Bank Liability Structure

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Abstract

Since the fiduciary duty of bank management is to maximize bank value and not social welfare, we analytically solve for the liability structure that maximizes the value of a bank leveraged by deposits and subordinated debt. Our analysis offers a perspective on privately-rational bank capital structure and its interaction with regulatory environment. Absent deposit insurance and regulation, banks use high leverage. The drivers of bank leverage are the low volatility of bank assets and the income from serving deposits, besides corporate taxes. The optimal level of subordinated debt makes its endogenous default coincide with bank run. In response to deposit insurance and regulatory closure, banks increase their value and leverage by expanding deposits, even if they are charged fair insurance premium. They however reduce subordinated debt to keep endogenous default and regulatory closure concurrent. These optimal responses from banks counteract the objective of regulators in lessening expected bankruptcy costs.

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1 Introduction

Bank leverage has drawn much attention from regulators and the public after the crises experienced by banking industry. Regulators around the world have gradually rolled out regulations on bank capital structure, and the shape of bank regulation is still evolving.\(^1\) Banks have been readjusting their capital structure, and academics have been grappling with the questions of the level and composition of capital that banks should hold.\(^2\) Proposals of further regulation are abundant in the literature. Some have argued for restricting bank leverage to a level similar to non-financial firms.\(^3\) Some others have antithetical views on whether banks should hold subordinated debt.\(^4\) Others have proposed cutting corporate tax because it reduces incentives for leverage.\(^5\)

The debate on bank capital regulation calls for a better understanding of bank leverage and the consequence of regulation. Each regulatory mandate intends to fix a particular broken factor observed in bank liability structure.\(^6\) Arguments for a regulatory policy often implicitly assume that other factors are unaffected, ignoring the overall response of banks that optimally adjust many parts of its liability structure. For instance, deposit insurance intends to address bank runs caused by the fear of losing deposits en masse. With deposit insurance, a bank may find financing with more deposits increase its value despite being more exposed to the risk of failure. More broadly, it is unclear whether banks’ optimal responses will undo or significantly diminish the intended effects of a regulatory mandate. It is even possible that a regulation may result in unintended consequences.

To work out banks’ optimal responses to a regulation on leverage, one needs to understand how banks choose leverage and liability structure when they act to

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1. After the frequent bank runs during the Great Depression, the Banking Act of 1933 created the Federal Deposit Insurance Corporation (FDIC). After the financial crisis during the Great Recession of 2007–2009, the Dodd-Frank Act of 2010 brought sweeping regulatory reforms ranging from FDIC deposit insurance to stress tests of banks’ capital adequacy. Worldwide regulators agreed on Basel III in 2011 to strengthen restrictions on bank leverage.
2. See Thakor (2014) for a review of the debate on bank capital.
4. Bulow and Klemperer (2013) argue that banks should hold no subordinated debt, besides equity and securities convertible to equity. The Fed governor, Daniel Tarullo (2013), goes in the opposite direction by arguing for requirement of holding long-term subordinated debt, which he thinks will improve capital structure and resolution of banks.
5. For example, Schandlhuber (2013) finds that a tax rate increase causes average bank to increase subordinated debt by about 5.9% prior to the enactment of the tax change. Citing academic studies, Fleischer (2013) propose cutting corporate tax rate for banks to make them safer.
6. For this reason, Santos (2000) motivates regulation as a policy arising out of market failure.
maximize their value. We develop a general model of optimal liability structure for banks. Value maximization is a fiduciary responsibility of bank management: acting in the interest of its claim-holders.\(^7\) Banks do not maximize social welfare, such as reducing systemic risk or increasing banking services. Analysis of social welfare issues of bank leverage is unquestionably important, but understanding the optimal choice of liability structure by value-maximizing banks is a necessary for a proper social welfare analysis of bank regulation.

Banks distinguish themselves from other firms by taking deposits. Deposits are different from other forms of debt partly because banks earn income from the provision of account services to depositors.\(^8\) Other important features of deposits are that depositors can run, deposits may be FDIC insured, and deposit-taking banks are subject to special regulations. Bank run by rational depositors and bank closure by rule-following regulators are reflected in equity holders’ endogenous choice of default on its subordinated debt in order to maximize equity value. The risk exposure of deposits is also reflected in the insurance premium charged by the FDIC. Our model incorporates these institutional features explicitly.

Extending the framework pioneered by Merton (1974, 1977) and Leland (1994), we analytically solve for the optimal bank liability structure for banks either with or without deposit insurance and regulation. The solutions offer a perspective on the liability structure of banks when they act to maximize value. Absent regulation, value-maximizing banks take high leverage, which combines deposits and subordinated debt. The special driving forces of bank leverage are the low asset volatility and the income from serving deposit accounts, apart from taxes. The subordinated debt plays a salient role in optimal bank liability structure: the default of subordinated debt coincides exactly with the point when depositors choose to run. With optimal choice of subordinated debt, the distance to default is the same as the distance to bank run.

The above property of optimal subordinated debt has economic motives. Because of income from liquidity service, deposits are cheaper than subordinated debt as financing sources. A bank should generally prefer deposits when balancing tax and liquidity benefits with bankruptcy cost. Given deposits, however, subordinated debt

\(^7\)The focus on bank value maximization sets aside the principal-agent problem such as management’s conflict of interests with other stakeholders. This problem may play a role in bank choices of liability structure. For example, Admati, DeMarzo, Hellwig, and Pfleiderer (2013) discuss how conflict of interests leads bank management to use excessive leverage even if it destroys bank value.

\(^8\)In the literature, account service to depositors is also referred to as production of liquidity. The income from this service is sometimes referred to as the liquidity premium of deposits.
does not affect bankruptcy risk as long as bank run happens before default. The bank should then take as much subordinated debt as possible for availing of the tax benefits but avoid making default happen before bank run. As a result, the optimal subordinated debt sets default and bank run concurrent.

The introduction of FDIC raises both the leverage and market value of banks, even if banks are charged a fair insurance premium. With deposit insurance, a bank issues more deposits but uses less subordinated debt. The optimal mix ensures that the endogenous default of subordinated debt coincide with the bank closure and avoid protecting FDIC from losses in its insurance obligation. Although the bank holds less subordinated debt, it takes more deposits, leading to a higher total leverage. This optimal response from banks prevents FDIC insurance program from reducing expected costs of bankruptcy.

A new perspective in our model is the endogenous link of leverage to FDIC’s insurance premium and charter authority policies on bank closure. The insurance premium depends on the leverage and liability structure, and the banks’ decisions on leverage and liability structure depend on the premium. We explicitly model this feedback channel, which is crucial in assessing regulatory policies pertaining to bank capital structure.

Our analysis reveals an issue with subordinated debt. Since subordinated debt is a claim ranked lower than deposits, it is naturally viewed as a capital that protects deposits. Reflecting this view, regulators treat subordinated debt as Tier 2 regulatory capital. Since a bank can adjust its liability structure so that optimal default coincides with bank run or bank closure, it may remove the benefits of deposit protection. A large body of academic literature debates whether subordinated debt provides a market discipline on bankruptcy risk. The role of subordinated debt in optimal liability structure is especially relevant to this debate and should not be ignored.

Since banks use much higher leverage than non-financial firms does, corporate tax is particularly important for bank liability structure. Incorporating the framework of Goldstein, Ju and Leland (2001), our model offers a coherent framework to link bank optimal leverage to corporate tax rate. Apart from showing that bank leverage is lower in an economy with lower corporate tax rate, our model provides detailed analysis on how banks respond to tax changes in designing their liability structure. In response to a reduction in tax rate, banks shrink subordinated debt but expand deposits. Deposit expansion dampens the effects of a tax cut on leverage. Our model

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9Flannery and Serescu (1996) contend that subordinated debt pricing rationally reflects the risk in a bank. Gorton and Santomero (1990), however, opine the opposite.
lays a stepping stone for the welfare analysis of the benefit and cost of tax policy reforms.

The road-map for this paper is as follows. Section 2 develops the model of bank liability structure in alternative regulatory environments. Section 3 characterizes bank optimal liability structures in each regulatory environment, along with the risk-based FDIC premium in the presence of endogenous liability and leverage choice. Section 4 examines quantitatively the leverage of banks and how the introduction of FDIC insurance, and other factors such as taxes, affects bank leverage. Section 5 relates our work to the literature and discusses further applications and extensions.

2 Liability Structure

Banks share some common characteristics with non-financial firms: both have access to cash flows generated by their assets and both finance their assets by issuing debt and equity. Banks, however, differ from non-financial firms in that they take deposits and provide liquidity services to their depositors through check writing, ATMs, and other transaction services such as wire transfers, bill payments, etc. The banking business of taking deposits and serving accounts is heavily regulated in most countries. In the U.S., a large part of deposit accounts is insured by the FDIC, which charges insurance premium and imposes additional regulations on banks. Deposits and the associated account services, FDIC insurance, and bank regulation distinguish banking business from other non-financial corporate business and set the capital decision of banks apart from that of other firms.

Firms operate in a market with two frictions: corporate taxes and bankruptcy costs. These frictions are crucial for firms in their choice of capital and liability structure, as recognized in the literature originating from Modigliani and Miller (1963) and Baxter (1967) and analyzed with structural models by Leland (1994). Banks face these frictions too, but they have to simultaneously incorporate other considerations, such as the potential of a run by depositors, FDIC deposit insurance premium, and charter authority’s closure of banks, in determining their optimal capital and liability structure. Figure 1 illustrates the liability structure of a typical bank. In Section 2.1, we discuss each part of the structure in detail.
2.1 Assets and Liabilities

A typical bank owns a portfolio of risky assets that generate cash flows. The portfolio of assets is valued at \( V \), which is the major part on the asset side of Figure 1. The asset is risky, and its value follows a stochastic process.\(^{10}\) The instantaneous cash flow of the asset is \( \delta V \), where \( \delta \) is the rate of cash flow. In a non-financial firm, \( \delta V \) is the total earnings, but in a bank, \( \delta V \) represents only the earnings from bank assets such as loans, not including the income from serving deposit accounts. The risk of the asset portfolio is represented by the volatility of asset value and denoted by \( \sigma \). Notice that \( \sigma \) is also the volatility of asset cash flow. We assume that the portfolio of assets is given exogenously.\(^{11}\) Following Merton (1974) and Leland (1994), we assume investors have full information about asset value.\(^{12}\)

Banks take short-term deposits from households or businesses and provide account services for deposits. Deposits, the first part on the liability side in Figure 1, are the

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\(^{10}\)Following Merton (1974) and Leland (1994), we assume the stochastic process is a geometric Brownian motion, which is described by equation (18) in Appendix.

\(^{11}\)This assumption rules out interesting issues of endogenous asset substitution. The literature has pointed out that debt may create incentives to substitute assets with higher risk (e.g., Green, 1984, and Harris and Raviv, 1991) and FDIC insurance may also make for such incentive (e.g., Pennacchi, 2006, and Schneidar and Tornell, 2004).

\(^{12}\)In reality, active investors use all available information to assess bank asset value and cash-flows although only accounting values of assets are directly observable in quarterly frequency. The full-information assumption sets aside the disparity between accounting value and intrinsic value. We may therefore interpret \( V \) as the fair accounting value. If the assets are of same risk category, we may interpret \( V \) as the value of risk-weighted assets.
most important source of funds for banks to finance their assets. Let $D$ denote the amount of deposits that a bank takes. Deposits are rendered safe through two channels: (1) depositors withdraw their money early enough to ensure that the bank has enough assets to redeem their deposits in full; (2) the bank purchases insurance that guarantees the depositors in full. The first channel may precipitate financial distress costs and potential financial insolvency associated with a bank run. The second channel requires the bank to pay insurance premium, denoted by $I$. We will discuss bank run and deposit insurance in the next subsection.

If deposits are risk-free, the fair interest rate on deposits is the risk-free rate, denoted by $r$. Banks typically pay a lower interest rate on deposits. Depositors accept a lower interest rate, say $r - \eta$ for $\eta > 0$, because they receive the service of maintaining accounts and transacting certain normal payments. Banks also charge fees for services such as money transfers, overdrafts, etc. Let $\eta_2$ be the banks’ fee incomes on each dollar of deposits. A bank’s net cost on deposits is $C = (r - \eta_1)D - \eta_2D$, excluding deposit insurance premium. Let $\eta = \eta_1 + \eta_2$, which is the net income on each dollar of deposits. The net deposits liability is $C = (r - \eta)D$, excluding deposit insurance premium. The parameter $\eta$ plays a crucial role in our model of banks. It represents a sacrifice to the required rate of return that the households are willing to accept for the services provided by the bank. This sacrifice distinguishes deposits from other form of debt. If deposits are risk-free because of deposit insurance, the bank’s total cost on deposits is $I + C$. If deposits are not insured but deposits are risk-free because of depositors’ ability to run, we have $I = 0$.

Another important form of debt issued by banks is subordinated debt, the second part on the liability side in Figure 1. Subordinated debt pays coupon until bankruptcy, at which it has a lower priority than deposits in claiming the liquidation value of bank assets. The lower priority potentially protects deposits at bankruptcy. For that reason, long-term subordinated debt is treated as Tier 2 capital in bank capital regulation. Subordinated debt comes with a cost: its yield contains a credit spread, denoted by $s$, over the risk-free rate to compensate debt holders for bearing the risk of bankruptcy. The credit spread arises endogenously in our model; it depends on the risk of assets and the leverage of the bank. Thus, a bank’s choice of liability structure affects the credit spread which we will solve in our model endogenously along with the value of subordinated debt, denoted by $D_1$. The coupon on subordinated debt is $C_1 = (r + s)D_1$, where $D_1$ is the value of the debt at issuance (face value).
A typical bank is owned by its common equity holders, who garner all the residual value and earnings of the bank after paying the contractual obligations on deposits and subordinated debt. The first slice of value that equity owners lay claim to is the difference between assets and debt: \( V - (D + D_1) \), also on the liability side in Figure 1. This slice, referred to as the tangible equity or book-value of equity, is the value equity holders would receive if bank assets are liquidated at fair value and all debt is paid off at par. A larger book-value of equity means a smaller loss for depositors and subordinated debt holders at after liquidation. Hence, regulators regard it as bank capital of the highest quality, the core Tier 1 capital.

Equity holders are also rewarded by all future earnings of the bank. The present value of future earnings is the bank’s charter value, the bottom part on asset side in Figure 1. Part of the earnings comes from the service income on deposit accounts: \( \eta D \), but these are the earnings before paying insurance premium \( I \) if deposits are insured. Another part of the earnings is the savings from corporate tax. Since the costs of debt financing are deductible from earnings for tax purposes, the flow of tax saving is \( \tau(I + C + C_1) \). The dividend paid to equity holders is the difference between the asset cash flow and the after-tax liability associated with deposits and subordinated debt: \( \delta V - (1 - \tau)(I + C + C_1) \). Since equity value depends on its dividend, it is affected by the liability structure. In a bank with deposit insurance, the liability structure is characterized by the triplet \((I, C, C_1)\). In an uninsured bank that faces bank run, the pair \((C, C_1)\) typifies the liability structure.

### 2.2 Bank Run, FDIC, and Charter Authority

A consequence of borrowing using deposits is the risk that depositors may run, a major challenge commonly faced by banks but not by non-financial firms. As experienced in the crises of U.S. banking history and theorized by Diamond and Dybvig (1983), depositors may run from a bank if they believe it has difficulty in repaying their deposits promptly and timely upon their demand.\(^{13}\) When depositors run, the bank will be closed, unless it is recapitalized to stop the run, and its assets will be liquidated. Suppose the market liquidation costs through bankruptcy courts, which includes dead-weight losses due to liquidation process and legal expenses, is a fraction \( \alpha \) of the asset value \( V_a \) at bankruptcy. The liquidation value is \((1 - \alpha)V_a\). Deposits come with the risk of bank run. With full information, it is rational for depositors

\(^{13}\) In September 2007, Northern Rock, a U.K. Bank, experienced a run on its deposits, and had to be nationalized in 2008. See Shin (2008) for a cogent analysis of the Northern Rock bank run.
to run before the bank value drops below $D/(1 - \alpha)$. It is reasonable to assume that depositors may actually wish to run earlier than $D/(1 - \alpha)$, worrying about a delay of payments when the bank files for bankruptcy. In our model, depositors run, and the bank is closed, when asset value drops to a level $V_a$ with $V_a \geq D/(1 - \alpha)$. Letting $\kappa \geq 1/(1 - \alpha)$, the threshold for bank to close due to a bank run is $V_a = \kappa D$.

The establishment of the FDIC is to deter bank runs by insuring that deposits (up to a limit) be paid when a bank closes. With FDIC insurance, a bank is closed by its charter authority, which is typically either the bank’s state banking commission or the Office of the Comptroller of the Currency (OCC). The charter authority closes a bank if the bank is insolvent or even if the bank’s capital is deemed to be too low to be sustainable. For example, a bank is categorized by regulators as critically under-capitalized when the total capital that protects deposits drops to a threshold (say, 2% of asset value). The total capital is the sum of Tier 1 and Tier 2 capitals. In our model, it is the sum of tangible equity and subordinated debt and amounts to $[V - (D + D_1)] + D_1 = V - D$. Let $V_a$ be the threshold when the charter authority closes the bank. Then, $V_a - D = 2\%V_a$ implies $V_a = D/0.98$. In general, charter authority closes a bank when its asset value reaches $V_a = \kappa D$, where $\kappa \geq 1$.

The FDIC functions both as a receiver of the closed bank and an insurer of the deposits. As a receiver, the FDIC liquidates the assets of the closed bank in its best effort to pay back the creditors. Suppose the liquidation cost is $\beta V_a$, proportional to the asset value $V_a$ when the bank is closed. It is possible that costs associated with the liquidation by the FDIC is different from the costs of liquidation through bankruptcy court, and thus we allow $\beta \neq \alpha$. Since the FDIC does not go through the lengthy procedure of bankruptcy, it is likely that $\beta \leq \alpha$.

As an insurer, the FDIC pays $D$ to depositors when the bank is closed. The insurance corporation loses $D - (1 - \beta)V_a$ if $(1 - \beta)V_a < D$ and nothing otherwise. The loss function is $[D - (1 - \beta)V_a]^+$, where $[x]^+ = x$ if $x \geq 0$ and $[x]^+ = 0$ if $x < 0$. Since $V_a = \kappa D$, the loss function is positive if $\kappa < 1/(1 - \beta)$, in which case the FDIC expects to suffer a loss after bank closure. To cover the loss, the FDIC charges insurance premium on banks. In 2006, Congress passed reforms that permits the FDIC to charge risk-based premium. For deposit insurance assessment purposes, an insured depository institution is placed into one of four risk categories each quarter.

\[14\] To deter bank runs during the credit crisis of 2007–2009, the FDIC deposit insurance limit was raised from $100,000 to $250,000 on October 3, 2008.

\[15\] In practice, the FDIC always expects a chance of loss because liquidation cost is uncertain. To keep analysis tractable, we assume a fixed $\beta$ and $\kappa < 1/(1 - \beta)$ so that the FDIC expects a loss.
determined primarily by the institution’s capital levels and supervisory evaluation. Hence a riskier bank pays higher insurance premium than a safer bank does. Recall that $I$ denotes the deposit insurance premium a bank pays.\footnote{Until 2010, the FDIC assesses the insurance premium based on total deposits. The assessment rate of the insurance is $a$ such that $I = aD$. There have long been concerns that banks shift deposits out of account temporally at quarter-ends to lower the assessment base. Since April 2011, the FDIC has changed the assessment base to the difference between the risk-weighted assets and tangible equity, as required by the Dodd-Frank Act (Section 331). If $V$ equals the value of risk-weighted assets, the new assessment base equals $D + D_1$, which implies the new assessment rate is $b$ such that $I = b(D + D_1)$. The actual premium calculations may also depend on credit rating and the proportion of long-term debt to deposits. See Federal Deposit Insurance Corporation (2011) for more details.} 

The economic role of FDIC and charter authority in our model can be explained as follows. Depositors run at the right time to make their deposits risk-free if deposits are not insured. Deposit insurance prevents a bank run and lets the charter authority to close a bank later than the time depositors would have chosen to run were there no deposit insurance. The prevention of a bank run increases the expected life of the bank. The bank pays insurance premium to FDIC in “good states” when it is solvent. Keeping a fixed liability structure, the transfer of payments across the states improve the overall value of the bank by receiving more account service income, increasing the tax shields, and reducing the expected cost of default. Analysis in Section 4.3 will show that the combined actions of the charter authority and the FDIC creates additional value for the bank, but the dead-weight losses associated with bankruptcy actually increases after the bank adjusts its liability structure optimally.

Equity holders can choose to default before a bank run or a closure by charter authority. Absent bank run and closure, there is an optimal point for equity holders to default. The default decision maximizes equity value, given a liability structure. The optimal default of debt is referred to as endogenous default and derived by Leland (1994) for firms with long-term debt but without deposits. Section 3.1 provides the formula of endogenous default in the presence of deposits. Let $V_d$ be the point of endogenous default, i.e., equity holders choose to default if and only if asset value $V$ reaches or goes below $V_d$, in the absence of a bank run or bank closure. Then bankruptcy happens if either the debt is defaulted by equity holders endogenously or the bank is closed due to bank run or by charter authority. In other words, the point of bankruptcy is $V_b = \max\{V_d, V_a\}$. 

In summary, banks face three types of bankruptcy. The first type is endogenous default chosen by equity holders. In this type bankruptcy, liquidation of assets has to go through private-sector bankruptcy procedure, and the cost associated with the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Allowed range</th>
<th>Baseline value</th>
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</thead>
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<td>Asset value</td>
<td>$V$</td>
<td>$(0, \infty)$</td>
<td>1.00 $billion</td>
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<td>$\eta$</td>
<td>$(0, r)$</td>
<td>0.50%</td>
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<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>$(0, \infty)$</td>
<td>3.00%</td>
</tr>
<tr>
<td>Corporate tax rate</td>
<td>$\tau$</td>
<td>$(0, 1)$</td>
<td>30.00%</td>
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<td>Court bankruptcy cost</td>
<td>$\alpha$</td>
<td>$(0, 1)$</td>
<td>25.00%</td>
</tr>
<tr>
<td>Closure by bank run</td>
<td>$\kappa$</td>
<td>$[1 / (1 - \alpha), \infty)$</td>
<td>$1 / (1 - \alpha)$</td>
</tr>
<tr>
<td>FDIC liquidation cost</td>
<td>$\beta$</td>
<td>$(0, 1)$</td>
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</tr>
<tr>
<td>Closure by charter authority</td>
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<td>$[1, 1/(1 - \beta))$</td>
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<td>Insurance subsidy</td>
<td>$\omega$</td>
<td>$[0, 1]$</td>
<td>100.00%</td>
</tr>
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Table 1: Exogenous parameters, which are pre-specified, not determined by either valuation or optimization in the model. The allowed ranges are assumptions of the model. The baseline values are used as benchmark in Section 4.

The second type is a bank run, which happens before endogenous default. Bankruptcy cost at bank run is $\alpha V_a$, as it also goes through bankruptcy procedure. The last type is bank closure by charter authority. The cost of closing a bank is $\beta V_a$ as the FDIC liquidates the assets. In order to keep our formulation simple, we denote the recovery value of assets after bankruptcy by $(1 - \phi)V_a$, where $\phi$ equals $\alpha$ or $\beta$, depending on the type of bankruptcy. When bank assets are liquidated after bankruptcy, depositors are paid first, and the subordinated debt holders are paid the next if there is value left. Consequently, the payoff to debt holders is $[(1 - \phi)V_b - D]^+$.  

## 3 Valuation and Optimization

Table 1 summarizes the exogenous parameters in the model and the assumptions on them. In the table, account service income is positive but doesn’t cover the entire cost of taking deposits: $0 < \eta < r$. Corporate tax is present: $0 < \tau < 1$. The bankruptcy and FDIC liquidation are both costly: $0 < \alpha < 1$ and $0 < \beta < 1$. These assumptions are not only realistic but also the requisite mathematical conditions to carry out valuation and optimization.

### 3.1 Bank Valuation and Endogenous Insurance Premium

Since deposits are either withdrawn with full value or insured by FDIC, the value of deposits is its par value $D$. Because the net liability on deposits is $C = (r - \eta)D$, the value of deposits is related to its liability by $D = C / (r - \eta)$.

The values of subordinated debt and equity are affected by the risk of bankruptcy.
The Arrow-Debreu state price of bankruptcy plays a key role in bank valuation. Consider a security that pays $1 when bankruptcy occurs, and pays nothing otherwise. The price of this security is the state price of bankruptcy, also the risk-neutral probability of bankruptcy. The state price is $P_b = [V_b/V]^{\lambda}$, where $\lambda$ is the positive root of a quadratic equation, which is given by (20) in Appendix. The quadratic equation implies $\lambda$ is an increasing with $r$ and decreasing with $\delta$ and $\sigma$. If the cash flow of assets is zero, $\delta = 0$, we have $\lambda = 2r/\sigma^2$, which is proportional to $r$ and inversely proportional to $\sigma^2$. The state price $P_b$ is a solution to Merton’s (1974) no-arbitrage pricing equation (19); we provide the details in Appendix even though the derivation is standard in the literature.

Bank value depends on its liability structure $(I, C, C_1)$ because the liabilities affect bankruptcy boundary and its state price. The following theorem, proved in Appendix A.1, summarizes the relationship between bank value and liability structure.

**Theorem 1** Given a liability structure $(I, C, C_1)$, the bank run, or closure, boundary and the endogenous default boundary are, respectively,

$$V_a = \kappa C/(r - \eta)$$

$$V_d = (1 - \tau)[\lambda/(1 + \lambda)](I + C + C_1)/r .$$

The bankruptcy boundary is $V_b = \max\{V_a, V_d\}$. The equity, subordinated debt, and bank values are, respectively,

$$E = V - (1 - \tau)(1 - P_b)(I + C + C_1)/r - P_b V_b$$

$$D_1 = (1 - P_b)C_1/r + P_b[(1 - \phi)V_b - D]^{+}$$

$$F = V - P_b \min\{\phi V_b, V_b - D\}$$

$$+ (1 - P_b)[C\eta/(r - \eta) + \tau(I + C + C_1) - I]/r .$$

In equation (3), equity value is the residual asset value after subtracting the expected after-tax liabilities on insurance, deposits, and subordinated debt and the expected value lost to bankruptcy. In equation (4), subordinated debt value is the sum of expected coupon value before bankruptcy and expected recovery value at bankruptcy. In equation (5), which is for bank value, the first term is the assets value, the second term reflects the value expected to lose to bankruptcy, and the last term shows the value of expected account service income and tax benefit after paying insurance premium.

Theorem 1 shows the role of account service income and deposit insurance in bank valuation. Along with tax savings on debt, account service income ($\eta$) increases bank
value as shown by the last term on the right-hand side of equation (5). The ability of a bank to attract deposits at a rate lower than the risk-free rate comes at a price: the bank has to close and incur bankruptcy cost if depositors run or if the charter authority closes the bank. For a bank with deposit insurance, insurance premium reduces bank value, which is evidenced by the last term of the equation. Although the liability structure in the theorem includes insurance premium, these formulas in the theorem apply to banks without deposit insurance if we set $I = 0$.

One may obtain the endogenous credit spread of subordinated debt from Theorem 1. The endogenous credit spread is $s = C_1/D_1 - r$, where $D_1$ is a function of $C_1$ as given in equation (4). The credit spread takes the probability of bankruptcy into account through state price $P_b$; at the same time, the state price is affected by the liability structure. Insurance premium affects the credit spread even though $I$ does not appear in equation (4) explicitly. The insurance premium affects the endogenous default boundary in equation (2), which in turn affects bankruptcy boundary $V_b$ and its state price. The last two affect the credit spread.

A comparison of the model of banks with the model of firms in Leland (1994) shows the connection and distinction between banks and non-financial firms. If we set $C = I = 0$ but $C_1 > 0$, the formulas in Theorem 1 reduce to those in Leland (1994) for a firm with an unprotected debt—the word “subordinated” drops when there is no deposit. If we set $I = \eta = C_1 = 0$ but $C > 0$, the formulas in Theorem 1 coincide with Leland’s for firms with a debt protected at level $\kappa D$. Leland’s seminal capital structure theory is for non-financial firms. Nevertheless, it does not apply to banks that take deposits, earn account service income, pay deposit insurance premium, and face the risk of bank run or closure. The theory developed in this paper extends Leland’s to banks and offers a consistent framework for understanding the connections and distinctions between banks and other firms.

The deposit insurance premium is exogenously given in Theorem 1, but it should endogenously depend on the amount of deposits under insurance and the risk involved. In principle, the insurance corporation should charge each bank a fair insurance premium. A fair premium makes the insurance contract worth zero to each party of the contract. The next Theorem, proved in Appendix A.2, characterizes the fair insurance premium.

**Theorem 2** Given $D$ dollars of deposits, the fair insurance premium is

$$I^0 = r[1 - (1 - \beta)\kappa]D P_a/(1 - P_a),$$

where $P_a = [\kappa D/V]^\lambda$ is the state price of bank closure.
An alternative way to write the insurance pricing equation is

$$(1 - P_a)(I^o/r) = P_a[1 - (1 - \beta)\kappa]^+ D,$$  \hspace{1cm} (7)$$

which says that the expected present value of insurance premium paid to the insurance corporation equals the expected present value of the insurance obligations at bank closure. If $\kappa < 1/(1 - \beta)$, the fair premium $I^o$ is positive. It converges to zero if $\kappa$ rises to $1/(1 - \beta)$. If $\kappa \geq 1/(1 - \beta)$, the fair premium is zero because the bank will be closed with enough asset value to cover the deposits in full.

The fair insurance premium $I^o$ increases with $D$. If deposits increase, not only the insurance premium increases, the assessment rate of insurance premium, which is the premium on each dollar of deposits, also increases. By Theorem 2, the assessment rate is

$$h = I^o/D = r[1 - (1 - \beta)\kappa]^+ P_a/(1 - P_a).$$  \hspace{1cm} (8)$$

The rate is increasing with $D$ because $P_a$ is bigger for larger $D$. The positive relationship between $h$ and $D$ makes sense because expansion of deposits exposes the insurance corporation to bigger risk.

Some academics have argued that the FDIC does not charge enough insurance premium to cover its risk exposure.\textsuperscript{17} A premium lower than the fair rate provides subsidized insurance to banks. To allow for subsidized insurance premium, we assume that the FDIC insurance premium is $I = \omega I^o$, where $\omega = 1$ represents a fair premium and $\omega < 1$ represents a subsidized premium. Relating to the net cash outflow on deposits by $D = C/(r - \eta)$, we have $I = iC$, where

$$i = \omega[1 - (1 - \beta)\kappa]^+[r/(r - \eta)]P_a/(1 - P_a).$$  \hspace{1cm} (9)$$

If the FDIC subsidizes deposit insurance, it increases the bank value because the bank pays lower insurance premium for enjoying the risk-free value of deposits. Even with the subsidy, the assessment rate and the total premium a bank pays still endogenously depends on the amount of deposits and the bank's risk profile.

With endogenous insurance premium, a liability structure is characterized by the pair $(C, C_1)$ because $C$ determines $I$. Imposing $I = iC$ in the bank value formula (5), we obtain

$$F = V + (1 - P_b)[\eta/(r - \eta) + \tau - (1 - \tau)i]C/r + (1 - P_b)\tau C_1/r - P_b \min\{\phi V_0, V_0 - D\}.$$  \hspace{1cm} (10)$$

\textsuperscript{17}See Duffie, Jarrow, Purnanandam, and Yang (2003). On the other hand, one may argue that a lower premium is necessary to compensate the insured banks for the costs of reporting requirements and tight regulation.
On the right-hand side of equation (10), the second term is the value of tax deduction and account service income, netted off against the insurance premium. The third term is the value of tax benefits to the bank for its interest expense on subordinated debt. The last term is the expected loss of value due to bankruptcy, for which bankruptcy cost $\phi$ takes the value of $\alpha$ or $\beta$, depending on the type of bankruptcy.

3.2 Optimal Liability Structure

Now we proceed to examine how a value-maximizing bank chooses its liability structure. We first consider an unregulated bank, which is neither under FDIC deposit insurance nor subject to any other regulation. The bank takes into account the possibility that depositors run to protect their deposits. The unregulated bank is important for understanding the inherent difference between banks and other firms because we want to know whether the leverage in bank liability structure is driven by banking business or government regulation, or both.

The unregulated bank operating in a free market serves as a benchmark for evaluating the effects of regulatory mandates such as FDIC insurance and charter authority’s closure of troubled banks. The benchmark is useful for examining how a bank might arrange its liability structure differently in alternative regulatory environments. In Section 4.3, we are able to gain insights into the optimal responses of banks to the FDIC. The optimally-responding banks adjust their leverage, liability structure, and default decisions, relative to the benchmark in which banks make their choices of liability structure unfettered by any government interventions.

As pointed out earlier, a liability structure of an unregulated bank is described by the pair $(C, C_1)$. An optimal liability structure is the deposit liability $C^*$ and subordinated debt liability $C_1^*$ that maximize bank value. The next theorem, in Appendix A.3, provides a characterization of the optimal liability structure for an unregulated bank without deposit insurance or regulation.

**Theorem 3** Suppose $0 < \eta < r$, $0 < \tau < 1$, and $\kappa \geq 1/(1 - \alpha)$. The optimal liability structure of an unregulated bank is unique. In the optimal liability structure, $V_a^* = V_d^*$, and the state price of bankruptcy is

$$P_b^* = \frac{1}{1 + \lambda} \cdot \frac{\eta(1 - \tau)\kappa + r\tau(1 + \lambda)\kappa}{\eta(1 - \tau)\lambda + r\tau(1 + \lambda)\kappa + r(1 - \tau)\alpha\lambda\kappa}.$$  \hspace{1cm} (11)
The optimal deposit and subordinated-debt liabilities are

\[ C^* = (r - \eta)V P_b^{1/\lambda} / \kappa \]

\[ C_{1*} = r V P_b^{1/\lambda} \left[ \frac{1 + \lambda}{(1 - \tau)\lambda} - \frac{r - \eta}{r \kappa} \right] . \]

Equation (11) gives the exact formula of the optimal state price \( P_b^* \); it is an elementary algebraic function of the following exogenous parameters: \( r, \sigma, \delta, \tau, \eta, \) and \( \alpha \).

The theorem characterizes the optimal liability structure of a bank that faces corporate tax, bears the risk of costly bankruptcy, and takes deposits to earn service income. Combining Theorem 3 with Theorem 1, we obtain analytical solutions, which we omit to save space, for deposit value \( D^* \), subordinated debt value \( D_{1*} \), equity value \( E^* \), bank value \( F^* \), bankruptcy boundary \( V_b^* \), and credit spread \( s^* \) in the optimal capital structure of the unregulated bank. The ratio of subordinated debt liability to deposit liability is a characterization of the liability structure and is thus referred to as liability ratio. By the theorem, the optimal liability ratio is \( x^* = C_{1*}/C^* = r(1 + \lambda)/(\lambda(1 - \tau)(r - \eta)) \).

The theorem states that the optimal amount of subordinated debt makes the endogenous default boundary coincide with the bank-run boundary. Deposits come with the discount of interest rate and service income, in addition to tax savings. The cost of taking deposits is the expected loss due to a bank run. In contrast, subordinated debt brings tax savings but produces no account service income; its cost is the expected loss due to bankruptcy. Therefore, at the margin, the bank should use deposits, not subordinated debt, to balance the tax and liquidity benefits with the cost of bankruptcy. With this balance, the bank should take as much subordinated debt as possible for availing the tax benefits but should avoid the expected bankruptcy cost resulting from endogenous default. To avoid the expected cost associated with endogenous default, the bank should not set the endogenous default boundary above the bank-run boundary. As a result, the optimal subordinated debt should make default occur at exactly the same point as bank run.

The assumption of a rational bank run, \( \kappa \geq 1/(1 - \alpha) \), is essential for Theorem 3. We have found that \( V_d^* > V_a^* \) may happen in the optimal liability structure if \( \kappa < 1/(1 - \alpha) \). In the situation of \( \kappa < 1/(1 - \alpha) \), depositors run from the bank at a point when the bank does not have enough assets to refund the deposits. Such a late bank run is not rational for depositors.

The account service associated with deposits is important for Theorem 3. If \( \tau \in (0, 1) \) and \( \kappa \geq 1/(1 - \alpha) \) but \( \eta = 0 \), the optimal liability structure is not unique.
If we set $\eta = 0$ in formulas (11)–(13), we obtain the optimal structure with the maximum deposits, but for every liability ratio $x \geq x^*$, there exists an optimal structure. Thus, when $\eta = 0$, the optimal liability ratio is not unique. The optimal capital structure is not unique because deposits and subordinated debt have the same tax benefits and bankruptcy costs in the absence of account service. The multiple optimal structures corresponding various $x$ may be thought as optimal structures for a non-financial firm that faces corporate tax but does not receive account service income.\textsuperscript{18} A lower liability ratio in the optimal structure corresponds to more deposits. Holding no deposits ($C = 0$ and $x = \infty$) is one of the optimal liability structures. Obviously, a firm holding no deposits and providing no account service is not a bank. Hence, a structural model without considering deposits and bank account service is not appropriate for understanding banks’ optimal liability structure and leverage.

Corporate tax plays an important role in Theorem 3. If $\eta \in (0, r)$ and $\kappa \geq 1/(1-\alpha)$ but $\tau = 0$, the optimal structure is not unique. Setting $\tau = 0$ in the theorem gives the optimal structure with the least deposits, but for every liability ratio $x \leq x^*$, there exists an optimal structure. A bank with $\tau = 0$ provides account service but receives no tax benefit. Tax benefit is typically ignored in the literature that focuses on bank account services. Without tax benefit, banks have no incentive to issue subordinated debt because it may expose banks to bankruptcy cost. With zero tax benefit, as long as the level of subordinated debt is low enough so that default does not happen before bank run, default risk is irrelevant to bank valuation. Therefore, as long as the liability ratio sets the default boundary below the bank-run boundary, any liability structure with subordinated debt is optimal. The indeterminacy of subordinated debt in the absence of tax benefit suggests that a model ignoring tax savings may not be appropriate for understanding bank liability structure.

Theorem 3 ignores FDIC deposit insurance and regulation of charter authority, but it serves as a useful benchmark so that we can examine the effects of deposit insurance and bank regulation. For an FDIC insured bank, the optimal liability structure is a pair of $C^*$ and $C^*_i$ that maximizes the bank value in equation (10) subject to equation (9). The value-maximizing bank in our framework is fully aware that any decision pertaining to leverage and liability structure has a consequence on the FDIC insurance premium. The bank should therefore be mindful of the channel

\textsuperscript{18}Without account service income, deposits in our model bears resemblance to the secured debt in Leland’s (1994) because deposits is protected by bank run. Leland considers the optimal capital structure of firms that take either secured or unsecured debt but not the optimal mix of the two. He analytically solves the optimal capital structure of firms that take unsecured debt, but for a firm that takes secured debt, he solves the optimal structure numerically.
in its choice of leverage and liability structure. The endogenous determination of FDIC premium and liability structure captures the feedback channel from FDIC to the banks and vice versa.

The next theorem, proved in Appendix A.4, characterizes the conditions for a liability structure to be optimal in a bank under FDIC insurance.

**Theorem 4** Suppose $0 < \eta < r$, $0 < \tau < 1$, $1 < \kappa < 1/(1 - \beta)$, and the FDIC insurance premium is $I = \omega I^\circ$, where $I^\circ$ is defined in Theorem 2. A liability structure with $V_d < V_a$ is never optimal for an FDIC-insured bank. There exists $\kappa^* \in [1, 1/(1 - \beta))$ such that for all $\kappa \in (\kappa^*, 1/(1 - \beta))$, the optimal structure is unique and satisfies $V_d^* = V_a^*$. In such optimal structure, the state price of bankruptcy is

$$P^*_b = \frac{1}{1 + \lambda} \cdot \frac{\eta(1 - \tau)\lambda + r\tau\kappa(1 + \lambda)}{\eta(1 - \tau)\lambda + r\tau\kappa(1 + \lambda) + r(1 - \tau)\lambda(\kappa - 1 + \omega[1 - (1 - \beta)\kappa]^+)}.$$  \hspace{1cm} (14)

The optimal deposits and subordinated debt liabilities are

$$C^* = (r - \eta)V P_b^{1/\lambda}/\kappa$$ \hspace{1cm} (15)

$$C_1^* = rV P_b^{1/\lambda} \left[ \frac{1 + \lambda}{(1 - \tau)\lambda} - \frac{r - \eta}{r\kappa} - \omega[1/\kappa - (1 - \beta)]^+ \frac{P_b^*}{1 - P_b^*} \right].$$ \hspace{1cm} (16)

Formula (14) shows that the optimal state price $P_b^*$ is a function of the following exogenous parameters: $r$, $\sigma$, $\delta$, $\tau$, $\eta$, $\beta$, $\kappa$, and $\omega$. Combining this theorem with Theorems 1 and 2, we obtain analytical solutions, which we omit to save space, for the deposits value $D^*$, subordinated-debt value $D_1^*$, equity value $E^*$, bank value $F^*$, bankruptcy boundary $V_b^*$, credit spread $s^*$, and insurance premium $I^*$ in the optimal capital structure of the FDIC insured bank.

Theorem 4 shows that it is optimal for banks to leverage so that the endogenous default of subordinated debt is as late as the closure of banks. FDIC insurance and regulation of charter authority do not directly affect how a bank optimally chooses subordinated debt, but they affect through the choice of closure boundary. The optimal subordinated debt still maximizes tax benefit and avoids protecting deposits. In particular, Theorem 4 shows that zero subordinated debt $C_1 = 0$ is not optimal for a bank as long as the insurance premium is not too high. The economic intuition is simple. When the insurance premium is not too high, it is optimal to use a positive level of deposits. By the optimal condition that the bank closure boundary must equal the default boundary, it is optimal for the bank to issue a positive amount of subordinated debt to meet the condition. It is also easy to reason analytically. If
If \( i \) is not too large, the above equations imply \( V_a > V_d \), indicating that the structure is suboptimal by Theorem 4.

In theory, if asset volatility \( \sigma \) and liquidation cost \( \beta \) are high enough, it is possible for a liability structure with \( V_d > V_a \) to be optimal for some low \( \kappa \) close to 1. We have confirmed this possibility by both mathematical derivations and numerical optimization. If \( \sigma \) and \( \beta \) are very high and \( \kappa \) is very low, the fair insurance premium rate \( i \) may be very high, making deposits too expensive as funds compared to subordinated debt. When that happens, reducing deposits to have \( V_a < V_d \) may be optimal. Preventing \( i \) from being too high is the reason for \( \kappa \) to be higher than a threshold \( \kappa^* \) in the theorem. Nevertheless, for all asset volatility and liquidation cost we consider in Section 4, we find \( \kappa^* = 1 \). That is, \( V_a^* = V_d^* \) and the formulas in Theorem 4 hold for all \( \kappa \in (1, 1/(1 - \beta)) \).

Theorem 4 incorporates the endogenous insurance premium in optimal liability structure. Besides considering the tradeoff between tax benefits, account service income, bank closure, and bankruptcy costs, banks take the cost of deposit insurance into account. If we set \( \kappa \geq 1/(1 - \alpha) \) and assume that \( \alpha \geq \beta \), the insurance premium becomes zero. Consequently, the formulas in this theorem reduce to those in Theorem 3. With positive \( \omega \) and more general \( \kappa \) in this theorem, the assessment rate \( \omega h \) of insurance premium is increasing with \( D \), and thus \( i \) increases with \( C \). Therefore, under the assumption of Theorem 4, banks consider both the increase in insurance premium caused directly by the expansion of \( D \) as well as the increase caused indirectly through the rise of assessment rate. In Section 4.3, we will show the impact of endogenous insurance premium on banks’ optimal choice of capital structure.

### 4 Quantifying Optimal Bank Leverage

The model just developed paves a way to characterize quantitatively the optimal bank liability structure. As discussed earlier, we consider two types of banks. The first type operates in freely competitive market. Deposits in these banks are not insured and thus face the risk of a bank run. In practice, non-U.S. banks face the risk of a bank run because their governments do not provide deposit insurance.\(^{19}\) There are

\(^{19}\)Bank runs have happened even after the recent financial crisis. In 2010, depositors “ran” from two Swedish banks, Swedbank and SEB, and a Chinese bank, Jiangsu Sheyang Rural Commercial
also many U.S. banks uninsured by the FDIC. In theory, an unregulated bank serves as a counterfactual for a bank covered by FDIC insurance and regulation, which is the second type we consider. These are the majority of banks in the U.S.

4.1 Exogenous Parameters

Our model inherits the major advantage of structural models that coherently connects the risk of debt and equity to the risk of assets. The risk comes from asset volatility (\( \sigma \)), which is an important parameter. Since asset volatility is not directly observable, investors infer asset volatility from accounting data and market prices. Moody’s KMV provides estimates of asset volatility for a large number of companies across a wide range of industries. In Figure 2, we present the average, median, and the 10/90-percentiles of Moody’s estimates for banks in panel A. As a comparison, we present the estimates for manufacturing firms in panel B. The figure shows a difference between the assets held by banks and those owned by manufacturing firms: bank assets have much lower volatility. The average bank asset volatility is around 10% for banks, whereas the average asset volatility is 40~50% for manufacturing firms. Although bank asset volatility fluctuates over time, the median is around 5% for 2001–2012. The 90 percentile of bank asset volatilities is well below 15% for 2001–2007, and it stays well below 25% even for the difficult period of 2007–2012. In view of these stylized facts, we use \( \sigma = 10\% \) as the baseline value for asset volatility, as shown in Table 1, but explore alternative values in a range between 5% and 25%.

Bank. In 2013, depositors ran from Cypriots banks and forced the country to close its banks for many days. In 2014, depositors ran from two Bulgarian banks, Corporate Commercial Bank and First Investment Bank.
Another important parameter of bank assets is its rate of cash flow (δ). In the risk neutral world, the growth rate of asset value is r − δ. If the assets contain only commercial and consumer loans, the cash flow are interest and principal payments of the loans. Typically, the value of a loan declines over time, but banks may retain some of the cash flows for new lending so that bank assets grow. As a simple starting point, we assume the cash flow rate is the same as the interest rate δ = r. Under this assumption, the asset value does not grow in risk neutral probability, but it may still grow in real probability measure. With zero growth rate in risk neutral probability, the state price of bankruptcy does not vanish over time in Leland’s (1994) model. To compare the risk of bankruptcy and leverage across banks, it is common to measure each component of the capital structure as a percent of asset value. We normalize bank asset value to 1. To think in the context of a practical bank, one may regard the bank asset value as $V = 1$ billion.

The income from deposit account service is important in bank liability structure. The net income from deposit account service should be determined in the competitive market of deposits. In a perfect competitive market, the net income would be driven to zero in an unregulated industry with free entry, or it should just cover the insurance premium if a bank has deposit insurance. At least in the U.S., new entry of banks into the market is regulated by charter authorities. Without free entry, deposit rents arise from the market power enjoyed by the bank (De Nicolo and Rurk Ariss, 2010). The profitability should depend on the amount of deposits and the bank. Thus, parameter η may be differ across banks and should be a function of D. We do not explicitly model the equilibrium of deposits or the demand function of deposits, D(η), in order to keep the model tractable and focus on the choice of liability. We instead assume η to be a constant but allow it to be different across banks. A range of values for η are examined later. For the baseline number, we set η to be 50 bps (i.e., 0.50%), based on the average deposit rates and service charges of largest commercial banks.

Since a major benefit of leverage is the tax deductibility of interest on debt, corporate tax rate is an important parameter in capital structure. The statutory corporate tax rate in the U.S. ranges up to 35%. The U.S. Department of Treasury (2007) reports that the effective marginal tax rate on investment in business is 25.5 percent, but it varies substantially by business sectors. The academic literature suggests that the effective corporate tax rate is around 10% for non-financial firms (Graham, 2000) but about 30% for banks (Heckemeyer and Mooij, 2013). We choose τ = 30% as the baseline value for corporate tax rate and later vary it to examine the sensitivity of
optimal structure to the rate.

Bankruptcy cost counters the tax benefit of debt, but the task of measuring it has always been a challenge. A well-known reference is the study of Altman (1984), which examines a sample of 19 industrial firms which went bankrupt over the period of 1970–1978. The estimated bankruptcy cost is 19.7% of a firm value just prior to its bankruptcy. Bris, Welch and Zhu (2006), however, show that bankruptcy cost varies across firms and ranges between 0% and 20% of firm assets. Banks experienced higher bankruptcy costs. Based on 791 FDIC-regulated commercial banks failed during 1982–1988 (the Savings and Loan Crisis), James (1991) estimates that bankruptcy cost is 30% of a failed bank’s assets. Based on 325 insured depository institutions failed during 2008–2010 (the Great Recession), Flannary (2011) estimates that bankruptcy cost is 26.6% of a failed bank’s assets. In light of these estimates, we set $\alpha = \beta = 25\%$ as the baseline value and then examine alternative assumptions about bankruptcy costs.

The baseline values of other exogenous parameters are as follows. For an unregulated bank, we assume bank run happens at the point when the bank has exactly enough assets to repay deposits after liquidation in bankruptcy, i.e., $\kappa = 1/(1 - \alpha)$. For an FDIC insured bank, state banking regulatory agency closes it when it is unable to meet its obligations to depositors. The parameter $\kappa$ for bank closure should thus be at least 1. When a bank’s total capital is less than 2% of its assets, the FDIC classifies it as “critically undercapitalized,” and the charter authority typically closes the bank. Given these institutional arrangements, we set closure boundary as $\kappa = 1/(1 - 0.02) \approx 102\%$. For insurance subsidy, we use $\omega = 1$ (no subsidy) as the starting point, in view of the 2011 FDIC reform, but later examine the consequence of subsidized deposit insurance. All the baseline values are listed in Table 1.

The optimal liability structure is characterized by a set of variables endogenously determined by bank management and market valuation. The first endogenous variable of our interest is the charter value of the bank, which is the difference between bank value and asset value and expressed as percent of assets, $(F - V)/V$. The next three endogenous variables are the deposit, subordinated debt, and equity values relative to assets, i.e., $D/V$, $D_1/V$ and $E/V$. The sum of deposit and subordinated debt values relative to assets equals the leverage ratio, $(D + D_1)/V$, the ratio of the total debt to total assets.\textsuperscript{20} The amount of deposits determines the closure bound-

\textsuperscript{20} To avoid confusion, we should note that the ratio of debt to assets is often referred to as “debt ratio” in corporate finance. Instead, “leverage ratio” is often referred to the ratio of debt to equity. In bank regulation, however, “leverage ratio” is the ratio of Tier 1 capital to bank total assets. All
ary, and the leverage affects the endogenous default boundary. These two boundaries relative to asset value, $V_a/V$ and $V_d/V$, are interesting to analyze. The higher of the two is the bankruptcy boundary $V_b/V$, which influences the loss in bank value to bankruptcy cost, $P_b@V_b/V$, another endogenous variable of our interest. The possibility of bankruptcy causes the bank to pay a credit spread, $s$, on subordinated debt. The credit spread is observable in the market. For banks with deposit insurance, the insurance premium $I$ is endogenously determined in optimal liability structure.

4.2 Optimal Leverage of Unregulated Banks

Table 2 presents optimal values of endogenous variables for unregulated banks. Consideration of unregulated banks helps answer an important question: are banks fundamentally different from non-financial firms regardless of the regulations imposed on banks? This question is important for understanding the effects of deposit insurance, which will be delayed to the next subsection. The calculations in the table are based on the analytical solutions in Theorem 3. We calculate the optimal structure for the exogenous parameters assumed earlier and report the results in the middle column of each panel. Then, we perturb the account service income rate $\eta$, asset volatility $\sigma$, and bankruptcy cost $\alpha$ to show the optimal structures for alternative parameters.

Table 2 reveals some distinctive characteristics of bank optimal liability structure. The most striking is the high leverage ratio, which are unusual in non-financial firms but typical in banks. This high level of leverage is optimal for a bank even without deposit insurance. With the baseline values of exogenous parameters, the sum of deposits and subordinated debt amounts to 94.58 percent of the asset value, leaving only 5.42 percent of tangible equity (see the middle column of each panel). High bank leverage is likely related to one or both of the two fundamental factors that distinguish banks from other firms: (1) taking deposits to provide account services and (2) earn cash flows from low-volatility assets such as loans. We examine the effects of these two factors by considering a range of values of $\eta$ and $\sigma$.

Account service income appears to be a driver of bank leverage, as shown in panel A of Table 2. The optimal liability structure consists of more deposits if the account service income rate is higher. If we interpret the account service income as the premium on providing liquid cash accounts, the result is consistent with DeAngelo and Stulz (2013), who suggest that premium on liquidity production is a reason for high leverage in banks that take deposits. In their analysis, assets are risk-free. Our these alternative of ratios are related and measure the leverage of a company.
### A. Effects of Account Service Income

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<td>Charter value</td>
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<td>Deposit value</td>
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<tr>
<td>Sub-debt value</td>
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<tr>
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<tr>
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<tr>
<td>Default boundary</td>
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<tr>
<td>Bankruptcy loss</td>
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<tr>
<td>Credit spread</td>
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### B. Effects of Asset Volatility

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<tr>
<td>Bankruptcy loss</td>
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<td>Credit spread</td>
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### C. Effects of Bankruptcy Cost

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Table 2: Optimal Structure of Unregulated Banks. Numerical values of endogenous variables in the optimal liability structures of unregulated banks are calculated for various assumptions on account-service income rate, asset volatility, and bankruptcy cost. Other exogenous parameters are set to the values in Table 1. All numerical values are presented in percentage points.
result shows that liquidity service is still a driver of leverage if we relax DeAngelo and Stulz’ assumption to allow bank assets to be volatile.

Bank asset volatility is typically low, as noted previously, and low asset volatility appears to be an important driver of bank leverage, as shown in panel B of Table 2. If volatility is as low as 5%, the optimal leverage ratio is even higher than 100%, letting tangible equity be negative in the bank. In contrast, for an asset volatility of 25%, the leverage ratio sharply reduces to 76.90%, giving the bank nearly 33% tangible equity. A bank with 25% asset volatility takes much less deposits, only 25.39% of asset value. In view of the inverse relationship between volatility and leverage, a manufacturing company with asset volatility higher than 30% or 40% is unlikely to use the level of leverage that banks use. Thus, low asset volatility is crucial for bank to use high leverage. If a high leverage consists of large amounts of deposits, it allows a bank to provide liquidity service on deposits. Therefore, low volatility is also important for banking service with deposits.

Another noticeable characteristic of the optimal liability structures in Table 2 is the significance of subordinated debt.\textsuperscript{21} It is 56.97% of asset value for the baseline parameter values. As noted in Theorem 3, banks optimally set subordinated debt to a level such that default boundary is exactly the same as closure boundary. With this strategy, banks maximize tax deduction but do not affect the probability of bankruptcy. The tax benefit increases the bank charter value. In panels A and B, higher account service income rate and lower asset volatility are associated with more subordinated debt. Again, the increase in subordinated debt is a consequence of keeping default boundary at the same level as closure boundary. When a bank takes more deposits because of higher account service income or lower asset volatility, the closure boundary is higher, which allows higher level of subordinated debt to take advantage of additional tax benefit.

Leverage causes loss in bank value due to bankruptcy costs. In Table 2, the lost value is 3.15% of asset value for the baseline parameters. Because of potential bankruptcy loss, subordinated debt holders demand 101 bps of credit risk premium. Since higher account service income is associated with higher leverage, it drives up the probability of bankruptcy and credit spread. The last two rows of panel A illustrate the positive relation of bankruptcy loss and credit spread to the account service income rate. The last two rows in panel B demonstrate the relation of bankruptcy

\textsuperscript{21}In practice, subordinated debt has always been an important source of funding for banks. Avdjiev, Katasheva and Bogdanova (2013) report that during the 2009–2013 period alone, banks around the world have issued $4.1 trillion of unsecured long-term debt.
loss and credit spread to asset volatility. For an asset volatility as high as 25%, the credit spread is 328 bps, three times as high as the spread for 10% volatility.

To understand the effect of bankruptcy cost, which limits leverage, it is important to characterize banks’ response to changes in bankruptcy costs. If a bank does not adjust its liability structure for change in bankruptcy cost, we expect the bank-run boundary to be higher for higher bankruptcy cost because the bank-run boundary is $V_a = \frac{\kappa D}{1 - \alpha}$. This relation is plotted as the dashed line in panel A of Figure 3. In the plot, we first assume the unregulated bank optimizes its liability structure and then keep it fixed when we alter bankruptcy cost $\alpha$. Since the liability structure is fixed, the default boundary (as marked by circles) is independent of $\alpha$ in the figure. Nevertheless, a bank’s optimal response completely changes the relation between bank bank-run boundary and bankruptcy cost. When the bank optimally responds to increase in bankruptcy cost, it reduces deposits, resulting a inverse relation between $V_a^*$ and $\alpha$ (plotted as the solid line). Since the optimal default boundary is the same as closure boundary, default boundary also decreases as $\alpha$ increases.

Therefore, when the liability structure optimally responds, the relation between bank bankruptcy boundary and bankruptcy cost is different from the relation when a liability structure is fixed. Mathematically, $V_a = \frac{D}{1 - \alpha}$ is an increasing function in $\alpha$ if we fix $D$, but $V_a^* = \frac{D^*}{1 - \alpha}$ is a decreasing function in $\alpha$. Panel C of Table 2 demonstrates that the bank-run boundary is lowered from 50.15% to 47.69% of asset
value when $\alpha$ rises from 25% to 35%. As a result of the optimal response, the value lost to bankruptcy increases only moderately: from 3.15% to 3.80% of asset value. Without considering bank optimal response, one would expect the credit spread to rise when bankruptcy cost goes up. To the contrary, the credit spread in the optimal structure drops from 101 to 88 bps, when the bank cuts deposits down from 37.61% to 31.00% of asset value. At the same time, the bank boosts subordinated debt up from 56.91% to 58.99%. The increase of subordinated debt associated with the increase of bankruptcy cost would have been difficult to understand without considering banks’ optimal responses.

### 4.3 Optimal Leverage of FDIC-Insured Banks

Optimal response of bank liability structure is especially important in understanding the effects of FDIC and charter authority on banks because it may counteract the intended objectives of FDIC. The major function of FDIC is to reduce the probability of bank failure by preventing bank runs. For the baseline parameters, the optimal bank closure boundary with FDIC insurance is 50.44% of asset value, as shown in Table 3. This boundary is slightly higher than the 50.15% closure boundary of unregulated banks, which is presented again in this table for convenience of comparison. The higher closure boundary leads to a larger loss of value to bankruptcy. In Table 3, the value lost to bankruptcy in the FDIC-insured bank is 3.21% of asset value, slightly larger than the 3.15% in the comparable unregulated bank. The larger expected loss results in 1 bps increase in credit spread. The increases in closure boundary, bankruptcy loss, and credit spread are not necessarily consonant with the mandates of FDIC and charter authority. This outcome is result of the bank’s optimally response to FDIC insurance when it takes more deposits, undoing the intended impact of FDIC on the probability of bankruptcy. In Table 3, the amount of deposits in the FDIC-insured bank is 49.45% of asset value, 18.84 percentage points higher than the deposits in the comparable unregulated bank.

The optimal response of an FDIC-insured bank changes the relationship between its closure boundary and the closure rule of charter authority. In panel B of Figure 3, an insured bank first optimizes its liability structure for $\kappa = 102\%$ as presented in Table 3, and then $\kappa$ changes. As the closure rule gets tougher (say, $\kappa$ increases to 105%), the closure boundary gets lower ($V_a$ lowers to 50.41%, as shown in the second last column of the Table 3), instead of getting higher. If the bank keeps the liability structure fixed, the closure boundary should be $V_a = \kappa D$, which is a linear function.
of $\kappa$ plotted as the dashed line in the figure. The default boundary $V_d$ should be independent of $\kappa$, as marked by the circles in the figure. When the bank optimally responds to change of $\kappa$, however, the relationship between closure boundary and $\kappa$ is completely different; it almost does not increase as $\kappa$ increases. The optimal closure boundary $V_a^* = \kappa D^*$ is not a linear function of $\kappa$ anymore because the bank optimally reduces deposits in response to the increase of $\kappa$. The optimal default boundary $V_d^*$ is related to $\kappa$ the same way as $V_a^*$ because $V_d^* = V_a^*$ for all $\kappa$, as plotted in the figure.

FDIC deposit insurance to play a part in driving the leverage of a bank’s optimal structure higher. In Table 3, the leverage ratio of the insured bank is 96.36%, higher than the 94.58% leverage ratio of the comparable unregulated bank. With deposit insurance, the bank holds more deposits and less subordinated debt, but its total debt is higher. Intuitively, deposit insurance and the closure rule of charter authority allow the insured bank to take more deposits and keep the closure boundary from becoming too high. Thus, a major benefit banks receive from the FDIC is that it prevents bank from suffering from sharp increase in the probability of bank run or closure when increasing deposits. With this benefit, the banks use leverage higher. The higher optimal leverage due to FDIC insurance leads to higher bank value. The charter value of the FDIC-insured bank in Table 3 is 27.11% of asset value, 1.44 percentage points higher than the value of a comparable unregulated bank.

The bank value attributed to the tax benefit and liquidity income of an optimal liability structure may offer an explanation for the empirical results documented by Mehran and Thakor (2011) about bank acquisition. They find that an acquiring bank
bids with higher premium for a target bank if the target has more equity. If a high equity ratio in the target is not optimal to the acquirer and thus gives the acquiring bank more room to take advantage of tax benefit and liquidity income with additional leverage, the acquirer should be willing to place higher value on the target.

The FDIC insurance increases bank value even if it charges fair insurance premium. The insurance premium of the bank in Table 3 is 12 bps of asset value, which is about 24 bps of deposits, as the amount of deposits is about half of the asset value. Figure 4 shows how the insurance premium for the optimal structure depends on its two important factors: the closure rule and asset volatility. The range of insurance premium in the figure is broadly in line with the assessment rates published by the FDIC.\(^{22}\) The insurance premium is a cost to the bank, but the insured bank is able to benefit from FDIC insurance because of its optimal response, which in turn reduces the effect of FDIC on the bank’s probability of bankruptcy. If the FDIC subsidizes deposit insurance, it may make the bank value even higher and cause the insured bank to take more deposits and choose higher leverage. In the last column of Table 3, the insurance premium is 20 percent lower than the fair premium; that is, \(\omega = 0.80\). The leverage ratio of the bank with the subsidized insurance is 98.23%, 1.87 percentage points higher than leverage ratio of a comparable bank with unsubsidized insurance. As a result, the bank’s charter value is 0.62 percentage points higher.

As characterized by Theorem 4, the FDIC-insured bank adjusts the level of subordinated debt so that the default boundary is the same as the closure boundary. In Table 3, the insured bank issues less subordinated debt: 46.91% of asset value, which is 10.06 percentage points lower than the subordinated debt in a comparable unregulated bank. Thus, FDIC insurance reduces subordinated debt in banks, giving room for more deposits. While the literature has noted that FDIC insurance may have caused banks to increase deposits, it has not paid attention to the effects on the

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\(^{22}\)Federal Deposit Insurance Corporation (2011) reports that the initial base assessment rates are 12–26, 22, 32, 45 bps for banks in four risk categories.
banks’ desire to reduce subordinated debt. Nonetheless, subordinated debt is still a substantial part of the liability structure in an insured bank, through which the bank takes advantage of tax benefits.

FDIC insurance does not alter the effect of account service on optimal leverage. In panel A of Table 4, leverage ratio is higher for higher account service income rate. When the account service income rate changes from 30 bps to 70 bps, the leverage ratio increases from 93.32% to 99.33%. Leverage ratio increases with the account service income because the optimal amount of deposits goes up with it. Additional deposits raise the closure boundary, giving more room for subordinated debt. As a result, the relationship between leverage and account service income in an insured bank is similar to what we have seen in an unregulated bank.

Neither does the FDIC insurance put a dent in the dominant role of asset volatility as a main driver of leverage. As shown in Panel B of Table 4, the optimal leverage is sensitive to asset volatility in the FDIC-insured bank. If asset volatility is as low as 5%, leverage ratio goes up to 111.81%, giving negative tangible equity. If asset volatility is 25%, the leverage ratio reduces to 77.87%, giving 22.23% tangible equity. The effect of asset volatility on leverage, as shown in this table, is similar to the effect we have seen for unregulated banks. Thus, the FDIC insurance does not diminish the importance of asset volatility for bank leverage.

In the above analysis, we assume that FDIC liquidation of assets has the same cost as a bankruptcy in private sector. That is, $\alpha = \beta$. There are views that the FDIC can lower liquidation cost. For example, the U.S. lawmakers believe that the FDIC can implement a more efficient and orderly liquidation that protects the value of bank and thus grant the “orderly liquidation authority” to the FDIC by Title II of the Dodd-Frank Act. If this view is correct, the reduced liquidation cost may further affect the leverage of banks. Panel C of Table 4 demonstrates the effects of FDIC liquidation cost on bank optimal liability structure. We vary the FDIC liquidation cost $\beta$ in a range from 15% to 35% but keep the private sector bankruptcy cost $\alpha$ at 25%. Both deposits and subordinated debt vary inversely with FDIC liquidation cost. The leverage ratio is higher if FDIC liquidation costs less. The higher leverage ratio has consequence in credit spread. Without considering the optimal response of banks, one would expect credit spread to be lower if liquidation cost is lower. To the

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23 Book equity of a bank can be negative in practice. In its December 2011 filing, last time as a bank holding company, the U.S. operations of Deutsche Bank had total assets of $355 billion and Tier 1 capital of negative $5.68 billion.

24 On the other hand, some suspect the FDIC has no incentive to maximize liquidation value and thus may result in larger loss of assets value (James, 1991).
### A. Effects of Account Service Income

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### B. Effects of Asset Volatility

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### C. Effects of FDIC Liquidation Cost

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Table 4: Optimal structure of FDIC-insured banks. Numerical values of endogenous variables in the optimal liability structures of FDIC-insured banks are calculated for various assumptions on account-service income rate, asset volatility, and FDIC liquidation cost. Other exogenous parameters are set to the values in Table 1. All numerical values are presented in percentage points.
contrary, the credit spread goes up. The apparently counterintuitive change of credit spread is due to the optimal increase of leverage.

4.4 Relation of Bank Leverage to Corporate Tax

Tax deduction of interest expenses on debt is a major reason for leverage in all firms, not just in banks. Although the tax benefit of leverage is not a unique reason for banks to be different from other companies, corporate tax is more important for bank liability structure because banks use more leverage than non-financial firms do. Observing the importance of tax benefit to banks, several recent papers have attempted to measure empirically the link between leverage and tax rate based on linear regressions. These papers include Heckemeyer and de Mooij (2013), Keen (2011), Schepens (2013), and Schandlbauer (2013).

Our model offers a coherent framework for the link between tax rate and leverage after an adjustment. According to Goldstein et al. (2001), the asset value $V$ in Leland’s (1994) model, and also in our model, is the value of an all-equity firm that owns the assets and faces corporate tax. In other words, $V$ is the after-tax value of assets. If the before-tax value of assets is $V^*$, then the after-tax value is $V = (1 - \tau)V^*$. If the government lowers corporate tax rate from $\tau$ to $\tau'$, the after-tax value of assets should be higher. Let $V'$ be the after-tax value under corporate tax $\tau'$. Then, the after-tax values of assets in the two tax regimes are related by $V' = V(1 - \tau')/(1 - \tau)$. Incorporating the effect of a tax change on asset value is necessary in using our model to analyze the relationship between tax and capital structure, as Goldstein et al. point out. Otherwise, one would erroneously conclude that government can increase firm value by raising corporate tax rate.

Table 5 shows the optimal response of bank liability structure to changes of corporate tax rate. The table reports the after-tax values of assets in alternative tax regimes, incorporating the framework of Goldstein et al. (2001). The assets valued at $1$ billion under 30% tax rate is worth $1.0714$ billion under 25% tax rate. Although a lower corporate tax rate means a smaller tax saving and thus a lower charter value, the total bank value would be higher if government charges a lower tax rate. The bank value is the sum of asset value and charter value. In Table 5, the unregulated bank value is $1.2567 (= 1.00 + 0.2567)$ billion when tax rate is 30% and $1.2892 (= 1.0714 + 0.2178)$ billion when tax rate is 25%. The FDIC-insured bank value is also higher when tax rate is lower. The higher bank value makes sense because the bank, whether it is unregulated or FDIC-insured, keeps a larger part of the earnings
If corporate tax rate is lowered from 30% to 25%, both the unregulated and FDIC-insured banks hold less subordinated debt but take more deposits, as shown in Table 5. The unregulated bank reduces subordinated debt by $22 million, and the insured bank reduces it by 25.8 million. In comparison, the unregulated bank increases deposits by only $16 million, and the insured bank by $22.3 million. The differential reactions of deposits and subordinated debt are rational. Since the main purpose of subordinated debt is to take advantage of tax benefit, banks reduce subordinated debt in response to the reduction of tax benefit, giving room for more deposits, which earn more income from account service.

Banks use less leverage in a regime of lower tax rate because decrease of subordinated debt is larger than the increase of deposits. When tax rate is lowered, the increase of asset value further helps cutting down leverage ratio. In Table 5, leverage ratio is cut down by nearly 7 percentage points in either the unregulated bank or the FDIC-insured bank. Because of the lessening of leverage, the value lost to bankruptcy and the credit spread both drop if government cuts corporate tax rate. Nevertheless, the expansion of deposits in the optimal response dampens the effects of tax cut on leverage ratio. Without deposit expansion, tax cut would have lowered leverage ratio by more than just 7 percentage points.

Our results appear to support those who argue that lowering corporate tax will
help stabilize banks, but we need to cautious about this policy proposal. If lowering corporate tax rate leads to a loss in tax revenue; it may be an expensive policy change for the public to achieve a greater stability of banking industry. An alternative is to lower corporate tax rate just for banks as suggested by Fleischer (2013). Lowering tax for banks will make banking a subsidized industry, begging the question of fairness of corporate tax policy and the question of potential distortions that may occur in the economy. These important issues are beyond the scope of this paper, but the link between the tax rates and bank leverage in our model lays a stepping stone for a welfare analysis of the benefit and cost of tax policy reforms.

5 Conclusion

Our theory of bank liability structure explicitly models an array of factors that determine the leverage and debt composition of banks. The factors include tax deductibility of debt liability, account service of deposits, bank-run risk, interaction of subordinated debt with deposits, FDIC insurance and its risk-adjusted premium, and closure of bank by charter authorities. Since our model is structural and solved analytically, it provides a convenient setting for quantifying bank capital structure and designing practical capital strategies. An important aspect of our model is the interaction between deposits and subordinated debt: a bank takes as much subordinated debt as possible to benefit from tax deduction but not as much to protect the deposits.

The literature pertinent to our theory falls under three categories: bank run, bank leverage, and capital structure theory. The bank run literature was pioneered by Diamond and Dybvig (1984), who constructed a formal model in which bank run emerges as one equilibrium. The FDIC enables the economy to avoid this equilibrium. The literature has subsequently been extended significantly by Allen and Gale (1998) and others. In our model, depositors at unregulated banks can run in order to make their claims risk-free. There is no panic risk in our full-information model as depositors observe asset value and can implement an optimal withdrawal strategy without loss. The model does not have a situation in which a fraction of the depositors withdraw deposits late and suffer losses. Nevertheless, bankruptcy cost at a bank run forces

\[25\text{Models with a few time steps and discrete asset values are common in banking studies. Those models are useful for certain conceptual issues, but not for the issues investigated in this paper or for practical applications.}\]
banks to decide the right amount of deposits to hold.\textsuperscript{26}

Our theory contributes to the literature of bank leverage. A number of papers have studied the reason for high leverage of banks, going back to Buser, Chen and Kane (1981), who conceptually discuss banks that optimize deposits in the presence of FDIC. Song and Thakor (2007) examine banks’ choice between deposits and non-deposits as financing sources. Harding, Liang and Ross (2009) set out a structural model for banks whose debt consists of only deposits. They treat banks as firms in Leland (1994) and set aside special features of banks. DeAngelo and Stulz (2013) provide a rationale for bank leverage by positing that households and firms value a hedge against liquidity shocks and are willing to pay a premium. Focusing on liquidity service of deposits, they assume that banks hold risk-free assets, issue no subordinated debt, and have no FDIC insurance. Garnall and Strebulaev (2013) posit that high leverage of banks arises from low volatility of bank assets due to diversification. To keep their model solvable, their banks pay no premium on deposit insurance and have an exogenously-given mix of deposits and long-term debt. Allen and Carletti (2013) focuses on bank holding only deposits as debt. They assume deposits are cheaper than equity as financing source because they are traded in segmented markets. Our work goes beyond these insights to theorize the endogenous decision of leverage, which combines deposits and subordinated debt optimally, for banks that face bank-run risk or have FDIC insurance.

Our model is also an extension to the structural framework of Merton (1974, 1977) and Leland (1994). Unprotected long-term debt and secured debt are considered in Leland (1994), who interprets secured debt as short-term debt. Leland considers each class of debt separately, whereas we model the endogenous choice of deposits and long-term debt simultaneously. In addition, we explicitly model deposit insurance and closure policy of charter authority. In a setting with these detailed institutional features, we derive the endogenous FDIC insurance premium, taking into account the optimal liability structure. A number of papers, notably Merton (1977), Ronn and Verma (1986) have derived the risk-adjusted FDIC insurance policies, and our work extends their insights to endogenous decision on default and closure boundary when banks optimally respond to FDIC insurance.

The theory of bank liability structure has important implications to bank regulation. Since the financial crisis of 2007–2009, regulators have decided to raise capital requirement for banks. Basel III lifts the required equity ratio from 4%, which was

\textsuperscript{26}Auh and Sundaresan (2013) consider the run of repo in a similar setting, in which repo investors have full information.
set in Basel II, to 7% for all banks and to nearly 10% for large banks that are designated as systemically important. As we have noted in footnote 1, European and U.S. regulators have laid out higher capital requirements for banks, and academics have proposed raising equity ratio to as high as 20%. Two important questions for regulators are: how will banks adjust their liability structure in response to higher equity ratio requirement, and what are the unintended consequences? It is important to know whether a value-maximizing bank cuts down deposits or subordinated debt, or both, when reducing leverage to meet a capital requirement. Cutting back deposits reduces banking service, and lowering subordinated debt shrinks bank funding source. In light of the finding in this paper, it will be interesting to know whether a capital requirement makes subordinated debt protect deposits. The theory of optimal bank liability structure is fundamental for analyzing these issues, as shown in Sundaresan and Wang (2014a).

The model of bank liability structure provides a tool for studying other securities held by or proposed to banks. Perhaps the most controversial security since the crisis of 2007–2009 is reverse convertible debt, which converts to equity when bank is not well capitalized, as proposed by Flannery (2009). Sundaresan and Wang (2010) analyze the intricate issues in designing contingent capital. Some regulators in Europe are interested, but regulators in the U.S. have held back and expressed doubt (Financial Stability Oversight Counsel, 2012). The disagreement among regulators calls for better understanding the potential role of reverse convertible debt in bank liability structure. Albul, Dwight and Tchistyi (2010) and Chen, Glasserman and Nouri (2012) add contingent capital to the capital structure of firms but not banks. Extension of our model to include reverse convertible debt as in Sundaresan and Wang (2014b), can shed light on the interactions of convertible debt with deposits, subordinated debt, bank run, and the FDIC. These interactions are critical for figuring out whether convertible debt helps stabilize banks.

Our model can be extended to a setting in which banks dynamically change the liability structure as well as its composition of assets. Goldstein et al. (2001) have developed a dynamic framework for a corporate borrower. In their model, firms with a single debt consider the opportunity of issuing additional debt when they optimize today’s capital structure. For banks, an important issue is the ability to de-lever when it becomes poorly capitalized after losses. Another important issue for banks is their

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27 Hirtle (2011a, 2011b) explains the rationale for these capital requirements.

28 Subramanian and Yang (2013) consider the question of prudential regulation in a structural model, in which firms issue only perpetual debt, as in Leland’s (1994a) model.
dynamic adjustment of liability structure in response to changes of risk. Extension of our model to dynamic setting will be a useful, although challenging, project that warrants further research.

A Appendix

Our model of banks builds on the theory of firms pioneered by Merton (1974) and extended by Leland (1994). Firms issue long-term debt but do not take short-term deposits. Banks take deposits, offer account service, pay for deposit insurance, and face regulations. The bank asset value follows a geometric Brownian motion. In the risk-neutral probability measure, it is

\[ dV = (r - \delta)Vdt + \sigma VdW. \]

(18)

where \( r \) is the risk-free interest rate, \( \delta \) is the rate of cash flow, \( \sigma \) is the volatility of asset value, and \( W \) is a Wiener process. Following Leland (1994), \( V \) denotes the after-tax value of the assets, and thus \( \delta V \) is the after-tax cash flow.

For a given bankruptcy boundary \( V_b \), consider a security that pays one dollar if and only if \( V \) hits \( V_b \) for the first time. The price of this security, denoted by \( P_b \), is referred to as the state price of \( V_b \). According to Merton (1974), \( P_b \) satisfies a differential equation:

\[ \frac{1}{2}\sigma^2V'^2P''_b + (r - \delta)V P'_b - rP_b = 0, \]

(19)

where \( P'_b \) and \( P''_b \) are the first and second partial derivatives of \( P_b \) with respect to asset value \( V \). The general solution to the equation is \( P_b = a_1V^{-\lambda} + a_2V^{-\lambda'} \), where \( \lambda > 0 \) and \( \lambda' < 0 \) are the two roots of quadratic equation

\[ \frac{1}{2}\sigma^2\lambda(1 + \lambda) - (r - \delta)\lambda - r = 0. \]

(20)

The boundary conditions are \( P_b(V_b) = 1 \) and \( \lim_{V \to \infty} P_b(V) = 0 \). The conditions imply \( a_2 = 0 \) and \( a_1 = V_b^\lambda \), which give \( P_b = (V_b/V)^\lambda \).

Equity holders earn the dividend, \( \delta V - (1 - \tau)(I + C + C_1) \), until bankruptcy, for

29 Adrian and Shin (2010) document that financial intermediaries adjust balance sheets to their forecast of risk.

30 Notice that the cash flow \( \delta V \) also follows a geometric Brownian motion with volatility \( \sigma \). One may start with the assumption that asset cash flow follows a geometric Brownian motion with volatility \( \sigma \) and then show that asset value follows process (18).

31 Alternatively, one may specify the before-tax value of the assets, \( V^* \), as in Goldstein et al. (2001). Then, \( \delta V^* \) is earnings before interests and tax (EBIT), and the after-tax asset value is \( V = (1 - \tau)V^* \). Recovery value of bankruptcy is then \( (1 - \phi)(1 - \tau)V_b^* \), which is equivalent to \( (1 - \phi)V \). All the results in this paper can be derived and presented accordingly.
given insurance premium $I$, deposit liability $C$, and subordinated debt liability $C_1$. The pricing equation of equity value before bankruptcy is

$$\frac{1}{2}\sigma^2V^2E'' + (r - \delta)VE' - rE + \delta V - (1 - \tau)(I + C + C_1) = 0,$$

(21)

where $E'$ and $E''$ are derivatives with respect to asset value $V$. We assume $I \geq 0$ in general, but setting $I = 0$ gives the valuation without FDIC. There are two boundary conditions. Since bankruptcy wipes out equity, we have $E(V_b) = 0$. If $V \to \infty$, bankruptcy is remote, and $E(V)$ approximately equals to $V - (1 - \tau)(I + C + C_1)/r$.

The pricing equation of subordinated debt $D_1$ is

$$\frac{1}{2}\sigma^2V^2D_1' + (r - \delta)VD_1' - rD_1 + C_1 = 0,$$

(22)

where $D_1'$ and $D_1''$ are the derivatives with respect to $V$. There are also two boundary conditions. Debt holder receives $[(1 - \phi)V_b - D]^+$ at bankruptcy. If $V \to \infty$, the debt almost risk-free, and $D_1$ approaches $C_1/r$.

Theorem 1 in Section 3.1 presents the solutions to equations (21) and (22) and their boundary conditions. The solutions can be derived similarly to those in Leland (1994) and Goldstein et al. (2001). Sub-section A.1 provides the details.

To simplify the derivation of optimal liability structure, we introduce the following notations:

$$x = C_1/C, \quad c = C/(rV), \quad v_a = rV_a/C, \quad v_d = rV_d/C, \quad v_b = rV_b/C$$

(23)

$$\iota = \eta/(r - \eta), \quad \theta = (1 - \tau)\lambda/(1 + \lambda).$$

(24)

We refer to $x$ as the liability ratio and $c$ as the deposit liability scaled by asset. The state price of bankruptcy is simply $P_b = (v_b c)^\lambda$. By Theorem 1, the rescaled boundaries are

$$v_a = \kappa(1 + \iota), \quad v_d = \theta(1 + \iota + x), \quad v_b = \max\{\kappa(1 + \iota), \theta(1 + \iota + x)\}.$$

(25)

Notice that $V_a < V_d$ if and only if $v_a < v_d$. Furthermore, equation (9) can be written as a function of $c$

$$i = \omega[1 - (1 - \beta)\kappa]^+(1 + \iota)(v_a c)^\lambda/[1 - (v_a c)^\lambda].$$

(26)

We can also express the bank value in Theorem 1 as a ratio to asset value:

$$f(x, c) = F/V = 1 + [1 - (v_b c)^\lambda]\iota + \tau(1 + \iota + x) - i|c$$

$$- (v_b c)^\lambda \min\{\phi v_b, v_b - (1 + \iota)\}c.$$

(27)

Choosing $(C, C_1)$ to maximize bank value $F$ is equivalent to choosing the duplet $(x, c)$ to maximize $f$. Once we obtain the optimal $(x^*, c^*)$, the optimal liabilities $(C^*, C_1^*)$ can be obtained easily as $C^* = c^*rV$ and $C_1^* = x^*C^*$. 38
A.1 Proof of Theorem 1

The general solution to pricing equation (21) is
\[ E = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + V - (1 - \tau)(I + C + C_1)/r \]  
(28)
where \( \lambda > 0 \) and \( \lambda' < 0 \) are the two solutions to equation (20), and \( a_1 \) and \( a_2 \) are arbitrary constants. The boundary conditions of \( E \) imply \( a_2 = 0 \) and \( a_1 = -[V_b - (1 - \tau)(I + C + C_1)/r]V_b^\lambda \), which give equation (3).

If \( V_b = V_a = \kappa D \), \( C = (r - \eta)D \) gives equation (1). To prove \( V_d \) in equation (2) is the endogenous default boundary, we prove \( V_b \) maximizes the equity value when \( V_b = V_d \). Partial differentiation of equation (3) with respect to \( V_b \) gives
\[ \partial E/\partial V_b = [(1 + \lambda)/V_b](V_b/V)^\lambda (V_a - V_b) \]  
(29)
Since the above is positive if \( V_b < V_d \) and negative if \( V_b > V_d \), \( V_b = V_d \) maximizes the equity value. Notice that \( V_d \) is independent of \( V \). Equity holders choose to default before closure if \( V \) drops to \( V_d \) before \( V_a \). The bank is closed before default if \( V \) drops to \( V_a \) before \( V_b \). Therefore, the bankruptcy boundary is \( V_b = \max\{V_a, V_d\} \).

The general form of solution to pricing equation (22) is
\[ D = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + C_1/r \]  
(30)
where \( a_1 \) and \( a_2 \) can be any constants. The boundary conditions of \( D_1 \) imply \( a_2 = 0 \) and \( a_1 = \{(1 - \phi)V_b - D\}^+ - C_1/r \) \( V_b^\lambda \), which give equation (4).

Bank value is \( F = D + D_1 + E \). We obtain equation (5) by substituting equations (4) and (3), and using \( D = C/(r - \eta) \).

A.2 Proof of Theorem 2

Let \( Q \) be the value of deposit insurance to banks. Its pricing equation is
\[ \frac{1}{2} \sigma^2 V^2 Q'' + (r - \delta)VQ' - rQ - I = 0 \]  
(31)
where \( Q' \) and \( Q'' \) denote the first and second partial derivatives of \( Q \) with respect to \( V \). The general solution to the equation is \( Q(V) = -I/r + a_1 V + a_2 V^{-\lambda} \), where \( a_1 \) and \( a_2 \) can be any constants. The boundary conditions of the value of the insurance product are \( \lim_{V \to \infty} Q = -I/r \) and \( Q(V_a) = [D - (1 - \beta)V_a]^+ \), where \( V_a = \kappa D \). They imply \( a_1 = 0 \) and \( -I/r + a_2 V_a^{-\lambda} = [D - (1 - \beta)V_a]^+ \), which give \( Q(V) = -(1 - P_a)I/r + [D - (1 - \beta)V_a]^+ P_a \), where \( P_a = [V_a/V]^\lambda \). The insurance premium \( I^o \) is fair iff \( Q(V) = 0 \). It follows that \( (1 - P_a)I^o = r[D - (1 - \beta)V_a]^+ P_a \). Then, we obtain equation (6) by substituting \( V_a = \kappa D \) and factoring \( D \) out.
A.3 Proof of Theorem 3

It follows from $i = 0$, $\phi = \alpha$, and $\kappa \geq 1/(1 - \alpha)$ that $v_b = \max\{(1 + \iota)\kappa, \theta(1 + x)\}$. This last equation implies $\omega v_b \leq v_b - (1 + \iota)$, which simplifies equation (27) to

$$f = 1 + \left\{i + \tau(1 + x) - (v_b c)^\lambda[v + \tau(1 + x) + \alpha v_b]\right\}c$$

(32)

The first-order condition for $c$ to be optimal is

$$i + \tau(1 + x) - (1 + \lambda)[v + \tau(1 + x) + \alpha v_b](v_b c)^\lambda = 0.$$  (33)

Let $x^* = (1 + \iota)\kappa/\theta - 1$. Notice that $v_b = \theta(1 + x)$ if $x \geq x^*$, and $v_b = (1 + \iota)\kappa$ otherwise. If $x < x^*$, then $f'_x(x, c) = \tau[1 - (v_b c)^\lambda]c > 0$, which means $f$ increases with $x$. If $x > x^*$, then

$$f'_x(x, c) = \left\{\tau - \left[\lambda i/(1 + x) + (1 + \lambda)(\tau + \alpha \theta)\right](v_b c)^\lambda\right\}c.$$  (34)

Let $c_x$ be the $c$ that satisfies condition (33) for given $x > x^*$. Imposing the condition in equation (34) gives

$$f'_x(x, c_x) = -\frac{i}{1 + x}[1 - (v_b c_x)^\lambda]c_x < 0.$$  (35)

Thus, $f$ decreases with $x$ for $x > x^*$ if $c$ is always kept optimal relative to $x$.

Therefore, $x^* = (1 + \iota)\kappa/\theta - 1$ is the optimal point for $x$. At the optimal point, $v_b^* = v_a^* = v_a = (1 + \iota)\kappa$. We can solve $(v_b^* c^*)^\lambda$ from equation (33). Substituting out $1 + x^* = (1 + \iota)/\theta$ and $v_b^* = (1 + \iota)\kappa$, we obtain $(v_b^* c^*)^\lambda = \pi$, where $\pi$

$$\pi = \frac{1}{1 + \lambda} \cdot \frac{i \theta + \tau(1 + \iota)\kappa}{i \theta + \tau(1 + \iota)\kappa + (1 + \iota)\kappa \alpha \theta},$$

(36)

From $(v_b^* c^*)^\lambda = \pi$, we obtain $c^* = \pi^{1/\lambda}/[(1 + \iota)\kappa]$. We obtain equations (12)–(13) after replacing $i$, $\theta$, and scaled variables by the original parameters and variable. The proof is complete.

A.4 Proof of Theorem 4

We first show that $v_d < v_a$ is not optimal. If $v_d < v_a$, then $\theta(1 + i + x) < (1 + \iota)\kappa$, $v_b = v_a = (1 + \iota)\kappa$, and

$$f(x, c) = 1 + c\left\{[i - \tau(1 + i + x)][1 - (v_a c)^\lambda] - (\kappa - 1)(1 + \iota)(v_a c)^\lambda\right\}.$$  (37)

We then obtain $f'_x(x, c) = \tau[1 - (v_a c)^\lambda]c > 0$, which implies the current $x$ is not optimal.

It follows from equation (26) that $i'_c = \partial i/\partial c = \lambda i c^{-1}/[1 - (v_a c)^\lambda]$. Since $\kappa < 1/(1 - \beta)$, both $i$ and $i'_c$ are positive. We also have $ci'_c[1 - (v_a c)^\lambda] \leq \lambda i$ because $v_b \geq v_a$, with equality to hold when $v_b = v_a$. Both $i$ and $i'_c$ converge to zero when
\( \kappa \) rises to \( 1/(1 - \beta) \) but other parameters and variables are fixed. Thus, given any \( v > v_a \), there exists \( \kappa^* \in [1, 1/(1 - \beta)] \) such that \( \kappa \in (\kappa^*, 1/(1 - \beta)) \) implies \( i + c_i'c < \iota \) for all \( c \) in \([0, 1/v]\).

If \( v_d > v_a \), we have \( \theta(1 + i + x) > (1 + \iota)\kappa \), \( v_b = \theta(1 + i + x) \), and \( \phi = \alpha \). Notice that

\[
\min \{ \phi v_b, v_b - (1 + \iota) \} = \begin{cases} 
    v_b - (1 + \iota) & \text{if } v_a < v_b \leq (1 + \iota)/(1 - \alpha) \\
    \alpha v_b & \text{if } v_a < v_b > (1 + \iota)/(1 - \alpha).
\end{cases}
\]

If \( v_a < v_d \leq (1 + \iota)/(1 - \alpha) \), equations (38) and (27) give

\[
f'_x(x, c) = c \left[ \tau - (\tau + \lambda)(v_d c)^\lambda + \lambda(v_d c)^\lambda \frac{1 + i}{1 + i + x} \right] \]

\[
f'_c(x, c) = 1 + \iota - (1 + i + c_i'c) [1 - (v_d c)^\lambda] \]

\[
+ (1 + i + x + c_i'c) \left\{ \tau - (\tau + \lambda)(v_d c)^\lambda + \lambda(v_d c)^\lambda \frac{1 + i}{1 + i + x} \right\}. \tag{40}
\]

Let \( c_x \) be the optimal \( c \) for given \( x \), then equation (40) implies

\[
\tau - (\tau + \lambda)(v_d c_x)^\lambda + \lambda(v_d c_x)^\lambda \frac{1 + i}{1 + i + x} = -\frac{1 + \iota - (1 + i + c_x i'_c)[1 - (v_d c_x)^\lambda]}{1 + i + x + c_x i'_c}. \tag{41}
\]

For \( \kappa \in (\kappa^*, 1/(1 - \beta)) \), we have \( i + c_x i'_c < \iota \), which implies that the numerator is positive. Substitution of the above into equation (39) shows \( f'_x(x, c_x) < 0 \). Thus, \( v_a < v_d < (1 + \iota)/(1 - \alpha) \) is not optimal because lowering \( x \) and \( v_d \) increases \( f(x, c_x) \).

If \( v_b \geq (1 + \iota)/(1 - \alpha) \), equation (38) and (27) give

\[
f'_x(x, c) = c \left\{ \tau - [(1 + \lambda)(\tau + \alpha \theta) + \lambda(\iota - i)/(1 + i + x)](v_d c)^\lambda \right\}. \tag{42}
\]

\[
f'_c(x, c) = (\iota - i - c_i'c)[1 - (v_d c)^\lambda] \]

\[
+ (1 + i + c_i'c + x) \cdot \left\{ \tau - [(1 + \lambda)(\tau + \alpha \theta) + \frac{\lambda(\iota - i)}{1 + i + x}](v_d c)^\lambda \right\}. \tag{43}
\]

Let \( c_x \) be the optimal \( c \) relative to \( x \). Then, equation (43) implies

\[
\tau - (1 + \lambda)(\tau + \alpha \theta) + \frac{\lambda(\iota - i)}{1 + i + x} = -\frac{[\iota - i - c_x i'_c][1 - (v_d c_x)^\lambda]}{1 + i + x + c_x i'_c}. \tag{44}
\]

For \( \kappa > \kappa^* \), we have \( i + c_i'c \leq \iota \). Then, the numerator is positive. Substitution of the above into equation (42) shows \( f'_x(x, c_x) < 0 \), which means the current \( x \) is not optimal because lowering \( x \) increases \( f(x, c_x) \).

The above two cases show that there exists \( \kappa^* \) such that for \( \kappa^* < \kappa < 1/(1 - \beta) \), we have \( f'_x < 0 \) for \( x \) satisfying \( \theta(1 + i + x) > (1 + \iota)\kappa \), if \( c \) is kept to be optimal relative to \( x \). Therefore, \( \theta(1 + i + x) > (1 + \iota)\kappa \) cannot be optimal because reducing \( x \) adds value to the bank. Consequently, the optimal \( x^* \) and \( c^* \) must satisfy \( \theta(1 + i + x^*) = (1 + \iota)\kappa \), which implies \( v^*_d = (1 + \iota)\kappa \) and thus \( v^*_b = v^*_d = v_a \).

With \( v^*_a = v^*_d = v^*_b \), the state price of bankruptcy is: \( \pi = (v^*_a c^*)^\lambda = (v^*_b c^*)^\lambda \). This
equation implies \( v_a^* = \pi^{1/\lambda}/c^* \). In view of equation (25), we have \((1 + \iota)\kappa = \pi^{1/\lambda}/c^*\).

It follows that \( c^* = \pi^{1/\lambda}/[(1 + \iota)\kappa] \), \( i^* = (1 + \iota)[1 - (1 - \beta)\kappa]^+\pi/(1 - \pi) \), and \( x^* = (1 + \iota)\{\kappa/\theta - \omega[1 - (1 - \beta)\kappa]^+\pi/(1 - \pi)\} - 1 \).

Let \( x_c = v_a/\theta - (1 + \iota) \) for any \( c \in [1, 1/v_a] \) and \( g(c) = f(x_c, c) \). It follows from equation (37) that

\[
g(c) = 1 + c\{[\iota - \iota + \tau v_a/\theta][1 - (v_a c)^\lambda] - (\kappa - 1)(1 + \iota)(v_a c)^\lambda\}.
\]

This function is differentiable in \( c \), and

\[
g'(c) = [\iota - \iota + \tau v_a/\theta][1 - (1 + \lambda)(v_a c)^\lambda]
- (\kappa - 1)(1 + \iota)(1 + \lambda)(v_a c)^\lambda - c\iota'[1 - (v_a c)^\lambda].
\]

With equation (26) and the formula of \( i'_c \), the above simplifies to

\[
g'(c) = \iota + \tau v_a/\theta - \{\iota + \tau v_a/\theta + (\kappa - 1)(1 + \iota)
- \omega[1 - (1 - \beta)\kappa]^+(1 + \iota)[1 + \lambda](v_a c)^\lambda
+ (1 + \iota)\{\kappa - 1 + \omega[1 - (1 - \beta)\kappa]^+\}\theta\}.
\]

If \((x^*, c^*)\) is maximum, \( c^* \) must maximizes \( g(c) \), and thus \( g'(c^*) = 0 \). Letting \( \pi = (v_a c^*)^\lambda \) and setting equation (47) to zero, we obtain

\[
\pi = \frac{1}{1 + \lambda} \cdot \frac{i\theta + \tau(1 + \iota)\kappa}{i\theta + \tau(1 + \iota)\kappa + (1 + \iota)\{\kappa - 1 + \omega[1 - (1 - \beta)\kappa]^+\}\theta}.
\]

Finally, we complete the proof by substituting the original parameters into (48) and the original variables into the formulas for \( c^* \), \( i^* \), and \( x^* \).

References


DeAngelo, H., and R. Stulz, 2013, Why high leverage is optimal for banks, Working paper, University of Southern California.


Gornall, W., and I. Strebulaev, 2013, Financing as a supply chain: the capital struc-


Hirtle, B., 2011a, How were the Basel 3 minimum capital requirements calibrated?, Liberty Street Economics, Federal Reserve Bank of New York.


Schandlbauer, A., 2013, How do financial institutions react to a tax increase, Working paper, Vienna Graduate School of Finance.


