

# How Much Does Household Collateral Constrain Regional Risk Sharing?

Separate Appendix for Non-Separable Preferences

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## 1 Introduction

In the main text, we focus on the case of preferences that are separable between non-housing and housing consumption streams  $\{c_t(s^t)\}$  and  $\{h_t(s^t)\}$ . Here we extend the analysis to the case of non-separable preferences.

We use two approaches to derive testable implications from the model. The first approach links our model to the traditional risk-sharing tests based on linear consumption growth regressions, the workhorse of the consumption insurance literature. That is, we make assumptions on the regional consumption share processes needed to derive a linear consumption growth equation from the model. This equation linearly relates regional consumption share growth and regional income share growth, *conditional on the housing collateral ratio*. We refer to this as the *linear model* (section 4). We estimate the linear risk-sharing regressions for the case of non-separable preferences, both using aggregate collateral measures and regional collateral measures. We confirm the findings of the main text: even with non-separable preferences do we find evidence for a time-varying degree of risk-sharing between U.S. metropolitan areas.

In the second approach (section 5) we fully incorporate the model's non-linear dynamics in the estimation, and we use a simulated method of moments estimation to estimate the non-linear law of motion for consumption shares. We refer to this as the *non-linear model*. This is new approach to structural estimation of non-linear risk-sharing models.

## 2 Setup

The households have power utility over a CES-composite consumption good:

$$u(c_t, h_t) = \frac{1}{1-\gamma} \left[ c_t^{\frac{\varepsilon-1}{\varepsilon}} + \psi h_t^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}},$$

The preference parameter  $\psi > 0$  converts the housing stock into a service flow,  $\gamma$  is the coefficient of relative risk aversion, and  $\varepsilon$  is the intra-temporal elasticity of substitution between non-durable and housing services consumption. Separable utility is a special case where  $\gamma\varepsilon = 1$ . Here we generalize this treatment.<sup>1</sup>

## 3 Data

We additionally use regional rental prices in the analysis. The regional relative price of housing services, defined as the ratio of the Bureau of Labor Statistics' price index for rent to the price index for food, is denoted by  $\rho^i$ .

## 4 Linear Model for Regional Consumption Growth Wedges

Recall our assumption that the growth rate of the regional consumption wedge is linear in the product of the housing collateral ratio and the regional income share shock:  $\Delta \log \hat{\kappa}_{t+1}^i = -\gamma \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i$ , where  $\hat{\kappa}^i$  is region  $i$ 's consumption wedge, in deviation from the cross-sectional average. All growth rates of hatted variables denote the growth rates in the region in deviation from the cross-regional average, and the averages are population-weighted. If we allow for non-separability, this assumption delivers a linear consumption growth equation:

$$\Delta \log \hat{c}_{t+1}^i = \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \left( \frac{\gamma\varepsilon - 1}{\gamma(\varepsilon - 1)} \right) \Delta \log \hat{\alpha}_{t+1}^i - \left( \frac{\gamma\varepsilon - 1}{\gamma(\varepsilon - 1)} \right) \widetilde{m}y_{t+1} \Delta \log \hat{\alpha}_{t+1}^i.$$

where  $\alpha_t^i$  is the ratio of non-housing consumption to total consumption in region  $i$ . The interaction term of regional income share growth with the collateral ratio is familiar from the separable preference case. When preferences are non-separable between non-housing and housing consumption, the region-specific component of expenditure share growth affects region-specific consumption share growth, even if the consumption wedge is zero. When preferences exhibit complementarity ( $\varepsilon < 1$ ), a rental price increase decreases the non-durable expenditure share ( $\Delta \log \hat{\alpha}_t^i < 0$ ). When the region's willingness to substitute over time is bigger than its willingness to substitute between goods ( $\gamma\varepsilon < 1$ ), the region

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<sup>1</sup>These preferences belong to the class of homothetic power utility functions of Eichenbaum and Hansen (1990). Another special case is Cobb-Douglas preferences ( $\varepsilon = 1$ ). The cross-derivative  $u_{ch} > 0$  for  $\gamma\varepsilon < 1$  and  $u_{ch} < 0$  for  $\gamma\varepsilon > 1$ . There is complementarity between non-housing and housing consumption when  $\varepsilon < 1$ . Substitutability arises when  $\varepsilon > 1$ .

decreases its non-durable consumption share. Vice versa, when preferences exhibit substitutability ( $\varepsilon > 1$ ), a rental price increase increases the non-durable expenditure share ( $\Delta \log \hat{\alpha}_t^i > 0$ ). Finally, when the region's willingness to substitute over time is lower than its willingness to substitute between goods ( $\gamma\varepsilon > 1$ ), the region increases its consumption share.

Before we can estimate the linear consumption growth equation, two more steps are needed. First, we restate the consumption growth regression in terms of rental price growth, because we do not have regional expenditure share data. This involves a Taylor approximation. Before we can estimate the linear consumption growth equation, we introduce measurement error in consumption.

#### 4.1 Linear Model

We derive a linear consumption growth equation from our model. For simplicity, we use the region as a unit of analysis. The main text shows how to aggregate up from the household level.

We start from the FONC for optimality of region  $i$ 's optimization problem:

$$\beta^t \pi(s^t | s^0) \xi_t^i(s^t) u_c(c_t^i(s^t), h_t^i(s^t)) = p_t(s^t),$$

where  $p(s^t)$  is the time-zero price of a unit of consumption in node  $s^t$ , and  $\xi_t^i(s^t) = \frac{1}{\kappa_t^i(s^t)}$  is the regional consumption wedge at node  $s^t$ . We define a regions's expenditure share  $\alpha^i$  as the ratio of non-housing expenditures to non-housing plus housing services expenditures.

$$\alpha_t^i \equiv \frac{c_t^i}{c_t^i + \rho_t^i h_t^i}. \quad (1)$$

The equilibrium relative price of housing services  $\rho$  equals the marginal rate of substitution between consumption and housing services:

$$\rho_t^i(s^t) = \frac{u_h(c_t^i(s^t), h_t^i(s^t))}{u_c(c_t^i(s^t), h_t^i(s^t))}. \quad (2)$$

Using the definition of the non-housing expenditure share (1) and the expression for the regional rental price (2), we can write the marginal utility of non-durable consumption as

$$u_c(c_t^i, h_t^i) = (c_t^i)^{-\gamma} (\alpha_t^i)^{\frac{\gamma\varepsilon-1}{\varepsilon-1}}.$$

Substituting this expression in the FONC, and dividing the time  $t+1$  by the time  $t$  FONC, we get:

$$\left( \frac{\xi_{t+1}^i}{\xi_t^i} \right) \left( \frac{c_{t+1}^i}{c_t^i} \right)^{-\gamma} \left( \frac{\alpha_{t+1}^i}{\alpha_t^i} \right)^{\frac{\gamma\varepsilon-1}{\varepsilon-1}} = \frac{p_{t+1}(s^{t+1})}{p_t(s^t) \pi(s^{t+1} | s^t)} \equiv \frac{\hat{p}_{t+1}}{\hat{p}_t} = \frac{m_{t+1}}{\beta}, \quad (3)$$

where  $m_{t+1}$  is the stochastic discount factor.

We conjecture the modified risk-sharing rule for the equilibrium consumption allocation:

$$c_t^i = \frac{\left(\frac{\tilde{\xi}_t^i}{\xi_t^i}\right)^{\frac{1}{\gamma}}}{\tilde{\xi}_t^a} c_t^a$$

Now the consumption weight is the product of the weight  $\tilde{\xi}(s^t)$  and the non-durable expenditure share  $\alpha(s^t)$  raised to the power  $\frac{\gamma\varepsilon-1}{\gamma(\varepsilon-1)}$ :  $\tilde{\xi}_t^i = \xi_t^i (\alpha_t^i)^{\frac{\gamma\varepsilon-1}{\varepsilon-1}}$ . The modified aggregate consumption weight is

$$\tilde{\xi}_t^a(z^t) = \int \sum_{y^t} \frac{\pi(y^t, z^t)}{\pi(z^t)} \tilde{\xi}_t^i(\mu_0, s^t) d\Phi_0. \quad (4)$$

This risk-sharing rule satisfies market-clearing by the definition of  $\tilde{\xi}^a$  in equation (4). Substituting the risk-sharing rule back in equation (3) and imposing that the region is unconstrained ( $\xi_{t+1}^i = \xi_t^i$ ) yields the expression for  $\frac{m_{t+1}}{\beta}$  in equation (5):

$$m_{t+1} = \beta \left(\frac{c_{t+1}^a}{c_t^a}\right)^{-\gamma} \left(\frac{\tilde{\xi}_{t+1}^a}{\tilde{\xi}_t^a}\right)^{\gamma} = \beta \left(\frac{c_{t+1}^a}{c_t^a}\right)^{-\gamma} (\tilde{g}_{t+1})^{\gamma}, \quad (5)$$

When preferences are non-separable, the SDF has the same generic form as for separable preferences, but the weight shock  $\tilde{g}$  in equation (5) is more complicated. It also depends on the entire cross-sectional distribution of expenditure shares (see equation 4).

As with separable preferences, the growth rate  $\tilde{g}_t$  of the aggregate weight process determines the consumption growth of the unconstrained households. If rental prices and expenditure shares were constant over time, it would still be the case that a household's consumption share only increases when it runs into a binding constraint. When expenditure shares move around, a region's consumption weight can also increase when the nondurable expenditure share decreases and housing services and nondurables are complements ( $\varepsilon < 1 < \gamma\varepsilon$ ), because regions want to maintain the optimal mix of non-durable consumption and housing services.

We take logs on both sides of equation (3) and collect the aggregate terms on the right hand side:

$$\Delta \log \xi_{t+1}^i - \gamma \Delta \log c_{t+1}^i + \left(\frac{\gamma\varepsilon-1}{\varepsilon-1}\right) \Delta \log \alpha_{t+1}^i = -\log \beta + \log m_{t+1}$$

We now take the cross-sectional average of the previous equation (by integrating over regions, or for a finite number of regions, summing over regions and dividing by the number of regions), and subtract the above equation from the cross-sectional average. The Euler equation, expressed as growth rates in deviation from the cross-sectional average growth

rate (denoted with hats), i.e. growth rates of shares, is:

$$\Delta \log \hat{\xi}_{t+1}^i - \gamma \Delta \log \hat{c}_{t+1}^i + \left( \frac{\gamma \varepsilon - 1}{\varepsilon - 1} \right) \Delta \log \hat{a}_{t+1}^i = 0, \quad (6)$$

where  $\hat{\xi}^i$  is region  $i$ 's consumption wedge, in deviation from the cross-sectional average.

This linear consumption growth equation is not yet empirically testable, because we do not observe the region-specific consumption wedges  $\hat{\xi}^i$ . In what follows we make simple assumptions that allow us to implement this equation. We assume the housing collateral ratio shifts the consumption wedges *monotonically* between the wedges for two polar cases: perfect risk sharing and autarchy. We now fill in the details. In autarchy, the first polar case,  $\Delta \log \hat{c}_{t+1}^i = \Delta \log \hat{\eta}_{t+1}^i$ . Substituting this into the Euler equation for nondurables, we get:

$$\Delta \log \hat{\xi}_{t+1}^{i,A} = \gamma \Delta \log \hat{\eta}_{t+1}^i - \left( \frac{\gamma \varepsilon - 1}{\varepsilon - 1} \right) \Delta \log \hat{a}_{t+1}^i.$$

In the case of perfect risk sharing, the consumption shares stay constant over time:  $\Delta \log \hat{\xi}_{t+1}^{i,FC} = 0$ . We recall the definition of the re-normalized housing collateral ratio:  $\widetilde{m}y_{t+1} = \frac{my_{t+1}^{max} - my_{t+1}}{my_{t+1}^{max} - my_{t+1}^{min}}$ . It is a measure of collateral scarcity, and it lies between zero and one. We assume the housing collateral ratio shifts the regional consumption wedges between those polar cases. Later, we actually show this is an accurate description of the regional consumption dynamics in the model. Specification (7) encompasses both limit cases and assumes that the shifting is linear:

$$\Delta \log \hat{\xi}_{t+1}^i = \gamma \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i - \left( \frac{\gamma \varepsilon - 1}{\varepsilon - 1} \right) \widetilde{m}y_{t+1} \Delta \log \hat{a}_{t+1}^i, \quad (7)$$

Substituting in this expression for consumption weight growth into the Euler equation, we get an expression in terms of observable variables only. The consumption growth equation to be estimated is:

$$\Delta \log \hat{c}_{t+1}^i = \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \left( \frac{\gamma \varepsilon - 1}{\gamma(\varepsilon - 1)} \right) \Delta \log \hat{a}_{t+1}^i - \left( \frac{\gamma \varepsilon - 1}{\gamma(\varepsilon - 1)} \right) \widetilde{m}y_{t+1} \Delta \log \hat{a}_{t+1}^i,$$

The consumption growth equation involves income growth interacted with the collateral ratio, expenditure share growth and expenditure share growth interacted with the collateral ratio. When preferences between non-housing and housing consumption are separable ( $\gamma \varepsilon = 1$ ), the marginal utility of non-durable consumption is  $u_c(c_t^i, h_t^i) = (c_t^i)^{-\gamma}$ , and the same steps lead to the expression

$$\Delta \log \hat{c}_{t+1}^i = \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i.$$

This is the equation that was the basis of our estimation in the main text.

**Rental Prices Instead of Expenditure Shares** Because we do not have regional data for expenditure share growth, but we do have rental price growth data, we use equation

(2) to reformulate the consumption Euler equation. We have

$$\Delta \log \alpha_{t+1}^i = -\Delta \log (1 + \psi^\varepsilon (\rho_{t+1}^i)^{1-\varepsilon}).$$

Using a linear expansion around  $\rho_{t+1}^i = \rho_t^i$ ,

$$\Delta \log (1 + \psi^\varepsilon (\rho_{t+1}^i)^{1-\varepsilon}) \approx (1 - \varepsilon) \frac{1}{1 + \psi^{-\varepsilon} (\rho_t^i)^{\varepsilon-1}} \Delta \log \rho_{t+1}^i.$$

To obtain a linear Euler equation we expand  $\frac{1}{1 + \psi^{-\varepsilon} (\rho_t^i)^{\varepsilon-1}}$  around  $\rho_t^i = 1$ :

$$\frac{1}{1 + \psi^{-\varepsilon} (\rho_t^i)^{\varepsilon-1}} \approx \frac{1}{1 + \psi^{-\varepsilon}} + (\varepsilon - 1) \frac{\psi^{-\varepsilon}}{(1 + \psi^{-\varepsilon})^2} (\rho_t^i - 1).$$

The resulting linear consumption growth regression is:

$$\Delta \log \hat{c}_{t+1}^i \approx a_1 \widetilde{m} y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + (a_2 + a_3 \widetilde{m} y_{t+1}) \Delta \log \hat{\rho}_{t+1}^i + (a_4 + a_5 \widetilde{m} y_{t+1}) (\hat{\rho}_t^i - 1) \Delta \log \hat{\rho}_{t+1}^i,$$

where it is understood that  $(\hat{\rho}_t^i - 1) \Delta \log \hat{\rho}_{t+1}^i$  is  $(\rho_t^i - 1) \Delta \log \rho_{t+1}^i$  in deviation from its cross-sectional average. The coefficients  $\alpha$  are functions of the underlying structural parameters. In particular:  $a_1 = 1$ ,  $a_2 = \frac{\gamma\varepsilon-1}{\gamma(1+\psi^{-\varepsilon})}$ ,  $a_3 = -a_2$ ,  $a_4 = \frac{(\varepsilon-1)(\gamma\varepsilon-1)\psi^{-\varepsilon}}{\gamma(1+\psi^{-\varepsilon})^2}$ , and  $a_5 = -a_4$ . The parameters satisfy the following non-linear restriction:  $\frac{a_5}{a_3} = \frac{a_4}{a_2} = \frac{\varepsilon-1}{\psi^\varepsilon+1}$ .

**Measurement Error** We take into account measurement error in non-durable consumption. We express observed consumption shares with a tilde and assume that income shares and rental prices are measured without error. Equation (8) is the empirical specification we estimate in section 4:

$$\Delta \log \tilde{c}_{t+1}^i \approx a_1 \widetilde{m} y_{t+1} \Delta \log \tilde{y}_{t+1}^i + (a_2 + a_3 \widetilde{m} y_{t+1}) \Delta \log \hat{\rho}_{t+1}^i + (a_4 + a_5 \widetilde{m} y_{t+1}) (\hat{\rho}_t^i - 1) \Delta \log \hat{\rho}_{t+1}^i + \nu_{t+1}^i. \quad (8)$$

All measurement error terms are absorbed in  $\nu_{t+1}^i$ . The measurement error term is  $\nu_{t+1}^i = -\Delta \hat{b}_{t+1}^i$ .

## 4.2 Estimation of the Linear Model under Non-separable Utility

In this section, we estimate the linear consumption growth regressions that were derived in the previous section for the case of non-separable preferences. We first use aggregate collateral measures, and then regional collateral measures, paralleling the treatment in the main text.

### 4.2.1 Aggregate Collateral Measures

Table 1 shows the estimation results for the consumption Euler equation under non-separability:

$$\Delta \log \tilde{c}_{t+1}^i = a_0^i + a_1 \tilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + (a_2 + a_3 \tilde{m}y_{t+1}) \Delta \log \hat{\rho}_{t+1}^i + (a_4 + a_5 \tilde{m}y_{t+1})(\hat{\rho}_t^i - 1) \Delta \log \hat{\rho}_{t+1}^i + \nu_{t+1}^i,$$

where  $\Delta \log \hat{\rho}^i$  is the rental price growth in region  $i$  in deviation from the cross-regional average (see appendix 4.1). We also estimate, but do not report, a specification with a separate income term. As with separable utility, this ‘Specification II’, uses  $my$  rather than  $\tilde{m}y$ . These results do not add new insights about the results in table 1.

[Table 1 about here.]

First, the point estimates for  $a_1$  are very similar to the ones in the main text for the linear model with separable preferences. Second, a fraction  $a_2 + \frac{a_3}{2}$  of rental price shocks end up in consumption growth, when  $my_{t+1} = 0$  and  $\hat{\rho}_t = 1$ . In row 1 (2), 72 (70) percent of rental price shocks are insured away. In general, the interaction term of rental price growth with the aggregate housing collateral ratio is not significant.

### 4.2.2 Regional Collateral Measures

In 2, we re-estimate the linear consumption wedge equation under non-separable preferences. We find that when regional collateral is more abundant, a larger fraction of regional rental price shocks is insured. For a region whose rental price is at the cross-sectional average ( $\hat{\rho} = 1$ ), an average collateral reading ( $my^i = 0$ ) implies that 51% of regional income shocks are insured away. A one standard deviation increase in  $my^i$  from 0 to 0.1 increases this fraction to 72%, whereas a one standard deviation reduces it to 30 %. Again we find that the fraction of rental price shocks and income shocks that are insured away varies substantially with the amount of housing collateral.

[Table 2 about here.]

## 5 Non-Linear Model

The goal of this section is to devise an estimation strategy that fully incorporates the non-linear consumption dynamics implied by the limited commitment model in the main text. This approach can be seen as an alternative to the calibration approach in the main text. Section 5.2 describes a conditional simulation algorithm that implements the non-linear law of motion for consumption shares. The algorithm is related to indirect inference estimation methods (e.g. Duffie and Singleton (1993), Gourieroux, Montfort and Renault (1992), Gourieroux and Monfort (1996)). It estimates the parameters of the model by

minimizing the distance between cross-sectional moments of regional consumption shares in the model and in the data. We also match a few key asset pricing moments in model and data in order to estimate the preference parameters.

As in section 4, we find that the correlation between income shocks and consumption shocks depends on the housing collateral ratio. The interaction term of the collateral ratio with region-specific income shares is crucial to match the implied consumption moments to the data.

## 5.1 Regional Consumption Share Dynamics

We recall that the equilibrium consumption share process  $\{\hat{c}_t^i\}$  follows a non-linear law of motion: Region  $i$ 's consumption share increases to the cutoff level when one of its households enters a state with a binding constraint. This household's new consumption share is the consumption share for which its collateral constraint holds with equality. The region's consumption share is the sum of the two households' shares, and jumps up to the cutoff level. If both households in region  $i$  are unconstrained, the region's consumption share decreases at a rate dictated by the aggregate weight shock  $g_t$ . The cutoff level  $\underline{\varpi}_t$  itself crucially depends on the housing collateral ratio  $my_t$  and the current region-specific income shocks  $\Delta \log \hat{\eta}_{t+1}^i$ , but not the history of these shocks  $\Delta \log \hat{\eta}^{i,t}$ . Nor does it depend on the household level shocks. (Recall section 5.5 in the main text). The household's individual history is erased when it runs into a binding constraint. This is the *amnesia* property, shared by many limited commitment models (e.g. Albarran and Attanasio (2001) and Ljungqvist and Sargent (2004) for a general discussion). The cut-off consumption share also depends on the aggregate history of the economy  $\Delta \log(c^a)^t$  (captured by  $g$ ).

We first focus on the simplest case of separable preferences. The law of motion of each region's consumption weight at the start of the next period,  $\hat{c}'$ , follows a simple cutoff rule:

$$\begin{aligned}\varpi'^{i,i} &= \hat{c}^i \text{ if } \hat{c}^i > \underline{\varpi}_{t+1}^i(\widetilde{my}_{t+1}, \Delta \log \hat{\eta}_{t+1}^i, \Delta \log c^{a,t}) \\ &= \underline{\varpi}_{t+1}^i(\widetilde{my}_{t+1}, \Delta \log \hat{\eta}_{t+1}^i, \Delta \log c^{a,t}) \text{ elsewhere.}\end{aligned}$$

and the new consumption shares are:

$$\hat{c}'^{i,i} = \frac{\varpi'^{i,i}}{g},$$

where the aggregate weight shock is computed as the cross-sectional average  $g = \frac{1}{N} \sum_{i=1,\dots,N} \varpi'^{i,i}$ .

**Cutoff Rule under Separable Preferences** The cut-off  $\underline{\varpi}_{t+1}$  is a function of the aggregate consumption growth shocks  $\Delta \log(c^a)^{t+1}$ , the aggregate housing collateral scarcity measure  $\widetilde{my}_{t+1}$  and the current regional income share shock  $\Delta \log \hat{\eta}_{t+1}^i$ . Keeping track of



the current aggregate consumption growth shock and one lag ( $\Delta \log(c^a)_{t+1}$  and  $\Delta \log(c^a)_t$ ) closely approximates the full history dependence of  $\xi_{t+1}$  on the entire aggregate history.

We have to approximate the cutoff function  $\varpi_{t+1}^i(\widetilde{m}y_{t+1}, \Delta \log \hat{\eta}_{t+1}^i, \Delta \log c^a, t)$ . We start with a cut-off rule that is a *linear* function of the state variables:

$$\begin{aligned} \varpi_{t+1}^i = & \theta_0 \bar{\eta}^i + \theta_1 \Delta \log c_{t+1}^a + \theta_2 \widetilde{m}y_{t+1} \Delta \log c_{t+1}^a + \\ & \theta_3 \Delta \log \hat{\eta}_{t+1}^i + \theta_4 \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \theta_5 \Delta \log c_t^a. \end{aligned} \quad (9)$$

Denote the vector of cutoff parameters by  $\Theta$ . The first term is the average income share of a region over the entire sample; it acts as a regional fixed effect. The interaction terms capture the collateral effect. First, for a given aggregate consumption growth shock, the cutoff level is higher when housing collateral is scarcer ( $\theta_2 > 0$ ). Second, a given income share shock raises the cutoff more when collateral is scarce ( $\theta_4 > 0$ ). The last term, associated with  $\theta_5$ , captures the dependence of the cutoff weight on the aggregate consumption growth history.

**Cutoff Rule under Non-Separable Preferences** First, even in the absence of binding constraints, a region's consumption weight will change when its expenditure share changes. If not, the unconstrained households would not be equalizing their intertemporal marginal rates of substitution. We adjust last period's consumption share for the change in region-specific expenditure share growth, in the following way:

$$\Delta \log \hat{c}_{t+1}^i = \frac{\gamma\varepsilon - 1}{\gamma(\varepsilon - 1)} \Delta \log \hat{\alpha}_{t+1}^i. \quad (10)$$

Because we do not have data for region-specific expenditure share growth, we exploit the mapping in equation (2) and use data on region-specific rental price growth instead. The adjustment in equation (10) becomes:  $\Delta \log \hat{c}_{t+1}^i = \frac{1-\gamma\varepsilon}{\gamma} \frac{1}{1+(\hat{\rho}_t^i)^{\varepsilon-1}} \Delta \log \hat{\rho}_{t+1}^i$ .

Second, the cutoff additionally depends on aggregate and region-specific rental price growth.

$$\begin{aligned} \varpi_{t+1}^i = & \theta_0 \bar{\eta}^i + \theta_1 \Delta \log c_{t+1}^a + \theta_2 \widetilde{m}y_{t+1} \Delta \log c_{t+1}^a + \\ & \theta_3 \Delta \log \hat{\eta}_{t+1}^i + \theta_4 \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \theta_5 \Delta \log c_t^a \\ & \theta_6 \bar{\rho}^i + \theta_7 \Delta \log \rho_{t+1}^a + \theta_8 \widetilde{m}y_{t+1} \Delta \log \rho_{t+1}^a + \\ & \theta_9 \Delta \log \hat{\rho}_{t+1}^i + \theta_{10} \widetilde{m}y_{t+1} \Delta \log \hat{\rho}_{t+1}^i + \theta_{11} \Delta \log \rho_t^a \end{aligned} \quad (11)$$

The first two lines are identical to specification (9). Non-separability contributes parallel rental price terms. The average regional rental price is part of the fixed effect ( $\theta_6$ ). The aggregate rental price share growth ( $\Delta \log \alpha_{t+1}^a$ ) and its interaction with the housing collateral ratio are the new aggregate state variables ( $\theta_7$  and  $\theta_8$ ). The cutoff also depends on the region-specific rental price share growth ( $\Delta \log \hat{\alpha}_{t+1}$ ) and its interaction with the housing collateral ratio ( $\theta_9$  and  $\theta_{10}$ ). To allow for aggregate history dependence, we add

last period's aggregate expenditure share growth ( $\theta_{11}$ ).

Finally, as is apparent from equations (4) and (5), the cutoff also depends on cross-regional distribution of individual expenditure share growth. In some specifications, we include the cross-sectional dispersion of rental price growth and its interaction with  $\widetilde{m}y$  as additional variables in the cutoff specification ( $\theta_{12}$  and  $\theta_{13}$ ). To keep the number of terms manageable, we omit the aggregate history dependence terms in that specification, i.e. we set  $\theta_5 = 0$  and  $\theta_{11} = 0$ .

## 5.2 Simulation Algorithm

We use a simulated method of moments estimation that: (1) searches over cutoff parameters  $\Theta$  to match the correlation of consumption shares, generated by simulating the model, with observed income shares and the correlation of consumption shares in the data with observed income shares and (2) that searches over parameters  $(\gamma, \varepsilon)$  to minimize pricing errors on an aggregate stock market return, a long bond return and a risk-free rate. Throughout, we hold the time discount factor  $\beta$  fixed.

Actual consumption is tainted by measurement error (section 4). We take this into account by drawing several paths of measurement error innovations, indexed by  $s$ , and average over these paths.

The simulation method is conditional simulation: The actual consumption share  $\hat{c}_{t+1}^{i,s}$  is computed conditionally on the simulated value in the previous period  $\hat{c}_t^{i,s}$  (Gourieroux and Monfort (1996), p.17).

1. We start with an initial guess for the parameter vector  $\Theta$ , and we evaluate the cutoff function  $\underline{\varpi}_t^i(\cdot)$  at the *observed* aggregate state variables and region-specific state variables according to equation (9) when preferences are separable and (11) when preferences are non-separable. This produces a  $T - 1$  by  $N$  matrix of cutoff realizations  $\Xi = [\underline{\varpi}_t^i]_{t=1, n=1}^{t=T, n=N}$ .
2. For a fixed standard deviation  $\sigma_b$ , we draw a panel of log-normally distributed random variables for measurement error:  $\left\{ b_t^{i,s} \right\}_{t=1}^T$  for  $i = 1, 2, \dots, N$ . We draw  $S$  such panels, where  $s = 1, 2, \dots, S$  denotes the simulation index.
3. For each  $s \in S$  we build a time series of length  $T$  of predicted consumption shares  $\{\hat{c}_t^{i,s}\}$ .
  - For each simulation run  $s \in S$ , we set the observed initial consumption share of region  $i$  equal to the first observation on the consumption share in the data, and the true consumption share  $\hat{c}$  is constructed from the observed share  $\tilde{c}$ :  $\hat{c}_1^{i,s} = \tilde{c}_1^{i,s} \exp(-\hat{b}_1^{i,s})$ .

- At each  $t$ , given  $\hat{c}_t^{i,s}$ , we find next period's consumption share  $\hat{c}_{t+1}^{i,s}$  for every region by comparing last period's consumption share  $\hat{c}_t^{i,s}$  to the current cutoff level (the  $(t, i)$  entry of the cutoff matrix  $\Xi$ ).
  - We compute the aggregate weight shock  $g_{t+1}^s$  as the cross-sectional average of the new consumption weights.
  - At the end of every period we re-normalize the consumption shares by the aggregate weight shock. This re-normalization guarantees that the population-weighted average of shares is one.
  - At  $t + 1$ , the *observed* simulated consumption share is constructed by adding measurement error to the  $\hat{c}_{t+1}^{i,s}$  obtained in the previous step:  $\tilde{c}_{t+1}^{i,s} = \hat{c}_{t+1}^{i,s} \exp(\hat{b}_{t+1}^{is})$ .
4. We repeat this last step  $S$  times to end up with  $S$  panels of size  $T \times N$  model-predicted observed consumption shares.
  5. For each region, we build the model-predicted observed consumption share at time  $t$  by averaging over the  $S$  different measurement error realizations:

$$k_t^i(\Theta) = \frac{1}{S} \sum_{s=1}^S \tilde{c}_t^{i,s},$$

and the model-predicted aggregate weigh shock at time  $t$

$$g_t(\Theta) = \frac{1}{S} \sum_{s=1}^S g_t^s,$$

for every period  $t = 2, \dots, T$ .

6. Given the initial guess for  $(\gamma, \varepsilon)$  and the fixed value for  $\beta$ , we construct the stochastic discount factor  $\{m_t\}$ , using the aggregate weight shock  $g_{t+1}$ , according to equation (5) or its equivalent under separable preferences (set  $\gamma\varepsilon = 1$ ). We form the pricing error at time  $t$ :

$$p_t^j(\Theta, \beta, \gamma) = m_t R_t^j - 1,$$

where  $R^j$  is the gross return on asset  $j \in J$ .

### 5.3 Moment Conditions

Given the simulated consumption data, the aggregate weight shocks and the pricing errors:

$$\{k_t^i(\Theta), h_t(\Theta), p_t^j(\Theta, \beta, \gamma)\}, \quad (12)$$

we can construct the moments used in the actual estimation. We use two different sets of moments. The benchmark set matches the unconditional correlation between income

and consumption shares, in addition to some asset pricing moments. The robustness set matches these same correlations, conditional on the housing collateral ratio.

**Benchmark Set of Moments** The algorithm matches three sets of moments: (1) the correlation between consumption and income share for each region (N moments):

$$h^i = \frac{1}{T-1} \sum_{t=1}^{T-1} (\tilde{c}_t^i - k_t^i(\Theta)) \hat{y}_t^i, \quad i = 1 \dots N.$$

(2) the cross-sectional dispersion in consumption (3 moments):

$$\begin{aligned} h^{N+1} &= \frac{1}{T-1} \sum_{t=1}^{T-1} (\sigma^i(\tilde{c}_t) - \sigma^i(k_t^N(\Theta))) \\ h^{N+2} &= \frac{1}{T-1} \sum_{t=1}^{T-1} \left( \frac{\sigma^i(\tilde{c}_t)}{\sigma^i(\hat{y}_t)} - \frac{\sigma^i(k_t^N(\Theta))}{\sigma^i(\hat{y}_t)} \right) \\ h^{N+3} &= \frac{1}{T-1} \sum_{t=1}^{T-1} \left( \frac{\sigma^i(\tilde{c}_t)}{\sigma^i(\hat{y}_t)} - \frac{\sigma^i(k_t^N(\Theta))}{\sigma^i(\hat{y}_t)} \right) my_t, \end{aligned}$$

where  $\sigma^i(\cdot)$  stands for the cross-sectional standard deviation, and, (3)  $J$  asset pricing moments:

$$\begin{aligned} h^{N+4} &= \frac{1}{T-1} \sum_{t=1}^{T-1} p_t^1(\Theta, \beta, \gamma, \varepsilon) \dots \\ h^{N+3+J} &= \frac{1}{T-1} \sum_{t=1}^{T-1} p_t^J(\Theta, \beta, \gamma, \varepsilon). \end{aligned}$$

The objective is to minimize the loss function  $W$ :

$$W = [g]^\prime \Psi [h],$$

where  $\Psi$  is the weighting matrix; we use the identity matrix. To find the parameters  $(\Theta, \gamma, \varepsilon)$  that minimize the distance between the moments in the data and the moments simulated from the model, we use a hill-climbing algorithm.

**Robustness Set Moments** In the second specification, we replace the first  $N$  moments, matching the unconditional correlation between consumption shares and income shares in model and data for each region, by  $N$  moments which match the *conditional* mean consumption share in model and data for each region. The conditioning variable is the

scaled housing collateral ratio  $\widetilde{m}y_{t-1}$ . The moment for the  $i^{th}$  region is

$$h^i = \frac{1}{T-1} \sum_{t=1}^{T-1} (\widetilde{c}_t^i - k_t^i(\Theta)) \widetilde{m}y_{t-1}, \quad i = 1 \dots N. \quad (13)$$

The other  $3 + J$  moments remain unchanged.

## 5.4 Estimation Results Non-Linear Model

The simulated moments estimation recovers the structural parameters: the degree of relative risk aversion  $\gamma$  (separable preferences), the intratemporal elasticity of substitution between non-durables and housing services  $\varepsilon$  (non-separable preferences) and the parameters in the parametric cutoff specification  $\Theta$  in equation (9 or 11). We discuss the results for separability and non-separability separately in the next sections. The estimates lend considerable support to the collateral channel.

### 5.4.1 Separable Preferences

Table 3 reports the estimates of the collateral model under separable preferences. In specification 1 in the first column, the history dependence in the cutoff rule is shut down;  $\theta_5$  in equation (9) is set to zero. The coefficients in the cutoff specification have the expected sign: in a recession the cutoff goes up ( $\theta_1 < 0$ ). This effect is stronger in a period with scarce housing collateral ( $\theta_2 > 0$ ).

More importantly, households with positive income innovations are more likely to run into binding constraints because these raise the cutoff consumption weight. This effect is stronger when collateral is scarce: A positive income share shocks increases the cutoff when housing collateral is scarce ( $\theta_4 > 0$ ). In an average period,  $\widetilde{m}y = .5$  and a regional income shock increases the cutoff level ( $\theta_3 + \frac{\theta_4}{2} > 0$ ).  $\theta_4$  is positive as predicted by the collateral mechanism, and measured precisely.

[Table 3 about here.]

**Predicted vs. Actual Consumption Dispersion** Figure 1 illustrates the risk-sharing dynamics and how they relate to the evolution of the housing collateral ratio. It plots our measure of collateral scarcity  $\widetilde{m}y_{rw}$  on the left axis (solid blue line) against actual cross-regional consumption share dispersion (dashed green line) and estimated consumption share dispersion (dotted red line). The collateral scarcity measure and the consumption share dispersion in the data show a strong positive co-movement over time. The estimated cross-sectional consumption share dispersion tracks the actual dispersion very well, at least at low frequencies, with the exception of the 1990s. The correlation between the predicted cross-regional consumption share dispersion and our measure of collateral scarcity is 0.51.

[Figure 1 about here.]

**Time-varying Consumption and Income Correlations** The shocks to the risk sharing technology also show up in a time-varying correlation between regional and aggregate consumption growth. For rolling 5-year windows, we compute the correlation between each region’s consumption growth and aggregate consumption growth. We average these correlations across regions. The resulting average correlation with aggregate consumption growth is high when housing collateral is scarce. This is shown in figure 2, which plots the observed and predicted correlation of regional consumption growth with aggregate consumption growth against the housing collateral ratio 5 years in the future. As collateral becomes relatively more abundant between 1974 and 2002, regional consumption growth became more correlated with aggregate consumption growth. In the data the correlation goes up from 0.3 to 0.6, in the model estimation it goes up from 0.1 to almost 0.9.

[Figure 2 about here.]

Figures 3 and 4 display this increase in risk sharing between 1974 and 2002 in another way. For both the data and the model simulation evaluated at the estimated parameters, they show a histogram of the cross-sectional distribution of consumptions shares in levels (figure 3) and in growth rates (figure 4). In both the data and the model, the cross-sectional distribution is much wider in 1974, when collateral scarcity is at its highest level, than in 2002, when collateral is the most abundant. The distribution is also less dispersed when collateral is abundant.

[Figure 3 about here.]

[Figure 4 about here.]

**Without Collateral Channel** If we shut down the collateral channel in the non-linear model, that is if the cutoff specification does not depend on the housing collateral ratio ( $\theta_2 = 0 = \theta_4$  in specification 2), the model performs worse. The  $J$ -test statistic is 2.19 ( $\chi^2$ -distributed with 2 degrees of freedom). The null that the non-linear model with the collateral mechanism does not outperform the model without the collateral mechanism can be rejected at the 66 percent probability level. This is not as high as standard p-values, mainly because of the short sample size. However, the two models are economically significantly different: The predicted consumption shares do not line up with the observed consumption shares without the collateral mechanism. Figure 5 drives this point home. Just as in figure 1, it plots actual cross-regional consumption share dispersion (dashed green line) and estimated consumption share dispersion (dotted red line). The actual and estimated (predicted) consumption share dispersion are much further apart than in the non-linear model with the collateral channel (see Figure 1).

[Figure 5 about here.]

**Aggregate Weight Shock** The aggregate weight shocks, plotted in Figure 6, provide a direct measure of the shocks to the risk sharing technology. The figure plots the estimated aggregate weight shock, observed aggregate consumption growth (dashed green line) and the estimated consumption share dispersion (dotted red line). Aggregate weight shocks usually happen in recessions, but they tend to be larger when the collateral ratio is low. When there is a positive aggregate weight shock, the consumption share dispersion across regions increases.

[Figure 6 about here.]

**Risk Premia** The aggregate weight shock is the multiplicative contribution of the collateral constraints to the SDF. This term increases the volatility of the SDF. The aggregate weight shock  $g$  is 1.37 at its maximum. Recall that  $g$  would be one if constraints are never binding. In the estimation, the multiplicative component of the stochastic discount factor due to the presence of housing collateral constraints accounts for three quarters of the volatility in the SDF. As a result, the estimated Sharpe ratio for the collateral model is 3 times as volatile as the one in the standard consumption asset pricing model. The coefficient estimate for the degree of risk aversion is much smaller than the traditional estimates from the complete markets consumption-based asset pricing model. A smaller coefficient of relative risk aversion can reconcile volatile asset prices with smooth aggregate consumption.

**History Dependence** Cutoff specification 2 in the second column of table 3 adds limited history dependence. The estimate for  $\theta_4$  is still positive and measured precisely. The new term, lagged consumption growth enters negatively ( $\theta_5 < 0$ ).

**Measurement Error** We estimate the same model for different magnitudes of measurement error volatility. The last column of Table 4 replicates the last column of table 3. The other columns estimate the same model but for  $\sigma_b = .005, .010, .015$ . The value of the objective function is non-monotonic in the standard deviation of measurement error. It is lowest for  $\sigma_b = .02$ . More importantly, all parameter estimates have the same sign, similar magnitudes, and are similarly precisely estimated.

[Table 4 about here.]

## 5.5 Non-Separable Preferences

Under non-separability between non-housing and housing consumption, both the pricing kernel and the law of motion for individual consumption shares are different. The *representative agent* pricing kernel additionally depends on aggregate rental price growth and the cutoff consumption share also depends on rental price changes (equation 11). Table 5 reports four different specifications for the cutoff level.

[Table 5 about here.]

The new terms in specification 3 are the average region-specific rental price level (fixed effect), the aggregate rental price growth rate and the aggregate rental price growth rate interacted with the housing collateral ratio. In specification 4, we add region specific rental price changes and their interaction term with the housing collateral ratio. In the third column, we add limited history dependence by including one lag of aggregate consumption growth and aggregate rental price growth. Finally, specification 6 includes the cross-regional dispersion of rental prices and its interaction with the housing collateral ratio. These terms capture the dependence of the cutoff weight on the distribution of rental prices. For parsimony we omit the lagged aggregate consumption growth and aggregate rental price growth variables in this specification.

The parameter  $\varepsilon$  is the intratemporal elasticity of substitution between non-durable and housing services consumption. This estimate ranges from -.4 to .9 and has mostly a large standard error. The non-linear model has a hard time pinning down this parameter. The coefficients on the new rental price growth terms in the cutoff specification coming from the non-separability ( $\theta_6 - \theta_{13}$ ) are measured less precisely than the coefficients on the terms from the separable specification. However, the parameter estimates for  $\theta_3$  and  $\theta_4$  (columns 2-5) have the right sign and are still precisely estimated. When regions witness a positive relative income shock, the cutoff level increases more when housing collateral is relatively scarce ( $\theta_4 > 0$ ). This indicates the presence of the collateral effect under non-separable preferences, just as in the model with separable preferences. The correlation between the aggregate weight shock is the measure of collateral scarcity is large and positive (.3-.5). The main collateral effect is strongly present. In addition the non-separable model matches simulated and observed consumption paths more closely: The objective function  $W$  is lowest in specification 4.

**Robustness Set of Moments** The parameter estimates are very similar when we replace the first  $N$  moments, which match the unconditional correlation between consumption shares and income shares in model and data for each region, by  $N$  moments which match the conditional mean consumption share in model and data. The conditioning variable is  $\widetilde{m}y_t$ , which indexes the risk-sharing capacity, or equivalently, the investment opportunity set. The last 6 moments are unchanged. Table 6 shows the parameter estimates for specification 1 and 2, corresponding to columns 1 and 2 of table 3 and for specification 4 and 5, corresponding to columns 2 and 3 of table 5. Again, the key parameter  $\theta_4$  is positive and measured precisely. The parameter estimates imply that consumption is relatively more dispersed than income in times when collateral is scarce.

[Table 6 about here.]



### 5.5.1 Regional Collateral Measures

Finally, we revisit the non-linear model and use the regional housing collateral ratio  $my^i$  instead of the aggregate housing collateral ratio in the cutoff specification (9). The regional collateral measures are described in the data appendix in the main text. Table 7 reports estimation results for the separable preference specification with the benchmark set of moments (as in table 3). The coefficients  $\theta_2$  and  $\theta_4$ , which capture the collateral mechanism, have the correct sign and are estimated precisely. When the economy goes through a period of low aggregate consumption growth, risk sharing becomes more difficult when regional collateral is scarce ( $\theta_2 > 0$ ). Likewise, when a region has high region-specific income growth, its consumption share increases by more when its collateral is scarce ( $\theta_4 > 0$ ).

[Table 7 about here.]

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Figure 1: Housing Collateral Ratio and Cross-Regional Consumption Share Dispersion. On the left axis is the collateral scarcity measure  $\widetilde{myrw}$  (solid blue line). On the right axis is the observed (dashed green line) and predicted (dotted red line) cross-sectional consumption share dispersion. The predicted dispersion is the one that corresponds to the model under separability (specification 2).

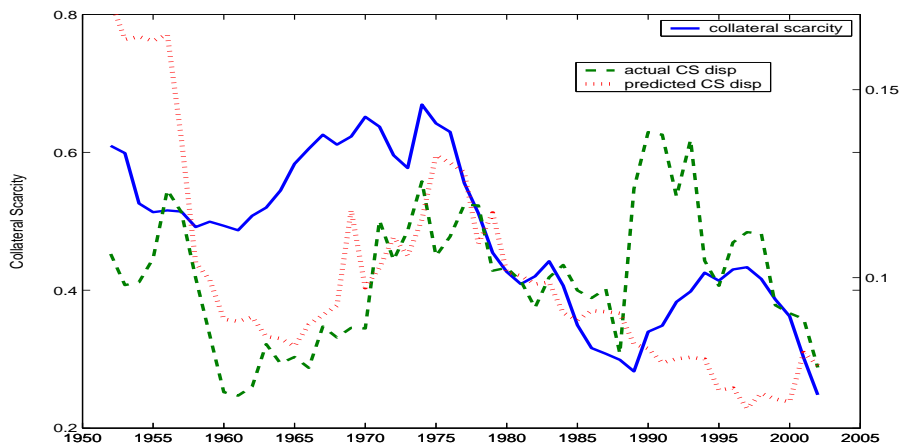


Figure 2: Housing Collateral Ratio and Correlation between Regional and Aggregate Consumption Growth. On the left axis is the collateral scarcity measure  $\widetilde{myrw}$  (solid blue line). On the right axis is the observed (dashed green line) and predicted (dotted red line) average correlation between the past 20 years over regional consumption growth and aggregate consumption growth. The average is taken across regions. The predicted dispersion is the one that corresponds to the model under separability (specification 2).

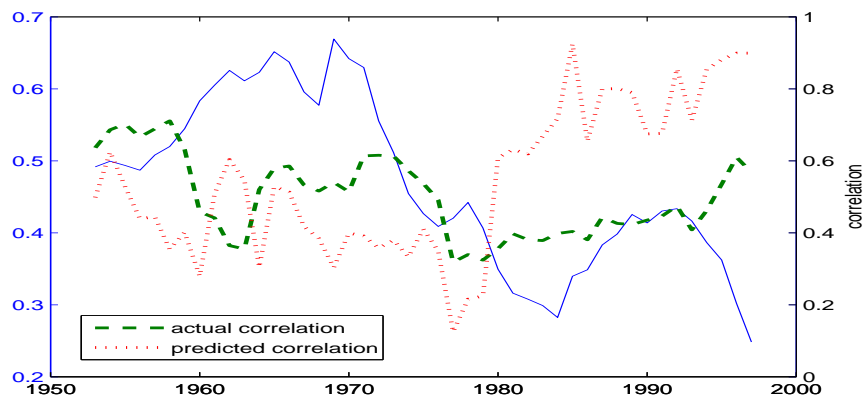


Figure 3: The Cross-Sectional Distribution of Consumption Shares

The left column shows the cross-section distribution of consumption shares *in levels* in 1974, the year with the lowest reading of the housing collateral ratio (high scarcity) and in 2002, the year with the highest reading of the collateral ratio (low scarcity). The right column displays the corresponding distributions as implied by the model estimation in Table 3, specification 2. On the horizontal axes are the levels of the consumption share, which are one on average for each region. On the vertical axes are the number of regions in each bin. Bins are defined by midpoints ranging from 0.8 to 1.4 with increments of 0.05.

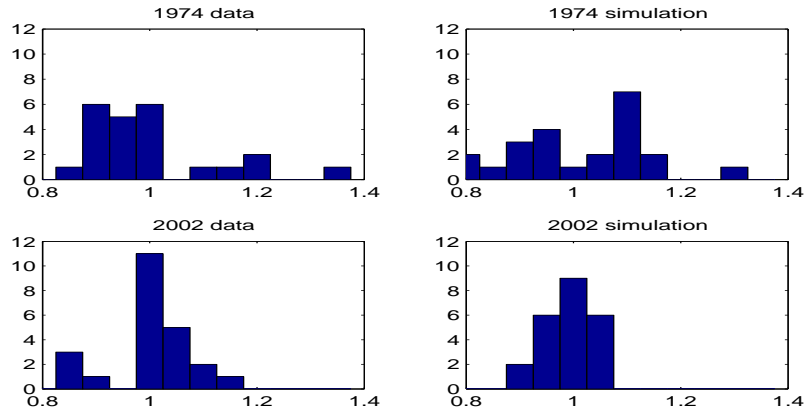


Figure 4: The Cross-Sectional Distribution of Consumption Share Growth Rates

The left column shows the cross-section distribution of consumption shares *in growth rates* in 1974, the year with the lowest reading of the housing collateral ratio (high scarcity) and in 2002, the year with the highest reading of the collateral ratio (low scarcity). The right column displays the corresponding distributions as implied by the model estimation in Table 3, specification 2. On the horizontal axes are the growth rates of the consumption share, which are zero on average for each region. On the vertical axes are the number of regions in each bin. Bins are defined by midpoints ranging from -0.3 to 0.3 with increments of 0.05, except for additional bins with midpoints -.025, -.01, .01 and .025. The stars denote the bin midpoints.

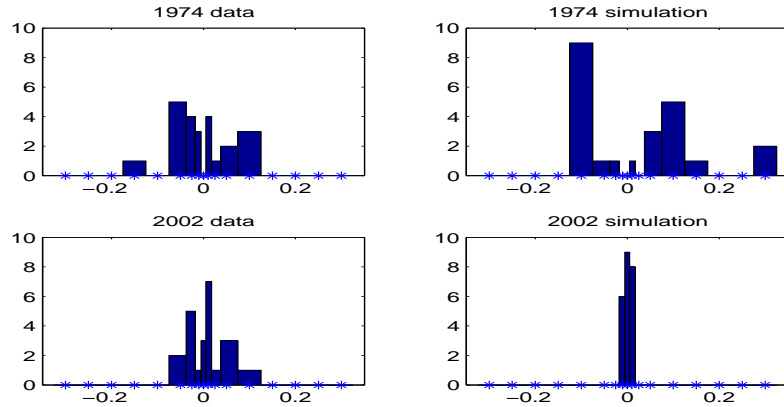


Figure 5: Housing Collateral Ratio and Cross-Regional Consumption Share Dispersion: No Collateral Effect.

On the left axis is the collateral scarcity measure  $\widetilde{myrw}$  (solid blue line). On the right axis is the observed (dashed green line) and predicted (dotted red line) cross-sectional consumption share dispersion. The predicted dispersion is the one that corresponds to the model under separability (specification 2) but with  $\theta_2 = 0 = \theta_4$  in the cutoff specification.

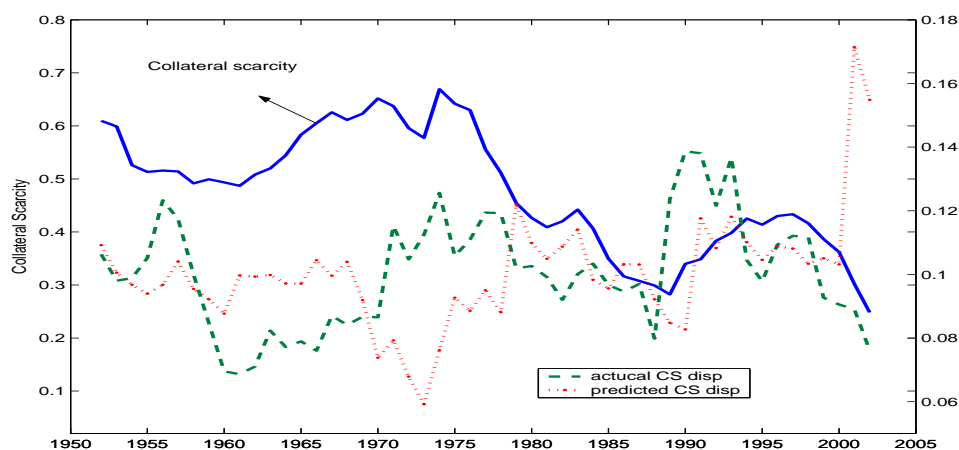


Figure 6: Estimated Aggregate Weight Shock and Consumption Share Dispersion, and Observed Aggregate Consumption Growth

On the left axis is the estimation-implied aggregate weight shock  $g$  (solid blue line) and observed aggregate consumption growth (dashed green line). On the right axis is the predicted cross-sectional consumption share dispersion (dotted red line). The predicted dispersion is the one that corresponds to the model under separability (specification 2).

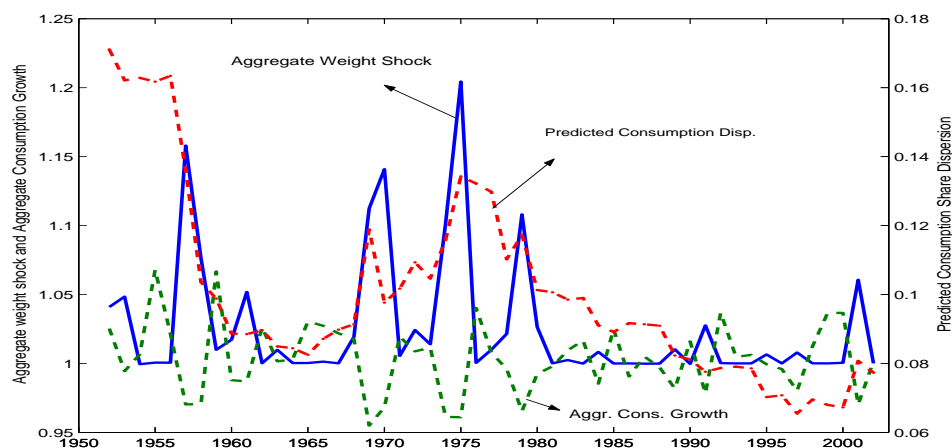


Table 1: Linear Model - Non-Separable Preferences - Aggregate Collateral Measures

We estimate:  $\Delta \log \tilde{c}_{t+1}^i = a_0^i a_1 \tilde{m} y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + (a_2 + a_3 \tilde{m} y_{t+1}) \Delta \log \hat{\rho}_{t+1}^i + (a_4 + a_5 \tilde{m} y_{t+1}) (\hat{\rho}_t^i - 1) \Delta \log \hat{\rho}_{t+1}^i + \nu_{t+1}^i$ . Rows 1-2 are for the period 1952-2001 (1166 observations). The measure of idiosyncratic income is disposable personal income. Rows 3-4 are identical to rows 1-2 but are for the period 1970-2000 (713 observations). Regressions 5-6 use labor income plus transfers, available only for 1970-2000 (704 observations). All rows use the rescaled *myrw* and *myfa*, estimated for the period 1925-2002. *my<sup>max</sup>* (*my<sup>min</sup>*) is the sample maximum (minimum) in 1925-2002. The coefficients on the fixed effect,  $a_0^i$ , are not reported. Estimation is by feasible Generalized Least Squares, allowing for both cross-section heteroscedasticity and contemporaneous correlation. Rows 7-8 are the results for the instrumental variable estimation by 3SLS. Instruments are a constant,  $\log(\hat{\eta}_{t+2}^i)$ ,  $\log(\hat{\eta}_{t+3}^i)$ ,  $\log(\hat{\eta}_{t+4}^i)$ ,  $\Delta \hat{\rho}_{t+2}^i$ ,  $\Delta \hat{\rho}_{t+3}^i$ ,  $\Delta \hat{\rho}_{t+4}^i$ ,  $\log(\tilde{c}_{t+2}^i)$ ,  $\log(\tilde{c}_{t+3}^i)$ ,  $\log(\tilde{c}_{t+4}^i)$ , and  $my_{t+2}$ ,  $my_{t+3}$ ,  $my_{t+4}$ . The period is 1952-1998 (1074 observations). All results are for 23 US metropolitan areas.

Coll. Measure	$a_1$	$\sigma_{a_1}$	$a_2$	$\sigma_{a_2}$	$a_3$	$\sigma_{a_3}$	$a_4$	$\sigma_{a_4}$	$a_5$	$\sigma_{a_5}$	$R^2$
1 <i>myrw</i>	.68	(.05)	.98	(.18)	-1.40	(.39)	1.13	(.96)	-1.95	(1.74)	9.3
2 <i>myfa</i>	.74	(.06)	.53	(.21)	-.46	(.48)	1.72	(1.41)	-3.48	(2.81)	9.5
3 <i>myrw</i>	.75	(.04)	.64	(.05)	-.57	(.11)	1.90	(.49)	-2.69	(.92)	9.0
4 <i>myfa</i>	.81	(.04)	.53	(.07)	-.40	(.15)	.34	(.82)	.35	(1.66)	9.0
5 <i>myrw</i>	1.01	(.04)	.80	(.09)	-.97	(.22)	.66	(.67)	-.77	(1.23)	14.3
6 <i>myfa</i>	1.09	(.05)	.62	(.14)	-.60	(.32)	.61	(1.18)	-1.03	(2.40)	13.8
7 <i>myrw</i>	.55	(.07)	.87	(.24)	-1.31	(.53)	.30	(1.69)	-.84	(2.96)	
8 <i>myfa</i>	.62	(.08)	1.09	(.31)	-1.66	(.71)	2.18	(1.83)	-4.23	(3.67)	

Table 2: Linear Model - Non-Separable Preferences - Regional Collateral Measures

This table reports estimation results under non-separability. The consumption growth regression is:  $\Delta \log \tilde{c}_{t+1}^i = b_0^i + b_1 \Delta \log \hat{\eta}_{t+1}^i + b_2 X_{t+1}^i \Delta \log \hat{\eta}_{t+1}^i + b_3 \Delta \log \hat{\rho}_{t+1}^i + b_4 X_{t+1}^i \Delta \log \hat{\rho}_{t+1}^i + b_5 (\hat{\rho}_t^i - 1) \Delta \log \hat{\rho}_{t+1}^i + b_6 X_{t+1}^i (\hat{\rho}_t^i - 1) \Delta \log \hat{\rho}_{t+1}^i + \nu_{t+1}^i$ , where  $X^i = my^i$  is the region-specific housing collateral ratio (569 observations). It is measured as the residual from a regression of the log ratio of real per capita regional housing wealth to real per capita labor income,  $\log(hv_t^i) - \log(\eta_t^i)$ , on a constant and a time trend. A higher  $my^i$  means more abundant collateral in region  $i$ . In all regressions  $\eta$  is disposable income. The coefficients on the fixed effect,  $\beta_0^i$ , are not reported. Estimation is by feasible Generalized Least Squares allowing for both cross-section heteroscedasticity and contemporaneous correlation. All regressions are for the period 1975-2000 for 23 US metropolitan areas, the longest period with metropolitan housing data.

Coll. Measure	$b_1$	$\sigma_{b_1}$	$b_2$	$\sigma_{b_2}$	$b_3$	$\sigma_{b_3}$	$b_4$	$\sigma_{b_4}$	$b_5$	$\sigma_{b_5}$	$b_6$	$\sigma_{b_6}$	$R^2$
<i>my<sup>i</sup></i>	.39	(.02)	-.33	(.18)	.10	(.02)	-1.80	(.20)	1.38	(.23)	-9.74	(2.70)	7.7

Table 3: Non-Linear Model - Separable Preferences.

The estimation is by simulated method of moments (S=20), using the N+6 moments described in the text. Estimation of the cutoff policy function:  $\varpi_{t+1}^i = \theta_0 \bar{\eta}^i + \theta_1 \Delta \log c_{t+1}^a + \theta_2 \widetilde{m}y_{t+1} \Delta \log c_{t+1}^a + \theta_3 \Delta \log \hat{\eta}_{t+1}^i + \theta_4 \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \theta_5 \Delta \log c_t^a$ . The measurement error volatility is fixed at  $\sigma_b = .02$ . The discount factor is fixed at  $\beta = .95$ . The weighting matrix is the identity matrix in 3 iterations and the Newey-West HAC matrix for the computation of standard errors. The housing collateral ratio is *myrw*. The parameter  $\gamma$  is a preference parameter; the parameters in the consumption weight cutoff specification parameter are  $\Theta$ . The two columns correspond to two different specifications of the cutoff process, discussed in the text. The last three rows report the aggregate weight shock and collateral scarcity, the highest realization of the aggregate weight shock, and the Simulated Method of Moments function value  $W$ . Data are for 1951-2002 for 23 US metropolitan areas.

Parameter	Specification 1	Specification 2
$\gamma$	.51 (.34)	.27 (.17)
$\theta_0$	.69 (.07)	.53 (.12)
$\theta_1$	-14.24 (5.17)	-8.85 (21.60)
$\theta_2$	.05 (8.62)	28.53 (50.23)
$\theta_3$	-7.36 (4.31)	-29.53 (2.79)
$\theta_4$	25.72 (9.87)	60.10 (5.54)
$\theta_5$		-23.61 (5.48)
$corr(g, \widetilde{m}y)$	.36	.16
$max(g)$	1.37	1.81
$W$	0.054	.0313

Table 4: Non-Linear Model - Separable Preferences - Size of Measurement Error

The estimation is by simulated method of moments (S=20), using the N+6 moments described in the text. Estimation of the cutoff policy function:  $\varpi_{t+1}^i = \theta_0 \bar{\eta}^i + \theta_1 \Delta \log c_{t+1}^a + \theta_2 \widetilde{m}y_{t+1} \Delta \log c_{t+1}^a + \theta_3 \Delta \log \hat{\eta}_{t+1}^i + \theta_4 \widetilde{m}y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \theta_5 \Delta \log c_t^a$ . Same as table 3, for four different values of the volatility of the measurement error  $\sigma_b$ . The cutoff specification corresponds to the second column of table 3 (specification 2).

Parameter	$\sigma_b = .005$	$\sigma_b = .010$	$\sigma_b = .015$	$\sigma_b = .020$
$\gamma$	.71 (.42)	.45 (.29)	.60 (.38)	.27 (.17)
$\theta_0$	.85 (.02)	.70 (.09)	.79 (.05)	.53 (.12)
$\theta_1$	-5.82 (11.76)	-13.33 (9.79)	-9.59 (4.01)	-8.85 (21.60)
$\theta_2$	.49 (22.02)	-2.72 (16.97)	.13 (9.45)	28.53 (50.23)
$\theta_3$	-8.75 (2.86)	-5.48 (3.69)	-7.85 (4.72)	-29.53 (2.79)
$\theta_4$	27.54 (6.98)	21.96 (8.47)	26.30 (10.63)	60.10 (5.54)
$\theta_5$	1.95 (1.42)	1.66 (.53)	.90 (1.09)	-23.61 (5.48)
$corr(g, \widetilde{m}y)$	.41	.33	.37	.16
$max(g)$	1.20	1.39	1.27	1.81
$W$	.0548	.0511	0.0546	.0313

Table 5: Non-Linear Model - Non-Separable Preferences

The estimation is by simulated method of moments (S=20), using the N+6 moments described in the text. The measurement error volatility is fixed at  $\sigma_b = .02$ . Estimation of the cutoff policy function:  $\underline{\varpi}_{t+1}^i = \theta_0 \bar{\eta}^i + \theta_1 \Delta \log c_{t+1}^a + \theta_2 \widetilde{my}_{t+1} \Delta \log c_{t+1}^a + \theta_3 \Delta \log \hat{\eta}_{t+1}^i + \theta_4 \widetilde{my}_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \theta_5 \Delta \log c_t^a + \theta_6 \bar{\rho}^i + \theta_7 \Delta \log \rho_{t+1}^a + \theta_8 \widetilde{my}_{t+1} \Delta \log \rho_{t+1}^a + \theta_9 \Delta \log \hat{\rho}_{t+1}^i + \theta_{10} \widetilde{my}_{t+1} \Delta \log \hat{\rho}_{t+1}^i + \theta_{11} \Delta \log \rho_t^a$ . The discount factor is fixed at  $\beta = .95$ . The weighting matrix is the identity matrix in 3 iterations and the Newey-West HAC matrix for the computation of standard errors. The housing collateral ratio is *myrw*. The parameters  $\gamma$  and  $\varepsilon$  are preference parameters, the parameters in the consumption weight cutoff specification parameter are  $\Theta$ . The last three rows report the aggregate weight shock and collateral scarcity, the highest realization of the aggregate weight shock, and the Simulated Method of Moments function value  $W$ . The four columns denote different specifications for the cutoff rule discussed in the text. Data are for 1951-2002 for 23 US metropolitan areas.

Parameter	Specification 3	Specification 4	Specification 5	Specification 6
$\gamma$	.40 (.57)	1.09 (.54)	.51 (.34)	.94 (.51)
$\varepsilon$	.90 (3.47)	-.38 (.43)	.16 (1.76)	.02 (.91)
$\theta_0$	.48 (.15)	.36 (.25)	.59 (.14)	.48 (.23)
$\theta_1$	-3.70 (8.00)	-10.56 (9.25)	-1.70 (5.37)	-4.72 (8.85)
$\theta_2$	2.19 (15.34)	16.44 (17.73)	-.36 (11.25)	-.18 (17.03)
$\theta_3$	-12.73 (3.17)	.47 (2.33)	-9.41 (4.57)	-5.24 (4.88)
$\theta_4$	35.44 (7.96)	10.01 (3.97)	27.91 (9.92)	20.61 (11.89)
$\theta_5$			-2.90 (1.77)	
$\theta_6$	.43 (.14)	.35 (.23)	.35 (.14)	.23 (.14)
$\theta_7$	3.83 (12.84)	-15.66 (22.07)	3.00 (17.12)	-6.33 (25.99)
$\theta_8$	.75 (31.01)	.81 (45.87)	-6.90 (33.09)	.16 (51.34)
$\theta_9$		-2.99 (8.86)	-1.34 (13.55)	-2.51 (9.60)
$\theta_{10}$		5.19 (20.83)	1.48 (32.70)	.30 (29.86)
$\theta_{11}$			1.00 (5.43)	
$\theta_{12}$				1.28 (1.82)
$\theta_{13}$				.14 (4.31)
$corr(g, \widetilde{my})$	.49	.33	.46	.46
$max(g)$	1.21	1.34	1.25	1.25
$W$	.0546	.0245	.0550	.0363

Table 6: Non-Linear Model - Robustness Set of Moments.

Same as in table 3 columns 1 and 2 and table 5 columns 2 and 3, expect that the first  $N$  moments in the estimation match the conditional mean consumption share in model and data for each region, conditional on the lagged housing collateral scarcity measure  $\widetilde{my}_t$ . The last 6 moments are unchanged.

Parameter	Separable Preferences		Non-Separable Preferences	
	Specification 1	Specification 2	Specification 4	Specification 5
$\gamma$	.62 (.33)	.68 (.45)	.93 (.48)	.89 (.51)
$\varepsilon$			-.50 (.52)	-.15 (.74)
$\theta_0$	.72 (.07)	.83 (.03)	.47 (.18)	.47 (.13)
$\theta_1$	-10.84 (5.83)	-6.44 (9.00)	-6.15 (17.72)	-3.57 (16.44)
$\theta_2$	-2.07 (11.63)	.37 (17.41)	7.98 (31.62)	.47 (29.63)
$\theta_3$	-4.86 (3.18)	-5.17 (2.31)	-4.27 (2.98)	-4.88 (3.30)
$\theta_4$	20.34 (6.21)	20.57 (4.54)	17.69 (4.71)	20.69 (6.91)
$\theta_5$		1.94 (.57)		1.66 (1.85)
$\theta_6$			.29 (.19)	.32 (.10)
$\theta_7$			-13.33 (34.69)	-9.00 (30.91)
$\theta_8$			-.59 (61.38)	-4.11 (53.54)
$\theta_9$			-3.54 (11.60)	-1.16 (4.24)
$\theta_{10}$			1.29 (23.58)	-.18 (9.73)
$\theta_{11}$				4.34 (4.59)
$corr(g, \widetilde{my})$	.38	.36	.38	.46
$max(g)$	1.31	1.18	1.38	1.26
$W$	.0138	.0135	.0073	.0079

Table 7: Non-Linear Model - Separable Preferences - Regional Collateral Measures.

The estimation is by simulated method of moments (S=20), using the N+6 moments described in the text. Estimation of the cutoff policy function:  $\underline{\omega}_{t+1}^i = \theta_0 \bar{\eta}^i + \theta_1 \Delta \log c_{t+1}^a + \theta_2 \bar{m} y_{t+1} \Delta \log c_{t+1}^a + \theta_3 \Delta \log \hat{\eta}_{t+1}^i + \theta_4 \bar{m} y_{t+1} \Delta \log \hat{\eta}_{t+1}^i + \theta_5 \Delta \log c_t^a$ . The measurement error volatility is fixed at  $\sigma_b = .02$ . The discount factor is fixed at  $\beta = .95$ . The weighting matrix is the identity matrix in 3 iterations and the Newey-West HAC matrix for the computation of standard errors. The housing collateral ratio is region-specific and measures the collateral scarcity (0 is highest level of  $my^i$ , 1 is the lowest level of  $my^i$ ). The parameter  $\gamma$  is a preference parameter; the parameters in the consumption weight cutoff specification parameter are  $\Theta$ . The two columns correspond to two different specifications of the cutoff process, discussed in the text. The last three rows report the highest realization of the aggregate weight shock and the Simulated Method of Moments function value  $W$ . Data are for 1975-2000 for 23 US metropolitan areas.

Parameter	Specification 1	Specification 2
$\gamma$	-.04 (.27)	-.17 (1.50)
$\theta_0$	.74 (.03)	.65 (.11)
$\theta_1$	11.15 (6.32)	1.02 (7.63)
$\theta_2$	9.47 (5.79)	13.51 (6.27)
$\theta_3$	-12.31 (2.39)	-3.48 (2.60)
$\theta_4$	12.94 (2.99)	12.14 (4.13)
$\theta_5$		7.11 (1.78)
$max(g)$	1.46	1.21
$W$	0.0605	.0589