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Reduced Quality and an Unlevel Playing Field Could Make Consumers Happier

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We study a model of imperfect competition and limited production capacity in which a key feature is the trade-off between quality and quantity. In particular, lowering product quality enables firms to increase total production. We illustrate that, in the presence of limited capacity, the choice of lower quality often results in increased social welfare. We also explore the relation between the extent of competition and the choice of quality. We find that, in some cases, reduced competition leads to increased production, decreased average quality, increased total welfare, and makes consumers better off. Finally, we consider the possibility of regulator-mandated quality standards. Imposing high-quality standards never improves welfare in our model. On the other hand, mandating an upper bound on quality could either increase or decrease welfare in either a monopoly or a duopoly market.

Key words: trade-off between quality and quantity; limited capacity; oligopoly; market concentration; competitiveness measure; quality; social welfare; consumer surplus; regulation; Cournot
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1. Introduction

Over the last three decades the topic of quality has received significant attention in the academic and practitioner literature (e.g., De Vany and Saving 1983, Evans and Lindsay 1993, Spence 1975). The literature has advanced the idea of viewing quality as an important strategic and competitive tool (e.g., Taguchi and Clausing 1990). Improving quality is, of course, costly. Hence, it is not surprising that the discussion in the accounting and other literatures has often stressed the importance of identifying relevant benefits (direct and indirect) and costs of quality.[1]

In this paper, we focus on one particular component of the cost of quality, applicable to industries facing limited capacity, usually characterizing the early, but most profitable, stages in the life cycle of products.[2] We study a model of imperfect competition and limited production capacity, in which the choice of low product quality enables firms to increase total production.[3] We find that in the presence of limited capacity, firms' decisions to reduce quality often results in increased social welfare.

We also explore the relation between the extent of competition and the choice of quality, adopting the conventional measure of concentration as a proxy for competitiveness. We show that the average product quality in the market might either increase or decrease with increased competition. Furthermore, we find that in some cases reduced competition results in increased total welfare. This finding is related to earlier results in the industrial organization (IO) literature. In particular, Farrell and Shapiro (1990a, b) show, in the contexts of mergers, that increased market concentration sometimes results in increased total welfare. That increase in total welfare is due to an increase in firms' total profits that exceeds a concurrent decline in consumer surplus (see, also, Levin 1990, Perry and Porter 1985). In contrast, in our setting the increase in total welfare is a consequence

[1] Examples include, on the one hand, Albright and Roth (1992), Dailla et al. (1995), Miller (1992), and Schilit (1994). On the other hand, Greising (1994) writes, "Quality devotees grew obsessed with methodology-cost cutting, defect reduction, quicker cycle times, continual improvement…Quality became its own reward. Standards were more important than sales. And companies appeared more interested in prizes than profits" (p. 56).

[2] Terwiesch and Bohn (2001) characterize the ramp-up stage where "high demand arises when the product is still 'relatively fresh' and customers are ready to pay a premium price" (p. 1).

[3] Terwiesch and Bohn (2001) also study the interaction among capacity utilization, yield, and yield improvement during ramp up. A related trade-off addressed is the one between yield (quality in our setting) and speed (production level in our setting).

of an increase in consumer surplus. That is, reduced competition leads to increased production, decreased average quality, and better-off consumers. We next show that when the duopolists have different capacities, the small firm would always be willing to pay more than the large firm for additional capacity offered by an outside party. In contrast, for an internal capacity sale the large firm would sometimes be willing to pay more than would the small firm. Finally, we consider the possibility of regulated quality standards. We demonstrate that such an intervention could either increase or decrease welfare, in either a monopoly or a duopoly market.

Of course, our results should not be interpreted as arguments against quality initiatives. There is little doubt that such initiatives have resulted in significant gains to many organizations. It is also clear that there are several important quality attributes not captured by our model. Our point is that, in the presence of capacity limitations, when there exists a nontrivial trade-off between stricter quality requirements and quantity, improving quality involves (opportunity) costs. The benefits, private or social, from quality improvement are sometimes insufficient to offset those costs.[4]

The remainder of this paper is organized as follows. In §2, we present our model. Section 3 provides an analysis of the choice of quality under alternative market structures. In §4, we consider extensions to our model. Section 5 provides the conclusion. Highlights of the proofs appear in the appendix.

2. The Model

We consider a Cournot duopoly market with a single product of two possible quality levels: high (H) and low (L).[5] Quality pertains to the likelihood of a malfunction; a product of low quality is more likely to break. For simplicity, we assume that the two firms incur, and subsequently report in their accounting systems, identical constant marginal production costs, α, regardless of the average quality of their products.[6] In addition, there are economic costs, such as the shadow price of using a unit of capacity, that are not considered by accounting systems. A low-quality product could involve certain costs for both the consumer and the producer. We model the consumer's and producer's incremental expected damage amounts, due to a low-quality unit (relative to a high-quality unit) by CD and PD, respectively.[7]

Production capacity is limited. Limited capacity often characterizes early stages in the life cycle of products. However, these early stages are often much more important (profitable) than later stages in the product life cycle because in later stages, as competition intensifies, profits fall (see Terwiesch and Bohn 2001). Examples of markets characterized by periods of limited capacity include the cyclical resins segment of the plastics industry, oil refining during the summer, the 18″ satellite dish in the late 1990s, cellular telephone service, the early stages of the new computer chips, and gas turbine-based power systems between 1999–2002. Whereas, in general, capacity could be a choice variable, increasing capacity can take a long time. Until the time when the capacity increase is completed, firms operate in a limited capacity world. Also, in certain industries, due to significant uncertainty regarding future demand and large amounts of investments required for expansion, firms make capacity choices below current demand (e.g., gas turbine-based power systems and the paper industry). In our model, each firm can utilize its limited capacity to produce high-quality, low-quality, or a combination of high- and low-quality products. We assume that the choice of a low product quality enables firms to increase total production. As an example, think of the case of quality assurance for a given production process. As we decrease the level of required quality, we reject fewer units, the quantity available for sale increases, and the average quality of output decreases. Increased production, however, comes at a cost, because reduced quality can result in costs (or damages) to consumers, producers, or both. Another example consistent with our setting is the choice of processing speed for an adjustable production process, where labor and automation costs are fixed but materials costs are variable.[8] By increasing

[4] For other forms of quality—such as product design and process quality control—the relation between quality and quantity is opposite to the one present in our setting, e.g., improving the quality of a production process results in fewer defective units, or a larger quantity of acceptable output.

[5] Alternatively, we can assume that the manager chooses the average quality from a continuous bounded subset. Our analysis and most of the results would be unaffected. We prefer the two-point representation because it enables us to simplify the exposition.

[6] This assumption enables us to focus attention on the trade-off between quantity and quality. Our results easily generalize to cases where direct production costs are different, that is, $\alpha_L \neq \alpha_H$.

[7] For consumers, those costs could include the time spent to manage a repair (such as calling for assistance and technical support, disassembling the product and shipping it for repair or replacement and reassembling it later on), the time spent with no product (e.g., a computer that has a limited technological life, a favorite TV show that was missed, or a production plan that was delayed), or actual damages (for example, lost data, defective products, losing a bonus for not meeting a production deadline, or water damage due to a defective hose). For producers, those costs could include repair costs, the cost of producing a replacement product (including the opportunity cost of using capacity units), shipping and handling, and reputation effects.

[8] When labor and automation costs are variable, the direct production costs of a low-quality unit are lower than those of a high-quality unit, because less time is spent on each unit. Our model can be easily extended to such a case. See also footnote 6.

processing speed, the firm can produce more units for a given period, but runs a greater risk of a production error (see also Terwiesch and Bohn 2001).[9]

We assume that production technology is exogenously given and involves quality-related capacity substitution denoted by β, $0 \le \beta \le 1$. Formally, $x_{Hi} + \beta \cdot x_{Li} \le K_i$, where x_{Hi} (x_{Li}) is the quantity of the high- (low-) quality product produced by firm i.[10] The industry's total available capacity is K. Let X_H (X_L) denote the aggregate quantity of high- (low-) quality units produced, X_1 (X_2) denote the total quantity of high- and low-quality units produced by Firm 1 (Firm 2), and TX denote the total quantity produced (i.e., $TX = X_1 + X_2 = X_H + X_L$). Then, $X_H + \beta \cdot X_L \le K$. Denote Firm 1's capacity, K_1, by ηK, and Firm 2's capacity, K_2, by $(1-\eta)K$. Without loss of generality, we confine attention to $\eta \in [0.5, 1]$, so that Firm 1 is the larger rival.

We define firm i's average quality as $AQ_i = x_{Hi}/(x_{Hi} + x_{Li})$. Similarly, we let AQ denote the average quality in the market. It is easy to verify (see appendix, Lemma A1) that, unless the capacity constraint is binding, a firm would always produce only high-quality units.[11]

Market demand is given by $P_i = A - CD(1 - AQ_i) - (X_1 + X_2)$, where P_i is the price paid to firm i (as a function of firm i's average quality), and $X_1 + X_2$ is the aggregate quantity sold in the market.[12] The demand function, production technology, and available capacity are common knowledge, therefore consumers rationally anticipate the average quality chosen by each firm. (Other sources of information could be publications such as *Consumer Reports* that distinguish among similar products produced by different firms along the quality dimension, or firms' reputations.) However, consumers cannot initially distinguish among units of different quality sold by a given firm. Hence, the market price paid to firm i reflects its choice of its average product quality. Because the duopoly rivals might choose different levels of quality, the market prices for the two firms can differ, reflecting different average quality offered by the two firms.[13]

3. Analysis

We first study two benchmark cases: the monopoly case, that is, $\eta = 1$, and the symmetric Cournot duopoly case, that is, $\eta = 0.5$. The superscripts m and sd denote the monopoly and the symmetric duopoly cases, respectively.

3.1. The Monopoly Case

The monopolist chooses the production level of high- and low-quality units to maximize profits, subject to the capacity constraint

$$\max_{x_H, x_L} \Pi_M \equiv [A - CD \cdot (1 - AQ) - x_L - x_H] \cdot (x_L + x_H) - \alpha \cdot (x_L + x_H) - PD \cdot x_L$$

$$\text{s.t. } x_H + \beta \cdot x_L \le K, \quad x_H \ge 0, \quad x_L \ge 0.$$

Observation 1. The solution to the monopolist's optimization problem is unique, with the following optimal production decision: (i) For

$$K \ge \frac{A-\alpha}{2} - \frac{CD + PD}{2(1-\beta)}$$

the monopoly produces only high-quality products. (ii) For

$$\frac{A-\alpha}{2} - \frac{CD+PD}{2(1-\beta)} \ge K \ge \frac{\beta(A-\alpha)}{2} - \frac{\beta(CD+PD)}{2(1-\beta)},$$

the monopoly produces both high- and low-quality products. (iii) For

$$K \le \frac{\beta(A-\alpha)}{2} - \frac{\beta(CD+PD)}{2(1-\beta)},$$

the monopoly produces only low-quality products. Average quality, AQ^m, is always increasing in the available capacity, K.

[9] Another example is the choice of airplane seating capacity. By decreasing leg-room space (an attribute of quality), the airline can increase the number of seats installed, i.e., it faces a trade-off between quality and quantity (however, this example does not capture the notion of damages). Similarly, hospitals that have a limited number of beds (patient days) face a trade-off between a higher-quality treatment (which might require a longer hospitalization) and the number of patients they can treat. See §4 for a discussion of governmental quality control in this industry.

[10] In the production-line speed example, β represents the ratio of speed of the production line when producing high-quality units to the speed of the line when producing low-quality units.

[11] This observation is due to our assumption that the direct production costs and expected damage-related costs of a low-quality unit exceed the direct costs of a high-quality unit. This might not always be the case. When this is not the case, a firm can choose to produce below-maximum quality level even at the absence of production constraints: "The optimal number of faulty TV sets for Sony to sell is 'not zero,' even if Sony promises to repair all faulty Sony sets that break down…[because] it is cheaper…to repair a few sets than to have such stringent quality control that the manufacturing process produces zero defectives" (Stickney and Weil 2003, p. 520). Recall footnote 6 above.

[12] Intuitively, the term $A - (X_1 + X_2)$ represents the marginal consumer's benefit from an additional unit sold. The sum of $P_i + CD \cdot (1 - AQ_i)$ represents the expected amount that a consumer would pay for that unit. In equilibrium, the marginal costs equal the marginal benefit. Also, the assumption of a unit slope for the case of a linear demand is with no loss of generality.

[13] An interesting extension is the case in which consumers cannot identify the producer (free riding). Here, the market price reflects the equilibrium total average quality. Another related extension would allow for monitoring of quality by the buyers (as in Baiman et al. 2000). Another scenario of interest is where consumers can distinguish between products of different quality levels produced by the same firm. In this case, our analysis would remain intact, because there will be a price difference of exactly CD between the two products (otherwise, arbitrage opportunities would prevail).

3.2. The Symmetric Cournot Duopoly Case

In this case, each firm chooses the production quantities of high- and low-quality products to maximize its profits, Π_i, subject to a capacity constraint, taking into account the quantity and average quality chosen by its rival, where

$$\Pi_1 \equiv [A - CD\cdot(1-AQ_1) - x_{L1} - x_{H1} - x_{L2} - x_{H2}] \cdot (x_{L1}+x_{H1}) - \alpha\cdot(x_{L1}+x_{H1}) - PD\cdot x_{L1}$$

for Firm 1. Firm 2 has a similar profit function.

Observation 2. The symmetric Cournot duopoly game has a unique equilibrium, in which the firms choose identical production levels. The solution is one of the following: (i) For

$$K \ge \frac{2(A-\alpha)}{3} - \frac{2(CD+PD)}{3(1-\beta)},$$

each firm produces only high-quality products. (ii) For

$$\frac{2(A-\alpha)}{3} - \frac{2(CD+PD)}{3(1-\beta)} \ge K \ge \frac{2\beta(A-\alpha)}{3} - \frac{2\beta(CD+PD)}{3(1-\beta)},$$

each firm produces both low- and high-quality products. (iii) For

$$K \le \frac{2\beta(A-\alpha)}{3} - \frac{2\beta(CD+PD)}{3(1-\beta)},$$

each firm produces only low-quality products. Average quality, AQ^{sd}, is always increasing in the available capacity, K.

We note the following: (i) When firms can produce unlimited quantity of low-quality units ($\beta = 0$), firms always produce some high-quality units; and (ii) for a production substitution rate sufficiently large ($\beta \ge 1 - (CD+PD)/(A-\alpha)$), neither firm produces low-quality units for any capacity level. The cut-off value is strictly less than one, because when capacity is binding an increase in total output requires a reduction in the quantity of high-quality units and an increase in the quantity of low-quality units that is higher than the total increase in output. Thus, the incremental cost of increasing output includes the resulting damages associated with the low-quality units. When β is sufficiently high, the incremental damage cost, $(CD+PD)/(1-\beta)$, exceeds the benefit from the increase in total output, which is necessarily smaller than $A-\alpha$.

Next, we compare the two benchmarks. Let consumer surplus be denoted by

$$CS \equiv \Sigma_i[A - CD(1-AQ_i) - P_i]\cdot(x_{Li}+x_{Hi})/2,$$

and total welfare be denoted by $W \equiv \Pi_1 + \Pi_2 + CS$. Then,

Corollary to Observations 1 and 2. *Comparing the cases of the monopoly and the symmetric duopoly,*[14]

$$TX^m \le TX^{sd}, \quad P^m \ge P_i^{sd}, \quad AQ^m \ge AQ^{sd},$$
$$CS^m \le CS^{sd}, \quad \Pi^m \ge \Pi_1^{sd} + \Pi_2^{sd}, \quad \text{and} \quad W^m \le W^{sd}.$$

Most of the comparisons are immediate and are in line with standard results in the literature. A new comparison is that of the average quality in the two markets. The reason that average quality in the monopoly market exceeds that of the duopoly market is that the monopolist wants to produce a smaller quantity than do the two duopolists, and therefore substitutes fewer low-quality units for the high-quality ones.[15] Nonetheless, consumer surplus, as well as total welfare, increases with the increased competition in spite of the lower average quality. This result is particularly interesting in light of the emphasis on improving quality in the popular press, practitioner literature, and other writings on management practices (e.g., Dailla et al. 1995, Taguchi and Clausing 1990).

The literature often makes the point that the benefits from increased quality should be compared with the cost of quality. In our setting, the direct production cost is independent of quality, but low-quality products involve the additional cost of expected damages. Nonetheless, in the presence of limited capacity, firms might choose to produce low-quality products to increase total production. The reason is the existence of additional differential costs that are never reported in any accounting system. Because the production of a unit of a high-quality product consumes more capacity, the opportunity costs of producing it are higher. For example, assume that a firm produces strictly positive quantities of both quality levels. In that case, the shadow price of capacity equals $(CD+PD)/(1-\beta)$ (see the proof of Proposition 3 in the appendix). The

[14] The inequalities are strict in most cases. Equalities arise in some cases where the outputs in the monopoly and the symmetric duopoly markets are equal, and the entire capacity is used either to produce only low- or only high-quality products. Necessary and sufficient conditions for having equalities between the cases of the monopoly and the symmetric duopoly are

$$K \le \frac{\beta(A-\alpha)}{2} - \frac{\beta(CD+PD)}{2(1-\beta)} \quad \text{or}$$
$$\frac{2(A-\alpha)}{3} - \frac{2(CD+PD)}{3(1-\beta)} \le K \le \frac{A-\alpha}{2}.$$

For the set of parameters used for Figure 1, $A = 100$, $\alpha = 10$, $CD = 9$, $PD = 6$, and $\beta = 0.4$, those conditions are met when $K \le 13$ or $43.33 \le K \le 45$.

[15] This result is in line with Leffler (1982) who shows, in a different setting, that a monopolist can choose higher quality than the quality prevailing in a competitive market. See also Leland (1977).

full (a portion, β, of the) shadow price of a capacity unit is included in the economic production costs of a high-quality (low-quality) unit. When β is relatively small, the difference in capacity utilization is significant, and the firm will be inclined to produce more low-quality units.[16] What is especially interesting is that when low-quality units are produced consumers gain from the increase in product quantity, and the reduction in price outweighs their expected damage associated with the lower average product quality. Furthermore, the increase in consumer surplus exceeds the reduction in total producers' profits which implies an overall increase in welfare.

3.3. The Case of Asymmetric Duopoly

The impression one might have from the corollary to Observations 1 and 2 is that a social planner would find increased competition desirable because of its impact on consumer surplus and on total welfare, in spite of the resulting reduced quality. To examine this intuition, we study the impact of reduced competition (or, more precisely, increased concentration) on average quality, profits, consumer surplus, and total welfare.

To model concentration, we hold fixed the total available market capacity, K, and transfer production capacity from one duopoloist to the other.[17] Intuitively, as one firm becomes larger the duopoly market becomes less competitive. Interpreting Firm 1's share of total market capacity, η, as a measure of concentration is consistent with measures of concentration commonly used in the economics literature, such as the Herfindahl, m-firm, or entropy measures.[18]

We first characterize the equilibrium in the asymmetric duopoly setting. Recall that, without loss of generality, we have assumed that Firm 1 is the large firm ($\eta \geq 0.5$).

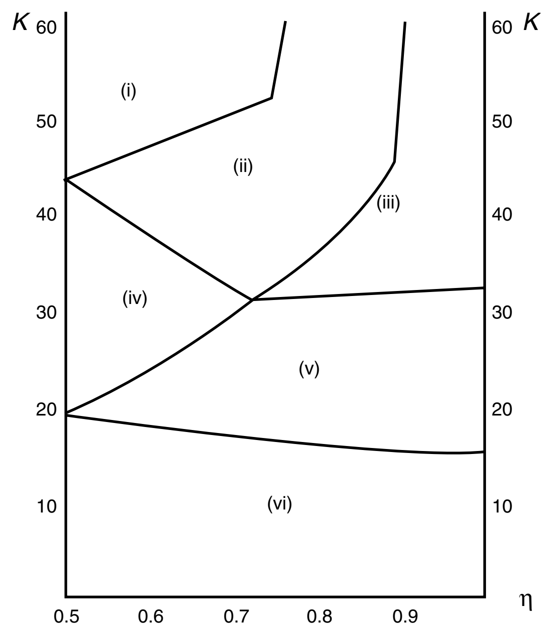
Proposition 1. *The asymmetric duopoly model has a unique equilibrium, characterized by one of the following cases*:[19] (i) *both firms produce only high quality*; (ii) *Firm* 1 *produces only high quality and Firm* 2 *produces both low and high quality*; (iii) *Firm* 1 *produces only high quality and Firm* 2 *produces only low quality*; (iv) *Firms* 1 *and* 2 *produce both low and high quality*; (v) *Firm* 1 *produces both low and high quality and Firm* 2 *produces only low quality*; (vi) *both firms produce only low quality*.

Figure 1 illustrates the relation between the prevailing equilibrium and total market capacity and capacity concentration.

Consider now increasing Firm 1's share of total capacity, η, while maintaining the total capacity, K, fixed. The interpretation of this comparative static is increasing the level of capacity concentration (or decreasing competition). Specifically, we examine the impact of allocated capacity on total welfare (i.e., the sum of consumer surplus and the duopolists' profits), consumer surplus, and average quality as we move from the symmetric duopoly ($\eta = 0.5$) to the monopoly case ($\eta = 1$).

Proposition 2. *Examining total welfare as a function of allocated capacity, two alternative patterns could emerge*. (1) *Total welfare is* (*weakly*) *decreasing in* η, *i.e.*, $\partial W/\partial\eta \leq 0\ \forall\, \eta \in [0.5, 1]$. (2) *Total welfare is increasing in* η *for lower levels of* η, *and then decreasing in* η *for higher levels of* η. *Both the increase and decrease are strict for certain ranges of* η*'s values*.

Figure 1 Prevailing Equilibrium as a Function of Total Market Capacity and Capacity Concentration



Case (i): Both firms produce only high quality
Case (ii): Firm 1 produces only high quality and firm 2 produces both low and high quality
Case (iii): Firm 1 produces only high quality and Firm 2 produces only low quality
Case (iv): Firms 1 and 2 produce both high and low quality
Case (v): Firm 1 produces both low and high quality and Firm 2 produces only low quality
Case (vi): both firms produce only low quality.
Parameters: $A = 100$, $\alpha = 10$, $CD = 9$, $PD = 6$, and $\beta = 0.4$.

[16] Intuitively, if a firm replaces a unit of a high-quality product with a unit of a low-quality product, it incurs additional costs of $CD + PD$, and frees $1 - \beta$ units of capacity. If the firm produces both quality levels, the value of the freed capacity should equal its costs.

[17] This shift of capacity is exogenous. Section 4 considers endogenous capacity transactions.

[18] For a description and critique of these measures, see Tirole (1988) and Curry and George (1983).

[19] In the proof (in the appendix) we provide the necessary and sufficient conditions for existence of each one of the cases.

The first pattern described in Proposition 2 is an extension of the corollary to Observations 1 and 2: Total welfare decreases when the large firm is allocated a greater share of the (fixed) market capacity.

The second pattern described in Proposition 2 is more interesting. Under certain circumstances, when capacity is transferred to the large firm, total welfare increases. The impact of increased concentration on total welfare is reversed (i.e., welfare decreases), once the large firm becomes sufficiently large. At the limit, when only one firm remains in the market, $\eta = 1$, the total welfare is (weakly) lower than the total welfare corresponding to the symmetric duopoly. This pattern exists for a large set of parameters, when the capacity constraint is binding, but capacity is not too small. For example, in environments where demand is cyclical—and as a result, capacity is binding in certain periods—we predict that this would be the prevailing pattern. Formally, this pattern requires total capacity such that Case (ii) of Proposition 1 is obtained for some $0.5 < \eta < 1$. Necessary and sufficient conditions for existence require that total capacity would be at an intermediate level, formally,

$$\frac{(A-\alpha)(1+\beta)}{3} - \frac{(CD+PD)(1+\beta)}{3(1-\beta)} \le K \le \frac{2(A-\alpha)}{3} - \frac{(CD+PD)}{3(1-\beta)}.^{20}$$

The second pattern emerges in two cases: (i) The first case is where both symmetric duopolists choose to produce both high and low quality, and the quantity of high-quality products exceeds the quantity of low-quality products.[21] Figure 2 illustrates welfare changes for this case. (ii) The second case is where both symmetric duopolists choose to produce only high-quality products and the total capacity is not too large, such that when the small firm starts producing low-quality products the large firm still finds its capacity constraint binding.[22] In the

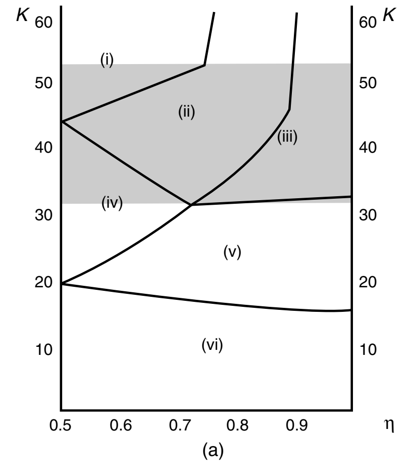
[20] For example, for the set of parameters used for Figure 1, $A = 100$, $\alpha = 10$, $CD = 9$, $PD = 6$, and $\beta = 0.4$, necessary and sufficient conditions for the existence of this pattern are $30.33 \le K \le 51.66$. The shaded area in Figure 1a demonstrates all cases where this pattern exists.

[21] This is the case, for example, for the parameters of Figure 1, where $30.33 \le K \le 43.33$.

[22] This is the case, for example, for the parameters of Figure 1, where $43.33 \le K \le 51.66$. The cut-off between the case presented in this footnote and the one presented in the prior footnote is

$$K = \frac{2(A-\alpha)}{3} - \frac{2(CD+PD)}{3(1-\beta)}.$$

Figure 1a **Sets of Parameters Where Pattern 2 of Proposition 2 Prevails**



Note. In the shaded area, total welfare is increasing in η for lower levels of η, and then decreasing in η for higher levels of η.

Figure 1b **Sets of Parameters Where Observation 3, the Simpson's Reversal Paradox, Prevails**

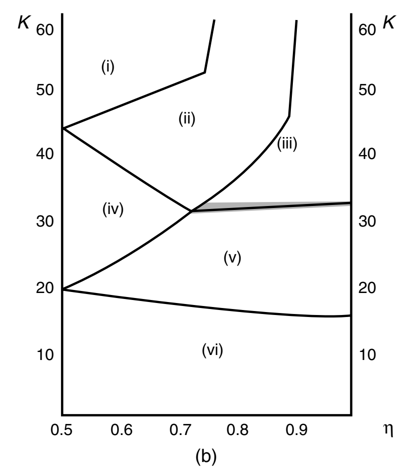


Figure 1c **Sets of Parameters Where Enforcing Production of Low-Quality Units Reduces Welfare in a Duopoly Setting, While It Improves Welfare in a Monopoly Setting**

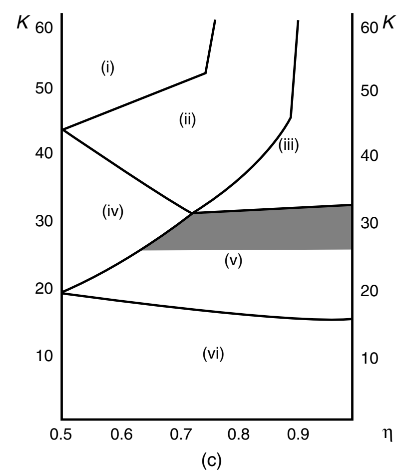
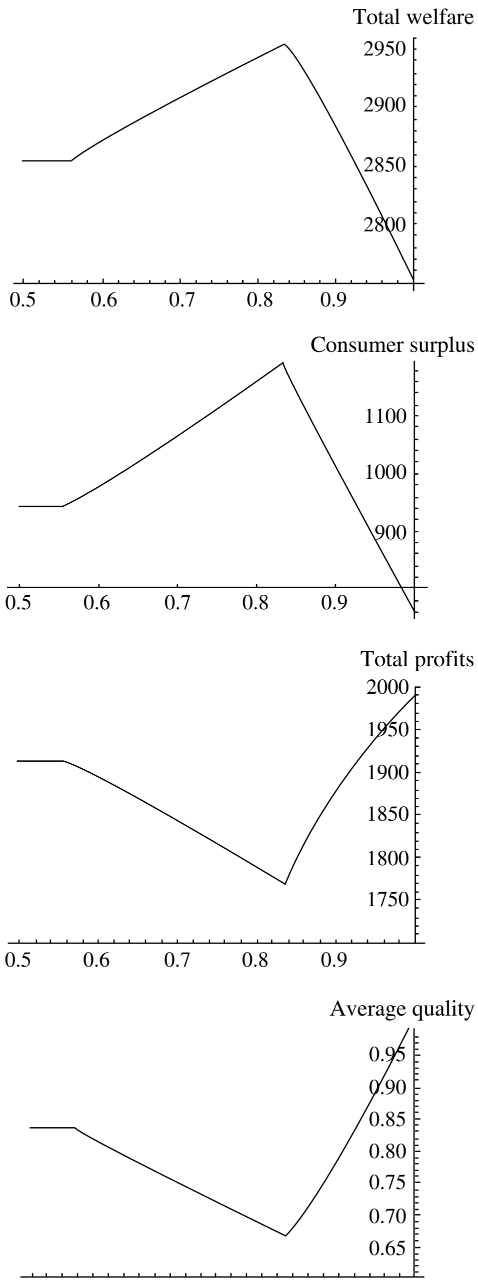


Figure 2 The Impact of Concentration on Total Welfare, Consumer Surplus, and Average Quality



Notes. Total welfare and consumer surplus are first increasing and then decreasing in concentration, η. Total profits and average quality, on the other hand, are first decreasing and then increasing in η.
Parameters: $A = 100$, $\alpha = 10$, $CD = 9$, $PD = 6$, $\beta = 0.4$, $K = 39$.

appendix we provide full characterization of total welfare.[23]

Intuitively, welfare increases as η increases because Firm 2, which produces a smaller quantity, is less concerned than Firm 1 about the adverse impact of market price reduction resulting from increased production. Therefore, Firm 2 increases its production of low-quality units. Firm 1, however, produces only high-quality units and uses the additional capacity for the production of additional high-quality units. The net effect of the shift in capacity is an increase in the total quantity of units, which amounts to an increase in total welfare. Once the small firm produces only low-quality units, an increase in η implies a decrease in the production of low-quality units (by Firm 2) and a (smaller) increase in the production of high-quality units (by Firm 1). The net effect in this range is a smaller total output and reduced welfare.

The result that welfare in a duopoly market could be maximized when one firm is larger than the other has interesting policy implications. It suggests that in cases where a regulator is compelled to restrict competition to give firms an incentive to invest in infrastructure (which was the case with regional wireless communication services and cable television), maximizing competition within an oligopoly market by trying to create equal size rivals might be undesirable.

Corollary 1 to Proposition 2. *For the set of parameters supporting pattern* (2) *of Proposition* 2, *total welfare is maximized when one duopolist is strictly larger than the other, i.e.*, $0.5 < \eta < 1$.

A related result that concentration can increase welfare has been established in the industrial organization literature by Farrell and Shapiro (1990a), who study mergers in oligopoly settings. The reason for the increase in welfare in their setting is that increased concentration results in an increase in total profits, which exceeds the concurrent decline in consumer surplus. In contrast, in our setting consumer surplus moves together with total welfare, whereas total profits usually follow an opposite path to that of the consumer surplus and welfare.[24] The following corollary

[23] An interesting case is the one where total welfare corresponding to the symmetric duopoly and the monopoly cases are identical, and the total welfare under any asymmetric duopoly exceeds that level. Necessary conditions for this case are that (i) the market capacity is just below the monopolist's unconstrained optimal production level, and (ii) at that capacity level, the symmetric Cournot case involves production of only high-quality units. Given the parameters of Figure 1, these conditions are met when $45 \geq K \geq 43.33$. More generally, the conditions are met for

$$\frac{2(A-\alpha)}{3} - \frac{2(CD+PD)}{3(1-\beta)} \leq K \leq \frac{A-\alpha}{2}.$$

[24] The one exception to the pattern of profits moving in an opposite direction to consumer surplus is when the large firm has excess capacity (and produces only high-quality products), and when the small firm has limited capacity and produces both high- and low-quality products. As η increases, the small firm shifts production and produces more low-quality products, while maintaining its total output constant. The large firm does not change its production. As a result, total profits decrease, consumer surplus is unchanged, and total welfare decreases. Necessary and sufficient

examines the impact of changes in allocated capacity on consumer surplus.

Corollary 2 to Proposition 2. *For certain parameters, consumer surplus increases as market concentration increases.*

The implication of the corollary is that consumers might prefer a duopoly market with one large and one small firm to a duopoly market with equal size rivals.

Finally, we examine the properties of the average quality as a function of the market share, η. We first note that the large firm always chooses average quality higher than that chosen by the small firm, i.e., $AQ_1 \geq AQ_2$. Hence, the market share of the large firm is smaller than the ratio of its capacity to the total market capacity, $X_1/(X_1+X_2) \leq \eta$.[25] As market share changes, the average quality obtains the same pattern as total profits (and opposite to the pattern of consumer surplus and total welfare). It is interesting to note in passing the following:

Observation 3. It is possible that when η increases each firm's average quality decreases (with a strict decrease for one firm), while the market average quality, AQ, strictly increases.

Observation 3 could be explained by Simpson's reversal Paradox (see Simpson 1951; also see Sunder 1983 for an application of this paradox to cost allocation). When capacity is transferred to Firm 1, it uses this capacity to increase the number of both its low- and high-quality units in proportions that strictly reduce its average quality. At the same time, Firm 2 produces only low-quality units, hence its average quality remains the same (at zero). The total market average quality is a weighted average of the two firms' average qualities. As η increases, the weight on Firm 1's (higher) average quality increases by relatively more than the reduction in its average quality, such that the overall average quality increases. Necessary conditions for the paradox to arise are (a) Case (v), where the large firm produces both high- and low-quality products and the small firm produces only low-quality products, and (b) capacity level is sufficiently large, formally,

$$K > \beta \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right].$$

conditions for this case are

$$\frac{A-\alpha}{3} + \frac{CD+PD}{3(1-\beta)} \leq \eta K \quad \text{and}$$

$$\frac{\beta(A-\alpha)}{3} - \frac{2\beta(CD+PD)}{3(1-\beta)} \leq (1-\eta)K \leq \frac{A-\alpha}{3} - \frac{2(CD+PD)}{3(1-\beta)}.$$

Using the parameters of Figure 1, this happens when $38.33 \leq \eta K$ and $5.33 \leq (1-\eta)K \leq 13.33$.

[25] Our market share result is in contrast with Koenigsberg (1980) who shows, in a different model, an opposite result. In his model, consumers are concerned about waiting time, and the large firm is more efficient than is the small firm in controlling waiting time.

Note that these two necessary conditions could be jointly satisfied only when $\beta < 0.5$ and $\eta > 2/3$.[26]

We conclude this section by providing another nonmonotonicity result, where increased concentration can cause the small firm to behave more aggressively relative to an identical firm in a competitive market. Consider, as a benchmark, a perfect competition equilibrium where all firms behave as price takers. Assume that total market capacity is sufficiently high, such that only high-quality units are produced. Compare this case with an asymmetric duopoly setting, with identical total capacity, where one firm has identical capacity to a firm in the competitive market, and the other firm owns the remaining capacity. As the following observation demonstrates, the small duopolist might produce a higher quantity compared with a firm with identical capacity and technology that operates in a perfectly competitive market.[27]

Observation 4. Assume that $\beta \leq 1-(2\cdot(CD+PD))/(A-\alpha)$. Then, for any market capacity, K, the small firm produces some low-quality units when its share of total capacity, $1-\eta$, is sufficiently small. Intuitively, under limited competition, the market price exceeds marginal cost of production, α. In contrast, under perfect competition, with no capacity constraint, the price equals α, so no firm ever produces a low-quality unit. Hence, only under limited competition, the market price is consistent with the production of low-quality units, and a firm in this environment might produce more units than would a firm with an identical capacity in a competitive market.

4. Extensions

In this section, we consider two extensions of our model. We first consider the possibility of small capacity purchases. We show, on the one hand, that the small firm is always willing to pay a third party more than the large firm would for incremental additional capacity (increasing total market capacity). On the other hand, an incremental capacity transaction between the two firms could be more valuable to either the small or the large firm.

[26] For example, consider $A=100, \alpha=10, \beta=0.4, CD=9, PD=6, K=30$, then for $\eta=0.85, x_{H1}=24.58, x_{L1}=2.29, x_{H2}=0, x_{L2}=11.25, AQ_1=0.915, AQ_2=0, AQ=0.645$, whereas for $\eta=0.86, x_{H1}=24.83, x_{L1}=2.42, x_{H2}=0, x_{L2}=10.5, AQ_1=0.911, AQ_2=0, AQ=0.658$. Using the parameters of Figure 1, necessary conditions for Observation 3 are $K>26$ and $\eta>2/3$. The shaded area in Figure 1b represent all parameters where we obtain Observation 3.

[27] In a perfectly competitive market, when $K \geq A-\alpha-(CD+PD)/(1-\beta)$ only high-quality units are produced, when $K \leq \beta(A-\alpha)-\beta(CD+PD)/(1-\beta)$, only low-quality units are produced. When $A-\alpha-(CD+PD)/(1-\beta) \geq K \geq \beta(A-\alpha)-\beta(CD+PD)/(1-\beta)$, both low- and high-quality units are produced, the market price is $P=\alpha+(CD+PD)/(1-\beta)$, and all firms are indifferent to the quality of their products.

The second extension we consider involves quality standards imposed by a welfare-maximizing regulator. We show that if the regulator is limited to setting quality standards (but it cannot dictate price or quantity), then the only intervention that can enhance welfare is imposing a maximum quality standard, i.e., restricting the production of high-quality units. An example for such standards is the Certificate-of-Need requirement in the health care industry, where hospitals are restricted in their purchases of expensive equipment; this can be viewed as restricting the quality of care provided.[28] Imposing a maximum quality standard is not always optimal. In some cases it results in welfare reduction. This suggests that the regulator's limit on quality should be considered on a case-by-case basis.

4.1. Capacity-Related Transactions

We first examine an incremental exogenous capacity infusion for one of the duopolists.

Proposition 3. *Consider an incremental exogenous increase in capacity. The small firm always values the additional capacity more than does the large firm.*

An increase in capacity has two offsetting effects: a direct increase in profit due to the incremental increase in sales, and the indirect decrease in revenues due to the reduced price for the total quantity sold prior to the increase in capacity. The first effect is not a function of the firm's capacity (although it depends on the firm's choice of quality). The second (adverse) effect is larger for the large firm. Thus the small firm, more than the large firm, values an exogenous, firm-specific increase in capacity.

Consider now selling capacity between the two firms. It is obvious that, in the absence of a legal restriction, one firm will buy the entire capacity of the other firm, because the buyer's monopolistic profits would exceed the total profits under a duopoly. However, when the only capacity sale permitted is a given (small) increment (e.g., due to legal restrictions), a result different from that of Proposition 3 emerges.

Proposition 4. *Consider a sale of incremental production capacity between the two firms, such that we remain in the same equilibrium class as described in Proposition* 1. *Then, in some cases, the small firm is willing to pay more than the large firm is willing to pay, whereas the reverse holds in other cases. Either way, consumer surplus and total welfare decrease as a result of a profit-maximizing sale.*

Unlike under Proposition 3, the large firm might be willing to pay more for incremental capacity because buying capacity from a rival has the additional effect of reducing the rival's capacity (and production). In some cases (e.g., Cases (iii) and (v) of Proposition 1), when the small firm produces aggressively, the large firm benefits more than does the small firm from its rival's buying capacity.

4.2. Policy Implications

Our discussion so far has focused on the welfare implications of the choice of product quality. We conclude our analysis by examining the welfare consequences of a regulator's intervention. Obviously, a regulator could maximize welfare by dictating the production quantity and product quality. Although this kind of intervention would be most effective in our case, it is not descriptive of most regulated industries for several political and other reasons. An example of a similar kind of regulatory intervention arises in the automobile industry: Regulators, concerned about air pollution, impose mile per gallon (MPG) standards (a form of quality standards), rather than imposing a direct quantity standard on the industry.[29] Below we consider one case of restricted regulatory power—imposing quality standards. We start our analysis by showing that the only quality-standard intervention that might be desirable for a welfare-maximizing regulator involves limiting maximum product quality. This result is consistent with Leland (1979), who shows that minimum quality standard chosen by a professional group exceeds the socially optimal level.

Proposition 5. (i) *A standard allowing only the production of high-quality products never improves welfare. For some parameter values, it implies a strict reduction in total welfare*. (ii) *A standard that allows only the production of low-quality products can either increase or decrease total welfare*.

Intuitively, as shown in §3, consumer surplus and total welfare increase in total quantity. A standard

[28] A natural alternative explanation is that Certificate-of-Need prevents hospitals from excessive spending in a Prisoner's Dilemma–type Nash equilibrium. Assume that the benefits from new equipment are below its costs, then, the Prisoner's Dilemma story is more plausible. (In particular, once the equipment is there physicians will use it even if they know it does not help much, possibly due to patient pressure.) However, even if the new equipment is very helpful, regulators might not want hospitals to purchase it. An immediate explanation would be a budget constraint. Alternatively, given that there is a limited capacity in hospitals (number of beds or patient days), and assuming that more tests require longer hospitalization, such equipment purchase could imply that fewer patients will receive better treatment. A regulator who is concerned with long-term capacity requests, as well as with the well being of those who would not be able to receive any treatment, can limit the availability of some useful equipment, regardless of the level of competition.

[29] Note that the regulator could also use the tax system to effectively enforce any policy. However, such a regulation could be overinclusive because it might affect more than this specific market (see Scholes and Wolfson 1992). We ignore this possibility here.

that mandates high quality, in a case where some low-quality units would be produced in the absence of such a standard, results in a decrease in total quantity and total welfare. A standard that mandates low quality, however, has two effects on the production decision. First, a firm that would have produced high-quality units will substitute low-quality units for high-quality units. Second, the marginal costs are changed. If without intervention only high-quality units are produced, then imposing a low-quality standard increases the marginal cost from α to $\alpha + CD + PD$, and results in a decrease in total quantity. Hence, total welfare decreases. If without intervention both high- and low-quality units are produced, then imposing a low-quality standard decreases the marginal cost from $\alpha + (CD + PD)/(1-\beta)$ to $\alpha + CD + PD$, and results in an increase in total quantity; the net welfare effect can be either an increase or a decrease. Such regulator's intervention should be judged on a case-by-case basis. An interesting question is whether there exists a systematic difference between the monopoly and the duopoly regarding the impact of regulatory intervention. Conventional wisdom suggests that a regulator should be more inclined to regulate a monopoly than a duopoly, because the monopoly market is less competitive. Interestingly, in most cases this intuition does not hold in our setting, as the following corollary establishes.

Corollary 1 to Proposition 5. *For a large set of parameters, mandating production of low-quality units reduces total welfare in a monopoly, whereas it would increase welfare in the parallel duopoly setting. In other cases, the reverse holds.*

The case where limiting product quality reduces welfare in a monopoly setting while it increases welfare in a duopoly setting is intuitive in our setting. Recall that such standards reduce welfare when the firms produce only high-quality units before the regulation. Because the monopoly is more likely to produce only high-quality units, whereas for the same capacity level duopolists often produce both high and low quality, the result follows. The case where intervention reduces welfare in a duopoly setting while it improves welfare in a monopoly setting occurs when the monopolist produces both quality levels, and the larger duopolist produces only high-quality units.[30]

[30] In general, the impact of regulatory intervention is reversed when the prevailing equilibrium for the asymmetric duopoly is Case (iii), and the substitution rate between high- and low-quality units is high ($\beta < 0.5$). Using the parameters of Figure 1, necessary (but not sufficient) conditions for such a reversal are $30.33 < K < 32.5$ and $\eta > 0.71$. The shaded area in Figure 1c represents all cases where the reversal occurs.

5. Conclusion

In this paper, we endogenize the choice of quality under imperfect competition and limited production capacity. In our setting, the choice of low product quality enables firms to increase total production. We have adopted a parsimonious model for our analysis that enabled us to isolate the impact of quality choice on welfare. Several possible generalizations could be considered, including adopting more general cost, damage, and demand functions. We expect the basic economic forces identified here will continue to hold in a more general setting. In fact, we can show examples of more general settings where our results are qualitatively the same.

Our key result is that, in the presence of limited capacity, reduced quality could be socially desirable, and that the extent of competition is an important consideration in the choice of quality. We find that, in some cases, reduced competition results in lower quality, but also in increased consumer surplus and total welfare. An interesting implication of this result is that a regulator overseeing an oligopolistic market with limited capacity should not necessarily try to maximize competition by allowing the competitors to have equal power. Instead, the regulator could find it desirable to make the playing field unleveled by allowing one firm to dominate the market.

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Appendix

Sketch of Proofs

Proof of Observation 1. The monopoly maximizes:

$$\max_{x_H, x_L} \Pi_M \equiv \left[A - CD \cdot (1 - AQ) - x_L - x_H\right] \cdot (x_L + x_H) - \alpha \cdot (x_L + x_H) - PD \cdot x_L$$

$$\text{s.t. } x_H + \beta \cdot x_L \le K, \quad x_H \ge 0, \quad x_L \ge 0$$

(multipliers $\lambda_0, \lambda_H, \lambda_L$, respectively).

The first-order and the complementary-slackness conditions are (in addition to the conditions that constraints are satisfied and that the multipliers are nonnegative)

$$A - \alpha - CD - PD - 2 \cdot x_L - 2 \cdot x_H - \beta \cdot \lambda_0 + \lambda_L = 0,$$

$$A - \alpha - 2 \cdot x_L - 2 \cdot x_H - \lambda_0 + \lambda_H = 0,$$

$$\lambda_0 \cdot (x_H + \beta \cdot x_L - K) = 0, \qquad \lambda_L \cdot x_L = 0, \qquad \lambda_H \cdot x_H = 0.$$

Before we present the solution, we show the following:

Lemma A1. *If the capacity constraint is not binding, then* $x_L = 0$.

Proof. Suppose not. Replace some low-quality units with high-quality units, such that total quantity remains the same. Now, observe that revenues are higher (as AQ increases) and total costs are lower (as $PD \cdot x_L$ decreases); hence, the monopolist's profits are higher and the original production plan cannot be optimal. □

First, consider the case where the capacity constraint is binding, and both quality levels are produced, i.e., $x_L > 0$, $x_H > 0$, and $\lambda_L = \lambda_H = 0$. The solution to the monopolist's problem is

$$x_H = K - \frac{\beta}{2\cdot(1-\beta)} \cdot \left[A - \alpha - 2\cdot K - \frac{CD+PD}{1-\beta}\right], \quad \text{(A1)}$$

$$x_L = \frac{1}{2\cdot(1-\beta)} \cdot \left[A - \alpha - 2\cdot K - \frac{CD+PD}{1-\beta}\right]. \quad \text{(A2)}$$

The constraint that $x_H > 0$ in Equation (A1) is binding when

$$K - \frac{\beta}{2\cdot(1-\beta)} \cdot \left[A - \alpha - 2\cdot K - \frac{CD+PD}{1-\beta}\right] < 0 \quad \text{or}$$
$$K < \frac{\beta\cdot(A-\alpha)}{2} - \frac{\beta\cdot(CD+PD)}{2\cdot(1-\beta)}.$$

In this range, $x_H = 0$ and the monopolist produces only low-quality units, $x_L = K/\beta$. The constraint that $x_L > 0$ in Equation (A2) is binding when

$$\left[A-\alpha-2\cdot K-\frac{CD+PD}{1-\beta}\right]<0 \quad \text{or} \quad K > \frac{(A-\alpha)}{2} - \frac{CD+PD}{2(1-\beta)}.$$

In this range, $x_L = 0$, and the monopolist produces only high-quality units, $x_H = K$.

Finally, when the capacity constraint is not binding, the monopolist's solution is

$$x_H = \frac{A-\alpha}{2}, \qquad x_L = 0. \quad \square$$

Proof of Observation 2. This is a special case of the proof of Proposition 1 below, where $\eta = 0.5$. Note that if $\beta > 1-(CD+PD)/(A-\alpha)$, no firm produces any low-quality units. □

Proof of Corollary to Observations 1 and 2. Use the definitions provided for profits, consumer surplus, and welfare. The results represent straightforward comparisons of the terms for the monopoly and for the symmetric duopoly. □

Proof of Proposition 1. Each firm maximizes profits, taking its rival's quantities as given. Firm 1's optimization is

$$\underset{x_{H1},\, x_{L1}}{\text{Max}} \quad \Pi_1 \equiv \left[A - CD\cdot(1-AQ_1) - x_{L1} - x_{H1} - x_{L2} - x_{H2}\right] \cdot (x_{L1}+x_{H1}) - \alpha\cdot(x_{L1}+x_{H1}) - PD\cdot x_{L1}$$
$$\text{s.t.} \quad x_{H1} + \beta\cdot x_{L1} \le \eta\cdot K, \qquad x_{H1}\ge 0, \qquad x_{L1}\ge 0$$
(multipliers λ_{01}, λ_{H1}, λ_{L1}, respectively),

with Firm 2's optimization problem similarly defined (with a capacity level of $(1-\eta)\cdot K$). For Firm 1, the five first-order conditions and the complementary slackness conditions are

$$A-\alpha-CD-PD-2\cdot x_{L1}-2\cdot x_{H1}-x_{L2}-x_{H2}-\beta\cdot\lambda_{01}+\lambda_{L1}=0,$$
$$A-\alpha-2\cdot x_{L1}-2\cdot x_{H1}-x_{L2}-x_{H2}-\lambda_{01}+\lambda_{H1}=0,$$
$$\lambda_{01}\cdot(x_{H1}+\beta\cdot x_{L1}-\eta\cdot K)=0, \quad \lambda_{L1}\cdot x_{L1}=0, \quad \lambda_{H1}\cdot x_{H1}=0.$$

Similar five conditions hold for Firm 2. Below we characterize the solution to the system of 10 equations (where all constraints are satisfied).

First, consider the set of parameters where the capacity constraint is binding for both firms, and both firms produce high- and low-quality units. In this case

$$x_{H1} = \frac{1}{1-\beta}\cdot\eta\cdot K - \frac{\beta}{1-\beta}\cdot\left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right], \quad \text{(A3)}$$

$$x_{L1} = -\frac{1}{1-\beta}\cdot\eta\cdot K + \frac{1}{1-\beta}\left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right], \quad \text{(A4)}$$

$$x_{H2} = \frac{1}{1-\beta}\cdot(1-\eta)\cdot K - \frac{\beta}{1-\beta}\left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right], \quad \text{(A5)}$$

$$x_{L2} = -\frac{1}{1-\beta}\cdot(1-\eta)\cdot K + \frac{1}{1-\beta}\left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right]. \quad \text{(A6)}$$

It is easy to verify

$$\frac{\partial x_{H1}}{\partial\eta} > 0, \quad \frac{\partial x_{L1}}{\partial\eta} < 0, \quad \frac{\partial x_{H2}}{\partial\eta} < 0, \quad \frac{\partial x_{L2}}{\partial\eta} > 0,$$
$$\frac{\partial(x_{H1}+x_{L1})}{\partial\eta} = 0, \quad \frac{\partial(x_{H2}+x_{L2})}{\partial\eta} = 0.$$

As long as (A3)–(A6) are strictly positive, they constitute a solution to the asymmetric duopoly case. The relevant constraints are (A4) and (A5), or

$$\frac{\beta}{1-\eta}\cdot\left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right] \le K \le \frac{1}{\eta}\cdot\left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right].$$

The pairs of (η, K) that satisfy these two inequalities constitute Case (iv) of the proposition.

Next, suppose that (A5) is binding, that is, Firm 2 produces only low-quality units. It is easy to verify that this occurs when

$$K < \frac{\beta}{1-\eta}\cdot\left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right],$$

and that Firm 2's production level is $x_{H2} = 0$, and $x_{L2} = ((1-\eta)\cdot K)/\beta$. If Firm 1 produces both quality levels,

$$x_{L1} = \frac{1}{1-\beta}\cdot\left[\frac{A-\alpha}{2} - \frac{CD+PD}{2\cdot(1-\beta)}\right] - \frac{2\cdot\beta\cdot\eta+(1-\eta)}{2\cdot\beta\cdot(1-\beta)}\cdot K,$$
$$x_{H1} = -\frac{\beta}{1-\beta}\cdot\left[\frac{A-\alpha}{2} - \frac{CD+PD}{2\cdot(1-\beta)}\right] + \frac{(1+\eta)}{2\cdot(1-\beta)}\cdot K.$$

As long as both $x_{L1}\ge 0$ and $x_{H1}\ge 0$, they constitute equilibrium output levels. This is the case when

$$\frac{\beta}{1+\eta}\cdot\left[A-\alpha-\frac{CD+PD}{(1-\beta)}\right] \le K \le \frac{\beta}{2\cdot\beta\cdot\eta+(1-\eta)}\cdot\left[A-\alpha-\frac{CD+PD}{(1-\beta)}\right].$$

The pairs of (η, K) that satisfy these two inequalities, as well as the necessary condition that $x_{H2}=0$, that is

$$K < \frac{\beta}{1-\eta} \cdot \left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right],$$

constitute Case (v) of the proposition.

Next, suppose that (A3) is binding, that is, Firm 1 produces only low-quality units, $x_{H1}=0$, which also implies $x_{H2}=0$ as $\eta \geq 0.5$. It is easy to verify that this occurs when

$$K < \frac{\beta}{1+\eta} \cdot \left[A-\alpha - \frac{CD+PD}{(1-\beta)}\right].$$

The pairs of (η, K) that satisfy this condition constitute Case (vi) of the proposition.

Next, suppose that (A4) is binding, that is, Firm 1 produces only high-quality units. It is easy to verify that this occurs when

$$K > \frac{1}{\eta} \cdot \left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right],$$

and that Firm 1's production level is $x_{L1}=0, x_{H1}=\eta\cdot K$. If Firm 2 produces both quality levels,

$$x_{L2} = \frac{1}{1-\beta}\cdot\left[\frac{A-\alpha}{2} - \frac{CD+PD}{2\cdot(1-\beta)}\right] - \frac{(2-\eta)}{2\cdot(1-\beta)}\cdot K,$$

$$x_{H2} = -\frac{\beta}{1-\beta}\cdot\left[\frac{A-\alpha}{2} - \frac{CD+PD}{2\cdot(1-\beta)}\right] + \frac{2-2\cdot\eta+\eta\cdot\beta}{2\cdot(1-\beta)}\cdot K.$$

As long as $x_{L2}>0$ and $x_{H2}>0$, they constitute the equilibrium output levels. This is the case when

$$\frac{1}{2-2\cdot\eta+\eta\cdot\beta}\cdot\left[A-\alpha-\frac{CD+PD}{(1-\beta)}\right] \leq K \leq \frac{1}{2-\eta}\cdot\left[A-\alpha-\frac{CD+PD}{(1-\beta)}\right].$$

The pairs of (η, K) that satisfy these two conditions, as well as $x_{L1}=0$, that is

$$K > \frac{1}{\eta} \cdot \left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right],$$

constitute Case (ii) of the proposition. Note that there exist additional pairs of (η, K) that are included in Case (ii) for parameter values where Firm 1's capacity constraint is not binding.

Next, suppose that (A6) is binding, that is, Firm 2 produces only high-quality units. It is easy to verify that this occurs when

$$K > \frac{1}{2-\eta}\cdot\left[A-\alpha-\frac{CD+PD}{(1-\beta)}\right].$$

The pairs of (η, K) that satisfy this condition constitute Case (i) of the proposition. Again, Case (i) also contains pairs of (η, K), where the capacity constraint is not binding either for Firm 1 or for both firms.

Finally, suppose that (A5) is binding, that is, Firm 2 produces only low-quality units. It is easy to verify that this occurs when

$$K < \frac{1}{2-2\cdot\eta+\eta\cdot\beta}\cdot\left[A-\alpha-\frac{CD+PD}{(1-\beta)}\right].$$

If it is also true that

$$K > \frac{\beta}{2\cdot\beta\cdot\eta+(1-\eta)}\cdot\left[A-\alpha-\frac{CD+PD}{(1-\beta)}\right]$$

(see conditions related to Case (v) above), then Firm 1 produces only high-quality units. The pairs of (η, K) that satisfy these two conditions constitute Case (iii) of the proposition.

So far, we focused on parameter values for which the capacity constraint is binding for Firm 1 (and consequently for the smaller, Firm 2). Where the capacity constraint is not binding, the cut-off between Case (ii) and Case (iii) is changed, as well as the cut-off between Case (i) and Case (ii) (see the kinks in the respective lines depicted in Figure 1). We do not present the derivation of the solution to this case (it is similar to the derivations reported above). Note that the capacity constraint might not be binding for Firm 1 only in Cases (i), (ii), and (iii), whereas for Firm 2 the capacity constraint might not be binding only in Case (i).

In Table A1, we examine the impact of small changes in allocated capacity for which the resulting equilibrium remains within the original equilibrium class. □

Proof of Proposition 2. In Table A1 we summarize the impact of small changes in η on the welfare measure for each one of the six cases described in Proposition 1. Furthermore, we note that $\partial x_{H1}/\partial\eta \geq 0$ and $\partial x_{H2}/\partial\eta \leq 0$ for all K, η. Recall, also, that in the symmetric duopoly case both firms adopt the same production plan.

Table A1 Summary of the Impact of Increasing Allocated Capacity $(\partial(\cdot)/\partial\eta)$, Within an Equilibrium Class

Variable/case	x_{H1}	x_{L1}	x_{H2}	x_{L2}	X_1	X_2	X_H	X_L	TX	AQ_1	AQ_2	AQ	P_1	P_2	Π_1	Π_2	$T\Pi$	CS	W
i	≥ 0	$=0$	≤ 0	$=0$	≥ 0	≤ 0	≤ 0	$=0$	≤ 0	$=0$	$=0$	$=0$	≥ 0	≥ 0	≥ 0	≤ 0	≥ 0	≤ 0	≤ 0
ii	≥ 0	$=0$	<0	>0	≥ 0	≤ 0	<0	>0	≥ 0	$=0$	<0	<0	≤ 0	<0	≥ 0	<0	<0	≥ 0	>0 or <0
iii	>0	$=0$	$=0$	<0	>0	<0	>0	<0	<0	$=0$	$=0$	>0	>0	>0	>0	<0	>0	<0	<0
iv	>0	<0	<0	>0	$=0$	$=0$	$=0$	$=0$	$=0$	>0	<0	$=0$	>0	<0	>0	<0	$=0$	$=0$	$=0$
v	>0	$\gtreqless 0$	$=0$	<0	>0	<0	>0	<0	<0	$\gtreqless 0$	$=0$	>0	>0	>0	>0	<0	>0	<0	<0
vi	$=0$	>0	$=0$	<0	>0	<0	$=0$	$=0$	$=0$	$=0$	$=0$	$=0$	$=0$	$=0$	>0	<0	$=0$	$=0$	$=0$

Note. Cases: (i) Both firms produce only high quality; (ii) Firm 1 produces only high quality and Firm 2 produces both low and high quality; (iii) Firm 1 produces only high quality and Firm 2 produces only low quality; (iv) Firms 1 and 2 produce both low and high quality; (v) Firm 1 produces both low and high quality and Firm 2 produces only low quality; (vi) both firms produce only low quality.

Assume that the symmetric duopoly case is such that both firms produce only low-quality units, i.e.

$$K < \frac{\beta}{1+\eta} \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right].$$

Then, the only possible scenarios as η increases are the following: (i) Both firms always produce low quality (Case (vi))—welfare never changes; Pattern (1) emerges. (ii) At some point the larger firm (Firm 1) produces both low- and high-quality units (Case (vi) then Case (v)); Pattern (1) emerges. (iii) Firm 1 switches from only low-quality units first, to both low- and high-quality units and then to only high-quality units (Cases (vi), (v), and (iii)); Pattern (1) emerges.

Next, assume in the symmetric duopoly that both firms produce only high-quality units, i.e.,

$$K > \frac{1}{2-\eta} \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right].$$

The only possible patterns are
(iv) Firm 2 first produces only high-quality units, then both high- and low-quality units, and then only low-quality units. Firm 1's capacity constraint is always binding (Cases (i), (ii), and (iii)); Pattern (2) emerges. (v) Same as (iv) but firm 1's capacity is not binding: (a) For some or all parameter values that support Case (iii), Pattern (2) emerges. (b) For all parameter values that support Case (iii), and some (but not all) parameter values that support Case (ii), Pattern (2) emerges. (c) For all parameter values that support Cases (ii) and (iii), Pattern (2) emerges.

Finally, assume that both firms produce both high and low quality under the symmetric duopoly, i.e.,

$$\frac{\beta}{2-\eta} \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right] < K < \frac{1}{2-\eta} \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right].$$

Then, the following cases could occur: (vi) Firm 2 switches to low quality only (Cases (iv) and (v)); Pattern (1) emerges. (vii) Firm 2 switches to low quality only, then Firm 1 switches to high quality only (Cases (iv), (v), and (iii)); Pattern (1) emerges. (viii) Firm 1 switches to high quality only, then Firm 2 switches to low quality only (Cases (iv), (ii), and (iii)); Pattern (2) emerges. (ix) Firm 1 switches to high quality only, then Firm 2 switches to low quality only, then Firm 1 switches back to high and low quality (Cases (iv), (ii), (iii), and (v)); Pattern (2) emerges. □

Proof of Corollary 1 to Proposition 2. Immediate; see description of welfare pattern (2) in Proposition 2. □

Proof of Corollary 2 to Proposition 2. The proof is omitted; it would be similar to the proof of Proposition 2. □

Proof of Observation 3. We first note, by observing Table A1, that the only case where the average quality of both firms can go down (with at least one strictly) at the same time that the average quality goes up is Case (v), where the large firm produces both high- and low-quality products, and the small firm produces only low-quality products. The necessary conditions for this case are

$$\frac{\beta}{1+\eta} \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right] \le K \le \frac{\beta}{2\cdot\beta\cdot\eta + (1-\eta)} \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right] \quad \text{and}$$

$$K < \frac{\beta}{1-\eta} \cdot \left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right].$$

Within Case (v), we need to specify the set of parameters where the average quality of the large firm is decreasing. The average quality of the large firm is

$$\frac{\beta}{1-\beta}\,\frac{\beta[A-\alpha-(CD+PD)/(1-\beta)] - (1+\eta)K}{(1-\eta)K - \beta[A-\alpha-(CD+PD)/(1-\beta)]}.$$

This term is decreasing in η when

$$K > \beta \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right].$$

All the conditions derived above are necessary and sufficient conditions for the observation. To get a nonempty set of parameters supporting the observation, we need

$$K < \frac{\beta}{1-\eta} \cdot \left[\frac{A-\alpha}{3} - \frac{CD+PD}{3\cdot(1-\beta)}\right] \quad \text{and}$$

$$K > \beta \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right],$$

which implies $\eta > 2/3$, as well as

$$K > \beta \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right] \quad \text{and}$$

$$K \le \frac{\beta}{2\cdot\beta\cdot\eta + (1-\eta)} \cdot \left[A - \alpha - \frac{CD+PD}{(1-\beta)}\right],$$

which implies $\beta < 0.5$. □

Proof of Observation 4. Assume that K is large enough, such that Firm 1's capacity constraint is not binding, i.e.,

$$x_{L1} = 0, \quad x_{H1} = \frac{A-\alpha}{2} - \frac{x_{L2}+x_{H2}}{2}.$$

(The case where Firm 1's capacity constraint is binding is straightforward.) Assume, furthermore that Firm 2 fully utilizes its capacity; hence, its first-order conditions imply

$$A - \alpha - x_{H1} - 2\cdot x_{L2} - 2\cdot x_{H2} - \frac{CD+PD}{1-\beta} = 0 \quad \text{and}$$

$$x_{H2} + \beta\cdot x_{L2} = (1-\eta)\cdot K.$$

Solving for x_{H2} and x_{L2}, we get

$$x_{L2} = \frac{1}{1-\beta} \cdot \left[\frac{A-\alpha}{3} - \frac{2\cdot(CD+PD)}{3\cdot(1-\beta)}\right] - \frac{(1-\eta)}{(1-\beta)}\cdot K,$$

$$x_{H2} = -\frac{\beta}{1-\beta} \cdot \left[\frac{A-\alpha}{3} - \frac{2\cdot(CD+PD)}{3\cdot(1-\beta)}\right] + \frac{(1-\eta)}{(1-\beta)}\cdot K.$$

Now, for $x_{L2} > 0$ it must be

$$(1-\eta)\cdot K < \left[\frac{A-\alpha}{3} - \frac{2\cdot(CD+PD)}{3\cdot(1-\beta)}\right].$$

It is clear that for every K there exists η sufficiently close to 1 such that this inequality holds (note the right-hand side

(RHS) is positive due to our assumption that β is sufficiently small).

Furthermore, for

$$(1-\eta)\cdot K < \beta\cdot\left[\frac{A-\alpha}{3}-\frac{2\cdot(CD+PD)}{3\cdot(1-\beta)}\right].$$

Firm 2 produces only low-quality units. Again, recall that the RHS is positive; then, for any K, there is an η sufficiently close to 1 such that the inequality holds. □

Proof of Proposition 3. It is sufficient to show that $\lambda_{01}\le\lambda_{02}$ because these are the shadow prices of the capacity constraints. From the first-order conditions we get

$$\lambda_{0i}=\frac{1}{1-\beta}[CD+PD+\lambda_{Hi}-\lambda_{Li}],\quad \text{so}$$
$$\lambda_{02}-\lambda_{01}=\frac{1}{1-\beta}[\lambda_{H2}-\lambda_{H1}+\lambda_{L1}-\lambda_{L2}].$$

It is easy to use this condition for the following cases:

Case (ii). $\lambda_{H1}=\lambda_{H2}=\lambda_{L2}=0$, $\lambda_{L1}>0$, so $\lambda_{02}-\lambda_{01}=(1/(1-\beta))\cdot\lambda_{L1}>0$.

Case (iii). $\lambda_{H1}=\lambda_{L2}=0$, $\lambda_{L1}>0$, $\lambda_{H2}>0$, so $\lambda_{02}-\lambda_{01}=(1/(1-\beta))\cdot[\lambda_{L1}+\lambda_{H2}]>0$.

Case (iv). $\lambda_{H1}=\lambda_{H2}=\lambda_{L1}=\lambda_{L2}=0$, so $\lambda_{02}-\lambda_{01}=0$.

Case (v). $\lambda_{H1}=\lambda_{L1}=\lambda_{L2}=0$, $\lambda_{H2}>0$, so $\lambda_{02}-\lambda_{01}=(1/(1-\beta))\cdot\lambda_{H2}>0$.

For all other cases we prove the result directly from the first-order conditions:

Case (i). Recall that $\lambda_{H1}=\lambda_{H2}=0$. Now, $\lambda_{01}=A-\alpha-x_{H2}-2\cdot x_{H1}$, and $\lambda_{02}=A-\alpha-x_{H1}-2\cdot x_{H2}$, so $\lambda_{02}-\lambda_{01}=x_{H1}-x_{H2}>0$, as $\eta>0.5$.

Case (vi). Recall that $\lambda_{L1}=\lambda_{L2}=0$. Then, $\beta\cdot\lambda_{01}=A-CD-PD-\alpha-x_{L2}-2\cdot x_{L1}$, and $\beta\cdot\lambda_{02}=A-CD-PD-\alpha-x_{L1}-2\cdot x_{L2}$, so $\beta\cdot(\lambda_{02}-\lambda_{01})=x_{L1}-x_{L2}>0$, as $\eta>0.5$. □

Proof of Proposition 4. When one firm sells capacity to the other total capacity remains intact. Hence, we examine total firms' profits for a given capacity level. First, consider the case where capacity is binding for both firms. Then, as reported in Table A1, total firms' profits increase in η in Cases (iii) and (v), thus the large firm will be the buyer of a small increment of capacity from the small firm. The sum of the firms' profits decreases in η in Case (ii), thus the small firm will be the buyer of a small increment of capacity from the large firm. In Cases (i), (iv), and (vi), where both firms utilize their capacity in a similar way, such capacity transfer is of no profit consequences. When the capacity constraint is not binding (for the large firm), profits are always increased when the large firm acquires capacity from the small firm. □

Proof of Proposition 5 and Corollary 1 to Proposition 5. Follow the discussion in the text, and see Footnote 30 for necessary conditions. □

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