



# Investment timing, agency, and information <sup>☆</sup>

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## Abstract

This paper provides a model of investment timing by managers in a decentralized firm in the presence of agency conflicts and information asymmetries. When investment decisions are delegated to managers, contracts must be designed to provide incentives for managers to both extend effort and truthfully reveal private information. Using a real options approach, we show that an underlying option to invest can be decomposed into two components: a manager's option and an owner's option. The implied investment behavior differs significantly from that of the first-best no-agency solution. In particular, greater inertia occurs in investment, as the model predicts that the manager will have a more valuable option to wait than the owner.

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## 1. Introduction

One of the most important topics in corporate finance is the formulation of the optimal investment strategies of firms. The investment decision has two components: how much to invest and when to invest. The first is the capital allocation decision, and the second is the investment timing decision. The standard textbook prescription for the capital allocation decision is that firms should invest in projects only if their net present values (NPVs) are positive. Similarly, a standard framework for the investment timing decision is the real options approach. The real options approach posits that the opportunity to invest in a project is analogous to an American call option on the investment project, and the timing of investment is economically equivalent to the optimal exercise decision for an option. The real options approach is well summarized in [Dixit and Pindyck \(1994\)](#) and [Trigeorgis \(1996\)](#).<sup>1</sup>

However, both the simple NPV rule and the standard real options approach fail to account for the presence of agency conflicts and information asymmetries. In most modern corporations, shareholders delegate the investment decision to managers, taking advantage of managers' special skills and expertise. In such decentralized settings, there are likely to be both information asymmetries (e.g., managers are better informed than owners about projected cash flows) and agency issues (e.g., unobserved managerial effort, perquisite consumption, empire building). A number of papers in the corporate finance literature provide models of capital budgeting under asymmetric information and agency. (See [Stein \(2001\)](#) for a useful summary.) The focus of this literature is on the first element of the investment decision: the amount of capital allocated to managers for investment. Thus, this literature provides predictions on whether firms over- or underinvest relative to the first-best no-agency benchmark. The focus of this paper is on the second element of the investment decision: the timing of investment. We extend the real options framework to account for the issues of information and agency in a decentralized firm. Analogous to the notions of over- or underinvestment, our paper provides results on hurried or delayed investment.

No agency conflicts arise in the standard real options paradigm, as it is assumed that the option's owner makes the exercise decision.<sup>2</sup> However, in this paper, an owner delegates the option exercise decision to a manager. Thus, the timing of investment is determined by the manager. The owner's problem is to design an

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<sup>1</sup>The application of the real options approach to investment is broad. [Brennan and Schwartz \(1985\)](#) use an option pricing approach to analyze investment in natural resources. [McDonald and Siegel \(1986\)](#) provide the standard continuous-time framework for analysis of a firm's investment in a single project. [Majd and Pindyck \(1987\)](#) enrich the analysis with a time-to-build feature. [Dixit \(1989\)](#) uses the real option approach to examine entry and exit from a productive activity. [Triantis and Hodder \(1990\)](#) analyze manufacturing flexibility as an option. [Titman \(1985\)](#) and [Williams \(1991\)](#) use the real options approach to analyze real estate development.

<sup>2</sup>While our paper focuses on the agency issues that arise from the divergence of interests between owners and shareholders, similar issues exist between stockholders and bondholders. [Mello and Parsons \(1992\)](#), [Mauer and Triantis \(1994\)](#), [Leland \(1998\)](#), [Mauer and Ott \(2000\)](#), and [Morellec \(2001, 2003\)](#) examine the impact of agency conflicts on firm value using the real options approach.

optimal contract under both hidden action and hidden information. The true quality of the underlying project can be high or low. The hidden action problem is that the manager can influence the likelihood that the quality of the project is high. An optimal contract will have the property that the manager will be induced to provide costly (but unverifiable) effort. The hidden information problem is that the underlying project's future value contains a component that is only privately observed by the manager. Absent any mechanism that induces the manager to reveal his private information voluntarily, the manager could have an incentive to lie about the true quality of the project and divert value for his private interests. For example, the manager could divert privately observed project value by consuming excessive perquisites, building empires, or working less hard. An optimal contract induces the manager to deliver to the owner the true value of the privately observed component of project value, and thus no actual value diversion takes place in equilibrium.

We show that the underlying option can be decomposed into two components: a manager's option and an owner's option. The manager's option has a payout upon exercise that is a function of the contingent compensation contract. Based on this contractual payout, the manager determines the exercise time. The owner's option has a payout, received at the manager's chosen exercise time, equal to the payoff from the underlying option minus the manager's compensation. The model provides the solution for the optimal contract that comes as close as possible to the first-best no-agency solution.

The model implies investment behavior that differs substantially from that of the standard real options approach with no agency problems. In general, managers display greater inertia in their investment behavior, in that they invest later than implied by the first-best solution. In essence, this is a result of the manager (even in an optimal contract) not having a full ownership stake in the option payoff. This less than full ownership interest implies that the manager has a more valuable option to wait than the owner.

An important aspect of the model is the interaction of hidden action and hidden information. We find that the nature of the optimal contract depends explicitly on the relative importance of these two forces. While we focus on the economically most interesting case in which both forces play a role in the optimal contract, it is instructive to consider two extremes. If the cost–benefit ratio of inducing effort (a measure of the strength of the hidden effort component) is very low, then the hidden action component disappears from the optimal contract terms. Thus, if the nature of the underlying option is such that inducing effort is sufficiently inexpensive, then a simple problem of hidden information is left and the contract simply rewards the manager with information rents. This is similar to the setting of Maeland (2002), which considers a real options problem with only hidden information about the exercise cost.<sup>3</sup> Conversely, as the cost–benefit ratio of inducing effort becomes very

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<sup>3</sup>Bjerk Sund and Stensland (2000) provide a similar model to Maeland (2002), in which a principal delegates an investment decision to an agent who holds private information about the investment's cost. Brennan (1990) considers a setting in which managers attempt to signal the true quality of latent assets to investors through converting them into observable assets (e.g., exercising real options).

high, then the hidden action component dominates the optimal contract. The cost of inducing effort is so high as to no longer necessitate the payment of information rents. When the cost–benefit ratio of inducing effort is in the intermediate range, both forces are in effect, and the optimal contract must induce both effort and truthful revelation of private information. The interplay between hidden information and hidden action could reduce the inefficiency in investment timing, compared with the setting in which hidden information is the only friction. This is because the manager’s additional option to exert effort makes his incentives more closely aligned with those of the owner.

We further generalize the model to allow managers to display greater impatience than owners. Several potential justifications exist for such an assumption. First, various models of managerial myopia attempt to explain managers’ preferences for choosing projects with quicker paybacks, even in the face of eschewing more valuable long-term opportunities. (See Narayanan, 1985; Stein, 1989; Bebchuk and Stole, 1993.) Such models are based on information asymmetries and agency problems. Second, in our investment timing setting, greater impatience can represent the manager’s preference for empire building or greater perquisite consumption and reputation that comes from running a larger company sooner rather than later. Third, managers could have shorter horizons (because of job loss, alternative job offers, death, etc.). Phrased in real options terms, managerial impatience decreases the value of the manager’s option to wait. While the base case model predicts that investment will never occur sooner than the first-best case, in this generalized setting investment can occur either earlier or later than the first-best case.

The setting of our paper is most similar to that of Bernardo et al. (2001). In a decentralized firm under asymmetric information and moral hazard, they examine the capital allocation decision, while we examine the investment timing decision. In their model, the firm’s headquarters delegates the investment decision to a manager, who possesses private information about project quality. The manager can improve project quality through the exertion of effort, which is costly to the manager but unverifiable by headquarters. These two assumptions mirror our framework. In addition, managers have preferences for empire building in that they derive utility from overseeing large investment projects. This assumption is addressed in the generalized version of our model that appears in Section 3.3. Absent any explicit incentive mechanism, managers always claim that all projects are of high quality and worthy of funding, and then they provide the minimal amount of effort. As in our paper, they use an optimal contracting approach to jointly derive the optimal investment and compensation policies. An incentive contract is derived that induces truth-telling and minimizes agency costs. In equilibrium, they find that there will be underinvestment in all states of the world. Our model provides an intertemporal analogy to their equilibrium: in our base case model, we find that in equilibrium there is delayed investment as a result of the information asymmetries and agency costs.<sup>4</sup>

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<sup>4</sup>In a different setting, Holmstrom and Ricart i Costa (1986) provide a model that combines an optimal wage contract with capital rationing. In their model, the manager and the market learn about managerial

While our paper derives an optimal contract that best aligns the incentives of owners and managers, other papers in the corporate finance literature analyze the capital budgeting problem under information asymmetry and agency using other control mechanisms. Harris et al. (1982) consider the case of capital allocation in a decentralized firm with multiple division managers. Managers have private information about project values. In addition, managers have private interests in overstating investment requirements, and then diverting the excess cash flows to minimize effort or to consume greater perquisites. They focus on the role of transfer prices in allocating capital. Firms offer managers a menu of allocation/transfer price combinations. In equilibrium, truth-telling is achieved, and there can be both under- and overinvestment. (Antle and Eppen (1985) provide a model that is similar to that of Harris et al., 1982.) Harris and Raviv (1996) use a similar framework, but focus on a random auditing technology. By combining probabilistic auditing with a capital restriction, headquarters is able to learn the true project quality from the manager. In equilibrium there will be both regions of under- and overinvestment. Stulz (1990) considers a decentralized investment framework in which the manager has private information about investment quality and a preference for empire building. Absent any controls, the manager would always overstate the investment opportunities and invest all available cash. The owners of the firm use debt as a mechanism to align the interests of managers and shareholders. By increasing the required debt payment, managers have less free cash flow to spend on investment projects. The optimal level of debt is chosen to trade off the benefits of preventing managers from investing in negative NPV projects when investment opportunities are poor with the costs of rationing managers away from taking positive NPV projects when investment opportunities are good. Again, in equilibrium there will be both under- and overinvestment.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model. Section 3 simplifies the optimization program and solves for the optimal contracts. In Section 4, we analyze the implications of the model in terms of the stock price's reaction to investment, equilibrium investment lags, and erosion of the option value stemming from the agency problem. Section 5 generalizes the model to allow for managers to display greater impatience than owners. Section 6 concludes. The appendix contains the solution details of the optimal contracts.

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*(footnote continued)*

talent over time by observing investment outcomes. A conflict of interest arises because the manager wants to choose investment to maximize the value of his human capital while the shareholders want to maximize firm value. The optimal wage contract has the option feature that ensures the manager against the possibility that an investment reveals his ability to be of low quality, but allows the manager to capture the gains if he is revealed to be of high quality. This option feature of the wage contract encourages the manager to take on excessive risks. Rationing capital mitigates the manager's incentive to overinvest. As a result, in equilibrium both under- and overinvestment are possible.

## 2. Model

In this section, we begin with a description of the model. We then, as a useful benchmark, provide the solution to the first-best no-agency investment problem. Finally, we present the full principal-agent optimization problem faced by the owner.

### 2.1. Setup

The principal owns an option to invest in a single project. We assume that the principal (owner) delegates the exercise decision to an agent (manager). Once investment takes place, the project generates two sources of value. One portion is observable and contractible to both the owner and the manager, while the other portion is privately observed only by the manager. Let  $P(t)$  represent the observable component of the project's value and  $\theta$  the value of the privately observed component. Thus, the total value of the project is  $P(t) + \theta$ .<sup>5</sup>

In a standard call option setting, exercise yields the difference between the observable value  $P(t)$  of the underlying asset and the exercise price,  $K$ . Thus, the payoff from exercise is typically  $P(t) - K$ . However, in the present model, the payoff from exercise also includes a privately observed random variable,  $\theta$ , whose realization directly impacts the option payoff. Thus, in this model the net payoff from exercise is  $P(t) + \theta - K$ . The problem could be equivalently formulated as one in which the total value of the project is  $P(t)$  and the effective cost of exercising the option is  $K - \theta$ .

Let the value  $P(t)$  of the observable component of the underlying project evolve as a geometric Brownian motion:

$$dP(t) = \alpha P(t) dt + \sigma P(t) dz(t), \quad (1)$$

where  $\alpha$  is the instantaneous conditional expected percentage change in  $P(t)$  per unit time,  $\sigma$  is the instantaneous conditional standard deviation per unit time, and  $dz$  is the increment of a standard Wiener process. Let  $P_0$  equal the value of the project at time zero, in that  $P_0 = P(0)$ . Both the owner and the manager are risk neutral, with the risk-free rate of interest denoted by  $r$ . (We rule out the time-zero selling-the-firm contract between the owner and the manager. This could be justified, for example, if the manager is liquidity constrained and cannot obtain financing.) For convergence, we assume that  $r > \alpha$ .

The assumption that a portion of project value is observed only by the manager and not verifiable by the owner is common in the capital budgeting literature. This information asymmetry invites a host of agency issues. Harris et al. (1982) posit that managers have incentives to understate project payoffs and to divert the free cash

<sup>5</sup>For ease of presentation, we model the process  $P(t)$  for the present value of observable cash flows. We could back up a step and begin with an underlying process for observable cash flows. However, if observable cash flows follow a geometric Brownian motion, then the present value of expected future observable cash flows will also follow a geometric Brownian motion. Similarly, instead of modeling  $\theta$  as the present value of unobservable cash flows, we could begin with an underlying process for the unobservable cash flows themselves.

flow to themselves. In their model, such value diversion takes the form of managers reducing their level of effort. [Stulz \(1990\)](#), [Harris and Raviv \(1996\)](#), and [Bernardo et al. \(2001\)](#) model managers as having preferences for perquisite consumption or empire building. In these models, managers have incentives to divert free cash flows to inefficient investments or to excessive perquisites. In all of these models, mechanisms are used by firms (i.e., incentive contracts, auditing, required debt payments) to mitigate such value diversion.

The private component of value,  $\theta$ , could take on two possible values:  $\theta_1$  or  $\theta_2$ , with  $\theta_1 > \theta_2$ .<sup>6</sup> We denote  $\Delta\theta = \theta_1 - \theta_2 > 0$ . One could interpret a draw of  $\theta_1$  as a higher quality project and a draw of  $\theta_2$  as a lower quality project. Although the owner cannot observe the true value of  $\theta$ , the owner does observe the amount handed over by the manager upon exercise. While in theory the manager could attempt to hand over  $\theta_2$  when the true value is  $\theta_1$ , in equilibrium the amount transferred to the owner at exercise is always the true value. (Off the equilibrium path, the manager could attempt to hand over  $\theta_2$  when the true value is  $\theta_1$ . If the transferred value is less than  $\theta_1$  at the trigger  $P_1$ , a nonpecuniary penalty is imposed on the manager. This penalty will ensure that it will never be in the manager's interest not to hand over the true value of the project upon exercise.)

The effort of the manager plays an important role in determining the likelihood of obtaining a higher quality project. The manager could affect the likelihood of drawing  $\theta_1$  by exerting a one-time effort, at time zero. If the manager exerts no effort, the probability of drawing a higher quality project  $\theta_1$  equals  $q_L$ . (Without loss of generality, we could normalize the manager's lower effort level to zero.) However, if the manager exerts effort, he incurs a cost  $\xi > 0$  at time zero, but increases the likelihood of drawing a higher quality project  $\theta_1$  from  $q_L$  to  $q_H$ . Immediately after his exerting effort at time zero, the manager observes the private component of project quality. To ensure a positive net exercise price, we restrict  $\theta_1 < K$ .

Although the owner cannot contract on the private component of value,  $\theta$ , he can contract on the observable component of value,  $P(t)$ . Contingent on the level of  $P(t)$  at exercise, the manager is paid a wage. (Wages here are payments contingent on the project's quality. They are analogous to a payment scheme in which a fixed wage is paid to the manager for exercising, plus a bonus for delivering a higher quality project.) The manager has limited liability and is always free to walk away. (The limited-liability condition is essential in delivering the investment inefficiency result in this context. Otherwise, with risk-neutrality assumptions for both the owner and the manager and no limited liability, the first-best optimal investment timing could be achieved even in the presence of hidden information and hidden action. For a related discussion of limited liability, see [Innes \(1990\)](#). An alternative mechanism of generating investment inefficiency in an agency context is to assume managerial risk aversion.)

In summary, the owner faces a problem with both hidden information (the owner does not observe the true realization of  $\theta$ ) and hidden action (the owner cannot

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<sup>6</sup>In Section 3.3 we generalize the model to allow  $\theta$  to have continuous distributions.

verify the manager’s effort level). The owner needs to provide compensation incentive both to induce the manager exert effort at time zero and to have the manager reveal his type voluntarily and truthfully, by choosing the equilibrium exercise strategy and supplying the corresponding unobservable component of firm value. Before analyzing the optimal contract, we first briefly review the first-best no-agency solution used as the benchmark.

2.2. First-best benchmark (the standard real options case)

As a benchmark, we consider the case in which there is no delegation of the exercise decision and the owner observes the true value of  $\theta$ . Equivalently, this first-best solution can be achieved in a principal-agent setting, provided that  $\theta$  is both publicly observable and contractible. Let  $W(P; \theta)$  denote the value of the owner’s option, in a world where  $\theta$  is a known parameter and  $P$  is the current level of  $P(t)$ . Using standard arguments (i.e., Dixit and Pindyck, 1994),  $W(P; \theta)$  must solve the differential equation:

$$0 = \frac{1}{2} \sigma^2 P^2 W_{PP} + \alpha P W_P - r W. \tag{2}$$

Eq. (2) must be solved subject to appropriate boundary conditions. These boundary conditions serve to ensure that an optimal exercise strategy is chosen as

$$W(P^*(\theta), \theta) = P^*(\theta) + \theta - K, \tag{3}$$

$$W_P(P^*(\theta), \theta) = 1 \tag{4}$$

and

$$W(0, \theta) = 0. \tag{5}$$

Here,  $P^*(\theta)$  is the value of  $P(t)$  that triggers entry. The first boundary condition is the value-matching condition. It simply states that at the moment the option is exercised, the payoff is  $P^*(\theta) + \theta - K$ . The second boundary condition is the smooth-pasting or high-contact condition. (See Merton (1973) for a discussion of the high-contact condition.) This condition ensures that the exercise trigger is chosen so as to maximize the value of the option. The third boundary condition reflects the fact that zero is an absorbing barrier for  $P(t)$ .

The owner’s option value at time zero,  $W(P_0; \theta)$ , and the exercise trigger  $P^*(\theta)$  are

$$W(P_0; \theta) = \begin{cases} \left(\frac{P_0}{P^*(\theta)}\right)^\beta (P^*(\theta) + \theta - K) & \text{for } P_0 < P^*(\theta), \\ P_0 + \theta - K & \text{for } P_0 \geq P^*(\theta) \end{cases} \tag{6}$$

and

$$P^*(\theta) = \frac{\beta}{\beta - 1} (K - \theta), \tag{7}$$

where

$$\beta = \frac{1}{\sigma^2} \left[ -\left(\alpha - \frac{\sigma^2}{2}\right) + \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2} \right] > 1. \tag{8}$$

Because the realized value of  $\theta$  can be either  $\theta_1$  or  $\theta_2$ , we denote  $P^*(\theta_1) = P_1^*$  and  $P^*(\theta_2) = P_2^*$ . We always assume that the initial value of the project is less than the lower trigger,  $P_0 < P_1^*$ , to ensure some positive option value inherent in the project.

The ex ante value of the owner’s option in the first-best no-agency setting is  $q_H W(P_0; \theta_1) + (1 - q_H) W(P_0; \theta_2)$ . We can therefore write this first-best option value,  $V^*(P_0)$ , as

$$V^*(P_0) = q_H \left(\frac{P_0}{P_1^*}\right)^\beta (P_1^* + \theta_1 - K) + (1 - q_H) \left(\frac{P_0}{P_2^*}\right)^\beta (P_2^* + \theta_2 - K). \tag{9}$$

It will prove useful in future calculations to define the present value of one dollar received at the first moment that a specified trigger  $\hat{P}$  is reached. Denote this present value operator by the discount function  $D(P_0; \hat{P})$ . This is simply the solution to Eq. (2) subject to the boundary conditions that  $D(\hat{P}; \hat{P}) = 1$  and  $D(0; \hat{P}) = 0$ . The solution can be written as

$$D(P_0; \hat{P}) = \left(\frac{P_0}{\hat{P}}\right)^\beta, \quad P_0 \leq \hat{P}. \tag{10}$$

### 2.3. A principal-agent setting

The owner offers the manager a contract at time zero that commits the owner to pay the manager at the time of exercise. (Renegotiation is not allowed. While commitment leads to inefficiency in investment timing ex post, it increases the value of the project ex ante.) The payment can be made contingent on the observable component of the project’s value at the time of exercise. Thus, in principle, for any realized value of  $P(t)$  obtained at the time of exercise,  $\hat{P}$ , a contracted wage  $w(\hat{P})$  can be specified, provided that  $w(\hat{P}) > 0$ . The contract will endogenously provide incentives to ensure that the manager exercises the option in accordance with the owner’s rational expectations and delivers the true value of the project to the owner.

The principal-agent setting leads to a decomposition of the underlying option into two options: an owner’s option and a manager’s option. The owner’s option has a payoff function of  $\hat{P} + \theta - K - w(\hat{P})$ , and the manager’s option has a payoff function of  $w(\hat{P})$ . Upon exercise, the owner receives the value of the underlying project ( $\hat{P} + \theta$ ), after paying the exercise price ( $K$ ) and the manager’s wage ( $w(\hat{P})$ ). The manager’s payoff is the value of the contingent wage,  $w(\hat{P})$ . The sum of these payoff functions equals the payoff of the underlying option. The manager’s option is a traditional American call option, because the manager chooses the exercise time to maximize the value of his option. However, in this optimal contracting setting, the

owner sets the contract parameters that induce the manager to follow an exercise policy that maximizes the value of the owner’s option. In addition, the manager possesses a compound option, because the manager has the option to exert effort at time zero to increase the total expected surplus. The properties of the manager’s option thus are contingent upon this initial effort choice.

Given that  $\theta$  has only two possible values, for any  $w(\hat{P})$  schedule, at most two wage/exercise trigger pairs are chosen by the manager. (We allow for the possibility of a pooling equilibrium in which only one wage/exercise trigger pair is offered. However, this pooling equilibrium is always dominated by a separating equilibrium with two wage/exercise trigger pairs.) Thus, the contract need only include two wage/exercise trigger pairs from which the manager can choose: one chosen by a manager when he observes  $\theta_1$ , and one chosen by a manager when he observes  $\theta_2$ . Therefore, the owner offers a contract that promises a wage of  $w_1$  if the option is exercised at  $P_1$  and a wage of  $w_2$  if the option is exercised at  $P_2$ . The revelation principle ensures that a manager who privately observes  $\theta_1$  exercises at the  $P_1$  trigger, and a manager who privately observes  $\theta_2$  exercises at the  $P_2$  trigger.

The owner’s option has a payout function of  $P_1 + \theta_1 - K - w_1$ , if  $\theta = \theta_1$ , and  $P_2 + \theta_2 - K - w_2$ , if  $\theta = \theta_2$ . Thus, using the discounting function  $D(\cdot; \cdot)$  derived in Eq. (10), conditional on the manager exerting effort, the value of the owner’s option,  $\pi^o(P_0; w_1, w_2, P_1, P_2)$ , can be written as

$$\pi^o(P_0; w_1, w_2, P_1, P_2) = q_H D(P_0; P_1)(P_1 + \theta_1 - K - w_1) + (1 - q_H) D(P_0; P_2)(P_2 + \theta_2 - K - w_2). \tag{11}$$

The manager’s option has a payout function of  $w_1$  if  $\theta = \theta_1$  and  $w_2$  if  $\theta = \theta_2$ . Conditional on the manager exerting effort, the value of the manager’s option,  $\pi^m(P_0; w_1, w_2, P_1, P_2)$ , can be written as

$$\pi^m(P_0; w_1, w_2, P_1, P_2) = q_H D(P_0; P_1)w_1 + (1 - q_H) D(P_0; P_2)w_2. \tag{12}$$

For notational simplicity, we sometimes drop the parameter arguments and write the owner’s and manager’s option values as  $\pi^o(P_0)$ , and  $\pi^m(P_0)$ , respectively.

The owner’s objective is to maximize its option value through its choice of the contract terms  $w_1$ ,  $w_2$ ,  $P_1$ , and  $P_2$ . Thus, the owner solves the optimization problem

$$\max_{w_1, w_2, P_1, P_2} q_H \left(\frac{P_0}{P_1}\right)^\beta (P_1 + \theta_1 - K - w_1) + (1 - q_H) \left(\frac{P_0}{P_2}\right)^\beta (P_2 + \theta_2 - K - w_2). \tag{13}$$

This optimization is subject to a variety of constraints induced by the hidden information and hidden action of the manager. The contract must induce the manager to exert effort, exercise at the trigger  $P_1$  and provide the owner with a project value of  $P_1 + \theta_1$  if  $\theta = \theta_1$ , and exercise at the trigger  $P_2$  and provide the owner with a project value of  $P_2 + \theta_2$  if  $\theta = \theta_2$ .

There are both ex ante and ex post constraints. The ex ante constraints ensure that the manager exerts effort and that the contract is accepted. These are the standard constraints as in a static moral hazard/asymmetric information setting.

The ex ante incentive constraint is

$$q_H \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_H) \left( \frac{P_0}{P_2} \right)^\beta w_2 - \xi \geq q_L \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_L) \left( \frac{P_0}{P_2} \right)^\beta w_2. \quad (14)$$

The left side of this inequality is the value of the manager's option if effort is exerted minus the cost of effort. The right side is the value of the manager's option if no effort is exerted. This constraint ensures that the manager will exert effort. Rearranging the ex ante incentive constraint Eq. (14) gives

$$\left( \frac{P_0}{P_1} \right)^\beta w_1 - \left( \frac{P_0}{P_2} \right)^\beta w_2 \geq \frac{\xi}{\Delta q}, \quad (15)$$

where  $\Delta q = q_H - q_L > 0$ .

The ex ante participation constraint is

$$q_H \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_H) \left( \frac{P_0}{P_2} \right)^\beta w_2 - \xi \geq 0. \quad (16)$$

This constraint ensures that the total value to the manager of accepting the contract is non-negative.

The ex post incentive constraints ensure that managers exercise in accordance with the owner's expectations. Specifically, managers exercise  $\theta_1$ -type projects at the  $P_1$  trigger and exercise  $\theta_2$ -type projects at the  $P_2$  trigger. To provide such a timing incentive, managers must not have any incentive to divert value. As discussed at the beginning of Section 2.1, managers with private information have an incentive to misrepresent cash flows and divert free cash flows to themselves. For example, the manager could have an incentive to lie and claim that a higher quality project is a lower quality project and then divert the difference in values. This could be done by diverting cash for private benefits such as perquisites and empire building (as in Stulz, 1990; Harris and Raviv, 1996; Bernardo et al., 2001). These incentive compatibility conditions ensure that this value diversion does not occur; such deception only occurs off the equilibrium path.

The ex post incentive constraints are

$$\left( \frac{P_0}{P_1} \right)^\beta w_1 \geq \left( \frac{P_0}{P_2} \right)^\beta (w_2 + \Delta\theta) \quad (17)$$

and

$$\left( \frac{P_0}{P_1} \right)^\beta (w_1 - \Delta\theta) \leq \left( \frac{P_0}{P_2} \right)^\beta w_2. \quad (18)$$

The second constraint is shown not to bind, so only constraint Eq. (17) is relevant to our discussion. The first inequality ensures that a manager of a higher quality project chooses to exercise at  $P_1$ . By truthfully revealing the private quality  $\theta_1$  through exercising at  $P_1$ , the manager receives the wage  $w_1$ . This inequality requires the payoff from truthful revelation to be greater than or equal to the present value of the payoff from misrepresenting the private quality by waiting until the trigger  $P_2$ . The

payoff from misrepresenting  $\theta_1$  as  $\theta_2$  is equal to the wage  $w_2$ , plus the value of diverting the private component of value  $\Delta\theta$ . These constraints are common in the literature on moral hazard and asymmetric information. For example, entirely analogous conditions appear in Bolton and Scharfstein (1990) and Harris et al. (1982).

While the two ex post incentive constraints ensure that the manager exercises in accordance with the owner's rational expectations, we also need to ensure that a manager of a  $\theta_1$ -type project will hand over  $P_1 + \theta_1$  in value and not divert the unobservable amount  $\Delta\theta$  of the project's value. (There is no need to worry about the opposite problem of a manager of a  $\theta_2$ -type project exercising at  $P_2$  and handing over  $P_2 + \theta_1$ , because that would never be in the manager's interest.) We assume that a nonpecuniary penalty of  $\kappa$  can be imposed on a manager who fails to deliver  $P_1 + \theta_1$  at the trigger  $P_1$ . (For nonpecuniary penalties and optimal contracting, see the seminal contribution of Diamond, 1984.) Specifically, we assume that the penalty,  $\kappa$ , is large enough to satisfy the condition  $\kappa \geq \Delta\theta - w_1$ . Thus, when the manager with a high quality project exercises at  $P_1$ , it is in their interest to deliver a value of  $P_1 + \theta_1$  and receive  $w_1$  instead of delivering only  $P_1 + \theta_2$  and receiving the penalty  $\kappa$ . (A manager could never transfer a value of  $\theta < \theta_2$ , because it is common knowledge that  $\theta_2$  is the lower bound of the distribution of  $\theta$ . See Bolton and Scharfstein (1990) for similar assumptions and justifications.) Such a penalty could be envisioned as a reputational penalty (i.e., managers who fail to deliver what they promise are given poor recommendations) or a job search cost (i.e., such managers are terminated and forced to seek new employment). (An alternative mechanism for ensuring ex post enforceability of the manager's claim is through a costly state verification mechanism as in Townsend (1979) and Gale and Hellwig (1985). Specifically, the owner could possess a monitoring technology that permits, at a cost, the determination of the true value of  $\theta$  after investment is undertaken. Provided that the cost is not too high, it can be easily shown that the owner would always choose to pay the monitoring cost for managers who signal high-quality projects and only hand over  $\theta_2$  in value.) Without such a penalty, any kind of contracting solution would likely break down because the manager would not have to live up to his claims.

The ex post limited-liability constraints are

$$w_i \geq 0, \quad i = 1, 2. \quad (19)$$

Non-negative  $w_1$  and  $w_2$  are necessary to provide an incentive for the manager to implement the exercise of the project. For example, if  $w_2 < 0$ , then upon learning that  $\theta = \theta_2$ , the manager would rather walk away from the contract than sticking around and receive a negative wage at  $P_2$ .<sup>7</sup> It is assumed that if the manager walks away, the investment opportunity is lost. Thus, the owner ensures that the manager has an incentive to invest ex post.

<sup>7</sup>Even if the manager decided to try to fool the owner by exercising at  $P_1$ , the net payout to the manager would be  $w_1 - \Delta\theta < 0$ , where this inequality is displayed in Proposition 4.

Therefore, the owner’s problem can be summarized as the solution to the objective function in Eq. (13), subject to a total of six inequality constraints: the ex ante incentive and participation constraints, and each of the two ex post incentive and limited-liability constraints. The problem can be substantially simplified in that we can reduce the number of constraints to two.

### 3. Model solution: optimal contracts

In this section, we provide the solution to the optimal contracting problem described in the Section 2: maximizing Eq. (13) subject to the six inequality constraints Eqs. (15)–(19). The nature of the solution depends on the parameter values. In particular, the solution depends explicitly on the magnitude of the cost–benefit ratio of inducing the manager’s effort. Depending on this magnitude, the optimal contract can take on three possible types: a pure hidden information type, a joint hidden information/hidden action type, and a pure hidden action type.

#### 3.1. A simplified statement of the principal-agent problem

Although the owner’s optimization problem is subject to six inequality constraints, the solution can be found through considering only two of the constraints. Appendix A proves four propositions, Proposition 1–4, that provide the underpinnings for this simplification.

Proposition 1 shows that the limited liability for the manager of a  $\theta_1$ -type project in constraint Eq. (19) does not bind, while Proposition 2 shows that the ex ante participation constraint Eq. (16) does not bind. Proposition 3 demonstrates that the limited liability for the manager of a  $\theta_2$ -type project binds, and thus we can substitute  $w_2 = 0$  into the problem. Proposition 4 implies that the ex post incentive constraint for the manager of a  $\theta_2$ -type project does not bind.

These four propositions jointly simplify the owner’s optimization problem as

$$\max_{w_1, P_1, P_2} q_H \left(\frac{P_0}{P_1}\right)^\beta (P_1 + \theta_1 - K) - q_H \left(\frac{P_0}{P_1}\right)^\beta w_1 + (1 - q_H) \left(\frac{P_0}{P_2}\right)^\beta (P_2 + \theta_2 - K), \tag{20}$$

subject to

$$\left(\frac{P_0}{P_1}\right)^\beta w_1 \geq \left(\frac{P_0}{P_2}\right)^\beta \Delta\theta \tag{21}$$

and

$$\left(\frac{P_0}{P_1}\right)^\beta w_1 \geq \frac{\xi}{\Delta q}. \tag{22}$$

In summary, we now have a simplified optimization problem for the owner. Eq. (20) is the owner’s option value. Constraint Eq. (21) is the simplified ex post incentive

constraint for the manager of the  $\theta_1$ -type project. Constraint Eq. (22) ensures that it is in the manager’s interest to extend his effort at time zero.

Proposition 5, proved in Appendix A, demonstrates that at least one of the two constraints must bind. The two constraints can be written more succinctly as

$$\left(\frac{P_0}{P_1}\right)^\beta w_1 \geq \max \left[ \left(\frac{P_0}{P_2}\right)^\beta \Delta\theta, \frac{\zeta}{\Delta q} \right]. \tag{23}$$

### 3.2. General properties of the solution

Before we provide the explicit solutions for the three contract regions, we discuss some general properties of contracts that hold for all regions.

The first property of the solution is that the manager of the higher quality project exercises at the first-best level. Intuitively, for any manager’s option value that satisfies constraint Eq. (23), the owner always prefers to choose the first-best timing trigger  $P_1^*$  and vary wage  $w_1$  to achieve the same level of compensation. On the margin, it is cheaper for the owner to increase the wage for the manager of a higher quality project than to have that manager deviate away from the first-best optimal timing strategy.

**Property 1.** *The optimal contracts have  $P_1 = P_1^*$ , for all admissible parameter regions.*

**Proof.** Consider any candidate optimal contract  $(\bar{w}_1, \bar{P}_1, \bar{P}_2)$  with  $\bar{P}_1 \neq P_1^*$ . The owner could improve his surplus by proposing an alternative contract  $(\hat{w}_1, P_1^*, \bar{P}_2)$ , in which  $\hat{w}_1$  is chosen such that the manager’s option has the same value as the first contract, in that  $(P_0/P_1^*)^\beta \hat{w}_1 = (P_0/\bar{P}_1)^\beta \bar{w}_1$ . The newly proposed contract is clearly feasible, as it will also satisfy constraints Eqs. (21) and (22). For all such constant levels of the manager’s option value, the owner’s objective function (20) is maximized by choosing  $P_1 = P_1^* = \operatorname{argmax}_x (P_0/x)^\beta (x + \theta_1 - K)$ .  $\square$

It is less costly for the owner to distort  $P_2$  away from the first-best level than to distort  $P_1$  away from the first-best level to provide the appropriate incentives to the manager. The next property of the solutions is that delay (beyond first-best) for the lower quality project is needed to create enough incentives for the manager of a higher quality project not to imitate the one of a lower quality project.

**Property 2.** *For all admissible parameter regions, the investment trigger for a manager of a  $\theta_2$ -type project is (weakly) later than the first-best, in that  $P_2 \geq P_2^*$ .*

**Proof.** Suppose  $P_2 < P_2^*$ . This contract is dominated by the contract with  $P_2 = P_2^*$ .  $P_2$  can always be increased without violating constraint Eq. (21). Moreover, the objective function Eq. (20) is increasing in  $P_2$ , for  $P_2 < P_2^*$ , irrespective of which constraint binds. Thus, any contract with  $P_2 < P_2^*$  is dominated by one with  $P_2 = P_2^*$ .  $\square$

Intuitively, the necessity of ensuring that the manager of a higher quality project not imitate one of a lower quality project leads the manager of a lower quality project to display a greater option to wait than the first-best solution. To dissuade the manager of a higher quality project from exercising at the trigger  $P_2$ , the contract must sufficiently increase  $P_2$  above  $P_2^*$  to make such lying unprofitable.

The extent to which  $P_2$  exceeds  $P_2^*$  depends explicitly on the relative strengths of the forces of hidden information and hidden action. The amount of suboptimal delay varies across the three regions.

### 3.3. Optimal contracts

We first define the three regions that serve to determine the nature of the optimal contract. As a result of Proposition 5, the solution depends on which of the two constraints Eqs. (21) and (22) bind. The key to the contract is the cost–benefit ratio of inducing the manager’s effort, defined by  $\xi/\Delta q$ . The numerator is the direct cost of extending effort, and the denominator is the change in the likelihood of drawing a higher quality project  $\theta_1$  as a result of effort. The regions are then defined by where this cost–benefit ratio falls relative to the present value of receiving a payment of  $\Delta\theta$  at three particular trigger values:  $P_1^* = P^*(\theta_1)$ ,  $P_2^* = P^*(\theta_2)$ , and  $P_3^* = P^*(\theta_3)$ , where

$$\theta_3 = \theta_2 - \frac{q_H}{1 - q_H} \Delta\theta < \theta_2. \quad (24)$$

These present values are ordered by  $(P_0/P_3^*)^\beta \Delta\theta < (P_0/P_2^*)^\beta \Delta\theta < (P_0/P_1^*)^\beta \Delta\theta$ . Another region exists in which  $\xi/\Delta q > (P_0/P_1^*)^\beta \Delta\theta$ , however in this range the costs of effort are so high as to no longer justify the exertion of effort in equilibrium. Thus, we do not consider this region.<sup>8</sup>

Because optimal contracts specify  $P_1 = P_1^*$  and  $w_2 = 0$  across all three regions, we may focus on  $P_2$  and  $w_1$  when we describe the optimal contracts in each of the three regions. The proofs detailing the solution are provided in Appendix A.

*Hidden information only region:*  $\xi/\Delta q < (P_0/P_3^*)^\beta \Delta\theta$

In this region, we have

$$P_2 = P_3^* = P^*(\theta_3) > P_2^* \quad (25)$$

and

$$w_1 = \left( \frac{P_1^*}{P_3^*} \right)^\beta \Delta\theta, \quad (26)$$

where  $\theta_3$  is given in Eq. (24).

The net costs of inducing effort are low enough so that the firm has no need to compensate the manager for extending effort. In this range, the ex ante incentive constraint does not bind, and therefore the cost of effort does not find its way into the optimal contract. (In a different setting where the hidden information is the cost of exercising, [Maeland \(2002\)](#) shows a similar result.) The payments that the

<sup>8</sup>A proof of this result is available from the authors by request.

manager of the  $\theta_1$ -type project receives are purely information rents that induce the manager to exercise at the first-best trigger  $P_1^*$ , in accordance with the revelation principle. Because  $w_1$  is relatively low in this region, the  $P_2$  trigger needs to be high (relative to the first-best trigger  $P_2^*$ ) to dissuade the manager of the  $\theta_1$ -type project from deviating from the equilibrium first-best trigger  $P_1^*$ .

We can use these contract terms to place a value on the owner’s and manager’s option values. The owner’s and manager’s option values,  $\pi^o(P_0)$  and  $\pi^m(P_0)$ , respectively, can be written as

$$\pi^o(P_0) = q_H \left(\frac{P_0}{P_1^*}\right)^\beta (P_1^* + \theta_1 - K) + (1 - q_H) \left(\frac{P_0}{P_3^*}\right)^\beta (P_3^* + \theta_3 - K) \tag{27}$$

and

$$\pi^m(P_0) = q_H \left(\frac{P_0}{P_3^*}\right)^\beta \Delta\theta. \tag{28}$$

The solution for the owner’s option value is observationally equivalent to the first-best solution in which one substitutes  $\theta_3$  for the lower project quality  $\theta_2$ . In such a setting, the owner will choose to exercise at  $P_1^*$  if  $\theta = \theta_1$  and at  $P_3^*$  if  $\theta = \theta_3$ . Thus, the impact of the costs of hidden information is fully embodied by a reduction of project quality in the low state.

*Joint hidden information/hidden action region:*  $(P_0/P_3^*)^\beta \Delta\theta \leq \xi/\Delta q \leq (P_0/P_2^*)^\beta \Delta\theta$

In this region, we have

$$P_2 = P_J = P_0 \left(\frac{\Delta q \Delta\theta}{\xi}\right)^{1/\beta} > P_2^* \tag{29}$$

and

$$w_1 = \left(\frac{P_1^*}{P_J}\right)^\beta \Delta\theta = \frac{\xi}{\Delta q} \left(\frac{P_1^*}{P_0}\right)^\beta. \tag{30}$$

Here, both the ex ante and ex post constraints bind. Given that now the manager must be induced into providing effort,  $w_1$  must be high enough to provide enough compensation for the ex ante incentive constraint Eq. (22) to bind. This reflects the hidden action component of the contract. In addition, the exercise trigger  $P_2$  must be high enough to dissuade the manager of the  $\theta_1$ -type project from deviating from the equilibrium first-best trigger  $P_1^*$ . Thus, in this region,  $P_2$  is set so that the ex post incentive constraint Eq. (21) binds. This requires that  $P_2$  be above the full-information trigger  $P_2^*$ . This deviation from the full-information trigger reflects the hidden information component of the contract.

$P_2$  is lower in this region than in the hidden information only region. This is because this joint region  $w_1$  is now higher to induce effort. This higher wage makes it easier to satisfy the ex post incentive constraint, and the deviation from  $P_2^*$  required to prevent managers of the  $\theta_1$ -type project from pretending to have a  $\theta_2$ -type project becomes smaller. Surprisingly, moral hazard serves to increase investment timing efficiency because the increased share of the firm that must go to compensate the

manager leads the manager to more fully internalize the benefits of relatively more efficient investment timing.

The owner’s and manager’s option values,  $\pi^o(P_0)$  and  $\pi^m(P_0)$ , respectively, can be written as

$$\pi^o(P_0) = q_H \left(\frac{P_0}{P_1^*}\right)^\beta (P_1^* + \theta_1 - K) + (1 - q_H) \left(\frac{P_0}{P_J}\right)^\beta (P_J + \theta_3 - K) \tag{31}$$

and

$$\pi^m(P_0) = q_H \frac{\xi}{\Delta q}. \tag{32}$$

The owner’s option value deviates from the first-best value,  $V^*(P_0)$  in Eq. (9) in two ways. First, the hidden information rents effectively make the manager mark down his privately observed component of project value from  $\theta_2$  to  $\theta_3$ , similar to that in the pure hidden information region. Second, the exercise trigger for a manager of a  $\theta_2$ -type project is equal to  $P_J$ , which is larger than  $P_2^*$ . The only difference between  $\pi^o(P_0)$  in this region and in the pure hidden information region is the different terms for the exercise trigger:  $P_J$  versus  $P_3^*$ . Here, the trigger is lower because of the hidden action component.

*Hidden action only region:*  $(P_0/P_2^*)^\beta \Delta\theta < \xi/\Delta q < (P_0/P_1^*)^\beta \Delta\theta$

In this parameter range, we have

$$P_2 = P_2^* \tag{33}$$

and

$$w_1 = \frac{\xi}{\Delta q} \left(\frac{P_1^*}{P_0}\right)^\beta. \tag{34}$$

The equilibrium triggers equal those of the first-best outcomes. The moral hazard costs are so high that rents needed for motivating effort (via the ex ante incentive constraint) are sufficiently large so that the ex post incentive constraints do not demand additional rents. That is, the wage needed to motivate the manager to extend effort ends up being high enough so that the manager of the  $\theta_1$ -type project no longer needs  $P_2$  to exceed  $P_2^*$  to dissuade him from deviating from the equilibrium trigger  $P_1^*$ . Thus, the contract is entirely driven by the need to motivate ex ante effort, as the ex post incentive constraint that reflects hidden information does not bind.

The owner’s and manager’s option values,  $\pi^o(P_0)$  and  $\pi^m(P_0)$ , respectively, can be written as

$$\pi^o(P_0) = V^*(P_0) - q_H \frac{\xi}{\Delta q} \tag{35}$$

and

$$\pi^m(P_0) = q_H \frac{\xi}{\Delta q}. \tag{36}$$

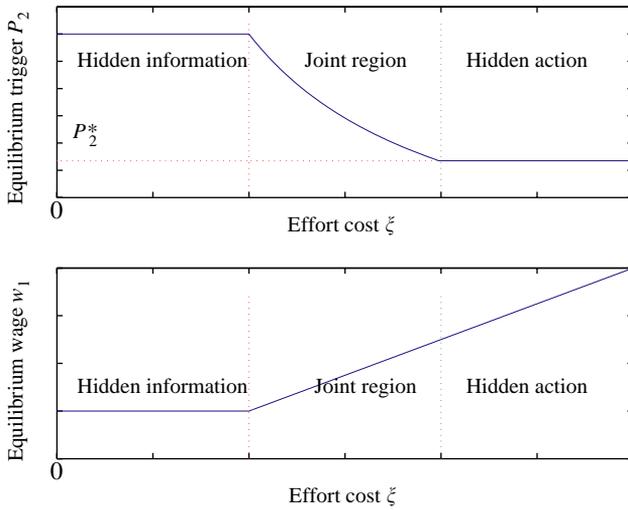


Fig. 1. Optimal incentive contracts across the three parameter regions. The upper and lower graphs plot the equilibrium trigger strategy  $P_2$  and wage payment  $w_1$  in terms of effort cost  $\xi$ , respectively. As the cost of effort increases, the hidden action problem becomes more pronounced. The upper graph demonstrates that, as the cost of effort increases, the equilibrium trigger strategy  $P_2$  decreases, as it approaches the first best trigger  $P_2^*$ . The lower graph demonstrates that, as the cost of effort increases, the wage payment must increase to induce effort from the manager. In summary, as the cost of inducing hidden effort increases, the timing of investment becomes more efficient while the value of the compensation package increases.

The owner’s option value is equal to the first-best solution  $V^*(P_0)$  characterized in Eq. (9), minus the present value of the rent paid to the manager to induce effort.

Fig. 1 summarizes the details of the optimal contracts through the three regions. The upper and lower graphs plot the equilibrium trigger strategy  $P_2$  and wage payment  $w_1$  in terms of effort cost  $\xi$ , respectively. The upper graph shows that the trigger strategy for the manager of the  $\theta_2$ -type project is flat and equal to  $P_3^*$  for  $\xi$  in the pure hidden information region; is decreasing and convex in  $\xi$  for the joint hidden action/hidden information region; and is flat and equal to the first-best trigger level  $P_2^*$  for  $\xi$  in the pure hidden action. The equilibrium trigger  $P_2$  is closer to the first-best level, for higher level of  $\xi$ , ceteris paribus. The lower graph plots corresponding wage contracts for a manager of the  $\theta_1$ -type project. For low levels of  $\xi$  (pure hidden information region), he needs to be compensated only with pure information rents. As a result, wage is insensitive to effort cost  $\xi$  and is flat in this region. In both the joint hidden information/hidden action region and the pure hidden action region,  $w_1$  increases linearly in  $\xi$ .

3.4. An extension to cases with continuous distributions of  $\theta$

For ease of presentation, our basic model uses a simple two-point distribution for  $\theta$ . To check the robustness of our results, we generalize our model to allow for

admissible continuous distributions of  $\theta$  on  $[\underline{\theta}, \bar{\theta}]$  in appendix. In this setting, the principal designs the contract such that the manager finds it optimal to exert effort at time zero and then reveal his  $\theta$  truthfully by choosing the recommended equilibrium strategy  $P(\theta)$  and  $w(\theta)$ . As in the basic setting, we also suppose that the owner could impose a nonpecuniary penalty  $\kappa$  on the manager if the manager fails to live up to his signaled (and true) value of the unobservable component  $\theta$ .<sup>9</sup> The manager is protected by ex post limited liability in that  $w(\theta) \geq 0$  for all  $\theta$ . Also, the manager's participation is voluntary at time zero. We show two key results remain valid.

1. Agency problems (hidden information and hidden action) lead to a delayed investment timing decision, compared with first-best trigger levels.
2. Introducing hidden action into the model at time zero lowers investment timing distortions, because the manager has an option to align his incentives better with the owner by exerting effort at time zero. This leads to an investment timing trigger closer to the first-best level.

In addition, the model predicts that the manager with the lowest privately observed project value  $\underline{\theta}$  receives no rents, in that  $w(\underline{\theta}) = 0$  as in our basic setting (the manager with  $\theta_2$  receives no rents). The ex ante participation constraint does not bind, because the limited liability condition for the manager and ex ante incentive constraint together provide enough incentive for the manager with any ex post realized  $\theta$  to participate, as in our basic setting. For technical convenience, we have assumed that the distribution of  $\theta$  under effort first-order stochastically dominates that under no effort. Intuitively, the manager is more likely to draw a better distribution of  $\theta$  after exerting effort than not exerting effort. Under those conditions, managers of higher quality projects exercise at lower equilibrium trigger strategies and receive higher equilibrium wages. (See appendix for other technical conditions.)

We may further generalize our model by allowing for multiple discrete choices of effort levels. One can solve this problem by following a similar two-step procedure. First, solve for the optimal contract for each given level of effort; and second, choose the optimal level of effort for the owner by searching for the maximum among owner's option value across all effort levels. Subtle technical issues arise when we allow for effort choice to be continuous.<sup>10</sup> However, the basic approach and intuition remain valid.

<sup>9</sup>A sufficient condition to deter the manager from diverting the unobservable incremental part of value  $\theta - \underline{\theta}$  is to require that the nonpecuniary cost  $\kappa$  is large enough to deter the manager with the highest type  $\bar{\theta}$ , in that  $\kappa \geq \bar{\theta} - \underline{\theta} - w(\bar{\theta})$ .

<sup>10</sup>We need to verify the validity of first-order approach, which refers to the practice of replacing an infinite number of global incentive constraints imposed by ex ante incentive to exert effort, with simple local incentive constraints as captured by first-order condition associated with the global incentive constraints. See Rogerson (1985) and Jewitt (1988) for more on the first-order approach.

#### 4. Model implications

In this section, we analyze several of the more important implications of the model. Section 4.1 examines the stock price reaction to investment (or failure to invest). The stock price moves by a discrete jump because the information released at the trigger  $P_1^*$ . Investment at  $P_1^*$  signals good news about project quality and the stock price jumps upward; failure to invest at  $P_1^*$  signals bad news about project quality and the stock price falls downward. A clear prediction of our model is that the principal-agent problem introduces inertia into a firm's investment behavior, in that investment on average is delayed beyond first-best. Section 4.2 considers the factors that influence the expected lag in investment. Specifically because the timing of investment differs from that of the first-best outcome, the principal-agent problem results in a social loss and reduction in the owner's option value. Section 4.3 analyzes the comparative statics of the social loss and owner's option value with respect to the key parameters of the model.

We focus our analysis on the contract that prevails in the joint hidden information/hidden action region. The incentive problems are the richest and most meaningful in this region. Therefore, when we refer to contracting variables such as  $w_1$  and  $P_J$ , we are referring to the values of those variables that hold in this joint hidden information/hidden action region. The terms of the contract and resulting option values in this region are displayed in Eqs. (29)–(32).

##### 4.1. Stock price reaction to investment

In this section, we analyze the stock price reaction to the information released via the manager's investment decision.<sup>11</sup> The manager's investment decision signals to the market the true value of  $\theta$ , and the stock price reflects this information revelation. This allows for the manager's compensation contract to be contingent on the firm's stock price. That is, while in the model we have made the wages in the incentive contract contingent on the manager's investment decision, the wages can also be made contingent on the stock price.

The equity value of the firm is equal to the value of the owner's option value given in Eq. (31). Prior to the point at which  $P(t)$  reaches the threshold  $P_1^*$ , the market does not know the true value of  $\theta$ . The market believes that  $\theta = \theta_1$  with probability  $q_H$  and  $\theta = \theta_2$  with probability  $1 - q_H$ .

Once the process  $P(t)$  hits the threshold  $P_1^*$ , the manager's unobserved component of project value is fully revealed. The manager's investment behavior signals to the market the true value of  $\theta$ . If the manager exercises the option at  $P_1^*$ , then the manager reveals to the market that the privately observed component of project value is high. Therefore, the firm's value instantly jumps to  $S_u$ , given by

$$S_u = P_1^* + \theta_1 - K - w_1 = P_1^* + \theta_1 - K - \left(\frac{P_1^*}{P_J}\right)^\beta \Delta\theta. \quad (37)$$

<sup>11</sup>We thank the referee for suggesting this discussion.

If the manager does not exercise his option at  $P_2^*$ , then the market infers that the manager's privately observed component of project value is low. Then, the firm's value instantly drops to  $S_d$ , given by

$$S_d = \left( \frac{P_1^*}{P_J} \right)^\beta (P_J + \theta_2 - K). \quad (38)$$

Fig. 2 plots the stock price  $S$  as a function of  $P$ , the current value of the process  $P(t)$ . For all  $P < P_1^*$ ,  $S(P) = \pi^o(P)$ , where  $\pi^o$  is given in Eq. (31). For  $P = P_1^*$ ,  $S(P) = S_u$  if investment is undertaken, and  $S(P) = S_d$  if investment is not undertaken. The jump in the stock price at  $P_1^*$  is a result of the information revealed by the manager's investment decisions.

This result is consistent with the empirical findings in McConnell and Muscarella (1985). They find that announcements of unexpected increases in investment spending lead to increases in stock prices, and vice versa for unexpected decreases.

Given that the stock price movement at the trigger  $P_1^*$  reveals the true value of  $\theta$ , the manager's incentive contract can be made contingent on the stock price. For example, the manager could be paid a bonus  $w_1$  if the stock price jumps upward to

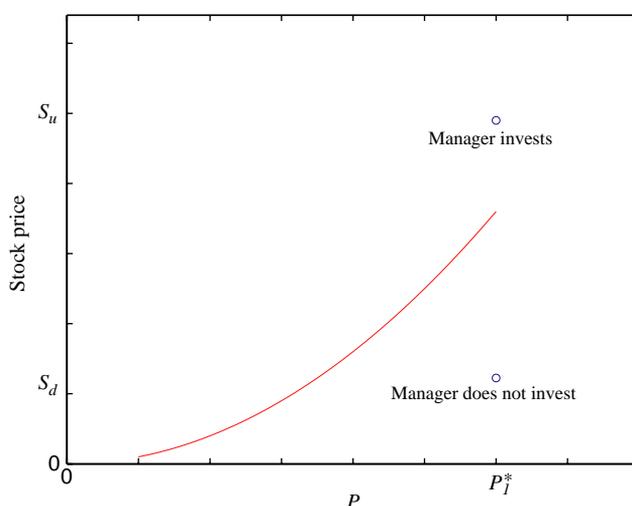


Fig. 2. Stock price reaction to investment. This graph plots the stock price as a function of  $P$ , the present value of the observed component of cash flows. Whenever the level of  $P$  is below the lower investment trigger  $P_1^*$ , the market does not know the true value of  $\theta$ , the present value of the unobserved component of cash flows. Thus, for all  $P$  below  $P_1^*$ , the stock price equals the value of the owner's option given in Eq. (31). At the moment the process  $P$  hits the trigger  $P_1^*$ , the true value of  $\theta$  is revealed through the manager's action: if the manager invests, then the value of  $\theta$  is the higher value  $\theta_1$ ; if the manager does not invest, then the value of  $\theta$  is the lower value  $\theta_2$ . Thus, the stock price is discontinuous at  $P_1^*$ . Investment signals good news and the stock price jumps to  $S_u$ , while failure to invest signals bad news and the stock price drops to  $S_d$ , where  $S_u$  and  $S_d$  are given in Eqs. (37) and (38), respectively.

$S_u$ . Because  $w_2 = 0$ , no bonus is paid if the stock price falls to  $S_d$ . Similarly, such a contingent payoff could be implemented through a properly parameterized stock option grant.

4.2. Agency problems and investment lags

In the standard real options setting, investment is triggered at the value maximizing triggers,  $P_1^*$  and  $P_2^*$ , for the higher and lower project quality outcomes, respectively. However, in our setting, while the trigger for investment in the higher quality state remains at  $P_1^*$ , investment in the lower quality state could be triggered at  $P_J$ , which is higher than the first-best benchmark level  $P_2^*$ .

Let  $T$  and  $T^*$  be the stopping times at which the option is exercised, in our model and the first-best setting, respectively. We denote  $\Gamma = E(T - T^*)$  as the expected time lag stemming from the principal-agent problem. A solution for such an expectation can be derived using Harrison (1985, Chapter 3). The expected lag is given by

$$\Gamma = \left( \frac{1 - q_H}{\alpha - \sigma^2/2} \right) \ln \left( \frac{P_J}{P_2^*} \right) \tag{39}$$

$$= \left( \frac{1 - q_H}{\alpha - \sigma^2/2} \right) \left[ \ln \left( \frac{P_0}{K - \theta_2} \right) + \frac{1}{\beta} \ln \left( \frac{\Delta q \Delta \theta}{\xi} \right) - \ln \beta + \ln(\beta - 1) \right], \tag{40}$$

where we assume that  $\alpha > \sigma^2/2$  for this expectation to exist.

An important insight from Section 3 is that increases in the cost–benefit ratio of inducing effort lead to less distortion in investment timing. That is, as the ratio  $\xi/\Delta q$  increases, the equilibrium trigger  $P_J$  becomes closer to the first-best trigger  $P_2^*$ . This is confirmed by the comparative static

$$\frac{\partial \Gamma}{\partial (\xi/\Delta q)} = - \left( \frac{1 - q_H}{\alpha - \sigma^2/2} \right) \frac{\Delta q}{\beta \xi} < 0. \tag{41}$$

An increase in the volatility of the underlying project,  $\sigma$ , has an ambiguous effect on the expected time lag  $\Gamma$ . This can be seen from the comparative static

$$\frac{\partial \Gamma}{\partial \sigma} = - \left( \frac{1 - q_H}{\alpha - \sigma^2/2} \right) \frac{1}{\beta^2} \left[ \ln \left( \frac{\Delta q \Delta \theta}{\xi} \right) - \frac{\beta}{\beta - 1} \right] \frac{\partial \beta}{\partial \sigma} + \frac{(1 - q_H)\sigma}{(\alpha - \sigma^2/2)^2} \ln \left( \frac{P_J}{P_2^*} \right), \tag{42}$$

where  $\partial \beta / \partial \sigma < 0$ . An increase in  $\sigma$  raises the option value and makes waiting more worthwhile, implying that both  $P_2^*$  and  $P_J$  are larger, ceteris paribus. However, if the cost–benefit ratio for exerting effort is relatively high, in that

$$\ln \left( \frac{\xi}{\Delta q} \right) > \frac{\beta - 1}{\beta} + \ln(\Delta \theta), \tag{43}$$

then the change of  $P_J$  relative to the change in  $P_2^*$  is larger. Therefore, under such conditions the expected time lag increases in volatility  $\sigma$ .

An increase in the expected growth rate of the project,  $\alpha$ , also has an ambiguous effect on the expected time lag  $\Gamma$ . This can be seen from the comparative static

$$\frac{\partial \Gamma}{\partial \alpha} = -\frac{1 - q_H}{(\alpha - \sigma^2/2)^2} \left[ \ln\left(\frac{P_J}{P_2^*}\right) - \frac{1}{\beta} \left( \ln\left(\frac{\Delta q \Delta \theta}{\xi}\right) - \frac{\beta}{\beta - 1} \right) \frac{\alpha - \sigma^2/2}{\sqrt{(\alpha - \sigma^2/2)^2 + 2r\sigma^2}} \right]. \tag{44}$$

However, if Eq. (43) holds, then expected time lag decreases with drift  $\alpha$ .

### 4.3. Social loss and option values

Although the owner chooses the value-maximizing contract to provide an incentive for the manager to extend effort, the agency problem ultimately still proves costly. In an owner-managed firm, the manager extends effort and exercises the option at the first-best stopping time. However, in firms with delegated management, a social loss results from the firm’s suboptimal exercise strategy.

By a social loss, we are referring to the difference between the values of the first-best option value,  $V^*(P_0)$  in Eq. (9), and the sum of the owner and manager options,  $\pi^o(P_0)$  and  $\pi^m(P_0)$  in Eq. (31) and Eq. (32). Thus, define the social loss stemming from agency issues as  $L$ , where  $L = V^*(P_0) - [\pi^o(P_0) + \pi^m(P_0)]$ . Simplifying, we have

$$L = (1 - q_H) \left[ \left(\frac{P_0}{P_2^*}\right)^\beta (P_2^* - K + \theta_2) - \left(\frac{P_0}{P_J}\right)^\beta (P_J - K + \theta_2) \right]. \tag{45}$$

This social loss is likely to have economic ramifications on the structure of firms. For firms in industries with potentially large social losses stemming from agency costs, powerful forces push them to be privately held, or to be organized in a manner that provides the closest alignment between owners and managers.

There are two effects of a later-than-first-best exercising trigger ( $P_J > P_2^*$ ) on the social loss  $L$ : a larger payout ( $P_J + \theta_2 - K$ ) reduces social loss, ceteris paribus, and a lower discount factor [ $(P_0/P_J)^\beta < (P_0/P_2^*)^\beta$ ] increases the social loss. The latter dominates the former, because  $P_J > P_2^*$  and  $P_2^* = \arg \max (P_0/x)^\beta (P_0 + \theta_2 - K)$ . Eq. (45) suggests that social loss is driven by the distance of the equilibrium trigger  $P_J$  from  $P_2^*$ . The firm’s exercise timing becomes less distorted as the net cost–benefit ratio of inducing effort increases. That is, as the ratio  $\xi/\Delta q$  increases, the equilibrium trigger  $P_J$  gets closer to the first-best trigger  $P_2^*$ , and thus

$$\frac{\partial L}{\partial (\xi/\Delta q)} < 0. \tag{46}$$

With or without an agency problem, the owner’s value decreases as the cost of effort  $\xi$  increases, in that  $d\pi^o(P_0)/d\xi < 0$ . Without an agency problem (e.g., the firm’s

owner also manages the investment decisions), the owner’s value falls one for one with an increase in effort cost; the owner simply must increase his effort outlay. In the case of delegated management with agency costs, the owner’s value  $\pi^o(P_0)$  also falls as the cost of effort increases. A question that we ask below is whether or not  $\pi^o(P_0)$  falls by more or less than the first-best value does when the cost of effort increases.

In terms of the owner’s option value, the incentive problem represents a trade-off between timing efficiency and the surplus that must be paid to the manager to extend effort. One can obtain better intuition on the forces at work in the agency problem through the following decomposition. In the first-best solution, the owner pays the cost of effort  $\xi$  and obtains the first-best option value  $V^*(P_0)$ . In the agency equilibrium, the owner delegates the cost of effort to the manager, but then holds the suboptimal option value  $\pi^o(P_0)$ . The loss in the owner’s option value resulting from the incentive problem is therefore given by

$$\Delta\pi^o(P_0) \equiv V^*(P_0) - \xi - \pi^o(P_0) = L + V^m, \tag{47}$$

where  $L$  is the total social loss given in Eq. (45), and  $V^m$  is the ex ante expected surplus paid to the manager to exert effort and is given by

$$V^m = \pi^m(P_0) - \xi = q_H \frac{\xi}{\Delta q} - \xi = \frac{q_L}{\Delta q} \xi. \tag{48}$$

Decomposing the loss in the owner’s option value given in Eq. (47) into the sum of the timing component ( $L$ ) and the compensation component ( $V^m$ ) is useful for providing intuition. When the owner delegates the option exercise decision to the manager, the owner’s option value is lowered for two reasons: the exercising inefficiency induced by agency and information asymmetry; and the surplus needed to pay the manager to induce him to extend effort and reveal his private information. The impact of a higher effort cost  $\xi$  represents a trade-off in terms of the timing and compensation components. As shown in Eq. (46), a higher effort cost results in more efficient investment timing. This must be traded off against the increased compensation that must be paid to provide appropriate incentives to the manager, as seen in Eq. (48). Therefore, the total effect on the loss of owner’s option value stemming from an increase in  $\xi$  depends on whether the timing effect or compensation effect is larger, in that

$$\frac{\partial}{\partial \xi} \Delta\pi^o(P_0) = -(1 - q_H)(\beta - 1) \left(\frac{P_0}{P_J}\right)^\beta \left(1 - \frac{P_2^*}{P_J}\right) \frac{P_J}{\beta \xi} + \frac{q_L}{\Delta q} \tag{49}$$

$$= \frac{\beta - 1}{\beta} \frac{1}{\Delta q \Delta \theta} [-(1 - q_H)(P_J - P_2^*) + q_L(P_2^* - P_1^*)]. \tag{50}$$

If the investment trigger  $P_J$  is significantly larger than  $P_2^*$ , in that

$$(1 - q_H)(P_J - P_2^*) > q_L(P_2^* - P_1^*), \tag{51}$$

then an increase in  $\xi$  leads to a smaller loss in the owner’s option value, as the gain in timing efficiency overshadows the loss resulting from the manager’s increased

compensation.<sup>12</sup> That is, while the owner's option value under agency falls as  $\zeta$  increases, it could fall by less than the full amount of the increase in  $\zeta$  as a result of the gain in timing efficiency.

## 5. Impatient managers and early investment

So far, we have assumed that both owners and managers value payoffs identically. However, managers could be more impatient than owners. Several potential justifications exist for such an assumption. First, various models of managerial myopia attempt to explain a manager's preference for choosing projects with quicker paybacks, even in the face of eschewing more valuable long-term opportunities. For example, Narayanan (1985) and Stein (1989) argue that concerns about either the firm's short-term performance or labor market reputation could give the manager an incentive to take actions that pay off in the near term at the expense of the long term. Second, in our investment timing setting, greater impatience can represent the manager's preference for empire building or greater perquisite consumption and reputation that comes from running a larger company sooner rather than later. Third, managerial short-termism could be the result of the manager facing stochastic termination. (We assume that the owner can costlessly replace the manager in the event of separation.) This termination, for example, could be the result of the manager leaving for a better job elsewhere or being fired. We can model such stochastic termination by supposing that the manager faces an exogenous termination driven by a Poisson process with intensity  $\zeta$ . The addition of stochastic termination transforms the manager's option to one in which his discount rate  $r$  is elevated to  $r + \zeta$  to reflect the stochastic termination. (We suppose that the manager receives his reservation value (normalized to zero), when the termination occurs. See Yaari (1965), Merton (1971) and Richard (1975) for analogous results on stochastic horizon.)

Phrased in real options terms, managerial impatience decreases the value of the manager's option to wait. Thus, this generalization leads to very different predictions about investment timing. While the basic model predicts that investment never occurs earlier than the first-best case, in this generalized setting investment can occur earlier or later than the first-best case. This is similar to the result found in Stulz (1990) when there is both over- and underinvestment in the capital allocation decision, as shareholders use debt to constrain managerial empire-building preferences.

The owner discounts future cash flows by the discount function  $D(P_0; \hat{P}) = (P_0/\hat{P})^\beta$  for  $P_0 < \hat{P}$ . We can therefore represent greater managerial impatience by

<sup>12</sup>The condition is nonempty. This can be seen as follows. Condition Eq. (51) is equivalent to

$$P_J > \frac{1}{1 - q_H} [(1 - q_H)P_2^* + q_L(P_2^* - P_1^*)] = P_3^* - \frac{\Delta q}{1 - q_H} (P_2^* - P_1^*).$$

The joint hidden action/hidden information region is characterized by  $P_2^* \leq P_J \leq P_3^*$ . Therefore, the condition is met for some  $P_J$ .

defining a managerial discount function  $D^m(P_0; \hat{P}) = (P_0/\hat{P})^\gamma$ , where  $\gamma > \beta$  ensures that  $D^m(P_0; \hat{P}) < D(P_0; \hat{P})$ . That is, a dollar received at the stopping time described by the trigger strategy  $\hat{P}$  is worth less to the manager than to the owner.<sup>13</sup>

This generalized problem is similar to that of Section 3.3, with the exception that the constraints all use  $\gamma$  instead of  $\beta$ . Much of the solution methodology is the same. For example, Propositions 1 and 2 apply as before, using the same proof. In addition, Propositions 3 and 4 remain valid and are demonstrated in appendix. Thus, the optimal contracting problem in the generalized setting can be written as

$$\begin{aligned} \max_{w_1, P_1, P_2} & q_H \left(\frac{P_0}{P_1}\right)^\beta (P_1 + \theta_1 - K) - q_H \left(\frac{P_0}{P_1}\right)^\beta w_1 \\ & + (1 - q_H) \left(\frac{P_0}{P_2}\right)^\beta (P_2 + \theta_2 - K), \end{aligned} \tag{52}$$

subject to

$$\left(\frac{P_0}{P_1}\right)^\gamma w_1 \geq \left(\frac{P_0}{P_2}\right)^\gamma \Delta\theta \tag{53}$$

and

$$\left(\frac{P_0}{P_1}\right)^\gamma w_1 \geq \frac{\xi}{\Delta q}. \tag{54}$$

Similar to Proposition 5, at least one of Eq. (53) and Eq. (54) binds. Otherwise, the owner could strictly increase his payoff by lowering the wage payment  $w_1$  without violating any constraints.

Just as in Section 3.3, there are three contracting regions: a hidden information region, a joint hidden information/hidden action region, and a hidden action region, depending on the level of cost–benefit ratio  $\xi/\Delta q$ . In this section, we focus on the joint hidden information/hidden action region. (The derivations for the optimal contracts in the other regions are shown in appendix.)

The joint hidden information/hidden action region is defined by  $(P_0/\hat{P}_3^*)^\gamma \Delta\theta < \xi/\Delta q < (P_0/P_2^*)^\gamma \Delta\theta$ , where  $\hat{P}_3^*$  is defined in Eq. (107) and shown to be greater than the trigger  $P_2^*$ . In this region the optimal contract can be written as

$$P_1 = \hat{P}_1, \tag{55}$$

$$P_2 = \hat{P}_J = P_0 \left(\frac{\Delta q \Delta\theta}{\xi}\right)^{1/\gamma}, \tag{56}$$

$$w_1 = \left(\frac{\hat{P}_1}{\hat{P}_J}\right)^\gamma \Delta\theta < \Delta\theta \tag{57}$$

<sup>13</sup>This is also consistent with the interpretation that the manager has a higher discount rate than the owner. Because  $\partial\beta/\partial r > 0$ , the manager's higher discount rate is embodied by the condition  $\gamma > \beta$ .

and

$$w_2 = 0, \tag{58}$$

where  $\hat{P}_1$  is the root of  $H(x) = 0$ , defined by

$$H(x) = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) \left( \frac{x}{P_0} \right)^\gamma \frac{\xi}{\Delta q} \right] - x. \tag{59}$$

Unlike the results of the basic model, we now have the possibility of investment occurring before the first-best trigger is reached, in that  $P_1 = \hat{P}_1 < P_1^*$ . To see this, note that  $H(0) = P_1^*$  and

$$H(P_1^*) = \frac{\beta}{\beta - 1} \left( 1 - \frac{\gamma}{\beta} \right) \left( \frac{P_1^*}{P_0} \right)^\gamma \frac{\xi}{\Delta q} < 0. \tag{60}$$

The derivative of  $H(\cdot)$  is

$$H'(x) = \frac{\beta}{\beta - 1} \gamma \left( 1 - \frac{\gamma}{\beta} \right) \left( \frac{x}{P_0} \right)^{\gamma - 1} \frac{1}{P_0} \frac{\xi}{\Delta q} - 1 < 0. \tag{61}$$

Therefore, there exists a unique solution  $P_1 = \hat{P}_1 < P_1^*$ .

As in the basic model, the trigger strategy for the manager of a  $\theta_2$ -type project is greater than the first-best trigger,  $P_2^*$ .  $\hat{P}_J > P_2^*$  in the region  $(P_0/\hat{P}_3^*)^\gamma \Delta\theta < \xi/\Delta q < (P_0/P_2^*)^\gamma \Delta\theta$ , where  $P_3^*$  is given in Eq. (107). However, for  $\gamma > \beta$ , the trigger is closer to the first-best trigger than for the standard case in which  $\gamma = \beta$ . This is true, because for  $\gamma > \beta$ ,

$$\hat{P}_J = P_0 \left( \frac{\Delta q \Delta \theta}{\xi} \right)^{1/\gamma} < P_0 \left( \frac{\Delta q \Delta \theta}{\xi} \right)^{1/\beta} = P_J. \tag{62}$$

Thus, when the manager is more impatient than the owner, equilibrium investment occurs sooner than it does in the standard principal-agent model. In particular, investment occurs prior to when the first-best trigger is reached for the  $\theta_1$ -type project. The greater impatience on the part of the manager implies that it is in the owner’s interest to offer a contract that motivates earlier exercise. This results in both costs and benefits to the owner. By motivating investment for the  $\theta_2$ -type project earlier than the standard principal-agent model, investment timing trigger moves closer to the first-best one. Because the manager receives no surplus for the  $\theta_2$ -type project, the owner is the sole beneficiary of this timing efficiency. However, investment for the  $\theta_1$ -type project occurs earlier than that in the model of Section 2, which is the first-best outcome. Therefore, the owner is worse off with respect to the  $\theta_1$ -type projects for two reasons: investment occurs too early, and the wage paid to the manager in this state must be higher (than in the standard model) to motivate earlier investment. The net effect on ex ante owner’s option value is ambiguous and is driven by the relative parameter values.

## 6. Conclusion

This paper extends the real options framework to account for the agency and information issues that are prevalent in many real-world applications. When investment decisions are delegated to managers, contracts must be designed to provide incentives for managers both to extend effort and to truthfully reveal their private information. This paper provides a model of optimal contracting in a continuous-time principal-agent setting with both moral hazard and adverse selection. The implied investment behavior differs significantly from that of the first-best no-agency solution. In particular, there is greater inertia in investment, as the model predicts that the manager has a more valuable option to wait than the owner. The interplay between the twin forces of hidden information and hidden action leads to markedly different investment outcomes than when only one of the two forces is at work. Allowing the manager to have an effort choice that affects the likelihood of getting a high quality project mitigates the investment inefficiency resulting from information asymmetry. When the model is generalized to include differing degrees of impatience between owners and managers, we find that investment could occur either earlier or later than optimal.

Some extensions of the model would prove interesting. First, the model could allow for repeated investment decisions. This richer setting would permit owners to update their beliefs over time, and managers to establish reputations. Second, the model could be generalized to include competition in both the labor and product markets. As shown by Grenadier (2002), the forces of competition greatly alter the investment behavior implied by standard real options models.

## Appendix A. Solution to the optimal contracting problem

This appendix provides a derivation of the optimal contracts detailed in Section 3.

First, we simplify the optimal contracting problem by presenting and proving the following four propositions. Proposition 1 shows that the limited liability for the manager of a  $\theta_1$ -type project in constraint Eq. (19) does not bind, while Proposition 2 shows that the ex ante participation constraint Eq. (16) does not bind.

**Proposition 1.** *The limited-liability condition for a manager of a  $\theta_1$ -type project does not bind. That is,  $w_1 > 0$ .*

**Proof.**

$$w_1 \geq \left(\frac{P_1}{P_2}\right)^\beta (w_2 + \Delta\theta) \geq \left(\frac{P_1}{P_2}\right)^\beta \Delta\theta > 0.$$

The first and second inequalities follow from Eqs. (17) and (19), respectively.  $\square$

To motivate the manager to exert effort, we need to reward the manager with an option value larger than zero, which is the manager's reservation value. This leads to the following result.

**Proposition 2.** *The ex ante participation constraint Eq. (16) does not bind.*

**Proof.**

$$\left(\frac{P_0}{P_1}\right)^\beta w_1 + \frac{1 - q_H}{q_H} \left(\frac{P_0}{P_2}\right)^\beta w_2 - \frac{\xi}{q_H} \geq \frac{\xi}{\Delta q} - \frac{\xi}{q_H} > 0,$$

where the first inequality follows from the ex ante incentive constraint Eq. (15) and the limited liability condition for the type- $\theta_2$  project.  $\square$

Propositions 1 and 2 allow us to express the owner’s objective as maximizing the value of his option, given in Eq. (13), subject to Eqs. (15), (17), and (18) and  $w_2 \geq 0$ . Using the method of Kuhn-Tucker, we form the Lagrangian

$$\begin{aligned} \mathcal{L} = & \left(\frac{P_0}{P_1}\right)^\beta (P_1 + \theta_1 - K - w_1) + \frac{1 - q_H}{q_H} \left(\frac{P_0}{P_2}\right)^\beta (P_2 + \theta_2 - K - w_2) \\ & + \lambda_1 \left[ \left(\frac{P_0}{P_1}\right)^\beta w_1 - \left(\frac{P_0}{P_2}\right)^\beta (w_2 + \Delta\theta) \right] + \lambda_2 \left[ \left(\frac{P_0}{P_2}\right)^\beta w_2 - \left(\frac{P_0}{P_1}\right)^\beta (w_1 - \Delta\theta) \right] \\ & + \lambda_3 \left[ \left(\frac{P_0}{P_1}\right)^\beta w_1 - \left(\frac{P_0}{P_2}\right)^\beta w_2 - \frac{\xi}{\Delta q} \right] + \lambda_4 w_2, \end{aligned} \tag{63}$$

with corresponding complementary slackness conditions for the four constraints.

The first-order condition with respect to  $w_1$  gives

$$\lambda_1 - \lambda_2 + \lambda_3 = 1. \tag{64}$$

The first-order condition with respect to  $w_2$  implies

$$\left(-\lambda_1 + \lambda_2 - \lambda_3 - \frac{1 - q_H}{q_H}\right) \left(\frac{P_0}{P_2}\right)^\beta + \lambda_4 = 0. \tag{65}$$

Simplifying Eq. (65) gives  $\lambda_4 = (P_0/P_2)^\beta/q_H > 0$ . Therefore, the complementary slackness condition  $\lambda_4 w_2 = 0$  implies that  $w_2 = 0$ . This is summarized in Proposition 3.

**Proposition 3.** *The limited liability for a manager of a  $\theta_2$ -type project binds, in that  $w_2 = 0$ .*

The intuition is straightforward. Giving the manager of a  $\theta_2$ -type project any positive rents implies higher rents for managers of  $\theta_1$ -type projects to meet the ex post incentive constraint of the manager of a  $\theta_1$ -type project. To minimize the rents subject to the manager’s participation and incentive constraints, the owner gives the manager of a  $\theta_2$ -type project no ex post rents.

The first-order conditions with respect to  $P_1$  and  $P_2$  imply

$$P_1 = \frac{\beta}{\beta - 1} (K - \theta_1 - \lambda_2 \Delta\theta) \tag{66}$$

and

$$P_2 = \frac{\beta}{\beta - 1} \left( K - \theta_2 + \frac{q_H}{1 - q_H} \lambda_1 \Delta\theta \right). \quad (67)$$

The following proposition states that the ex post incentive constraint for the manager of a  $\theta_2$ -type project, Eq. (18), does not bind, in that  $\lambda_2 = 0$ . We verify this conjecture, formalized in Proposition 4 for each region. Proposition 4 allows us to ignore Eq. (18) in the optimization problem.

**Proposition 4.** *Optimal contracts imply  $w_1 \leq \Delta\theta$ .*

Intuitively, if  $w_1 > \Delta\theta$ , then the manager of a  $\theta_2$ -type project would never accept the equilibrium contract with  $w_2 = 0$ . This would clearly be inconsistent with Proposition 3.

Propositions 1–4 jointly simplify the owner’s optimization problem as the solution to the objective function Eq. (20), subject to constraints Eqs. (21) and (22).

The following proposition demonstrates that at least one of the two constraints binds.

**Proposition 5.** *At least one of Eqs. (21) and (22) binds.*

The argument is immediate. If Proposition 5 did not hold, then reducing  $w_1$  increases the owner’s value strictly without violating any of the constraints. With  $\lambda_2 = 0$ , then Eq. (64) could be written as  $\lambda_1 + \lambda_3 = 1$ . Therefore, it must be the case that at least one of Eqs. (21) and (22) binds.

The problem summarized in Eqs. (20)–(22) is solved below.

#### A.1. The hidden information only region

Suppose that the ex ante incentive constraint Eq. (22) does not bind. Because Eq. (21) must hold as an equality and  $\lambda_1 = 1$ , Eq. (67) implies  $P_2 = P_3^*$  where  $P_3^*$  is given in Eq. (25) and  $w_1$  is given in Eq. (26). The inequality  $P_1^* < P_3^*$  implies Eq. (18) does not bind, consistent with Proposition 4. Finally, to be consistent with the assumption that Eq. (22) does not bind, we require that  $\xi/\Delta q < (P_0/P_3^*)^\beta \Delta\theta$ , the parameter range defining this hidden information only region.

#### A.2. The joint hidden information/hidden action region

We derive the optimal contract in this region by conjecturing that both Eqs. (21) and (22) bind. Solving these two equality constraints gives Eqs. (29) and (30). The inequality  $P_J > P_1^*$  confirms that Eq. (18) does not bind, consistent with Proposition 4. The solution for  $P_2$  implies that  $\lambda_1$  can be written as

$$\lambda_1 = \frac{\beta - 1}{\beta} (P_J - P_2^*) \frac{1 - q_H}{q_H \Delta\theta}. \quad (68)$$

The only possible region under which both constraints could bind is characterized by<sup>14</sup>

$$\left(\frac{P_0}{P_3^*}\right)^\beta \Delta\theta < \frac{\xi}{\Delta q} < \left(\frac{P_0}{P_2^*}\right)^\beta \Delta\theta. \tag{69}$$

We now show that both Eqs. (21) and (22) bind throughout this entire region. The region characterized by Eq. (69) can be equivalently expressed as  $P_2^* < P_J < P_3^*$ . Because Eq. (68) implies that  $\lambda_1$  is monotonically increasing in  $P_J$ , therefore,  $0 < \lambda_1 < 1$  in this region. Because  $\lambda_3 = 1 - \lambda_1$ , we also have  $0 < \lambda_3 < 1$ . By complementary slackness conditions, both Eqs. (21) and (22) bind in this joint region, confirming the result that Eq. (69) is the whole region, with both constraints binding. (We need additional technical conditions to ensure that inducing the manager to extend high effort is in the interest of the owner.)

### A.3. The hidden action only region

Suppose that Eq. (21) does not bind and Eq. (22) binds, then  $\lambda_1 = 0$  by complementary slackness, and  $\lambda_3 = 1$ . Therefore,  $P_2 = P_2^*$  given in Eq. (33) and  $w_1$  is given in Eq. (34). We need to verify that Eqs. (21) and (18) do not bind. The constraint Eq. (18) is non-binding (consistent with Proposition 4) if and only if  $P_1^* < P_J$ . The constraint Eq. (21) is non-binding if and only if  $P_J < P_2^*$ . Thus, together these imply that  $P_1^* < P_J < P_2^*$ , which is identical to the condition  $(P_0/P_2^*)^\beta \Delta\theta < \xi/\Delta q < (P_0/P_1^*)^\beta \Delta\theta$  that defines this region.

If the parameters do not fall in any of the three regions, namely,  $\xi/\Delta q > (P_0/P_1^*)^\beta \Delta\theta$ , then it can be shown that the owner does not choose to motivate the manager to exert effort. The cost of effort is so high as to overwhelm any potential benefits of motivating effort. A proof of this result is available from the authors upon request.

## Appendix B. Optimal contracting with a continuous distribution of $\theta$

This appendix contains the derivation of the optimal contracts when the distribution of the project’s unobserved component  $\theta$  of value is continuous.

Denote the manager’s time-zero expected utility as  $u(\hat{\theta}, \theta)$ , if he reports that his privately observed component of project value is  $\hat{\theta}$ , and the true level of his privately observed value is  $\theta$ . His time-zero expected utility is then given by

$$u(\hat{\theta}, \theta) = \left(\frac{P_0}{P(\hat{\theta})}\right)^\beta \left(w(\hat{\theta}) + \theta - \hat{\theta}\right). \tag{70}$$

<sup>14</sup>If  $(P_0/P_1^*)^\beta \Delta\theta > \xi/\Delta q > (P_0/P_2^*)^\beta \Delta\theta$ , then only the third constraint binds. If  $(P_0/P_3^*)^\beta \Delta\theta > \xi/\Delta q$ , then only the first constraint binds. If  $\xi/\Delta q > (P_0/P_1^*)^\beta \Delta\theta$ , then supporting high effort is no longer in the owner’s interest.

We denote  $U(\theta)$  as the value function of the manager whose privately observed component of project value is  $\theta$ . That is,

$$U(\theta) = u(\theta, \theta) = \left( \frac{P_0}{P(\theta)} \right)^\beta w(\theta). \tag{71}$$

As in Section 3.3, we denote  $\xi$  as the cost of extending effort at time zero. Let  $F_H(\theta)$  and  $F_L(\theta)$  be the cumulative distribution functions of  $\theta$  drawn if the manager extends effort and if he does not extend effort, respectively. Using the revelation principle, we can write the principal’s optimization problem as

$$\max_{P(\cdot), w(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \left( \frac{P_0}{P(\theta)} \right)^\beta (P(\theta) + \theta - K - w(\theta)) dF_H(\theta) \tag{72}$$

subject to an

*ex post incentive-compatibility condition:*

$$U(\theta) \geq u(\hat{\theta}, \theta) \quad \text{for any } \hat{\theta} \text{ and } \theta; \tag{73}$$

*limited-liability condition:*

$$w(\theta) \geq 0 \quad \text{for any } \theta; \tag{74}$$

*ex ante incentive compatibility condition:*

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF_H(\theta) - \xi \geq \int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF_L(\theta) \tag{75}$$

and

*ex ante participation constraint:*

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF_H(\theta) - \xi \geq 0. \tag{76}$$

**Proposition 6.** *The ex ante participation constraint Eq. (76) does not bind.*

**Proof.** The ex ante incentive constraint (75) and the limited-liability condition (74) together imply that the ex ante participation constraint (76) does not bind.  $\square$

First, we simplify the ex ante incentive constraint Eq. (75) using integration by parts. This gives

$$\int_{\underline{\theta}}^{\bar{\theta}} M(\theta) dU(\theta) \geq \xi, \tag{77}$$

where  $M(\theta) = -(F_H(\theta) - F_L(\theta))$ .

Next, we simplify the ex post incentive-compatibility condition Eq. (73) by totally differentiating  $U(\theta)$ , the value function for a manager of a type- $\theta$  project, with

respect to  $\theta$ . This gives

$$\frac{dU(\theta)}{d\theta} = u_1 \frac{d\hat{\theta}}{d\theta} + u_2, \tag{78}$$

where

$$u_1 = \left. \frac{\partial u(\hat{\theta}, \theta)}{\partial \hat{\theta}} \right|_{\hat{\theta}=\theta} \quad \text{and} \quad u_2 = \left. \frac{\partial u(\hat{\theta}, \theta)}{\partial \theta} \right|_{\hat{\theta}=\theta}. \tag{79}$$

Because the manager optimally reveals his project quality by choosing recommended equilibrium strategy, we have  $u_1(\theta, \theta) = 0$ . Therefore, we have  $U'(\theta) = u_2$ . Integration gives

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_2(s, s) ds = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} ds. \tag{80}$$

We note that the Spence-Mirrlees condition is satisfied.<sup>15</sup>

A standard result in the contracting literature with asymmetric information states that the limited-liability condition for a manager of a  $\underline{\theta}$ -type project binds, in that  $U(\underline{\theta}) = w(\underline{\theta}) = 0$ . Therefore, the information rents  $U(\theta)$  that accrues to the manager of a  $\theta$ -type project is given by

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} ds. \tag{81}$$

The relationship between  $U(\theta)$  and the equilibrium wage implies that

$$w(\theta) = \left( \frac{P(\theta)}{P_0} \right)^{\beta} U(\theta) = \int_{\underline{\theta}}^{\theta} \left( \frac{P(\theta)}{P(s)} \right)^{\beta} ds. \tag{82}$$

Using Eq. (82), we simplify the present value of expected wage payment as

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} \left( \frac{P_0}{P(\theta)} \right)^{\beta} w(\theta) dF_H(\theta) &= \int_{\underline{\theta}}^{\bar{\theta}} \left[ \int_{\underline{\theta}}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} ds \right] dF_H(\theta), \\ &= \left[ \int_{\underline{\theta}}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} ds \right] F_H(\theta) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} F_H(\theta) \left( \frac{P_0}{P(\theta)} \right)^{\beta} d\theta, \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \lambda_H(\theta) \left( \frac{P_0}{P(\theta)} \right)^{\beta} dF_H(\theta), \end{aligned} \tag{83}$$

<sup>15</sup>Details are available upon request.

where

$$\lambda_H(\theta) = \frac{1 - F_H(\theta)}{f_H(\theta)} \tag{84}$$

is the inverse of the hazard rate under  $F_H(\cdot)$ .

Using Eq. (83) allows us to simplify the principal’s optimization problem as

$$\max_{P(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \left( \frac{P_0}{P(\theta)} \right)^\beta (P(\theta) + \theta - K - \lambda_H(\theta)) dF_H(\theta), \tag{85}$$

subject to the ex ante incentive constraint Eq. (77) and ex post limited-liability condition Eq. (74). The equilibrium wage is then obtained by using Eq. (82).

Similar to the model with discrete values for  $\theta$ , optimal contracts, characterized by the pair of trigger strategy and wage payment functions, depend on the region in which effort cost  $\xi$  lies. Section B.1 solves for the optimal contracts in the region under which the ex ante incentive constraint does not bind. Section B.2 solves for the optimal contracts in the region under which both the ex ante incentive constraint and the ex post incentive constraint bind.

*B.1. The hidden information only region*

If effort cost  $\xi$  is low enough, then no additional rent is needed to induce the manager to extend effort. The following condition ensures that the ex ante incentive constraint Eq. (77) does not bind.

**Condition 1.**

$$\int_{\underline{\theta}}^{\bar{\theta}} M(\theta) \left( \frac{P_0}{P_3^*(\theta)} \right)^\beta d\theta \geq \xi, \tag{86}$$

where

$$P_3^*(\theta) = \frac{\beta}{\beta - 1} [K - \theta + \lambda_H(\theta)]. \tag{87}$$

Maximizing Eq. (85) could be done point by point. This gives the candidate optimal trigger level  $P(\theta) = P_3^*(\theta)$ , where  $P_3^*(\theta)$  is given in Eq. (87). Because  $\lambda_H(\theta) > 0$ , the exercise trigger is larger than the first-best level, confirming the intuition delivered in Section 3.4 using the two-point distribution of  $\theta$ . A verification easily confirms that Eq. (77) does not bind under Condition 1.

The following condition ensures that the candidate trigger strategy is positive for any  $\theta$ .

**Condition 2.** For all  $\theta$  on the support,  $\theta - \lambda_H(\theta) < K$ .

Finally, we ensure that the candidate trigger strategy decreases in  $\theta$  by requiring Condition 3:

**Condition 3.**  $d\lambda_H(\theta)/d\theta < 1$ .

Conditions 2 and 3 also imply that wage is positive and increases in the project quality  $\theta$ , as seen from Eq. (82).

*B.2. The joint hidden information/hidden action region*

When the effort cost is higher, both the ex ante incentive constraint Eq. (75) and the ex post incentive constraint Eq. (73) bind. The condition governing the parameters for this region when  $\theta$  is drawn from a continuous distribution is given as follows.

**Condition 4.**

$$\int_{\underline{\theta}}^{\bar{\theta}} M(\theta) \left( \frac{P_0}{P_3^*(\theta)} \right)^\beta d\theta \leq \xi \leq \int_{\underline{\theta}}^{\bar{\theta}} M(\theta) \left( \frac{P_0}{P_2^*(\theta)} \right)^\beta d\theta, \tag{88}$$

where  $P_3^*(\theta)$  is given in Eq. (87) and

$$P_2^*(\theta) = \frac{\beta}{\beta - 1} \left( K - \theta + \frac{1 - F_L(\theta)}{f_H(\theta)} \right). \tag{89}$$

Denote  $l$  as the Lagrangian multiplier for Eq. (77). Then, the candidate equilibrium trigger strategy is given by

$$P(\theta) = P_J(\theta) = \frac{\beta}{\beta - 1} \left( K - \theta + \lambda_H(\theta) - l \frac{M(\theta)}{f_H(\theta)} \right). \tag{90}$$

The Lagrangian multiplier  $l$  is positive under Condition 4. Therefore, the optimal trigger with the exception of the one for the manager of the lowest quality project  $\underline{\theta}$  is larger than the first-best  $P^*(\theta) = \beta(K - \theta)/(\beta - 1)$ . Because Eq. (77) holds with strict equality, we could combine Eqs. (77), (81) and (90) to obtain

$$\xi = \int_{\underline{\theta}}^{\bar{\theta}} M(\theta) \left( \frac{P_0}{P_J(\theta)} \right)^\beta d\theta. \tag{91}$$

Solving the equation gives the Lagrangian multiplier  $l$ . The Lagrangian multiplier  $l$  increases in effort cost  $\xi$ , in that

$$\frac{dl}{d\xi} = \frac{P_0}{\beta - 1} \left[ \int_{\underline{\theta}}^{\bar{\theta}} \frac{M^2(\theta)}{f_H(\theta)} \left( \frac{\beta}{\beta - 1} \right)^2 \left( \frac{P_0}{P_J(\theta)} \right)^{\beta+1} d\theta \right]^{-1}. \tag{92}$$

Therefore, as effort cost  $\xi$  increases, the optimal trigger  $P_J(\theta)$  decreases, as shown by

$$\frac{dP_J(\theta)}{d\xi} = - \frac{\beta}{\beta - 1} \frac{M(\theta)}{f_H(\theta)} \frac{dl}{d\xi} < 0. \tag{93}$$

This is consistent with our intuition and results in Section 4 that a higher effort cost mitigates investment inefficiency by pushing the exercise trigger toward the first-best level.

The following two conditions ensure that the conjectured candidate solutions  $P_j(\theta)$  is positive and decreasing in  $\theta$ .

**Condition 5.**

$$\frac{d}{d\theta} \left( \theta - \lambda_H(\theta) + l \frac{M(\theta)}{f_H(\theta)} \right) > 0, \tag{94}$$

for  $0 \leq l \leq 1$ .

**Condition 6.** *The distribution  $F_H(\cdot)$  first-order stochastically dominates  $F_L(\cdot)$ , in that*

$$F_H(\theta) \leq F_L(\theta) \text{ for all } \theta. \tag{95}$$

*This implies  $M(\theta) \geq 0$ , for all  $\theta$ .*

Under Conditions 5 and 6, wage is positive and increasing in  $\theta$ , in that

$$\begin{aligned} w(\theta_2) &= \int_{\underline{\theta}_2}^{\theta_2} \left( \frac{P(\theta_2)}{P(s)} \right)^\beta ds > \int_{\underline{\theta}}^{\theta_2} \left( \frac{P(\theta_1)}{P(s)} \right)^\beta ds \\ &> \int_{\underline{\theta}}^{\theta_1} \left( \frac{P(\theta_1)}{P(s)} \right)^\beta ds = w(\theta_1) \end{aligned} \tag{96}$$

for  $\theta_2 > \theta_1$ . The first inequality follows from the monotonicity of  $P(\theta)$ .

**Appendix C. Derivations of optimal contracts in Section 5**

This appendix provides a derivation of the optimal contracts for the generalized model of Section 5. Propositions 1 and 2 apply as in Appendix A. Using the method of Kuhn-Tucker, we form the Lagrangian

$$\begin{aligned} \mathcal{L} &= \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K - w_1) + \frac{1 - q_H}{q_H} \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K - w_2) \\ &+ \lambda_1 \left[ \left( \frac{P_0}{P_1} \right)^\gamma w_1 - \left( \frac{P_0}{P_2} \right)^\gamma (w_2 + \Delta\theta) \right] + \lambda_2 \left[ \left( \frac{P_0}{P_2} \right)^\gamma w_2 - \left( \frac{P_0}{P_1} \right)^\gamma (w_1 - \Delta\theta) \right] \\ &+ \lambda_3 \left[ \left( \frac{P_0}{P_1} \right)^\gamma w_1 - \left( \frac{P_0}{P_2} \right)^\gamma w_2 - \frac{\xi}{\Delta q} \right] + \lambda_4 w_2, \end{aligned} \tag{97}$$

with corresponding complementary slackness conditions for the four constraints. As in appendix, we also conjecture that the ex post incentive constraint does not bind, in that the Lagrangian multiplier  $\lambda_2$  associated with the constraint below is zero:

$$\left( \frac{P_0}{P_2} \right)^\gamma w_2 \geq \left( \frac{P_0}{P_1} \right)^\gamma (w_1 - \Delta\theta). \tag{98}$$

We will verify this conjecture for each region.

The first-order conditions with respect to  $w_1$  and  $w_2$  imply

$$0 = (\lambda_1 + \lambda_3) \left(\frac{P_0}{P_1}\right)^\gamma - \left(\frac{P_0}{P_1}\right)^\beta \tag{99}$$

and

$$0 = -(\lambda_1 + \lambda_3) \left(\frac{P_0}{P_2}\right)^\gamma - \frac{1 - q_H}{q_H} \left(\frac{P_0}{P_2}\right)^\beta + \lambda_4. \tag{100}$$

Using Eq. (99) to simplify Eq. (100) gives  $\lambda_4 > 0$ . The complementary slackness condition implies that  $w_2 = 0$ . The first-order conditions with respect to  $P_1$  and  $P_2$  are given by

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left(1 - \frac{\gamma}{\beta}\right) w_1 \right] \tag{101}$$

and

$$P_2 = \frac{\beta}{\beta - 1} \left[ K - \theta_2 + \lambda_1 \frac{q_H}{1 - q_H} \frac{\gamma}{\beta} \left(\frac{P_0}{P_2}\right)^{\gamma - \beta} \Delta\theta \right]. \tag{102}$$

Therefore, it must be the case that at least one of Eqs. (53) and (54) binds (similar to Proposition 5 of Appendix A). Depending on the cost–benefit ratio  $\xi/\Delta q$ , we have three disjoint regions to be analyzed.

### C.1. The hidden information only region

Suppose that the constraint Eq. (54) does not bind and thus  $\lambda_3 = 0$ . Then,  $\lambda_1 = (P_0/P_1)^{\beta - \gamma} > 1$ . A binding ex ante incentive constraint Eq. (53) implies that the wage payment is  $w_1 = (P_1/P_2)^\gamma \Delta\theta$ . The first-order conditions Eqs. (101) and (102) give the coupled equations

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left(1 - \frac{\gamma}{\beta}\right) \left(\frac{P_1}{P_2}\right)^\gamma \Delta\theta \right] \tag{103}$$

and

$$P_2 = \frac{\beta}{\beta - 1} \left[ K - \theta_2 + \frac{q_H}{1 - q_H} \frac{\gamma}{\beta} \left(\frac{P_1}{P_2}\right)^{\gamma - \beta} \Delta\theta \right]. \tag{104}$$

With  $\gamma > \beta$ , we immediately have  $P_1 < P_1^*$  and  $P_2 > P_2^*$ . Therefore,  $w_1 < \Delta\theta$ , as conjectured, confirming that Eq. (98) does not bind and  $\lambda_2 = 0$ .

Define the ratio  $x = P_1/P_2$ . The coupled equations Eqs. (103) and (104) allow us to first solve for ratio  $x^*$ , in that

$$G(x^*) = 0, \tag{105}$$

where

$$G(x) = x \left[ K - \theta_2 + \frac{\gamma}{\beta} \left( \frac{q_H}{1 - q_H} \right) x^{\gamma - \beta} \Delta\theta \right] - \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) x^\gamma \Delta\theta \right] = 0. \tag{106}$$

First,  $G(0) = -(K - \theta_1) < 0$  and  $G(1) = \gamma\Delta\theta / (\beta(1 - q_H)) > 0$ . Second,

$$G'(x) = K - \theta_2 + \frac{\gamma + \gamma(\gamma - \beta)}{\beta} \frac{q_H}{1 - q_H} x^{\gamma - \beta} \Delta\theta + \gamma x^{\gamma - 1} \left( \frac{\gamma}{\beta} - 1 \right) \Delta\theta > 0,$$

for  $\gamma > \beta$ . Therefore, there exists a unique  $x^* \in (0, 1)$  solving Eq. (105).

Therefore, for the region defined by  $\xi / \Delta q < (P / \hat{P}_3^*)^\gamma \Delta\theta$ , where

$$\hat{P}_3^* = \frac{\beta}{\beta - 1} \left[ K - \theta_2 + \frac{q_H}{1 - q_H} \frac{\gamma}{\beta} (x^*)^{\gamma - \beta} \Delta\theta \right] \tag{107}$$

the optimal contract can be written as

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) (x^*)^\gamma \Delta\theta \right], \tag{108}$$

$$P_2 = \hat{P}_3^*, \tag{109}$$

$$w_1 = \left( \frac{P_1}{P_2} \right)^\gamma \Delta\theta \tag{110}$$

and

$$w_2 = 0. \tag{111}$$

Finally, if  $\xi / \Delta q < (P_0 / \hat{P}_3^*)^\gamma \Delta\theta$ , constraint Eq. (54) is not binding, consistent with our conjecture.

C.2. The joint hidden information/hidden action region

We derive the optimal contract in this region by conjecturing that both Eqs. (53) and (54) bind. Solving these two equality constraints gives Eqs. (56) and (57). Plugging Eqs. (56) and (57) into the first-order condition Eq. (101) gives

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) \left( \frac{P_1}{\hat{P}_J} \right)^\gamma \Delta\theta \right]. \tag{112}$$

The solution for  $P_1$  is  $\hat{P}_1$ , the same solution for  $P_1$  as in the hidden action region. In Section 5 we proved that a unique  $\hat{P}_1$  exists, where  $\hat{P}_1 \in (0, P_1^*)$ . Naturally, we have  $w_1 = (P_1 / \hat{P}_J)^\gamma \Delta\theta$ . As before, we have verified that Eq. (98) does not bind in this region, because  $\hat{P}_1 < \hat{P}_J$  implies that  $w_1 < \Delta\theta$ .

We know that the only possible regions in which both Eqs. (53) and (54) bind is  $(P_0 / \hat{P}_3^*)^\gamma \Delta\theta \leq \xi / \Delta q \leq (P_0 / P_2^*)^\gamma \Delta\theta$ , because we have already shown that in the other regions only one of these constraints binds.<sup>16</sup> Equivalently stated in terms of  $\hat{P}_J$ , this

<sup>16</sup>In the region  $\xi / \Delta q > (P_0 / \hat{P}_1)^\gamma \Delta\theta$ , it can be shown that effort cannot be induced. This result is available upon request.

region is characterized by  $P_2^* < \hat{P}_J < \hat{P}_3^*$ . We now verify that the above solutions are optimal for this entire region. Recall that  $\lambda_1 + \lambda_3 = (P_0/\hat{P}_1)^{\beta-\gamma}$ . Therefore, if we show that  $\lambda_1$  lies within the range defined by

$$0 < \lambda_1 < \left(\frac{P_0}{\hat{P}_1}\right)^{\beta-\gamma}, \tag{113}$$

then we have shown both Eqs. (53) and (54) bind ( $\lambda_1, \lambda_3 \neq 0$ ).

The first-order condition with respect to  $P_2$  implies that

$$\lambda_1 = \left[ \frac{\beta}{\beta-1} \frac{q_H}{1-q_H} \frac{\gamma}{\beta} \left(\frac{P_0}{\hat{P}_J}\right)^{\gamma-\beta} \Delta\theta \right]^{-1} (\hat{P}_J - P_2^*). \tag{114}$$

Because  $\hat{P}_J > P_2^*$ , we have confirmed that  $\lambda_1 > 0$ . We next prove that  $\lambda_1 < (P_0/\hat{P}_1)^{\beta-\gamma}$ .

Expressing  $\lambda_1$  as a function of  $\hat{P}_J$ , we can rewrite Eq. (114) as

$$\lambda_1(\hat{P}_J) = \left(\frac{P_0}{\hat{P}_1(\hat{P}_J)}\right)^{\beta-\gamma} \left(\frac{\hat{P}_1(\hat{P}_J)}{\hat{P}_J}\right)^{\beta-\gamma} \left[ \frac{q_H}{1-q_H} \frac{\gamma}{\beta} (P_2^* - P_1^*) \right]^{-1} (\hat{P}_J - P_2^*). \tag{115}$$

Note that from Eq. (112),  $\hat{P}_1$  is a function of  $\hat{P}_J$ ; we make this functional dependence explicit in the above equation. Proving that  $\lambda_1 < (P_0/\hat{P}_1)^{\beta-\gamma}$  over the region  $\hat{P}_J \in (P_2^*, \hat{P}_3^*)$  is equivalent to showing that

$$N(x) > 0 \quad \text{for } x \in (P_2^*, \hat{P}_3^*),$$

where  $N(x)$  is defined by

$$N(x) = P_2^* + \left(\frac{\hat{P}_1(x)}{x}\right)^{\gamma-\beta} \frac{q_H}{1-q_H} \frac{\gamma}{\beta} (P_2^* - P_1^*) - x. \tag{116}$$

Using implicit differentiation in Eq. (112), we can write

$$\frac{d\hat{P}_1(x)}{dx} = \frac{\hat{P}_1(x)}{x} \frac{\gamma(\hat{P}_1(x) - P_1^*)}{\gamma(\hat{P}_1(x) - P_1^*) - \hat{P}_1(x)} > 0, \tag{117}$$

because  $\hat{P}_1(x) < P_1^*$  in this region. Therefore,

$$\frac{dL(x)}{dx} = (\gamma - \beta) \frac{q_H}{1-q_H} \frac{\gamma}{\beta} (P_2^* - P_1^*) \left(\frac{\hat{P}_1(x)}{x}\right)^{\gamma-\beta-1} \frac{1}{x^2} \left(x \frac{d\hat{P}_1(x)}{dx} - \hat{P}_1(x)\right) - 1. \tag{118}$$

From Eq. (117),

$$x \frac{d\hat{P}_1(x)}{dx} - \hat{P}_1(x) = \frac{(\hat{P}_1(x))^2}{\gamma(\hat{P}_1(x) - P_1^*) - \hat{P}_1(x)} < 0, \tag{119}$$

because  $\hat{P}_1(x) < P_1^*$  in this region. Therefore,  $N'(x) < 0$ , for  $x \in (P_2^*, \hat{P}_3^*)$ . Because  $N(\hat{P}_3^*) = 0$ , we thus have  $N(x) > 0$  for  $x \in (P_2^*, \hat{P}_3^*)$ . This confirms that  $\lambda_1, \lambda_3 > 0$  in this entire region, and therefore both Eqs. (53) and (54) bind.

### C.3. The hidden action only region

Suppose that Eq. (54) binds, while Eq. (53) does not. Thus,  $\lambda_1 = 0$  and  $\lambda_3 = (P_0/P_1)^{\beta-\gamma} > 1$ . With  $\lambda_1 = 0$ , Eq. (102) implies that  $P_2 = P_2^*$ .

A binding Eq. (54) implies that the wage payment is

$$w_1 = \left(\frac{P_1}{P_0}\right)^\gamma \frac{\xi}{\Delta q} = \left(\frac{P_1}{\hat{P}_J}\right)^\gamma \Delta\theta, \quad (120)$$

where  $\hat{P}_J$  is given in Eq. (56). Substituting Eq. (120) into Eq. (101) gives  $P_1 = \hat{P}_1$ , the root of the expression given in Eq. (59). Section 5 proves that a unique  $\hat{P}_1$  exists, where  $\hat{P}_1 \in (0, P_1^*)$ .

To ensure that our conjecture that Eq. (98) is not binding, Eq. (120) implies that we need to check if  $\hat{P}_1 < \hat{P}_J$  holds. This inequality can be written as  $\xi/\Delta q < (P_0/\hat{P}_1)^\gamma \Delta\theta$ , which is assured to hold in this region. To be consistent with the fact that Eq. (53) does not bind, we need  $(P_0/P_2^*)^\gamma \Delta\theta > \xi/\Delta q$ , which again holds in this region.

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