Revenue Management Under a General Discrete Choice Model of Consumer Behavior

Kalyan Talluri
Department of Economics and Business
Universitat Pompeu Fabra, Barcelona

Garrett van Ryzin
Graduate School of Business
Columbia University, New York
gjv1@columbia.edu

Abstract

We analyze an airline yield management problem on a single flight leg in which the buyers' choice of fare classes is modeled explicitly. The choice model we use is very general and includes a wide range of discrete choice models of practical interest. The optimization problem is to find, at each point in time, the optimal subset of fare classes to offer. We characterize the optimal policy for this problem exactly and show it has a surprisingly simple form. The analysis also provides insights into when so-called "nested allocation policies" (a popular form of control in practice) are optimal. (This paper is an abreviated version of a paper by the same title submitted to *Management Science*.)

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1 Introduction and Overview

Yield (or revenue) management is a practice that dates back to the deregulation of the U.S. airline industry in the late 1970's. It was developed as an outgrowth of the need to manage capacity sold at discounted fares, which were targeted to leisure travelers, while simultaneously minimizing the dilution of revenue from business travelers willing and able to pay full fares.

Concurrent with the evolution of industry practice, a considerable amount of management-science literature on yield management has been published over the last twenty years. See Littlewood's [17], Belobaba [5], [6], [7], Brumelle and McGill [12], Curry [14], Lee and Hersh [16], Robinson [20] and Wollmer [21] for work on managing capacity on a single flight leg. A key result of this work is that the optimal policy can be implemented using a set of so-called *nested allocations.* (See Brumelle and McGill [12].) Recent surveys of yield management research are provided by McGill and van Ryzin [18] and Talluri and Barnhart [3].

Despite the success of this body of work, most of the above-mentioned models make a common, simplifying assumption; namely, that consumer demand for each of the fare products is completely independent of the controls being applied by the seller. However, casual observation - and a brief reflection on one's own buying behavior as a consumer - suggests that this is not the case in reality. The likelihood of selling a full fare ticket may very well depend on whether a discount fare is available at that time; the likelihood that a customer buys at all may depend on the lowest available fare, etc. Clearly, such behavior could have important revenue management consequences and should be considered when making control decisions.

We lay no claim to uncovering this deficiency. Indeed, many researches have tried to address "buy-up" (buying a higher fare when lower fares are closed) and "buy-down" (substituting a lower fare for a high fare when discounts are open) effects in the context of traditional models. See Phillips [19], Belobaba [5], Andersson [2] and Algers and Besser [1], and Belobaba and Hopperstad [9]. However, to date there is no methodology that directly and completely addresses this problem.

In summary, while many attempts have been made to understand the impact of choice behavior on traditional yield management methods and to develop heuristics that partially capture buy-up and buy-down behavior, to date there is no methodology that directly and completely addresses the problem. In this paper, we develop a methodology that we believe substantially fills this void. We analyze a single-leg yield management problem in which we explicitly model consumer choice behavior using a general choice model, which specifies the probability of purchasing each fare product as a function of the set of available fare products. The model includes nearly every choice model of practical interest.

Given this general model of consumer choice behavior, we then formulate the single-leg, multiple-fare-class yield management problem as one of selecting a subset of fare products to offer at each point in time. We derive optimality conditions for the resulting dynamic program. While the policy might appear to be potentially complex under this model, we show that it has a simple form. First, we show that the optimal subsets can be reduced to an ordered family, $S_1, ..., S_m$, of *nondominated* subsets (the definition of a nondominated subset is defined precisely below). Typically, this family of subsets is much smaller than the number of total possible subsets. The optimal policy then consists of opening one of the sets S_k in the sequence, where the optimal index k is increasing in the remaining capacity x. That is, the more capacity we have available at any point in time, the further the optimal set is along the sequence. Moreover, we show that the optimal policy is a nested allocation policy (defined precisely below) if and only if the family of nondominated subsets is increasing - that is $S_1 \subseteq S_2 \subseteq ... \subseteq S_m$. This provides a very complete and general characterization of the cases in which nested allocation policies are optimal. We also provide conditions that gaurantee the nesting is by fare class order. We use these conditions to show that for the traditional, independent-demand model, the optimal policy is nested by fare class order.

The same conditions show that for the classical multinomial logic (MNL) choice model, the optimal policy is nested by fare class order as well.

We also develop a practical estimation procedure for our model. One major difficulty in estimating choice models in the yield management setting is that one typically cannot observe no-purchase decisions. In many industries, sale are conducted remotely and anonymously and the only available data are purchase transactions. Thus, it is often impossible to distinguish between periods with no arrival and periods in which there was an arrival and the arriving customer decided not to purchase. (An exception is when sales are direct, e.g. from the firm's own web site, in which case considerable information on no-purchases can potentially be gathered). We overcome this incomplete data problem by applying the expectation-maximization (EM) method of Dempster et al. [15] to the traditional maximum-likelihood discrete-choice parameter estimation. The method allows us to simultaneously estimate both the parameters of the choice model and the arrival rates using only transaction data on sales. Together, our estimation procedure and optimization model provide a theoretically sound and quite complete approach to the single-leg problem with choice behavior. numerical examples.

Our analysis provides a quite complete characterization of optimal policies under a general choice model of demand. The fact that the optimal policy consists of selecting a set from a sequence of nondominated sets - and that the optimal set to select is further along the sequence the more capacity one has available - is strikingly simple given the prima facia complexity of the problem. Moreover, the analysis based on nondominated sets provides insight into when nested and nested-by-fare-class policies are optimal, which is useful in understanding both the traditional independent demand model, as well as new demand models such as the MNL.

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