Separate Appendix for **The Returns on Human Capital:**

Good News on Wall Street is Bad News on Main Street

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May 25, 2006

B.1 Habits

We consider the possibility that habit formation in the household's preferences is responsible for the discrepancy between consumption innovation moments in the model and the data. If the log surplus consumption ratio follows an AR(1) with coefficient $0 < \phi < 1$ and a constant sensitivity parameter $\lambda > 0$ that multiplies the consumption growth innovations, then news about consumption is given by:

$$c_t - E_{t-1}c_t = \left[\frac{1-\phi\rho}{1-\phi\rho+\lambda\rho(\phi-1)}\right] \left\{ \begin{array}{c} (r_t^m - E_{t-1}r_t^m) + \\ (1-\sigma)\left(E_t - E_{t-1}\right)\sum_{j=1}\rho^j r_{t+j}^m \end{array} \right\} \tag{1}$$

Clearly, the habit cannot fix the volatility and correlation puzzles because, when $\phi < 1$, the term in brackets is larger than 1. Rather, the puzzle in a model with habits is even larger.

We now derive equation (1). Denote the log surplus consumption ratio by sp_t, and assume it follows an AR(1) as in Campbell and Cochrane (1999):

$$sp_t = \phi sp_{t-1} + \lambda(sp_{t-1})\left(c_t - E_{t-1}c_t\right),$$

where $\lambda, \phi > 0$ and $\phi < 1$. Lowercase letters denote logs. The consumption Euler equation is standard

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for $\theta = 1$:

$$1 = E_{t-1}\left[\beta\left\{\left(\frac{C_t}{C_{t-1}}\frac{Sp_t}{Sp_{t-1}}\right)^{-1/\sigma} R_t^m\right\}^{\theta}\right]$$

where Sp_t is the surplus consumption ratio in levels. We do not allow for non-separability of utility in current and future consumption goods. Taking logs and assuming log-normality produces the following equation:

$$0 = \frac{\theta}{\sigma}\mu_{t-1}^m - \frac{\theta}{\sigma}\left(E_{t-1}\Delta c_t + E_{t-1}\Delta sp_t\right) + \theta E_{t-1} r_t^m$$

where the intercept is time-varying because of sp_t:

$$\begin{aligned}
\mu_{t-1}^m &= \sigma \log\beta + \frac{1}{2}\frac{\theta}{\sigma} var_{t-1}[\Delta c_t + \Delta sp_t - \sigma r_t^m] \\
&= \sigma \log\beta + \frac{1}{2}\frac{\theta}{\sigma} var_{t-1}[\Delta c_t + (\phi - 1)\, sp_{t-1} + \lambda(sp_{t-1})\Delta c_t - \sigma r_t^m] \\
&= \sigma \log\beta + \frac{1}{2}\frac{\theta}{\sigma}\left\{\begin{array}{c} (1+\lambda(sp_t))^2 \;\; var_{t-1}[\Delta c_t] \\ -\sigma\left(1+\lambda(sp_{t-1})\right) cov_{t-1}\left[\Delta c_t, r_t^m\right] \\ +\sigma^2 var_{t-1}[r_t^m] \end{array}\right\}
\end{aligned}$$

This implies expected consumption growth can be restated as:

$$E_{t-1}\Delta c_t = \mu_{t-1}^m + \sigma E_{t-1} r_t^m - E_{t-1}\Delta sp_t$$

We fix the sensitivity parameter, because check for heteroscedasticity in section **??**: $\lambda(sp_t) = \lambda$ is constant. In that case the intercept is constant:

$$\mu_m = \sigma\log\beta + \frac{1}{2}\frac{\theta}{\sigma}\left\{(1+\lambda)^2 \;\; V_c - \sigma\left(1+\lambda\right) V_{cm} + \sigma^2 V_m\right\}$$

This can be substituted back into the consumption innovation equation to produce the following expression:

$$\begin{aligned}
c_t - E_{t-1}c_t &= r_t^m - E_{t-1}r_t^m + (1-\sigma)\left(E_t - E_{t-1}\right)\sum_{j=1}\rho^j r_{m,t+j} \\
&\quad - \left(E_t - E_{t-1}\right)\sum_{j=1}\rho^j \Delta sp_{t+j}
\end{aligned}$$

First, note that $(E_t - E_{t-1})\,\Delta sp_{t+j} = (\phi - 1)\,(E_t - E_{t-1})\,sp_{t-1+j}$. Second, note that

$$(E_t - E_{t-1})\,sp_{t+j} = \lambda\phi^{j-1}\,(c_t - E_{t-1}c_t)\,.$$

All of this implies in turn that:

$$\begin{aligned} c_t - E_{t-1}c_t &= r_t^m - E_{t-1}r_t^m + (1-\sigma)\,(E_t - E_{t-1})\sum_{j=1}\rho^j r_{t+j}^m \\ &\quad -(\phi-1)\,(E_t - E_{t-1})\sum_{j=1}\phi^{j-1}\rho^j\lambda\,(c_t - E_{t-1}c_t)\,, \end{aligned}$$

which can be simplified further into:

$$\begin{aligned} c_t - E_{t-1}c_t &= r_t^m - E_{t-1}r_t^m + (1-\sigma)\,(E_t - E_{t-1})\sum_{j=1}\rho^j r_{t+j}^m \\ &\quad -\frac{(\phi-1)\lambda\rho}{1-\phi\rho}\,(c_t - E_{t-1}c_t)\,. \end{aligned}$$

Finally, note that

$$1 + \frac{(\phi-1)\lambda\rho}{1-\phi\rho} = \frac{1-\phi\rho+(\phi-1)\lambda\rho}{1-\phi\rho},$$

so that

$$c_t - E_{t-1}c_t = \frac{1-\phi\rho}{1-\phi\rho+\lambda\rho(\phi-1)}\left\{\begin{array}{c} (r_t^m - E_{t-1}r_t^m) + \\ (1-\sigma)\,(E_t - E_{t-1})\sum_{j=1}^{\infty}\rho^j r_{m,t+j} \end{array}\right\}$$

This is the equation we set out to derive. The implied variance of consumption innovations and their covariance with financial return innovation follow immediately from this expression.

B.2 Model with Housing Wealth

This appendix augments the model to include housing wealth. We re-derive the consumption innovation equations in our benchmark case with cointegration and time-varying wealth shares. The moments of the data are somewhat changed when the returns on housing are included into the VAR. However, our main results continue to hold. We conclude that the residual does not capture housing wealth, rather it captures human wealth.

Budget Constraint The representative agent's budget constraint is:

$$W_{t+1} = R^m_{t+1}\left(W_t - C_t - P^h_t H_t\right) = R^m_{t+1}\left(W_t - \frac{C_t}{S_t}\right). \tag{2}$$

where P^h_t is the relative price of housing services, C is non-housing consumption, and $S_t = \frac{C_t}{C_t + P^h_t H_t}$ is the non-housing expenditure share. This can be rewritten in logs, denoted by lowercase variables:

$$\Delta w_{t+1} = r^m_{t+1} + \log\left(1 - \exp(c_t - s_t - w_t)\right).$$

We follow Campbell (1993) and linearize the budget constraint:

$$\Delta w_{t+1} = k + r^m_{t+1} + \left(1 - \frac{1}{\rho}\right)(c_t - s_t - w_t),$$

where $\rho = 1 - \exp(\overline{c - s - w})$ and k is a linearization constant. A second way of writing the growth rate of wealth is by using the identity:

$$\Delta w_{t+1} = \Delta c_{t+1} - \Delta s_{t+1} + (c_t - s_t - w_t) - (c_{t+1} - s_{t+1} - w_{t+1}).$$

Combining these two expressions, iterating forward, and taking expectations, we obtain the linearized budget constraint (Campbell, 1991):

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j r^m_{t+1+j} + (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta s_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \Delta c_{t+1+j} \tag{3}$$

Preferences The representative household has non-separable preferences over housing and non-housing consumption. We model the period utility kernel as CES with intratemporal substitution parameter ε:

$$u(C_t, H_t) = \left[(1-\alpha)C_t^{\frac{\varepsilon-1}{\varepsilon}} + \alpha H_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Intertemporal preferences are still of the Epstein-Zin type:

$$U_t = \left((1-\beta)u(C_t, H_t)^{(1-\gamma)/\theta} + \beta\left(E_t U_{t+1}^{1-\gamma}\right)^{1/\theta}\right)^{\theta/(1-\gamma)},$$

where γ is the coefficient of relative risk aversion and σ is the intertemporal elasticity of substitution, henceforth IES. Finally, θ is defined as $\theta = \frac{1-\gamma}{1-(1/\sigma)}$. Special cases obtain when $\varepsilon = 1$ (Cobb-Douglas) and $\varepsilon = \sigma$.

The Euler equation with respect to the market return takes on the form

$$1 = E_t[\exp(sdf_{t+1} + r^m_{t+1})],$$

where the log stochastic discount factor is:

$$sdf_{t+1} = \theta \log \beta - \frac{\theta}{\sigma}\Delta c_{t+1} - \frac{\theta}{\sigma}\left(\frac{\sigma-\varepsilon}{\varepsilon-1}\right)\Delta s_{t+1} + (\theta-1)r^m_{t+1}$$

We then assume that non-housing consumption growth, non-housing expenditure share growth and the market return are conditionally homoscedastic and jointly log-normal. This leads to the consumption Euler equation:

$$E_t\Delta c_{t+1} = \mu_m + \sigma E_t r^m_{t+1} - \left(\frac{\sigma-\varepsilon}{\varepsilon-1}\right)E_t\Delta s_{t+1}, \tag{4}$$

where μ_m is a constant that includes the variance and covariance terms for non-housing consumption, non-housing expenditure share, and market innovations, as well as the time preference parameter.

Substituting out Consumption Growth We can now substitute equation (4) back into the consumption innovation equation in (3), to obtain an expression with only returns on the right hand side:

$$c_t - E_{t-1}c_t = r^m_t - E_{t-1}r^m_t + (1-\sigma)(E_t - E_{t-1})\sum_{j=1}^{\infty}\rho^j r^m_{t+j} + \left(\frac{\sigma-1}{\varepsilon-1}\right)(E_t - E_{t-1})\sum_{j=0}^{\infty}\rho^j\Delta s_{t+j}, \tag{5}$$

Innovations to the representative agent's non-housing consumption are determined by (1) the unexpected part of this period's market return (2) the innovation to expected future market returns, and (3) innovations to current and future expenditure share changes. In the realistic parameter region $\sigma < 1$, $\varepsilon < 1$, the last term is more important the more $\sigma < \varepsilon$.

Housing Return Data We construct data on the log change in the value of the aggregate housing stock (Δp^h_{t+1}) and the log change in the dividend payments on the aggregate housing stock (Δd^h_{t+1}).

The aggregate housing stock is measured as the value of residential real estate of the household sector (Flow of Funds, series FL155035015). The dividend on aggregate housing is measured as housing services consumption (quarterly flow, from NIPA Table 2.3.5). We construct a log price index p^h by fixing the 1947.I observation to 0, and using the log change in prices in each quarter. Likewise, we choose an initial log dividend level, and construct the dividend index using log dividend growth. The log dividend price ratio $d^h - p^h$ is the difference of the log dividend and the log price index. The initial dividend index level is chosen to match the mean log dividend price ratio to the one on stocks (-4.6155). In the model the mean dividend price ratios are the same on all assets. We construct housing returns from the Campbell-Shiller decomposition:

$$r_t^h = k + \Delta d_t^h + (d_{t-1}^h - p_{t-1}^h) - \rho(d_t^h - p_t^h)$$

where ρ and k are Campbell Shiller linearization constants. In the model, these constants must be the same for all assets (financial wealth, housing wealth and human wealth). We use stock market data to pion down ρ and k: $\rho = \frac{1}{1+\overline{d^a - p^a}} = .9901$ and $k = -\log(\rho) - (1-\rho)\log(\rho^{-1} - 1) = .0556$. To get the log real return, we deflate the nominal log return by the personal income price deflator, the same series used to deflate all other variables. The procedure results in an average quarterly housing return of 2.22% with a standard deviation of 1.30%. For comparison, the log real value weighted CRSP stock market return is 1.92% on average with a standard deviation of 8.26%. The correlation between the two return series is .11.[1]

VAR Additions To keep the state space as small as possible, we define a new variable, $\tilde{r}^a = \varphi r^a + (1-\varphi) r^h$, which denotes the return on a portfolio of financial assets and housing. The portfolio weight φ_t is the ratio of financial income (dividends, interest and proprietor's income) to financial income plus housing income (measured by housing services). This weight varies over time and is 0.67 on average. Likewise, we define the log dividend-price ratio $\widetilde{dp}^a = \varphi dp^a + (1-\varphi) dp^h$. From the return series and the dividend-price series, we construct the wealth growth series $\widetilde{\Delta a}$, as in the main text. The

[1] Those numbers are broadly consistent with the small literature on housing returns. Case and Shiller (1989) find that the volatility of house price changes is mostly idiosyncratic. The regional component of housing prices only explains between 7 and 27 percent of individual house price variation for the four cities in their study. They also report a zero correlation between housing returns and stock returns. Regional repeat sales price indices from Freddie Mac for 50 US states between 1976 and 2002 show a low volatility. The median region has a real annual house price appreciation (ex-dividend return) with a standard deviation of 5.1%. Across regions, the volatility varies between 2.4% and 12.8% per year (own calculations). For nation-wide data, the annual volatility of the ex-dividend return is 3.3%.

Table 1: Moments from Data - Model With Housing

Moments	**Panel A: Firm Value**	**Panel B: Stock Market**
$Std(DR^a_t)$	.092	.108
$Std(CF^y_{t,\infty})$	.032	.042
$Std(DR^a_\infty)$	.113	.103
$Corr(DR^a_t, DR^a_\infty)$	-.797	-.956
$Corr(DR^a_t, CF^y_{t,\infty})$	.423	.074
$Corr(DR^a_\infty, CF^y_{t,\infty})$	-.611	-.208
$Std(c_t)$	.011	.011
$Corr(c_t, DR^a_t)$	.184	.204
$Corr(CF^a_t, CF^y_t)$	-.203	.192
$Corr(CF^a_{t,\infty}, CF^y_{t,\infty})$	-.439	-.425

Notes: This Table reports the same moments as in Tables **??** and **??**, except that DR^a_t and DR^a_∞ pertain to the return on a portfolio of financial asset returns and housing returns. In the left panel, the financial asset returns in the portfolio are firm value returns; in the right column they are stock returns.

variables $\widetilde{\Delta a}$ and $\widetilde{dp}^a$ take the place of Δa and dp^a in the VAR. The labor income share s is defined as the ratio of labor income to total income, where total income consists of labor income, financial income and housing income. To the 7 elements in the VAR without housing we add the log growth rate in the non-housing expenditure share (Δs, element 8). Once the VAR has been estimated, we can construct the new series for news about current and future growth rates on the non-housing expenditure share $\{CF^s_{t,\infty}\}$:

$$CF^s_{t,\infty} = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta s_{t+j} = e_8'(I - \rho A)^{-1} \varepsilon_t.$$

The procedure with time-varying wealth shares goes through as in the main text. The expression for consumption innovations with time-varying human wealth share is identical to equation (**??**), except for the additional term $\frac{\sigma-1}{\varepsilon-1}(CF^a)_{t+1,\infty}$.

Moments of the Data Table 1 summarizes the moments from the data using the firm value returns and stock returns. The main change with the model without housing is that the combined financial asset - housing return innovations $\tilde{r}^a$ are 33% less volatile than financial assets alone. News about changes in the non-housing expenditure share d^a has a very low variance ($Std(CF^s_{t,\infty}) = 0.08$ compared to $Std(c) = .34$). This term will play a negligible role in the analysis.

Consumption Growth Accounting The results with time-varying wealth shares are close to the results without housing (2 shows the case with time-varying human wealth shares). Matching the

Table 2: Human Wealth and Market Discount Rate News - Model With Housing.

	Panel A: Firm Value				**Panel B: Stock Market**			
Model	*II*	*III*	*IV*	*V*	*II*	*III*	*IV*	*V*
$Std(DR^y_\infty)$	.113	0	.022	.130	.104	0	.031	.113
$Corr(DR^a_t, DR^y_\infty)$	-.797	0	.546	.524	-.955	0	.062	.530
$Corr(CF^a_{t,\infty}, DR^y_\infty)$	.575	0	-.151	.101	-.002	0	-.199	-.147
$Corr(DR^a_\infty, DR^y_\infty)$	1.000	0	-.538	-.368	1.000	0	-.126	-.601
$Std(DR^y_t)$	.136	.032	.017	.105	.120	.042	.017	.080
$Corr(DR^y_t, DR^a_t)$	.767	.423	.094	-.522	.853	.074	.070	-.710
$Corr_{DR^y_t, DR^a_\infty}$	-.982	-.611	-.462	.268	-.939	-.208	-.285	.740
$Std(c)$	.044	.028	.024	.011	.047	.031	.026	.012
$Corr(c, DR^a_t)$	.843	.650	.648	.184	.804	.535	.627	.204
$Std(DR^m_t)$	.121	.039	.026	.074	.114	.042	.028	.051
$Corr(DR^m_t, DR^a_t)$	.835	.805	.853	-.312	.902	.594	.849	-.461
$Corr(DR^m_t, DR^y_t)$	.993	.875	.590	.971	.994	.837	.561	.947
$Corr(DR^m_t, DR^m_\infty)$	-.983	-.816	-.526	-.995	-.965	-.683	-.408	-.976

Notes: This Table reports the same moments as in Tables **??** and **??**, except that DR^a_t and DR^a_∞ pertain to the return on a portfolio of financial asset returns and housing returns. Computations are done for the model with time-varying human wealth share and quarterly data. Standard deviations are annualized. The *EIS* is $\sigma = .28$ and the intratemporal elasticity of substitution between housing and non-housing consumption is $\varepsilon = 0.5$. The average wealth shares are $\bar{\nu} = .7761$ in Panel A and $\bar{\nu} = .7923$ in Panel B.

moments of consumption requires financial cum housing wealth returns and human wealth returns to be negatively correlated. The resulting market return is negatively correlated with returns on financial cum housing wealth, and strongly positively correlated with returns on human wealth. This is true for both measures of financial assets (both panels).

The failure of the benchmark models to match the consumption moments derives from a failure to generate $Corr(DR^a_t, DR^y_t) < 0$. Consumption is still much too highly correlated with financial cum housing asset returns, but the failure in the consumption variance is less pronounced than before. In sum, the properties of the human wealth process in the model with housing are virtually unaffected, relative to the model without housing.

B.3 Heterogeneity: Aggregation

Our objective is start from the household consumption innovations and aggregate these innovations to get an expression for aggregate consumption innovations. To keep it simple, we assume all households share the same IES and the same mean log consumption/wealth ratio, and hence, the same ρ.

We assume each household's consumption Euler equation is satisfied. If this is the case, each household $i's$ consumption innovations can be stated as follows:

$$c_{t+1}^i - E_t^i c_{t+1}^i = r_{t+1}^m - E_t^i r_{t+1}^{m,i} + (1-\sigma)(E_{t+1}^i - E_t^i) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}^{m,i},$$

where E^i denotes the conditional expectation operator, conditional on household $i's$ information set. We use E to denote expectations conditional on the econometrician's (smaller) information set. We let $\widetilde{E}$ denote the cross-sectional expectation operator: $\widetilde{E}(x^i) = \frac{1}{I}\sum_i x^i$.

First, note that the weighted consumption innovations are (roughly) equal to the aggregate consumption innovations:

$$\widetilde{E}\left(\frac{C_t^i}{C_t}\right)\left(c_{t+1}^i - E_t c_{t+1}^i\right) \simeq \left(c_{t+1} - E_t c_{t+1}\right),$$

and, that the weighted household return innovations are equal to the market return innovations:

$$\widetilde{E}\left(\frac{W_t^i}{W_t}\right)\left(r_{t+1}^{m,i} - E_t r_{t+1}^{m,i} + (1-\sigma)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}^{m,i}\right) \simeq$$

$$r_{t+1}^m - E_t r_{t+1}^m + (1-\sigma)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}^m.$$

To aggregate the household consumption innovations and obtain an expression in terms of the market return on the right hand side, we need to weight these household return innovations by the wealth shares of each household:

$$\widetilde{E}\left(\frac{W_t^i}{W_t}\right)\left(c_{t+1}^i - E_t^i c_{t+1}^i\right) =$$

$$\widetilde{E}\left(\frac{W_t^i}{W_t}\right)\left(r_{t+1}^{m,i} - E_t^i r_{t+1}^{m,i} + (1-\sigma)(E_{t+1}^i - E_t^i) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}^{m,i}\right)$$

On the left hand side, however, we want an expression in terms of aggregate consumption. So, we split the wealth share into a consumption wealth ratio term and a consumption share term:

$$
\begin{aligned}
\widetilde{E}\left(\frac{W_t^i/C_t^i}{W_t/C_t}\right)\left(\frac{C_t^i}{C_t}\right)\left(c_{t+1}^i - E_t c_{t+1}^i\right) + \widetilde{E}\left(\frac{W_t^i/C_t^i}{W_t/C_t}\right)\eta_t^i\left(c_{t+1}^i\right) &= \\
\widetilde{E}\left(\frac{W_t^i}{W_t}\right)\left(r_{t+1}^{m,i} - E_t r_{t+1}^{m,i} + (1-\sigma)(E_{t+1}-E_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}^{m,i}\right) & \\
+\widetilde{E}\left(\frac{W_t^i}{W_t}\right)\left(\eta_t^i(r_{t+1}^{m,i}) + (1-\sigma)\left(\eta_{t+1}^i - \eta_t^i\right)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}^{m,i}\right), &
\end{aligned}
$$

where $\eta_{t+1}^i(x_{t+1}^i)$ denote the econometrician's prediction errors for some random variable x_{t+1}^i:

$$\eta_{t+1}^i(x_{t+1}^i) = (E_t^i - E_t)(x_{t+1}^i).$$

We make the following assumptions we need to obtain the aggregation result:

Assumption 1. *The average consumption-wealth ratio equals the aggregate consumption-wealth ratio:*

$$\widetilde{E}\left(\left(\frac{W_t^i/C_t^i}{W_t/C_t}\right)\right) = 1. \tag{6}$$

Assumption 2. *The consumption/wealth ratio deviations at t are orthogonal to weighted consumption innovations at $t+1$:*

$$\widetilde{E}\left(\left(\frac{W_t^i/C_t^i}{W_t/C_t} - 1\right)\left(\frac{C_t^i}{C_t}\right)\left(c_{t+1}^i - E_t c_{t+1}^i\right)\right) = 0. \tag{7}$$

Assumption 3. *The cross-sectional average of the econometrician's prediction errors are zero:*

$$\widetilde{E}\left(\left(\frac{W_t^i}{W_t}\right)\eta_t^i\left(x_{t+1}^i\right)\right) = 0$$

and

$$\widetilde{E}\left(\left(\frac{W_t^i/C_t^i}{W_t/C_t}\right)\left(\frac{C_t^i}{C_t}\right)\eta_t^i\left(x_{t+1}^i\right)\right) = 0. \tag{8}$$

Given the assumptions in (7) and (6), it is immediate that the cross-sectional average of the weighted consumption innovations satisfies:

$$\widetilde{E}\left(\left(\frac{W_t^i/C_t^i}{W_t/C_t}\right)\left(\frac{C_t^i}{C_t}\right)\left(c_{t+1}^i - E_t c_{t+1}^i\right)\right) = \widetilde{E}\left(\left(\frac{C_t^i}{C_t}\right)\left(c_{t+1}^i - E_t c_{t+1}^i\right)\right) \simeq (c_{t+1} - E_t c_{t+1})$$

On the right hand side, we know that:

$$\widetilde{E}\left(\left(\frac{W_t^i}{W_t}\right)\left(r_{t+1}^{m,i} - E_t r_{t+1}^{m,i} + (1-\sigma)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}^{m,i}\right)\right)$$
$$\simeq r_{t+1}^m - E_t r_{t+1}^m + (1-\sigma)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}^m.$$

Combining this result with the zero average prediction error assumption in (8), produces the desired result. The expression in equation (6) simplifies to the aggregate consumption innovation in the text:

$$c_{t+1} - E_t c_{t+1} = r_{t+1}^m - E_t r_{t+1}^m + (1-\sigma)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}^m$$

What do these assumption imply? In logs, the consumption/wealth ratio deviations are:

$$(c_t^i - w_t^i) - (c_t - w_t) = (1-\sigma)\left(E_t^i \sum_{j=1}^{\infty}\rho^j r_{t+1+j}^{m,i} - E_t \sum_{j=1}^{\infty}\rho^j r_{t+1+j}^m\right).$$

The second assumption implies that these deviations cannot be correlated with consumption-weighted household consumption innovations at $t+1$. The assumption can be somewhat weakened by having the right-hand side of 7 be a constant instead of zero. The aggregation consumption innovation equation then also contains a constant, but this does not affect the consumption variance and correlation moment of interest. The third assumption implies that for (cross-sectional) average variables, the econometrician does as well at forecasting consumption and returns as the household.

Borrowing Constraints

What about borrowing constraints? Binding constraints add a third component to aggregate consumption innovations, news about future average multipliers on these constraints:

$$c_{t+1} - E_t c_{t+1} = r_{t+1}^m - E_t r_{t+1}^m + (1-\sigma)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}^m - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \lambda_{t+1+j},$$

where λ_{t+j} denotes the cross-sectional weighted-average multiplier at $t+j$ (see section of the separate appendix for a derivation). Clearly, it does not help to have very binding constraints all the time. For model-implied consumption innovations to be smooth and only mildly correlated with financial asset returns, a positive innovation in financial returns must be associated with more binding constraints in the future. The collateral constraints of Lustig and Van Nieuwerburgh (2005) have this feature.

If the households were to encounter some binding constraints, the household's consumption innovations would be determined by:

$$c^i_{t+1} - E^i_t c^i_{t+1} = r^m_{t+1} - E^i_t r^{m,i}_{t+1} + (1-\sigma)(E^i_{t+1} - E^i_t)\sum_{j=1}^{\infty}\rho^j r^{m,i}_{t+1+j} - (E^i_{t+1} - E^i_t)\sum_{j=1}^{\infty}\rho^j \lambda^i_{t+1+j},$$

where λ^i_t is the Lagrange multiplier on household i's constraint at time t. Repeating the same aggregation exercise produces the following result:

$$c_{t+1} - E_t c_{t+1} = r^m_{t+1} - E_t r^m_{t+1} + (1-\sigma)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r^m_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \lambda_{t+1+j},$$

where the aggregate multiplier at $t+j$ is the cross-sectional weighted-average of the individual multipliers: $\lambda_{t+j} = \widetilde{E}\left(\left(\frac{W^i_t}{W_t}\right)\lambda^i_{t+j}\right)$.

B.4 Additional Tables

Table 3: VAR Estimation under Cointegration

Variable	Δa_{t-1}	Δy_{t-1}	dp^a_{t-1}	rtb_{t-1}	ysp_{t-1}	s_{t-1}	Δc_{t-1}	cay_{t-1}	R^2
Panel A: Firm Value									
Δa_t	0.062	0.814	0.079	−0.001	0.372	0.558	−3.742	−0.047	0.106
$(t-stat)$	(0.413)	(0.646)	(1.038)	(−0.062)	(0.135)	(0.372)	(−1.399)	(−0.089)	
Δy_t	0.091	0.259	−0.015	−0.005	−0.037	−0.090	0.155	0.036	0.476
$(t-stat)$	(4.396)	(1.493)	(−1.405)	(−1.708)	(−0.098)	(−0.438)	(0.419)	(0.492)	
dp^a_t	−0.220	−1.850	0.729	0.028	4.905	1.520	5.700	0.932	0.773
$(t-stat)$	(−1.043)	(−1.045)	(6.810)	(0.907)	(1.271)	(0.722)	(1.517)	(1.249)	
rtb_t	4.108	−2.257	−1.386	0.510	46.849	24.451	15.112	5.435	0.426
$(t-stat)$	(3.609)	(−0.236)	(−2.402)	(3.052)	(2.252)	(2.155)	(0.746)	(1.351)	
ysp_t	−0.028	−0.041	0.007	−0.003	0.378	−0.124	−0.016	−0.007	0.706
$(t-stat)$	(−3.589)	(−0.625)	(1.681)	(−2.989)	(2.672)	(−1.607)	(−0.116)	(−0.258)	
s_t	−0.002	0.088	−0.003	−0.002	−0.112	0.881	−0.012	0.004	0.926
$(t-stat)$	(−0.398)	(1.991)	(−1.187)	(−2.605)	(−1.165)	(16.736)	(−0.130)	(0.188)	
Δc_t	0.025	0.052	0.007	−0.003	0.267	0.066	0.064	−0.092	0.572
$(t-stat)$	(2.639)	(0.670)	(1.549)	(−2.210)	(1.572)	(0.714)	(0.385)	(−2.790)	
cay_t	−0.059	−0.329	0.001	0.001	0.223	0.014	0.784	0.887	
Panel B: Stock Market									
Variable	Δa_{t-1}	Δy_{t-1}	dp^a_{t-1}	rtb_{t-1}	ysp_{t-1}	s_{t-1}	Δc_{t-1}	cay_{t-1}	R^2
Δa_t	−0.048	0.420	0.106	−0.005	1.897	−0.732	−2.668	−0.061	0.136
$(t-stat)$	(−0.318)	(0.266)	(1.063)	(−0.198)	(0.593)	(−0.450)	(−0.894)	(−0.152)	
Δy_t	0.080	0.237	0.011	−0.003	0.263	−0.038	−0.013	−0.082	0.540
$(t-stat)$	(4.793)	(1.345)	(0.958)	(−1.181)	(0.736)	(−0.208)	(−0.038)	(−1.826)	
dp^a_t	0.025	−0.022	0.861	0.022	−0.022	1.445	1.261	0.314	0.797
$(t-stat)$	(0.191)	(−0.016)	(9.836)	(0.983)	(−0.008)	(1.015)	(0.483)	(0.897)	
rtb_t	3.477	−0.541	0.673	0.579	63.907	28.895	0.336	−3.163	0.412
$(t-stat)$	(3.494)	(−0.052)	(1.016)	(3.416)	(3.014)	(2.682)	(0.017)	(−1.191)	
ysp_t	−0.022	−0.012	−0.008	−0.004	0.264	−0.147	−0.028	0.038	0.718
$(t-stat)$	(−3.335)	(−0.178)	(−1.846)	(−3.462)	(1.889)	(−2.073)	(−0.218)	(2.163)	
s_t	−0.003	0.101	−0.002	−0.002	−0.090	0.891	−0.070	−0.009	0.927
$(t-stat)$	(−0.562)	(2.118)	(−0.577)	(−2.544)	(−0.931)	(18.069)	(−0.772)	(−0.751)	
Δc_t	0.026	0.024	−0.003	−0.003	0.161	0.099	0.181	−0.033	0.556
$(t-stat)$	(3.154)	(0.283)	(−0.561)	(−2.155)	(0.923)	(1.121)	(1.118)	(−1.526)	
cay_t	−0.027	−0.249	−0.034	0.001	−0.435	0.285	0.753	1.043	

Notes: This table reports the VAR coefficient estimates obtained using OLS. Panel A uses firm value returns. Panel B uses stock returns. All results are for the full sample 1947-2004 (annual data). The rows denote the time t variables and the columns the time $t-1$ variables). The t-statistics are reported in parentheses. The VAR contains 7 variables plus the consumption-wealth ratio measure *cay*. The dynamics of *cay* are implied by the VAR coefficients.

Table 4: Human Wealth and Market Discount Rate Innovations - Quarterly Data.

Model	*II*	*III*	*IV*	*V*	*II*	*III*	*IV*	*V*
	Panel A: Firm Value Returns				*Panel B: Stock Returns*			
$Std(DR^y_\infty)$	.165	0	.024	.117	.150	0	.027	.122
	(0.036)	(−−)	(0.008)	(0.035)	(0.015)	(−−)	(0.012)	(0.051)
	[0.042]	[−−]	[0.012]	[0.036]	[0.019]	[−−]	[0.012]	[0.050]
$Corr(DR^y_\infty, DR^a_t)$	-.857	0	.563	.660	-.972	0	.255	.550
	(0.058)	(−−)	(0.279)	(0.214)	(0.025)	(−−)	(0.325)	(0.298)
	[0.071]	[−−]	[0.291]	[0.213]	[0.021]	[−−]	[0.330]	[0.280]
$Corr(DR^y_\infty, CF^a_{t,\infty})$	.540	0	-.196	.172	-.149	0	-.079	.015
	(0.258)	(−−)	(0.330)	(0.312)	(0.253)	(−−)	(0.330)	(0.313)
	[0.265]	[−−]	[0.334]	[0.324]	[0.258]	[−−]	[0.339]	[0.327]
$Corr(DR^y_\infty, DR^a_\infty)$	1.000	0	-.575	-.337	1.000	0	-.292	-.598
	(−−)	(−−)	(0.321)	(0.289)	(0.000)	(−−)	(0.312)	(0.290)
	[−−]	[−−]	[0.319]	[0.312]	[0.000]	[−−]	[0.315]	[0.264]
$Std(DR^y_t)$	.185	.030	.017	.102	.164	.034	.017	.093
	(0.041)	(0.007)	(0.001)	(0.035)	(0.019)	(0.013)	(0.001)	(0.048)
	[0.049]	[0.011]	[0.002]	[0.030]	[0.023]	[0.013]	[0.002]	[0.044]
$Corr(DR^y_t, DR^a_t)$	.842	.493	.079	-.614	.934	.232	.054	-.641
	(0.059)	(0.223)	(0.062)	(0.220)	(0.055)	(0.292)	(0.050)	(0.282)
	[0.076]	[0.240]	[0.081]	[0.222]	[0.051]	[0.290]	[0.071]	[0.263]
$Std(c)$	.055	.029	.025	.011	.056	.032	.027	.011
	(0.009)	(0.005)	(0.005)	(0.005)	(0.009)	(0.009)	(0.007)	(0.008)
	[0.014]	[0.008]	[0.007]	[0.004]	[0.010]	[0.010]	[0.007]	[0.007]
$Corr(c, DR^a_t)$	.918	.694	.685	.181	.880	.576	.617	.197
	(0.033)	(0.208)	(0.213)	(0.057)	(0.094)	(0.210)	(0.184)	(0.052)
	[0.045]	[0.207]	[0.204]	[0.075]	[0.108]	[0.257]	[0.224]	[0.069]
$Std(DR^m_t)$	.170	.048	.035	.066	.161	.049	.038	.059
	(0.032)	(0.008)	(0.004)	(0.024)	(0.016)	(0.009)	(0.004)	(0.032)
	[0.040]	[0.011]	[0.005]	[0.022]	[0.021]	[0.010]	[0.005]	[0.029]
$Corr(DR^m_t, DR^a_t)$	.893	.886	.906	-.295	.958	.812	.902	-.273
	(0.041)	(0.054)	(0.043)	(0.292)	(0.034)	(0.116)	(0.022)	(0.354)
	[0.054]	[0.061]	[0.040]	[0.312]	[0.036]	[0.152]	[0.034]	[0.356]
$Corr(DR^m_t, DR^y_t)$	.995	.820	.438	.932	.997	.739	.427	.902
	(0.004)	(0.131)	(0.069)	(0.053)	(0.005)	(0.139)	(0.059)	(0.057)
	[0.004]	[0.141]	[0.081]	[0.054]	[0.003]	[0.138]	[0.083]	[0.085]
$Corr(DR^m_t, DR^m_{t,\infty})$	-.992	-.853	-.712	-.988	-.990	-.856	-.695	-.982
	(0.007)	(0.089)	(0.137)	(0.011)	(0.010)	(0.102)	(0.173)	(0.013)
	[0.008]	[0.098]	[0.180]	[0.013]	[0.016]	[0.127]	[0.172]	[0.020]

Notes: Panel A uses firm value returns. Panel B uses stock returns. All results are for the full sample 1947.II-2004-IV (quarterly data). In each panel, the first column is *Model II*, with $C' = \frac{1}{\rho}\left(e_1'\rho A + (1-\rho)e_3'\right)$. The second column is *Model III* with $C' = 0$, and the third column is *Model IV* with $C' = e_2' A$. The last column is *Model V* with C chosen to minimize the distance between the model-implied and actual consumption news standard deviation and correlation. Computations are done for $\bar{\nu} = 0.7761$ in panel A and $\bar{\nu} = 0.7923$ in panel B, and $\sigma = .28$. The standard errors in ( ) are generated by bootstrapping with replacement from the VAR residuals. The standard errors in [ ] are generated by a wild bootstrap (robust to heteroscedasticity).

Table 5: Human Wealth and Market Discount Rate News - Long Sample

Moments	*Model II*	*Model III*	*Model IV*	*V*
	Stock Market			
$Std(DR^y_\infty)$	.130	0	.019	.147
	(0.024)	(−−)	(0.008)	(0.032)
	[0.028]	[−−]	[0.009]	[0.034]
$Corr(DR^a_t, DR^y_\infty)$	-.892	0	.205	.591
	(0.060)	(−−)	(0.325)	(0.185)
	[0.066]	[−−]	[0.334]	[0.239]
$Corr(CF^a_{t,\infty}, DR^y_\infty)$	-.326	0	.082	.377
	(0.239)	(−−)	(0.293)	(0.240)
	[0.250]	[−−]	[0.323]	[0.289]
$Corr(DR^a_\infty, DR^y_\infty)$	1.000	0	-.224	-.558
	(−−)	(−−)	(0.355)	(0.223)
	[−−]	[−−]	[0.352]	[0.260]
$Std(DR^y_t)$	.148	.042	.034	.119
	(0.028)	(0.009)	(0.004)	(0.025)
	[0.031]	[0.011]	[0.005]	[0.028]
$Corr(DR^y_t, DR^a_t)$	.806	.079	-.016	-.698
	(0.085)	(0.220)	(0.112)	(0.167)
	[0.099]	[0.233]	[0.149]	[0.228]
$Std(c)$	.060	.040	.038	.018
	(0.009)	(0.007)	(0.006)	(0.003)
	[0.011]	[0.009]	[0.008]	[0.005]
$Corr(c, DR^a_t)$	.793	.614	.626	.217
	(0.088)	(0.176)	(0.153)	(0.113)
	[0.099]	[0.181]	[0.162]	[0.156]
$Std(DR^m_t)$	.149	.055	.049	.076
	(0.023)	(0.008)	(0.004)	(0.019)
	[0.028]	[0.010]	[0.008]	[0.022]
$Corr(DR^m_t, DR^a_t)$	.880	.765	.795	-.388
	(0.055)	(0.112)	(0.058)	(0.229)
	[0.063]	[0.117]	[0.074]	[0.288]
$Corr(DR^m_t, DR^y_t)$	.990	.687	.568	.915
	(0.007)	(0.120)	(0.088)	(0.033)
	[0.009]	[0.135]	[0.110]	[0.057]
$Corr(DR^m_t, DR^m_\infty)$	-.981	-.824	-.687	-.973
	(0.013)	(0.109)	(0.204)	(0.009)
	[0.013]	[0.113]	[0.201]	[0.024]

Notes: This table uses stock market data for the long sample of annual 1929-2004 data. The first column is *Model II*, with $C' = \frac{1}{\rho}\left(e_1'\rho A + (1-\rho)e_3'\right)$. The second column is *Model III* with $C' = 0$, and the third column is *Model IV* with $C' = e_2'A$. The last column is *Model V* with C chosen to minimize the distance between the model-implied and actual consumption news standard deviation and correlation. Computations are done for $\bar{\nu} = 0.7923$ and $\sigma = .28$. The standard errors in ( ) are generated by bootstrapping with replacement from the VAR residuals. The standard errors in [ ] are generated by a wild bootstrap (robust to heteroscedasticity).

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