Endogenous Liquidity and the Business Cycle

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Abstract

I present a model in which asymmetric information in capital quality endogenously determines the amount liquid funds in an economy. Liquidity is key to relax financial constraints that affect investment and employment decisions. Liquidity is determined by the wedges induced by financial frictions and, in turn, how these wedges depend on liquidity.

Aggregate fluctuations can be attributed to mean preserving spreads in capital quality. These shocks increase the cost of obtaining liquid funds and consequently aggravate financial frictions. Quantitatively, the model generates sizable recessions similar to the patterns to the financial crisis of 2008-2009.

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1 Introduction

The recent financial crisis began with an abrupt collapse in liquidity. A common view is that the crisis arose when lenders found it difficult to distinguish the value of collateral assets. This shortfall in liquidity would have spread to the real economy as firms were restrained from accessing funds to hire workers and finance investment. The overall outcome was the deepest recession of the post-war era.

This paper develops a theory to formalize and quantify this chain of events. The theory builds on the interaction of two frictions: limited enforcement in contractual agreements and asymmetric information about the quality of capital. Limited enforcement prevents agents from carrying out transactions solely by promising repayment. This market imperfection imposes constraints on the possible contractual agreements that can be reached. These constraints are only relaxed if transactions are paid on the spot. To make spot payments, the firm can sell or collateralize its capital. Asymmetric information induces a shadow cost on selling or collateralizing these assets. These costs arise because highly valued capital is transacted at a low price as it cannot be distinguished from capital of lower quality.

The first contribution of the paper is to characterize, in general equilibrium, the firm’s strategic decision to use assets of privately known quality to obtain liquidity in order to relax enforcement constraints. In particular, the paper shows how, at the margin, the cost of selling (or collateralizing) assets must equal the marginal benefit of relaxing enforcement constraints. Through the lens of the theory, recessions occur after an exogenous increase in the dispersion of asset qualities. This occurs because dispersion increases the cost of obtaining liquidity and, consequently, enforcement constraints are tighter.

The second contribution is to provide a quantitative assessment of this theory. In particular, a calibrated version of the model is fed with a sequence of dispersion shocks that is capable of generating a pattern for output, consumption, investment and hours losses comparable to the ones observed during the 2008-2009 recession. Interestingly, these asymmetric-information driven recessions occur despite that dispersion shocks have no effect on the production possibility frontier or the distribution wealth. Beyond replicating patterns observed during the Great Recession, the quantitative analysis shows also that increases in the dispersion of asset quality can generate economic fluctuations consistent with several other business cycle features. [1] The model explains sizeable liquidity-driven recessions which operate primarily through movements in the labor wedge. This is a salient feature of the business cycle decomposition of Chari et al. [2007]. [2] Liquidity driven recessions are characterized by increases in labor productivity. This feature cannot be generated through total factor productivity (TFP) shocks but was characteristic of the 2008-2009 crisis (see Ohanian
The model accounts for a negative correlation between investment and labor wedges. This supports the view in Justiniano et al. [2010b] that financial factors are responsible for this co-movement. The model is also in line with evidence on counter-cyclical capital reallocation documented by Eisfeldt and Rampini [2006]. The model produces two forces that counterbalance the relation between Tobin’s Q and investment: TFP shocks induce a positive correlation (as in standard Q-theory) but dispersion shocks reverse the correlation. This second force explains how capital reallocation slows down in periods where its benefits seem greater as described in Eisfeldt and Rampini [2006]. Finally, the model can explain why the risk-free interest rate falls whereas the cost of borrowing by firms increases during recessions.

The heart of the paper is an endogenous liquidity mechanism by which increases in asset-quality dispersion translate into negative impacts on real activity. To explain the intuition, it is useful to draw an analogy with Akerlof [1970]’s lemons problem. In the classic lemons problem, a seller owning an asset with private information sells it only if its price is above his valuation. Instead, an uninformed buyer buys it only if its price is below his valuation of the quality he expects to receive. Equilibria are summarized by a marginal asset such that all assets of inferior quality are sold. The spread between the quality of the marginal asset and the expected quality sold equals the spread between valuations of buyers and sellers. Shocks that reduce the average of qualities under a given threshold quality reduce the average of sold qualities, the price and the volume of trade.

In the paper, the source of asymmetric information is the depreciation rate (the quality) of different capital units in the portfolio of entrepreneurs. Knowing their quality, entrepreneurs select and sell units to financial intermediaries that ignore their quality. The fundamental difference with Akerlof [1970] is that the relative valuation by buyers and sellers is an endogenous outcome of the limited enforcement problems faced by entrepreneurs.

Limited enforcement induces entrepreneurs to value capital units less than intermediaries which is essential for trade under asymmetric information. The reason is that entrepreneurs receive an additional benefit from liquid funds. Liquidity allows entrepreneurs to relax their enforcement constraints and scale up their operations. Thus, like in Akerlof [1970], an entrepreneur sells a capital unit if the value of obtaining liquidity is greater than his valuation of that unit. Intermediaries pay a pooling price equal to the full-information price of the average quality sold. Thus, on the threshold asset, the entrepreneur receives a pooling price which is necessarily lower than the full-information price it would have to pay to repurchase that unit. The only reason why the entrepreneur is willing to take that loss is the additional benefit obtained by relaxing his enforcement constraints. Hence, the endogenous spread in valuations that stem from limited enforcement is what supports trade under asymmetric
information. Again, dispersion shocks that worsen adverse selection, lead firms to sell capital of lower quality and face tighter constraints which bears aggregate consequences.

To gain realism, I also study the effects of dispersion shocks when entrepreneur can issue collateralized debt (or repos) contracts.\(^1\) Allowing for these richer type of contracts alleviates the asymmetric information problem but does not alter the essence of the mechanism. When there is adverse selection, high-quality capital is not sold because the pooling price is too low compared to its replacement cost. If an intermediary could commit to return a high-quality asset at a pre-specified price, earning a default premium, entrepreneurs could be made better off. The entrepreneur would pay the premium if it can retain a high-quality asset and obtain the needed liquidity. For a given distribution of capital quality, I show that entrepreneurs do obtain more liquidity by collateralizing capital rather than by selling it. However, I also show that dispersion shocks can be increased such that allocations by obtained collateralizing capital are the same if one only restricts to selling contracts (with less dispersion). Thus, through the lens of the model, collateralized debt and assets sales are observationally equivalent if dispersion is not part of the set observables.

To generate quantitatively relevant effects, the model requires enforcement constraints to operate on labor contracts.\(^2\) Limited enforcement in labor contracts induce entrepreneurs to finance a portion of their payroll with liquid funds. Dispersion shocks cause an endogenous reduction in liquid funds which tightens these constraints. A contraction in labor demand follows. This feature is crucial to explain substantial movements in output and has an empirical support in the work of Chodorow-Reich [2013]. This feature distinguishes this model from the majority of models with financial frictions which focus on distortions on investment. This distinction is important because, it is known by now that investment frictions alone cannot generate strong output responses to fundamental shocks.\(^3\)

I also show that without additional frictions that distort capital accumulation, the model cannot deliver pro-cyclical investment. As labor demand contracts there is wealth shift

\(^{1}\)I thank an anonymous referee for suggesting this section of the paper. Throughout the paper I will use the term collateralized debt and repo contracts interchangeably. I show that these contracts are isomorphic in this environment.

\(^{2}\)I follow Hart and Moore [1994] in modeling the limit enforcement in labor contracts as Kiyotaki and Moore [2008] do for investment claims.

\(^{3}\)The reason is that big changes in investment flows have only a small impact on the total capital stock. Capital and not investment is is ultimately what determines output. Shocks that exacerbate financial frictions on investment resemble investment shocks (see Chari et al. [2007]). Barro and King [1984] found that investments shocks are not an important source of business cycle fluctuations. Only in combination with some other mechanism can these shocks cause variations in hours . In Greenwood et al. [1988] or in Greenwood et al. [2000] this mechanism is variable capital utilization. New-Keynesian are another alternative (see Bernanke et al. [1999], Christiano et al. [2009], del Negro et al. [2010] or Justiniano et al. [2010a]. Since then, this result has been rediscovered in the form of the irrelevance of liquidity shocks that distort investment to explain output movements (e.g. Bigio [2009]).
from workers to entrepreneurs via a monopsony effect. Since wealth shifts towards the entre-
preneurial sector, investment shifts as a response to this wealth transfer. The enforcement
problem in investment reverts this issue because it also distorts investment.

The paper is novel in several dimensions. Until recently, there were only a few number
models that incorporated asymmetric information into general equilibrium. A notable excep-
tion is Eisfeldt [2004] who studies a stationary model where agents sell assets under private
information for self-insurance motives. A closely related paper is Kurlat [2009] that was
developed independently. The ideas in our papers build on earlier insights in Kiyotaki and
Moore [2008] (henceforth KM). KM argue that the combination of asymmetric information
could explain aggregate fluctuations by inducing endogenous changes in liquidity. However,
liquidity in KM is determined by exogenous shocks. Although, KM motivated these shocks
by a problem of asymmetric information, the contributions of Kurlat [2009] and this paper
are to formalize these ideas.

Kurlat [2009] and this paper differ in important modeling aspects. However, there is a
focus here on the effects on labor that is important for quantitative reasons. Following KM,
Kurlat’s paper focuses only on frictions that affect investment but labor market frictions are
key to deliver strong movements in GDP. In this dimension, the paper also connects to the
recent empirical work by Jermann [2012] (henceforth JQ) who also stress the importance
of financial conditions for the labor market. JQ studies a business cycle model where en-
trepreneurs face shocks to an enforcement coefficient that limits their debt holdings. Both
papers share in common that entrepreneurs need to obtain liquid funds to finance work-
ning capital. The key distinction between JQ and this paper is that here fluctuations are
originated by increases in the dispersion of asset-qualities.

Why is it important then to model asymmetric information to explain liquidity? First,
asymmetric information can relate to the literature on financial frictions to the growing liter-
ature on counter-cyclical dispersion. Bloom [2009], Christiano et al. [2012], and Bloom et al.
[2009] provide evidence that the dispersion of profits and revenues increase during recessions.
Thus, the paper provides a link between these literatures via adverse-selection. A second
reason is that asymmetric information imposes restrictions on the time-series properties of
allocations and liquidity. For example, I show that adverse selection is less severe when
the return to capital is high. Third, the analysis uncovers an inefficiency result: if efficient
allocations require some liquidity, allocations are never efficient. All of these features are

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4 Asymmetric information is the natural market imperfection that motivates liquidity constraints because,
as known at least since the work of Stiglitz and Weiss [1981], it can induce credit rationing outcomes.

5 Both papers share the insight of the early work on working capital constraints to generate output
responses. See for Christiano and Eichenbaum [1992] where this constraint induces strong real effects
of monetary policy.
testable implications. Finally, the model can teach us whether asymmetric information is a quantitative relevant friction for financial markets, something that relates to many recent policy debates.

The paper also makes some technical contributions. I introduce asymmetric information into a dynamic general equilibrium model with aggregate shocks. I show how to solve for the full dynamics of the model without the need to keep track of trade histories. Also, thanks to the observational equivalence between sales and collateralized debt, I show how to solve very easily a model with collateralized debt and default. The idea is to solve the model using sales contracts only first. Then, one can reverse engineer the dispersion shocks that deliver the same allocations with collateralized debt. This allows me to provide a rich description about loan sizes, interest rates, default rates and the fraction of collateralized assets which I use to assess the model.

The rest of the paper is organized as follows. Section 2 describes a static model of a firm that needs to raise liquid funds by selling capital under asymmetric information and relax enforcement constraints. This exercise describes the key tradeoffs in the determination of liquidity and how this affects labor demand and output. That section also describes a similar problem that distorts investment. Section 3 shows the relationship between selling under asymmetric information and collateralized debt under asymmetric information. Section 4 presents the dynamic model. Section 5 provides some further characterizations. For this, it uses the solutions to the problems of Section 2. Section 6 presents some quantitative exercises and Section 7 concludes. A detailed discussion of the literature is contained in the online Appendix. The rest of the appendices include proofs and extensions.

2 Forces at Play

This section presents two static models to illustrate the key forces in the dynamic model. Both models are subcomponents of the dynamic model so also serves as an intermediate step in the analysis of the dynamic model.

2.1 Endogenous Liquidity, Output and Hours

Consider a static economy in partial equilibrium. The economy is populated by workers that only labor, financial firms that buy and sell capital and entrepreneurs. An entrepreneur maximizes the value of his firm which is the sum of current profits and the value of his capital stock. The entrepreneur holds k units of capital.

Production. Production is carried out using k, combined with labor, l, using a Cobb-
Douglas technology $F(k, l) \equiv k^{\alpha}l^{(1-\alpha)}$ to produce output. The entrepreneur’s profits are $AF(k, l) - wl$. The entrepreneur hires workers from an elastic supply schedule $w = l^\nu$. Wages are given.

Limited enforcement in labor contracts. Before production, an entrepreneur hires an amount of labor promising to pay $wl$. It is possible that the entrepreneur reneges on this promise and defaults on his payroll. In that case, workers are able to seize a fraction $\theta L$ of production and the entrepreneur diverts $(1 - \theta L)$ for himself.

To relax this problem, the entrepreneur can pay a fraction $(1 - \sigma)$ of the wage bill upfront. Of course, he has to obtain working capital to make this payment before production although he has no output yet. The way he obtains this working capital is by selling some capital units. Sold capital units are reallocated after production takes place so they are used for production. Thus, capital serves two purposes. It is a production input and it is also used to obtain working capital. Due to asymmetric information about its quality, selling capital will induce a cost.

Heterogeneous Capital. The capital stock held by the entrepreneur is composed of a continuum of pieces. Pieces are identified by their quality $\omega \in [0, 1]$. Qualities determine the depreciation of each unit. In particular, there is an increasing, bounded and continuous function $\lambda(\omega) : [0, 1] \to \mathbb{R}_+$ that determines the efficiency units that will remain from a given piece of quality $\omega$. The distribution of $\omega$ in that continuum is given by some $f_\phi(\omega)$ with c.d.f. denoted by $F_\phi$. For now, $\phi$ is a parameter.

Pieces can be sold separately. I use the indicator function $\iota(\omega) : [0, 1] \to \{0, 1\}$ to indicate the decision of selling a unit of quality $\omega$. That is, the entrepreneur transfers

$$k \int \lambda(\omega) \iota(\omega) f_\phi(\omega) \, d\omega$$

efficiency units to the buyer. The efficiency units that remain with his capital stock are

$$k \int \lambda(\omega) [1 - \iota(\omega)] f_\phi(\omega) \, d\omega.$$  

Information. When a given piece is sold, its $\omega$ (quality) cannot be identified by a buyer. This implies that only the entrepreneur knows the efficiency units that will remain from a particular sold unit. The buyers of these units are financial intermediary firms. Intermediaries only the quantity of units they buy, $k \int_0^1 \iota(\omega)f_\phi(\omega) \, d\omega$. However, since they ignore

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6Think of $\lambda(\omega)$ as $1 - \delta(\omega)$, where $\delta$ is a $\omega$–specific depreciation that occurs after production takes place. Using the notation $\lambda(\omega)$ is done for convenience. Shocks to the efficiency units are commonly used in continuous time settings. A recent example is Brunnermeier and Sannikov [2009].

7Each quality has 0 measure so focusing on all-or-nothing sales is without loss of generality.
behind a sale, they also ignore how many efficiency units will remain from this portfolio, \( k \int_0^1 \lambda(\omega) \iota(\omega) f_\phi(\omega) d\omega \). Since they ignore this, they also ignore the market value of the capital bought.

For now, I assume that units sold to financial intermediaries are sold at the same price \( p \). Later, I impose conditions such that there is a unique pooling price. Thus, when units are sold, the entrepreneur obtains liquidity \( pk \int_0^1 \iota(\omega) f_\phi(\omega) d\omega \). Let \( x = p \int_0^1 \iota(\omega) f_\phi(\omega) d\omega \) be the liquidity per unit of capital \( k \). I assume that financial firms sell efficiency units at an exogenous price \( q \).\(^8\) A non-profit condition for the financial firms requires them to equate the value of efficiency units bought to the amount of liquidity given to the entrepreneur. Thus, equilibrium requires,

\[
pk \int_0^1 \iota(\omega) f_\phi(\omega) d\omega = qk \int_0^1 \lambda(\omega) \iota(\omega) f_\phi(\omega) d\omega.
\]

This expression yields a relationship between the price under asymmetric information and the perfect information price of efficiency units \( q \):

\[
p = q \mathbb{E}_\phi [\lambda(\omega) | \iota(\omega) = 1]
\]

where \( \mathbb{E}_\phi \) is the conditional expectation under \( f_\phi \). This relationship says that the pooling price equals the value of the expected quality sold. Let’s now define the problem of the entrepreneur.

**Problem 1 (Producer)** *The entrepreneur solves:*

\[
W^p (k; p, q, w) = \max_{\sigma, \check{\iota}(\omega), l} [Ak^{\alpha} l^{1-\alpha} - \sigma wl] + (xk - (1 - \sigma)wl) + q \int_0^1 (1 - \iota(\omega)) \lambda(\omega) k f_\phi(\omega) d\omega
\]

subject to:

\[
Ak^{\alpha} l^{1-\alpha} - \sigma wl \geq (1 - \theta^L) Ak^{\alpha} l^{1-\alpha}
\]

\[
(1 - \sigma)wl \leq xk
\]

\[
x = p \int_0^1 \iota(\omega) d\omega.
\]

Recall that \( \sigma \) is the fraction of the wage bill that is paid after production. The first constraint in this problem, (1), is an incentive compatibility constraint that states that the outout that remains with the entrepreneur after it pays the \( \sigma \)-fraction of the wage bill must exceed the amount of funds he can divert. Rational workers require this incentive

\(^8\)This price will be an equilibrium object in the following section.
compatibility because they could otherwise provide work to other entrepreneurs at the market wage without risking a default. The second constraint, (2), is a working capital constraint and it says that the fraction of the wage bill paid in advance, \((1 - \sigma) \omega l\), cannot exceed the liquid funds, \(xk\), held by the entrepreneur.

To solve this problem, I employ a version of the envelope theorem and exploit the fact that this problem is homogeneous in capital. The strategy consists of breaking the problem into two subproblems. The first subproblem is an optimal labor choice subject to the enforcement and working capital constraints for given an amount of liquidity. The value of this problem yields an indirect profit function of liquidity. The second subproblem determines what qualities are sold given this indirect profit function. Notice that

Hence, let's hold \(\iota (\omega)\), and therefore \(x\) at its optimal. Once \(x\) is fixed, the objective of the entrepreneur must be to choose employment subject to the enforcement constraint (16) and the working capital constraint (18). I solve this problem for \(k = 1\) and because the objective is linear in \(k\) if we maximize the over the labor to capital ratio and all the constraints can be written in terms of this ratio.

**Problem 2 (Optimal Labor)** Given \(x\), the entrepreneur solves

\[
\begin{align*}
  r(x; \omega) &= \max_{l, \sigma} \left[ Al^{1-\alpha} - \sigma \omega l \right] + x - (1 - \sigma) \omega l \\
  \text{subject to} \quad &Al^{1-\alpha} - \sigma \omega l \geq (1 - \theta^L) Al^{1-\alpha} \\
  \text{and} \quad & (1 - \sigma) \omega l \leq x.
\end{align*}
\]

The optimal employment decision is given by:

**Proposition 1 (Optimal Labor)** The solution to Problem 2 is \(l^*(x) = \min \{ l^{\text{cons}}(x), l^{\text{unc}} \} \) where \(l^{\text{cons}}(x) = \max \{ l : \theta^L Al^{1-\alpha} + x = l \} \) and \(l^{\text{unc}}\) is the unconstrained labor choice. Constraints are always slack if \(\theta^L \geq (1 - \alpha)\).

This proposition states that the entrepreneur is constrained to hire less labor than the unconstrained optimal if liquidity does not reach a certain level. When this is the case, the enforcement and the working capital constraints bind. The entrepreneur is bound to choose employment so that his wage bill equals his liquid funds plus the pledgeable fraction of income.\(^9\) An immediate corollary from Proposition 1 is that if the pledgeable amount of

\(^9\)The max simply takes care of not choosing \(l = 0\) when \(x = 0\). The entrepreneur can still hire workers even if \(x = 0\).
output is less than the efficient labor share, \( \theta^L < (1 - \alpha) \), efficient employment requires a positive amount of liquid funds. The condition is intuitive: \( \theta^L \) is the fraction of output that can be fully pledged to workers and since \( (1 - \alpha) \) is the efficient labor share of output, liquid funds must fill the gap.

Now using the envelope theorem, the problem of choosing \( \iota (\omega) \) can be solved using the indirect profit of liquidity \( r (x; w) \), the objective of Problem 2:

**Lemma 1 (Producer’s Problem II)** Problem 1 is equivalent to:

\[
WP(k; p, q, w) = \max_{\iota(\omega) \geq 0} r(x; w) k + xk + qk \int \lambda(\omega) (1 - \iota(\omega)) f_{\phi}(\omega) \, d\omega
\]

\[
x = p \int \iota(\omega) d\omega
\]

where \( r(x; w) \) is the value of Problem 2.

By reducing the problem I can solve for the optimal selling decision \( \iota(\omega) \) directly and obtain an equilibrium expression for the pooling price \( p \).

**Proposition 2 (Producer’s Equilibrium Liquidity)** An equilibrium is characterized by a threshold quality function \( \omega^* \). All qualities under \( \omega^* \) are sold by the producer. The equilibrium liquidity \( x \) and the pooling price \( o \) are given by

\[
x = p F_{\phi}(\omega^*) \text{ and } p = q E_{\phi} [\lambda(\omega) | \omega < \omega^*].
\]

In addition, \( \omega^* \) is either: [1] Interior solution: \( \omega^* \in (0, 1) \) and solves,

\[
(1 + r_x(x)) E_{\phi} [\lambda(\omega) | \omega < \omega^*] = \lambda(\omega^*), \quad (4)
\]

[2] Fully liquid: \( \omega^* = 1 \) if \( r_x(q E_{\phi} [\lambda(\omega)]) \geq 0 \), or [3] Market Shutdown: \( \omega^* = \emptyset \) with \( p = 0 \).

Proposition 2 establishes that all equilibria are characterized by a threshold quality \( \omega^* \). All qualities below this are sold. The interesting cases are the interior solutions. Equation (4) resembles the equilibrium condition in Akerlof [1970]'s classical lemons example where a marginal quality valued by a seller equals the expected quality valued by the buyer. However, there is a key distinction. Whereas in Akerlof [1970] valuations by buyers and sellers are exogenously given, here those valuations depend on the shadow value of an extra unit of liquidity.

The shadow value of additional liquidity is \( (1 + r_x(x)) \). By selling a given unit, the entrepreneur obtains \( p \) liquid funds. Those liquid funds are used to pay for the entrepreneur’s
pay-roll in advance. Those funds are eventually returned to the entrepreneur in the form of less wages after production plus the benefit of additional workers \( r_x (x) \). Hence, the overall, marginal benefit of a given quality of capital is \( p (1 + r (x)) \). Naturally, costs and benefits must be equal at the margin. When the entrepreneur sells the threshold unit \( \lambda (\omega^*) \), he loses this in efficiency units. Those units are worth to him their opportunity cost, \( q\lambda (\omega^*) \).

Substituting market clearing condition and clearing out \( q \) from both sides gives us the corresponding expression for the interior solutions.

**Wealth.** A corollary to Proposition 2 shows that the entrepreneur’s problem is linear in his capital stock. This corollary will be used when we introduce the dynamic model.

**Proposition 3** (Value of the Firm) \( W_p (k; p, q, w) = \tilde{W}_p (p, q, w) k \) where

\[
\tilde{W}_p (p, q, w) \equiv r (x; w) + q\bar{\lambda}.
\] (5)

Here, \( r (x; w) \) is the solution to problem 1 and \( x, p \) and \( \omega^* \) are given by Proposition 2.

The propositions in this section are proven jointly in the appendix. \( W_p \) is the sum of profits per unit of capital given \( x \) and the value of hist initial capital stock. The entrepreneur’s financial wealth is \( xk + qk \int_0^1 \lambda (\omega) f_\phi (\omega) d\omega \) but the zero-profit condition for intermediaries implies \( x = q \int_0^\omega^* \lambda (\omega) f_\phi (\omega) d\omega \). Hence, his financial wealth equals \( q\bar{\lambda} \).

It is worth discussing some features of this model before jumping to the investment problem.

**Comparative Statics on \( \phi \).** To say more about this problem, let’s assume the following about the advantage rate of \( f_\phi \):

**Assumption 1.** \( f_\phi \) satisfies that \( \frac{\lambda (\omega^*)}{E_\phi [\lambda (\omega) | \omega < \omega^*]} \) is increasing in \( \omega^* \).

This assumption is useful to guarantee:

**Proposition 4** (Interior Solutions) Assume 1 and that \( \lambda (0) > 0 \). Then, there always exists single positive \( \omega^* \) in Proposition 2.

**Proof.** Note that \( \frac{\lambda (\omega^*)}{E_\phi [\lambda (\omega) | \omega < \omega^*]} \) is increasing. Under the assumptions, the advantage rate is 1 when \( \omega^* = 0 \). At \( \omega^* = 1 \), the advantage rate is greater than 1. In contrast, \( 1 + r_x (qE_\phi [\lambda (\omega) | \omega < \omega^*]) \) is decreasing in \( \omega^* \), starts at a number greater then 1. Thus, if the two curves cross, they must cross at a single point. Otherwise, or if they don’t cross \( \omega^* = 1 \) is an admissible solution.

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We are now ready to perform comparative statics about \( f_\phi \), the distribution of capital qualities. This exercise provides the main intuition behind the endogenous liquidity mechanism. Consider a family of distributions \( \{ f_\phi \} \) indexed by \( \phi \). I impose some structure on the quality distributions \( \{ f_\phi \} \) to provide an interpretation to \( \phi \):

**Assumption 2.** The set \( \{ f_\phi \} \) satisfies:

1. **Mean preservingness:** \( \int \lambda(\omega) f_\phi(\omega) \, d\omega = \bar{\lambda} \) for any \( \phi \in \Phi \).

2. **Monotone adverse selection:** \( \mathbb{E}_\phi[\lambda(\omega) | \omega < \omega^*] \) is weakly decreasing in \( \phi \) for any \( \omega^* \).

The first condition states that for any \( \phi \), the mean of \( f_\phi \) is always \( \bar{\lambda} \). This implies that the changes in \( \phi \) are mean-preserving. The implication of this condition is that the aggregate amount of capital does not change with \( \phi \). The second condition is more important. It provides an order to the \( \phi \) terms. In particular, it implies that adverse selection is necessarily worse with a greater \( \phi \). This follows because for any threshold quality \( \omega^* \), a financial firm will receive less efficiency units. Hence, the pooling price must fall after \( \phi \) is increases for any initial equilibrium cutoff \( \omega^* \). Since the second property can be often obtained by an increase in the variance of \( f_\phi \) I refer to an increase in \( \phi \) as an increase in dispersion or a mean-preserving spread.

In the dynamic model that follows, a Markov process will draw a values for \( \phi \) along the business cycle. periods with higher \( \phi \) will be periods of worse adverse selection although the production possibilty will remain unchanged. This will have dramatic effects on output. The intuition is captured by the static forces discussed here. Take a given value of \( \phi \). Equilibrium requires to solve equation (4) which combines the 0 profit condition of intermediaries with the entrepreneur’s incentive to sell capital under asymmetric information:

\[
\text{Marginal Value of Liquidity} = \frac{\lambda(\omega^*)}{\mathbb{E}_\phi[\lambda(\omega) | \omega < \omega^*]} \quad (6)
\]

Now consider an increase in \( \phi \). Since by assumption, \( \mathbb{E}_\phi[\lambda(\omega) | \omega < \omega^*] \) must fall with \( \phi \), it will be the case that the marginal benefit of liquidity, \( (1 + r_x(x)) \), must increase to restore equilibrium. With this, the threshold quality \( \omega^* \) also falls. Equation (6) reveals the worse adverse selection as \( \phi \) increases. For any given \( \omega^* \), financial firms will pay a lower price because they expect worse average qualities under that \( \omega^* \). If the entrepreneur does not reduce choose a lower cut-off quality, he will face a loss on the margin. He must react. This means that increases in \( \phi \) cause a reduction in the equilibrium amount of liquidity. By Proposition 1 we know this will drag labor demand.
Figure 1 plots the increasing labor supply schedule against three labor demands corresponding to different values of $\phi$ in this static model. These demand schedules are consistent with the endogenous choices of liquidity that the entrepreneur chooses to hold for given wages in the y axis. For each wage, an increase $\phi$ reduces the labor demand because the cost of obtaining liquidity is higher. This induces a contraction in labor demand for any wage rate.

Figure 2 describes the effects of $\phi$ on the rest aggregate outcomes. As $\phi$ induces worse adverse selection, the $\omega^*$ jointly with $p$ and $x$ decline. Hours per unit of capital fall in response to the reduction in liquidity. The contraction in hours is responsible for explaining the contraction of output. Moreover, wages fall as labor is moving downwards along the supply schedule. A final observation is that the entrepreneur’s profits increase. This effect depends critically on initial condition for $\phi$ because the sign of the response of profits depends on whether hours are lower than their monopsonic equilibrium amount.

Figure 3 illustrates how liquidity is endogenously determined at different levels of dispersion. The top panels show the value loss on the marginal quality sold and the marginal return to liquidity. These amounts satisfy equation 6. The marginal value of liquidity has two components, the additional increase in the labor force with more liquidity and the additional profits obtained by hiring an additional worker depicted in the bottom panel.

2.1.1 Discussion

Limited Enforcement on Labor Contracts. The option to default on labor contracts imposes a constraint on the entrepreneur’s employment decision that depends on his liquid funds. This form of limited enforcement has a similar effect to the working capital constraint that requires the wage bill to be payed up-front. In fact, it corresponds to the limiting case where $\theta^L$ to 0. Working capital constraints, originally introduced by Christiano and Eichenbaum [1992], explain labor demand responses to borrowing costs. Quantitative work by Christiano et al. [2005] or Jermann [2012] has shown that these are important features of business cycle models. A similar force operates under limited enforcement. However, under limited enforcement, the fraction of the wage bill payed up front, $(1 - \sigma)$ here, is not constant.

Under decreasing returns to scale, production costs become a higher proportion of output as more labor is used. Since the fraction of output that can be pledged is constant and but costs are an increasing proportion, the entrepreneur needs a higher proportion of liquid funds to operate at a bigger scale. This feature has the observable implication that working capital

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10The figure corresponds to the actual demand and supply schedules at a point of the state space of the dynamic model of the following section. This point is the mean of the invariant distribution of the model’s state.
Figure 1: **Labor Demand and Supply**. The figure plots the response of labor demand to moves in the dispersion of capital quality.
Figure 2: **Comparative Statics on** $\phi$. The figure plots the response to variables (in partial equilibrium) as $\phi$ is increased.
Figure 3: $\omega^*$ as a function of $\phi$. The figure plots variables that affect the trade off between the benefit of liquidity and costs of obtaining it as a function of $\phi$. 
over total costs must be increasing in output. Hence, in the model, the working capital to
costs ratio falls as $\phi$ induces higher costs of obtaining liquidity. In the empirical assessment
of the model, I present evidence that the working capital over costs ratio fell during the
Great Recession, which may allow the model to dispersion shocks to $TFP$ shocks.

Figure 4 replicates partial equilibrium exercise of Figure 2 when $\sigma$ is constant (as with a
standard working capital constraints). The main takeaway from the figure is that a standard
working capital constraint would amplify the impact of $\phi$ despite that liquidity holdings are
higher as entrepreneurs have more incentives to obtain liquidity. The overall qualitative
implications are similar.

The idea that workers are lenders of hours to their employers is shared with Michelacci
and Quadrini [2005] and Michelacci and Quadrini [2009]. The difference is that in those
papers, worker-employer relations are long-term. If labor is a complementary input to other
variable inputs, the response of hours to liquidity may be still large even without enforcement
problems in labor. If variable inputs are obtained by short-term easy-to-default contracts
with suppliers, the lack of liquidity may affect labor demand via the complementarities with
other variable inputs.\footnote{Recent work by \textsuperscript{?} formalize this point and argue that if production requires interfirm linkages, the
elasticity of output to liquidity can much larger than in models with a representative firm. In other words,
the macro-elasticity of output to liquidity is much larger than its micro counterpart.}

\textit{Wage Rigidity.} The model abstracts from any form of wage rigidity. The empirical
assessment of the dynamic model shows that it fails because wage are more responsive than
in the data (see \textsuperscript{?}). Figure 2 presents the behavior of the model holding real wages fixed to
their equilibrium level at the lowest value of dispersion. The figure shows that wage rigidity
amplifies the effects of $\phi$ for several reasons. First, as liquidity falls with more dispersion,
entrepreneurs are constrained to higher less workers than if wages could adjust. Second, if
wages do not adjust, total costs over output do not decrease with less liquidity. Hence, the
enforcement problem becomes worse. This will reduce the benefits of obtaining liquidity, so
liquidity will be less than otherwise. These forces make output more sensible to $\phi$ under wage
rigidities. In addition, wage rigidities also lead to a reduction in profits after an increase in
$\phi$ as the monopsony effect vanishes.

\subsection{Endogenous Liquidity and Investment}

In this section, we study the problem of an entrepreneur that produces capital. He also lacks
the input for his production and faces a similar enforcement problem to the one studied
before. I call this entrepreneur the i-entrepreneur to distinguish him from the p-entrepreneur
of the previous section. Everything else is the same and the essence of his problem is the
Figure 4: Comparative Statics on $\phi$ with wage rigidity and constant working capital constraints. The figure plots the response to variables (in partial equilibrium) to increases in $\phi$ for fixed $w$ and $\sigma$. 
same as in KM.

Production of investment goods. The i-entrepreneur has a constant returns to scale technology that transforms a unit of consumption into a unit of capital.

Limited enforcement in investment claims. The i-entrepreneur can sell claims on capital goods in exchange for consumption goods. Following KM, an i-entrepreneur has access to a technology to divert a fraction \((1 - \theta^I)\) of his investment projects for personal use. This imposes a constraint on the issuance of claims. A similar restriction is obtained in models of hidden-effort (e.g., Holmstrom and Tirole [1997] and Holmstrom and Tirole [1997]).

Information. The entrepreneur holds a capital stock which he only uses to obtain liquid funds. The i-entrepreneurs has the same private information about \(\omega\) as before. In contrast, investment projects are homogeneous so there is no asymmetric information problem for newly produced capital. Financial intermediaries buy capital under asymmetric information and resell them at an exogenous price \(q\). They earn zero profits.

An i-entrepreneur’s problem is similar to the p-entrepreneurs except that he chooses an optimal financial structure for investment projects. He also maximizes the value of his assets. Note that like his p-counterpart, he also lacks the input to his production technology. To finance his production the i-entrepreneur obtains inputs by either selling capital under asymmetric information or issuing claims against his output.

Problem 3 (Investor) The i-entrepreneur solves:

\[
\begin{align*}
\mathcal{k}^{t,i}(k;p,q) &= \max_{i^d,i^s,i(\omega)} i - i^s + k^b + q^{-1} (x_k - i^d) + \int_0^1 (1 - \ell(\omega)) \lambda(\omega) k f_\phi(\omega) d\omega \\
\text{subject to:} & \\
& i = i^d + qi^s \\
& qk^b + i^d \leq x_k \\
& i - i^s \geq (1 - \theta^I) i \\
& x = p \int_0^1 \ell(\omega) f_\phi(\omega) d\omega.
\end{align*}
\]

This problem says that the liquid funds available to the entrepreneur are \(x_k\). These are obtained by selling capital \(\int i^s(\omega) f_\phi(\omega) d\omega\) at a price \(p^i\). The entrepreneur uses liquid funds to buy \(k^b\) capital produced by others at \(q\) or produces capital investing \(i^d\). In addition, he obtains production inputs issuing \(i^s\) claims at the market price \(q\). Thus, his output is \(i = i^d + qi^s\) since production is linear in inputs. Thus, \(i^d = i - qi^s\) is the portion of an i-entrepreneur’s investment that is financed internally (I call this the down payment). The
downpayment plays a similar role as the portion of the wage bill paid upront. Given $i^d$, the entrepreneur optimally chooses $i^*$, the number of investment claims it issues. Finally, (22), is an incentive compatibility condition that prevents the entrepreneur from defaulting on his claims. The constraint states that investment, net of claims, should be larger than the capital kept upon default $(1 - \theta^i) i$. This constraint is introduced in KM. We follow the same steps as with the $p$-entrepreneur and solve the $i$-entrepreneur’s problem is given $x$:

**Proposition 5 (Optimal Financing)** When $q > 1$, any solution to Problem 9 requires $i^* = \theta^i i$, $k^b = 0$ and $i^d = xk$. When $q = 1$, the solution for $i^*$, $i^d$ and $k^b$ is indeterminate. If $q < 1$, $k^b = xk$ and $i^d = i^* = 0$.

The interesting case is when $q > 1$. The proposition says that if $q > 1$, the entrepreneur will issue as many claims as he can. The reason is that he is exploiting an arbitrage opportunity: creating capital costs one unit of consumption but by selling claims he obtains $q$. Thus, for any unit of investment, he finances the $(1 - \theta q)$ the fraction of his output but he owns the $(1 - \theta)$ fraction. Thus, his replacement cost is $q^R = \frac{(1-\theta q)}{(1-\theta)}$. Replacement costs determines his choice of liquid funds. Proposition 6 is the analog of Proposition 2 which describes the equilibrium liquidity chosen by the $i$-entrepreneur:

**Proposition 6 (Investors Equilibrium Liquidity)** An equilibrium is characterized by a threshold quality function $\omega^i$ such that all qualities under $\omega^i$ are sold by the $i$-entrepreneur. The equilibrium liquidity and price for $i$-entrepreneurs are given by:

$$x^i = p^i F(\omega^i) \text{ and } p^i = q \mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^i] .$$

In addition $\omega^i$ is either: [1] Interior solution: $\omega^i \in (0, 1)$ and solves,

$$\frac{q}{q^R} \mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^i] = \lambda(\omega^i) , \quad (7)$$

[2] Fully liquid: $\omega^i = 1$ if $\frac{q}{q^R} \geq \lambda(1) / \lambda$ or [3] Market Shutdown: $\omega^i = \emptyset$ with $p^i = 0$.

As with producers, Proposition 6 states that the solution is also characterized by a threshold quality. However, in this case, the exogenous valuations in the lemons problem are replaced by Tobin’s $Q$, the market price of capital $q$ over the replacement cost $q^R$. Thus, this entrepreneur equates the marginal cost of liquidity to the marginal benefit of obtaining liquidity which is given by his arbitrage opportunity in the creation and financing of capital:

$$\frac{q}{q^R} = \frac{\lambda(\omega^i)}{\mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^i]}$$
As with the p-entrepreneur, an increase in $\phi$ will increase the marginal cost of obtaining liquid funds. This will lead to a fall in the threshold quality of capital sold, $\omega^i$. The consequences are a reduction in liquidity and, by Proposition 9, a contraction in aggregate investment.

3 Collateralized Debt with Default

I now allow p-entrepreneur’s to issue collateralize debt CD contracts without recourse. These contracts are equivalent to repurchase contracts (repos) so for the rest of the paper I use the terms interchangeably. A repo contract is characterized by a pair of prices $(p^S, p^F)$ and a unit of capital. When an entrepreneur enters a repo contract, it transfers the property of the capital unit to the intermediary. The quality of that unit is private information to its owner. The intermediary provides the entrepreneur $p^S$ liquid funds for that unit. Thus, $p^S$ is the purchase price. Under a repo contract, the entrepreneur must repurchase the capital unit paying a pre-specified price $p^F$. The entrepreneur may choose not to repurchase the capital unit, defaulting on the repo contract, in which case the intermediary retains the unit. If the entrepreneur pays $p^F$ he is returned the capital unit. Since there is no recourse, the contract is in fact a call option with private information. I assume that financial intermediaries can credibly commit to return the asset if the entrepreneur pays $p^F$.

One can see this contracts as collateralized debt with default. Let $p^S$ represent the loan size and $R = \frac{p^F}{p^S}$ the gross interest rate of a loan. The unit of capital represents the collateral of this loan. Collateral is pledged to obtain $p^S$ and is returned only if $Rp^S$ paid back to the intermediary.

Description of Equilibria. For simplicity of exposition, I focus on equilibria with a single contract $(p^S, p^F)$ and where intermediaries earn zero profits.\textsuperscript{12} I only present the description for p-entrepreneur’s. Given the symmetry in both problems, outcomes should be expected to be similar for i-entrepreneurs.

Recall that Lemma 1 shows that we could summarize the p-entrepreneur’s problem (Problem 1) without reference to the labor choice and the working capital choice. Once liquidity is determined, 2 takes care of this choice and yields an inderect value for liquidity. With repo contracts, we obtain an analogue problem to the one in Lemma 1 which determines the optimal liquidiy choice.

\textsuperscript{12}In fact, the unique contract studied in this section is the outcome of a competitive repo market with free entry of intermediaries is allowed. The is presented in Bigio [2013].
**Problem 4 (Producer with repo)** The p-entrepreneur maximizes:

\[ W^p(k; p^S, p^F, q, w) = \max_{I(\omega), \iota(\omega)} r(x) k + xk + \ldots \]

subject to:

\[ k \int_0^1 (1 - I(\omega)) \iota(\omega) (q\lambda(\omega) - p^F) + (1 - \iota(\omega)) q\lambda(\omega) f(\omega) d\omega \]

In this problem, \( r(x) \) is again the value of Problem 2. As before, \( \iota(\omega) \) is the indicator that is 1 when a unit of quality \( \omega \) participates of a contract. The indicator function \( I(\omega) \) turns into 1 when a repo contract is defaulted when its collateral is of quality \( \omega \). Note that when this indicator is off, the entrepreneur retains \( \lambda(\omega) \) efficiency units, which he values at a price \( q \), but he pays \( p^F \). When he defaults, this indicator turns into a 1 and the unit is no longer with his capital stock. In this case, he saves \( p^F \).

Profits to the intermediary are:

\[ \Pi(p^F, p^S, \iota(\omega), I(\omega)) = \int_0^1 \iota(\omega) \left[ ((1 - I(\omega))p^F + I(\omega)q\lambda(\omega)) - p^s \right] f(\omega) d\omega. \]

Profits are the sum of the profits on all contracts with collateral \( \omega \) brought to the intermediary (hence the \( \iota(\omega) \) at the outset of the integral). The intermediary earns \( (p^F - p^s) \) on the units that are not defaulted indicated by \( (1 - I(\omega)) \). The intermediary keeps the units that are defaulted and resells them at a price \( q \) per efficiency units. Thus, he earns \( q\lambda(\omega) \) minus \( p^s \) on defaulted units which are indicated by \( I(\omega) \).

**Equilibrium liquidity with CD.** The equilibrium liquidity with repo contracts is given by a pair of prices \( (p^S, p^F) \) and policy functions \( (I(\omega), \iota(\omega)) \) such that (1) \( (I(\omega), \iota(\omega)) \) are solutions to Problem 4 given prices, and (2) intermediaries are competitive; \( \Pi(p^F, p^S, \iota(\omega), I(\omega)) = 0 \). The equilibrium amount of liquidity may be summarized by by the following proposition.

**Proposition 7 (Repo Equilibria)** Any equilibrium with a single repo contract by prices \( (p^S, p^F) \) and two threshold qualities \( (\omega^p, \bar{\omega}^p) \). These satisfy the following set of conditions:

\[ p^S \int_0^{\omega^p} f(\omega) d\omega = \int_0^{\omega^p} q\lambda(\omega) f(\omega) d\omega + p^F \int_{\omega^p}^{\bar{\omega}^p} f(\omega) d\omega \]  \hspace{1cm} (9)

and

\[ q\lambda(\omega^p) = p^F \]  \hspace{1cm} (10)
and
\[ r_x \left( p^S \int_0^{\bar{\omega}^p} f(\omega) \, d\omega \right) p^S = (p^F - p^S). \] (11)

Then, \( \iota(\omega) \) equals 1 for \( \omega < \bar{\omega}^p \) and \( I(\omega) \) equals 1 for \( \omega < \omega^p \).

The proof is relegated to the appendix but the idea is intuitive. If an agent defaults on a given quality, he must also default on all units of inferior quality brought to the contract. Otherwise, the agent could be better off by defaulting on lower qualities and keeping the higher quality while not affecting his liquidity. This means that there is a threshold \( \omega^p \) such that all qualities under this are defaulted. This explains the form of the zero profit condition (9). The threshold quality for defaults must be such that the entrepreneurs is indifferent between defaulting and not which gives us (10). Since some qualities may be defaulted, the repurchase price is (weakly) higher than the selling price: \((p^F - p^S) \geq 0\). Hence, the agent bringing a given quality to a repo contract will experience a loss. Hence, it better be that the value of having additional liquid compensates for that loss. That is the intuition behind equation (11).

According to this proposition, equilibrium liquidity is characterized by 3 equations and 4 unknowns. Thus, potentially there is a continuum of solutions. For a particular example, Figure 5 depicts the entire set of equilibria. Each equilibrium is indexed by some \( \omega^* \) corresponding to a participation threshold \( \bar{\omega}^p \). The figure depicts the properties of the set of equilibria indexed by \( \omega^* \). The upper panels the equilibrium liquidity and the implied interest rate for a participation cutoff \( \omega^* \). The bottom panels show the implied default rate, \( F(\omega^p) / F(\bar{\omega}^p) \) and the loan size \( p^S \) of each contract. There are three contracts of particular interest: the sales contract (wide circle), the contract where \( \bar{\omega}^p = 1 \) (square), and the contract with the largest loan size \( p^S \) (diamond).

Properties of CD contracts. The first property, which can be checked easily, is that a particular case of repo contracts corresponds to the selling contracts of the earlier. This contract is the one for which \( \bar{\omega}^p = \omega^p \), that is, the one for which all contracts are defaulted. It is also the equilibrium with the lowest participation because contracts with lower participation would also have a full default but profits would be higher than 0. Second, liquidity is increasing in the participation cutoff \( \omega^* \). When more units are collateralized, the higher the quality of the pool and the lower the default rate. However, because higher participation rates require incentives to participate, \( p^S \) may be decreasing in \( \omega^* \). Hence, the third property, \( p^S \) is possibly non-monotone \( \omega^* \). In the quantitative section, I will select the equilibrium with the highest loan size.\(^{13}\) However, there is an observational equivalence result that is worth

\(^{13}\)This equilibrium would arise in an environment where intermediaries compete for costumer as in the shown by Bigio [2013]. The is that if an agent is defaulting on a given quality, he would sign the repo with
Observational Equivalence. Observe that if we substitute the 0 profit condition for the intermediary into the entrepreneur’s budget constraint, the value for the entrepreneur is:

$$W^p(k; p^S, p^F, q, w) = \left( r(x) + q\lambda \right) k.$$

This is the same value function of Proposition 3. This is a useful property of the problem studied. It says that as long as a sales contract and a repo contract yield the same amount of liquidity, they yield the same allocations. A corollary of this results is that for a given equilibrium with sales obtained for a given $\phi$, one can find another $\phi'$ such that equilibrium with repo contracts yield the same allocations. The workings of this algorithm is shown in Figure ???. The figure takes the construction in Figure 5 and computes the equilibria for the sales contracts and the highest loan size contracts for different values of dispersion. In the top panel, one can observe given an initial value of liquidity with sales, one can increase the dispersion to obtain the same amount of liquidity with repo contracts. The allocations of $l$ and $\sigma$ will be the same. The connotation of this process is that if $f_\phi$ is unobservable, both contracts are indistinguishable from aggregate data. I use this observational equivalence as an algorithm to compute equilibria with repos in the quantitative analysis of the model.

4 Dynamic Model

We now turn to the dynamic version of the model. This section embeds the problems studied in Section 2.

4.1 Environment

The dynamic model is formulated in discrete time and infinite horizon. There are two goods: a perishable consumption good (the numeraire) and capital. Every period there are two aggregate shocks realized: a TFP shock $A_t \in \mathbb{A}$ and a shock $\phi_t \in \{\phi_1, \phi_2, ..., \phi_N\}$ that selects a member among the family of capital quality distributions $\{f_\phi\}$. A Markov process for $(A_t, \phi_t)$ evolves according to a transition probability $\Pi$.

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the highest possible spot prices $p^S$ since he knows he won’t repay. Competition drives all all units to be brought to the same contract.
Figure 5: Set of Repo Contracts.

**x - liquidity**

**Interest Rates**

**Default Rates**

**p^L loan-size**
Figure 6: Set of Repo Contracts. xxx
4.2 Demography and Preferences

The three agents are workers, entrepreneurs and financial firms. The measure of entrepreneurs is normalized to a unit and the mass of workers to $\varpi$ and the measure of financial firms is irrelevant.

**Workers.** Workers choose consumption and labor only. They don’t have access to savings technologies. Their period utility is given by:

$$U^w(c, l) \equiv \max_{c \geq 0, l \geq 0} c - \frac{l^{1+\nu}}{(1 + \nu)}$$

where $l$ is the labor supply and $c$ consumption. $\nu$ is the inverse of the Frisch-elasticity. Workers satisfy a static budget constraint in every period: $c_t = w_t l_t$ where $w_t$ is their wage in period $t$. The only role for workers in the model is to provide an elastic labor supply schedule.\(^{14}\)

**Financial Firms.** Financial firms are intermediaries that purchase capital under asymmetric information and resell it under full disclosure. These firms are competitive profit maximizers. As in Prescott and Townsend [1984] and Bisin and Gottardi [1999], financial firms simplify the definition of equilibria but the can be replaced by a market clearing condition as in in Section 2.

**Entrepreneurs.** The attention is on the entrepreneur. An entrepreneur is identified by a number $z \in [0, 1]$. Every period, entrepreneurs are randomly assigned one of two possible types: investors and producers. I refer to these types as i-entrepreneurs and p-entrepreneurs because they will face the same problems as in Section 2. At the beginning of each period, entrepreneurs draw a type where the probability of becoming an i-entrepreneur is always equal to $\pi$. Thus, every period there is a mass $\pi$ of i-entrepreneurs and $1 - \pi$ of p-entrepreneurs.\(^{15}\)

The entrepreneur’s preferences over consumption streams are given by an expected utility criterion:

$$E \left[ \sum_{t \geq 0} \beta^t U(c_t) \right]$$

where $U(c) \equiv \frac{c^{1-\gamma}}{1-\gamma}$ and $c_t$ is the entrepreneur’s consumption at date $t$.

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\(^{14}\)The model could be modified to allow workers to save. The effect of this change would be to make the labor supply more elastic to temporal changes in wages without altering the results considerably. Also, we could have let entrepreneurs have a labor endowment. With GHH preferences the labor supply schedule would be the same.

\(^{15}\)Randomization is convenient to avoid keeping track of wealth distributions across groups. This feature reduces the dimension of the state space.
4.3 Technology

*Technology of p-entrepreneurs.* A p-entrepreneur, produces consumption goods with the same technology of Section 2. Thus, his profits are $A_t F (k_t(z), l_t) - w_t l_t$. Again, he has the technology to divert $\theta^p$ of their output for his personal benefit. Thus, he can default on his workers.

*Technology of i-entrepreneurs.* The i-entrepreneurs has access to the same constant returns to scale technology that transforms a consumption goods into capital of Section 2. In his case, he can issue investment claims and divert $\theta^i$ of the capital they create.

Thus, the economy operates like a two sector economy with sector’s producing according to the technologies of the static models presented before.

*Evolution of capital.* At the beginning of every period, capital is divisible into a continuum of pieces. Each piece is identified by a quality $\omega$. The differentiable function $\lambda (\omega)$ determines the corresponding efficiency units that remain from a quality $\omega$ by the end of the period. Efficiency units can also be interpreted as random depreciation shocks. Thus, $\omega$ and $\lambda$ correspond to the same objects of Section 2.

The distribution among qualities assigned to each piece is randomly changing over time. In particular, at a given point in time the distribution of capital qualities is determined by a density function $f_\phi$, which, in turn, depends on the current realization of $\phi_t$. The distribution is the same for all entrepreneurs although it differs through time. Therefore, the measure of units of quality $\omega$ out of a capital stock $k$ is $k (\omega) = kf_\phi (\omega)$. Between periods, each piece is transformed into future efficiency units by scaling qualities by their corresponding $\lambda (\omega)$. Thus, $\lambda (\omega)k (\omega)$ efficiency units remain from the $\omega$—qualities. Once capital units are scaled, they form homogeneous quantities of capital that can be merged or divided for form larger or smaller pieces. Thus, by the end of the period, the capital stock that remains from $k$ is,

$$\tilde{k} = \int \lambda (\omega) k (\omega) d\omega = k \int \lambda (\omega) f_\phi (\omega) d\omega. \quad (12)$$

In the following period, the capital held by every entrepreneur is again divided in the same way and the process is repeated indefinitely. This does not mean that an entrepreneur with $k$ capital at $t$ will necessarily hold all of $\tilde{k}$ at $t + 1$. On the contrary, entrepreneurs can choose to sell particular qualities. Using the earlier notation, $\iota^s (\omega)$ is indicator for the decision on

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Shocks to the efficiency units are commonly used in continuous time settings as Brunnermeier and Sannikov [2009], for example. In fact, any permanent shock that also affects the mean of the quality distribution can be decomposed into a permanent TFP shock and mean-preserving shock to the distribution. Both representations yield the same allocations so the environment accounts for mean and variance shocks. Isolating MPS from TFP shocks has the connotation that if $\phi$ has any effect on allocations, it is because these shocks affect equilibrium but not the feasible set of allocations.
selling the capital units of quality $\omega$.

In equilibrium, financial firms purchase all the units sold by entrepreneurs. Thus, an entrepreneur transfers $k \int t^s(\omega) f_\phi(\omega) d\omega$ units of capital to the financial sector. Accounting shows that the efficiency units that remain with the entrepreneur after his sale are $k \int \lambda(\omega) (1 - t^s(\omega)) f_\phi(\omega) d\omega$. Including investments and purchases of capital, the entrepreneur’s capital stock evolves according to:

$$k' = i - i^s + k^b + k \int \lambda(\omega) (1 - t^s(\omega)) f_\phi(\omega) d\omega, \quad (13)$$

where $i - i^s$, is the total investment $i$ carried out by the entrepreneur net issued claims, $i^s$. Finally, this stock is augmented by capital purchases, $k^b$.

Again, I impose Assumption 2 on the family \{f_\phi\} which state that (a) the average depreciation across qualities is always $\bar{\lambda}$ and (b) that the $\phi$-shocks lead to more adverse selection. The implication of the first condition is that the production possibility of the economy is invariant to this shock since $\tilde{k} = \bar{\lambda}k$ for any $\phi$. The implication of this property is that if $\phi$ has any effects on allocations, it is because it affects the actual but no the possible set of allocations.

### 4.4 Timing, Information and Markets

**Information.** Aggregate capital, $K_t \in \mathbb{K} \equiv [0, \bar{K}]$, is the only endogenous aggregate state variable. The aggregate state of the economy is therefore summarized by the vector $X_t = \{A_t, \phi_t, K_t\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$. At the beginning of each period, $X_t$ and the entrepreneurs’ type become common knowledge. This means that financial firms can discriminate between an entrepreneur’s activity.

The $\omega-$qualities are only known to the entrepreneur. In turn, financial firms can observe the amount of capital being transferred to them, $k \int t^s(\omega) f_\phi(\omega) d\omega$. However, they ignore the efficiency units that remain from that purchase, $k \int \lambda(\omega) t^s(\omega) f_\phi(\omega) d\omega$. The remaining efficiency units $\tilde{k}$ from a capital stock $k$ are independent of the entrepreneur’s choice of $t^s(\omega)$.

Using equation (12) we have:

$$\tilde{k} = k \int \lambda(\omega) (1 - t^s(\omega)) f_\phi(\omega) d\omega + k \int \lambda(\omega) t^s(\omega) k(\omega) d\omega.$$

Hence, the choice of $t^s(\omega)$ only affects the distribution of $t+1$ capital between entrepreneurs and intermediaries. Since the following period, $f_\phi$, affects every entrepreneur no matter how they come up to build $k'$, entrepreneurs only cares about maximizing the fraction of capital
that remains with him. This modeling choice is essential to solve the model without keeping track of the history of trades or the distribution of capital qualities. It also makes the lemons problem static.

**Timing.** The sequence of actions taken by the agents in this economy is as follows. At the beginning of each period all the relevant information is revealed. Then, p-entrepreneurs choose which qualities to transfer to financial firms in exchange for claims to consumption goods (liquid funds) denoted by x. Financial firms credibly guarantee to deliver these goods to its owner by the end of the period. Entrepreneurs transfer consumption claims to workers as an upfront payment of the fraction $1 - \sigma$ of salaries.

Workers then provide labor and production is carried out. The consumption goods created by this activity are then used by p-entrepreneurs to: (i) pay for the remaining fraction of the wage bill $(1 - \sigma)w_t$, (ii) to consume, and (iii) to purchase (or repurchase) capital to be delivered by financial firms.

When selling capital to p-entrepreneurs, financial firms obtain consumption goods which partially settle the claims on x and, in addition, are used to transact with i-entrepreneurs. In exchange for consumption goods, i-entrepreneurs sell capital qualities and claims to new investment projects, $i^*$, to financial firms. After the production of capital, all capital claims are settled.

This sequence of events is consistent with the physical requirement that consumption goods must be created before capital goods. For the rest of the paper, I treat these actions as occurring simultaneously.

**Markets.** Labor markets are competitive. I impose the following:

**Assumption 3.** Financial firms are **competitive** and the capital market is **anonymous** and **non-exclusive**.

Competition ensures financial firms earn 0 profits. Anonymity and non-exclusiveness guarantees that the market for capital is has pooling price. Without anonymity and exclusivity, financial firms pay a different price depending on the quantity of capital traded by each entrepreneur. For example, they can recover the full information outcomes if they offer a price schedule proportional to the cumulative distribution of $f_\phi$.

**Notation:** For the remainder of the paper, I append terms like $j(k, X)$ to indicate the policy function of an entrepreneur of type $j$ in state $(k, X)$. I use $\nu(\omega, k, X)$ to refer to the decision to sell a quality $\omega$ in that state. I denote by $E_\phi$ the expectations over the quality distribution $f_\phi$ and $E$ the expectations about future states.
4.5 Entrepreneur Problems and Equilibria

Entrepreneurs solve distinct problems according to their types. Both problems are recursive so from now I drop time subscripts. I begin with the description of the p-entrepreneur’s problem:

**Problem 5** (Producer’s Problem) *The p-entrepreneur solves*

\[
V^p(k, X) = \max_{c \geq 0, k^b \geq 0, \iota(\omega), \sigma \in [0, 1]} \left( U(c) + \beta \mathbb{E} \left[ V^j(k', X') \right] \right), \ j \in \{i, p\}
\]

*subject to*

**(Budget constraint)** \[c + q(X)k^b = AF(k, l) - \sigma wl + xk - (1 - \sigma)wl \] (14)

**(Capital accumulation)** \[k' = k^b + k \int \lambda(\omega) (1 - \iota(\omega)) f_\phi(\omega) \, d\omega \] (15)

**(Incentive compatibility)** \[AF(k, l) - \sigma wl \geq (1 - \theta^L) AF(k, l) \] (16)

**(Liquid funds)** \[x = 1 \int \iota^*(\omega) f_\phi(\omega) \, d\omega \] (17)

**(Working capital constraint)** \[(1 - \sigma)wl \leq xk \] (18)

The first constraint is his budget constraint. The right hand side of the budget constraint corresponds to the entrepreneur’s profits minus the amount of liquid funds he holds after paying for the \(\sigma\) fraction of the wage bill. The entrepreneur uses these funds to consume \(c\), and to purchase \(k^b\) at the full-information price \(q(X)\). The second constraint corresponds to the evolution of the entrepreneur’s capital stock with the restriction that p-entrepreneurs cannot produce capital or issue claims. The last 3 constraints are the constraints described in Section 2: the incentive compatibility constraint (16), the accounting of liquid funds per unit of capital (17), and the use of liquidity as working capital (18).

An i-entrepreneur’s problem is similar.

**Problem 6** (Investor’s Problem) *The i-entrepreneur solves*

\[
V^i(k, X) = \max_{c \geq 0, i^* \geq 0, k^b \geq 0, \iota(\omega) \geq 0} \left( U(c) + \beta \mathbb{E} \left[ V^j(k', X') \right] \right), \ j \in \{i, p\}
\]
subject to

(Budget constraint) \[ c + i + q(X)k^b = xk + q(X)i^s \] (19)

(Capital accumulation) \[ k' = k^b + i - i^s + k \int \lambda(\omega)(1 - t^s(\omega))f_\phi(\omega)d\omega \] (20)

(Liquid funds) \[ x = p^i(X) \int t^s(\omega)f_\phi(\omega)d\omega \] (21)

(Incentive compatibility) \[ i - i^s \geq (1 - \theta^i)i \] (22)

The right hand side of the i-entrepreneur’s budget constraint is the sum of funds obtained by selling capital, \( xk \), and by by issuing \( i^s \) claims to investment at price \( q(X) \) (because new units are of known quality). Funds are used to consume \( c \), to purchase \( k^b \), or to fund investment projects \( i \). The incentive compatibility condition (22) is the same as in Section 2.

**Financial firms.** Financial firms purchase capital units of different qualities from both entrepreneur types at pooling prices \( p^p \) and \( p^i \). They also purchase claims to investment projects at the full information price \( q \). Units are resold as homogeneous capital. Competition in financial markets ensures zero profits from trading with either entrepreneur type since there is no risk in this operation. I assume and later verify that the decision to sell a unit of quality, \( \omega \), is only a function of the entrepreneur’s type and the aggregate state \( X \) but independent of the size of his capital stock. Hence, we have the same zero-expected-profit conditions as before:

\[ p^p(X) = q(X)E_\phi [\lambda(\omega) | \text{quality } \omega \text{ is sold by a p-entrepreneur}] \] (23)

and

\[ p^i(X) = q(X)E_\phi [\lambda(\omega) | \text{quality } \omega \text{ is sold by a i-entrepreneur}] \] (24)

In every period, there is a measure over capital holdings and entrepreneur types. I denote this measure by \( \Gamma (k, j) \) for \( j \in \{i,p\} \). By independence, this distribution satisfies:

\[ \int \Gamma (dk,i) = \pi K \quad \text{and} \quad \int \Gamma (dk,p) = (1 - \pi) K . \] (25)

The total aggregate demand for efficiency units and the supply of investment claims are respectively:

\[ D(X) \equiv \int k^{b,p}(k,X) \Gamma (dk,p) \quad \text{and} \quad D^s(X) \equiv \int i^s(k,X) \Gamma (dk,i) . \]

Capital demand of p-types    Capital demand of i-types    Supply of new units by i-types
Finally, transfers of efficiency units from both groups to the financial sector are obtained by integrating over the corresponding qualities and capital stocks.

\[ S(X) \equiv \int k \left[ \int \iota^s(k, X, \omega) \lambda(\omega) f_\phi(\omega) d\omega \right] \Gamma(dk, i) \]

Effective units supplied by i-types

\[ + \int k \left[ \int \iota^s(k, X, \omega) \lambda(\omega) f_\phi(\omega) d\omega \right] \Gamma(dk, p) \]

Effective units supplied by p-types

Capital market clearing is given by \( D(X) = I^s(X) + S(X) \). Finally, the labor market clearing requires: \( \int l(k, X) \Gamma(dk, p) = \varpi l^w(X) \). The definition of equilibria does not depend on the distribution of capital because this economy admits aggregation, as shown later.

**Definition** (Recursive Competitive Equilibrium). A recursive competitive equilibrium is (1) a set of price functions, \( \{q(X), p^i(X), p^p(X), w(X)\} \), (2) a set of policy functions \( \{c^j(k, X), k^b^j(k, X), i^s^j(\omega, k, X)\} \), (3) a pair of value functions, \( \{V^j(k, X)\}\), and (4) a law of motion for the aggregate state \( X \) such that for distribution of over capital holdings \( \Gamma \) satisfying (25), the following hold: (1) taking price functions as given, the policy functions solve the entrepreneurs’ and worker’s problem and \( V^j \) is the value of the \( j \)-entrepreneur’s problem. (2) \( p^p(X) \) and \( p^i(X) \) satisfy the zero profit conditions (23) and (24). (3) The labor market clears. (4) The capital market clears. (5) Capital evolves according to \( K' = \int i(k, X) \Gamma(dk, i) + \bar{\lambda} K \). (6) The law of motion for the aggregate state is consistent with the individual’s policy functions and the transition function \( \Pi \).

5 Characterization

**Producer’s employment and liquidity.** I begin the characterization of equilibria by describing the solution to the p-entrepreneur’s problem. The strategy consists on breaking his problem into two subproblems. The first subproblem solves the entrepreneur’s labor, working capital and quality-sales decision. Once these solutions are found, the original problem is collapsed into a standard consumption-savings problem with linear stochastic returns.

Once \( k^b \) is substituted from the capital accumulation equation (15) into the p-entrepreneurs budget equation (14), his problem gives us a consolidated budget constraint:

\[ c + q(X) k' = AF(k, l) - w t + x k + q(X) k \int \lambda(\omega) (1 - \iota^s(\omega)) f_\phi(\omega) d\omega. \]
The choice of $\iota^s(\omega), l, \sigma$ only affect the right hand side of this consolidated budget constraint and not his objective function. The entrepreneur’s constraints only affect $\iota(\omega), l, \sigma$, not his consumption or savings decision. This means that the entrepreneur’s problem can be broken into two subproblems. The first solves for $\iota^s(\omega), l, \sigma$ to maximize the right hand side of the p-entrepreneur’s budget constraint. Then one solves for $c, k^b$ given the solution to the maximal budget. The first subproblem corresponds to Problem 1 in Section 2.

The solutions to $l(X)$ and $\sigma(X)$ are given by Proposition 1. Moreover, the equilibrium qualities sold are given by Proposition 2. Hence, any recursive competitive equilibrium is characterized by a threshold quality function $\omega^p(X)$ such that in state $X$, all qualities under $\omega^p(X)$ are sold by all p-entrepreneurs. The amount of liquid funds available to the p-entrepreneur, $x^p(X)$, is also determined.

Once we substitute the optimal policy for $\iota^p(k, X, \omega)$ into the p-entrepreneur’s problem, we collapse his decisions into a standard consumption-savings problem where the return and price of capital depend on equilibrium liquidity.

**Problem 7 (Producer’s Reduced Problem)** The

$$V^p(k, X) = \max_{c \geq 0, k' \geq 0} U(c) + \beta \mathbb{E} [V^j(k', X')|X], \ j \in \{i, p\}$$

subject to $c + q(X) k' = W^p(X) k$

where $W^p(X) \equiv [r(x^p(X), X) + q(X) \bar{\lambda}]$ (28)

$W^p(X)$ is the entrepreneur’s virtual wealth per unit of capital described in Proposition 3. Since all decisions are linear in the capital stock, the economy admits aggregation. This property delivers the tractability of the entrepreneurs problem.

**Investor’s financing and liquidity decisions.** The investor’s problem can be solved with the same method as for p-entrepreneurs. The idea is to break their problem into a consumption savings problem that uses the solution to their financing and quality sales decisions. A recursive equilibrium is characterized also by a threshold quality function $\omega^i(X)$ all qualities below $\omega^i(X)$ are sold by all i-entrepreneurs. Their financing decisions are the same as in Proposition 6. The proofs are relegated to the appendix. Substituting these results, the i-entrepreneur’s problem simplifies to:

**Problem 8 (Investor’s Reduced Problem)**

$$V^i(k, X) = \max_{c \geq 0, k' \geq 0} U(c) + \beta \mathbb{E} [V^j(k', X')|X], \ j \in \{i, p\}$$

subject to $c + q(X) k' = W^i(X) k$
where \( W^i (X) \equiv \left[ q(X) \int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) \, d\omega + q^R(X) \int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) \, d\omega \right] \) \hspace{1cm} (29)

For the investors, the virtual wealth per unit of capital \( W^i (X) \) takes a different form than for p-entrepreneurs. This quantity is a weighted sum over the entrepreneur’s capital qualities. The first term is the value of liquid funds: liquid funds correspond to the price of capital sold by i-entrepreneurs, \( p^i (X) \), times the volume of capital sold, \( F_\phi(\omega^p(X)) \). In equilibrium, \( p^i (X) = q(X) \mathbb{E}[\lambda(\omega) | \omega < \omega^p(X)] \). Substituting this condition, we obtain the first term in \( W^i (X) \). The second term corresponds to the illiquid units are valued at their replacement cost, \( q^R(X) \).

Optimal consumption-savings decisions. So far I have shown that the entrepreneurs’ problems can be summarized by two standard consumption-savings problems, problems 7 and 8. These problems are standard consumption-savings problems with homogeneous preferences and constant returns to scale. It is immediate to show that the policy functions are linear functions of the capital stocks and therefore, invoking Gorman’s aggregation result, we have the necessary conditions for the existence of a representative agent. This result guarantees the internal consistency of the definition of competitive recursive equilibrium without any reference to distributions. The optimal consumption-savings decisions are given by:

Proposition 8 (Optimal Policies) The policy functions for p-entrepreneurs are \( c^p (k, X) = (1 - \varsigma^p(X)) W^p(X) k \) and \( k^p (k, X) = \frac{\varsigma^p(X) W^p(X)}{q(X)} k \). For i-entrepreneurs these are: \( c^i (k, X) = (1 - \varsigma^i(X)) W^i(X) k \) and \( k^i (k, X) = \frac{\varsigma^i(X) W^i(X)}{q^R(X)} k \).

The functions \( \varsigma^p(X) \) and \( \varsigma^i(X) \) are marginal propensities to save for p-entrepreneurs and i-entrepreneurs. These functions solve a pair of functional equations. Proposition ?? presented in the Appendix shows that marginal propensities to save of both types solve a non linear functional equation which can be easily solved by repeated iteration.\(^{17}\) When \( \gamma = 1 \), one can verify that \( \varsigma^p = \varsigma^i = \beta \).

Full-information price of capital. The last object to be characterized is \( q \). One can rearrange the i-entrepreneur’s capital accumulation equation, substitute in the capital policy functions obtained from Proposition 8, and integrate across individuals to obtain their aggregate demand for investment net of claims:

\[
I^i(X) - I^s(X) = \left[ \frac{\varsigma^i(X) W^i(X)}{q^R(X)} - \int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) \, d\omega \right] \pi K. \hspace{1cm} (30)
\]

\(^{17}\)A similar operator is found in Angeletos [2007]. I cannot provide a direct proof for a theorem that guarantees that the repeated iteration of this operator converges for \( \gamma > 1 \). Nevertheless, Alvarez and Stokey [1998] show that the standard dynamic programming properties of this problem are guaranteed. Thus, if the operator converges, it converges to its unique fixed point.
Considering that in equilibrium only producers purchase capital, similar steps lead to an expression for the aggregate demand for capital purchases:

\[
D(X) = \left[ \frac{\tau^p(X) W^p(X)}{q(X)} - \int_{\omega > \omega^p(X)} \lambda(\omega) f_{\phi}(\omega) \, d\omega \right] (1 - \pi) K. \tag{31}
\]

The total sales of used capital under asymmetric information is obtained by aggregating over the capital sales of both types:

\[
S(X) = \left[ \int_{\omega \leq \omega^p(X)} \lambda(\omega) f_{\phi}(\omega) \, d\omega \right] (1 - \pi) K + \left[ \int_{\omega \leq \omega^i(X)} \lambda(\omega) f_{\phi}(\omega) \, d\omega \right] \pi K \tag{32}
\]

Market clearing requires \( D(X) = S(X) + I^s(X) \). To satisfy this expression, producers must repurchase all the units they sold in the period. In addition, it must be the case that investors satisfy their constraints on the issuance of investment claims (22). Since these constraints are linear in the entrepreneur’s capital stock, an aggregate version of this condition, \( I(X) - I^s(X) \leq (1 - \theta) I(X) \), holds if and only if there exists an allocation such that all the individual constraints are satisfied. Thus, any equilibrium must be characterized by a \( q(X) \) such that \( D(X) = S(X) + I^s(X) \) and \( \theta I(X) \leq I^s(X) \).

Solving for \( q(X) \) is not immediate from market clearing since enforcement constraints must also be met. Its solution and the rest of the equilibrium conditions that close the model are described in Appendix.

**Economic properties.** A distinguishing feature of this environment is that, in equilibrium, albeit they can do so, entrepreneurs will not choose to acquire the amount of liquidity that would allow them to entirely relax their enforcement constraints. This result follows from an existing tension between the enforcement constraints and the incentives to sell capital under asymmetric information. On one hand, selling a marginal unit of capital under asymmetric information is costly to the entrepreneur because he receives a pooling price for an object that he values above that price. On the other hand, when financial frictions are active, they provide the incentives that support trade under asymmetric information because relaxing these constraints is valued by the entrepreneur. When constraints are entirely relaxed by acquiring sufficient funds, liquidity has no value on the margin because there is no point in having additional funds. Nevertheless, to obtain this amount, the entrepreneur must incur a loss from selling a marginal asset in a pooling market.

Take for example a p-entrepreneur. If employment in his firm is efficient, the marginal loss of reducing liquidity is negligible because the marginal profit from labor is 0. Because selling the marginal quality asset is costly, the entrepreneur is better-off if he reduces part
of his capital sales. A similar consideration is true for i-entrepreneurs.  

Thus, the takeaway is that financial frictions must be active in order to support trade under asymmetric information. In other words, when liquidity is needed to enforce efficient employment or investment, the economy will feature under-employment and under-investment. The conditions on parameters that generate these inefficiencies are summarized by:

**Proposition 9 (Inefficiency)** Employment is always sub-efficient if \( \theta^L < (1 - \alpha) \). Investment \((I > 0)\) is always sub-efficient iff \( \theta^I \leq (1 - \pi) \).

**PROOF. FORMALLY.**

When \( \theta^L < (1 - \alpha) \), producers cannot credibly pledge workers the labor share of output unless they use liquid funds. Thus, in order to attain efficient employment, liquidity is needed. Yet, if employment is efficient, liquidity has no marginal value. By Proposition 2, this implies \( x^p = 0 \). A contradiction. Similarly, efficient investment requires the price of capital to equal its physical cost, \( q(X) = 1 \). When \( q(X) = 1 \), Proposition ?? implies that \( \varsigma^p(X) = \varsigma^i(X) \) and that \( W^p(X) \geq W^i(X) \). Market clearing conditions then imply that \( (I(X) - I^*(X))/I^*(X) \geq \pi/(1 - \pi) \). Hence, if \( \theta^I \leq (1 - \pi) \) holds, efficiency cannot be attained and investment frictions must be active.

Figure 7 provides a summary of the conditions on parameters that activate either friction. When \( \theta^L < (1 - \alpha) \), the financial frictions that affect labor markets are active so dispersion shocks impact the labor wedge. \( \theta^L = 0 \) corresponds to the working capital constraints in Christiano et al. [2005] or ?. When \( \theta^I < (1 - \pi) \), investment frictions are active, so dispersion shocks cause fluctuations that resemble fluctuations caused by investment specific shocks as in KM. The case where \((\theta^I, \theta^L) = (0, 1)\) is the specification in Kurlat [2009].

### 6 Results

We know study the quantitative performance of the model. We begin by describing the calibration strategy and then analyze the state space of the model. We then discuss the size of dispersion shocks needed to generate a recession as important as the Great Recession and what are the key parameters.

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18 If investment is efficient, then the physical cost of creating capital units equal its price \( q(X) = 1 \). Nevertheless, if \( q(X) = 1 \), trade under asymmetric information cannot be supported because liquidity has no value for the entrepreneur.
6.1 Calibration

The model period is a quarter. The calibration of preference parameters is standard to business cycle theory. I use log-utility for the calibration. Numerically, for any choice of $(\gamma, \beta)$ one can find a corresponding alternative value for $\beta$ such that the marginal propensities to consume under log-preferences are roughly the same.\(^{19}\) Therefore, I set $\beta = 0.97$ and $\gamma = 1$ to approximate policy functions corresponding with a risk aversion of 2 and a discount factor of 0.991. Log-utility is a convenient choice because the stochastic process that determines the quality distributions does not affect intertemporal decisions. This allows me to calibrate the dispersion shock to target a particular reduction in liquidity and guarantees that responses are independent of the process of $\phi$. We calibrate the model to obtain a Frisch-elasticity of 2 ($\nu = 1/2$). This elasticity is within the range of calibrations used in macro models. This parameter is key to determine the response of output to dispersion shock. We discuss this issue below.

Technology shocks are modeled so that their log follows an AR(1) process where the autoregressive coefficient, $\rho_A$, is set to 0.975 and the standard deviation of the innovations, $\sigma_A$, is set to 0.008. These numbers are obtained by estimation using quarterly data since 1970.\(^{20}\) $\lambda$ is set to 0.9873 so that the annualized depreciation rate is 5%, a typical number in the literature. $\pi$ is set to 0.1 to match the plant investment frequencies in Cooper et al.

\(^{19}\)I perform these numerical experiments in Bigio [2009]. This result is also related to findings in Tallarini [2000] that show that under CRRA preferences, risk aversion does not affect allocations in a standard growth model.

\(^{20}\)XXX TIME SERIES USED.
The calibration of the capital share, $\alpha$, is indirect. We set $\alpha$ to 0.27 to match a capital share of output to $1/3$.\(^{22}\)

Two parameter sets remain to be calibrated. The first is the pair of parameters, $\theta^L$ and $\theta^I$, which govern the extent of the limited enforcement problem in the investment and consumption goods sectors. The final parameter is the family of quality distributions $\{f_\phi\}$ and the stochastic process for $\phi$. I do not have any microeconomic support to calibrate these parameters directly so I take an indirect approach and pick the parameters to match some aggregate features of U.S. data.

I choose the family $\{f_\phi\}$ to be a set of log-normal distributions. $\phi$ indexes a variance and a support that keeps the average depreciation rate constant. The particular choice of log-normals is immaterial for the results but is convenient because this family is the parametric form chosen by many of the papers in the continuous time literature.\(^{23}\)

I assume that the stochastic process for $\phi$ is i.i.d. With log-utility, only the current realization of this shock and not its stochastic process determines consumption savings decisions. Since the experiments that I analyze don’t discuss their persistence or unconditional moments, they are not affected by this choice. Now, the choice of $\{f_\phi\}$ and $\theta^L$ will affect the labor market parameters. I jointly calibrate the average the dispersions of each $f_\phi$ and $\theta^L$ to match to moments. The first moment is an average for the labor wedge of 0.35. This number is close to the estimates of Shimer [2009] and Chari et al. [2007]. The second an average of working capital/costs ratio, corresponding to $\sigma$ in the model, of $1/3$. This figure is obtained using firm level data from COMPUSTAT.\(^{24}\) As an outcome of this calibration, the fraction of the wage bill secured in advance is on average close to 60%.$^{25}$ I obtained values of $\theta^L$ equal to xxx and a mean standard deviations of $f_\phi$ set to xxx. To get a rough idea, of what this figure means, the calibration requires a xxx% increase in capital quality dispersion to generate a reduction of $xxx\%$ in liquidity. This shock is more moderate than

\(^{21}\)The data suggests that around 20% to 40% plants augment a considerable part of their physical capital stock in a given year. These figures vary depending on plant age. By, setting $\pi$ to 0.1, the arrival of investment opportunities is such that about 30% of firms invest in a given year.

\(^{22}\)Acemoglu and Guerrieri [2008] estimate the capital share of output to be roughly constant over the last 60 years. Most of the literature sets $\alpha$ to the capital share. However, this accounting exercise is based on a frictionless labor market benchmark. In a frictionless environment, $\alpha$ has to be a lower number to account for the labor wedge.

\(^{23}\)I performed a robustness check for th choice of $\{f_\phi\}$: All of the exercises were corroborated using mean preserving families of log-Normal, Beta and Gamma distributions for the distribution of capital quality. Only minor changes in the quantitative results are found upon different choices.

\(^{24}\)XXX What data.

\(^{25}\)This figure is consistent with a production cycle of a quarter and wages paid at a monthly basis.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>2.5% risk free rate and risk aversion of 2.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.97</td>
<td>2.5% risk free rate and risk aversion of 2.</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1/2</td>
<td>Frisch-elasticity of 1/2.</td>
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<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1/3</td>
<td>Standard.</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.1</td>
<td>To match investment frequencies in Cooper et al. [1999].</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.9781</td>
<td>10% annual depreciation.</td>
</tr>
<tr>
<td><strong>Enforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta^L$</td>
<td>0.375</td>
<td>Labor wedge in Shimer [2009] and Chari et al. [2007].</td>
</tr>
<tr>
<td>$\theta^I$</td>
<td>0.4</td>
<td>Investment regressions in Gilchrist and Himmelberg [1998].</td>
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<tr>
<td><strong>Aggregate Shocks</strong></td>
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<tr>
<td>$\mu_A$</td>
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<td>Standard.</td>
</tr>
<tr>
<td>$\rho_A$</td>
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<td>Standard.</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.016</td>
<td>Standard.</td>
</tr>
<tr>
<td>$f_\phi$</td>
<td></td>
<td>Exponential family with adjusted support.</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>[0.4 - 0.9]</td>
<td>To match 60% fall in liquidity after 35% increase in MPS.</td>
</tr>
</tbody>
</table>

Table 1: FIX VALUES xxx.

the twofold dispersion increase studied by Bloom [2009].

Once the I set the family $f_\phi$ to match this labor moments I can calibrate $\theta^I$ usind investment data. I follow Lorenzoni and Walentin [2009] and set $\theta^I$ to 0.4 in order to match the regression coefficients obtained by running $I/K$ against the return to capital and Tobin’s $Q$ (as in the investment regressions of Gilchrist and Himmelberg [1998]). I use the invariant distribution of the model to obtain analytical estimators for these coefficients. The resulting coefficients are close to those obtained by Lorenzoni and Walentin [2009]. This parameter implies that the average leverage rate of investment projects is 1/2, which is close to the fraction of investment internally funded in the U.S..

A summary of the calibration is reported in Table 1. I use global methods in the computation of equilibria and impulse responses. All the exercises use a grid of 6 elements for both $A$ and $\Phi$ and 120 for the aggregate capital state. Increasing the grid size does not affect results.

### 6.2 A Glance at Equilibria

This section describes the calibrated recursive competitive equilibrium throughout the state-space. THIS SECTION IS ABOUT AGGREGATES. STATIC SECTION XXX SHOULD
GIVE THE MAPPING BETWEEN QUALITIES.

*Endogenous liquidity.* Figure 2 presents several equilibrium objects reported in different panels. Within each panel, there are four curves corresponding to a combination of aggregate TFP (high and low) and a dispersion shock (high and low). The x-axis of each panel is the aggregate capital stock.

The top panels describe the equilibrium liquid funds per unit of capital, $x$, for both entrepreneur types. I begin by describing the liquid funds of $p$-entrepreneurs. For a given combination of TFP and dispersion shocks, liquidity per unit of capital decreases with the aggregate capital stock (although the total value is increasing). This negative relation follows from decreasing marginal profits in the aggregate capital stock. With lower marginal benefits from increasing liquidity, $p$-entrepreneurs sell less capital. Comparing the curves that correspond to low and high dispersion shocks, we observe that liquidity shrinks with dispersion. This happens because quality dispersion increases the shadow cost of selling capital under asymmetric information as adverse selection becomes more severe. In contrast, TFP has the opposite effect. It improves liquidity due to a positive effect on marginal profits, which explains the increased benefit from selling capital. An analog pattern is found for the $i$-entrepreneur’s liquidity. The shadow cost of selling capital increases with dispersion, is decreasing in the capital stock and increasing in TFP.

One way to understand the increase in the shadow cost of selling capital with dispersion is through the loss of value incurred by selling a threshold quality. This cost is reflected in the percent difference between the replacement cost of the threshold quality and its pooling price. These terms are the ratios $\left(\frac{q^p(X)\lambda^p(X) - p^p(X)}{q^p(X)}\right)\%$ and $\left(\frac{q^i(X)\lambda^i(X) - p^i(X)}{q^i(X)}\right)\%$ for $i$ and $p$-entrepreneurs respectively. The figure reveals that dispersion increases these shadow costs substantially. TFP, on the other hand, increases the loss entrepreneurs are willing to bear because the benefits of additional liquid funds are greater.

*Hours, output, and investment.* As dispersion reduces the liquidity of producers, their effective demand for hours leads to a reduction in output. However, when either TFP or the capital stock are large, employed hours and output increase. The figure also shows the perverse effects of dispersion shocks on investment. With less liquidity at disposal, the supply of investment claims shrinks in response to more severe enforcement problems. The reduction in the liquidity of $p$-entrepreneurs has ambiguous effects on their profits because this reduces the amount of labor hired but wages also fall. The ambiguous effect on profits implies that the demand for capital may increase even after liquidity shortages. For the calibration, the overall effect involves a strong reduction in investment and an increase in $q$.

The analysis shows how the correlation between Tobin’s $Q$ and investment is determined

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26With a larger capital stock, labor demand is higher pressing wages upwards.
by two counterbalancing forces. The first force is TFP which produces a positive correlation between $Q$ and investment. The second force is dispersion which causes an increase in Tobin’s $Q$ along with a reduction in investment. Similar countervailing forces are also obtained in other models with financial frictions as, for example, Lorenzoni and Walentin [2009].

**Robustness.** It is important to note that the monotonic relation between dispersion and liquidity shown in this exercise crucially depends on Assumption 2. More general results cannot be established between $\phi$ and the equilibrium objects because disposing of this assumption breaks the ordering of conditional expectations. Without this ordering, the direction of adverse selection effects can become state-dependent.

### 6.3 Evaluating the Model

The first experiment evaluates the ability of the model to explain the patterns of the Great Recession. To do so, we fit a sequence of dispersion shocks into the model and compare the model’s outcome with its data counterparts. The path for the dispersion shocks is constructed by tracking several measures of dispersion such as the sales dispersion measure of publicly traded firms and the VIX index (see Data appendix for details).footnote[Although these measures of dispersion are not evidence of worse asymmetric information, the working assumption is that these measures are correlated with the dispersion of assets across the economy. This is an exercise in the same spirit of other recent empirical work on uncertainty shocks (Vavra, Schaal., Bloom).] I choose the highest value of $\Phi$ so that the drop in output during the trough of the recession, but the sequence and magnitudes of shocks follows the microdata.

To evaluate the model, we feed the model with a sequence of dispersion shocks that replicate the pattern of our measures of dispersion in the data. The model will deliver different times series for variables initialized at their values corresponding to initials point drawn from the invariant distribution of the model. Those sequences are averaged to obtain the average behavior of the variables of interest to a sequence of dispersion shocks that resemble those in the data. The macroeconomic data analogues are the percent deviation from the levels of each variable during the third quarter of 2007. I report the behavior of aggregate output, consumption, investment, hours and the marginal product of labor. I substract the estimate of the growth rate of potential U.S. output from the path of consumption, investment and output to correct for trends in the data to account for the fact that the model is stationary. The model’s fit compared to the data is described in Figure 8. A conclusion from this Figure is that the model tracks, both in magnitude and in shape the behavior of consumption, investment, hours and the marginal product of labor remarkably well. It is true, however,
<table>
<thead>
<tr>
<th>Variable</th>
<th>Model on Impact</th>
<th>U.S. Data - 2008I-2009I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-5.16%</td>
<td>-4.72%</td>
</tr>
<tr>
<td>Hours</td>
<td>-6.87%</td>
<td>-9.14%</td>
</tr>
<tr>
<td>Wage</td>
<td>-3.84%</td>
<td>2.12%</td>
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<tr>
<td>MPL</td>
<td>1.92%</td>
<td>5.35%</td>
</tr>
<tr>
<td>Investment</td>
<td>-16.31%</td>
<td>-20.12%</td>
</tr>
<tr>
<td>Consumption</td>
<td>-3.45%</td>
<td>-3.34%</td>
</tr>
</tbody>
</table>

Table 2: Model Response and Great Recession Data. Source: The data is obtained from the Federal Reserve Bank of St. Louis (see appendix). Data figures correspond to the percent change between the levels in the first quarter of 2008 and the levels the first quarter of 2009.

that the model reacts faster than the data during the first periods of the recession, possibly because it misses stronger non-linearities in the response to dispersion shocks. However, at the trough of the recession, the model is quite close to the data. Table 2 reports the deviations at the trough of the recession. The table shows that the model delivers quantitative responses on par with the data. The model only fails to account for the increase in the hourly wage which can be explained by reasons that are left out of the analysis such as sectoral differences in unemployment rates.

The model does not do a good job fitting the recovery in output. Notice that the output recovers to its average growth rate whereas hours, and investment remain at depressed levels. It follows that consumption and the marginal product of labor increase in the aftermath of the Great Recession, features of the jobless recovery experienced by the U.S. economy. Note that from the perspective of a business cycle model, this implies that gdp is increasing while capital and hours are falling. This represents a puzzle that the model is not designed to address because the only additional shock, TFP could not explain the recovery in output without an increase in hours once the dispersion shocks are gone. It is important to acknowledge that although the model fits the data well, this exercise is not designed to address causality. For example, Bloom presents another mechanism where the same sequence of shocks can explain this patterns. Although the exercise cannot validate or reject the theory it is useful to evaluate the potential importance of the endogenous liquidity channel.
Figure 8: Model Fit and Data for the U.S. Economy.
Implied credit conditions. What does the model have to say about the credit conditions during the Great Recessions? To answer this questions, I draw on the observational equivalence between selling contracts and collateralized debt explained in Section 2. Recall that in the static problem of the producer in that section, we could reconstruct equilibrium with collateralized debt such that it replicated the equilibrium allocations when only selling capital is allowed. All that was required was finding an increased dispersion shock. Recall also that that problem is a static subproblem of the one faced by the p-entrepreneur in the dynamic model. Thus, the same procedure can be applied to recover an equivalent allocation with collateralized debt contracts in the dynamic model. We do this by computing an equilibrium with sales only, and then backing out the dispersion shock that delivers the same amount of liquidity. As a result of this procedure, we obtain equilibrium values for interest rates, default rates, collateralization rates and loan sizes, liquidity and the working capital over costs ($\sigma$ in the model) for each point in the state space. We can use these equilibrium objects to reconstruct the model’s expected response for these variables to the implied sequence of shocks of Figure xxx to describe the credit market implications of the model. The resulting paths are reported in Figure 9 together with some micro-data analogues.

The upper panel presents four paths. The dashed line is the implied dispersion shock used as an input in the simulations of this figure and Figure 8. The scale of this path is chosen to replicate the behavior of GDP in the U.S. data whereas its shape is drawn from the time series of sales dispersion from COMPUSTAT. The dashed–dot line our reverse-engeneeired time series for dispersion from the model with collateralized debt. This dispersion is higher, as collateralized debt improves the more endogenous amount of liquidity. Naturally, higher dispersion is required to deliver the same amount of equilibrium liquidity. Both implied measures of dispersions are higher for the model than for the data. Finally, the more erratic time series (solid line) corresponds to the VIX index options implied stock volatility. Although magnitudes differ, the timing of peaks and troughs is close for all of the measures. Appendix xxx discusses the sources of dispersion and justifies this match.

The middle and bottom panels present the outcomes for several variables that capture the credit conditions in the model (dashed) and in its data counterparts (solid lines). The middle-left panel shows the path for loan sizes, $p^s$, in the model against the normalized time-series for the average size of all the syndicated loans in the U.S.. The loan size in the model falls by about 20% during the trough of this recession. This occurs as financial intermediaries lend less per unit of collateral as they expect a worse composition of collateral in their loan portfolios. A pattern of similar magnitude is observed in the Syndicated Loans data which fell about 25%. FOOTNOTE[An alternative measure are the reported increases in loan size from the Survey to Loans Officials xxx. However, there is no clear pattern. See
The middle panel describes the behavior of interest rates. I use corporate bond yields as a measure of interest rates in the data. The solid lines correspond to the Merrill Lynch Bank of America rates of their AAA and BBB index of bonds. Once again, the patterns in the model and in the data are similar shape and timing although the magnitude of the responses of interest rates are twice as high in the model. Interest rates increase in the model because higher dispersion leads to a higher number of defaults in the collateralized debt contracts. With more dispersion, the average value of the defaulted collateral will be lower also. Naturally, these leads financial intermediaries to charge a higher interest rate and to lend less per unit of collateral. FOOTNOTE[The data on syndicated loans provides data on interest for only a subset of the loans. Alternative measures to the one reported are obtained from the Survey to Loans Officials. The pattern of interest rate increases is clear during the great recession for loans of all maturities and risk measures. A similar pattern is observed from the Commercial Paper data. See Bigio crisis slides. I used data on corporate bonds for comparison with xxx Ghilchrist and others. Bai, Kehoe, etc.]

Default rates in the model increase substantially by 13% after the dispersion shocks reach their peak. This is shown in the middle right-panel. These magnitudes are several times higher than those corresponding Commercial to the and Industrial Loan write-offs obtained from Bank Regulatory data. These rates increased from less than 1% to slightly above 4% in the crises so the model overstates the effects of defaults. There is also slight delay in the behavior of the data that could reflect delayed accounting. The difference in these default rates explains the differences in the interest rate. An interpretation of these discrepancies is that the model requires additional financial disruptions than those that can be rationalized purely from adverse selection. FOOTNOTE[The model is missing additional ingredients such as disruptions in the intermediary sector or high transactions costs in dealing with corporate defaults.]

As an outcome of the fall in the average loans size, we also observe a drop in the investor’s and producer’s liquidity (bottom left). The drop in liquidity for the latter reaches 20% in the model. Data on syndicated loans reveals that the dollar value of these loans fell by approximately 25% at the peak of the Great Recession. Both series for total loans have a similar pattern as the fall in loan sizes. This would suggest that both in the model and in the data, most of the drop in liquidity responds to reductions in loan sizes. In the model $\omega^*$ is close to one so the fall in Liquidity is not affected by the amount of collateral. I also report the change in the stock of debt from COMPUSTAT firms which initially increases during the crises but eventually declines. FOOTNOTE[There are problems in using credit data from the Flow of Funds accounts and bank regulation data as they do not distinguish
new loan creation from drawing down on formerly written lines of credit.]

The lower-middle panel shows the behavior of \( \omega^d \) in the model after the shock. The plot shows that the threshold quality that triggers a default actually falls. This happens in spite of a higher interest rate because the replacement cost of defaulted unit also increases. Although the default quality threshold falls, the mass a units under that cutoff actually increases.

Finally, the model has a prediction about, \( \sigma \), the fraction of working capital over total costs. The model predicts that as producers hold less liquidity, limited enforcement is worse. However, the model also predicts that the relative importance of the effect diminishes as there is less liquidity. That is, the fraction of trade credit over total costs (from workers to entrepreneurs) increases as entrepreneurs have less liquidity. This means \( (1 - \sigma) \) increases. The pattern is true in the model as in the data from COMPUSTAT firms as shown in the bottom-right panel.

**Risk-free rates.** There is no risk-free asset in the model. If a risk-free asset in zero-net supply is introduced, one could also let the model deliver predictions about this variable. However, in the model, the pattern of the response of the risk-free rate to dispersion shocks depends on who has access to the asset. If workers have access to the risk-free asset together with entrepreneurs, the risk-free rate increases after a drop in liquidity. The reason is that workers want to borrow against future consumption which pushes the price of a risk-less bund downwards. If workers are excluded from this market, the model is also capable of replicating the patterns of the risk-free rate observed during recessions. Numerical exercises show that the movements in the model are substantially more dramatic than in the data though. This excess sensibility stems from the substantial variation in the entrepreneur’s consumption in the model. We return to this point in the following section.

**Evaluating the model.** The summary of the results of this section is that the endogenous liquidity channel is a potential mechanism to explain both movements in real quantities and credit market conditions patterns during the Great Recession. The model fits the quantities better than credit market (price) data. However, the model does require a strong increase in dispersion to match quantities. It is important to acknowledge that the results do not address causality.

The next section does an impulse response analysis of the model to explain the mechanics of the model and discusses the role of different parameters.
Figure 9: Model Fit and Credit Data for the U.S. Economy. The data counterparts are discussed in Appendix XXX.
6.4 Impulse Response Analysis

The first experiment quantifies the effects of a once-and-for-all mean-preserving spread in the dispersion of capital quality. Figure ?? reports responses to the highest dispersion shock in $\Phi$. Recall that this shock is designed to a reduction in output equivalent to the reduction in output close to the trough of the Great Recession. The point of this exercise is to explain the key dynamics of the model.

In response to the shock, the value of liquid funds available to both entrepreneurs contracts immediately. This reaction is triggered by adverse selection effects that explain larger premia between pooling and spot prices. Larger premia imply a greater implicit cost of selling capital. As a consequence, entrepreneurs opt to scale down liquidity. The collapse in liquid funds has real effects on output. Aggregate output falls by xxx% in the impact quarter due to the reduction in hours. In response to the lack of working capital funds, hours and wages fall by xxx% and xxx% respectively.

The liquidity crisis also affects investors who face tighter limits in their capacity to self-finance investment projects which Consequently, issued claims fall to meet incentive constraints which leads aggregate investment to drop by xxx%. With fewer investment projects carried out, the post-impact capital stock shrinks. In the subsequent periods after the shock, the capital stock’s dynamics drive the response of the entire system.

The plots at the bottom present the responses of different measures of the inefficiencies caused by the financial frictions in the model. There are important responses in the investment wedges, $q$ and $q^R$ (xxx% and xxx% respectively). This is the corrected investment wedge in Buera and Moll. This contrasts with the negligible response in the investment wedge computed from an aggregate consumption-euler equation (henceforth, CKM-wedge following from Chari et al. [2007]). The labor wedge, however, increases xxx%. A higher labor wedge follows from the increase in the marginal product of labor (MPL) and the decrease in wages.

The experiment shows that contrary to TFP shocks, output contractions can be explained by liquidity crisis resulting from dispersion shocks that reduce labor productivity along with increases in MPL. Labor wedge increases are common to many of the modern recessions documented by Chari et al. [2007] and Shimer [2009]. Moreover, as shown by Ohanian [2010] or Hall [2010], the 2008-2009 crisis was an episode in which hours fell in parallel with increases in MPL. Note also that this happens without a response in the CKM-wedge. This last feature is incorrectly interpreted as evidence against the presence of financial frictions when, in fact, investment is being distorted dramatically.

Magnitudes for Liquidity Loss. This magnitude is close to the fall in the issuance of outstanding asset-backed commercial paper or syndicated loans during the financial crises
of 2008-2009 (from peak to trough) as documented by Anderson and Gascon [2009] and Ivashina and Scharfstein [2010].

Which frictions matter? What is the relevant importance of the enforcement constraints on investment and labor for the results? Figure ?? presents responses to dispersion shocks when (a) only the investment friction is active, (b) only the labor friction is active, and (c) both frictions are active. The rest of the parameters keep the target moments constant.

This exercise shows that the enforcement constraint on labor is key to generate a strong output response. The contemporaneous output response is virtually the same with and without the investment friction but vanishes once the labor friction is turned off. The reason for this discrepancy is that output fluctuations can only be explained by hours since the capital stock is fixed in the short run. In contrast, the investment friction distorts the accumulation of capital so the investment friction is responsible for the output dynamics after the shock is realized. Without the labor friction, however, the response of output is mild for the simple reason that investment is a small portion of the total capital stock. In fact, without investment, the capital stock can drop at most by the depreciation rate. This limits the output response unless there is a change in hours or TFP.

The key lesson from this exercise is that investment frictions are responsible for the propagation of the shock but, without the labor market friction, they cannot generate a strong recession. Nevertheless, introducing the investment friction is important to explain the dynamics of aggregate investment. Indeed, in the absence of investment frictions, investment can react positively to a dispersion shock because profits can increase from the reduction in wages. Finally, it is important to note that with limited enforcement in the labor market there is no compromise between magnitude and persistence of shocks as in Cordoba and Ripoll [2004].

Labor supply elasticity. The key parameter for the magnitude of the output response is the labor supply elasticity. Figure ?? presents the response to a $\phi$—shock for different values of the Frisch-elasticity. As explained before, reductions in producers’ liquidity affect the labor demand. This is met, in equilibrium, via a reduction of hours and wages. The relative response of either margin depends on the labor supply elasticity. Not surprisingly, the response of hours and, consequently, output is stronger as the Frisch-elasticity is increased. The overall magnitude in the responses of output varies from -2.5% (when the Frisch-elasticity is 2) to -0.5% (when it is equal to 1/2). One can argue in favor of a large Frisch-elasticity since the model abstracts from features such as wage rigidities or worker

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27 The reduction in issued asset-backed commercial paper is smoother in the data than in the model. The model generates a smoother and more persistent responses in liquidity by fitting a particular sequence of dispersion shocks at the expense of making the analysis less transparent.
savings that would magnify the responses.

TFP shocks. Figure ?? presents the response to a positive TFP shock. The figure shows that the producers’ liquidity increases immediately in the period after the shock is realized. With additional liquid funds, p-entrepreneurs relax their constraints on employment. This effect magnifies the response of hours. In addition, TFP strengthens the demand for capital, causing an increase in q. This provides i-entrepreneurs with incentives to obtain more liquid funds. The increase relaxes the investor’s enforcement problem, which magnifies the response of investment. Figure ?? contrasts the response to technology shocks in the frictionless version of the model with the response activating the two frictions. The response of aggregate quantities is very similar in both models. Although there is an amplification mechanism within the model with active frictions, this does not mean that TFP shocks are magnified in relation to the RBC counterpart as the occupation times in states are also affected. This result is consistent with prior findings by Kocherlakota [2000] and Cordoba and Ripoll [2004] that suggest that financial frictions are weak amplification mechanisms of aggregate TFP shocks.

7 Conclusions

Summary. This paper describes how asymmetric information about capital quality endogenously determines the amount of liquid funds in capital markets. The economy features non-enforceable contracts, so liquidity is key to relax financial frictions. The paper shows that the dispersion of capital quality increases the cost of using it as collateral and has real effects by exacerbating financial frictions.

The main lessons are: [1] In equilibrium, liquidity is always insufficient to relax enforcement constraints completely. [2] A dispersion shock to capital quality can cause a collapse in liquidity. For example, a shock that generates a 60% reduction in liquidity may provoke a 2.5% output contraction. [3] The key friction to explain this large impact is limited enforcement in labor markets.

Research directions. A first direction for future research is to introduce other assets that are not subject to asymmetric information into the model. This may explain the relevant tensions that affect the use of assets as collateral when these differ in their information properties. The present analysis suggests that an increase in the supply of cash or bonds that targets liquidity could “crowd-out” the liquidity of other assets that feature asymmetric information. This could be caused if the incentives to use capital as collateral are reduced. This line of research is followed by Rocheteau [2009] in the context of money-search theory.

A richer model could also let financial firms play a more active role. The 2008-2009 crisis
began with events that caught the financial system by surprise and impacted on its balance-sheets. This extension is important because balance-sheets are key in explaining the provision of liquidity by financial institutions. Moreover, adverse selection can be exacerbated by weaker balance sheets which, in turn, can slow the recovery of bank profits. This aspect could explain the persistence of shocks and the slow recovery of economies after a liquidity crises. I pursue this extension in Bigio [2010].

This paper explains a collapse in liquidity through perturbations that exacerbate adverse selection effects. Strategic delays and learning can lead to dynamic effects left out from the model (see Kurlat [2009] or Daley and Green [2010]). In Caballero and Krishnamurthy [2008], liquidity crises are explained by Knightian uncertainty. Yet, other “non-fundamental” shocks could explain financial collapses in liquidity. Lorenzoni [2009] or Angeletos and La’O [2010] provide frameworks in which shocks that affect information structures have real effects by leading agents to take decisions away from full information responses. This phenomena could also operate through a liquidity channel.

Finally, the results of this paper hinge on tensions between two market imperfections: (a) asymmetric information and (b) financial frictions. In the aftermath of the financial crisis, there has been a fair amount of anecdotal evidence in both dimensions. Nevertheless, our profession lacks conclusive evidence that corroborates that these frictions are not a myth in a way that disentangles them from other phenomena interpreted as demand shocks.
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8 Relation with the literature

This paper relates to a number of prior studies on financial frictions. The seminal contributions of Bernanke and Gertler [1989], Carlstrom and Fuerst [1997] or Bernanke et al. [1999] explain how shocks that affect the relative wealth of borrowers have effects on lending because they exacerbate agency costs. The endogenous liquidity channel has a different property because dispersion shocks do not affect the relative wealth of constrained agents. Instead they affect the fraction of wealth that is chosen as collateral.\textsuperscript{28}

This paper also relates to Kiyotaki and Moore [1997] because there is a similar feedback effect from prices to constraints.\textsuperscript{29} That paper shows how shocks that impact the price of collateral tighten enforcement constraints and, moreover, induce a multiplier effect. This multiplier follows from the reduction in future returns to capital which, in turn, further depress current prices magnifying the initial effect. A similar feedback occurs here: dispersion shocks tighten enforcement constraints having effects on future returns. In anticipation, the price of capital adjusts affecting the incentives to use capital as collateral.\textsuperscript{30}

Another paper that introduces asymmetric information into a fully-fledged business cycle model is Christiano et al. [2012]. In this paper, reductions in liquidity occur as the riskiness of investment projects increases. The financial friction in that environment are costly state verification and it works in a similar way as the reduction in investment in this paper. A recent number of papers in the money-search literature also use private information in the quality of capital to explain the use of money and equity as mediums of exchange. Rocheteau [2009] shows how money dominates equity when the quality of the latter is private information. As in the present model, the dispersion of asset quality reduces liquidity. Lester et al. [2009] provide a similar environment in which agents are also allowed to acquire information about assets. A lesson from these studies is that liquidity is not invariant to policy.

As argued in the introduction, there are several reasons to formalize liquidity shocks through asymmetric information which relate to other studies. First, the theory of endogenous liquidity provides a set of testable implications on the stochastic process behind liquidity. For example, Proposition 9 in the main text shows that liquidity is always inefficient. The implication of this result is that the liquidity shocks studied by KM should be

\textsuperscript{28}In these papers, agency costs follow from a problem of costly state verification rather than from limited enforcement. Nonetheless, this distinction is immaterial.

\textsuperscript{29}In both papers, capital is a form of collateral. In Kiyotaki and Moore [1997] entrepreneurs use all their capital stock as collateral. Here, entrepreneurs sell a fraction of capital to obtain liquid funds that are used as collateral.

\textsuperscript{30}This feedback effect is only present for investing entrepreneurs and disappears completely under log-preferences.
restricted to regions where they are always or never binding. Second, and most important, policy experiments that treat liquidity as exogenous may be subject to the Lucas critique, as argued by Rocheteau [2009] in the context of money-search models. Several recent studies, for example Gertler and Karadi [2009], Curdia and Woodford [2009], Gertler and Kiyotaki [2010] or del Negro et al. [2010], analyze the effects of alternative government policies directed at replenishing liquidity. In those models, the disturbances that interrupt financial markets originate exogenously. The experiments studied in these papers are extremely valuable to assess the implications of the many policies undertaken during the financial crises of 2008-2009.

It would be interesting to test the robustness of results in the context of endogenous liquidity. There are several reasons why assessments can depend on how the private sector reacts to policy. For example, policies that transfer liquid funds to constrained agents will, in parallel, reduce the benefit of selling capital under asymmetric information. Thus, a policy that targets liquidity may, in principle, be ineffective to reactivate the economy because it can be partially offset by inducing a reduction in the liquidity of assets that are sold under private information.

A key aspect of the quantitative power of the model is the effect of reductions in liquidity in the determination of labor demand. In fact, a key policy concern during the crisis was that the lack of liquid funds would cause firms to demand less employment. In reality, payroll and inventories are partly financed by short-term instruments. The collapse of asset-backed securities like was at the epicenter of the financial crisis. Anderson and Gascon [2009] argue that, as in the model, commercial paper is a major funding source in the U.S. and its main uses include funding payroll. Economists such as Brunnermeier [2009] and Gorton and Metrick [2009] in part attribute this collapse to problems of asymmetric information similar to the ones described here. Chodorow-Reich [2013] provides empirical evidence on the effects of low liquidity for the funding of employment during the crisis. The paper also argues that the results are consistent with frictions deriving from asymmetric information.

A natural question is where does dispersion come from? On theoretical grounds, we know from Long and Plosser [1987] that small sectoral productivity shocks can propagate to the entire economy. Among others, the recent works of Acemoglu et al. [2010] and Carvalho [2010] explain that small idiosyncratic shocks can lead to a non-stationary cross-sectional distribution of output depending on the structure of production networks. Although these

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31 This fact is very well documented in the corporate finance literature. For references, see Berk and DeMarzo [2007] or Anderson and Gascon [2009].

32 Caballero and Simsek [2010] construct an environment in which firms are involved in a network of financial transactions. Shocks in one sector of the system may have different propagation properties when firms are unaware of the exact network structure. Reality is further more complex because firms face different
papers interpret production units as firms, there is no reason why one shouldn’t interpret a firm as a collection of such units. Considering the extreme complexity of inter-firm production and financial links, dispersion shocks are a natural and parsimonious way to model the entangled effects.

Moreover, dispersion shocks may be interpreted as also stemming from beliefs. The model cast in such a way that these shocks have no effect on the production possibility frontier or the cross-sectional distribution of wealth. In principle, one can think of these perturbations as shocks to beliefs that affect the amount of liquidity without a physical counterpart. This route has not been explored by the literature yet.

9 Appendix

A thought experiment can clarify how market clearing and enforcement constraints are satisfied together. First, observe that if \( q(X) < 1 \), there would not be any supply of investment claims since \( i \)-entrepreneurs would find it cheaper to purchase capital than to invest. Thus, \( q(X) < 1 \) cannot occur in equilibrium. Assume then that \( q(X) = 1 \). Given prices and policy functions, \( I(X) - I^s(X) \) can be solved for from (30) whereas \( I^s(X) \) equals \( D(X) - S(X) \). Given that \( I^s(X) \) and \( I(X) - I^s(X) \) are known quantities, one can check whether they satisfy \( \theta I(X) \leq I^s(X) \). When this condition is guaranteed, then \( q(X) = 1 \) is an equilibrium. When it is violated, \( q(X) \) must be 1 to satisfy incentive compatibility.

Proposition 5 ensures that when \( q(X) > 1 \), enforcement constraints bind so \( I^s(X) = \theta I(X) \). Substituting this equality into (30) yields a supply schedule for claims that is increasing in \( q(X) \). In addition, the supply of capital \( S(X) \) is increasing and demand \( D(X) \) decreasing in \( q(X) \). Thus, \( q(X) \) is found by solving for the market clearing condition when enforcement constraints are binding. Proposition ?? in the Appendix provides an expression for \( q(X) \) based on this analysis.

Proof at appendix. We can solve his problem in an analogous way to the \( p \)-entrepreneur’s employing the principle of optimality. Thus, we begin studying the optimal amount of investment claims with \( i^d \) and \( x \) as given. This decision is equivalent to maximizing future capital holdings given a downpayment:

Problem 9 (Optimal Investment Financing) The optimal financing problem is \( \max_{i^s \geq 0} i - i^s \) subject to \( i^d = i - qi^s \) and \( i^s \leq \theta I \) taking \( i^d > 0 \) as given.

degrees of complementarities (as in Jones [2010]) and the size distribution is such that large firms have a non-vanishing impact on the system (as in Gabaix [2010]).
Substituting the definition of $i^d$ into the objective leads to $i^d + (q - 1)i^s$. This quantity is the sum of the entrepreneur’s down payment and his arbitrage profits: the entrepreneur issues claims to 1 investment unit at $q$, but generating that unit costs him only one unit of consumption. Therefore, in states where $q > 1$, the entrepreneur will want to issue as many claims as possible. The entrepreneur’s enforcement constraint can be written in terms of $i^d$: $(1 - \theta I q) i^s \leq \theta I i^d$. This constraint binds whenever $q > 1$, so the choice of claims is $i^s = \theta I i^d/(1 - \theta I q)$. Substituting this to the objective function renders an expression for the increment in the entrepreneur’s capital stock per unit of down payment $(1 - \theta)/ (1 - \theta q)$. The inverse of this term defines the cost of generating a unit of capital in terms of consumption goods by exploiting the optimal external financing structure:

$$q^R \equiv \frac{(1 - \theta q)}{(1 - \theta)} \leq 1 \text{ for } q \in [1, 1/\theta)$$

Therefore, $q^R$ is the internal cost of generating a unit of capital. Thus, $q/q^R$ represents the marginal Tobin’s Q. In states where $q = 1$, $q^R$ is also 1 and $i^s$ is indeterminate. A simple comparison of internal costs and the price of capital establishes also: