A Review of Merton’s Model of the Firm’s Capital Structure with its Wide Applications

Suresh Sundaresan

Finance & Economics Division, Columbia Business School, Columbia University,
New York 10027; email: ms122@columbia.edu

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Abstract
Since its publication, the seminal structural model of default by Merton (1974) has become the workhorse for gaining insights about how firms choose their capital structure, a “bread and butter” topic for financial economists. Capital structure theory is inevitably linked to several important empirical issues such as (a) the term structure of credit spreads, (b) the level of credit spreads implied by structural models in relation to the ones that we observe in the data, (c) the cross-sectional variations in leverage ratios, (d) the types of defaults and renegotiations that one observes in real life, (e) the manner in which investment and financial structure decisions interact, (f) the link between corporate liquidity and corporate capital structure, (g) the design of capital structure of banks [contingent capital (CC)], (h) linkages between business cycles and capital structure, etc. The literature, building on Merton’s insights, has attempted to tackle these issues by significantly enhancing the original framework proposed in his model to make the theoretical framework richer (by modeling frictions such as agency costs, moral hazard, bankruptcy codes, renegotiations, investments, state of the macroeconomy, etc.) and in greater accordance with stylized facts. In this review, I summarize these developments.
1. MERTON’S MODEL OF CORPORATE DEBT

Merton’s (1974) paper on the valuation of corporate debt securities is one of his many seminal contributions to finance. This paper has been at the fulcrum of two (interrelated) big questions in finance. First, how should one go about understanding and explaining credit spreads? Second, how should one think about the design of the firm’s capital structure. This review is more about the second question, but it is inevitably linked to the first: credit spreads of a firm’s debt liabilities are best studied in the context of its optimal liability and capital structure.

I begin by briefly reviewing Merton (1974) to set the stage for the evolution of the literature since the publication of this pathbreaking paper. The literature has evolved in several directions. Two key contributions that extended the framework of Merton in significant ways are Black & Cox (1976) and Leland (1994). I review these important papers at the very outset to set the stage for the review of the literature.² In their seminal options pricing paper, Black & Scholes (1973) suggest how their approach can be used in thinking about the valuation of credit-risky debt. In an important paper, Brennan & Schwartz (1978) modeled the valuation of credit-risky debt and the issue of optimal capital structure using numerical techniques.

The essence of Merton (1974) is its parsimonious specification to derive major insights about the determinants of credit spreads. The following key assumptions are set right at the outset:

- **A.1** There are no transactions costs, taxes, or problems with indivisibilities of assets.
- **A.2** There are a sufficient number of investors with comparable wealth levels such that each investor believes that he can buy and sell as much of an asset as he wants at the market price.
- **A.3** There exists an exchange market for borrowing and lending at the same rate of interest.
- **A.4** Short sales of all assets, with full use of the proceeds, are allowed.
- **A.5** Trading in assets takes place continuously in time.
- **A.6** The Modigliani-Miller (MM) theorem that the value of the firm is invariant to its capital structure obtains.
- **A.7** The term structure is flat and known with certainty; i.e., the price of a riskless discount bond that promises a payment of $1 at time $T$ in the future is $P(t, T) = e^{-r(T-t)}$, where $r$ is the (instantaneous) riskless rate of interest, the same for all time.
- **A.8** The dynamics for the value of the firm, $V$, through time can be described by a diffusion-type stochastic process.

Merton notes that the perfect market assumptions (the first four) are easily relaxed. Assumption A.7 is made to focus attention on default risk: Merton notes that introducing stochastic interest rates will make a fairly innocuous modification [wherein one replaces $e^{-r(T-t)}$ by the pure discount bond price $P(t, T)$, which will now depend on relevant state variables of the economy] of his main insights. We are then left with A.5, A.6, and A.8 as the key assumptions. Assumption A.5 is used in practically all the papers in this literature. Assumption A.8 has been relaxed in some papers, which I discuss below. Assumption A.6 is actually derived in the paper with bankruptcy,

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¹Merton’s paper was published in the Journal of Finance in 1974, but a working paper was available in 1970 containing all the major results (see, respectively, Merton 1974, 1970; see also Merton 1992, ch. 11, pp. 357–87).

²I do not review (a) reduced-form models of default, which is a key area of research in its own right. The primary focus of reduced-form models of default is not the determination of optimal capital structure, which is the thrust of this review. I do not review (b) structural models of default with stochastic interest rates either, as the primary focus of that strand of literature is not optimal capital structure.
but more importantly, Merton (1974, p. 460, section V) notes the following: “If, for example, due to bankruptcy costs or corporate taxes, the MM theorem does not obtain and the value of the firm does depend on the debt-equity ratio, then the formal analysis of the paper is still valid.” He then notes that the debt value and the firm value must be simultaneously obtained. It is clear from the foregoing statement that Merton was providing the analytical machinery needed to solve for the optimal capital structure, although he did not pursue it.

In the context of the above assumptions, and the observations surrounding them, Merton (1974), examined the valuation of corporate debt in three possible manifestations: 3 zero-coupon debt, coupon-bearing debt, and callable debt. In each case his paper provided tractable and intuitive closed-form solutions for debt prices.

Let me begin by summarizing Merton’s approach in the context of his zero-coupon bond formulation. This is a natural starting point to trace the progress of the literature. I alter slightly the original specification of Merton (1974) to connect it more easily to the papers that followed. The value of the assets of the firm is governed by the geometric Brownian motion (GBM) process, as shown here:

$$\frac{dV_t}{V_t} = \left(\frac{r - \delta}{C_0}\right)dt + \sigma dW_t$$

(1)

where the initial value of the assets $V_0 > 0$ and $\delta$ is the constant cash flow payout ratio. The process $\{W_t\}$ is a standard Brownian motion under the (risk-neutral) martingale measure $Q$. The firm issues a single class of debt, a zero-coupon bond, with a face value $B$ payable at $T$. Default may happen only at date $T$, and if default happens, creditors take over the firm without incurring any distress costs and realize an amount $V_T$. Otherwise, they receive $F$. In short, the payoff to the creditors at date $T$ is

$$D(V_T, T) = \min(V_T, B) = B - (B - V_T)^+.$$ 

(2)

The representation of the payoff to creditors makes it clear that the creditors are short a put option written on the assets of the borrowing firm with a strike price equal to $B$, the face value of debt. In addition, once we recognize that the borrower (equity holders in Merton’s model), (a) owns the firm, (b) borrowed the amount $B$ at $t = 0$, and (c) owns a put option on the assets of the firm with a strike price equal to $B$, it is immediate, by a put-call parity relationship, that equity is a call option on the assets of the borrowing firm with a strike price equal to $B$, the face value of debt. We can therefore express, respectively, debt and equity values as follows:

$$D(V_t, t) = P(t, T) - \text{Put}_{BS}(V_t, B, r, T - t, \sigma)$$ and

$$E(V_t, t) = \text{Call}_{BS}(V_t, B, r, T - t, \sigma).$$

Merton’s insight above makes it clear that the spread between credit-risky debt and an otherwise identical risk-free debt is simply the value of this put option. This remarkable insight is robust in thinking about the determinants of credit spreads: Maturity of the debt; leverage $B$; (strike price of the put); and the business risk of the assets of the firm, $\sigma^2$, are the key factors that influence credit spreads. Define $\tau = T - t$ and $N(\cdot)$ as the standard Gaussian cumulative distribution function as shown below:

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3Most of the literature has tended to focus on Merton’s risky zero-coupon debt formulation.

4I denote by $\text{Put}_{BS}(V_t, B, r, T - t, \sigma)$ the value of a put option on the assets of the firm at a strike price equal to the face value of debt as given by the Black & Scholes (1973) model. Likewise, $\text{Call}_{BS}(V_t, B, r, T - t, \sigma)$ is the value of a call option.
\[ N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy. \]

Then the corporate debt value is

\[ D(t, T) = V_t e^{-\delta(T-t)} N(-d_1) + BP(t, T) N(d_2), \]  

(3)

where

\[ d_1 = \frac{\ln \left( \frac{V_t}{B} \right) + \left( \frac{r - \delta + 1/2 \sigma^2}{\sigma \sqrt{T-t}} \right) (T-t)}{\sigma \sqrt{T-t}} \]

and

\[ d_2 = d_1 - \sigma \sqrt{T-t}. \]

Defining leverage as \( d = \frac{B e^{-\tau(T-t)}}{V_t} \), Merton characterized credit spreads, \( R(\tau) - r \), where the yield to maturity of the risky zero-coupon bond is defined as \( D(t, T) = B e^{-R(\tau) \tau} \). The explicit formula for credit spreads from Merton is shown below:

\[ R(\tau) - r = -\frac{1}{\tau} \ln \left[ V_t e^{-\delta(T-t)} N(-d_1) + BP(t, T) N(d_2) \right]. \]  

(4)

Merton’s model allows us to compute (in the risk-neutral probability measure), respectively, the probability of default and the expected (discounted) recovery rate as follows:

Probability of default \( = P(V_T < B) = \pi_Q = N(-d_2) \) and

Expected discounted recovery rate \( = E^Q \left[ e^{-\tau(T-t)} \frac{V_T}{B} \mid V_T < B \right]. \)

Assuming that the expected return of the assets is \( \mu \), we can compute in Merton’s model the probability of default, \( \pi_P \), in the physical measure as follows: \( \pi_P = N(-d_2') \), where

\[ d_2' = \frac{\ln \left( \frac{V_t}{B} \right) + \left( \frac{\mu - \delta - 1/2 \sigma^2}{\sigma \sqrt{T-t}} \right) (T-t)}{\sigma \sqrt{T-t}}. \]

As shown by Crouhy, Galai & Mark (2000), there is a simple relationship between risk-neutral probability of default and the physical probability of default:

\[ \pi_Q = N \left( N^{-1}(\pi_P) + \frac{\mu - r}{\sigma} \sqrt{\tau} \right). \]

The idea that equity is a call option is one of the key ingredients behind the success of Kealhofer, McQuown, and Vasicek (KMV) in producing expected default frequencies (EDF), which are in part implied by equity prices.\(^5\)

\(^5\)KMV use a slightly different model and an extensive database on actual defaults to estimate EDF in the real-world probability measure.
Although Merton (1974) did not derive the endogenous default boundary or explicitly introduce corporate taxes to study the optimal capital structure in his paper, few could argue that his framework did not lay down the analytical machinery to take these logical next steps. Indeed, in later sections, Merton lays out the determination of the optimal call boundary (for callable debt) and discusses the effect of bankruptcy costs and taxes on the value of the borrowing firm. In fact, Black & Cox (1976) demonstrate how the endogenous bankruptcy boundary could be derived in their paper, which was published shortly after Merton (1974). Much later, Leland (1994) introduced corporate taxes and developed a model that derived both the endogenous default boundary as well as optimal capital structure. Together, these three papers provide the basis for thinking about trade-off theory of capital structure, credit spreads and how they are informed by economic environment, corporate debt covenants, agency costs, investment decisions, bankruptcy codes, etc. I provide a brief review of Black & Cox (1976) and Leland (1994) in this section, before delving into the rest of the literature.

1.1. Black & Cox (1976)

Black & Cox (1976) represents a significant extension of Merton (1974) in many respects. First, they model safety covenants, which allow the creditors to take over the borrowing firm when its value falls below a certain threshold: \[ V_B = B e^{-\gamma(T-t)} \], where \( T - t \) is the stated time to maturity of the debt contract. This makes the default time uncertain, ex ante. Equity is no longer a European call option on the borrowing firm’s assets. Rather, equity is a down-and-out call option on the firm’s assets, implying that the presence of safety covenants transfers value from equity to creditors and allows the issuance of debt at a tighter spread, ex ante. Second, they consider senior and subordinated debt and interpret their valuations. Third, they develop an approach to valuing coupon-paying risky bonds with the endogenous default boundary, with and without asset sale restrictions, and demonstrate that safety covenants and asset sale restrictions can serve to improve the creditor’s rights and increase the debt values.\(^6\) Merton (1974) priced a risky coupon bond assuming that assets can be sold to fund coupon payments. Black & Cox (1976) explicitly compare Merton’s formula and make precise the valuation consequences of imposing safety covenants and asset sale restrictions. As in the Merton model, we can compute the risk-neutral probability of default at \( t \) for any finite horizon \( s < T \). This is shown below:

\[
\pi_Q = N \left[ \frac{\ln \left( \frac{V_B}{V_t} \right) - a_1(s-t)}{\sigma \sqrt{s-t}} \right] + \left( \frac{V_B}{V_t} \right)^{2a_0} N \left[ \frac{\ln \left( \frac{V_B}{V_t} \right) + a_1(s-t)}{\sigma \sqrt{s-t}} \right],
\]

where

\[ a_1 = r - \delta - \gamma - \frac{1}{2} \sigma^2 \]

and

\[ a_0 = \frac{a_1}{\sigma^2}. \]

\(^6\)It is somewhat surprising that, having found the endogenous default boundary and the firm value at that boundary, they did not take the logical next step to determine the value-maximizing leverage decision. Nearly 20 years later, Leland (1994) would take that important next step.
Black & Cox (1976), in a later section of their paper, provided a way to endogenize the default boundary, allowing for default prior to maturity, and formally analyzed the role of covenants on spreads. They show that the value of a perpetual debt, subject to default risk, can be expressed as follows:

\[ D(V) = \frac{C}{r} + \left[ V_B - \frac{C}{r} \left( \frac{V}{V_B} \right)^{-\frac{1}{2}} \right], \]

where \( V_B \) is the default boundary. They then find the optimal default boundary by maximizing the value of equity and show that

\[ V_B = \frac{C}{r + \frac{\sigma^2}{2}}. \tag{5} \]

It is instructive to compare the debt pricing formula of Black & Cox with that which Merton (1974) found for default-risky, coupon-paying perpetual debt, which is reproduced below:

\[ D(V) = \frac{C}{r} \left[ 1 - \frac{2 \left( \frac{V}{V_B} \right)^{-\frac{1}{2}}}{\Gamma \left( 2 + \frac{2r}{\sigma^2} \right)} M \left( 2 + \frac{2r}{\sigma^2}, 2 - \frac{2C}{V_B} \sigma^2 \right) \right], \]

where \( \Gamma(\cdot) \) is the gamma function and \( M(\cdot, \cdot, \cdot) \) is the confluent hypergeometric function. Intuitively, the Merton (1974) model says that the value of a default-risky coupon bond is simply the value of an otherwise identical default-free perpetual, \( \frac{C}{r} \), multiplied by the risk-neutral state price of a claim that pays $1 in states where there is no default. Once the coupon debt pricing problem is interpreted this way, it becomes very transparent that the Black & Cox (1976) formula for default-risky coupon bonds is a natural extension of Merton’s (1974) with an endogenous default boundary \( V_B \).

Bielecki & Rutkowski (2002) have generalized Black & Cox’s (1976) model to study the pricing of coupon bonds, optimal capital structure, and other types of safety covenants.

### 1.2. Leland (1994)

In an important contribution, Leland (1994) took the next major step in generalizing the structural models of default by explicitly introducing taxes and bankruptcy costs. This allowed, for the first time, a formal characterization of optimal capital structure, debt capacity, and credit spreads in a classic trade-off model. To do this, Leland explicitly introduced corporate taxes and bankruptcy costs, which may be interpreted as liquidation costs. Thus, he formalized the trade-off framework and provided a way to determine both (a) the optimal default boundary and (b) the value-maximizing optimal capital structure. Leland thus relaxed assumption A.6 in Merton’s model.

With taxes and bankruptcy costs, the value of the borrowing firm in Leland’s model can be expressed as follows:

\[ \nu(V_0) = V_0 + \frac{\tau C}{r} \left[ 1 - \left( \frac{V}{V_B} \right)^{-\frac{1}{2}} \right] - \alpha V_B \left( \frac{V}{V_B} \right)^{-\frac{1}{2}}. \]

The equity value in Leland’s model is given by

\[ E(V_0) = V_0 - (1 - \tau) \frac{C}{r} + \left[ (1 - \tau) \frac{\tau C}{r} - V_B \right] \left( \frac{V}{V_B} \right)^{-\frac{1}{2}}. \]
Leland then (conceptually) considered the problem in two steps: First, determine the optimal $V_B$ by maximizing the equity value of the firm. Second, determine the optimal leverage by maximizing the value of the firm. Leland shows the optimal default boundary as

$$V_B = \frac{(1 - \tau)C}{r + \frac{\sigma^2}{2}}.$$  \hfill (6)

At this boundary, equity value goes to zero. Kim, Ramaswamy & Sundaresan (1993) had proposed a default boundary when the firm runs out of cash, but this admits the possibility that the equity value at the default boundary may still be positive. Leland’s endogenous default boundary in Equation 6 is a natural generalization of Black & Cox’s (1976) boundary in Equation 5.

Leland (1994, p. 1230, section III) then derives the optimal capital structure in closed form, and for the case of unprotected debt, he concludes the following: “Leverage of about 75 to 95 percent is optimal for firms with low-to-moderate levels of asset value risk and moderate bankruptcy costs. Even firms with high risks and high bankruptcy costs should leverage on the order of 50 to 60 percent, when the effective tax rate is 35 percent.” Leland goes on to examine the case of debt that is protected by positive net worth covenant and the optimal leverage associated with such a debt contract. A key insight here is that the optimal leverage associated with protected debt is lower than the one associated with unprotected debt. Leland suggests that the agency costs in general and the asset substitution incentives in particular may encourage the use of protective debt.

### 2. STYLISTED EMPIRICAL FACTS AND THE EVOLUTION OF RESEARCH IN STRUCTURAL MODELS

The core of the structural models of default described in Section 1 is concerned with firm-specific variables that govern the decision to default. These variables, such as asset volatility, leverage, payout ratios, etc., are critical variables but not the only variables governing default. Moreover, the models are set in a risk-neutral framework, making the connection between risk aversion and credit spreads much less direct than should be the case. A substantial body of research in asset pricing has shown that risk aversion (and hence the risk premia) are time varying, and episodes of flight to safety occur when there is a macroeconomic crisis. It is also well known that there is a link between business cycles and defaults: The propensity of firms to default is much higher in recessions than in expansions. It is empirically the case that the recovery rates (conditional on default) are much lower in recessions than in expansions. The models described above treat liquidation as the only option available to the borrower. In reality, there are many more possibilities for debt recontracting.

The factors described above and several stylized facts on credit spreads, observed renegotiation of debt contracts, time-series, and cross-sectional variations in capital structure have motivated the evolution of structural credit-risk models. I summarize some of the key stylized facts in the following bullet points:

- Short-term credit spreads are not negligible. They can spike up dramatically in a crisis. A classic example would be the spread between London interbank offered rates (LIBOR) and the overnight indexed swap (OIS) rates. The normal short-term LIBOR-OIS spreads are of the order of a few basis points in normal times, but in a crisis can easily exceed 100 basis points, as was the case during the credit crisis of late 2007. This sets a tough standard for structural models: Not only must such models produce positive short-term spreads in normal times, but they must be flexible enough to generate fairly high short-term spreads in a crisis state, suitably defined.
Credit spreads implied by structural models (by calibrating these models to historical default rates and recovery rates), across rating categories, are much lower than what the data show. This is referred to as the credit spread puzzle. One of the early papers to document this was Jones, Mason & Rosenfeld (1984). More recently, Huang & Huang (2012) reached a similar conclusion after calibrating several structural models to historical data on defaults and recovery rates.

Debt contracts are frequently renegotiated, and often creditors settle for distressed exchanges rather than forcing borrowers into a bankruptcy process. Roberts & Sufi (2009) provide empirical evidence on actual renegotiations of debt contracts. One of the categories of defaults used by the rating agencies is distressed exchanges in which creditors accept a new set of claims in exchange for their original debt contracts.

Static structural models predict leverage ratios that are much higher than what one observes in real life. This has been noted by Leland (1994) and other scholars. The ability of the firm to alter its capital structure dynamically might be an important consideration.

Leverage ratios tend to be fairly stable over time. The usual determinants of leverage in structural models are not necessarily useful in explaining the time-series variations in spreads. See Lemmon, Roberts & Zender (2008) for an elaboration of this point.

There is a subset of firms in the economy that use very little or no leverage. Korteweg (2010) documents this evidence.

Firms tend to use cash and loan commitments (in addition to short-term and long-term debt) as an integrated capital structure strategy.

Firms may be combining their investment strategies with their capital structure strategies. For example, market makers in debt securities may tend to use repo financing more intensively than other forms of financing.

Firms make their decisions in a principal agent setting, wherein managers, equity holders (borrowers), and creditors may have very different objective functions. The resulting agency problems may have significant implications for the optimal capital structure decisions and optimal contracting decisions. Leland (1998), DeMarzo & Fishman (2007a, b), DeMarzo & Sannikov (2006), and others have pursued this line of attack.

A quick perusal of these bullet points suggests that the structural models reviewed above are in need of major modifications to bring them closer to the stylized facts present in the data. I now review the structural models that have attempted to explain these stylized facts.

In each section, I review a set of papers that describes the types of advances made and the additional insights provided. Additional references can be found in the Related Resources section.

3. THE ROLE OF JUMP RISK IN STRUCTURAL MODELS

The structural models described above generally imply that as the time to debt maturity approaches zero, the credit spreads approach zero as well. Although it seems a safe assumption that the option value of default goes to zero as the time to maturity approaches zero (in structural models, with the asset values evolving according to a GBM process), this is contrary to the empirical finding that short-dated debt securities do trade at significant spreads. Money market debt instruments such as commercial paper, bankers acceptances, repo agreements, LIBOR, etc., do trade at significant spreads. Moreover, the level of leverage suggested by these models is much higher than what one observes in real life. Merton (1976) developed a simple model of options pricing when the underlying stock prices are governed by a mixed diffusion-jump process. It is easy to combine that insight with Merton (1974) to develop a structural model of default in which
short-term spreads will be nontrivial. One branch of literature has done precisely this. This literature relaxes assumption A.8 in Merton’s model.

Models by Zhou (2001), Hilberink & Rogers (2002), and Chen & Kou (2009) fall into this category. Of these models, the latter two provide an explicit treatment of optimal capital structure. Hilberink & Rogers (2002) extend the structural model with endogenous default to Levy processes with one-sided (downward) jumps and characterize the credit spreads and optimal capital structure. Jumps allow the possibility of uncertain recovery upon default, and it succeeds in generating reasonable levels of short-term credit spreads.

Chen & Kou (2009) use a double-exponential jump process and study the behavior of credit spreads and optimal capital structure. The optimal capital structure implications of Chen & Kou (2009) in the presence of jump risk are intuitive: They show that with jumps, the optimal level of debt used can be rather low, especially for firms with high operating risk. This is consistent with the fact that many high growth firms tend to have very low levels of debt.

Another approach to reconcile significant short-term spreads is to rely on incomplete accounting information as done in Duffie & Lando (2001). In this approach, the underlying value process is not observable, and the investor must rely on imperfect accounting information. This approach, however, assumes that equity is not traded and thus may be more relevant for private firms.

4. CORPORATE CASH BALANCES AND LOAN COMMITMENTS AS A PART OF CAPITAL STRUCTURE

Another focus of structural models of capital structure is the empirical regularity that corporations hold cash (in some cases very large amounts) and cash equivalents and credit lines, while simultaneously borrowing in public debt markets. The leverage ratios of nonfinancial corporate entities turn out to be much lower than what very basic structural models of default tend to predict.

Anderson & Carverhill (2012) explore the issue of corporate cash balances in an integrated structural model of capital structure with mean-reverting earnings. They examine the dynamically optimal cash holding and leverage policy in a firm with given assets in place and long-term debt outstanding. In their model, the firm chooses its dividend, short-term borrowing, and share issuance policies. They show that the liquidity level is related to the long-term capital structure and is adjusted dynamically to reflect the state of the economy. Their paper’s key characterization of corporate liquidity is the following: The optimal cash balance reaches a peak near the long-term average earnings. When the conditions improve beyond the average, dividends are paid and the cash balance is kept at the target level. When conditions decline below the long-term rate, the optimal cash balance increases by trading off on dividend payouts. Equity issuance occurs in the lower limit.

Asvanunt, Broadie & Sundaresan (2011) argue that firms use cash balances and credit lines to enable access to liquidity in bad states when credit markets may be unavailable to the firm. They show that equity issuance as a strategy for getting liquidity is suboptimal. Ignoring future investment opportunities and share repurchases, this paper shows that carrying positive cash balances for managing illiquidity is in general inefficient relative to entering into loan commitments. This is because cash balances may (a) have agency costs; (b) reduce the riskiness of the firm, thereby lowering the option value to default; (c) postpone or reduce dividends in good states; and (d) tend to inject liquidity in both good and bad states. Loan commitments, however, (a) reduce agency costs and (b) permit injection of liquidity in bad states as and when needed. They show that loan commitments can improve overall welfare and reductions in spreads on existing debt for a broad range of parameter values.
DeMarzo & Sannikov (2006) demonstrate loan commitments’ integral role in the capital structure with their examples of principal-agent problems underlying firms, such as managers diverting resources for private benefit and/or not expending the optimal effort.

5. DYNAMIC CAPITAL STRUCTURE

An important class of models has examined the following question: If the firms are in a position to optimize their capital structure decision at each instant, how would they go about it? This is an important question, as the ability to modify the capital structure at a later point in time may serve to inform on the optimal capital structure now.

Fischer, Heinkel & Zechner (1989) (FHZ henceforth) provided an early characterization of dynamic capital structure. Their model is predicated on the following premises: First, the value of an optimally levered firm can be greater than an otherwise identical unlevered firm by the amount of the recapitalization costs needed to lever up. Second, an optimally levered firm should provide a fair rate of return, but an otherwise identical unlevered firm must earn a return lower than the fair rate. With these assumptions in place, they examined the dynamic capital structure problem facing the firm in a trade-off setting, by modeling recapitalization costs and treating the firm’s investment decision as being exogenous. A key insight of this paper is that a typical firm has a range of capital structure values within optimally chosen (upper and lower) boundaries. This range will naturally depend on the characteristics of the firms as well as their recapitalization costs. Their approach delivers an optimal policy that resembles the static \((S, s)\) inventory policies.

The literature has since evolved in the traditional framework, and Goldstein, Ju & Leland (2001) (GJL henceforth) offer reasons as to why the concerns expressed by FHZ may not be that significant. GJL consider a dynamic strategy in which firms can lever up when their earnings before interest and taxes (EBIT) go up, but may not lever down when EBIT goes down. GJL’s model also predicts that there will be a range of debt ratios for which the optimal response is to maintain the current debt level. But in GJL’s setting, in addition to the default boundary \(V_B\), there is also an upper boundary \(V_U\) at which the firm will optimally sell a larger debt issue. GJL invoke a scaling assumption, which works intuitively when the underlying assets are modeled through a GBM process. The scaling assumption means that if two otherwise identical firms differ in the size of their assets by a factor \(g\); then the optimal coupon and the optimal default boundary will also differ by the same factor \(g\). This scaling property is shown clearly in Figure 1 below, which is reproduced from GJL’s paper.

In GJL’s paper, no downward adjustments in debt levels were permitted. In reality, in bad states, firms may wish to de-lever either voluntarily through renegotiations of the sort that I discuss in the next section, or involuntarily when creditors refuse to roll short-term debt over. In addition, GJL assume that the investment policy is exogenous. GJL’s model delivers higher credit spreads and leverage as compared with its static counterpart.

Titman & Tsyplakov (2007) consider a model in which the firm can alter both its financial structure and its investment policies over time. One of the concerns of this paper is to take into account the interactions between the financial distress and the agency conflicts of creditors and equity holders. Due to the complexity of the issues that are analyzed, numerical methods are used to solve the model. The paper shows that the agency conflicts can lead the firm to choose a conservative, initial level of capital structure. In their model, agency conflicts push the firms away from their target debt ratios, whereas the presence of financial distress costs increases the tendency to move toward the target debt ratios. The authors calibrate the model parameters to stylized facts and provide additional empirical validation of the implications of their model.
Ju & Ou-Yang (2006) offer one of the few models of dynamic optimal capital structure with stochastic interest rates. Using a mean-reverting interest rate process, they contend that the firm will choose the maturity of the debt (that it will roll over when it is solvent) as a trade-off between the gains associated with dynamically adjusting the leverage and the transactions costs associated with that dynamic adjustment. They also show that the spot rate of interest is no longer a factor in determining the capital structure; rather, it is the long-run mean of the interest rate process that matters. Their model under some parametric assumptions delivers an optimal leverage of 35%, which is lower than many static models of capital structure.

The dynamic models of capital structure discussed here take the investment decisions as given. Below, I review models that allow the investment decisions to emerge endogenously in a structural setting. The models here generally have the following implications: The optimal leverage predicted by the dynamic models is generally lower than otherwise similar but static models. The credit spreads implied by these models are still well below what one observes in real life. Empirical work by Strebulaev (2007) provides tests of dynamic capital structure models and directions for further research in this area. Strebulaev (2007) shows that the model leverage and empirical leverage exhibit very different cross-sectional properties at refinancing triggers.

6. STRUCTURAL MODELS WITH RICHER REORGANIZATION POSSIBILITIES

Shortly after the publication of Merton (1974), several papers explored how his model could be extended to reconcile the fact that firms in real life have many ways of dealing with financial distress rather than filing for Chapter 7 under the bankruptcy code and thus resorting to liquidation. This assumption is maintained in Merton (1974), Black & Cox (1976), and Leland (1994). In fact, empirical evidence suggests that firms have a much richer strategy space in resolving financial distress. From an empirical standpoint, defaults are typically classified into (a) payment defaults or delinquencies, (b) distressed exchanges (wherein creditors exchange their original debt...
claims for a new package of securities with a lower value), and (c) bankruptcy (Table 1). The last category can include Chapter 11 filing, prepackaged bankruptcy filings, Chapter 7 filings, etc. In 2008, Moody’s reported the following statistics concerning the type of defaults.

Roberts & Sufi (2009) perform an empirical analysis using a large sample of private credit agreements between US publicly traded firms and financial institutions. They show that more than 90% of long-term debt contracts are renegotiated prior to their stated maturity. Further, they show evidence that such negotiations may result in large changes to the amount, maturity, and pricing of the contract and occur relatively early in the life of the contract. But they conclude that they are rarely a consequence of distress or default.

Several papers have developed models in which the lender and the borrower can operate within the bankruptcy code in which they can resolve financial distress through filing for Chapter 11, or Chapter 7, or by renegotiating their original debt claims under the shadow of either Chapter 7 or Chapter 11. These models have implications for both optimal capital structure and credit spreads. They also make structural models more realistic in terms of renegotiations that occur in real life for resolving financial distress. Anderson & Sundaresan (1996) present a discrete time model in which lenders and creditors behave in a noncooperative setting: Borrowers make take-it-or-leave-it debt service payments to creditors, and the endogenous bankruptcy boundary is determined in the context of an extensive form game defined by the terms of the debt contract and the applicable bankruptcy code. This model produces much higher credit spreads, even with moderate liquidation costs. In addition, it generates deviations from the absolute priority rule. The discrete time setting allows the authors to examine the design parameters of debt contracts (such as sinking funds) and to work with finite maturity debt. The equilibrium debt service and the bankruptcy trigger arise endogenously as outcomes of the noncooperative game between lenders and borrowers. Later, Anderson, Sundaresan & Tychon (1996) generalized this framework into continuous time.

Mella-Barral & Perraudin (1997) also develop a similar idea in continuous time with perpetual debt formulation: In their setting, they work with the optimal shutdown policy of the firm, which faces stochastic prices for its output and a constant flow rate of costs. They formulate strategic debt service in their setting.

Fan & Sundaresan (2000) develop a more generalized formulation of the bargaining problem, which is embedded into the structural framework of Merton (1974). In one bargaining formulation, they propose exchange offers: debt for equity swaps, wherein creditors exchange their old debt claims for equity. Here, the firm after the exchange offer becomes an all-equity firm. In the other formulation, they propose a strategic debt service, wherein the creditors and borrowers renegotiate the debt service payments, with the understanding that the borrowers lose tax benefits when they do not service the debt contract as promised. They define a bargaining power parameter \( \eta \) of borrowers, such that at the endogenously determined reorganization point \( V_B \), the firm is shared between the creditors and borrowers as follows: \( D(V_B) = [1 - \theta(\eta)]V_B \) and \( E(V_B) = \theta(\eta)V_B \). When equity holders have all the bargaining power, \( \eta = 1 \) and \( \theta(\eta) = \alpha \), where \( \alpha \) is the proportional liquidation costs under Chapter 7 of the bankruptcy code. However, if creditors have all the

<table>
<thead>
<tr>
<th>Initial default type</th>
<th>Percent of issuer accounts</th>
<th>Percent of dollar volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy</td>
<td>45.50%</td>
<td>81.70%</td>
</tr>
<tr>
<td>Distressed exchanges</td>
<td>22.80%</td>
<td>13.20%</td>
</tr>
<tr>
<td>Payment default</td>
<td>31.70%</td>
<td>5.20%</td>
</tr>
</tbody>
</table>
bargaining power, then $\eta = 0$ and $\theta = 0$. The Nash solution to this bargaining problem leads to the endogenous reorganization boundary as follows:

$$V_B = \frac{(1 - \tau)C}{r + \frac{\sigma^2}{2}(1 - \eta \alpha)}.$$  \hfill (7)

Comparing Equations 5, 6, and 7, we can see how the endogenous restructuring boundary can change with bargaining considerations. Clearly, when $\eta = 1$, equity holders get all the power, and they would restructure early by pushing $V_B$ high. In this case, $\alpha$, the costs of liquidation, matter, unlike in Equations 5 and 6. However, if $\eta = 0$, the boundary is the same as in Leland (1994), although the interpretations are very different, as in Leland (1994) $V_B$ represents the liquidation boundary and in this class of models $V_B$ is the restructuring boundary. The bargaining models described above examine restructuring under the shadow of liquidation.

Francois & Morellec (2004) examine the bargaining problem when the outside option facing the borrowing firm is filing for Chapter 11 rather than liquidation. They model the exclusivity period, $d$, in the Chapter 11 process, which is the length of time that the borrowing firm has to try to restructure its claims and emerge as a going concern. Their model offers the following insights: The optimal default threshold associated with private workouts (in which the exclusivity period is infinite) is greater than the default threshold associated with Chapter 11 filings [$V(d)$, $d \in (0, \infty)$], which in turn is greater than the default threshold associated with Chapter 7 filings [$V(d = 0)$]. They show that the liquidation probability decreases with $d$, and the shareholders’ incentives to default early increase with $d$. Their model implies that the optimal capital structure depends on the length of the exclusivity periods and the bargaining power conferred by the bankruptcy code to the borrowers: For low to moderate bargaining power enjoyed by borrowers, the higher the length of the exclusivity period $d$, the higher will be the leverage. Their model finds $V_B$ by maximizing the equity values and assumes that the contracted coupon payments are forgiven while the firm is under the exclusivity period.

Galai, Raviv & Wiener (2007) model the severity of default explicitly by making $V_B$ time dependent as in Black & Cox (1976). The firm liquidates when the weighted average of the distances from $V_t$ to $V_B$ during the history of the firms defaults exceeds a certain value.

Broadie, Chernov & Sundaresan (2007) admit both a reorganization boundary and a liquidation boundary and contrast the first-best outcomes, with the alternatives of equity maximization and debt-value maximization. Their model accounts for real-life features such as automatic stay, grace period, and debt relief. A key result is that the first-best outcome is different from the outcome that obtains when equity holders choose $V_B$ (the Chapter 11 boundary). Equity-maximizing $V_B$ is much higher and thus appropriates value ex post, at the expense of creditors. Their result is in contrast to Leland (1994), who shows that, with the liquidation option only, the first-best outcome coincides with the equity-maximizing outcome. Broadie, Chernov & Sundaresan (2007) argue that by giving creditors the right to select the length of the grace period once the firm is taken to Chapter 11 by the equity holders, the outcome can be moved much closer to the first-best outcome.

### 7. AGENCY COSTS IN STRUCTURAL MODELS

Mello & Parsons (1992) was one of the early papers to explicitly treat agency costs. The authors observe that in Merton (1974), the value of the firm is independent of its capital structure. They construct a model in which the value of the firm is an endogenous function of the firm’s product market and the firm’s operating and investment decisions. Managers are assumed to make
decisions based on equity maximization. This setup provides a link between the financial structure of the firm and the value of the firm, through the channel of operating decisions. The agency costs in their model can then be quantified by comparing the value of the firm under the first-best outcome with the values obtained under the alternative.

Leland (1998) examines the agency costs associated with the asset substitution incentives in a classic structural setting. By so doing, he integrates the investment decisions with the financing decisions. Leland uses a structural framework in which the borrowing firm may increase the risk of assets in place (i.e., perform asset substitution) after debt capital is raised. This is the classic risk-shifting propensity that leads the equity holders to have an incentive to over-invest in risky assets and transfer wealth from creditors to borrowers. Leland finds that the agency cost of over-investment is relatively small and does not have a strong effect on optimal leverage. Leland assumes that a firm’s asset value is exogenous and therefore independent of financial structure.

Mauer & Sarkar (2005) model a firm that has an option to invest in a project. The cost of exercising the option is partially financed with debt. The contractual terms of the debt contract are set in advance. Their model assumes that equity holders decide when to accept the project, and creditors set the terms of the debt contract. Creditors assume that equity holders will choose an exercise trigger that maximizes equity value. The optimal capital structure then depends on a trade-off between interest tax shields, expected bankruptcy costs, and the agency costs associated with a conflict between stockholders and creditors over the timing of the exercise of the investment option. They find that the over-investment effects are much stronger in their setting.

Recently, several papers have explicitly modeled the optimal contracting and capital structure problem in a principal-agent setting and the agency costs that arise in that setting. Notable papers in this context are ones by DeMarzo & Fishman (2007a,b). I illustrate the approach here with a brief summary of DeMarzo & Sannikov (2006). They study in a continuous-time setting the consequences of agency costs due to the ability of the borrower to take hidden actions (diverting cash flows from private consumption or not expending the requisite effort), which may distort the cash flows observed by outsiders. This implies that the outsiders cannot make a determination as to whether an observed low realization of cash flows is due to the hidden actions or due to a bad draw. They show that the optimal contract that the outsiders enter into with the borrower then features a line of credit, debt, and equity. In this sense, their paper extends the structural model to reflect moral hazard issues in deriving the optimal capital structure in the presence of contingent debt contracts (line of credit).

8. STRUCTURAL APPROACH TO BANKS’ CAPITAL STRUCTURE

In the aftermath of credit crisis of 2008, there is a renewed interest in how to design the capital structure of banks so that the costs associated with their financial distress and bankruptcy can be internalized by the creditors rather than by the taxpayers. Regulators in different countries have proposed that banks should evaluate the use of contingent convertible debt or CoCos, which convert to equity automatically when a prespecified trigger level is reached. This has been a hot topic from a regulatory perspective: The bank losses and the need to recapitalize banks (often at the expense of taxpayers) in the aftermath of a credit crisis has led legislators and regulators to suggest the use of contingent capital (CC) to internalize the losses of banks in a crisis. Such a trigger may be based on accounting information or market prices, or may be determined by regulators. The goal is to ensure that banks are automatically recapitalized in a crisis and are freed from debt service payments in bad states of the world. Recently, several papers have utilized the structural framework of Merton to examine the pricing and desirability of using CC.
Glasserman & Nouri (2012) use Merton’s framework to study the properties of CC when the trigger is based on an accounting measure, such as the book value of assets. They show that the conversion of CC to equity would occur each time the trigger is reached, by an amount just sufficient to meet the capital requirements. Consistent with the spirit of CC, they assume that the trigger point will be above the bankruptcy barrier and derive tractable formulations of yield spreads.

Albul, Jaffee & Tchistyi (2010) also use the structural framework to address CC. They also focus on capital structure decisions. Assuming that the trigger can be placed on asset values, they derive closed-form solutions for the capital structure, which includes CC. To keep matters simple, they assume that debt securities have infinite maturity. Their analysis suggests that CC may have the potential to provide most of the tax shield benefits of straight debt while providing the same protection as equity. They conclude that banks should substitute CC for straight debt in their capital.

Pennacchi (2010) uses a structural approach to model a bank that has short-term deposits, common equity, and CC. He abstracts from modeling long-term bonds for tractability reasons. The short-term deposits are always assumed to be priced at par. Pennacchi places the trigger on the ratio of the asset value to the combined value of the CC and equity, under the assumption that bankruptcy is costless. This ensures a unique equilibrium in stock and CC prices because the ratio is independent of the conversion of CC.

Pennacchi, Vermaelen & Wolff (2011) use the framework developed by Pennacchi (2010) but propose a new design called call option enhanced reverse convertible or COERC. COERC allows the bank’s original shareholders to buy back the shares at the par value of the bond. They assume that MM results hold, and hence do not model costs of financial distress, which is important in the design of bank debt.

Sundaresan & Wang (2012), argue that CC based on equity market triggers may either lead to multiple equilibria or no equilibrium, unless the design guarantees that there is no value transfer between bank equity and contingent debt at conversion. They use a continuous-time framework with discrete monitoring of conversion triggers to show that the no value transfer condition is both necessary and sufficient to guarantee a unique equilibrium. They also note that the IRS in the United States currently does not recognize contingent debt interest payments to be tax deductible.

9. MACROECONOMY, DEFAULTS, AND CAPITAL STRUCTURE

It is well known that default rates and recovery rates depend on the macroeconomic state of the economy. I reproduce below (Figure 2) the evidence in this regard from Chen (2010).

The relationship between the macroeconomic state of the economy on the one hand and the frequency and severity of defaults on the other hand comes through in this picture. Therefore, explicitly incorporating the macroeconomic state into Merton’s framework is clearly an important intellectual pursuit. Several papers have attempted to do precisely that.

Hackbarth, Miao & Morellec (2006) model firms’ cash flows to depend on the macroeconomic state of the economy (high and low). They show that the firm’s leverage decisions are then influenced by the business cycle state of the economy. Firms make their default decision in one macroeconomic state, cognizant of their optimal behavior in the other macroeconomic state. They predict that the market leverage of firms will be countercyclical.

Chen, Collin-Dufresne & Goldstein (2009), Chen (2010), and Bhamra, Kuehn & Strebulaev (2010) build models to explain credit spreads and/or capital structure by linking investor’s preferences explicitly with default risk. The approach of Chen, Collin-Dufresne & Goldstein (2009) is to use a consumption-based asset pricing model to understand credit spreads. They use
the habit formation model to generate strong countercyclical risk prices. To this recipe they add exogenously imposed countercyclical asset value default boundaries. Together, these modeling tools can generate high credit spreads. Their model does not explicitly link the macroeconomic state of the economy with firms’ financing and default decisions. This is the focus of Chen (2010) and Bhamra, Kuehn & Strebulaev (2010).

Chen (2010) builds a structural model of default with explicit linkages to business cycle conditions in the economy. His model has the following ingredients: It recognizes that recessions are intuitively to be associated with high marginal utilities. This has the implication that default losses that occur during recessions will have a bigger impact on the marginal investor. The other piece of the model is about the way recessions affect firms: In recessions, the cash flows of firms may be expected to grow slowly and may be expected to become more volatile and more correlated with the market. Chen (2010) is then able to show that these two factors can make defaults more likely in recessions. In his model, liquidating assets during recessions can be costly, which results in higher default losses. His model is able to generate high credit spreads and low leverage ratios of Baa-rated firms in the data.

Bhamra, Kuehn & Strebulaev (2010) examine the impact of macroeconomic risk on the aggregate dynamics of cross-sectional capital structure. Theirs is a representative agent model, which endogenizes the link between actual and risk-neutral default probabilities explicitly. One of

Figure 2
Default rates, credit spreads, and recovery rates over the business cycle. (a) Moody’s annual corporate default rates during 1920–2008 and the monthly Baa-Aaa credit spreads during 1920/01–2009/02. (b) The average recovery rates during 1982–2008. The long-term mean recovery rate is 41.4%, per Moody’s data. Shaded areas are NBER-dated recessions. For annual data, any calendar year in which a recession, as defined by NBER, occurred over at least five months is treated as a recession year. Figure reprinted with permission from Chen (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. J. Finance 65(6):2171–212. © 2010 by Wiley. All rights reserved.
their conclusions is that the optimal financing decisions are more conservative in bad times when firms refinance their obligations. They show that the default boundary is countercyclical. The aggregate dynamics of the capital structure is countercyclical.

10. ENDOGENOUS INVESTMENTS

Several papers have attempted to integrate investment decisions of firms with their financing decisions. This branch has integrated the real options literature with the capital structure theory to develop insights about how mature firms may differ in their capital structure choice from young firms with lots of growth options. These models examine the problem of debt overhang, priority structure of debt, and the possibility of renegotiations. Barclay & Smith (1995) find evidence that firms with more growth options tend to use less long-term debt and that large, regulated firms tend to use more long-term debt. Barclay, Smith & Morellec (2006) show an interesting link between growth options and debt capacity: They argue that when the value of the firm increases with additional growth options, the under-investment costs of debt increase and the free cash flow benefits of debt decline. As a result, growth options cause a reduction in the optimal amount of total debt. Sundaresan & Wang (2007) argue that the greater the borrower’s bargaining power, the lower is the debt capacity and less likely is the probability that growth options will be exercised. Tserlukevich (2008) assumes that financing is frictionless but investment decisions are costly. He shows that the leverage of the firm is negatively related to profitability. His model produces slow but lumpy leverage adjustments even in the absence of frictions on the financing side. Gomes & Schmid (2010) show that, in the presence of financial market imperfections, mature firms tend to be more highly levered. They also tend to have more safe assets. Morellec & Schürhoff (2010, 2011) explore the interactions between capital structure decisions and financing decisions in the presence of asymmetric information and taxation. Bolton, Chen & Wang (2011) emphasize the role of corporate liquidity and risk management in a dynamic model that captures both investment and financing decisions.

11. CONCLUSION AND A POSSIBLE FUTURE RESEARCH AGENDA

This review has attempted to trace the development of the capital structure literature since the publication of Merton (1974). In the main, the literature has evolved in the following ways: First, the literature has progressed in modeling real-life frictions to enhance Merton’s approach. Such frictions include bankruptcy costs, financial distress costs, asymmetric information, moral hazard, agency costs, etc. Second, the literature has integrated institutions, such as bankruptcy codes and private sector loan commitments, to model capital structure. Third, the importance of macroeconomic factors such as business cycles is being explicitly recognized in the modeling of capital structure. Finally, factors such as dynamic adjustments in capital structure, interactions between investment and financing decisions, jumps in the values of underlying firms’ assets, and effects of industry equilibrium considerations are being incorporated into models of capital structure. Finally, the empirical literature on capital structure has blossomed, affording theorists with challenging stylized facts.

What might be some important research questions that structural models of capital structure can address in the next decade? One avenue of research is to bring in more explicitly the shadow banking system that has rapidly evolved as a complementary channel of credit in global markets, but especially so in the United States. Integral parts of the shadow banking systems are debt instruments such as repurchase (repo) agreements (which are collateralized lending and borrowing, with the collateral protected by safe harbor provisions); asset-backed securities, which are...
ubiquitous in consumer finance (credit card receivables, auto loan receivables, student loan receivables, etc.); and so forth. Bringing this research agenda into structural models of default obliges scholars to come to grips with the run risk presented by such short-term lenders, and how that run risk is internalized in the choice of leverage and liability structure. Answering such questions also brings the structural models intellectually closer to the banking literature, while retaining the elegant dynamic aspects of structural models. This is a particularly challenging task as one moves from a single-firm decision to a model of industry equilibrium, which is an important step that must be taken to address the interaction between run risk and capital/liability structure of firms in an equilibrium context. Another task is to explore the manner in which structural models can be improved to address the credit-risk puzzle. Most scholars accept this as a failure of existing structural models of default. But the way this issue has been framed deserves additional scrutiny: The models used to calibrate to the data are single-debt models. In reality, firms use multiple classes of debt. Often the long-term, unsecured debt yield spreads are used in the calibration exercise to examine how well structural models square up with observed credit spreads. Once structural models are enhanced to include short-term, secured debt, the models are likely to produce a much higher spread for long-term unsecured debt as long-term bond investors must internalize the run risk, and the risk of subordination, in the bankruptcy process. Moreover, the calibration exercise places the burden on average recovery rates, without calibrating to the stylized facts such as the ratio of short-term to long-term debt and the variations in the recovery rates over time due to business cycle fluctuations. Finally, incorporating the insights of structural models explicitly into the asset pricing literature may be another ambitious task. Currently, most asset pricing models in economics bring in default through solvency constraints. In such models, there are no defaults in equilibrium! This is a major flaw that deserves to be addressed.

DISCLOSURE STATEMENT

The author is not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

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The literature on structural models of optimal capital structure is huge, and this review is constrained by journal space. It is therefore inevitable that I have overlooked some papers on this topic. I apologize to the authors of these papers in advance. I thank an anonymous reviewer for valuable comments and suggestions.

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