Integrating Managerial and Tax Objectives in Transfer Pricing

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Abstract

This paper examines transfer pricing in multinational firms when individual divisions face different income tax rates. Assuming that a firm decouples its internal transfer price from the arm’s length price used for tax purposes, we analyze the effectiveness of alternative pricing rules under both cost- and market-based transfer pricing. In a tax-free world, Hirshleifer (1956) advocated that the internal transfer price be set equal to the marginal cost of the supplying division. Extending this solution, we argue that the optimal internal transfer price should be a weighted average of the pre-tax marginal cost and the most favorable arm’s length price. When the supplying division sells the intermediate product in question also to outside parties, the external price becomes a natural candidate for the arm’s length price. We argue that for internal performance evaluation purposes firms should generally not value internal transactions at the prevailing market price if the supplying division has monopoly power in the external market. By imposing intracompany discounts, firms can alleviate attendant double marginalization problems and, at the same time, realize tax savings which take advantage of differences in income tax rates. Our analysis characterizes optimal intracompany discounts as a function of the market parameters and the divisional tax rates.
1 Introduction

Textbooks in managerial accounting usually portray transfer pricing as an instrument for achieving decentralization and coordination in multidivisional firms. Accordingly, the role of transfer prices is to provide valuations for intermediate products and services so as to facilitate transactions across profit centers within a firm.\(^1\) For multinational firms, textbooks also emphasize the importance of tax considerations in the choice of transfer prices. Yet, the choice of transfer prices for tax purposes is typically portrayed as a tax compliance issue that remains conceptually separate from the managerial and economic dimensions of transfer pricing.

In this paper we analyze the interrelation between the preferred managerial transfer price and the “arm’s length” price used for tax purposes.\(^2\) An immediate question therefore is whether firms separate their internal transfer prices from the ones used for tax purposes. While there is is no statutory conformity requirement, it appears that the majority of multinational firms insist on one set of prices both for simplicity and in order to avoid the possibility that multiple valuations become evidence in any disputes with the tax authorities.\(^3\) At the same time, a growing number of multinational firms decouple their internal transfer prices from the ones used for tax purposes. Proponents of decoupling argue that tax valuations are driven by regulations, yet these regulations will frequently not capture the underlying economics of internal transactions.\(^4\)

Our analysis first examines a setting in which the intermediate product is a propri-

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\(^1\)See, for example, Horngren, Foster and Datar (2002), Jiambalvo (2001) and Kaplan and Atkinson (1998).

\(^2\)Our analysis builds on several recent studies that have identified tradeoffs between managerial incentives and taxes. We first outline our approach and subsequently explain the relationship to these earlier studies.

\(^3\)See, for example, Halperin and Srindhi (1991) and Granfield (1995).

\(^4\)Wilson (1993) documents the practice of decoupling in several field studies. Further evidence is provided in the recent survey by Ernst & Young (1999). Springsteel (1999) reports that 77% of all respondents in a “best practices” group of large companies operate two sets of books, compared with only 25% of those firms outside that group.
etary component which is not sold to outside parties and for which there are no close substitutes on external markets. Most multinational firms then resort to a cost-based approach for internal valuation purposes.\textsuperscript{5} As a benchmark, we begin with Hirshleifer’s (1956) setting in which the product’s marginal cost is verifiable by the internal accounting system.

To re-examine the Hirshleifer solution derived in a tax-free world, we take the perspective that the firm has identified a range of prices that can be justified to the tax authorities as arm’s length prices. Such a range will typically be generated by applying alternative transfer pricing methods identified in Section 482 of the U.S. tax code.\textsuperscript{6} From a tax perspective, the most favorable arm’s length price then depends on the relative income tax rates of the two divisions. We argue that the optimal internal cost-based transfer price can be expressed as a weighted average of the pre-tax unit cost and the most favorable arm’s length price. When faced with this transfer price, the buying division will correctly internalize the overall after-tax cost of intracompany transfers, which includes both the pre-tax cost of production and the tax related cash outflows.

By definition, the most favorable arm’s length price minimizes the firm’s overall tax liability for given transfer quantities. If due to a conformity requirement the same price is used for managerial performance evaluation, the resulting intracompany transfers will be too low whenever the arm’s length price exceeds the pre-tax unit cost. The optimal single transfer price balances the conflicting goals of tax minimization and efficient resource allocation. We derive an upper bound for the tax related mark-up that a firm would seek to impose over and above its pre-tax cost when constrained to choose a

\textsuperscript{5}According to survey evidence reported in Horngren, Foster and Datar (2002), 78\% of all multinational firms resort to either to cost-based or market-based transfer pricing.

\textsuperscript{6}Aside from the comparable uncontrolled price (CUP) method (which will frequently not be applicable for specialized and proprietary products), Section 482 of the Internal Revenue Code identifies the following methods as permissible: cost-plus, resale-minus, profit split and comparable profits. Our model is also consistent with the notion that a firm has entered into an advance pricing agreement (APA) with the tax authorities. See Eden (1998) and King (1994) on the implementation of these alternative methods.
single transfer price. Provided this cost-plus price is below the most favorable arm’s length price, the expected corporate profit after taxes will be maximized by a transfer price that does not minimize the firm’s tax burden ex-post.\footnote{This finding is related to results in Narayanan and Smith (2000), as explained in Section 2 below.}

The second part of our analysis allows for the possibility that the intermediate product is also sold to external parties. Specifically, a foreign division is assumed to sell the intermediate product both to a domestic division and to a range of external customers, yet the domestic division does not have access to other external suppliers of this product. The volume of trade with external customers is sufficiently significant so that the external market price qualifies as a comparable uncontrolled price (CUP) and therefore this price also serves as the arm’s length price. We focus on the standard case in which the domestic division faces a higher tax rate than the foreign subsidiary.

If the foreign division is a price taker in the external market, then, consistent with Hirshleifer’s (1956) prescription in a tax-free world, intracompany transfers should be valued at the external market price. However, we focus on the more setting in which the foreign division is a price setter externally. A policy of valuing internal transfers at the external market price (which also serves as the arm’s length price) then results in inefficiently low quantity transfers. This inefficiency reflects two frictions. First, there will be a double marginalization effect reflecting that the supplying division does not fully internalize the net-revenue attainable by the buying division. Second, the resulting internal transfers will not be responsive to differences in the divisional income tax rates, even though they should be.

Surveys on transfer pricing practices show that many firms do not merely set the internal transfer price equal to the external market price but instead make adjustments in the form of intracompany discounts. Such discounts are frequently justified with cost differences between internal and external transactions.\footnote{Standard examples include the absence of bad debt expenses and lower selling and marketing expenses on internal transactions.} We find that even with identical
costs the imposition of intracompany discounts increases the firm’s expected after-tax profits. Discounts result in more efficient internal trade and, at the same time, result in tax savings since the foreign division will be induced to raise the external market price which in turn shifts taxable income from the (high-tax) domestic to the (low-tax) foreign division.

There is a sizable literature on transfer pricing for multinational enterprises operating in different tax jurisdictions. For instance, Capithorne (1971), Horst (1971), Samuelson (1982), Halperin and Srinidhi (1987) and Harris and Sansing (1998) have examined the impact of differential tax rates and transfer pricing regulations on production and pricing decisions. In all of these studies, a central agent is assumed to make the intracompany transfer decisions. In addition to operating decisions, Smith (2002a) and Sansing (1999) have studied how current IRS regulations affect firms’ incentives to make investments.9

Halperin and Srinidhi (1991) have characterized allocative distortions in decentralized firms resulting from tax considerations.10 Specifically, they focus on the resale-price and the cost-plus method for deriving the transfer price. Narayanan and Smith (2000) analyze the impact of taxes and alternative market structures on the desired transfer price. Both models assume that the same transfer price is used for tax and managerial performance evaluation purposes. In contrast, our analysis focuses on the choice of internal transfer prices that are decoupled from the ones used for tax reporting pur-

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9Smith (2002b) analyzes a moral hazard model in which the transfer price provides incentives for a manager (residing in a different tax jurisdiction) to work hard. While Smith identifies a tension between taxes and managerial incentives, the transfer price does not coordinate inter-divisional transactions in his model.

10Ignoring tax considerations, parts of the recent transfer pricing literature have examined principal-agent models. Papers in this category include Vaysman (1996, 1998), Christensen and Demski (1999), Anctil and Dutta (1999). Our model abstracts from moral hazard and compensation issues and instead takes it as given that divisional managers seek to maximize the after-tax income of their own divisions. This “neoclassical” framework is consistent with the approach taken in earlier tax-related studies on transfer pricing.
poses.\footnote{A recent study by Hyde (2002) also allows for internal transfer prices that are decoupled from arm’s length prices. In Hyde’s analysis the domestic division sets the managerial and the tax transfer price, but risks a penalty in case an audit finds the firm to have deviated from the (unique) tax-admissible price. By comparison, we take compliance with the tax rules as given and seek to identify the optimal internal transfer price under both cost-based and market-based transfer pricing.}

The remainder of the paper is organized as follows. Section 2 analyzes cost-based transfer pricing for both the decoupling and the conformity scenario. We turn to market-based transfer pricing in Section 3 and conclude in Section 4.

## 2 Cost-Based Transfer Pricing

We consider the interactions of two divisions (profit centers) in a multinational firm. A foreign division supplies an intermediate product to a domestic division which utilizes this intermediate product as a component in a final product sold externally in the domestic market. If \( q \) units of the intermediate product are transferred, the domestic division earns a net-revenue of \( R(q, \theta) \). This amount reflects gross sales revenues less all incremental costs incurred by the domestic division other than those for the intermediate product. The state variable \( \theta \) reflects information known only to the management of the domestic division.\footnote{The state variable \( \theta \) could be an \( n \)-dimensional vector of parameters in a compact set of possible states, \( \Theta \).}

Initially, we assume that production costs of the intermediate product are linear in the quantity produced. In keeping with earlier managerial accounting literature on transfer pricing, we first assume that the unit cost, \( c \), incurred by the foreign division is verifiable by the firm’s accounting system. The firm’s central office sets an internal transfer price \( TP \) based on the assessed cost, and the domestic division then chooses the quantity \( q \) of the intermediate product to be transferred.\footnote{The unit cost \( c \) should be interpreted as the marginal cost of production, i.e., it includes applicable opportunity costs (which must also be verifiable) in addition to the unit variable cost. Any fixed costs of production are considered sunk. Below, we consider extensions of our basic model with non-linear cost structures.}
The two divisions are assumed to be legally separate entities and taxation occurs “at the source”. Therefore the tax liability of each division is determined by the divisions’ taxable income. In particular, our setting is consistent with the standard scenario considered in the tax literature according to which the foreign division is a wholly owned subsidiary of the multinational corporation located in the domestic country.\(^{14}\) The income tax rate in the foreign country is denoted by \(t\), while the domestic tax rate is \(t + h\). Unless otherwise specified, we focus on the standard case in which the foreign subsidiary faces a lower tax rate and therefore \(h > 0\).

Our model also takes it as given that divisional managers seek to maximize the income of their divisions. While this specification appears quite descriptive, it is ad-hoc within our model. In effect, we suppress moral hazard problems which generate an explicit conflict between divisional managers and the overall corporate profit objective. By embedding our framework into an explicit principal-agent model, future research may endogenously derive the need for a divisionalized organization and local performance measurement.

Our main setting is one of a multinational enterprises that “decouples” its internal transfer price, \(TP\), from the arm’s length price used for taxation. We take it as given that for tax purposes the firm has identified a range of allowable arm’s length prices. Initially, we represent this range by the interval \([\underline{p}, \overline{p}]\).\(^{15}\) The interpretation is that any price in this interval can be justified to the tax authorities as based on an acceptable valuation method and a supporting set of data on comparables.\(^{16}\)

\(^{14}\)Some countries, including the U.S., will tax the profits of subsidiaries upon repatriation of those profits, i.e., when the subsidiary pays dividends to the parent. We ignore this aspect since repatriation may be postponed for a long time. Furthermore, as pointed out by Hartman (1985), taxation upon repatriation may effectively not be an issue if foreign direct investments are financed by the foreign subsidiary rather than through equity transfers from the domestic parent. Finally, the accumulation of “excess” foreign tax credits may effectively alleviate any further taxation of repatriated profits.

\(^{15}\)As shown below, some of our findings can be extended to settings in which the allowable range of admissible arm’s length prices changes with the volume of intracompany transfers.

\(^{16}\)In the U.S., these methods include the comparable uncontrolled price, resale price and various comparable profit and profit split methods. Our model is also consistent with the possibility that
Since $h > 0$, an arm’s length transfer price of $\bar{p}$ will minimize the multinational firm’s overall tax burden for any given transfer quantity $q$. If $\bar{p}$ is used for taxation purposes, the contribution to the foreign division’s after-tax income is equal to

$$\pi_f(q, TP) = (TP - c) \cdot q - t \cdot (\bar{p} - c) \cdot q,$$

while the contribution to the after-tax income of the domestic division is:

$$\pi_d(q, TP, \theta) = R(q, \theta) - TP \cdot q - (t + h) \cdot [R(q, \theta) - \bar{p} \cdot q]. \tag{1}$$

The managerial transfer price serves to align the objectives of the domestic division with the corporate objective. For a given quantity, the total after-tax corporate income is independent of the internal transfer price $TP$ and is given by:

$$\pi(q, \theta) = (1 - t) \cdot [R(q, \theta) - c \cdot q] - h \cdot [R(q, \theta) - \bar{p} \cdot q]. \tag{2}$$

In response to any transfer price, $TP$, the domestic division will choose the quantity transferred so as to maximize its performance measure in (1). Thus, the domestic division internalizes

$$TP - (t + h) \cdot \bar{p} \tag{3}$$

as the unit cost of internal transfers. The expression for corporate profit in (2) shows that the after-tax unit cost of transferring the intermediate product is the after-tax cost $(1 - t) \cdot c$ to the foreign division less the tax savings associated with transfers valued at $\bar{p}$:

$$(1 - t) \cdot c - h \cdot \bar{p}. \tag{4}$$

Our first result characterizes the managerial transfer price that achieves goal congruence between the domestic division and the corporate objective. Since the result follows directly by equating (3) and (4), we do not provide a formal proof.

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the acceptable arm’s length price has been agreed to with the tax authorities via a so-called Advance Pricing Agreement (APA). Unlike previous studies, e.g., Halperin and Srinidhi (1987, 1991), we do not model those IRS-admissible methods explicitly, but instead interpret the price range $[\underline{p}, \bar{p}]$ as the outcome of a search resulting from alternative valuation methods and comparables.
Proposition 1. Cost-based transfer pricing results in efficient quantity transfers if, and only if, the internal transfer price is given by:

\[ TP = (1 - t) \cdot c + t \cdot \bar{p} \]  

The finding in Proposition 1 provides a useful benchmark for several observations. For multinational enterprises facing different tax rates, the Hirshleifer (1956) rule of setting the transfer price equal to the marginal cost incurred by the supplying division will achieve efficiency only in the exceptional case when this cost coincides with the preferred arm’s length transfer price. Instead, this “textbook solution” induces underproduction if \( c > \bar{p} \) and overproduction if \( c < \bar{p} \).\(^{17}\)

The optimal managerial transfer price in (5) is independent of the difference in tax rates, \( h \), because this difference is internalized by the domestic division when it decides on the quantity to be transferred. One would expect the Hirshleifer solution to be appropriate in the limit when \( h = 0 \). While equation (5) suggests otherwise, it should be noted that any arm’s length price \( p \in [\underline{p}, \bar{p}] \) results in the same corporate tax liability (for a given quantity \( q \)) when \( h = 0 \). To obtain goal congruence the internal transfer price must be set so that the corporate after-tax cost \((1 - t) \cdot c\) is equal to the divisional after-tax cost of \( TP - t \cdot p \), for \( p \in [\underline{p}, \bar{p}] \). Thus, the transfer pricing rule in (5) still applies when \( h = 0 \), yet there are many other solutions. Provided \( c \in [\underline{p}, \bar{p}] \), one can set \( TP = c \) and obtain efficient decentralization, i.e., the Hirshleifer solution applies with identical tax rates.

In the managerial accounting literature, transfers at unit cost have been criticized as deficient because the supplying division does not receive any (economic) profit. For instance, in their discussion of cost-plus transfer pricing Eccles and White (1988) em-
phasize the fairness requirement of letting the supplying division earn a share of the total contribution margin. In contrast, economists have pointed to quantity distortions arising from markups.\footnote{In the analysis of Baldenius, Reichelstein and Sahay (1999) and Sahay (2000) markups are essential in order to give the supplying division incentives for cost-reducing investments.} Our result shows that, when $\bar{p} > c$ and income taxes are taken into account, markups are essential in order to \textit{eliminate} quantity distortions. However, this form of markup pricing does not yield a positive \textit{after-tax} profit for the supplying division. The entire surplus still accrues to the party that decides on the quantity to be traded, that is, the domestic buyer. If transfers were valued at the pre-tax cost $c$, the contribution to the foreign division’s pre-tax income would be zero, yet the contribution to after-tax income would be negative.

The finding in Proposition 1 presumes that divisional managers are evaluated on the basis of their divisional after-tax income. In some multinational firms, however, divisional performance evaluation is based on pre-tax income, yet others base their performance evaluation on “effective” tax rates. It is readily seen that with pre-tax income as the performance measure the transfer pricing rule in (5) can be modified to:

\[ TP = \frac{1}{1 - t - h} \cdot [(1 - t) \cdot c - h \cdot \bar{p}] \]  

(6)

in order to ensure efficient quantity transfers. The formula in (6) reflects that if management of the domestic division (seeking to maximize its pre-tax income) is to internalize the corporate objective, then the after-tax corporate cost in (3) must be marked-up by $(1 - t - h)^{-1}$. This mark-up reflects that all revenues obtained by the domestic division are ultimately taxed at the rate $1 - t - h$. \footnote{Some multinational firms, e.g., Siemens Corporation, base their divisional performance assessments on \textit{effective} tax rates which reflect the firm’s overall “average” tax rate. This practice is frequently justified by the desire for a simplified performance measure which merely approximates the tax consequences of managerial decisions. Denoting by $\hat{t}$ the effective tax rate, the transfer price in (5) would have to be adjusted to $TP = (1 - t) \cdot c + (\hat{t} - h) \cdot \bar{p}$. The implicit assumption here is that the transaction in question is “small” so that its effect on the average and the effective corporate tax rate is negligible.} 

Proposition 1 identifies a single transfer price which leads the domestic division to
internalize the “correct” valuation for intracompany transfers. This result relies on several restrictions some of which can be relaxed. First, we have assumed the unit cost $c$ to be constant. As noted above, this cost may include applicable opportunity costs but in many contexts such costs will vary with production volume. Also, the presence of capacity constraints at the supplying division may lead to non-linearities in the cost of production. This suggests the consideration of a general cost functions $c(q)$. We do not impose any restrictions on the shape of $c(\cdot)$ and in particular allow for the possibility of “jumps” which could reflect fixed costs associated with the acquisition of incremental capacity.

A second generalization of our basic result in Proposition 1 concerns the assumption that the range of admissible arm’s length prices is exogenous and fixed. For several of the transfer pricing methods acceptable in most OECD countries, it is plausible that the volume of internal transfers does not have an impact on the arm’s length price (for example, under the resale-minus and the CUP method). On the other hand, when transfer prices are cost-based, $\bar{p}$ may well be a function of $q$. Accordingly, we now allow for the range of admissible arm’s length prices to depend on the quantity transferred and we denote this range by $[\underline{p}(q), \bar{p}(q)]$.\(^{21}\)

\(^{20}\)When the the supplying division only makes the intermediate product in question and the most favorable arm’s length price is given by the full cost marked-up by some “comparable” factor µ, $\bar{p}$ will generally not be a function of $q$. To illustrate, suppose fixed costs are denoted by $FC$ and these costs are sunk for managerial purposes. With cost-plus transfer pricing, the total transfer payment becomes $p \cdot q = (c + \frac{FC}{q}) \cdot (1 + \mu) \cdot q$. Thus, for tax purposes each additional unit is valued at $c \cdot (1 + \mu)$. However, this reasoning does not extend to settings where the fixed cost $FC$ is allocated among two or more products. If the other products are sold externally and the allocation of fixed costs is proportional to the relative volume of each product, then the cost-based transfer price of the intermediate product will be increasing in its volume.

\(^{21}\)We thank an anonymous referee for suggesting this extension.
Corollary to Proposition 1. If the unit production cost and the admissible range of arm’s length prices depend on the quantity transferred, a transfer pricing schedule of the form

\[ TP(q) = (1 - t) \cdot c(q) + t \cdot \bar{p}(q) \] (7)

results in efficient transfers.

The transfer pricing schedule in (7) induces the domestic division to internalize the corporate after-tax cost of intracompany transfers for alternative levels of \( q \). As a consequence, the domestic division has an incentive to implement the volume of trade that maximizes the firm’s total after-tax profits. We note that this result does not depend on any curvature assumptions for the functions \( c(q) \) and \( \bar{p}(q) \).

Our analysis so far has assumed that the firm decouples its internal transfer prices from the ones reported for tax purposes. Many multinational enterprises, however, rely on a single transfer price for both tax and managerial purposes. Such practice avoids the cost of maintaining two “sets of books” and the attendant risk that tax authorities may subpoena internal records in case of transfer pricing disputes. Discrepancies between valuations used internally and those used for tax purposes could then become evidence in legal proceedings with the tax authorities.

Firms insisting on the requirement of conformity between internal and arm’s length prices face a fundamental tradeoff between tax minimization and coordination. To illustrate this tradeoff, we return to the basic setting in Proposition 1 where the unit cost, \( c \), is constant and the range of admissible arm’s length prices is fixed. If the transfer price is set equal to the most favorable price that can be justified to the tax authorities, i.e., \( p = \bar{p} \), the firm minimizes its corporate tax payment for given transfer quantities. Yet, at that price, the domestic division will internalize too high a cost and therefore demand a transfer quantity that is too low whenever \( \bar{p} > c \). For any transfer
price, $p$, the domestic division will demand a quantity $Q(p, \theta)$ such that

$$Q(p, \theta) \in \arg \max_q \{(1 - t - h) \cdot [R(q, \theta) - p \cdot q]\}. \quad (8)$$

As a consequence, the overall corporate profit after taxes equals:

$$\pi(p, \theta) \equiv (1 - t) \cdot [R(Q(p, \theta), \theta) - c \cdot Q(p, \theta)] - h \cdot [R(Q(p, \theta), \theta) - p \cdot Q(p, \theta)]. \quad (9)$$

The corporate objective is to maximize (9) in expectation over the possible states of the world, i.e., $E_\theta[\pi(p, \theta)]$, subject to the incentive compatibility condition in (8) and the requirement that $p \in [\underline{p}, \bar{p}]$.

It is instructive to consider the corporate objective function in (9) for extreme values of $h$. As observed in connection with Proposition 1, the transfer price should be set at the unit cost $c$ if the tax rates are identical in the two jurisdictions, i.e., $h = 0$. As $h$ becomes positive, the firm has an incentive to raise the transfer price above cost and one would expect the desired markup to be increasing in $h$. If $h$ were to become large so that $(t + h) \to 1$, the firm would put virtually no weight on the profits of the domestic division and therefore choose the price $p$ so as to maximize the “monopoly profit” of the foreign division. Formally, for a given $\theta$, the objective function in (9) then reduces to the monopoly problem $\pi^m(p, \theta) = (1 - t) \cdot (p - c) \cdot Q(p, \theta)$. We denote by $p^m(\theta) \in \arg \max_p \{\pi^m(p, \theta)\}$ the monopoly price that maximizes this objective.\(^{22}\)

For intermediate values of $h$, and a given realization of $\theta$, the optimal single transfer price to satisfies $c < p < p^m(\theta)$. This observation suggests a sufficient condition for the preferred single transfer price to be chosen below the most favorable arm’s length price.

**Proposition 2.** If the firm chooses a single transfer price for internal and tax valuation purposes, the optimal transfer price is lower than the tax-minimizing arm’s length price provided $p^m(\theta) < \bar{p}$ for all $\theta$.

\(^{22}\)The monopoly pricing problem $\pi^m(p, \theta)$ is assumed to be single-peaked in $p$ and, furthermore, $p^m(\theta)$ is assumed to be continuous in $\theta$. 

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Proof: All proofs are in the Appendix.

The monopoly price \( p^m(\theta) \) provides an upper bound on the markup that a central planner would impose on the unit cost \( c \) so as to balance the conflicting objectives of internal coordination and tax minimization. If this “cost-plus” price is below \( \bar{p} \), it would be too costly to adopt the tax-minimizing arm’s length price \( \bar{p} \). Assuming that \( c < \bar{p} \), this condition is more likely to be satisfied if the internal demand exhibits a high price elasticity and therefore \( p^m(\theta) \) is relatively close to \( c \) (since this price elasticity pertains to the final product sold by the domestic division, we do not expect a direct relation between this elasticity and the admissible \( \bar{p} \)). On the other hand, if \( p^m(\theta) > \bar{p} \) for all \( \theta \), then for sufficiently large differences in the tax rates, tax minimization will take precedence and hence the optimal transfer price will equal \( \bar{p} \). To make the basic tradeoff more explicit, we consider a setting in which the demand curve for the final product is linear, as in Narayanan and Smith (2000).

Suppose the domestic division faces a quadratic revenue function for its final product, i.e., \( R(q, \theta) = [a(\theta) - \frac{1}{2} \cdot b(\theta) \cdot q] \cdot q \). Its demand for the intermediate product therefore becomes \( Q(p, \theta) = \frac{1}{b(\theta)} [a(\theta) - p] \) and the resulting monopoly price equals \( p^m(\theta) = \frac{1}{2} [a(\theta) + c] \). The expected monopoly profit is then maximized by \( E_\theta[p^m(\theta)] \). In this quadratic setting we obtain a closed form expression for the optimal transfer price:

\[
p^* = \min\{\bar{p}, \ w \cdot c + (1 - w) \cdot E_\theta[p^m(\theta)]\}
\]

with the weight \( w \) given by

\[
w = \frac{1 - t - h}{1 - t + h}.
\]

In this linear-quadratic setting the optimal single transfer price is indeed a weighted average of the pre-tax cost, \( c \), and the expected monopoly price that maximizes the profits for the foreign division. Furthermore, the relative weights depend only on the tax parameters \( t \) and \( h \).\(^{23}\) We note that the preferred arm’s length price is less than

\(^{23}\)Equation (10) is consistent with Proposition 3 in Narayanan and Smith (2000) who also interpret the optimal single price as a cost-plus price.
\( \bar{p} \) even if \( E_a[p^m(\theta)] > \bar{p} \), provided the tax rate difference \( h \) is sufficiently small and, therefore, the weight \( (1 - w) \) on \( E_a[p^m(\theta)] \) becomes small.\(^{24}\)

The above results demonstrate that the conformity requirement between internal and arm’s length transfer prices generally imposes a cost in terms of expected after-tax profits.\(^{25}\) To conclude this section, we ask how the cost of conformity varies with the underlying parameters. Such analysis should be helpful in predicting which firms are more likely to incur the cost of implementing two sets of books and to bear the potential cost of disputes with the tax authorities.

The following result focuses on the case of primary interest where \( \bar{p} \geq c \). From the previous discussion one would expect that the cost of conformity increases both in the tax differential \( h \) and in the admissible \( \bar{p} \). We now show that these two factors do not merely contribute independently to the cost of conformity, but that they reinforce each other as cost factors. Formally, we define the cost of conformity as the expected after-tax profits with decoupled transfer prices less the expected after-tax profit under conformity. We invoke the usual definition that a function exhibits increasing differences in two of its arguments if the corresponding cross-partial derivative is positive.

**Corollary to Proposition 2.** The cost of conformity exhibits increasing differences in \( h \) and \( \bar{p} \).

This finding shows that the cost of conformity increases at a higher rate in \( h \) for larger values of \( \bar{p} \), and vice versa. Put differently, as the most favorable arm’s length price, \( \bar{p} \) increases, the incremental gain from decoupling and reporting \( \bar{p} \), is relatively large for high-\( h \) firms compared to low-\( h \) firms.

\(^{24}\)Proposition 2 can be extended to settings where the admissible arm’s length price depends on the quantity transferred, that is \( \bar{p}(Q(\cdot)) \). Under mild regularity conditions, it can be shown that \( p^* < \min_{\theta} \{ \bar{p}(Q(p^*(\theta), \theta)) \} \), if \( p^m(\theta) < \bar{p}(Q(p^m(\theta), \theta)) \) for all \( \theta \). The optimal transfer price will then be below the arm’s length price that is most favorable for a given quantity \( q \).

\(^{25}\)Halperin and Srinidhi (1991) also consider a single transfer price derived from a cost-plus method. In their analysis, however, the actual transfer quantity is determined through direct negotiations between the division managers.
3 Market-Based Transfer Pricing

According to survey data, multinational enterprises rely more heavily on market-based transfer pricing than their counterparts involved only in domestic transactions.\(^{26}\) One explanation for this preference is that the so-called comparable uncontrolled (market) price (CUP) method is acceptable for tax purposes in virtually all countries and tax jurisdictions. By calculating internal transfer prices based on information about external sales of comparable goods, firms can adopt a unified approach for calculating their transfer prices. At the same time, the use of so-called *intracompany discounts* leaves flexibility for calculating internal transfer prices that differ from (and are usually lower than) external arm’s length prices.\(^{27}\)

This section extends the above model by allowing for the possibility that the foreign division sells the intermediate product to a range of external customers. The market price charged in the external market then becomes the arm’s length price.\(^{28}\) Our main question is whether multinational firms can rely on the external market price for internal transfer pricing purposes or whether an adjustment to the external price, in the form of an intracompany discount, will improve the firm’s expected profit after taxes. We note that our approach here differs from that of Harris and Sansing (1998) where the firm also uses the CUP method for tax purposes, yet the quantity transferred between the divisions is determined by a central decision maker.\(^{29}\)

\(^{26}\)See, for example, Price Waterhouse (1984), Tang (1997) or Ernst & Young (1999).

\(^{27}\)The study by Price Waterhouse (1984) reports that 17% of the respondents value transfers at the prevailing market price, while 27% rely on adjusted market prices. The “adjusted” market price is characterized as “simply the prevailing market price adjusted for market imperfections that are avoided by selling internally”.

\(^{28}\)Two well publicized court cases, involving Compaq and Bausch & Lomb, have established the admissibility of arm’s length prices based on external sales. In both cases, the IRS charged that foreign subsidiaries sold intermediate products (soft contact lenses and printed circuit boards, respectively) at artificially high transfer prices to their U.S. parents, yet the courts ruled that these valuations were admissible for tax purposes because they were in line with well established market prices; see, for example, King (1994).

\(^{29}\)Harris and Sansing (1998) compare the performance of a vertically integrated firm with centralized
In extending the above model, we now allow for quantities $q_d$ and $q_f$ of the intermediate product to be sold to the domestic division and to external customers of the foreign division, respectively. The domestic division earns net revenues of $R_d(q_d, \theta)$, while the foreign division obtains revenues from external sales of the intermediate product, denoted by $R_f(q_f, \theta)$, plus the internal transfer payment. In contrast to the analysis in Section 2 above, we no longer assume that the unit cost of the intermediate product is verifiable. Instead this cost, $c(\theta)$, is now allowed to vary with the underlying state $\theta$. As before, the realization of the state variable $\theta$ is assumed to be known to the divisional managers but not to the firm’s central office.\(^\text{30}\)

The argument for setting transfer prices at the prevailing market price is usually based on the opportunity cost of internal transfers induced by capacity constraints at the supplying division. Accordingly, we assume that the total quantity of the intermediate product supplied by the foreign division cannot exceed some capacity constraint, $K(\theta)$, which may fluctuate with the current state:

$$q_d + q_f \leq K(\theta) .$$

(11)

As a benchmark, suppose first the foreign division is a price-taker, that is, it can sell arbitrary quantities (up to $K(\theta)$) of the intermediate product at a given price, $p^o$. For example, the foreign division may have a standing order to supply any quantity up to $K(\theta)$ to a large external customer at this price. The overall after-tax profit is then given by:

$$(1 - t) \cdot [R_d(q_d, \theta) + p^o \cdot q_f - c(\theta) \cdot (q_d + q_f)] - h \cdot [R_d(q_d, \theta) - p^o \cdot q_d] ,$$

decision making against the after-tax profits obtained by two independent parties who are subject to double marginalization problems. In our analysis, the vertically integrated firm also suffers from double marginalization when the internal transfer price is set equal to the external market price (Proposition 3 below). Intracompany discounts are shown to mitigate these double marginalization effects.

\(^{30}\)While our model supposes that the divisional managers have complete and symmetric information at the time of decision making, it is readily seen that our analysis would be unchanged if management of the supplying division knew all revenue and cost information, yet the domestic division only knew its own revenue function $R_d(\cdot, \theta)$. 

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subject to the capacity constraint in (11). The argument given in connection with Proposition 1 then applies with the relevant pre-tax marginal cost being $p^o$ rather than $c$. As a consequence, the after-tax cost of internal transfers is $(1 - t) \cdot p^o - h \cdot p^o$, and the domestic division internalizes this cost if the internal transfer price is set at $p^o$.

A more common scenario is one in which the foreign division has market power and therefore is a price-setter on the external market. Since the domestic division cannot buy the intermediate product from another source, the foreign division has effective monopoly power. This may reflect that the intermediate product is a proprietary component (possibly due to intellectual property rights).\(^{31}\) The external demand curve is assumed to be downward sloping and is denoted by $Q_f(p, \theta)$.

Throughout the following analysis we maintain the assumption that for any price $p$ the foreign division may choose, the volume of external sales $Q_f(p, \theta)$ is sufficiently significant so that the external market price can also serve as the arm’s length price for tax purposes, i.e., the CUP method is applicable.\(^{32}\) If the firm’s central office adopts a policy of valuing internal transfers also at the prevailing price, i.e., $TP = p$, the foreign division will solve the following pricing problem:

$$\pi_f(p, \theta) = (1 - t) \cdot (p - c(\theta)) \cdot [Q_d(p, \theta) + Q_f(p, \theta)] ,$$

subject to:

$$Q_d(p, \theta) \in \arg \max_q \{(1 - t - h) \cdot [R_d(q_d, \theta) - p \cdot q_d]\}$$

and

$$Q_d(p, \theta) + Q_f(p, \theta) \leq K(\theta).$$

When internal transfers are valued at the prevailing market price, the resulting price and quantity allocations are independent of the divisional tax rates. For each division

\(^{31}\)For example, in vertically integrated electronics companies, a frequently encountered situation is that one division manufactures hardware components that are sold externally and to one or several other divisions within the firm.

\(^{32}\)In some situations it is conceivable that a firm may sell parts of its scarce capacity externally with the primary goal of establishing an arm’s length price for tax purposes.
the pre-tax and after-tax valuations coincide and, as a consequence, the outcome is invariant to the individual tax rates and any differences thereof. Denoting by $p(\theta)$ the solution to the maximization problem in (12), we find that:

$$R'_d(Q_d(p(\theta), \theta), \theta) = p(\theta) > R'_f(Q_f(p(\theta), \theta), \theta)$$

(13)

where $R_{f}(q_{f}, \theta) = q_{f} \cdot P_{f}(q_{f}, \theta)$ and $P_{f}(q_{f}, \theta)$ is the external willingness-to-pay curve, that is, the inverse of the demand curve $Q_{f}(p, \theta)$. The equality in (13) reflects that the domestic division views the price $p(\theta)$ as its marginal cost (both before and after tax). The inequality $p(\theta) > R'_f(Q_f(p(\theta), \theta)$ in (13) holds for any $p$ whenever $Q_f(p, \theta)$ is downward sloping and therefore marginal revenue is always less than price.

An immediate implication of (13) is that a transfer price equal to the external market price will result in inefficiently low internal transfers when $h = 0$. With identical tax rates, efficiency requires that external and internal marginal revenue be the same. Yet, in setting the price the foreign division does not fully internalize the net-revenue earned by the domestic division but only that division’s willingness-to-pay as given by $Q_{d}(p, \theta)$. This bias reflects the familiar double-marginalization problem inherent in supplier-buyer relations.

With different tax rates (i.e., $h > 0$), the resulting prices and quantity allocations remain unchanged provided internal transfers are valued at the market price. The immediate question then is how the first-best quantities, which the central office would implement if it could observe $\theta$, vary with differences in the tax rates. Put differently, how does $(q^*_d(\theta | h), q^*_f(\theta | h))$ relate to $(q^*_d(\theta | h = 0), q^*_f(\theta | h = 0))$, assuming that the available capacity, $K(\theta)$, is sufficiently small for all states $\theta$ so that the constraint in (11) is always binding? We note that $(q^*_d(\theta | 0), q^*_f(\theta | 0))$ maximize total net revenues across the two divisions.

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33By definition, $(q^*_d(\theta | h), q^*_f(\theta | h))$ maximize $(1 - t)[R_d(q_d, \theta) + R_f(q_f, \theta) - c(\theta) \cdot (q_d + q_f)] - h \cdot [R_d(q_d, \theta) - R_f(q_f, \theta)]$, subject to the constraint that the total quantity not exceed $K(\theta)$. 

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Lemma 1. For any $h > 0$, the first-best quantities satisfy $q^*_d(\theta | h) > q^*_d(\theta | 0)$.

It may seem counterintuitive that the first-best solution calls for a larger share of the scarce capacity to be allocated to the domestic division when that division faces a higher tax rate. To understand this finding, consider initially the first-best allocation when $h = 0$, i.e., $(q^*_d(\theta | 0), q^*_f(\theta | 0))$. At these quantities the resulting marginal revenues must be equal, i.e., $R'_d(q^*_d(\theta | 0), \theta) = R'_f(q^*_f(\theta | 0), \theta)$. Suppose now that $h$ is positive and starting from this allocation an additional (small) unit of capacity is reallocated from the foreign to the domestic market. While this only has a second-order effect on pre-tax revenues, the impact on the firm’s tax liability represents a first-order effect because the arm’s length price increases in response to a lower external sales quantity. Formally, the marginal effect on the firm’s tax liability becomes:

$$-h \cdot [R'_d(q^*_d(\cdot), \theta) - P_f(q^*_f(\cdot), \theta) + P'_f(q^*_f(\cdot), \theta) \cdot q^*_d(\cdot)].$$

(14)

Since $R'_d(q^*_d(\cdot), \theta) = R'_f(q^*_f(\cdot), \theta) < P_f(q^*_f(\cdot), \theta)$ and $P_f(q, \theta)$ is downward sloping, it follows that the marginal effect on income taxes, as given in (14), is positive. In conjunction with Lemma 1, the preceding argument identifies the bias inherent in setting the internal transfer price equal to the arm’s length price. To state the formal result, we make the assumption that the capacity constraint is always binding under unadjusted market-based transfer pricing.

Assumption (A1): The function $\pi_f(\cdot, \theta)$ is decreasing in $p$ at $p(\theta)$ for all $\theta$.

This assumption says that the foreign division would do better by lowering the price below $p(\theta)$, yet the capacity constraint prevents it from doing so.

Proposition 3. Given (A1), a policy of setting the internal transfer price equal to the arm’s length price results in internal transfer quantities below the efficient level.

Our finding here is consistent with that obtained for cost-based transfer pricing in Section 2. Proposition 1 advocates a managerial transfer price equal to the tax-weighted
average of the pre-tax cost \( c \) and the most favorable arm’s length price \( \bar{p} \). Provided \( \bar{p} > c \), the resulting transfer quantities would be too low if \( TP = \bar{p} \). In that sense, we predict the same directional bias under market-based and cost-based transfer pricing if internal transfers are valued at the arm’s length price.

A common practice in connection with market-based transfer pricing is to impose intracompany discounts, that is, internal transfers are valued at the external market price less some discount. To examine the impact of intracompany discounts formally, it will be convenient to consider additive discounts such that \( TP(p) = p - \Delta \), where \( \Delta \) is a constant dollar amount set by the central office.\(^{34}\) In response to an intracompany discount \( \Delta \), the foreign division will choose \( p(\Delta, \theta) \) so as to maximize:

\[
\pi_f(p, \Delta, \theta) = (1 - t) \cdot (p - c) \cdot [Q_d(p, \Delta, \theta) + Q_f(p, \theta)] - \Delta \cdot Q_d(p, \Delta, \theta),
\]

subject to \( Q_f(\cdot) + Q_d(\cdot) \leq K(\theta) \), and subject to the restriction that \( Q_d(\cdot) \) is an optimal response by the domestic division:

\[
Q_d(p, \Delta, \theta) \in \arg \max_{q_d} \{R_d(q_d, \theta) - (p - \Delta) \cdot q_d - (t + h) \cdot [R_d(q_d, \theta) - p \cdot q_d]\}. \tag{16}
\]

In effect, we examine a three-stage mechanism in which the central office moves first by selecting the discount \( \Delta \), followed by the foreign division’s choice of \( p(\Delta, \theta) \), which, in turn, induces the internal transfer quantity \( Q_d(p(\Delta, \theta), \Delta, \theta) \). In choosing \( \Delta \), the objective of the central office is the expected corporate after-tax profit:

\[
\pi(\Delta) = E_\theta \{ (1 - t) \cdot [R_d(Q_d(\cdot, \theta) + p(\Delta, \theta) \cdot Q_f(\cdot) - c(\theta) \cdot (Q_d(\cdot) + Q_f(\cdot))] - h \cdot [R_d(Q_d(\cdot, \theta) - p(\Delta, \theta) \cdot Q_d(\cdot))] \}, \tag{17}
\]

where \( Q_d(\cdot) \equiv Q_d(p(\Delta, \theta), \Delta, \theta) \) and \( Q_f(\cdot) \equiv Q_f(p(\Delta, \theta), \theta) \). In response to a discount the foreign division must raise the market price in order to meet total demand as the

\(^{34}\)It will become clear that the Corollary to Proposition 3 below would also hold if discounts were calculated as constant percentages, i.e., \( TP(p) = (1 - \gamma) \cdot p \), where \( \gamma \) denotes the discount factor. The calculation of optimal discounts, however, is considerably simpler for constant additive discounts; see Proposition 4 below.
domestic division will increase its quantity. A discount will therefore lead to an efficiency enhancing reallocation of the scarce capacity. In addition, discounts will result in a tax gain since the attendant increase in the arm’s length price decreases the firm’s overall tax burden whenever the domestic division faces a higher tax rate.

**Corollary to Proposition 3.** Given (A1), an intracompany discount increases the firm’s expected after-tax profit.

The above corollary merely states a “local result” saying that total after-tax profit is increasing in $\Delta$ at $\Delta = 0$. While this finding supports the widespread use of intracompany discounts, it does not speak to the question of optimal discounts. To address this issue in an analytically tractable setting, we restrict the following analysis to linear demand functions. In particular:

$$Q_f(p, \theta) = \alpha_f(\theta) - \beta_f(\theta) \cdot p , \quad (18)$$

so that the inverse demand function for the foreign market is $P_f(q_f, \theta) = a_f(\theta) - b_f(\theta) \cdot q_f$, where $a_f(\theta) \equiv \frac{\alpha_f(\theta)}{\beta_f(\theta)}$ and $b_f(\theta) \equiv \frac{1}{\beta_f(\theta)}$. Similarly, the net-revenue function of the domestic division is assumed to take the form:

$$R_d(q_d, \theta) = \left[a_d(\theta) - \frac{1}{2} \cdot b_d(\theta) \cdot q_d\right] \cdot q_d .$$

For a positive discount $\Delta$, the demand by the domestic will depend on its tax rate $(t+h)$, since its pre-tax cost per unit is $p - \Delta$ while the unit cost for tax purposes is $p$. Specifically, we find that:

$$Q_d(p, \Delta, \theta) = \alpha_d(\theta) - \beta_d(\theta) \cdot (p - \tau_d \cdot \Delta) , \quad (19)$$

where $\alpha_d(\theta) \equiv \frac{\alpha_d(\theta)}{\beta_d(\theta)}$, $\beta_d(\theta) \equiv \frac{1}{b_d(\theta)}$, and $\tau_d = \frac{1}{1-t-h}$. Anticipating this demand, the foreign division seeks to maximize $\pi_f(p, \Delta, \theta)$ and therefore sets the market price at:

$$p(\Delta, \theta) = p(0, \theta) + \nu(\theta) \cdot \tau_d \cdot \Delta . \quad (20)$$
Here, \( \nu(\theta) \equiv \frac{\beta_d(\theta)}{\beta_d(\theta) + \beta_f(\theta)} \) measures the relative price sensitivities of the two markets and
\[
p(0, \theta) = \frac{\alpha_d(\theta) + \alpha_f(\theta) - K(\theta)}{\beta_d(\theta) + \beta_f(\theta)},
\]
is the price that would result in the absence of a discount.

**Proposition 4.** Suppose the internal and external demand functions are linear such that (A1) holds. The optimal discount is then given by:
\[
\Delta^* = (1-t) \cdot \frac{E_\theta \{ \beta_d(\theta) \cdot (1 - \nu(\theta)) [a_f(\theta) - p(0, \theta)] \}}{E_\theta \{ \beta_d(\theta) \cdot [1 - (\nu(\theta))^2] \}} + h \cdot \frac{E_\theta \{ \beta_d(\theta) \cdot \nu(\theta) [a_d(\theta) - p(0, \theta)] \}}{E_\theta \{ \beta_d(\theta) \cdot [1 - (\nu(\theta))^2] \}}.
\]

To understand the determinants of the optimal discount, suppose first that there are no taxes, i.e., \( t = h = 0 \) and the slope coefficients of the demand curves are independent of \( \theta \), i.e., \( \beta_d(\theta) \equiv \beta_d, \beta_f(\theta) \equiv \beta_f \) and \( \nu = \beta_d/(\beta_d + \beta_f) \). The optimal discount in Proposition 4 then reduces to:
\[
\Delta^* = \frac{E_\theta \{ a_f(\theta) - p(0, \theta) \}}{1 + \nu}.
\]

Straightforward algebra shows that, for a given \( \theta \), the expression \( \Delta_\theta \equiv \frac{a_f(\theta) - p(0, \theta)}{1 + \nu} \) is precisely the difference between the market price and the marginal revenue of foreign sales, evaluated at the optimal quantities \( (q^*_d(\theta, 0), q^*_f(\theta, 0)) \), i.e., \( \Delta_\theta = P_f(q^*_f(\theta, 0), \theta) - R'_f(q^*_f(\theta, 0), \theta) \). Suppose, hypothetically, that the central office could adjust the discount to the actual state \( \theta \). The foreign division would then set the price equal to \( P_f(q^*_f(\theta, 0), \theta) \) and the domestic division would choose \( q_d \) so as to equate its own marginal revenue with \( p(\Delta_\theta, \theta) - \Delta_\theta = P_f(q^*_f(\theta, 0), \theta) - \Delta_\theta = R'_f(q^*_d(\theta, 0), \theta) \). Suppose, hypothetically, that the central office could adjust the discount to the actual state \( \theta \). The foreign division would then set the price equal to \( P_f(q^*_f(\theta, 0), \theta) \) and the domestic division would choose \( q_d \) so as to equate its own marginal revenue with \( p(\Delta_\theta, \theta) - \Delta_\theta = P_f(q^*_f(\theta, 0), \theta) - \Delta_\theta = R'_f(q^*_d(\theta, 0), \theta) \). Hence, \( q_d = q^*_d(\theta, 0) \) and a discount in the amount of \( \Delta_\theta \) would indeed eliminate the double-marginalization problem.\(^{35}\)

\(^{35}\)In response to \( \Delta_\theta \), the foreign division would not want to choose a price higher than \( P_f(q^*_f(\theta, 0), \theta) \), and because of the capacity constraint it also could not choose a lower price. \(^{36}\)The formula in Proposition 4 shows that the optimal discount for a tax-free setting is deflated by
In choosing the optimal discount, the firm’s central office faces an “averaging” problem. For certain realizations of θ, Δ* will be too high and for others too low. We note, however, that this “averaging problem” becomes negligible if both inverse demand functions and the foreign division’s cost function are positively correlated, since the distance between the two demand curves determines the magnitude of the double-marginalization problem.

Finally, Proposition 4 predicts that ceteris paribus the optimal discount should increase with the tax differential h. Higher discounts result in a larger arm’s length price and, furthermore, the resulting tax gain increases with larger values of h. Proposition 4 shows that the optimal discount can be expressed as the sum of two components, the first one of which counteracts the double marginalization problem with identical tax rates, while the second one captures the tax advantages resulting from higher arm’s length prices. It seems likely, though, that this additive separability is specific to the class of linear demand functions.

It is instructive to compare the finding in Proposition 4 with the earlier results in Samuelson (1982) and Halperin and Srinidhi (1987). These authors have argued that under market-based transfer pricing firms have an incentive to reduce external sales of the product that serves as a comparable for the one transferred internally. As a consequence, the arm’s length transfer price increases and the corporate tax burden decreases, provided h ≥ 0. This prediction is consistent with the result in Proposition 4.

In our model, however, the central office does not make the pricing and transfer decisions directly, but instead seeks to facilitate intracompany transfers through the imposition of discounts which are based on imperfect knowledge of the divisional revenues and costs.

\[ \text{the factor } (1 - t), \text{ when } t > 0 \text{ but } h = 0. \] This may seem counterintuitive since the firm’s objective function is independent of t when both divisions face the same tax rate. The point to notice, though, is that the foreign division’s demand is “marked-up” by \( \tau_d = 1/(1 - t) \) (given \( h = 0 \)) because the discount applies to the internal transfer price but not to the arm’s length price. In order to counteract this effect, the optimal discount must be deflated by \( (1 - t) \).
The literature on transfer pricing explains the widespread use of intracompany discounts with cost differentials between internal and external sales. To address this issue, suppose the unit cost of internal transfers is \( c(\theta) - k \), with the cost difference \( k \geq 0 \) known to the firm’s central office.\(^{37}\) The optimal discount, \( \Delta^*(k) \), will then be a function of \( k \). By definition, \( \Delta^* \) is equal to \( \Delta^*(0) \).

**Corollary to Proposition 4.** If the unit cost of internal transfers is \( c(\theta) - k \), then the optimal discount is given by:

\[
\Delta^*(k) = \Delta^* + (1 - t) \cdot \frac{E_\theta \{ \beta_d(\theta)(1 - \nu(\theta)) \}}{E_\theta \{ \beta_d(\theta) [1 - (\nu(\theta))^2] \}} \cdot k.
\]

We find that the optimal discount does not fully pass on cost differences between internal and external transactions to the domestic buyer. Returning to the special case where \( \beta_d(\theta) \equiv \beta_d, \beta_f(\theta) \equiv \beta_f \) and therefore \( \nu(\theta) \equiv \nu \), we find that \( \Delta^*(k) = \frac{1 - \nu}{1 + \nu} \cdot k \). Only if the domestic demand curve becomes relatively “inelastic”, i.e., \( \beta_d \) becomes small relative to \( \beta_f \), we find that \( \Delta^*(k) \) converges towards \( (1 - t) \), so that cost differentials increase the optimal discount dollar-for-dollar on an after-tax basis.

To shed further light on the magnitude of optimal discounts and on the cost of conformity as \( h \) increases, we examine the following numerical example. For simplicity, suppose the demand functions as well as the unit cost are known so that \( (\alpha_d, \beta_d, \alpha_f, \beta_f, c) \) are independent of \( \theta \), and only the capacity constraint is state dependent with \( K(\theta) = K + \theta \). The parameter values are chosen as follows: \( (\alpha_d = 1, \beta_d = \beta_f = 0.01, c = 0, t = 0.2, K = 0.5) \), and \( \theta \sim U[0, \frac{3}{4}] \). Holding the size of the domestic market fixed, we consider an “identical markets” scenario, where \( \alpha_f = \alpha_d = 1 \), and a “larger foreign market” scenario, where \( \alpha_f = 1.5 > \alpha_d \).

\(^{37}\) For technical reasons, the following result assumes that the cost difference, \( k \), is sufficiently small relative to the other parameters of the problem. The formal restriction is given in the proof of the Corollary to Proposition 4.
Identical Markets ($\alpha_f = \alpha_d$)

Larger Foreign Market ($\alpha_f > \alpha_d$)

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<th>$\pi(\Delta^<em>)/\pi^</em>$</th>
<th>$\Delta^* E[p(\Delta^*,\theta)]/E[K(\theta)]$</th>
<th>$\pi(0)/\pi^*$</th>
<th>$\pi(\Delta^<em>)/\pi^</em>$</th>
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Table 1: Simulation Results

In Table 1, we denote by $\pi^*$ the first-best expected firm profit after taxes. The simulations show that unadjusted market-based transfer pricing performs better in the “identical markets” scenario ($\alpha_f = \alpha_d$) compared to a setting where $\alpha_f > \alpha_d$. In both scenarios, however, the performance of this transfer pricing policy (relative to first-best) decreases as the tax differential increases. In contrast, with optimal discounts the firm can consistently achieve an efficiency rate of over 99%. As one would have expected, the cost of conformity is increasing in $h$. The optimal discount ranges from 26.7% to 35.2% of the expected arm’s length price, and it is higher if $\alpha_f > \alpha_d$ than if $\alpha_f = \alpha_d$.

To conclude this section, we briefly discuss the possibility of effectively unconstrained capacity, i.e., the constraint in (11) does not bind. It can be shown that Proposition 3 and its Corollary continue to hold with unlimited capacity provided the external monopoly price (i.e., the price the foreign division would set in the external market as an unconstrained monopolist) exceeds the “internal monopoly price”. Under unadjusted market-based transfer pricing the resulting arm’s length price is a weighted average of the two monopoly prices. The imposition of an internal discount then makes both divisions better off since in response to discount the foreign division will charge a higher price externally while the domestic division buys the intermediate product at a lower transfer price closer to the incremental cost. On the other hand, in a model with-
out income taxes, Baldenius and Reichelstein (2002) have shown that intracompany discounts can be harmful if capacity is unconstrained and the above condition relating the two monopoly prices is not met.

4 Conclusion

This paper has examined the choice of administered transfer prices, i.e., cost- and market-based transfer prices, for multinational firms whose divisions face different income tax rates. We find that if firms decouple their internal transfer prices from the arm’s length price used for tax valuation purposes, the preferred internal price will be affected by differential tax rates and the admissible arm’s length price. This dependence reflects that intracompany transactions and their tax valuations induce economically relevant cash flows.

For both cost- and market-based transfer pricing, we conclude that an internal transfer price set equal to the arm’s length price will generally result in inefficiently low intracompany transfers. The optimal cost-based transfer price is a weighted average of the pre-tax unit cost and the most favorable arm’s length price. When the intermediate product is sold to external parties, we find that internal transfers should be subjected to intracompany discounts relative to the market price which also serves as the arm’s length price. We argue that the magnitude of such discounts should increase with the tax rate differential between the divisions.

Our analysis has taken divisional revenues and costs as given, subject to the variations induced by exogenous state uncertainty. A natural extension of our model would be to introduce upfront specific investments by the divisions which may increase capacity, lower unit cost or increase revenues. Such specific investments are at the heart of most studies in the incomplete contracting literature. For the theory of transfer pricing in multinational firms, it would be useful to explore how intracompany discounts and/or mark-ups on cost should be affected by tax considerations and the simultaneous need
for ex-ante investment incentives.

Another extension of our model concerns the possibility of competition in the final product market. For certain industries, it seems plausible that the external buyers of the intermediate product, after further processing it into a final product, will compete with the domestic division in the final product market. Such competition may take various forms, depending for instance on the degree of substitutability between the final products. One would expect that competitive interaction in the final product market will have an impact on the desired choice of internal transfer prices.

Finally, our model has focused on the allocation of some divisible scarce input across segments of the multinational firm. The literature on multinational transfer pricing also emphasizes the importance of transferring intangible assets such as intellectual property rights. The transfer of such assets not only poses significant challenges for identifying acceptable arm’s length prices but also for finding internal valuation rules which encourage the acquisition and subsequent transfer of those assets across separate profit centers.
Appendix

Proof of Proposition 2. Ignoring the constraint that the transfer price must be in the range \([\underline{p}, \bar{p}]\), the necessary first-order condition for the optimal \(p\) is:

\[
E_{\theta}\{\pi'(p, \theta)\} = E_{\theta}\{(1-t)[p-c] \cdot Q'(p, \theta) + h \cdot Q(p, \theta)\} = 0.
\]

Here we have used the fact that \(R'(Q(p, \theta), \theta) = p\). Rewriting this first-order condition, one obtains:

\[
E_{\theta}\{\pi'(p, \theta)\} = E_{\theta}\{(1-t)[(p-c) \cdot Q'(p, \theta) + Q(p, \theta)] - (1-t-h) \cdot Q(p, \theta)\} = 0.
\]

By definition, \(p^m(\theta)\) maximizes the monopoly problem \((1-t) \cdot (p-c) \cdot Q(p, \theta)\) and therefore \((p^m(\theta) - c) \cdot Q'(p^m(\theta), \theta) + Q(p^m(\theta), \theta) = 0\). Provided this monopoly problem is single-peaked, it follows that for any \(p\):

\[
E_{\theta}\{\pi'(p, \theta)\} < 0
\]

if \(p > p^m(\theta)\) for all \(\theta\). We conclude that the optimal single transfer price satisfies \(p < \bar{p}\) if \(\bar{p} > p^m(\theta)\) for all \(\theta\).

Proof of Corollary to Proposition 2. Proposition 1 has shown that the quantity under decoupling is the first-best quantity, \(Q^*(\bar{p}, h, \theta)\), which, in turn, maximizes the corporate objective function in (2). Let \(p^c = \min\{\hat{p}, \bar{p}\}\) denote the optimal transfer price under conformity, with \(\hat{p}\) as the optimal price without the admissibility constraint \(p \in [\underline{p}, \bar{p}]\). Therefore, \(\hat{p} \in \arg\max_p E_{\theta}\{\pi(p, \theta)\}\), where \(\pi(p, \theta)\) is the objective function in (9).

We show that the difference in expected after-tax profits,

\[
D(h, \bar{p}) \equiv E_{\theta}\{(1-t)[R(Q^*(\cdot), \theta) - c \cdot Q^*(\cdot)] - h \cdot [R(Q^*(\cdot), \theta) - \bar{p} \cdot Q^*(\cdot)]
- (1-t)[R(Q(p^c, \theta), \theta) - c \cdot Q(p^c, \theta)] + h \cdot [R(Q(p^c, \theta), \theta) - p^c \cdot Q(p^c, \theta)]\},
\]

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exhibits increasing differences in \((\bar{p}, h)\), i.e., \(D_{h\bar{p}} \geq 0\). Here \(Q^*(\bar{p}, h, \theta) \equiv Q^*(\cdot)\). We distinguish two cases.

**Case 1:** \(\hat{p} < \bar{p}\). The admissibility condition that \(p \in [\bar{p}, \tilde{p}]\) is then slack and therefore \(p^c = \hat{p}\). By the Envelope Theorem:

\[
D_h = E_\theta \{ \bar{p} \cdot Q^*(\bar{p}, h, \theta) - R(Q^*(\bar{p}, h, \theta), \theta) - [\hat{p} \cdot Q(\hat{p}, \theta) - R(Q(\hat{p}, \theta), \theta)] \} ,
\]

\[
D_{h\bar{p}} = E_\theta \left\{ Q^*(\bar{p}, h, \theta) + [\bar{p} - R'(Q^*(\bar{p}, h, \theta), \theta) \frac{\partial Q^*}{\partial \bar{p}}] \right\} .
\]

For \(D_{h\bar{p}}\) to be positive, it therefore suffices to show that \(Q^*(\bar{p}, h, \theta)\) is increasing in \(\bar{p}\) and that \(\bar{p} \geq R'(Q^*(\bar{p}, h, \theta), \theta)\) for all \(\theta\). The first-order condition for the firm-wide profit maximization in (2), slightly modified by including \(\bar{p}\) as an argument, yields

\[
\frac{\partial\pi(q, \bar{p}, \theta)}{\partial q}\bigg|_{q = Q^*(\bar{p}, h, \theta)} = (1 - t - h) \cdot R'(Q^*(\bar{p}, h, \theta), \theta) - (1 - t) \cdot c + h \cdot \bar{p} = 0 . \tag{22}
\]

By assumption, \(\bar{p} > c\), and hence (22) implies that \(\bar{p} > R'(Q^*(\bar{p}, h, \theta), \theta)\). Moreover, a sufficient condition for \(Q^*(\bar{p}, h, \theta)\) to be increasing in \(\bar{p}\) is that \(\pi(q, \bar{p}, \theta)\) exhibits increasing differences in \(q\) and \(\bar{p}\). We note that:

\[
\frac{\partial^2\pi(q, \bar{p}, \theta)}{\partial q \partial \bar{p}} = h \geq 0 .
\]

As a consequence, \(D_{h\bar{p}} \geq 0\).

**Case 2:** \(\hat{p} \geq \bar{p}\). Then \(p^c = \bar{p}\) and:

\[
D_h = E_\theta \{ \bar{p} \cdot Q^*(\bar{p}, h, \theta) - R(Q^*(\bar{p}, h, \theta), \theta) - [\bar{p} \cdot Q(\bar{p}, \theta) - R(Q(\bar{p}, \theta), \theta)] \} ,
\]

\[
D_{h\bar{p}} = E_\theta \left\{ Q^*(\bar{p}, h, \theta) + [\bar{p} - R'(Q^*(\bar{p}, h, \theta), \theta) \frac{\partial Q^*}{\partial \bar{p}}] - [Q(\bar{p}, \theta) + (\bar{p} - R'(Q(\bar{p}, \theta), \theta)) \cdot Q'(\bar{p}, \theta)] \right\}
\]

\[
\geq E_\theta \left\{ [\bar{p} - R'(Q^*(\cdot), \theta)] \frac{\partial Q^*}{\partial \bar{p}} \right\}
\]

\[
\geq 0 .
\]
Here we make use of the fact that for any transfer price \( p \) the domestic division will choose the quantity so that \( R'(Q(p, \theta), \theta) = p \), and that \( Q^*(\bar{p}, h, \theta) \geq Q(\bar{p}, \theta) \). This completes the proof of the corollary.

**Proof of Lemma 1.** For a given \( \theta \), the optimal quantities \( (q_d^*(\theta \mid h), q_f^*(\theta \mid h)) \) maximize the after-tax profit:

\[
\pi = (1 - t) \cdot [R_d(q_d, \theta) + R_f(q_f, \theta) - c \cdot (q_d + q_f)] - h \cdot [R_d(q_d, \theta) - P_f(q_f, \theta) \cdot q_d]
\]

subject to the capacity constraint \( q_d + q_f \leq K(\theta) \). If this constraint is binding, the optimal quantities \( (q_d^*, q_f^*) \equiv (q_d^*(\theta \mid h), q_f^*(\theta \mid h)) \) satisfy the first-order condition:

\[
R'_d(q_d^*, \theta) = R'_f(K(\theta) - q_d^*, \theta) + H \cdot [R'_d(q_d^*, \theta) + P'_f(K(\theta) - q_d^*, \theta) \cdot q_d - P_f(K(\theta) - q_d^*, \theta)]
\]  

(23)

where \( H \equiv \frac{h}{1-t} \). Since by definition \( R'_f(q_f, \theta) = P_f(q_f, \theta) + P'_f(q_f, \theta) \cdot q_f \), the above first-order condition reduces to:

\[
(1 - h) \cdot R'_d(q_d^*, \theta) = (1 - h) \cdot R'_f(K(\theta) - q_d^*, \theta) + H \cdot P'_f(K(\theta) - q_d^*, \theta) \cdot K(\theta)
\]

(24)

or equivalently,

\[
R'_d(q_d^*, \theta) = R'_f(K(\theta) - q_d^*, \theta) + \frac{H}{1-h} \cdot P'_f(K(\theta) - q_d^*, \theta) \cdot K(\theta).
\]

When \( h = 0 \), \( R'_d(q_d^*(\theta \mid 0), \theta) = R'_f(K(\theta) - q_d^*(\theta \mid 0), \theta) \). Since \( P'_f(\cdot, \theta) < 0 \), and marginal revenue is decreasing, we find that \( q_d^* \equiv q_d^*(\theta \mid h) > q_d^*(\theta \mid 0) \) for \( h > 0 \).

**Proof of Proposition 3:** As argued above, the double-marginalization problem results in transfer quantities \( Q_d(\cdot) \) that are already too small when \( h = 0 \), i.e., \( Q_d(p(\theta), \theta) < q_d^*(\theta \mid h = 0) \). For \( h > 0 \), the foreign subsidiary will choose the same price and the same capacity allocation, that is \( p(\theta) \) and \( Q_d(p(\theta), \theta) \) are invariant to \( h \). Lemma 1 shows that
for $h > 0$ the efficient quantities exceed those corresponding to $h = 0$ and therefore $Q_d(p(\theta), \theta) < q_3^*(\theta | h)$. 

\[ \text{Proof of Corollary to Proposition 3.} \] By (A1), the foreign division’s after-tax profit function, $\pi_f(\cdot | \Delta = 0, \theta)$ is decreasing in $p$ at $p(0, \theta)$, the price it will set when internal transfers are valued at the market price. This monotonicity property will still hold at $p(\Delta, \theta)$ for small discounts $\Delta$. For small $\Delta$ the resulting price change triggers a reallocation from external to internal trade, such that $Q_f(p(\Delta, \theta), \theta) + Q_d(p(\Delta, \theta), \Delta, \theta) \equiv K(\theta)$. To simplify notation, let $Q_d(\cdot) = Q_d(p(\Delta, \theta), \Delta, \theta)$, and $Q_f(\cdot) = Q_f(p(\Delta, \theta), \theta)$. We therefore have
\[
\frac{dQ_d(\cdot)}{d\Delta} + \frac{dQ_f(\cdot)}{d\Delta} \equiv 0. \tag{25}
\]
We note that $\frac{dQ_d(\cdot)}{d\Delta} = \frac{\partial Q_d(\cdot)}{\partial p} p'(\Delta, \theta) + \frac{\partial Q_d(\cdot)}{\partial \Delta}$ and $\frac{dQ_f(\cdot)}{d\Delta} = \frac{\partial Q_f(\cdot)}{\partial p} p'(\Delta, \theta)$. Since $\frac{\partial Q_d(\cdot)}{\partial \Delta} > 0$ and both demand functions are decreasing in $p$, we conclude that $p'(\Delta, \theta) \geq 0$ and $\frac{dQ_d(\cdot)}{d\Delta} \geq 0$.

Since the capacity constraint will continue to bind in response to a (small) discount, we can ignore the effect on total production cost, $c(\theta) \cdot K(\theta)$. Let $R_d(\cdot) = R_d(Q_d(\cdot), \theta)$, and $R_f(\cdot) = p(\Delta, \theta) \cdot Q_f(\cdot)$. For a given $\theta$, the derivative of the firm’s profit function in (17) with respect to $\Delta$ equals:
\[
\pi'(\Delta, \theta) = (1-t) \cdot [R_d'(\cdot) - R_f'(\cdot)] \cdot \frac{dQ_d}{d\Delta} - h \cdot \left[ (R_d'(\cdot) - p(\Delta, \theta)) \cdot \frac{dQ_d}{d\Delta} - p'(\Delta, \theta) \cdot Q_d(\cdot) \right].
\]
At $\Delta = 0$, $R_d'(Q_d(p(0, \theta), \theta), \theta) = p(0, \theta)$, so the above expression simplifies to
\[
\pi'(0, \theta) = (1-t) \cdot [R_d'(\cdot) - R_f'(\cdot)] \cdot \frac{dQ_d}{d\Delta} + h \cdot p'(0, \theta) \cdot Q_d(\cdot).
\]
Since $p'(\Delta, \theta) \geq 0$ and $\frac{dQ_d(\cdot)}{d\Delta} \geq 0$, $\pi'(\Delta, \theta) > 0$ at $\Delta = 0$ provided:
\[
R_d'(Q_d(p(0, \theta), \theta), \theta) - R_f'(Q_f(p(0, \theta), \theta), \theta) \geq 0.
\]
This inequality holds precisely due to double-marginalization, i.e.,
\[
R_d'(Q_d(p(0, \theta), 0, \theta), \theta) = p(0, \theta) > R_f'(Q_f(p(0, \theta), \theta), \theta).
\]
Proof of Proposition 4. For any discount \( \Delta \), the resulting market price set by the foreign division and the domestic trading quantity are as given in equations (19) and (20) in the main text. We will show below that Assumption (A1) implies that \( \pi_f(\cdot | \Delta, \theta) \) is decreasing in \( p \) for all \( \Delta \geq 0 \). As argued in the proof of Proposition 3, a variation in the discount triggers a reallocation of quantities from the external to the internal market. In the linear-quadratic setting, \( p' (\Delta, \theta) = \tau_d \cdot \nu(\theta) \) and therefore:

\[
\frac{dQ_d}{d\Delta} = \beta_d(\theta) \cdot \tau_d \cdot (1 - \nu(\theta)) \equiv -\frac{dQ_f}{d\Delta}. \tag{26}
\]

The corporate optimization problem then becomes:

\[
\Delta^* \in \arg \max_\Delta E_\theta \left\{ (1 - t)[R_d(Q_d(p(\Delta, \theta), \Delta, \theta), \theta) + R_f(K(\theta) - Q_d(p(\Delta, \theta), \Delta, \theta), \theta)] - h \cdot [R_d(Q_d(p(\Delta, \theta), \Delta, \theta), \theta) - p(\Delta, \theta) \cdot Q_d(p(\Delta, \theta), \Delta, \theta)] \right\}
\]

with the corresponding first-order condition:

\[
E_\theta \left\{ [(1 - t)[R_d'(\cdot) - R_f'(\cdot)]] \frac{dQ_d}{d\Delta} - h \left[ [R_d'(\cdot) - p(\Delta, \theta)] \frac{dQ_d}{d\Delta} - p'(\Delta, \theta) Q_d(\cdot) \right] \right\} = 0. \tag{27}
\]

Using (26) together with \( p'(\Delta, \theta) = \nu(\theta) \tau_d \) and the two equations:

\[
\begin{align*}
R_d'(\cdot) &= a_d(\theta) - b_d(\theta) \cdot Q_d(\cdot) = p(\Delta, \theta) - \tau_d \cdot \Delta, \\
R_f'(\cdot) &= a_f(\theta) - 2 \cdot b_f(\theta) \cdot [\alpha_f(\theta) - \beta_f(\theta) \cdot p(\Delta, \theta)] = 2 \cdot p(\Delta, \theta) - a_f(\theta),
\end{align*}
\]

we can rewrite (27) as:

\[
E_\theta \left\{ \beta_d(\theta) [(1 - t)(1 - \nu(\theta))[a_f(\theta) - p(\Delta, \theta) - \tau_d \Delta]] + h \cdot [(1 - \nu(\theta))\tau_d \cdot \Delta + \nu(\theta)(a_d(\theta) - p(\Delta, \theta) + \tau_d \Delta)] \right\} = 0.
\]

Finally, \( p(\Delta, \theta) = p(0, \theta) + \nu(\theta) \tau_d \Delta \), and therefore the optimal discount becomes:

\[
\Delta^* = (1 - t) \cdot \frac{E_\theta \left\{ \beta_d(\theta) \cdot (1 - \nu(\theta))[a_f(\theta) - p(0, \theta)] \right\}}{E_\theta \left\{ \beta_d(\theta) [1 - (\nu(\theta))^2] \right\}} + h \cdot \frac{E_\theta \left\{ \beta_d(\theta) \cdot \nu(\theta)[a_d(\theta) - p(0, \theta)] \right\}}{E_\theta \left\{ \beta_d(\theta) [1 - (\nu(\theta))^2] \right\}}.
\]

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We note that the optimization problem is concave in \( \Delta \) because the objective function is quadratic with a negative coefficient on \( \Delta^2 \). To complete the proof, we verify that if \( \pi_f(\cdot | \Delta = 0, \theta) \) is decreasing in \( p \) at \( p(0, \theta) \), as stipulated by Assumption (A1), then \( \pi_f(\cdot | \Delta, \theta) \) is decreasing in \( p \) at \( p(\Delta, \theta) \) for any \( \Delta \geq 0 \). Taking the derivative of the foreign division’s objective function in (15) with respect to \( p \) gives
\[
\pi'_f(p | \Delta, \theta) = (1-t)[\alpha_f(\theta) + \alpha_d(\theta) + \tau_d \cdot \beta_d(\theta) \cdot \Delta - (\beta_d(\theta) + \beta_f(\theta)) \cdot (2 \cdot p - c(\theta))] + \beta_d(\theta) \cdot \Delta.
\]
This expression is negative at \( p(\Delta, \theta) = \alpha_d(\theta) + \alpha_f(\theta) + \beta_d(\theta) \cdot (\tau_d - \tau_f) \cdot \Delta \), provided:
\[
K(\theta) \leq \frac{1}{2} [\alpha_d(\theta) + \alpha_f(\theta) - (\beta_d(\theta) + \beta_f(\theta)) \cdot c(\theta) + \beta_d(\theta) \cdot (\tau_d - \tau_f) \cdot \Delta],
\]
where \( \tau_f = \frac{1}{1-t} \leq \tau_d = \frac{1}{1-t-h} \). Thus, the right-hand side of the above inequality is increasing in \( \Delta \), and if the inequality holds at \( \Delta = 0 \), as postulated in (A1), it will hold a fortiori for any \( \Delta \geq 0 \). Hence, Assumption (A1) ensures that the foreign division will use all available capacity for any positive discount.

**Proof of Corollary to Proposition 4.** We first show that the arm’s length price set by the foreign division is independent of \( k \) for sufficiently small values of \( k \). For \( p = p(\Delta, \theta) \), the after-tax income of the foreign division is:
\[
(1-t)[(p(\Delta, \theta) - c(\theta)) \cdot K(\theta) + k \cdot Q_d(p(\Delta, \theta), \Delta, \theta)] - \Delta \cdot Q_d(p(\Delta, \theta), \Delta, \theta).
\]
If the foreign division were to lower the price slightly (in response to a positive \( k \)), the resulting change in its after-tax income equals:
\[
(1-t)[-K(\theta) + \beta_d(\theta) \cdot k] - \beta_d(\theta) \cdot \Delta = -(1-t) \cdot K(\theta) - \beta_d(\theta) [\Delta - (1-t) \cdot k],
\]
which will be negative for sufficiently small values of \( k \). Hence, \( p(\Delta, k, \theta) \equiv p(\Delta, \theta) \).
For a positive cost differential $k$, the corporate objective is to find $\Delta^*(k)$:

$$
\Delta^*(k) \in \arg \max_{\Delta} E_\theta \{(1 - t)[R_d(Q_d(\cdot)) + R_f(K(\theta) - Q_d(\cdot), \theta) + k \cdot Q_d(\cdot)] - h \cdot [R_d(Q_d(\cdot) - p(\Delta, \theta) \cdot Q_d(\cdot)]
$$

where $Q_d(p(\Delta, \theta), \Delta, \theta) \equiv Q_d(\cdot)$. Following the same steps as in the proof of Proposition 4, we obtain the optimal discount:

$$
\Delta^*(k) = \frac{E_\theta \{\beta_d(\theta) [(1 - t)(1 - \nu(\theta))[a_f(\theta) - p(0, \theta) + k] + h\nu(\theta)[a_d(\theta) - p(0, \theta)]\}}{E_\theta \{\beta_d(\theta) [1 - (\nu(\theta))^2]\}}.
$$
References


