

# Risk Management Framework for Hedge Funds

## Role of Funding and Redemption Options on Leverage

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### Abstract

We develop a model of hedge fund returns, which reflect the contractual relationships between a hedge fund, its investors and its prime brokers. These relationships are modelled as short option positions held by the hedge fund, wherein the “*funding option*” reflects the short option position with prime brokers and the “*redemption option*” reflects the short option position with the investors. Given an alpha producing human capital, the hedge fund’s ability to deploy leverage is shown to be sharply constrained by the presence of these short options. We show that the hedge funds typically have an optimal level of leverage. Optimal leverage is shown to differ across hedge funds reflecting their de-levering costs, Sharpe ratios, correlation of assets, secondary market liquidity of their assets, and the volatility of the assets. Using a minimum level of unencumbered cash level as a risk limit, we show how a hedge fund can optimally allocate its risk capital across different risk-taking units to maximize alpha in the presence of these short option positions. Implications of our analysis for hedge fund investors and policy makers are summarized.

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# 1 Introduction and Overview

Risk management issues relating to Hedge funds, which received increased scrutiny following the collapse of Long Term Capital Management (LTCM) in 1998, have once again been receiving renewed attention following the onset of credit crisis in mid-2007. This increased focus on risk management of hedge funds has arisen from two considerations: first, hedge funds themselves have come to more fully appreciate the risks associated with funding by prime brokers and investor redemptions. Second, the fact that hedge funds are counterparties to prime brokers, who are often international banks engaged in corporate and consumer lending has raised the spectre of propagation of systemic risk through hedge funds. The potential for hedge funds to transmit systemic risk (through their deleveraging processes) to the banking system has become a matter of concern as evidenced by Kambhu, Schuermann, and Stiroh (2007), Hildebrand (2007), Lo (2008) and Papademos (2007). President's working group (1999) has also emphasized this aspect.

In 2008 alone a record number of hedge funds have failed and the market positions of the hedge funds industry have shrunk dramatically. Lo (2008) reports that the estimated assets in the hedge fund industry grew from \$38 billion in 1990 to \$1.87 trillion in 2007. The estimated assets in the last quarter of 2008 stood at \$1.60 trillion. The market positions of hedge funds fell from \$5.23 trillion in 2007 to \$3.68 trillion as of the last quarter of 2008. This drop in market positions amounting to a little over \$1.5 trillion is the reduction in the overall leverage in the hedge fund industry, which is a result of extensive voluntary and involuntary de-leveraging undertaken by many hedge funds. Several important risk management lessons have emerged from the manner in which the credit crisis has impacted the hedge fund business. Two of these deserve special mention and form the focus of our study. First, hedge funds have realized that the prime brokers and counterparties can either significantly increase margin requirements and/or potentially withdraw their credit lines in periods of crisis<sup>1</sup>. Such actions dramatically increase the funding costs of hedge funds and in some cases impair their

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<sup>1</sup>For ease of exposition, we will refer to all counterparties that have a funding/margining relationship as PBs for the rest of the paper.

ability to maintain (potentially profitable) risky positions. This risk then forces hedge funds to de-lever in bad states of the world thereby imposing losses, and threatening their survival. Second, hedge fund investors, who become liquidity-constrained in a credit crisis, tend to withdraw their capital under precisely the same circumstances thereby increasing the risk of large-scale redemptions. These two sources of risk may be relevant even to a hedge fund that has been performing well by any benchmark of performance prior to the onset of (or even during) the credit crisis.

## 1.1 Funding and Redemption Options

We can think of these two risk factors in options parlance as the hedge fund being short in two types of very valuable options:

*The ability and the willingness of prime brokers to withdraw credit lines in bad states of the world is equivalent to the hedge fund being short an option to reduce leverage in bad states of the world. By virtue of this short option position, the fund agrees or commits to reduce leverage in bad states of the world.*

*The willingness of investors to redeem their partnership shares in bad states of the world is equivalent to the hedge fund being short in redemption option, which obliges the fund to agree to provide its investors liquidity precisely when it is needed most for the fund to protect its continuing investors and enhance its chances of survival.*

We will refer to the option held by the prime brokers as “*funding option*” and the option held by the investors as “*redemption option*”. Figure 1 highlights the nature of these contractual agreements that a typical hedge fund will have with its funding counterparties (*PBs*) and its investors. To focus attention on our principal questions, we abstract from other contractual issues, including the performance and management fees that the fund manager negotiates with investors.

These options operate through different channels. The funding options (implicitly) sold to prime brokers are exercised through increased margins and/or reduced credit lines in bad states of the world, which can lead to involuntary de-leveraging if the fund has not placed

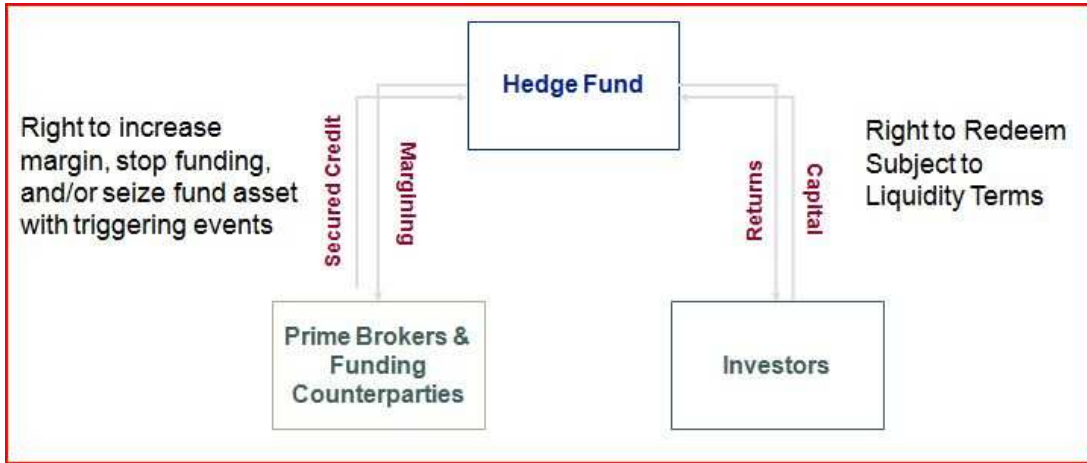


Figure 1: Hedge Fund's Contractual Short Options Positions

risk limits anticipating such a possibility. Table 1 illustrates vividly the manner in which the haircuts were increased against different classes of collateral in August 2008<sup>2</sup>. In the case of certain asset classes such as CDOs, prime brokers simply refused to accept them. In such instances as well as in those where the margin got multiplied by a factor of more than 10 (as in ABS in Table 1, for example), hedge funds found themselves in need of de-levering, often involuntarily.

If the cost of involuntary de-leveraging is very high, the funds may find themselves in a downward spiral threatening their survival. Moreover, prime brokers may also specify a *NAV* (net asset value) trigger for periodic (yearly, for example) declines below which they may terminate funding<sup>3</sup>. The funding option is especially potent, given the fact that most hedge funds (unlike banks) do not have access to equity or other capital markets for financing. Unlike banks, hedge funds cannot count on central bank facilities for emergency funding either. On the other hand, the redemption options operate via reduction of assets under management (*AUM*) which can lead to some or all risk limits to become binding. A flurry of major redemptions or draw-downs may cause the hedge fund to breach the *NAV* trigger

<sup>2</sup>Source: Global Financial Stability Report, IMF, October 2008.

<sup>3</sup>The *NAV* trigger tends to be much lower than investors redemption trigger and is less likely to be normally activated than investors redemption trigger.

Table 1: **Initial Margins (Haircuts) Imposed by Prime Brokers in %**

Type of Collateral	April 2007	August 2008
U.S. Treasuries	0.25%	3.0%
Investment-grade bonds	0-3%	8%-12%
High-yield bonds	10-15%	25% to 40%
Investment grade		
Corporate CDS	1%	5%
Senior Leveraged Loans	10%-12%	15%-20%
Mezzanine Leveraged Loans	18%-25%	35%
ABS CDO:		
AAA	2%-4%	95%
AA	4%-7%	95%
A	8%-15%	95%
BBB	10%-20%	95%
Equity	50%	100%
AAA CLO	4%	10%-20%
Prime MBS	2%-4%	10%-20%
ABS	3%-5%	50%-60%

with its prime brokers. A deluge of redemptions forces the hedge fund into involuntary de-leveraging, with significant impact on realized returns for both exiting and remaining investors. Despite the presence of gates, lockup periods, and notice periods, the hedge fund industry returned nearly \$400 billion of capital in 2008 to meet the requests for redemptions by investors<sup>4</sup>. In the year 2008, both poor performance and investor redemptions contributed to the massive declines in assets under management<sup>5</sup>.

Since both options are likely to be triggered in the states of world where the cost of de-leveraging is high, they can have a significant impact on the hedge fund performance. It follows that a proper evaluation of the expected hedge fund return should take into account of the likelihood that these options may be triggered and the likely impact on the funds performance in the event that they are triggered. *One of the main points that we wish to get across (to be elaborated in much greater detail later) is the fact that the expected return of a hedge fund, as well as the associated risk, depends not only on the ex ante evaluation of the hedge fund portfolio strategies per se, but also on how effectively the hedge fund manages the funding and redemption options.*

Prudent risk management practice at hedge funds requires first and foremost that these options are formally recognized and correctly understood by the managers, transparently communicated to all relevant counterparties, properly priced ex-ante, and actively managed ex-post. The fact that a hedge fund is short in these options informs on the level of leverage it should deploy under normal circumstances and how it should manage leverage over time as states of the world move from “normal states” to “abnormal states”. Finally, since these options tend to get exercised in bad states of the world, worst case losses must be estimated through appropriate stress scenario analysis, and incorporated ex ante in risk budgeting<sup>6</sup>.

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<sup>4</sup>Hedge Fund Liquidations, New York Law Journal, March 2, 2009.

<sup>5</sup>Since 2009, the hedge fund industry has recovered and started to see capital inflows.

<sup>6</sup>Normally stress tests tend to focus on extreme scenarios of market and counterparty credit risk. Such scenarios may also trigger the funding and redemption options. Unanticipated surges in future volatilities are not easy to model, and hence stress-scenario analysis are a way in which one can get a better understanding of how the potential exercise of these options should limit current risk-taking.

Our paper will focus on these important dimensions of risk management.

Hedge funds with good risk management practices attempt to deal with the funding option by having an ongoing and healthy (i.e., open and transparent) relationship with multiple prime brokers, and through these relationships building ample excess capacity in available credit lines. The availability of excess funding capacity effectively makes the funding option farther out of the money, which in turn provides reassurances to credit committees of relevant prime brokers. Having multiple prime brokers not only reduces the chance that the funding option will be exercised (it is much less likely that multiple prime brokers will pull credit lines at the same time unless it is performance-driven), but also diversifies and therefore reduces counterparty risk<sup>7</sup>. An “ever-green” facility is clearly desirable, but the price is usually prohibitive.

The redemption option held by the investors is usually dealt with through carefully articulated and investor-approved contractual provisions such as a reasonably long redemption cycle (say quarterly) and with a reasonable notification period (say 45 to 90 days), lock-up periods (can be hard or soft), early redemption penalties, investor-level or fund-level gates, etc. These contractual provisions are typically proposed to the investors and approved by the investors, ex-ante, so that all investors understand that liquidity in bad states of the world may only come at a price (effectively paid to those investors who are more patient and providing better liquidity to the fund). In designing such contracts, hedge funds must protect the interests of “long-term” investors, but agree to provide liquidity to short-term investors in bad states at a fair price. Designing different share classes is yet another way to address this short option position. Avoiding the co-mingled positions by short-term and long-term investors through contractual provisions helps to manage the liquidity profile better. The generation of alpha might require a minimum investment time horizon, and hence

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<sup>7</sup>Hildebrand (2007) argues for the need of the prime broker to have a complete risk metric of each hedge fund that the prime broker is exposed to. In fact he argues that the prime broker should be aware of the margining terms agreed by their hedge fund clients with other counterparties and clients. Hildebrand ignores the potential for prime brokers to engage in predatory behaviour. A fund employing multiple *PBs* may also have to work harder to achieve a higher degree of margining efficiency

there is a need to match this time horizon with the desire of the investors to have access to liquidity at frequent intervals. One of the purposes of the contractual provisions discussed is to try and minimize the gap between investment time horizon and the redemption cycles desired by investors. The main thrust of our paper is the manner in which these options influence optimal leverage, risk budgeting, and the active management of hedge fund risk. We examine the questions primarily from the perspective of a hedge fund risk manager. The framework, however, should be of interest to hedge fund investors and regulators of financial markets for reasons that we discuss later in the paper.

## 1.2 Placing the paper in the hedge fund literature

Academic research on hedge funds is extensive. Lo (2008) in his written testimony to the United States Congress provides a detailed treatment of hedge funds in terms of their potential contribution to exacerbating systemic risk in the economy<sup>8</sup>. Our papers contribution in this context is to show that funds may use conservative levels of leverage if they properly recognize the short option positions that are implied by their contractual arrangements with investors and prime brokers. The current crisis might have served to sharpen their focus on these options. One strand of literature has been focussed on the presence of nonlinearities in hedge fund returns. Some of the papers that have addressed this question include, Fung and Hsieh (1997a), Agarwal and Naik (2004), and Brunnermeier and Nagel (2004). Cheny, Getmansky, Shane and Lo (2005) proposes a specification in which there can be phase-locking behaviour in hedge fund returns, when with a small probability all hedge fund returns become exposed to common market-wide factors. Our paper identifies important sources of nonlinearities that are inherent in the way in which hedge funds contract with their investors and their funding counterparties. This is very distinct from the nonlinearities that arise from the portfolio strategies followed by hedge funds, which has been the focus of

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<sup>8</sup>Lo (2008) also contains a detailed list of papers and books that deal with systemic risk related issues pertaining to hedge funds.



these papers<sup>9</sup>. In fact, our paper suggests that nonlinearities can arise in hedge fund returns, due to the rational behaviour of hedge funds in managing their short option positions even if their portfolio strategies did not involve option-like positions. Another strand of literature has examined the presence of survivorship bias, selection bias, and back-filling bias in hedge funds databases. They include papers by Brown, Goetzmann, Ibbotson, and Ross (1992, 1997), Fung and Hsieh (1997b, 2000), and Brown, Goetzmann, and Ibbotson (1999). This is an important empirical question and does not pertain directly to our paper. The role of managerial contracts, high-water marks, lockups and gates have been addressed by Goetzmann, Ingersoll, and Ross (2003), Stavros, and Westerfield (2009), Brown, Goetzmann, Park (2001). Such performance contracts and high water marks may induce the hedge fund to alter endogenously its risk-taking behaviour<sup>10</sup>. Brown, Goetzmann, Park (2001) argue that career concerns may moderate excessive risk-taking even in the presence of such contracts. Ang and Bollen (2009) have examined the role of lockups assuming exogenous arrival rates of failures. The presence of lockups and gates will serve to lower the value of redemption options, *ceteris paribus*. Aragon (2004) uses monthly data to document a positive, concave relationship between a funds excess returns and its redemption notice period and minimum investment size. Hombert and Thesmar (2009) show empirically that funds with lockups outperform funds with no lockups, conditional on past bad performance. We can, in principle, model the gates and lockups by treating the redemption option as Bermudian, with restricted set of exercise dates and restricted sequential exercise. In our model, there is a parameter that allows us to examine the role of gates on hedge fund risk-return trade-offs: in the presence of gates, *AUM* grows relatively faster, on average, and the manager can use the unencumbered cash (which is a fraction of the *AUM*) as a risk management tool to choose the optimal leverage level. This in turn will determine endogenously the likelihood

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<sup>9</sup>Astute hedge funds often incorporate macro-hedging strategies, which often involve long positions in option-like instruments such as credit default swaps. Such long positions have the effect of at least partially offsetting the inherent short positions held by the hedge fund. A macro credit crisis that has the effect of pushing funding option in the money will also increase CDS spreads, for example.

<sup>10</sup>Such potential endogenous changes in risk-taking will be relevant to the questions that we address.

of liquidation. The relationship between gates and optimal leverage will be examined in the paper. Duffie, Wang and Wang (2008) come closest to the spirit of our paper in that they study the optimal use of leverage by a fund which trades off the costs of adjusting leverage with expected benefits as captured by the present values of the fees earned. Their paper focuses on conditions under which a constant proportion of assets under management are an incentive compatible fee structure and examines the implications of regulations on leverage. In contrast, our paper focuses on how short option positions held by prime brokers and investors influence the leverage decision and the allocation of risk capital across different units within a hedge fund.

### 1.3 Roadmap and a summary of results

In section 2 of the paper, we will lay out a hedge fund model which characterizes the short options with prime brokers and investors and study the interplay between leverage and these options. Section 3 works out the optimal leverage (risk capital) that a hedge fund should employ in order to maximize alpha<sup>11</sup>. This optimization problem facing the hedge fund explicitly accounts for short option position arising from its contractual commitments. Section 4 uses the framework developed in section 2 and the results in section 3 to characterize the distance to default or liquidation of a hedge fund. In section 5, we motivate the role of unencumbered cash level in managing the insolvency risk, and how that can be used as a risk management tool. The link between counterparty exposure, margining efficiency and unencumbered cash levels is discussed in this section. We formulate the risk budgeting problem when a the risk manager of the fund assigns risk capital to many risk-taking units within a hedge fund by maximizing the overall fund alpha subject to a) short option positions, and b) aggregate risk constraints specified through a restriction on the minimum level of unencumbered cash that the fund must always hold to mitigate the risk of insolvency. Section 6 outlines some implications for hedge fund investors and policy makers. Section 7 concludes.

Our main results cut to two important questions. First, we show that the hedge fund as

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<sup>11</sup>Risk capital is defined as the sum of all volatility adjusted leverage ratios.

a whole has an interior optimal level of aggregate risk capital, which is derived as follows. For each level of aggregate risk capital, which in the context of our model is the sum of the products of leverage and volatility of each risk-taking unit (“desks”) within the fund, we maximize the overall alpha of the fund to determine the risk-capital allocation. As the aggregate risk-capital level increases from zero, the optimal alpha increases until the effects of the short option positions drive the optimal alpha down. This allows us to uniquely identify the optimal level of aggregate risk-capital. Second, we compute the distance to default at the optimal leverage for a hedge fund and show that the distance to default is solely determined by Sharpe ratios, correlations, and de-leveraging costs. We show that each fund has a unique distance to default at its optimum level of risk-capital. In particular, it is no longer the case that the relationship between the excess return and risk is linear. This is due to the fact that at high enough leverage and volatility the short options go deep in the money and recognizing this, the desks are allocated less risk capital, ex-ante. Finally we show that the level of unencumbered cash held by the fund may serve as an important risk management tool in protecting the fund against sudden surges in volatility. In this context, we show how the Sharpe ratios of each risk-taking unit, the correlation of excess returns across risk-taking units, short option positions, and the de-leveraging costs influence the level and the allocation of risk capital when a risk limit on unencumbered cash level has to be met.

## 2 A Simple Model of Hedge Fund Returns

To lay the ground work, we consider a generic return generating process in the absence of any leverage:

$$R_{t+1} = \alpha + \sigma\epsilon_{t+1} \tag{1}$$

This return generating process assumes a market-neutral stance so that there are no

market-wide risk factors that are part of the return generating process<sup>12</sup>. We can therefore interpret  $\alpha$  as the excess return that can be earned by the hedge fund in the absence of any leverage (or a real-money investor for that matter). To make the problem non-trivial, we can assume that  $\alpha = S \times \sigma$ , where the Sharpe ratio  $S > 0$  so that it can be properly considered as a pure alpha strategy. The ability to identify such investment opportunities is the pre-requisite of a good hedge fund manager<sup>13</sup>.

We take for granted that the manner in which a hedge fund deploys the alpha strategy for investors will necessarily involve some degree of leverage. This is because in its pure form, an alpha strategy is cash-neutral or self-financing. The native level of leverage, conventionally defined as the notional size of the risky position divided by the initial cash outlay, is not infinite only because, in practice, a hedge fund must post initial margin to its *PBs* (more generally any clearing and/or trading counterparties) in order to mitigate its credit exposure to the *PBs*<sup>14</sup>. The overall leverage level of a hedge fund (henceforth  $L$ , defined as the notional size of the risky position as a multiple of the *AUM*) will be lower if the risk margin posted is only a small portion of the overall *AUM*.

The presence of the funding and redemption options means that, in general, the expected hedge fund return may be linear in  $L$  only if  $L$  is sufficiently low. Heuristically, the higher is the leverage, the more volatile is the levered return, and the more likely that the funding and redemption options are in the money. Since the hedge fund is short the options, its expected return will be negatively affected by the expected impact (in the form of expected de-leveraging costs) in the event that one or both options are exercised. At sufficiently (or excessively) high leverage levels, the nonlinearity associated with the funding and redemption

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<sup>12</sup>For ease of exposition, the return generating process abstract away from specific details of different styles in hedge fund strategy, such as the conditional directionality of a macro strategy or a CTA strategy. Such extensions should be immediate but distract away from the central themes. We also assume that this alpha generating process is fully scalable.

<sup>13</sup>Our paper therefore does not speak to hedge fund managers who generate return either by leveraging beta or by writing options.

<sup>14</sup>In addition, the fund may set aside additional cash to meet variation margin calls resulting from daily marking to market.

options can indeed dominate. We formalize this intuition in section 3. It is clear that given any alpha strategy, a hedge fund deploying such a strategy can be either a good or a bad proposition for investors. It can be a good proposition if the leverage level is appropriate and prudent risk limits are applied. It can be a bad proposition if the leverage level is excessive. Is there such a thing as an optimal leverage level? How can the leverage level be objectively measured, and judiciously managed? What is the manner in which the overall risk-capital should be allocated across different risk-taking units within a fund? We will address these questions below.

To formally address these questions, we consider a hedge fund manager who aims at deploying the strategy characterized by (1) cognizant of the funds short option positions with *PBs* and investors. For modeling purposes, we will assume that the *PBs* will shut down the fund when its returns fall below a certain threshold called the “funding trigger”. Likewise, we will assume that the investors will redeem when the returns fall below a certain threshold called the “redemption trigger”. The trigger is denoted by  $\underline{R}$ , which is a threshold level of negative returns at which either the funding option or the redemption option is exercised and the fund is liquidated. For simplicity, we will treat these triggers as being exogenous and solely determined by the performance of the fund. It should be clear that the triggers depend on contractual terms: for example, we may expect that the funding triggers to depend on hair cuts, and the redemption triggers to depend on the parameters of liquidity profile such as gates, lock-up periods, notice periods, etc. Clearly, these interesting issues warrant an independent study, in our opinion.

### **3 Optimal Level of Risk Capital (Leverage)**

We will derive in this section the optimal level of risk capital that the hedge fund will seek to deploy. Traditional approach to portfolio (risk) selection when the portfolio manager has no short option positions is to solve a volatility or *VaR* minimization problem subject to an expected return objective. In the new framework that we have proposed, it is necessary

to solve an optimization problem in which the expected return is maximized subject to the fact that the short option positions may be exercised with a high probability when the risk capital (leverage) is high.

In order to derive analytical results, we make the following simplifying assumptions. First, we consider a simple one period setting with two dates, used in mean-variance portfolio optimization models. We can regard the one period to be one year to aid intuition. The fund must choose its optimal risk capital at date 0, knowing that the short options may be exercised by either investors or prime brokers before date 1. We rewrite the generic return generating process (1) to reflect that there are many ( $N$ ) portfolio managers (risk-taking units) each of whom has an alpha generating mechanism. It is by aggregating across all those processes that we get the overall return generating process for the hedge fund (excluding the short option positions). To do this, we define the  $(N \times 1)$  vector of Sharpe ratios of all risk-taking units (“desks”) as follows:  $\underline{S} = \{S_i, i = 1, 2..N\}$

Similarly, we denote the  $(N \times 1)$  vector of the volatilities of different desks as follows:  $\underline{\sigma} = \{\sigma_i, i = 1, 2..N\}$ . Finally, we denote by  $\rho$  the  $(N \times N)$  matrix of correlation coefficients across different desks.

With these notations in place, we can write down the overall return generating process of the fund as follows<sup>15</sup>.

$$R_{t+1} = \underline{S} \circ \underline{\sigma} + \underline{\sigma} \circ \underline{\epsilon}_{t+1} \quad (2)$$

In equation (2), we have specified the return generating process with no funding or redemption options or leverage. We introduce these important elements now. Let  $\underline{L}$  be the vector of leverage employed by each desk. It is useful to think of risk-capital of each desk as the product of its leverage and volatility. Then the  $(N \times 1)$  vector of risk capital as  $x = \underline{L} \star \underline{\sigma}$  each element of which contains the product of leverage and volatility of a risk-taking unit.

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<sup>15</sup>In equation (2)  $\underline{S} \circ \underline{\sigma}$  denotes  $\sum_{i=1}^N S_i \sigma_i$ . Likewise,  $\underline{\sigma} \circ \underline{\epsilon}_{t+1}$  denotes  $\sum_{i=1}^N \sigma_i \epsilon_{i,t+1}$ .

Then the return generating process of the hedge fund may be written as follows<sup>16</sup>.

$$h_{t+1} = x' \underline{S} + x' \underline{\epsilon}_{t+1} - a \sqrt{x' \rho x} 1_{\underline{L} \circ \underline{R}_{t+1} \leq \underline{R}} \quad (3)$$

The return process for the fund is obtained by aggregating across all risk-taking units, taking into account the following features: first, each risk-taking unit of the fund must be assigned some risk capital  $x = \underline{L} \star \underline{\sigma}$  that they should deploy. Second, in making this decision, the fund must take into account the extent to which their risk capital will increase/decrease the likelihood that the short position in the option will be exercised. Third, different risk-taking units may subject the firm to de-leveraging costs if and when the short option position is exercised. This is reflected through the term  $a \sqrt{x' \rho x}$ . This term shows that the contribution of all risk-taking units to the overall de-leveraging costs of the firm is related to funding liquidity as measured by the leverage factor, the secondary market liquidity as measured by the volatility factor, and the macroeconomic circumstances as reflected by the parameter  $a$  and the correlation matrix  $\rho$ . In a period of aggregate liquidity shock, the parameter  $a$  will increase and  $\rho$  tends to a diagonal matrix of unities. Finally, the cross-correlation benefits of different risk-taking units engaging in different (and hopefully less-correlated) risk-taking activities under normal market conditions must be reflected in the decentralized decisions.

The optimality problem can be presented in the classical portfolio optimization framework with the explicit recognition of the short option position as follows. Taking expectations of equation (3) we get,

$$E(h_{t+1}) \equiv g(x' \underline{S}, x' \rho x) \equiv g(y, z) = y - a \sqrt{z} \times \left\{ N \left[ \frac{\underline{R} - y}{\sqrt{z}} \right] + e^{\frac{2y\underline{R}}{z}} N \left[ \frac{\underline{R} + y}{\sqrt{z}} \right] \right\} \quad (4)$$

In equation (4) we have used  $y = x' \underline{S}$  to denote the overall alpha of the fund, and  $z = x' \rho x$  as the aggregate risk-capital of the fund. In deriving Equation (4), we have made a few additional assumptions. First, we have assumed that the volatilities are constant. Second, we are treating Equation (4) as the discrete-time representation of a continuous-time return process and the option is triggered whenever the underlying continuous-time

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<sup>16</sup>In equation (3),  $1_{\underline{L} \circ \underline{R}_{t+1} \leq \underline{R}}$  is an indicator function, which takes a value of 1 when  $\underline{L} \circ \underline{R}_{t+1} \leq \underline{R}$  and 0 otherwise. Also,  $\underline{R}_{t+1} = \{R_{i,t+1}, i = 1, 2, \dots, N\}$

process breaches the trigger level,  $\underline{R}$  for the first time within the next year. In equation (4) we denote by  $N(\cdot)$  the normal distribution function. Our approach to modelling hedge fund risk-return tradeoffs is captured by equation (4), which looks similar to the classic mean-variance portfolio optimization problem, except that the implied risk aversion (the second term on the right hand side of equation (4)) itself is endogenous. Equation (4) also reflects one of the key contributions of our paper: the short option position that is faced by fund managers is an explicit part of their optimization program. As such the term within the curly bracket in equation (4) reflects the probability of the levered returns hitting the barrier before date  $t = 1$ . This follows from the results presented in Harrison (1985). For future reference, we denoted the probability of default (liquidation) of the hedge fund as

$$p_L \equiv \left\{ N \left[ \frac{R - y}{\sqrt{z}} \right] + e^{\frac{2yR}{z}} N \left[ \frac{R + y}{\sqrt{z}} \right] \right\}.$$

Note that the probability of default is amplified by the risk-factor  $\sqrt{z}$  and the parameter  $a$ . For the sake of simplicity, we will model just one option: the interpretation of our results will naturally depend on whether the option considered is the funding option or the redemption option. The combined treatment of both options is straightforward, and the qualitative results are likely to be similar to the ones that we report here. We are assuming that when the barrier is reached the fund is forced to liquidate at a cost of de-leveraging as modeled above.

It is useful to note the following properties of the objective function. First, note that the partial derivative with respect to the first variable ( $g_y$ ) is positive as shown below:

$$g_y(y, z) = 1 - \frac{2aR}{\sqrt{z}} \times \left\{ e^{\frac{2yR}{z}} N \left[ \frac{R + y}{\sqrt{z}} \right] \right\} > 0$$

Next, the partial derivative with respect to the second variable ( $g_z$ ) is negative as shown next.

$$g_z(y, z) = \frac{g - y}{2z} + \frac{aR}{2z} n \left( \frac{R - y}{\sqrt{z}} \right) + \frac{aR}{2z} n \left( \frac{R + y}{\sqrt{z}} \right) e^{\frac{2yR}{z}} + \frac{2ayR}{z^{\frac{3}{2}}} n \left( \frac{R + y}{\sqrt{z}} \right) e^{\frac{2yR}{z}} < 0$$

We denote by  $n(\cdot)$  the normal density function. These properties ensure that the risk-capital  $x$  has a finite optimal level. The optimal risk capital for unconstrained problem can be



written as follows.

$$x = -\frac{g_y}{2g_z}\rho^{-1}\underline{S} \quad (5)$$

To simplify the presentation of main results of the paper, we define the following notation:  $\hat{S} \equiv \sqrt{\underline{S}'\rho^{-1}\underline{S}}$ .  $\hat{S}$  denotes the correlation-weighted aggregate Sharpe ratio of the fund. We now state one of the main results of our paper.

• **Result 1**

The optimal aggregate risk-capital deployed by the hedge fund is determined by the following factors: a) Sharpe ratios of risk-taking units, b) de-leveraging costs, c) the location of the funding/redemption barriers governed by fund's contractual relationship with its *PBs* and investors, and d) the correlation across risk-taking units. Optimal level of risk capital can be expressed as an implicit closed form expression as shown below.

$$z = \frac{\frac{\hat{S}}{a} \left[ 1 - 2aN \left( \frac{R}{\sqrt{z}} + \hat{S} \right) e^{\frac{2\hat{S}R}{\sqrt{z}}} \frac{R}{\sqrt{z}} \right]}{z^{-\frac{1}{2}} \left[ N \left( \frac{R}{\sqrt{z}} - \hat{S} \right) + e^{\frac{2\hat{S}R}{\sqrt{z}}} N \left( \frac{R}{\sqrt{z}} + \hat{S} \right) \right] - 2z^{-1}Rn \left( \frac{R}{\sqrt{z}} - \hat{S} \right) - \frac{4R}{\sqrt{z}}\hat{S}e^{\frac{2\hat{S}R}{\sqrt{z}}} N \left( \frac{R}{\sqrt{z}} + \hat{S} \right)} \quad (6)$$

□

Result 1 demonstrates a) the non-linear relationship that exists between the optimal risk capital of a hedge fund and its underlying determinants, and b) the factors that underlie the nonlinear relationship. We further examine the main implications of Result 1 through specific examples below.

### 3.1 Effect of Fund's Sharpe Ratios

The effect of the differences in Sharpe ratios between two hedge funds, which are otherwise identical in terms of their short options positions is portrayed in Figure 2 below.

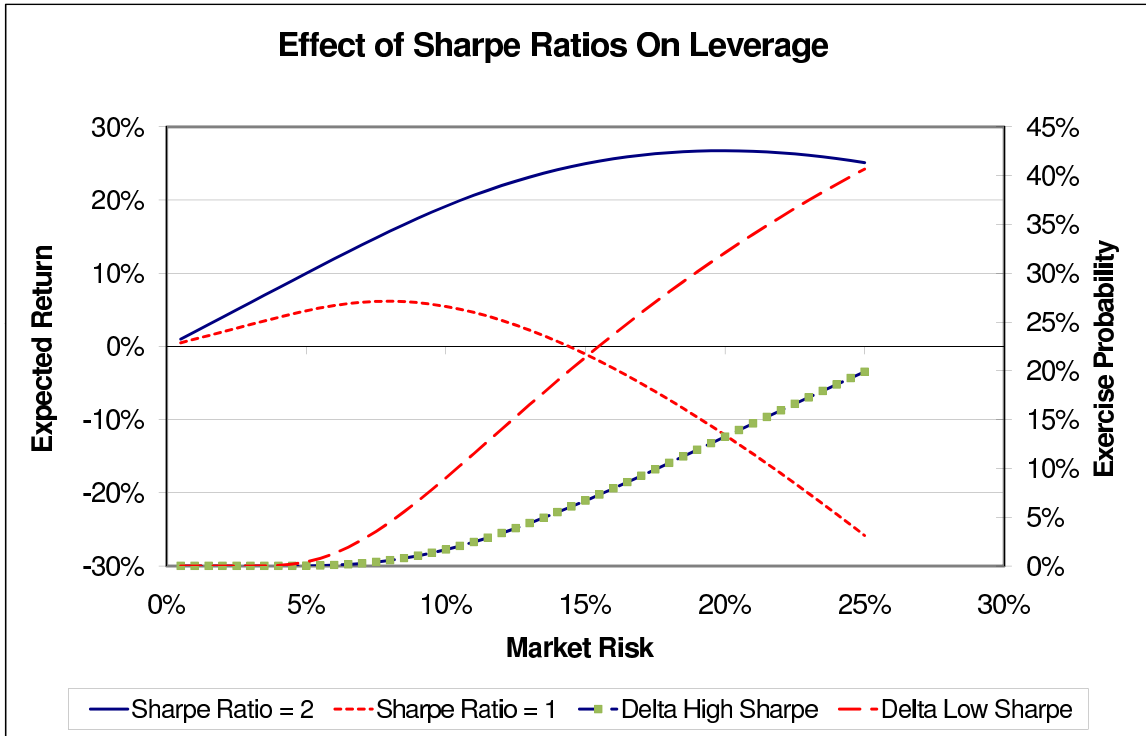


Figure 2: Effect of Sharpe Ratios on Leverage & Probability of Liquidation. Parameters:  $a = 5$ ,  $\underline{R} = -10\%$ , Asset volatility=0.50%.

In Figure 2, the X-axis represents “market risk”, which is  $\sqrt{z}$  in the context of our model. Clearly, higher the risk capital  $\sqrt{z}$  is, greater is the exposure of the fund to market risk. The key point is that the fund with a higher Sharpe ratio is able to take higher leverage and still keep the probability of exercise of short options (with *PBs* and investors) fairly low. Quantitatively, Figure 1 shows that the hedge fund with a Sharpe ratio of 2 can maintain a significantly higher optimal leverage without materially increasing the probability of liquidation, when compared to an otherwise identical hedge fund with a Sharpe ratio of 1.

A key implication of this figure is that the optimal leverage must be viewed in the context of the manager’s ability to produce a higher Sharpe ratio. Note that at low levels of leverage (for a given volatility), the expected returns increase for both funds. But the expected returns starts to decline quickly for the fund with lower Sharpe ratio. This is due to the fact that the fund with the lower Sharpe ratio is more prone to liquidation when it increases its risk capital. This is the key to interpreting Figure 2.

### 3.2 Effect of De-leveraging Costs

Note from figure 3 that the optimal aggregate risk capital is *decreasing* in the parameter  $a$ , which reflects the costs of de-leveraging due to the presence of short option positions. Thus the absolute level of risk capital (leverage) goes down as the de-leveraging costs associated with the short option positions increase. This is illustrated in figure 3 for the case of two hedge funds with different de-leveraging costs.

The pattern in Figure 3 suggests that at low levels of market risk, both funds experience an increase in expected return. But the fund with the higher de-leveraging cost faces a precipitous decline in expected returns once the risk capital exceeds 10%, whereas the fund with lower de-leveraging cost is able to increase its expected return further by adding to its leverage. Note that the fund with high de-leveraging cost reaches a expected return equal to zero when the probability of liquidation reaches a level of 40%. On the other hand, the fund with lower de-leveraging costs is able to generate an expected return of 25% at that level of

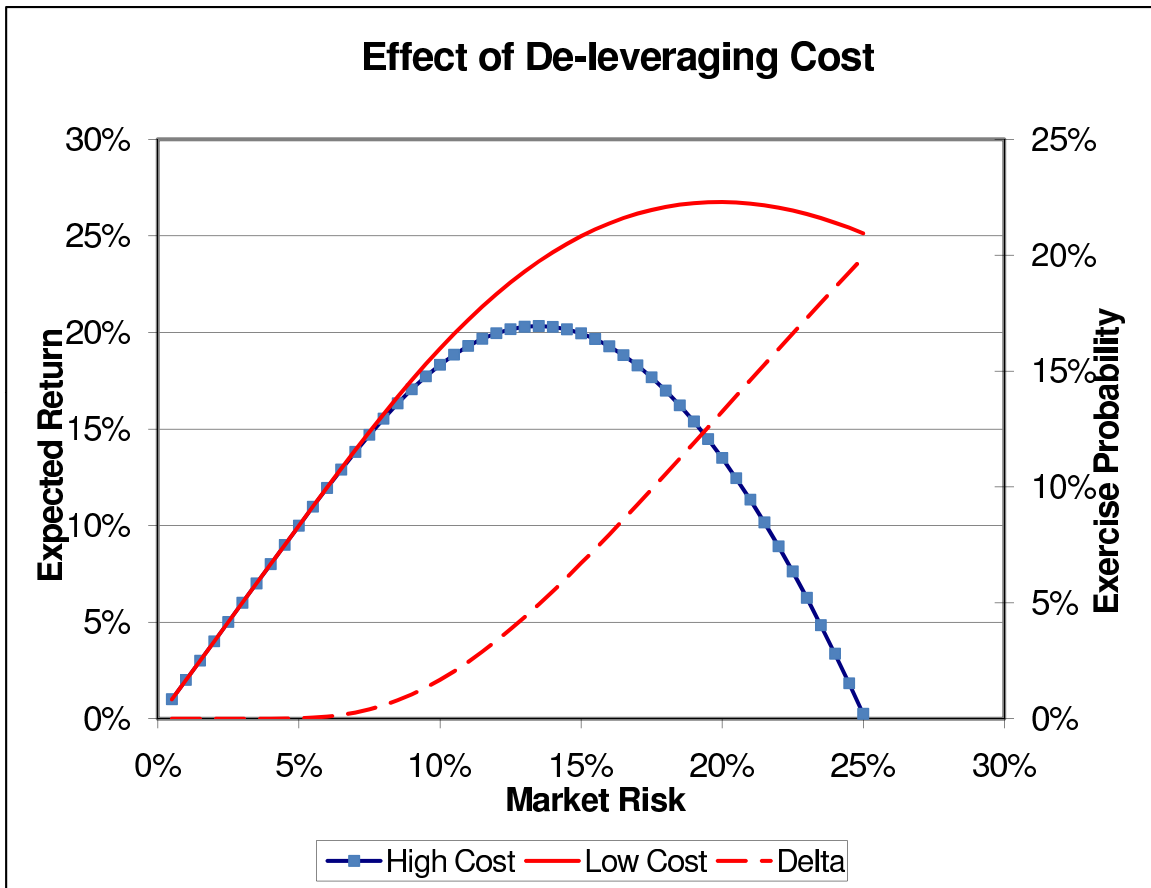


Figure 3: Effects of De-leveraging Costs. Parameters: Low cost ( $a = 5$ ), High cost ( $a = 8$ ),  $\underline{R} = -10\%$ , Sharpe ratio = 2, Asset volatility=0.50%.

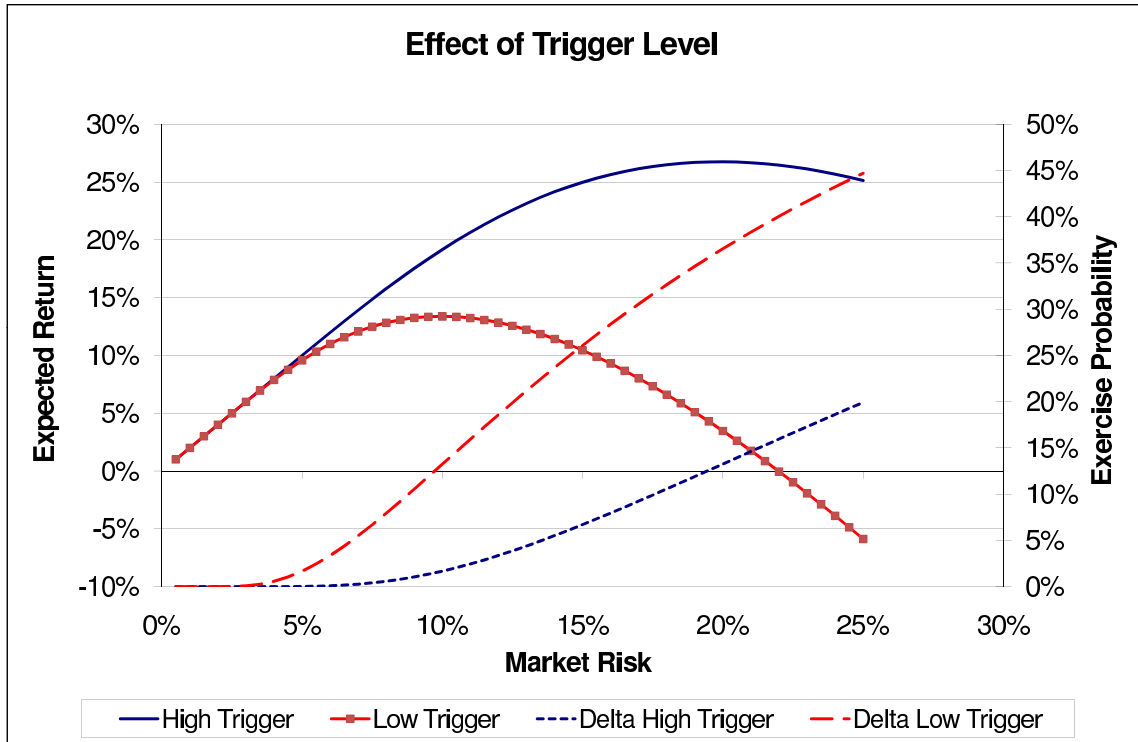


Figure 4: Effects of Trigger on Optimal Leverage. Parameters: De-leveraging cost ( $a = 5$ ), High Trigger:  $\underline{R} = -10\%$ , Low Trigger:  $\underline{R} = -5\%$ , Sharpe ratio = 2, Asset volatility = 0.50%.

market risk.

### 3.3 Effect of Trigger Level

Clearly, the optimal level of risk capital at every de-leveraging cost level is dependent on the liquidation barrier  $\underline{R}$ . If the liquidation barrier is closer, the leverage has to be lower. This result has an important risk management message: hedge funds will be well advised to factor in the effectiveness of their contractual relationships with prime brokers and investors in determining their aggregate risk capital.

Hedge fund investors should, in addition to evaluating the portfolio strategies, must obtain information about the effectiveness of the fund's agreements with its *PBs*.

## 4 Distance to Default or Liquidation

Equation (4) can be used to distinguish our formulation from the classic portfolio optimization problem. In the absence of any short options, hedge fund can allocate as much risk-capital as it wishes: such a strategy will increase the expected return of the portfolio without affecting the Sharpe ratio, assuming that the strategies are scalable. The presence of short options induces risk aversion as is clear from (4). This will in turn introduce a non-linearity in the relationship between expected returns and risk. For the funding or redemption option, we may regard the quantity  $\frac{\|R\|}{\sqrt{z}} \equiv DD$  as essentially the distance to default or distance to liquidation. Given a set of Sharpe ratios, correlations, option triggers, and de-leveraging costs, the distance to default for the fund is endogenously chosen. This is seen by writing (6) as follows.

$$\hat{S} = \frac{a \left[ N(-DD - \hat{S}) + e^{-2DD\hat{S}} N(-DD + \hat{S}) \right] - 2aDDn(-DD - \hat{S}) - 4ae^{-2DD\hat{S}} DD\hat{S} N(-DD + \hat{S})}{\left[ 1 - 2aN(-DD + \hat{S})e^{-2DD\hat{S}} DD \right]} \quad (7)$$

This concept of distance to default is a convenient way to think about risk limits: the optimal distance to default that emerges from (7) could be used as a guideline by the risk managers to set more conservative distance to default to cover unanticipated surges in volatility or unforeseen changes in triggers by investors and prime brokers. This more conservative level may give the necessary cushion for the funds to perform voluntary de-leveraging in the face of an unanticipated crisis. This is best understood by recasting figure 4 in terms of distance to default as shown below.

In Figure 5, note that the optimal distance to default is  $DD = 0.5$  when  $a = 4$ . One risk management policy might be to choose a more conservative level denoted by the dashed vertical line to the right of the unconstrained optimum at a  $DD = 1$ . This choice sacrifices some expected returns. The precise location of the more conservative policy is dictated by

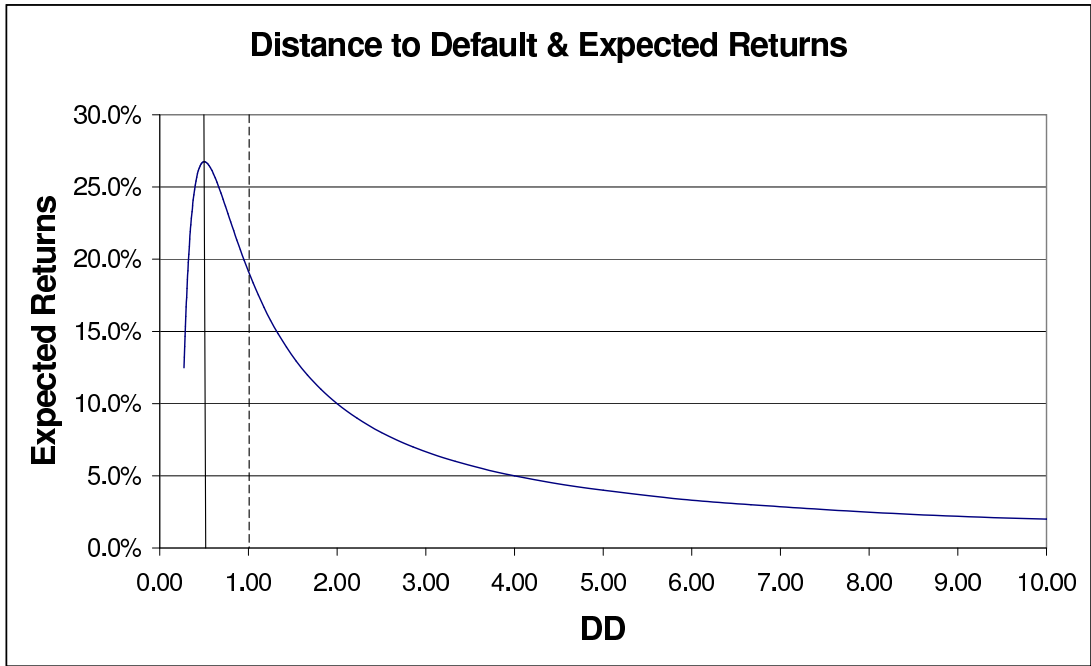


Figure 5: Distance to Default & Expected Returns

the funds desired level of  $DD$ , which is a function of the amount of flexibility it wants to be able to perform voluntary de-levering when there is an unanticipated shock to volatility and/or funding conditions with  $PBs$ . We show in section 5 how this can be accomplished using a risk limit on the level of unencumbered cash. Equation (7) places an important restriction on optimal risk-capital, and the implied distance to default for hedge funds. Given a distribution of Sharpe ratios (which capture the volatility of assets deployed by the fund and the correlation across different risk-taking units), and de-levering costs (which captures the secondary market liquidity of assets) the funds distance to default is pre-determined at the optimally chosen risk-capital. This is an important prediction: if the Sharpe ratios are the same for two funds, with one facing higher de-levering costs, then that fund must reduce its aggregate risk-capital and increase its distance to default.

Even if the fund negotiates a sufficiently low trigger level with investors and prime brokers, its endogenous (unconstrained) risk capital will leave the  $DD$  the same as any other hedge fund which is otherwise identical. Equation (7) also sets an upper bound on aggregate risk,

which is stated in Result 2.

- Result 2

The Sharpe ratio of a hedge fund,  $S_H$ , is its un-levered Sharpe ratio minus the quantity probability of liquidation multiplied by its potential de-leveraging costs. The aggregate risk upper bound that causes the distance to default to go to zero is at the point where the aggregate Sharpe ratio of the fund is exactly equal to its de-leveraging costs.

$$S_H = \hat{S} - a \times p_L(\hat{S}, \underline{R}, z)$$

When the probability of liquidation reaches  $p_L(\hat{S}, \underline{R}, z^*) = \frac{\hat{S}}{a}$ , the hedge fund's Sharpe ratio reaches zero. The quantity  $z^*$  is the upper limit of risk capital that the fund can employ.

□

The fund's aggregate Sharpe ratio depends on the extent of diversification provided by its risk-taking units (in terms of their correlation of returns with each other) as well as their ability to create superior returns, as measured by their respective Sharpe ratios. Another implication of Result 2 is that the survival probability of a fund is proportional to the amount by which the Sharpe ratio of the overall fund exceeds its de-leveraging costs. Result 2 also suggests that funds which operate in less liquid markets, and where the costs of de-leveraging can be high must have sufficiently high Sharpe ratios to justify any leverage at all. An implication of Result 2 is that funds which operate in liquid markets may support higher leverage so long as their Sharpe ratios are high enough.

The probability of exercise of the option is decreasing with the de-leveraging cost  $a$ . This result arises from the fact that a fund with lower de-leveraging cost for its short option positions will rationally choose a higher risk-capital. If the fund's contract with its prime brokers and investors leave it with very tight liquidation barriers, the fund will choose a conservative level of aggregate risk capital.



## 4.1 Survivorship bias

Result 2 allows us an explicit way to characterize the survivorship bias in hedge funds. Rewriting Result 2 in terms of expected returns we get:

$$E(R_H) = \hat{S}\sqrt{z} - a\sqrt{z} \times p_L(\hat{S}, \underline{R}, z)$$

The equation above makes it clear that the  $\alpha$  of the hedge fund must be discounted by the survivorship bias, which consists of three components: a) de-leveraging costs, b) trigger level  $\underline{R}$ , which is a function of the efficiency with which the fund has contracted with prime brokers and investors, and c) the aggregate Sharpe ratio of the fund. This bias has both time-series and cross-sectional implications. The cross-sectional implication is that funds differ in terms of their de-leveraging costs: holding all the other factors fixed, the survivorship bias should be greater for funds with higher ex-ante de-leveraging costs. A useful proxy for de-leveraging costs might be the secondary market liquidity. The cross-sectional variation in Sharpe ratios of funds leads to the second cross-sectional implication: holding other factors fixed, funds with higher Sharpe ratios should have lower survivorship bias in our model.

Over time, our model predicts that the survivorship bias can vary for a number of reasons: a) funds can improve their funding relationship with *PBs* and the liquidity terms with their investors. This is especially true of funds, which have survived a few years and are able to attract better funding terms and liquidity profile. The survivorship bias can also change over time for exogenous surges in volatilities. Although we have squarely formulated the problem as one facing the hedge fund, and as one in which optimal allocation of risk-capital is the variable of interest, it is very easy to see the generality of our approach: consider for example, a long-only fund with no leverage. For this fund, investors may still have the option to redeem, and this short option is precisely the one represented in equation (4) with the modification that the option is now triggered by investors when there is either a flight to quality or when the fund posts sub-par performances relative to its peers over a threshold period of time. The choice variable facing such a long-only fund is obviously not the leverage (risk-capital) level, but its asset allocation and hence the choice of its beta, as equation (1)

for the long-only fund will have a systematic (beta) component. The barrier level for the long-only fund is a threshold level of poor returns history. The formulation is therefore very general and can be applied to a broad range of optimal portfolio selection problems in which the funds are faced with varying degrees of short option positions with their investors.

Throughout this section we have analyzed the unconstrained choice of aggregate risk-capital by a hedge fund. Hedge funds are also concerned about the potential for liquidation due to unexpected surge in volatility, credit market dislocations, and large-scale investor redemptions. Such developments can drive the unencumbered cash levels to very low levels and drive the fund into liquidation, even when it may be operating at optimal leverage prior to the surge in volatility. While the short options framework addresses some of these concerns, prudent risk management may require explicit risk limits on unencumbered cash levels that further protect the hedge from insolvency by choosing a risk-capital which is below the unconstrained optimum that we have characterized in this section. This approach is presented in the next section

## 5 Unencumbered Cash and Risk-Capital Allocation

Unencumbered cash level is simply the fraction of  $AUM$  not posted as margin. Hence it is available to the fund manager to meet with unanticipated exercise of funding and redemption options, in addition to meeting margin calls that may arise from losses in the underlying portfolio.

Formally, we can define unencumbered cash (in the context of our formulation) as follows:

$$u = 1 - \lambda\sqrt{z} \tag{8}$$

We may think of  $\lambda\sqrt{z}$  as the margin demanded by  $PBs$  per unit of  $AUM$ , and  $\lambda$  as the multiplier used by  $PBs$  in setting the margins. We can think of  $\lambda$  as a margin multiplier imposed by  $PBs$ , which is risk-based (as it is proportional to  $\sqrt{z}$ ).

The unencumbered cash is typically the portion of the investor assets under independent

custodian (i.e., unencumbered by counterparty obligations). Mathematically, the unencumbered cash is simply the compliment of the margin posted with the counterparties. In order for this surplus portion of the cash to be completely unencumbered, however, it is important that a legal structure is put in place. The economic significance of the unencumbered cash is that this is the minimum amount that investors can get back if all counterparties were default on their obligations and all margin postings are lost (or nearly lost as the Lehman bankruptcy has demonstrated). In our view, the use of the unencumbered cash as a risk management tool has not been sufficiently emphasized. Indeed, we would argue that unencumbered cash is probably the most important risk management tool at the disposal of a hedge fund. There are several reasons why.

First, even though hedge fund investors are, by self selection, comfortable with the lack of principal protection, a commitment to a high level of unencumbered cash is the best that a hedge fund manager can do in providing some form of principal protection and ought to give comfort to most hedge fund investors. With a sufficiently high level of principal protection, the value of redemption option can be significantly mitigated.

Second, the unencumbered cash is not only a function of the portfolio risk perceived by the investment manager it is crucially also a function of the risk perceived by the counterparties. Since in most *PB* platforms the margin requirements are risk-based and margining terms are vigorously scrutinized and negotiated by both the hedge fund manager and the counterparties, the unencumbered cash gives a very objective measure of portfolio risk and there is usually a direct relationship with the amount of leverage that a hedge fund deploys.

Third, to the extent that lack of operational efficiency, or the lack of informational accuracy, or the sub-optimality of counterparty exposure tend to reduce unencumbered cash levels, a risk management framework centered on unencumbered cash provides a natural and measurable objective function for many crucial aspects of the operational platform of a hedge fund. The dynamic relationship between unencumbered cash and other key underlying variables are captured by equation (8). This simple specification suggests that there is an immediate response in unencumbered cash with changes in a) leverage, b) volatility of funds

assets, c) the trajectory of assets under management from one period to the next, and d) the inefficiencies associated with the funding arrangements with the prime brokers . A risk management framework anchored on unencumbered cash levels will therefore respond much more quickly to potential movements in the moneyness of the funding and redemption options than traditional measures such as VaR.

Note that the unencumbered cash level decreases with leverage as more risk capital must be allocated by the fund, reducing cash available. The unencumbered cash level is also decreasing in the volatility of funds return generating process. As the *AUM* decreases over time (either due to losses or due to withdrawals) the unencumbered cash level goes down. Finally, the parameter captures the inefficiency in funding arrangements due the ability and market conventions that the prime brokers use in setting funding parameters. Greater the inefficiency, higher will be the parameter and hence lower will be the unencumbered cash level.

## 5.1 Unencumbered Cash as a Risk Management Tool

One possible risk management policy is to maintain the same level of unencumbered cash through time. That is, under the risk management policy that the unencumbered cash is kept at certain level, the leverage level needs to be adjusted down when (i) volatility increases; and/or (ii) the margin multiplier increases; and/or (iii) the *AUM* decreases. If the adjustment of the leverage level is voluntary and/or preemptive, the cost of de-leveraging may be negligible. If the de-leveraging is involuntary, triggered by the exercise of the funding and/or redemption options, the cost of de-leveraging can be substantial, and in some cases devastating. To illustrate, let us consider the manner in which funding options can place the fund at the risk of liquidation and how unencumbered cash as a risk management tool may help the fund to survive. The margin requirement demanded by *PBs* may change dramatically depending on the performance of the fund and other macroeconomic developments. Each *PB* assigns a credit multiplier to the hedge fund as part of the margin agreement. To the extent that the hedge fund is perceived as a prudent risk taker and therefore a safer

credit, the *PBs* can agree to a lower credit multiplier. This component may be attributed to the informational differences between the fund and its *PBs*. In periods of poor performance, this multiplier may increase, resulting in higher haircuts. Second, if the hedge fund has multiple *PBs*, its positions are likely to be distributed across different *PBs* and therefore there is a loss of margin efficiency in that risk across *PBs* can not be netted. The aggregate risk perceived by each of the *PBs* will therefore be higher than the portfolio risk (namely ). As a result, will typically be much higher than the credit multiplier that each *PB* assigns to the hedge fund. Finally, the margin requirement by each individual *PB* is almost always linked to the market (systematic) risk exposure of the positions held at the *PB*. To the extent that the hedge fund does not have systematic risk exposure but have such exposure with individual *PBs*, there is inefficiency: the hedge fund is paying extra margin (in fact twice) for risk exposures that are offset between two *PBs*.

When the funding trigger is breached, the *PBs* can increase the margin requirement significantly as Table 1 illustrates. The dollar amount of the margin requirement is proportional to be a risk measure such as *VaR* (effectively the levered volatility). Higher the leverage, higher the volatility of assets, then greater will be the margin demanded of the hedge funds by the banking sector. This suggests that by keeping a higher level of unencumbered cash, the fund can lower leverage in periods of high volatility and effectively manage the risk of liquidation by *PBs*. We formalize this approach below.

We illustrate this below in Figure 6 by examining the unencumbered cash levels with optimal risk capital.

Note that when the *PBs* use a margin multiplier of 3, the fund is able to optimally lever itself and the un-encumbered cash level at the optimal leverage gives sufficient room for the manager to deal with de-leveraging voluntarily. When the margin multiplier increases to 6, then the manager cannot choose the desired leverage as it will drive the unencumbered cash level to zero. This implies that choosing a level of unencumbered cash level as a risk limit can, ex-ante, force the manager to reduce leverage and be able to voluntarily de-lever in future should there be a crisis.

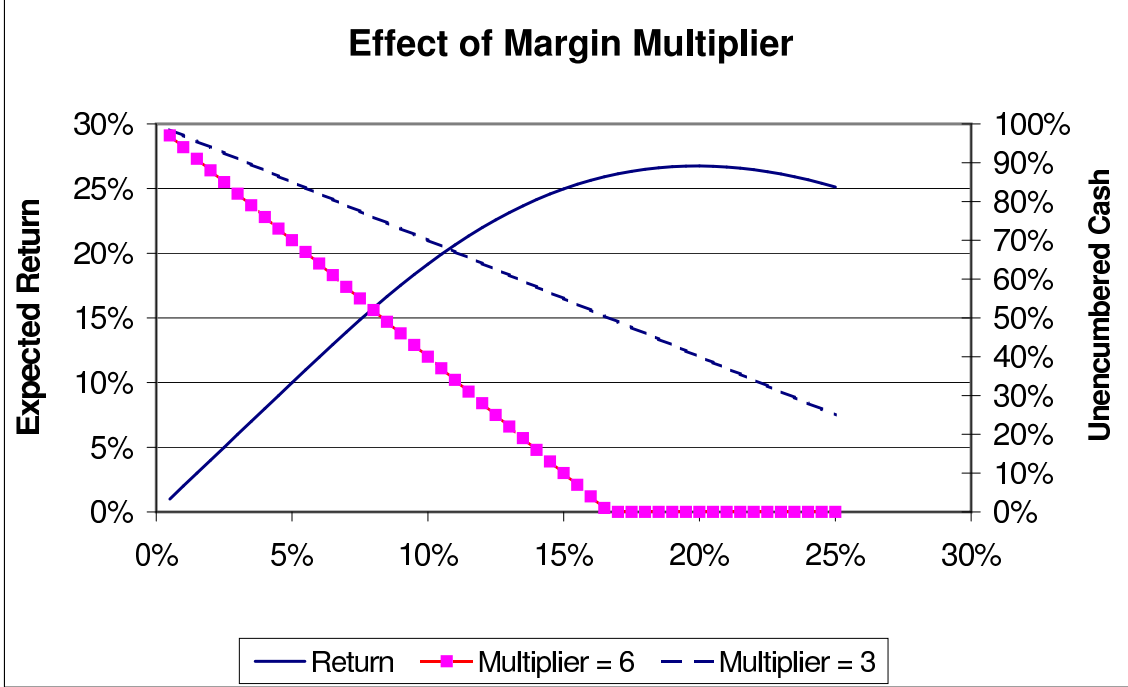


Figure 6: Effect of Margin Multiplier on Unencumbered Cash

## 5.2 Constraints on Unencumbered Cash

First, in the absence of any constraints on unencumbered cash, the aggregate risk-capital  $z$  gets allocated across different desks as follows:

$$x = \frac{\rho^{-1} \underline{S} \sqrt{z}}{\hat{S}} \quad (9)$$

Through  $z$ , the individual desks are assigned a penalty for de-leveraging costs associated with the potential exercise of short options. Note from equation (8) that setting a limit of unencumbered cash level  $u$  is equivalent to setting a limit on aggregate level of risk-capital itself. If set a risk limit on aggregate risk-capital as  $z \equiv x' \rho^{-1} x = \nu^2$ , then we are effectively setting a limit on unencumbered cash level itself. If this risk limit becomes binding, then the risk capital gets allocated as follows (based on equation (9) above).

$$x = \frac{\nu \rho^{-1} \underline{S}}{\hat{S}}$$

In general we can therefore set a limit on the minimum level of unencumbered cash that must be maintained (which translates to a limit on  $\sqrt{z}$ ) and solve the optimization problem in

(4) subject to that constraint. The solution to this constrained optimization problem yields the optimal risk capital allocation. We can now set up the optimization problem associated with the risk budgeting as follows.

$$Max_{\{x,\theta\}} \left[ g(y, z) - \theta(z - \nu^2) \right] \quad (10)$$

The first order conditions of optimality can now be derived from equation (10) as follows. The optimality condition corresponding to the choice of the risk-capital choice leads to:

$$g_y \underline{S} + 2g_z \rho x - 2\theta \rho x = 0 \quad (11)$$

$$z = \nu^2 \quad (12)$$

Solving the above equations we get the shadow cost  $\theta$  and the risk budget vector  $x$  as shown below.

$$x = \frac{1}{2} \frac{g_y}{\theta - g_z} \rho^{-1} \underline{S} \quad (13)$$

$$\theta = g_z + \frac{g_y}{2\nu} \hat{S} \quad (14)$$

Note that when the shadow cost  $\theta = 0$  we recover the unconstrained solution in equation (5) from equation (13). Given a risk limit  $\nu$  we can solve (13) and (14) simultaneously for the optimal level of risk budgeting subject to the constraint on risk capital. We provide an example of constrained risk capital allocation next.

### 5.2.1 Constrained Risk Capital Allocation

In Table 2, we consider a simple case in which the fund has two desks and the fund must decide on the risk-capital allocation problem. For simplicity we will suppose that the Sharpe ratios are:  $S_1 = 1.8000$ ,  $S_2 = 0.8717$ . We assume that the desks are independent of each other. This parameter configuration yields  $\hat{S} = 2.00$ , which is the value we have used in previous sections. As noted earlier, a constraint on  $\nu$  boils down to a constraint on unencumbered cash level, from equation (8) given a margin multiplier. In the tables that

Table 2: Risk Capital Allocation with Limits on Unencumbered Cash  $\rho = 0$

Risk Limit ( $\nu$ )	Implied Constraint On Unencumbered Cash	$E(R) = g$	$\theta$	$x_1$	$x_2$	$p_L$
10%	80%	19.3%	8.25	9.0%	4.4%	1.7%
12%	76%	22.4%	5.83	10.8%	5.2%	3.4%
14%	72%	24.9%	4.04	12.6%	6.1%	5.5%
16%	68%	26.9%	2.71	14.4%	7.0%	7.9%
18%	64%	28.4%	1.72	16.2%	7.8%	10.6%
20%	60%	29.4%	0.97	18.0%	8.7%	13.3%
22%	56%	30.0%	0.41	19.8%	9.6%	15.9%
24%	52%	30.1%	0.00	21.6%	10.4%	18.5%

follow, we assume a margin multiplier of  $\lambda = 2$ . Note that when the risk limit is set at a low level of 12% the expected return is 22% and the probability of liquidation is 3.4%. The unconstrained risk capital is given in the last row of the table to serve as a benchmark, and it entails a probability of liquidation of 18.5%. The tradeoff is the lowered expected return in the first row where the fund gives up 8% return to achieve a much lower probability of liquidation. Desk 1 which has a higher Sharpe ratio gets a greater allocation of risk capital, but desk 2, because of its independence to desk 1 also gets a reasonable share of the risk capital. Note also from Table 2 that when the risk limits are tight, the shadow costs as captured by  $\theta$  are high, and at this low level of capital allocated, the short option is out of the money. As we increase the risk capital, (either through high leverage, or ex-ante volatility, or both) maximum expected return achievable increases as the increase in short option value is more than offset by the increase in the expected return that would accrue to the fund. The unconstrained optimal risk-capital is 24% when the shadow costs go to zero.

We examine in Table 3 the implications when the correlation between the trading desks increase to 0.25. Note that the share of risk capital to desk 2, which has a lower Sharpe ratio declines because of its positive correlation with Desk 1. The unconstrained capital is



Table 3: Risk Capital Allocation with Limits on Unencumbered Cash  $\rho = 0.25$

Risk Limit ( $\nu$ )	Implied Constraint On Unencumbered Cash	$E(R) = g$	$\theta$	$x_1$	$x_2$	$p_L$
10%	80%	17.6%	7.08	9.1%	2.4%	2.2%
12%	76%	20.2%	4.72	10.9%	2.9%	4.2%
14%	72%	22.2%	3.01	12.8%	3.4%	6.7%
16%	68%	23.6%	1.77	14.6%	3.9%	9.5%
18%	64%	24.5%	0.87	16.4%	4.4%	12.3%
21%	58%	24.9%	0.00	18.9%	5.0%	16.4%

21% which is lower than the amount 24% in the zero correlation case.

## 6 Implications for Fund Managers & Policy Makers

Our analysis has some implications for hedge fund investors and policy makers. The extent to which due diligence is paid by hedge funds in managing their short option positions with investors and prime brokers should be of great interest to both hedge fund investors and policy makers. As we noted at the outset, these options should be fully understood by all parties ex-ante and vigorously managed, ex-post. Such a policy promotes transparency and helps to properly evaluate the economics of investing in hedge funds. From the investor's perspective a careful evaluation of hedge funds risk management policy in the management of these short option positions is at least as important as understanding the funds investment philosophy, market risk and counterparty risk. Such an understanding can help make investors to make more informed decisions, and minimizes surprises, ex-post. From a regulatory perspective, policy makers may benefit by focusing attention on these short option positions: in the event of a banking crisis, all prime brokers have tendency to withdraw their credit lines or increase significantly their margin requirements as we have documented. In some cases, prime brokers have been aggressive in exercising their funding options to terminate the hedge funds. In

such a case, most hedge funds are obliged to de-lever simultaneously precipitating secular declines in asset prices. To the extent that hedge fund risk management policies already set prudent risk limits anticipating the potential exercise of short options, such de-leveraging is more likely to be planned and voluntary, as the funds would have allocated less risk-capital, ex-ante. As a tool for hedge fund risk management, regulators may be better off focusing more on unencumbered cash levels to judge how well the funds are managing their risks, and how well they are placed to voluntarily de-lever. It is our view that this measure is much more transparent, relatively model-independent and easy to verify, unlike measures such as *VaR*, which are often dependent on models and assumptions. In policy discussion of gates, lockup periods and notification periods, it is useful to remember that these are usually agreed upon by investors, ex-ante in a transparent relationship between the fund and its investors. Regulators should make every effort to ensure that these contractual provisions are transparent to all investors. Their presence in the contractual agreements serves to mitigate systemic risk when large-scale redemptions ensue due to unanticipated banking crisis or other macroeconomic developments. These provisions enable orderly and planned liquidations to meet the liquidity demands of exiting investors while protecting continuing investors and the fund to continue to function. There is presently no formal coordination mechanism for orderly liquidations and workouts in the hedge fund industry that we are aware of. In the absence of a provisions such as automatic stay (which are part of chapter 11 proceedings under the bankruptcy code) it is important that the hedge fund industry has well-articulated contractual provisions that enable the fund its prime brokers and its investors to have an orderly resolution of redemptions and settlement of claims in the event of distress. To the extent that contracts allow investors to redeem at different points in calendar time (investor-level gates as opposed to fund-level gates tend to accomplish this better) it may mitigate systemic effects of investor redemptions.

## 7 Conclusions

We make the observation that hedge funds, by construction of their funding arrangements and contractual arrangements with their investors are short in two very valuable options. We argue that these options introduce significant nonlinearities in their return generating process, quite independent of any portfolio strategies that the funds may choose to follow. In this sense, we depart from many of the papers in the hedge funds literature which focus on nonlinearities in hedge funds returns arising from portfolio strategies followed by hedge funds. An important consequence of these short options position is that there is typically a well defined leverage or optimal risk-capital for hedge funds. Our paper makes the argument that setting prudent risk limits on unencumbered cash allows the hedge fund to stay well within the prudent leverage levels without undue sacrifice in expected return. We show, through explicit examples, how to develop ex-ante leverage constraints through such risk limits. A contribution of our paper is the integration of contractual short option positions into the risk management principles for hedge funds. We also stress the important role played by unencumbered cash as a tool for risk management.

The paper did not explicitly model the compensation structure of hedge fund managers to examine how they might interact with risk-taking behaviour and hence on prudent risk management structure. Aspects of compensation structure such as the management fee, performance fee, high water marks, etc. represent an important avenue for further research in hedge funds risk management.

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