks:\(^{17}\)

\[
I(X, X') = \beta \frac{p(X) \omega^*(X)}{q(X, X')} \pi K - D^k(X, X').
\]

Capital supplied by bankers during the second stage, \( S(X, X') \), are the units sold by k-producers during the first stage scaled by their average quality:

\[
S(X, X') = E[\lambda(\omega, \phi') | \omega < \omega^*(X)] \omega^*(X) \pi K.
\]

\( q(X, X') \) is obtained by using these expressions, and clearing out the price from the second-stage market-clearing equation. This price is characterized by:\(^{18}\)

**Proposition 6** In equilibrium \( q \) is given by,

\[
q(X, X') = \max \left\{ \left[ \frac{\beta A}{\pi \omega^*(X) E[\lambda(\omega, \phi') | \omega < \omega^*(X)] + (1 - \pi) (1 - \beta) \lambda (\phi')} \right], g(p(X), X') \right\}
\]

where

\[
g(p(X), X') = \min \left\{ 1, \left[ \frac{\beta (\pi p(X) \omega^*(X) + A (1 - \pi))}{\beta \omega^*(X) E[\lambda(\omega, \phi') | \omega < \omega^*(X)] + (1 - \beta) \lambda (\phi')} \right] \right\}.
\]

\(^{15}\)The first tranche is downwards sloping when \( q < 1 \). This is a region for values of \( q \) in which k-producers find it cheaper to buy capital than to produce it. The second region is a flat demand at \( q = 1 \) since k-producers are indifferent between investing or buying capital. Otherwise, k-producers do not participate in the market when \( q > 1 \) because it is less costly to produce capital directly.

\(^{17}\)From the expression, it is clear that investment is 0 when \( q < 1 \). When \( q = 1 \), \( D^k(X, X') \) is obtained as the residual of the difference between the supply of used units minus purchases by k-producers.

\(^{18}\)\( q(X, X') \) depends not only on the second stage state \( X' \) but also on \( X \). The dependence on \( X \) is because the supply of capital is a function of the cutoff function \( \omega^*(X) \), which is decided in the prior stage. This price may also depend on \( p(X) \) because this determines the wealth of k-producers who, in turn, purchase capital when \( q(X, X') < 1 \).
There are two things worth noting about this price function. First, it is immediate to show that it is decreasing in \( \omega^* (X) \). This is a natural outcome since larger cutoffs lead to more capital supplied by banks (and capital is a normal good). Secondly, the price is decreasing in \( \phi \). This shock lowers the average quality of capital for any possible cutoff, which is equivalent to lowering supply.

**Pooling-Market Price Function.** In equilibrium, market clearing in the first stage requires:

\[
Q^* (X) \int_0^\infty n^* (n, X) \, d\Lambda (n) = \omega^* (X) \int_0^\infty k\Gamma^k \, (dk) \iff Q^* (X) \kappa = \omega^* (X).
\]

Note that \( p(X) \) determines \( \omega^* (X) \) through the k-producer’s portfolio problem, which is independent of the capital stock. This is the reason why equilibria only depend on \( \kappa \) and not on the relative wealth of either sector. Conversely, any given pooling price \( p^* \) can be solved implicitly as a function \( \omega^* \) through the k-producer’s portfolio problem. To obtain the actual price in a given state, one must find the largest possible \( \omega^* \) for which the current \( \kappa \) can sustain the maximal losses given a level of intermediation. The equilibrium \( \omega^* \) corresponds to the value of \( p(X) \).

**Proposition 7** Given a state \( X \) and a law of motion for the state, there exists some \( p(X) \) satisfying market clearing. In addition, in any Pareto un-improvable equilibrium, \( p(X) \) is weakly increasing, and right continuous in the financial risk capacity \( \kappa \).

This result implies that the volume of intermediation is greater when the financial risk capacity is higher. In turn, \( \kappa \) is determined by the bankers’ financial decisions that ultimately depend on expected returns. Nothing precludes multiplicity though. The reason is that during the first stage, there could be two or more equilibrium triplets \((\omega, Q, p(X), q(X,X'))\) that satisfy the market clearing and limited liability. As prices increase, both the average quality of capital sold and the quantity increase. As a consequence, bank profits are possibly non-monotone in \( \omega \). Hence, worst-case profits for two different equilibrium prices may be the same. This source of multiplicity is common to all models with asymmetric information. Although multiplicity is an interesting phenomenon in and of itself, it is not the focus of this paper. Hence, for the rest of the paper, I introduce an equilibrium refinement: —

**Definition 2** (Pareto Un-improvable Equilibrium) A RCE is Pareto un-improvable if given the law of motion for \( X, \forall X, \) there does not exist any \( p^* > p(X) \), such that \( p^* \) satisfies market clearing in the first stage, and induces a second stage market clearing price \( \tilde{q}(p,X') \) that is consistent with the producer’s policy functions and the LLC.

This refinement selects the RCE where the volume of intermediation is greatest. A later section shows that such equilibria cannot be Pareto improved upon. We need to show some intermediate results first. Before proceeding to the characterization, I provide a description of alternative interpretations of the LLC constraint and financial intermediation. — **Bank Profits.** Replacing the functional form \( q \) obtained in Proposition 6, we find an analytic expression for marginal profits:

\[
\Pi(X,X') = \begin{cases} 
\frac{\beta (\pi p(X) \omega^* (X) + A (1-\pi))}{\beta \omega^* (X) \pi + (1-\beta) \mathbb{E} [\lambda (\omega, \phi') | \omega^* (X)]} - p(\omega^* (X)) & \text{if } q(X,X') < 1 \\
\mathbb{E} [\lambda (\omega, \phi') | \omega < \omega^* (X)] - p(\omega^* (X)) & \text{if } q(X,X') = 1 \\
\frac{\beta A}{\pi \omega^* (X) + (1-\pi) (1-\beta) \mathbb{E} [\lambda (\omega, \phi') | \omega < \omega^* (X)]} - p(\omega^* (X)) & \text{if } q(X,X') > 1
\end{cases}
\]

The behavior of \( \Pi \) is the heart of the model. Recall that \( p(X) \) is determined prior to the realization of \( \phi' \). Hence, this price does not respond to \( \phi' \). One can observe that \( \Pi(X,X') \) is decreasing in \( \phi' \) because \( \mathbb{E} [\lambda (\omega, \phi') | \omega < \omega^* (X)] \) is decreasing in the shock. Thus, \( \phi' \) affects marginal profits along two dimensions. On the quantity dimension, it reduces the holdings of capital by banks by affecting depreciation. On the price dimension, \( q(X,X') \), increases do to the reduction in supply. Marginal profits decrease because the first effect always dominates. Thus, larger \( \phi' \) are associated with lower intermediation profits. Thus, profits are decreasing in \( \phi \).

In turn, \( \Pi(X,X') \) is not necessarily monotone in \( \kappa \). This is the main feature that determines the evolution of \( \kappa \) and, consequently, the dynamics of this economy.
Why are marginal intermediation profits non-monotone in \( \kappa \)? From Proposition 7 we know that \( p(X) \) is increasing in \( \kappa \). Hence, any non-monotonicity must follow from the value of capital. Two effects oppose each other in the determination of this value. On one hand, there is substitution effect. Since capital is a normal good, the greater the volume of capital supplied, the lower its price \( q(X,X') \). The second effect is a composition effect. As the volume \( \omega^*(X) \) increases with \( \kappa \), so does its average quality. These two effects interact in a way that marginal profits are non-monotonic if \( \lambda(\omega,\phi) \) is sufficiently sensitive to \( \omega \). The non-monotonicity of profits in \( \kappa \) also makes \( \tilde{v}(X) \), non-monotone. This is ultimately the critical factor that prevents the recapitalization of the financial system in times of low \( \kappa \).

Since \( q(X,X') \) is decreasing in \( \kappa \) regardless of the state. This feature implies that if \( \lambda(\omega,\phi) \) is constant across qualities, that is, if asymmetric information is not present, \( \mathbb{E}[\lambda(\omega,\phi') | \omega > \omega^*(X)] \) is constant. Hence, marginal profits are decreasing in \( \kappa \). This proves,

**Proposition 8** Without asymmetric information, marginal profits \( \Pi(X,X') \) are decreasing in the financial risk capacity \( \kappa \). For sufficiently severe asymmetric information \( \Pi(X,X') \), is non-monotone.

### 4.3. Evolution of Financial Risk Capacity

**First-Stage Evolution of Financial Risk Capacity.** At the beginning of the first stage, \( \kappa \) evolves according to:

\[
(7) \quad \kappa' = (1 + e^*(X) - d^*(X)) \kappa.
\]

**Equity Injections and Dividends.** In equilibrium, \( \tilde{v}(X) \in [(1 - \tau),1] \) for any \( X \). If it were the case that \( \tilde{v}(X) > 1 \) for a given \( X \), equity would be injected until this value is one because bankers have the incentives to recapitalize their banks. If the value where above 1, this would imply that \( \kappa \) is not in equilibrium. Thus, states where \( \tilde{v}(X) > 1 \) are instantaneously reflected into a new state where \( \tilde{v}(A \times \phi \times \kappa') = 1 \) for some \( \kappa' > \kappa \).

The opposite occurs when \( \tilde{v}(X) < (1 - \tau) \). In such states, dividends are paid out until \( \tilde{v}(X) = (1 - \tau) \). This means that there is an inaction region, where \( \kappa \) is not altered by the financial policies, characterized by states where \( \tilde{v}(X) \in [1 - \tau,1] \).

**Multiplicity of Financial Policies.** Equation (7) is implicit in \( \kappa' \) since \( \kappa' \in X \). Proposition 5 imposes constraints on the set of equilibrium \( \kappa' \). However, since a banker’s decisions are functions of other banker’s decisions, there are multiple equity injection policies consistent with equilibrium.

**Equilibrium Selection.** For the rest of the paper, I use a refinement and refer to this equilibrium simply as the equilibrium. In particular, I select an equilibrium that depends on the current realization of \( \kappa \), which is otherwise an inessential state because it can be altered instantaneously. If \( \tilde{v}(A \times \phi \times \kappa) \in [(1 - \tau),1] \), then \( e^*(X) = d^*(X) = 0 \). Thus, I restrict attention to equilibria such that if the current state lies within an inaction region, then, the economy remains within that region.

Accordingly, if \( \tilde{v}(A \times \phi \times \kappa) > 1 \), I select equilibria for which \( e^*(X) \) takes \( \kappa' \) to the closest value of \( \kappa \) such that \( \tilde{v}(A \times \phi \times \kappa) = 1 \). Analogously, if \( \tilde{v}(A \times \phi \times \kappa) < (1 - \tau) \), then \( d^*(X) \) takes \( \kappa' \) to the closest value below \( \kappa \) such that \( \tilde{v}(A \times \phi \times \kappa) = 1 \).

I several reasons to select this equilibrium. First, I assumed that \( \bar{e} \) is never binding to simplify the solution to the model and to argue that bankers may fail to inject equity in spite of resources being available. However, if \( \bar{e} \) is binding, it could be the case that it is insufficient to take \( \kappa' \) to a point where \( \tilde{v}(X) \geq 1 \). Thus, this equilibrium selection, by minimizing the change in financial risk capacity, approximates an economy with limited resources. A second reason is that \( \kappa \) is a physical variable. It seems natural to coordinate an equilibrium over an observed physical variable than over sunspot variables. Finally, this equilibrium selection delivers persistent declines in output, intermediation and traded capital quality which is what I try to explain with this model.
**Second-Stage Evolution of Financial Risk Capacity.** Between the first and the second stages, $\kappa$ evolves depending on realized profits and the growth rate of the capital stock:

$$
\kappa' = R^b \left[ 1 + \Pi (X, X') \right] \frac{\kappa}{\gamma (X, X')}
$$

In this expression, $\gamma (X, X')$ is the growth rate of the capital stock:

$$
\gamma (X, X') = \pi \beta \left[ p (X) \omega^* (X) + q^* (X, X') \mathbb{E} \left[ \lambda (\omega, \phi') | \omega > \omega^* (X) \right] \omega^* (X) \right] + (1 - \pi) \beta \frac{A + q (X, X') \lambda (\phi)}{q (X, X')}.
$$

**Equity Value.** We can obtain a recursive expression for $\hat{v} (X)$ and $v_1^f (X)$ that depend on the transition function from $X$ to $X''$ by evaluating the Bellman equation at the optimal policies. The marginal value of bank equity at any state $X$ is:

$$
(8) \quad v_1^f (X) = \min \left\{ \max \left\{ \beta R^b \mathbb{E} \left[ \hat{v} (X) | X \right], (1 - \tau) \right\}, 1 \right\}.
$$

Combining this expression with the definitions of $\hat{v} (X)$ and $v_1^f (X)$ defines a self-map for $v_1^f (X)$.

### 4.4. States of the Financial Industry

In any RCE, the state space may be divided into several regions depending on the incentives to inject equity or pay dividends. For a given exogenous state, $(A, \phi)$, these incentives are summarized by the marginal value of equity and its change with respect to $\kappa$.

**Dividend-Payoff Reflecting Barrier.** Dividends are paid whenever $\hat{v} (X) < (1 - \tau)$. When $\kappa$ falls within this region, dividends payments instantaneously reduce $\kappa$ to its closest reflecting barrier. According to the equilibrium selection, that is the closest $\kappa$ satisfying $\hat{v} (X) = (1 - \tau)$. A dividend-payoff region associated with a sufficiently large $\kappa$ always exists. However, there may be dividend-payoff regions for intermediate values of $\kappa$ if the quality effect brings down marginal profits for low values of $\kappa$.

**Equity-Injection Reflecting Barrier** When $\kappa$ is within states where $v (X) > 1$, the value of equity attracts equity injections. These injections reflect $\kappa$ upwards towards the closest value satisfying $v (X) = 1$. Similarly, there may also be multiple intervals of $\kappa$ with this property. There may also be potentially multiple disconnected equity-injection and dividend-payoff regions.

**Competitive Inaction Regime.** The other two possible regions correspond to inaction regions. The first is a competitive inaction region defined as follows.

**Definition 3** (Competitive Inaction Region) A state $X$ is in a competitive inaction region if (a) $\hat{v} (X) \in [(1 - \tau), 1]$ and (b) when $v_\kappa (X)$ exists, $v_\kappa (X) \leq 0$ and, if defined, $v_\kappa (X) |_{\kappa} \leq 0$ for any $\hat{\kappa} > \kappa$.

Condition (a) implies this is indeed an inaction region. Condition (b) implies that the expected discounted marginal profits are decreasing in $\kappa$. This implies that there are less incentives to recapitalize banks as the financial risk capacity increases. In equilibrium $\hat{v} (X)$ may jump as a consequence of the Pareto refinement. For this reason, condition (b) is not equivalent to $v (X)$ being locally decreasing. Hence, the condition states that $v (X)$ is decreasing above $\kappa$, except at the finitely many points. This definition captures the idea that the quantity effect dominates the quality (adverse selection) in a competitive inaction region.

**Proposition 9** Every equilibrium has a competitive inaction region. Moreover, the inaction region with the largest volumes of intermediation is always competitive.

---

19 Blackwell's conditions cannot be checked immediately because it may fail to satisfy the discounting property. The operator is monotone, though. All numerical iterations lead to the same outcome. When $\rho = 1$, analytical expressions can be obtained.

20 This is because expected profits must be decreasing for large enough $\omega^*$. This occurs because the quantity effect always dominates the quality effect as $\omega^*$ approaches 1.
Financial Crisis (inaction) Regime. The remaining region are financial crises. In a financial crises regime, $\kappa$ is low in the sense that it triggers adverse selection. With low volumes of intermediation, market clearing requires low $p(X)$, but this brings down the quality of capital traded. Moreover, expected profits are low, and this discourages equity injections. By definition, higher levels of $\kappa$ increase the value equity in this region. This happens as more financial risk capacity leads to more intermediation, ameliorates adverse selection, and increases expected profits altogether. Bankers choose not to recapitalize because they lack the incentives for the given level of expected marginal profits.

Hence, bankers face a coordination problem in a financial crisis region. The reason is that equity injections could drive $\hat{v}(X)$ to 1, raising the value of every banker’s equity. However, since profits are low in a crisis regime, bankers fail to synchronize over the recapitalization that would end the financial crisis.

4.5. Solving Equilibria

This section outlines the strategy to compute equilibria. The solution involves two steps. The first is to compute first- and second-stage prices and expected profits for any (possibly off-equilibrium) volume of intermediation given exogenous states $(A, \phi, \phi')$. The second step uses these calculations to find equilibrium cutoffs given $\kappa$. With this, one obtains $\tilde{v}(X)$, and the inaction regions for $\kappa$.

Notation. Equilibrium objects are functions of the aggregate state. To compute equilibria, one needs to obtain prices and profits for any off-equilibrium $\omega$. I use bold letters to distinguish equilibrium from off-equilibrium objects. I use $p(\omega, \phi)$ to indicate the first-stage supply schedule given $\phi$ and a value of $\omega$ that is off equilibrium. Also, $q(\omega, p, A, \phi')$ denotes the price consistent with second-stage market clearing for the set of values of $(\omega, p, A, \phi')$. Finally, $\Pi(\omega, p, A, \phi')$ are the corresponding profits given arbitrary prices, volumes, and exogenous states.

Step 1: Prices and Profits for off-equilibrium cutoffs. Through Proposition 3, we can find a first-stage price $p(\omega, \phi)$ associated with $\omega$ by inverting the solution to the k-producer’s portfolio problem. We can also use Proposition 6 to obtain $q(\omega, p, A, \phi')$ for any $(\omega, p, A, \phi')$. With this, we can compute $\Pi(\omega, p, A, \phi')$. Computing this is done once.

Step 2.1: Equilibrium Volumes, Prices, and Profits. We begin with an initial guess for the marginal value for bank equity $\hat{v}(X)$. This value is updated in the following step. Using the calculations in Step 1, we find the equilibrium volume of intermediation given this guess. For each $X$, we look for the volumes yielding non-negative expected discounted profits and the largest $\omega$ such that the worst-case losses are at most $\kappa$:

$$\omega^*(X) = \max \left( \omega : \kappa \leq \min_{\phi} \Pi(\omega, p, A, \phi') \omega \text{ and } E[\hat{v}(X')] \Pi(\omega, p, A, \phi') |X| \geq 0 \right).$$

Since $\Pi(\omega, p, A, \phi')$ is continuous and $\omega \in [0, 1]$, this quantity is well defined. $\omega^*(X)$ is the largest volume of intermediation yielding non-negative expected profits such that there is enough capacity to sustain losses in the worst state. Thus, $\omega^*(X)$ is consistent with the banker’s problem and is the Pareto un-improvable equilibrium volume, the largest volume satisfying market clearing and optimal decisions by all agents.

Step 2.2: Equilibrium $\hat{v}(X)$. Given this $\omega^*(X)$, one can compute $\Pi(X, X') = \Pi(\omega^*(X), p(\omega^*(X), \phi), A, \phi')$. We use the functional equation (8) and the definition of $v_1^f(X)$ to update $\hat{v}(X)$. Steps 2.1 and 2.2 are iterated until convergence. When $\rho = 1$, $v_2^f(X) = (1 - \tau)$, so steps 2.1 and 2.2 are done only once. Appendix A provides details for the implementation of this strategy in a computer.

5. ANALYTIC EXAMPLES

This section provides two examples that illustrate how equity injections and dividends are stabilizing forces for in economy where intermediation is essential for growth. The first example describes a version of the model under symmetric information. The second introduces asymmetric information. In presence of severe asymmetric information,
recapitalization no longer stabilizes financial markets in response to large shocks. For the rest of this section, I assume \( \rho = 1 \) to use analytic expressions.

5.1. Example 1 - Risky intermediation without asymmetric information.

The first example is an economy where financial intermediation is risky but asymmetric information is not present. Assume that \( \lambda(\omega, \phi) = \lambda^*(\phi) \), so all units are of the same quality but decreasing in \( \phi \). In addition, \( \phi \) takes two values, \( \phi_B > \phi_G \). Draws are i.i.d. and \( A \) is constant. We have the following:

**Proposition 10** In any economy without asymmetric information, \( \kappa' \) fluctuates within a unique equilibrium interval \( [\kappa, \bar{\kappa}] \). If \( \kappa \leq \kappa \), then \( c^*(X) \) is such that \( \kappa' = \kappa \). If \( \kappa \geq \bar{\kappa} \), \( d^*(X) \) is such that \( \kappa' = \bar{\kappa} \). \( v(X) \) is decreasing and \( \omega(X) \) is increasing in \( \kappa \).

**Proof:** The proof to this proposition is straight forward. From Proposition 6, we know that \( \Pi(\omega, p, \phi) \) is decreasing in \( \omega \) since quality effects are not present without asymmetric information. Also, as noted earlier, \( \rho = 1 \), \( v_0(X) = (1 - \tau) \). We can use \( \Pi(\omega, p, \phi) \) and equation (6) to obtain an expression for the marginal value of equity in terms of any arbitrary \( \omega \). Call that value \( \tilde{v}(\omega, p) \). Without asymmetric information, \( \tilde{v}(\omega, p, A, \phi) \) is decreasing in \( \omega \). Consequently, by Propositions 5, there is a unique interval for \( \omega \) such that \( \tilde{v}(\omega, p) \in [(1 - \tau), 1] \). Correspondingly, since \( \Pi \) is decreasing in \( \omega \), there is a unique equilibrium interval for \( \kappa \) that determines a unique competitive inaction region.

\( Q.E.D. \)

Figure 3 shows the construction of equilibria graphically. The upper-left panel depicts four curves associated with an arbitrary \( \omega \). These correspond to the capital-supply schedule, \( p(\omega, \phi) \), the marginal value of bank assets in good and bad states, \( q(\omega, \phi_H) \lambda^*(\phi_H) \) and \( q(\omega, \phi_L) \lambda^*(\phi_L) \), and their expected value \( \mathbb{E}[q(\omega, \phi) \lambda^*(\phi)] \). The difference between \( \mathbb{E}[q(\omega, \phi) \lambda^*(\phi)] \) and \( p(\omega) \) is the expected marginal profit from financial intermediation. Multiplying this amount by \( \omega \) yields the total expected bank profits \( \mathbb{E}[q(\omega, \phi) \lambda^*(\phi) - p(\omega)] \omega \) normalized by the capital stock. Total expected bank profits are plotted at the bottom-left panel. The bottom-right panel plots worst-case profits, \( |q(\omega, \phi_H) \lambda^*(\phi_H)| - p(\omega)| \omega \). In equilibrium, \( \kappa \) must be sufficient to sustain the losses induced by a volume of intermediation associated with it. The panel in the top right plots the expected value of bank equity \( \tilde{v}(\omega, p) \) as a function of \( \omega \). The horizontal lines in the top-right panel are the marginal costs of injecting equity, 1, and the marginal benefit of dividend pay-offs \( (1 - \tau) \). In equilibrium, if a given \( \omega \) is the equilibrium volume of intermediation, bankers must not alter their net worth. The set of possible equilibrium \( \omega \) is characterized by volumes for which the value of equity falls within the marginal cost of injections and the benefit of dividend payments. The shaded areas in the graphs correspond to this set. Since, \( \tilde{v}(\omega, p) \) is decreasing in \( \omega \), the equilibrium set is a unique interval. For each \( \omega \) in that interval, there is an equilibrium \( \kappa \) corresponding to it. We obtain this equilibrium set by computing the maximal losses given each \( \omega \) in the equilibrium set. The bottom-right panel shows this interval for \( \kappa \) is obtained as the image of worst-case losses for the equilibrium \( \omega \)-set.

Figure 4 plots four equilibrium objects. These are obtained by following the second step of the solution method described earlier. The top-left panel plots \( \omega^* \) as a function of \( \kappa \) (that is, \( \kappa \) before equity injections or dividends). The panel on the top right depicts \( \tilde{v} \). In equilibrium, \( \kappa' \) must be in the inaction region where \( \tilde{v}(\kappa) \in [(1 - \tau), 1] \). The bottom panel depicts equity injections, dividends and \( \kappa' \) as functions of \( \kappa \). In equilibrium, \( e \) and \( d \) adjust to bring \( \kappa' \) to the equilibrium set depicted in Figure 3. The shaded area in the figure is the competitive inaction region. The regions to the right and left of the shaded area are the dividend payoff and equity injection regions, respectively.

**Dynamics.** Proposition 10 is useful to understand the dynamics of this economy. Recall that, in equilibrium, worst-case losses are always negative. The converse would imply infinite leverage and infinite expected profits. In contrast, expected profits must be non-negative or, otherwise, no intermediation would be provided. This implies that \( \kappa \) will increase or decrease depending on the realization of \( \phi \). When \( \phi_B \) is realized, profits are negative and drag \( \kappa \) down. Below \( \kappa \), profits attract equity injections that recapitalize banks and increase the financial risk capacity.
Figure 3.— Model without asymmetric information: Equilibrium objects as functions of $\omega$. 

Figure 4.— Model without asymmetric information: Equilibrium objects the as functions of $\kappa$. 
Thus, injections stabilize a financial system with low financial risk capacity. When $\phi_G$ is realized, dividends work in the opposite direction and $\kappa$ increases. When it increases beyond $\bar{\kappa}$, dividend payoffs reflect the financial risk capacity downwards. Hence, without asymmetric information, $\kappa$ fluctuates within a unique interval. Let me now show how asymmetric information changes things in a way that precludes this stabilizing force.

### 5.2. Example 2 - Risky intermediation with asymmetric information.

I modify $\lambda(\omega, \phi)$ to introduce asymmetric. I fix values for the lower and upper bounds of $\lambda(\omega, \phi)$, $\lambda_L$ and $\lambda_H$. I use the following functional form $E[\lambda(\omega, \phi) | \omega < \omega^*] = \lambda_L + (\lambda_H - \lambda_L) F_\phi(\omega^*)$. I assume $F_\phi$ is the CDF of a Beta distribution where $\phi$ indexes its parameters in a way that satisfies Assumption 1. The rest of the calibration is the same as in the previous example.

Figure 5 is the asymmetric-information analog of Figure 3. The upper-left panel shows four curves that correspond to $p(\omega, \phi)$, $q(\omega, p, \phi_H)$, $q(\omega, p, \phi_L) \lambda(\phi_L)$, and $E[q(\omega, p, \phi) \lambda(\phi)]$. Note that $q(\omega, p, \phi_H) \lambda(\phi_H)$ and $q(\omega, p, \phi_L) \lambda(\phi_L)$ are no longer decreasing in $\omega$ after asymmetric information is introduced. As volumes increase, the price of capital falls, but the quality improves. The relative strength of either effect governs the shape of the value of bank assets. These forces cause total expected and worst-case profits to be non-monotonic (bottom-left and right panels). Same levels of worst-case losses can result from multiple values of $\omega$. This implies that a given $\kappa$ can possibly sustain multiple levels of intermediation. The Pareto refinement implies that, in equilibrium, the largest volume consistent with bank optimality and the capacity constraint is intermediated. The top-right panel plots the marginal value of

![Figure 5: Model with asymmetric information: Equilibrium objects as functions of $\omega$. Parameters are set to: $\pi = 0.1$, $\beta = 0.98$, $\beta^f = 0.6$, $A = 1.45$, $\tau = 0.6364$, $\rho = 1$.](image)
This system will have much richer dynamics in this case. For this particular example, there are three equilibrium inaction intervals identified by the shaded areas. The equilibrium intervals for the financial risk capacity are obtained as the image of the worst-case losses for each equilibrium $\omega$-interval. The intervals corresponding to the largest sizes of $\kappa$ (intervals I and II in the figure) are competitive inaction regions. The upper bound of interval II in the figure has a distinctive property: If $\kappa$ is increased slightly at that point, it can support a much larger level of intermediation. This happens because there is a much larger level of intermediation that only increases worst-case losses infinitesimally.

Interval III is a financial crises regime. It is associated with low levels of financial intermediation. This region is an inaction region since bankers do not inject equity at these levels. Note that for larger values of $\omega$, equity injections are profitable since $\tilde{v}$ is above 1. As discussed earlier, this underscores the nature of the coordination failure faced by banks. Banks choose to maintain their net worth at current levels and engage in less intermediation.

Figure 6 plots the equilibrium objects as functions of $\kappa$. The upper-left panel plots the equilibrium financial intermediation. We can observe a discrete jump in the volume of intermediation from the second to the first region. This jump is the result of the fact that slightly larger $\kappa$ can support a much larger volume of intermediation and the Pareto refinement selects the equilibrium with largest volumes. The equity injection region between regions I and II is very small: regions I and II are close to each other. Note that to the left of the second region, the marginal value of equity $\tilde{v}$ is increasing for very low values of $\kappa$. The financial crisis regime can be barely observed because the volume of intermediation and losses associated with it are very small. This regime occurs when $\tilde{v}$ crosses the cost of equity injections. In presence of asymmetric information, $e$ and $d$ may cease to adjust $\kappa'$ for low values. By means of an example, we have shown:

**Proposition 11** For sufficiently severe asymmetric information, the return to financial intermediation is non-monotone in $\kappa$ and there exists a financial crises regime.

**Dynamics.** The immediate effects $\phi$ on $\kappa$ are the same as in the version without asymmetric information. However, the dynamics can be very different. With asymmetric information, a realization of $\phi = \phi_B$ can drive the financial system to the financial crises regime. As adverse selection effects are aggravated, profitability no longer justifies the
injection of capital into the system, and the economy may take long to recover. The economy eventually recovers as
banks slowly build equity through retained earnings. Once $\kappa$ reaches the equity injection regions between intervals III
and II, the banking system attracts equity injections, and $\kappa$ reaches the competitive inaction region II. The following
section of the paper studies some quantitative examples.

6. QUANTITATIVE EXAMPLES

6.1. Additional Features

I incorporate additional features to the model that do not the main insights but improve its performance.

Financial Management Costs. I assume bankers pay a constant amount of their equity every period and a
constant bonus if profits are positive. This parameter corresponds to $\psi$ in Table ??.

Capital Requirements. Capital requirements can be modeled by introducing a wedge into the LLC. Assuming
that the capital requirement is such that banks are not allowed to lose more than a fraction $\theta$ of equity in a given
period, the LLC reads:

$$-\Pi(X, X') Q \leq (1 - \theta) n'.$$

Physical Capital Cost. I introduce an additional parameter so that the cost of unit of capital is not one. I
introduce it so that I can use the estimated process for TFP described in the next section.

6.2. Parameters

Calibrated Parameters. I set $\beta$ and $\beta_f$ so that the risk free rate in the deterministic frictionless version of
the model is 3.0% in annual terms. The return to equity in stage two, $R^e$, is set to 1. I assume that the average
depreciation rate is a constant and equal to 0.9756 and gives a lower bound to the growth rate of the economy. The
fraction of capital good producers $\pi$ is set to 0.1. I set the banker’s exit rate, $\rho$, to 0 (infinite horizon). The tax $\tau$ is
calibrated to yield a marginal cost of equity of 10% and dividends taxes 30% following Hennessy and Whited (2005).
I set, $\theta$, to 8% to be consistent with Basel-II requirements.

Estimated Parameters. I estimate an AR(1) process for $\log(A_t)$ (details are presented in the data appendix).
The auto-correlation coefficient is estimated to be 0.993. Its mean is $-0.885$, and the standard deviation 0.0083. I
assume that $\phi$ follows a four-state Markov chain. The transition matrix for this Markov chain is calibrated using
a property of the model. Expected profits are positive, but worst-case profits must be negative in equilibrium. For
example, if $\phi$ takes only two values, positive profits are associated with the lowest value of $\phi$ and negative profits
with its largest value. I use the series for bank profits to estimate the transition probabilities for $\phi$ in that case. This
transition probability is determines expected profits and the value of equity injections. I use a four state Markov
chain in order to compare the responses to large and small shocks to $\phi$.

Matched Parameters. To calibrate the quality distributions associated with each $\phi$ I use the same parametric
form as in earlier examples. Under this form, $\lambda_L$ is the lowest depreciation and $F_\phi(\omega^*)$ is any CDF with support in
[0, 1]. I set $\lambda_L = 0$ to have the interpretation that some capital is worthless. For each of the four values of $\phi$, there
are parameters denoted by $(A_\phi, B_\phi)$ that summarize the quality distribution associated with that state. $A_\phi$ and $B_\phi$
are uniformly spaced between $[A_L, A_H]$ and $[B_L, B_H]$ respectively. Thus, there is a total of four parameters that
characterize a total of four quality distributions.

These parameters are set to yield four moments: historical and Great Recession levels for bank leverage and bank
ROA. There is mapping from these moments to the functional equations of the qualities. In the model, the difference
between the expected value of capital and the cost of capital are the bank’s ROA. The cost of capital is given by the
supply schedule obtained by solving the k-producer’s portfolio problem. This supply schedule is only a function of
$[A_L, A_H, B_L, B_H]$ and the transition matrix of $\phi$. The demand for capital is also a function of these parameters and the
real-side parameters. In turn, the bank’s leverage is the inverse of worst case losses, also function of $[A_L, A_H, B_L, B_H]$. Using this property, I search for values that lead to leverage and ROA as close as possible to the targets. There are four parameters for four moments (two unconditional moments, and two moments for crisis episodes). However, since these parameters affect the outcome of other moments, I am forced to make some compromises to obtain reasonable numbers.

In addition, I set the non-interest expense parameter $\psi$ to 8%. This matches the non-interest expenditures to equity ratios for the selected sample of BHCs. Table I summarizes the parameter values.

### 6.3. Results

The results in this and the following sections are meant as illustrative examples as much as Brunnermeier and Sannikov, Adrian and Boyarchenko and or He and Krishnamurthy. I have included some facts about financial crises to get a sense of how far the model is from the data. However, although I view this section as presenting illustrative examples, I believe the lessons here will survive in a more thorough quantitative analysis.

#### 6.3.1. Invariant Distribution and Historical Histograms

Figure 7 reports four histograms corresponding to the invariant distributions for $\kappa$ and the growth rate of $K$, both in the model and for the US. Bars represent occupation frequencies. The top-left panel depicts the marginal invariant distribution of $\kappa$. The top-right panel is the analog for the US data. In the model, the financial risk capacity spends most of time at the right of the distribution. However, there is some probability mass concentrated at the left end of the distribution. A similar concentration mass is found for the empirical counterpart. Both histograms show an empty region in the middle. In the model, occupations at the left of the distribution occur after a sufficiently bad combination of shocks. Red (darker) bars correspond to the distribution of occupation times in financial crises. Crises have high occupation times due to the long exit times from those regions. Intermediate intervals, for which $\kappa$ has no occurrence, correspond to equity injections regions. The two panels at the bottom show the corresponding growth rates of $K$. These distributions are similar.
6.3.2. Model and Historical Moments

Table II report model and empirical moments. These moments are computed from the invariant distribution and the distribution conditional on crisis states. The directions of change in the model’s moments from normal to crises times follow a very similar pattern as the movements from historical averages to statistics from the Great Recession. However, the model represents an exaggerated version of reality.

The occupation time on crises regimes is 32.6% in the model. The Great Recession, represents 14.6% of the time in the sample, clearly an overstatement of recent US history since the financial system was very stable prior to this episode. However, Reinhart and Rogoff (2009) calculate that during the national banking era banking crises occurred during 13% of the years in their sample (according to an arbitrary rule). The exit time in the model is 10 quarters whereas the great recession lasted 6 quarters, twice the length of the average recession.

The average growth rate of the economy in the model is close to the historical growth rate of the US. During a financial crisis, output growth falls to 10.0%, a dramatic description of reality when compared to the the Great Recession. However, Cerra and Saxena (2008) report a reduction in the growth rate of about 8% for a cross-country sample.

Financial crises are associated with a strong reductions in volumes of financial intermediation. One can notice this from the reduction in the loans to output ratio in the model. Volumes of intermediation and qualities fall in these episodes. This explains also the striking reduction in the investment to GDP. In the model, this ratio is high in comparison with the data. There are no further margins of improvement unless one lowers the discount factor. This follows from the AK-technology structure.

A second block of moments reports financial intermediation indicators. The most important of these is the value of $\kappa$. During a financial crises, $\kappa$ can fall up to 92%. This is a more dramatic swing than for the analogue of $\kappa$ during the Great Recession. Clearly, the dynamics of the model are more extreme than those in the data. The unconditional
financing premia in the model are also magnified. Financing premia during the crises falls or at least did not increase in the same amount in the data. This perhaps follows from a selection effect (credit rationing).

Bank ROE is higher within crisis episodes in the model. The direction of the drop depends on parameters. The equilibrium values of bank ROE will belong to an interval determined by the inaction regions. Nothing precludes the distribution bank ROE within financial crisis regimes to be smaller than their unconditional average. The model is successful in not delivering counterfactual increases in bank ROE.

Growth in the model is entirely explained by capital accumulation. A variance decomposition of the model shows that during normal times, most of the volatility of output growth is due to movements in lending conditions. During a financial crisis, financial intermediation is responsible for a smaller share of volatility because crises occur more often in low TFP states. TFP is mean reverting which is why it explains a greater portion growth volatility.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Occupation Times</th>
<th>Unconditional</th>
<th>Crisis</th>
<th>Historical</th>
<th>Great Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occupation Time</td>
<td>100%</td>
<td>32.6%</td>
<td>100%</td>
<td>14.5%</td>
<td></td>
</tr>
<tr>
<td>Duration (quarters)</td>
<td>-</td>
<td>10.26</td>
<td>20.8</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Economic Activity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Growth Rate</td>
<td>4.3%</td>
<td>-10.3%</td>
<td>2.98%</td>
<td>-2.36%</td>
<td></td>
</tr>
<tr>
<td>Investment/Output</td>
<td>39.9%</td>
<td>-0.935%</td>
<td>8.51%</td>
<td>5.64%</td>
<td></td>
</tr>
<tr>
<td>Investment/Capital</td>
<td>3.58%</td>
<td>-0.0644%</td>
<td>6.09%</td>
<td>3.92%</td>
<td></td>
</tr>
<tr>
<td>Financial Intermediation Indicators</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average ( \kappa )</td>
<td>0.0659</td>
<td>0.0048</td>
<td>0.0162</td>
<td>0.0148</td>
<td></td>
</tr>
<tr>
<td>Financial Leverage</td>
<td>6.56</td>
<td>1.99</td>
<td>9.87</td>
<td>11.3</td>
<td></td>
</tr>
<tr>
<td>Loans Output</td>
<td>6.68%</td>
<td>0.0832%</td>
<td>NaN</td>
<td>NaN</td>
<td></td>
</tr>
<tr>
<td>Return to Assets (ROA)</td>
<td>6.94%</td>
<td>16.9%</td>
<td>1.18%</td>
<td>-0.0839%</td>
<td></td>
</tr>
<tr>
<td>Return to Equity (ROE)</td>
<td>31.3%</td>
<td>48.1%</td>
<td>16.4%</td>
<td>-1.07%</td>
<td></td>
</tr>
<tr>
<td>Financing Premia</td>
<td>39.5%</td>
<td>106%</td>
<td>6.25%</td>
<td>5.89%</td>
<td></td>
</tr>
<tr>
<td>Financial Equity Indicators</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Dividend Rate</td>
<td>0.643%</td>
<td>0.0193%</td>
<td>1.12%</td>
<td>-18.8%</td>
<td></td>
</tr>
<tr>
<td>Financial Stocks Index</td>
<td>100%</td>
<td>8.31%</td>
<td>100%</td>
<td>42.9%</td>
<td></td>
</tr>
</tbody>
</table>

TABLE II

Model moments and reference statistics.

6.3.3. Response to Dispersion Shocks

Figure 10 shows the response of the economy to an increase in \( \phi \). The figures plot the expected responses of equilibrium objects after \( \phi \) takes its largest value. The system is initiated from states randomly drawn from the invariant distribution. The top-left panel shows the immediate effect on bank losses. The solid line in the panel corresponds to the banking system’s liabilities relative to their unconditional mean. The dashed line presents the corresponding series for bank assets. When the shock is realized, the value of assets falls under the value of liabilities because \( \phi \) affects the value of the capital bought by banks. This discrepancy induces a reduction in bank profits (dashed line in top-middle panel) during the period of impact. Whereas expected profits (solid line in top-middle panel) are constant, profits fall below their expected value. The reduction in profits causes a reduction in the financial risk capacity in the period after the shock (top-right panel). As a consequence, during the subsequent periods, the equilibrium \( \omega^* \) (middle-left panel) falls in response to the lower financial risk capacity. The fall in \( \omega^* \) represents both a decrease in financial intermediation and an exacerbation of adverse selection. After the period of impact, one can observe that banking activities decline as both assets and liabilities fall.

In reaction to this shock, on average, banks inject equity. However, these injections are small in relation to the drop in \( n \) and are far from replenishing bank equity immediately. The value of bank net worth also declines, almost entirely driven by the reduction in \( \kappa \) (middle-right panel).
The effects on real economic activity are depicted in the bottom row. The reduction in financial intermediation in the periods after the shock cause a drop in investment (bottom-left panel). Since this economy requires investment to grow, unsurprisingly, the growth rate of the economy declines (bottom-middle panel). The panel in the bottom right shows the level of output with and without the shock. Because this is a linear-production growth model, the reduction in growth rates have permanent effects.

The effects of $\phi$ have the hallmarks of the descriptions of financial crises where adverse selection is aggravated and where financial intermediation declines severely as in, for example, Calomiris and Gorton (1991) or Mishkin (2011).

One thing to note is that the reduction in the growth rate is very persistent despite the fact that the shock has virtually no memory. This persistence is a result of the internal behavior of the system. Several macroeconometric studies obtain highly persistent filtered time series in financial shocks that affect firm decisions (e.g., Christiano et al. (2009), Justiniano et al. (2010) or Jermann and Quadrini (2011)). Through the lens of the model, these long recoveries can be explained by the time banks take to rebuild net worth. However, this persistence only occurs in response to large shocks but not to small shocks.

**Inherent Non-linearities.** Let us consider now the response to a smaller shock $\phi$. Figure, 11 presents a superimposed impulse response to the system to a shock to this smaller size on the impulse response in Figure 10. It is clear that the responses to smaller shocks are much less persistent. In particular, the growth rate of the economy recovers almost immediately after the small shock. Naturally, bank losses are smaller for the smaller shock, and except for expected bank profits in the period after the shock, the directions of the responses are virtually the same for all the variables. So what explains this sharp differences in persistence? In contrast to what happens after large shocks, adverse-selection effects are not strong when small shocks occur. Hence, after small shocks, competition effects lead to higher marginal profits with less intermediation. This explains the differences in the directions of expected profits. With higher expected profits, equity injections are attracted. Notice that these are massive, almost twice the size of the average dividend payment. These equity injections mitigate the amplification of the shock. Higher profits imply that by retaining earnings, $\kappa$ will increase rapidly in the subsequent period after the small shock is realized, something that fails to occur after large shocks. This feature explains the inherent non linearity that this paper attempts to underscore.

### 7. Financial Stability Policies

This section discusses some potential policies aimed at restoring financial stability. I begin describing the the possible externalities that may arise in this environment and then discuss the effects of several policies.\(^{21}\)

#### 7.1. The Externalities

There are two externalities in this environment. The first stems from excessive intermediation in competitive inaction regions. The second externality is the coordination failure that occurs in financial crises regimes.

**Intermediation Externality.** Hanson et al. (2011) argue that macroprudential policy frameworks must identify why banks may provide excessive intermediation. Here, when banks purchase large amounts of capital from producers (when they are highly levered), they face greater potential losses in net worth. Banks lever up rationally: they take into consideration risks and benefits. However, because they are price takers, they fail to internalize this if they lose financial risk capacity together. They induce market responses on prices and on the capital qualities traded. Although this is also the case in a neoclassical growth model, the fact that net worth is an endogenous state variable alters things. A planner facing the same constraints as banks would take into account the impact on the aggregate net worth. In particular, in a competitive inaction region, a planner may choose to provide less intermediation to reduce extreme outcomes after bank losses. In turn, banks are better off as less intermediation in competitive inaction

\(^{21}\)Hanson et al. (2011) refer to similar externalities, but in their line of thought, these are caused by a combination of debt overhang problems and fire sales externalities. Nonetheless, they discuss the effects of the same policies that I refer to here.
regions leads to greater profits. Producers may also be better off because the economy can be more stable with less
intermediation, and they are risk averse. Thus, in these regimes, excessive financial intermediation induces a negative
externality.\footnote{Lorenzoni (2008) discovers a similar externality in the context of fire-sales.}

**Equity-Injections Externality.** In financial crisis regions, there is a coordination failure: when $\kappa$ is low, bankers
expect low future discounted profits. Given this projection, an individual banker will not inject equity because he
cannot affect $\kappa$. However, if all bankers were to inject capital simultaneously, by definition, the value of bank equity
would increase. This makes bankers better off and more intermediation makes producers better off also. The following
sections discuss the effects of alternative government policies on this environment.

\section*{7.2. Capital requirements}

**Effects of Capital Requirements.** Increasing the capital-requirement parameter $\theta$ has two effects. The first is
direct. Given a level of $\kappa$, this increase will lower the volume of intermediation in a given period. As discussed in the
previous section, this policy may be desirable if banks are intermediating excessively. Though, in general, a reduction
in their intermediation will have adverse effects on growth. The effect on the volume of intermediation is clear from
the LLC: $\Pi(X, X') Q \leq (1 - \theta) n', \forall X'$. This constraint is tighter as $\theta$ is increased. Depending on the region, the
reduction in $Q^*$ may decrease or increase bank profits (depending on whether competition or adverse selection effects
dominate), and the same is true about leverage.

The second effect is the indirect dynamic effect on $\kappa$ caused by the reduction in the profitability of intermediation.
Holding prices fixed, an increase in $\theta$ reduces bank ROE. This happens because it forces bank leverage down: from the
solution to $Q^*$, one can show that the shadow value of relaxing the bank’s LLC is $(1 - \theta) \max \left\{ \frac{E[v_2^f(X')\Pi(X, X')]}{n'} - \min_X \Pi(X, X'), 0 \right\}$. This expression is a linear negative function of $\theta$ if $\Pi(X, X')$ is fixed. This effects shows up in the conditions that
determine the marginal value of bank’s equity:

$$\hat{v}(X) = \beta^F \left[ E[v_2^f(X')] + (1 - \theta) \max \left\{ \frac{E[v_2^f(X')\Pi(X, X')]}{n'} - \min_X \Pi(X, X'), 0 \right\} \right]$$

Holding profits fixed, the value of bank equity is decreasing in $\theta$.

These two effects operate in different directions in determining the equilibrium intervals for $\kappa$. Reductions in
volumes may increase or decrease the value of bank equity. The reduction in bank leverage reduces bank ROE (by
lowering leverage) and this reduces the incentive to inject equity.

As argued earlier, a social planner will face a trade off between reducing probability of a financial crises at the
expense of a reduction in financial intermediation. Capital requirements may be a useful tool in inducing more
favorable outcomes. An optimal government policy potentially involves a state dependent $\theta$. However, the effects on
$\kappa$ will be hard to pin down without specific values of parameters. The following numerical exercises attempt to shed
light on how these effects balance out in the current calibration.

**Invariant Distribution and Moments under Basel-II and III.** This section describes the effects of an
increase in $\theta$ from 0.08 to 0.18.\footnote{A similar exercise is performed by Bianchi (2011) in the context of international capital flows.} These numbers are chosen to match the regulatory constraints imposed by the
Basel-II to Basel-III frameworks. Figure 8 presents the invariant distribution of $\kappa$ under both parameter values. Both
distributions are similar in that they show a bimodal shape with concentrated probability masses at the extremes.
However, there are important differences. First, the distribution under Basel-III has less concentration for lower values
of $\kappa$. This means that the unconditional probability of falling in financial crises regimes is lower. Second, towards the
right end of the support of these distributions, the probability mass under Basel-III is higher. Higher average levels of $\kappa$ can result from higher profitability from intermediation that compensate a lower degree of intermediation.
Table III reports moments under both policy regimes. The duration of a crisis is shorter under Basel-III. Why? Crises last less during Basel-III because this regime prevents large falls in $\kappa$. Since the financial risk capacity is higher on average, $\kappa$ can reach an equity injection region in less time. In addition, because declines in $\kappa$ are less extreme, financial crises are also less frequent. It takes less likely combinations of $\phi_t$ to cause a financial crisis. This figure is also consistent with higher levels of $\kappa$ observed both unconditionally and for crisis regimes. With higher levels of $\kappa$ the economy grows at faster rates.

![Invariant Distributions under Policy Regimes](image)

**Figure 8.** Invariant Distributions under Basel-II and III.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Basel-II Unconditional</th>
<th>Basel-II Crisis</th>
<th>Basel-III Unconditional</th>
<th>Basel-III Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occupation Time</td>
<td>100%</td>
<td>32.6%</td>
<td>100%</td>
<td>19.77%</td>
</tr>
<tr>
<td>Duration (quarters)</td>
<td>-</td>
<td>10.26</td>
<td>-</td>
<td>3.89</td>
</tr>
<tr>
<td>Average Growth Rate</td>
<td>4.3%</td>
<td>-10.3%</td>
<td>9.3%</td>
<td>-9.61%</td>
</tr>
<tr>
<td>Average $\kappa$</td>
<td>0.0659</td>
<td>0.0048</td>
<td>0.100</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**TABLE III**

Comparison of Moments Under Basel-II and III in the Model. The table plots some moments corresponding to values of $\theta$ equal to 0.08 (Basel-II) and 0.18 (Basel-III).

**The Timing of Basel-III.** This section studies the effects of increasing capital requirements unexpectedly and permanently once the economy is in a financial crisis. The purpose of this exercise is to study the recovery from a financial crises region under Basel-II and Basel-III values of $\theta$ once the economy is on a financial crises under Basel-II. To do so, I initialize the economy from random states drawn from the Basel-II distribution of financial crisis states. Then, I compare the growth rates if the economy remains under Basel-II or if Basel-III is introduced permanently. Figure 9 presents the paths for these growth rates under both policy regimes.

One can observe that the recovery under Basel-II is faster. The intuition behind this outcome is that once in a crisis, capital requirements may prolong the decline. Higher capital requirements will depress intermediation while exacerbate adverse selection and this will reduce the profitability from banking even more. As a consequence, this will postpone the recapitalization of banks. There is no inconsistency with the results in the previous section that
show that crises last less under Basel-III. This is because the economy starts at even lower levels of $\kappa$ under Basel-II.

Comparing this with the previous exercise, the model warns policy makers what not to do in bad times or what they should have done during good times. There is currently an ongoing debate between bank managers and policy makers (see Admati et al., 2011). Bank managers argue that increasing capital requirements immediately after a crises may be to stringent on financial intermediation. In the model, higher capital requirements during a financial crisis will carry two effects: (1) they will aggravate adverse selection and (2) they will reduce the incentives to recapitalize banks and thereby prolonging the crisis. The first effect is also shared by other models with fire-sale externalities (see, for example, Bianchi (2011) or Hanson et al. (2011)). However, if marginal profits from intermediation are decreasing, the additional constraints on intermediation will be offset by higher marginal profits (the effect on bank ROE could be indeterminate). If bank ROE increases, this would ultimately shorten the crisis by attracting more equity. In this model, bank ROE as a function of $\kappa$ must necessary fall within higher requirements, so the effect is necessarily damaging.

**Pro-Cyclical Capital Requirements.** The results in the previous sections suggest that pro-cyclical requirements are perhaps the optimal policy. In competitive inaction regions, higher requirements may prevent large swings in $\kappa$. However, during crisis regimes, by increasing bank profitability at times when adverse selection is severe, lower requirements may shorten the length of a crisis. Kashyap and Stein (2004) argue in favor of pro-cyclical requirements because they view that lending should be promoted during crisis times. In a related model, Bianchi (2011) performs this type of exercise.

### 8. Conclusions

This paper provides a theory about risky financial intermediation under asymmetric information. The main message is that financial markets where asymmetric information is a severe friction are likely to be more unstable. The source of this instability is caused by the non-monotonicity in bank profitability as a function of the net worth of the financial industry. This non-monotonicity implies that banks fail to recapitalize after large losses despite the availability of
resources. This is something that prolongs a financial crisis.
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APPENDIX A: ALGORITHM

The following algorithm was used to compute equilibria in the model.
1. Build a grid for the state space values of \( A \times \phi \) and the transition function \( \chi \).
2. Build a grid on the unit interval corresponding to volumes of intermediation.
3. For all possible values of \( A \times \phi \) and \( \omega \) on the grid solve \( \{ p(\omega, A, \phi), q(\omega, p, A, \phi), \Pi(\omega, A, \phi) \} \).
   
   - \( p(\omega, A, \phi) \) is discretized for values between \( p_L = \min_\omega \lambda(0, \phi) \) and \( p_H = \max_\phi \lambda(1, \phi) \) and solving the optimal portfolio problem assuming \( q^*(\omega, A, \phi) = 1 \).
   
   - Using this \( p \), one finds \( q(\omega, p, A, \phi), \Pi(\omega, p, A, \phi) \) and checks whether \( q(\omega, p, A, \phi) \geq 1 \).
   
   - For values where the condition fails, one solves \( p(\omega, A, \phi), q(\omega, A, \phi) \) jointly and then finds \( \Pi(\omega, A, \phi) \).
4. Guess a candidate function for \( \tilde{v} \).
5. Compute the set \( \omega^o \) using the candidate function \( \tilde{v} \).
   
   - Interpolate over the upper contour of \( E[\tilde{v}(X')]\Pi(\omega, p, \phi)|X] \).
6. Compute the set \( \omega^* \).
   
   - For each \( \omega \) in the grid, compute \( \kappa = \min_\phi \Pi(\omega, p, \phi) \omega \).
7. Compute \( \omega^* \) \( (X) \).
8. Define \( p(X) = p(\omega^* (X), A, \phi), \Pi(X) = \Pi(\omega^* (X), A, \phi) \) and \( q(X) = q(\omega^* (X), p(\omega^* (X), A, \phi), A, \phi) \).
9. Compute the transition function for \( X \).
10. Update the \( \tilde{v}(X) \) iterating the Bellman equation for \( \tilde{v}(X) \) until convergence.
11. Iterate steps 4-9 until convergence.
12. Compute \( v(X), d(X), e(X) \).

APPENDIX B: PROOFS

B.1. Proof of Propositions 1, 2 and 3

The proof of propositions 1, 2 and 3 is presented jointly. The idea of the proof is to transform the producer's problem into a consumption-savings problem with log-preferences and linear constraints. For this, one has to solve the asymmetric information problem showing that the problem is homogeneous. Once this is done, one can use the dynamic programming arguments for homogeneous objectives in Alvarez and Stokey (1998) to argue that all the Bellman equations have unique solutions. I proceed by guess and verify to find these solutions. Similar proofs appear in Bigio (2011).

Define \( W^p \equiv w^p k \equiv (A + q_\lambda(\phi')) k \) and \( W^q \equiv q^* k \equiv (p_\omega^* + q^* E[\lambda(\omega, \phi')|\omega < \omega^*]) k \) as in the main text. The guess for the \( c \)-producer's policy function is \( k^{c^*} = \beta \frac{W^p}{q^*} \) and \( c^* = (1 - \beta) W^p \) and that his value function is of the form \( V^c = \psi^c(X) + \frac{1}{1 - \beta} \log W^p \) where \( \psi^c(X) \) is a function of the aggregate state. For \( c \)-producers the guess is that \( k^{c^*} = \beta \frac{W^p}{q^*} \) and \( c^* = (1 - \beta) W^p \) and that their value function is of the form \( V^c_2 = \psi^c(X) + \frac{1}{1 - \beta} \log W^p \) where \( \psi^c(X) \) is, again, a function of the aggregate state.

Consider the \( k \)-producer’s problem during the first stage then. Substituting the guess for \( V^c_2 \) and his constraints yields:

\[
V^1_i = \max_{i(\omega) \in [0,1]} \mathbb{E} \left[ \psi^i(X) + \log \left( \left( p \int_0^1 \mathbb{I}(\omega) d\omega + q^i E[\lambda(\omega, \phi')|\mathbb{I}(\omega) = 1] \right) k \right) | X \right]
\]

From this expression, we show that choosing \( I(\omega) \) is identical to choosing a cutoff \( \omega^* \) under which all units of quality lower than this cutoff are sold. We show this by arguing that an optimal \( I^* (\omega) \) must be monotone decreasing. Suppose not and assume the optimal plan is given by some \( I'(\omega) \) whose value is not attained by any monotone decreasing policy. It is enough to show that the producer can find another candidate \( I(\omega) \) that integrates to the same number, that is monotone decreasing and that makes his value weakly greater. Thus, let’s compare \( I'(\omega) \) with some \( I(\omega) \) that integrate to the same number in \([0,1]\).

Since \( I'(\omega) \) and \( I(\omega) \) integrate to the same number, the amount of IOUs obtained by the \( k \)-producer during the first stage, is the same:

\[
p \int_0^1 \mathbb{I}(\omega) d\omega = p \int_0^1 I'(\omega) d\omega.
\]

Now, since \( I(\omega) \) is monotone decreasing and \( \lambda(\omega, \phi) \) is monotone increasing in \( \omega \),

\[
\int [1 - I'(\omega)] \lambda(\omega, \phi') d\omega \leq \int [1 - I(\omega)] \lambda(\omega, \phi') d\omega
\]

implying that any optimal can be attained by some \( I^* (\omega) \) monotone decreasing. This, shows that \( V^1_i \) is attained by some monotone decreasing function.
Since $\Pi^*(\omega)$ is monotone decreasing, it is also equivalent to choosing a threshold $\omega^*$. Substituting this threshold into the objective yields and expression for the optimal cutoff rule:

$$\omega^*(X) = \arg\max_{\omega} \mathbb{E} \left[ \log (p \tilde{\omega} + q^\theta (X, X')) \int_{\omega}^\infty \lambda (\omega, \phi') \, d\omega \right] |X|.$$  

This proves Proposition 3 provided that $V_1^0$ takes the shape of our guess. I now return to the second stage problems. Taking the solution to (9) as given, we know that the optimal plan for a k-producers sets $x = p(X) \omega^*(X) k$. Using the optimal policy for $\omega^*$ and these definitions, one can write the second stage Bellman equation without reference to the first stage Bellman equation. To do this, one can substitute in for $x$ and $k$ in the second stage Bellman equation to rewrite the k-producer’s problem as,

$$\max_{c \geq 0, k \geq 0} \log (c) + \beta \mathbb{E} \left[ \pi V_1^0 (k', X') + (1 - \pi) V_2^0 (k', X') |X| \right]$$  

$$c + i + q k = \pi\omega^*(X) k$$  

Now, since $(V_1^0, V_2^0)$ are increasing in $k'$, and $k^b$ and $i$ are perfect substitutes, an optimal solution will set $i > 0$ only if $q \geq 1$ and $k^b > 0$ only if $q \leq 1$. This implies that substituting the k-producer's capital accumulation equation into his budget constraint simplifies his problem to:

$$\max_{c \geq 0, k^b} \log (c) + \beta \mathbb{E} \left[ \pi V_1^0 (k^b, X') + (1 - \pi) V_2^0 (k^b, X') |X| \right] \text{ s.t. } c + q k^b = w^i k.$$

The same steps allow one to write the c-producer’s problem as,

$$\max_{c \geq 0, k^p} \log (c) + \beta \mathbb{E} \left[ \pi V_1^0 (k^p, X') + (1 - \pi) V_2^0 (k^p, X') |X| \right] \text{ s.t. } c + q k^p = w^p k.$$

Replacing the definitions of $V_1^0$ and $V_2^0$ into the objective above, and substituting our guess yields $V_2^0$ and $V_2^0$, we obtain:

$$\max_{c \geq 0, k^b} \log (c) + \beta \frac{1}{(1 - \beta)^{k^b + \tilde{\psi}^i (X)}} \text{ s.t. } c + q k^b = w^i k$$

and

$$\max_{c \geq 0, k^p} \log (c) + \beta \frac{1}{(1 - \beta)^{k^p + \tilde{\psi}^p (X)}} \text{ s.t. } c + q k^p = w^p k,$$

respectively. In these expressions, $\tilde{\psi}^i (X)$ and $\tilde{\psi}^p (X)$ are functions of $X$ which don’t depend on the policy decisions. Taking first order conditions for $(k^b, c)$ in both problems leads to:

$$c^i = (1 - \beta) w^i (\omega^*, X) k \quad \text{and} \quad k^b = \frac{\beta}{q} w^i (\omega^*, X) k$$

$$c^p = (1 - \beta) w^p (X) k \quad \text{and} \quad k^p = \frac{\beta}{q} w^p (X) k.$$  

These solutions are consistent with the statement of Propositions 1 and 2. To verify that the guess for our value functions is the correct one, we substitute in the optimal policies:

$$\log (1 - \beta) w^i (\omega^*, X) k + \frac{\beta}{(1 - \beta)^{k^b + \tilde{\psi}^i (X)}} \log \frac{\beta}{q} w^i (\omega^*, X) k + \tilde{\psi}^i (X)$$

$$= \log w^i (\omega^*, X) k + \frac{\beta}{(1 - \beta)^{k^b + \tilde{\psi}^i (X)}} + \tilde{\psi}^i (X)$$

for some function $\tilde{\psi}^i (X)$, The same steps lead to a similar expression for c-producers. This verifies the initial guess.

B.2. Proof of Lemma 4 and Proposition 5

Lemma 4 and Proposition 5 are proven jointly here. We begin by guessing that $V_1^1 (n, X) = v_1^1 (X) n$, and $V_2^1 (n, X) = v_2^1 (X) n$ where $v_1^1 (X) = \beta^2 \mathbb{E} \left[ v_1^1 (X) R^0 \right]$ if the banker remains alive and $v_2^1 (X) = \beta^2 R^0 n$ if he dies.

Plugging this guess into the bankers problem yields:

$$\max_{Q \geq 0, e \in [0, d], \omega \in [0,1]} (1 - \tau) d - e + \mathbb{E} \left[ v_2^1 (X) \left( \Pi (X, X') Q + n' \right) |X| \right]$$

subject to,

$$- \min_{X'} \Pi (X, X') Q \leq n'$$

Assume that the optimal solution to this problem is characterized by some $e^* (n, X)$ and $d^* (n, X)$ to be determined. In equilibrium, $\Pi (X, X')$ is finite. Hence, $E \left[ v_2^1 (X) \Pi (X, X') \right]$ is also finite, provided that the problem has a finite solution. If $E \left[ v_2^1 (X) \Pi (X, X') \right] > 0$ and $- \min_{X'} \Pi (X, X') \leq 0$, the banker would set $Q^* = \infty$. But this would imply that in equilibrium $\Pi (X, X') \leq 0$ for any $X'$ because there cannot be a future state where firms provide infinite intermediation and there are positive profits. Hence, it is the case that
Finally, if \( \tilde{e} \) and equal 0 if these are not satisfied. By assumption which equals, which is a linear function of \( E \). Thus, one can substitute this expression into the objective of the firm and express it without reference to \( Q \):

\[
E \left[ v^*_2 (X') \Pi \left( X, X' \right) \right] = \max \left\{ v^*_2 (X') \Pi \left( X, X' \right) n', -\min_{X'} \Pi \left( X, X' \right), 0 \right\},
\]

Thus, one can substitute this expression into the objective of the firm and express it without reference to \( Q \):

\[
\max_{e \in [0, \bar{e}], d \in [0, n]} (1 - \tau) d - e + \beta F R^b \mathbb{E} \left[ v^*_2 (X') + \max \left\{ v^*_2 (X') \Pi \left( X, X' \right), 0 \right\} \right] |
\]

where the second line uses the definition of \( n' \). Now, it is clear from this expression that any optimal financial policy satisfies,

\[
e > 0 \text{ only if } \beta F \left[ \mathbb{E}[v^*_2 (X')] + \max \left\{ \frac{\mathbb{E}[v^*_2 (X') \Pi (X, X')] - \min_{X'} \Pi (X, X')}{0} \right\} \right] \geq 1
\]

\[
d > 0 \text{ only if } \beta F \left[ \mathbb{E}[v^*_2 (X')] + \max \left\{ \frac{\mathbb{E}[v^*_2 (X') \Pi (X, X')] - \min_{X'} \Pi (X, X')}{0} \right\} \right] \leq (1 - \tau).
\]

If the inequalities are strict, it is clear that, \( e = \bar{e} \) and \( d = n \). By linearity, \( e \) and \( d \) are indeterminate when the relations hold with equality, and equal 0 if these are not satisfied. By assumption \( e = \bar{e} \) is never binding. If \( d = n \), for some state, \( d \) is linear in \( n \). This implies that \( e (n, X) = e^* (X) n \), \( d (n, X) = d^* (X) n \) are solutions to the producer’s problem.

We now use this results to show that the value function is linear in \( n \). Plugging in the optimal policies into the objective we obtain:

\[
\left( 1 - \tau \right) d^* (X) - e^* (X) + (1 + e^* (X) - d^* (X)) \beta F R^b \mathbb{E} \left[ v^*_2 (X') + \max \left\{ v^*_2 (X') \Pi (X, X'), 0 \right\} \right] = n
\]

which is a linear function of \( n \).

Returning to the optimal quantity decision, then it is clear that \( Q \) can be written as,

\[
Q = \frac{1 + e^* (X) - d^* (X)}{-\min_{X'} \Pi (X, X')} n = Q^* (X) n.
\]

and clearly, \( Q^* (X) = \arg \max_Q \mathbb{E} \left[ v^*_2 (X') \Pi (X, X') \right] \) subject to \( \Pi (X, X') \bar{Q} \leq n' \). This proves, Proposition 6.

We are ready to show that \( v^*_1 (X) \) solves a functional equation. Define

\[
\tilde{\nu} (X) = \beta F R^b \mathbb{E} \left[ v^*_2 (X') + v^*_2 (X') \max \left\{ \frac{\Pi (X, X')}{-\min_{X'} \Pi (X, X')}, 0 \right\} \right]
\]

as the marginal value of equity in the bank and note that:

\[
v^*_1 (X) = \max_{d^* (X) \in [0, 1], e \geq 0} (1 - \tau) d^* (X) - e^* (X) + (1 + e^* (X) - d^* (X)) \tilde{\nu} (X).
\]

If \( \tilde{\nu} (X) \in ((1 - \tau), 1) \), then \( v^*_1 (X) = \tilde{\nu} (X) \) because \( (d^* (X), e^* (X)) = 0 \). If \( \tilde{\nu} (X) \leq (1 - \tau) \), then \( e^* (X) = 0 \) and we have that,

\[
(1 - \tau) d^* (X) + (1 - d^* (X)) \tilde{\nu} (X) = (1 - \tau).
\]

Finally, if \( \tilde{\nu} (X) = 1 \), then \( v^*_1 (X) = 1 \). This information is summarized in the following functional equation for \( v^*_1 (X) \):

\[
v^*_1 (X) = \min \left\{ \max \left\{ \beta F R^b \left[ v^*_2 (X') \left( 1 + \max \left\{ \frac{\Pi (X, X')}{-\min_{X'} \Pi (X, X')}, 0 \right\} \right] \right] \right\} , (1 - \tau) \right\}, 1 \}
\]

which equals,

\[
\min \left\{ \max \left\{ \mathbb{E} \left[ \rho + (1 - \rho) v^*_1 (X') \right] \beta F R^b \left[ 1 + \max \left\{ \frac{\Pi (X, X')}{-\min_{X'} \Pi (X, X')}, 0 \right\} \right] \right\} \right\} , (1 - \tau) \right\}, 1 \}
\]
This functional equation determines the slope of the bankers value function, $v'_f(X)$. It can be shown that the solution to this functional equation is unique. Assumptions 9.18-9.20 of Stokey et al. (1989) are satisfied by this problem. It remains to show that Assumption 9.5 (part a) is also satisfied. By assumption, $X$ is compact so the only piece left is that $X$ is countable. Because the transition function for the state is an endogenous object, as it depends on an aggregate state, $\kappa$. It will be shown that although $(d,e)$ are not uniquely defined, there is unique mapping from $\phi$ to $\kappa'$. By exercise 9.10 in Stokey et al. (1989), together, these assumptions ensures that there is a unique solution to the this functional equation.

B.3. Proof increasing $p(x)$

Let $q(X,X')$, is the market price that solves the market clearing condition given a price under asymmetric information of $p$. In equilibrium, this price is a function of the previous state, $p(X)$. Thus, through the equilibrium price $p(X)$, $q(X,X')$ defines an equilibrium $q$, implicitly, as function of the current state $X'$ and the previous state $X$: $q(X',X) \equiv \{p(X),X')\}$. Given, $X$, and the law of motion for $X_t$, $\tilde{q}(X',X)$ determines the profits for the financial sector given and amount of trade.

Proof: To characterize the key objects $Q(X), p(X)$ and $q(X)$. we need some we need to define some objects. The supply for financial contracts is $Q^f(p,X) = \tilde{\omega}(p,X)\kappa$ and $Q^d = \{\}$ and $\Pi(p,X') = \tilde{q}(p,X')\mathbb{E}_{\phi'}[\lambda(\omega)| \omega > \tilde{\omega}(p,X)] - p$. I provide some further characterization of this supply.

$$Q^f > 0, Q^d(0,X) = 0$$

and

$$Q^d (E[q^f(X')]\mathbb{E}_{\phi'}[\lambda(\omega)]), X) = k.$$ 

Thus, the supply schedule is invertible and bounded, and thus, we define:

$$P(Q) \equiv p \text{ such that } Q^f(p,X) = Q$$

and

$$P_Q(Q) > 0, P(0) = 0 \text{ and } P(X) = E[q^f(X')\mathbb{E}_{\phi'}[\lambda(\omega)]]$$ 

This proposition establishes the existence of a well behaved supply function: $P(Q)$ is bounded, differentiable and increasing. $Q.E.D.$

B.4. Proof of Proposition 6

To pin down $q$, fix any sequence of states $(X,X')$, and let $\tilde{\omega} = \omega(X)$. We begin the proof assuming $q > 1$ so that $D^s = 0$. Market clearing in stage 2 requires $D^p(X,X') = S(X) = E\{\lambda(\omega,\phi')| \omega \leq \tilde{\omega}\} \tilde{\omega}\pi K$. By Proposition 1, we can integrate across the c-producer’s policy functions to obtain an expression for $D^p(X,X')$ as a function of $q$:

$$\beta \int \left[\frac{W^p(k,x,X)}{q} - \bar{\lambda}(\phi') k\right] \Gamma^c(dk) = \beta \frac{A + q\bar{\lambda}(\phi')}{q} (1 - \pi) K$$

By market clearing, $q$ be such that:

$$\left[\beta\frac{A + q\bar{\lambda}(\phi')}{q} - \bar{\lambda}(\phi')\right](1 - \pi) K = E\{\lambda(\omega,\phi')| \omega \leq \tilde{\omega}\} \tilde{\omega}\pi K.$$

Manipulating this expression leads to the value of $q$ that satisfies market clearing:

$$q = \frac{\beta A(1-\pi)}{E[\lambda(\omega,\phi')| \omega \leq \tilde{\omega}] \omega \pi + (1 - \pi)(1 - \beta) \bar{\lambda}(\phi')}$$

Recall now that this expression is valid only when $q > 1$, because capital good producer’s are not participating in the market. Thus, the expression is only true for value of

$$\beta A \left[\frac{\pi}{(1 - \pi)} E[\lambda(\omega,\phi')| \omega \leq \tilde{\omega}] \omega + (1 - \beta) \bar{\lambda}(\phi')\right]^{-1} > 1.$$  

If $q = 1$, then it must be the case that the total demand for capital must be larger than the supply provided by financial firms. $D^s(X,X')$ in this case is obtained also by integrating across the demand for capital of k-producer’s given in Proposition 1. Thus, for a stage one price $p$, this demand is given by

$$D^s = \beta p \tilde{\omega}\pi K - (1 - \beta) E[\lambda(\omega,\phi')| \omega \leq \tilde{\omega}] (1 - \omega) \pi K \text{ for } q = 1.$$
The solution is given by:
\[ \kappa \]

\( \epsilon \) don't bind, and therefore, the equilibrium price remains constant. If given \( \kappa \), because otherwise there would have existed a larger equilibrium with a higher price. Thus, \( \epsilon \) still holds. Since the inequality implies that capacity constraints bind in equilibrium, then, market clearing implies that there exists some

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\( q \) satisfying the market clearing condition.

The formula in Proposition 6 corresponds. Moreover, the demand function is weakly decreasing so for each \( \beta p \) \( (12) \)

\[ \text{Since both functions are increasing in } \epsilon, \text{the shock } \phi' \text{ in the conditional expectation, we know by Assumption A1, that this functions are decreasing in the shock } \phi'. \text{ Thus, } \Pi(X, X') \text{ is decreasing in } \phi'. \]

\[ \text{B.5. Proof of monotone relation between } \kappa \text{ and } \omega \]

Observe that \( \Pi[X', p] > 0 \), is continuous in \( p \). Thus, there is a sufficiently small \( \epsilon > 0 \) increase in \( p \) such that the inequality still holds. Since the inequality implies that capacity constraints bind in equilibrium, then, market clearing implies that there exists some \( \epsilon(\epsilon) \), such that \( p(X) + \epsilon = P \left( \frac{[\kappa + \epsilon(X)]}{X_{\max}} \right) \). In an un-improvable, it must be the case that \( \pi \left( p(X) + \epsilon, X_{\max}' \right) \) is decreasing, because otherwise there would have existed a larger equilibrium with a higher price. Thus, \( \epsilon(\epsilon) \) must increase. This, implies that for a given \( \kappa \), we can find a small enough increase in \( \epsilon \), so that the equilibrium price increase.

If \( \Pi[X', p] = 0 \) and constraint does not bind, then, there is always a small enough increase in \( \kappa \), such that capacity constraints don’t bind, and therefore, the equilibrium price remains constant. If \( \Pi[X', p] = 0 \) and capacity constraints bind, then increase in \( \kappa \) will relax the binding constraint. Either \( Q \) remains the same or increases. Thus, \( p \) must increase.
B.6. Proof of efficiency

Proof: The necessary condition: \((1 - \tau_d) \leq \beta^F R^b\). Suppose not, then for any state \(\Pi (X', X) = 0\), then, it is convenient to pay dividends.

\[
\tilde{v}(X) = \beta^F R^b \left[ \mathbb{E}[\tilde{v}(X')] + \max \left\{ \frac{\mathbb{E}[\tilde{v}(X') \Pi (X, X')]}{-\min_{X} \Pi (X, \bar{X})}, 0 \right\} \right]
\]

\[
< \beta^F R^b \left[ \mathbb{E}[\tilde{v}(X')] + \max \left\{ \frac{\mathbb{E}[\Pi (X', X)]}{-\min_{X} \Pi (X, \bar{X})}, 0 \right\} \right]
\]

\[
= \beta^F R^b \mathbb{E}[\tilde{v}(X')]
\]

\[
< \beta^F R^b \mathbb{E}[\tilde{v}(X')]
\]

where the first line follows from the definition of \(\tilde{v}(X)\) and \(v_f^2\), the second follows from the fact that \(\tilde{v}(X') \leq 1\) and the third from the assumption that an efficient equilibrium satisfies \(\mathbb{E}[\Pi (X, X')] = 0\) and the last inequality uses \(\tilde{v}(X') \leq 1\) once more. Then, if the condition is not satisfied, \(\tilde{v}(X) < \beta^F R^b < (1 - \tau_d)\). This fact in turn implies that any state with financial risk capacity \(\kappa\) consistent with efficient intermediation is reflected to another state with \(\mathbb{E}[\Pi (X, X')] > 0\).

The sufficient condition is obtained reversing the equalities.

\[
\tilde{v}(X) = \beta^F R^b \left[ \mathbb{E}[\tilde{v}(X')] + \max \left\{ \frac{\mathbb{E}[\tilde{v}(X') \Pi (X, X')]}{-\min_{X} \Pi (X, \bar{X})}, 0 \right\} \right]
\]

\[
> \beta^F R^b \left[ \mathbb{E}[\tilde{v}(X')] + \max \left\{ (1 - \tau_d) \mathbb{E}[\Pi (X', X')], 0 \right\} \right]
\]

\[
> \beta^F R^b \mathbb{E}[\tilde{v}(X')]
\]

The first line follow from the definition of \(\tilde{v}(X)\) . The second uses that \((1 - \tau_d) \tilde{v}(X)\) has an upper bound. The third uses The hypothesis that \(\mathbb{E}[\Pi (X, X')] = 0\). With this inequality, it is enough to argue that there will always exist some \(\kappa\) such that \(\mathbb{E}[\tilde{v}(X')]\) is sufficiently above \((1 - \tau_d)\) such that state is not reflected.

Q.E.D.

APPENDIX C: ADDITIONAL FIGURES
Figure 10.— **Impulse response function to small** $\phi$. The figure plots the response of the system to a realization of the largest value of $\phi$. 
Figure 11.— Impulse response function to large and small $\phi$. 