Integration of Unemployment Insurance with Retirement Insurance*

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ABSTRACT
This paper analyzes a social insurance system that integrates unemployment insurance with a pension program, allowing workers to borrow against their future wage income to finance consumption during an unemployment episode and thus improving search incentives while reducing the risks arising from unemployment. This paper identifies the conditions under which integration improves welfare and the factors which determine the optimal degree of integration. We show that when the duration of unemployment is very short compared to the period of employment or retirement, the optimal system involves exclusive reliance on pension-funded self-insurance. This system imposes a negligible risk burden for workers while avoiding attenuating search incentives. We also argue that joint integration of several social insurance programs with a pension program through an individual account is desirable unless the risks are perfectly correlated with each other.

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I. Introduction

The East Asian crisis brought home to much of the developing world a lesson that the Great Depression brought home to more advanced countries seventy years ago—the importance of a safety net. But as countries like Korea go about constructing their safety nets, they are cognizant of the complaints that have been raised against unemployment insurance systems: they attenuate incentives. To be sure, there are adverse incentive effects (or, as they are today generally referred to, moral hazard effects) in all insurance programs. What worries critics is that the risk reduction benefits might, on the face of it, be outweighed by the adverse incentive effects. For most individuals, a typical spell of unemployment is less than six months (and that spell would presumably be shorter, possibly much shorter in the absence of unemployment insurance.) Over a working time of, say, forty-five years, an individual with three such spells would lose perhaps 4 percent of his lifetime income—a risk which presumably the individual could easily absorb if he had sufficient savings or could borrow against future earnings. With the bulk of savings used for retirement, and mostly dedicated to social security programs, the amount individuals have to buffer themselves against these income shocks is limited; and well documented limitations in capital markets make it difficult for individuals to borrow much against future earnings. Thus compulsory old age public pension programs, while they help resolve one problem—the tendency of individuals not to save enough for their old age, because of the possibility of public “bail-outs”1—exacerbate another.

This naturally leads to the suggestion of an integrated unemployment and pension program, which we will call the integrated unemployment insurance (UI) system. Such integration makes particular sense with individual accounts, which are increasingly forming the basis of even public pension

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1 Individuals who find their incomes in retirement insufficient because of a failure to save, or because they have invested in risky assets which have failed, will receive some form of assistance from the government. That is, if society feels that it cannot let those with inadequate income to support themselves in their retirement suffer excessively, individuals know that if they do not save, or if they invest in risky assets that fail to yield a return, they will be ‘bailed out.’ This creates a moral hazard problem, which can be mitigated through forced savings programs which restrict the extent of risk taking. Typical public pension (social security) programs represent a simplified version of such restrictions; individuals must ‘save’ a certain amount, and the proceeds are invested in treasury bills. See J. E. Stiglitz,(1993).
programs. In such programs, benefits are related to contributions by simple formulae; in the simplest form, there is no redistribution. Such programs are like defined contribution pension programs, though some of the contributions can be used to “purchase” insurance (e.g. against inflation or interest rate fluctuations) which is not available on the market. Although it would be simple to impose redistributions on top, for simplicity, this paper ignores the redistributive components.

Under the integrated UI program, an individual who is unemployed can have his unemployment payments taken out of his individual account. Thus, the individual obtains the liquidity to maintain his standard of living; the compulsory and universal nature of the contributions provides, in effect, perfect collateral, so that early on in his life, his account balance could become negative. Because normally the risk is small, the individual can bear this risk—when it is spread out over his entire life; and since the individual bears the risk, there is no attenuation of job search incentives.

If, however, the loss from unemployment is substantial, it is optimal to have some true unemployment insurance—the individual should not bear the cost even over his lifetime. In general, individuals should not rely exclusively on the pension-funded self-insurance provided by an integrated UI system. (In this paper we take this ‘lifetime’ unemployment insurance to be tax-funded, but the results would be identical with a mandatory private insurance program with competitively determined premia.)

We characterize in this paper the optimal mix of the two types of unemployment benefits—tax-funded UI benefit and pension-funded borrowing (a form of self-insurance), and to identify the circumstances in which integration is relevant (that is, welfare enhancing). The lower the risk-aversion, 


3 Note that any redistribution imposed involves some efficiency costs. See J. E. Stiglitz (1999). Some of the detailed redistribution issues associated with the integration will be discussed later in this paper.

4 One might worry: doesn’t the ability to borrow effectively against future pension benefits undermine the rationale provided earlier for a public pension system. The answer is no. The moral hazard issues alluded to there do not arise, both because the resultant reduction in consumption is controlled by the government, and because borrowing is only allowed under the stipulated circumstances. There remains the moral hazard problem associated with search, which is the subject of this paper.

5 Since in our model, there are no adverse selection problems. It should thus be clear that in our model, though with UI there is a transfer of income from those who are lucky (who are not unemployed) to those that are unlucky, this is no different from the redistribution which occurs in any insurance market.

6 Individuals also “self-insure” through savings accumulated prior to unemployment. See below.
or the greater the elasticity of reemployment with respect to the insurance benefit, the less reliance should be placed on tax-funded insurance as opposed to (what might be viewed as implicit) pension-funded self-insurance. In an extreme case, if a worker is risk-neutral, then there should be no need for tax-funded unemployment insurance, and if there is no incentive problem associated with unemployment insurance, there is no need to rely on pension-funded self-insurance. Not surprisingly, the larger the risk, which in turn is related to the length of the period of unemployment relative to the working period, the greater the need for tax-funded insurance. In the limit, if the period of unemployment is vanishingly minute, then the individual can bear all the risk through pension-funded self-insurance with no welfare loss. The fact that unemployment risk is small compared to one’s lifetime payoff may suggest a heavy reliance upon self-insurance.

Even if individuals must rely on self-insurance, it may be possible to finance the required amount of self-insurance by savings made prior to unemployment. Thus, whether integration would enhance welfare depends upon the amount of pre-unemployment savings an individual makes, as well as upon the amount of “required” self-insurance. We identify in this paper the circumstances under which integration is welfare-enhancing. Integration is more likely to be welfare enhancing as the optimal mix of unemployment benefits entails a smaller amount of UI benefit, i.e., in the case of lower risk-aversion, higher search elasticity and lower unemployment risk.

The effects of self-insurance through savings and borrowings upon optimal unemployment insurance have recently been discussed in the optimal UI literature. Earlier literature focusing on the effect of UI on search intensity, had discussed how benefits and wage tax should vary over time. For simplicity, these models had, however, ignored the possibility of savings and borrowing, which clearly

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7 See, in particular, Hopenhayen and Nicolini(1997) Shimer and Werning(2003), Werning(2002) and Costain(1999). Later work (Kocherlakota(2004), Werning (2002)) take up savings. In particular, Werning (2002) who shows that earlier results were highly sensitive to the assumption that there is no savings and borrowing. Because of the complexity of the models, most of the literature employs higher parametric approaches, with some relying on solving recursively dynamic programming problems using first order conditions. But as Arnott and Stiglitz (1988) point out, problems with moral hazard are intrinsically non-concave, so the first order approach cannot be relied upon. Kocherlakota(2004) verified this for the problem at hand. For our purposes, most of the earlier literature referred to here suffers from one further problem: individuals do not have a period of employment prior to becoming unemployed. These papers offer one advantage over that presented here—in our basic model, the individual faces a risk of unemployment in only one period. While we explain how our model can be generalized to encompass multiple episodes of unemployment, eventually, hopefully, later literature will address both sets of issues within the compass of a single model.
can affect search intensity. These authors, however, do not address the central questions which are of concern here: when can the first and second best solutions be decentralized, when does self-insurance make any unemployment insurance unnecessary, when does private savings make any borrowing against pension savings unnecessary, and, more generally, when is there a social gain from allowing borrowing against such pension savings. The one set of papers that touch more closely on these issues recognizes that improving capital markets reduces the need for UI. Hansen and Imrohoroglu (1992) and Hamermesh (1982) empirically examined the welfare effect of a borrowing constraint for an unemployed worker by estimating how much UI increases his welfare, while Flemming (1979) showed by simulation that the optimal replacement rate of UI would be reduced in the presence of a perfect capital market.

Unemployment is, of course, only one of many risks that individuals face. There are, for instance, the risks of disability and health, as well as unemployment. The idea of integration can be applied to each of these risks, leading us to consider what we call an integrated lifetime insurance program. With integrated lifetime insurance, pension savings can be used to provide cover for these risks. The Provident Funds of Malaysia and Singapore provide prototypes of such an integrated program. The problem is that while the loss from any one of these risks may be small, there is some chance the individual may experience all of these losses. In that case, the funds available to an individual at the time of retirement may be reduced to an unacceptably low level. If the government has to provide lifetime insurance to cover such contingencies—in effect, bailing out the individual account because it would provide an unacceptably low level of pension—the borrowing by an individual to smooth consumption to cover any one of these risks would have an adverse disincentive effect much as the tax-funded benefit does. These considerations might appear to diminish the relevance of joint integration.

We argue in this paper, however, that so long as the risks are not perfectly correlated then it always pays to integrate all the social insurance programs with the pension program rather than to have

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9 Shavell and Weiss (1979) also examined the optimal UI benefit structure in the presence of perfect capital market, but in the absence of moral hazard on the part of workers.
separate insurance programs covering each risk. The key point is that the integrated lifetime insurance system allows a given amount of pension savings of an individual to be used for benefits under all the risks. This benefit of joint integration—having a common pool from which to draw upon—gets larger as the correlation gets smaller.

In the next section a basic model is presented to characterize the constrained optimum and to examine its properties, and Section III shows how the integration of UI with an (optimally designed) pension program can support the optimum. Section IV identifies the circumstances under which the constrained optimum entails no unemployment insurance and under which an integrated UI program improves welfare. Section V extends and generalizes the analysis of the previous sections in several directions, and an informal presentation of an integrated lifetime insurance system incorporating multiple risks is provided in Section VI.

II. The Model

Consider a worker who spends (M+2) periods working and retires for the remaining N periods, as depicted in Figure 1. An individual may experience unemployment shock in each period of working after the first with probability $q_t$ for $t = 2, \ldots, M+2$. Unemployment tends to occur earlier rather than later in an individual’s career, and accordingly for analytical simplicity we assume that $q_2 = q (> 0)$ and $q_t = 0$ for $t \geq 3$.10 The assumption of a single unemployment spell also allows the model to analyze unemployment benefits as a lifetime unemployment insurance in a simple way, i.e., to clearly analyze the factors determining the desired mix between UI benefit and self-insurance in the lifetime context.11

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10 This model thus makes simplifying assumptions about unemployment incidence and spells that ignore the recurrent nature of the incidence and stochastic nature of the spell. As will be discussed later in this paper (section V), the multiplicity of unemployment shocks and the stochastic nature of unemployment spells would not affect the main argument of this paper on the welfare advantage of integration. (It would, of course, affect the extent of relative advantage of pension-funded borrowing and thus optimal benefit mix of an integrated UI program.)

11 In a model with multiple unemployment spells the benefit structure of a lifetime UI would be very complicated because the unemployment benefit for one unemployment spell would be different from that for another as the expected lifetime income for an unemployed individual and thus risk and incentive effect of unemployment are different between the two unemployment spells. Thus the lifetime UI would be different from
The length of unemployment depends upon the search intensity of an unemployed worker. In this paper we assume that a worker can choose either ‘no search’ or ‘search’. If he searches, he gets reemployed immediately after an unemployment shock. If he does not search, he remains unemployed for one period. Thus, depending upon his search decision, a worker with an unemployment shock can either be employed or unemployed in period 2. The cost of search, $e$, is a random variable, which is distributed with distribution function $F(e)$. There is a threshold search cost $e'$, such that he chooses to search (or not to search) if $e < (or >) e'$. Thus the probability of being unemployed in period 2 is $q(1-F(e'))$.

We suppose there is a public pension program, under which an individual worker saves a certain portion of his income. In an integrated UI system, the money an individual has to spend if he is unemployed comes from two sources: from a formal unemployment program, funded by an unemployment insurance tax, and from past and prospective savings. If the period of unemployment is short enough and/or the mandated level of savings is high enough and if the individual is allowed to borrow as much as he wants from his pension account, there would be no need for additional precautionary savings to cover the risk of unemployment.

The objective of the integrated UI system is to provide efficient lifetime consumption smoothing and risk absorption, while minimizing the adverse search incentives. We will ascertain in our paper what this entails, e.g. identifying when integration is welfare improving, and when with integration there is no need for a supplemental formal unemployment insurance program.

We approach the problem in several stages. We first pretend that the government could control perfectly consumption at every date in every state and circumstance (that is, whether the the regular UI system we see in reality, which entails the same benefit structure over time during any unemployment spell.

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12 Although it does not matter in our model (since the only risk is associated with unemployment) whether the pension program is of a defined-contribution or of a defined-benefit type, here we will assume for simplicity that an individual’s pension is determined simply by his contributions.

14 Note that it is not always the case that removing a borrowing constraint to yield consumption smoothing over time for an individual is welfare-increasing in the second-best world. Consumption smoothing over time, for example, could have reduced the cost of being unemployed so as to induce unemployed individuals to choose less search.
individual does or does not face an unemployment shock) as well as search (which will presumably depend on the cost of search $e$) to characterize the *unconstrained* optimum.\textsuperscript{15} We then analyze the *constrained* optimum, where the government cannot *directly* control whether individuals search, although it does know that their search decisions will depend on the benefits provided. We characterize the set of unemployment insurance tax and benefits as well as savings and retirement benefits the government needs to achieve this constrained optimum, examining, in particular the circumstances in which integration is welfare enhancing.

\textbf{II.1. Unconstrained Optimum vs. Constrained Optimum}

Let $C_1$ be the consumption in period 1 for an individual. From period 2 on, there are two different states possible for an individual: ‘unemployed in period 2’ (U) and ‘employed in period 2’ (N). An individual in one state chooses a different consumption profile from an individual in the other state.

Let $C_{u,t}$, $C_{u,t}$ (for $t > 1$) be the amount of consumption at time $t$ for an individual in state N, U, respectively. Then, assuming that consumption $C$ and search effort $e$ are separable in the utility function, we can represent his expected utility as follows:

\begin{equation}
V(C_1, C_{u,t}, C_{u,t}, q) = U(C_1) + (1 - q(1 - F(e'))) \sum_{t=2}^{M+N+2} U(C_{u,t}) \\
+ q(1 - F(e')) \sum_{t=2}^{M+N+2} U(C_{u,t}) - q \int_0^e dF(e)
\end{equation}

where \{$C_1, C_{u,t}, C_{u,t}\}_t$ satisfies the following constraint:

\begin{equation}
C_1 + (1 - q(1 - F(e'))) \sum_{t=2}^{M+N+2} C_{u,t} + q(1 - F(e')) \sum_{t=2}^{M+N+2} C_{u,t} = (M + 2 - q(1 - F(e')))w.
\end{equation}

\textsuperscript{15} The unconstrained optimum may be different from the first-best optimum in that it does not take into account the possible externality of individual search activity. This externality is likely to be greater when there is a greater scarcity of jobs, i.e. the economy is in recession.
In the above formulation, we have made two simplifying assumptions: No discounting and constant wages. These assumptions will be relaxed in section IV. Constraint (2) says simply that the sum of the expected consumption over a lifetime should be equal to the expected lifetime wage income earned. This can also be interpreted as an aggregate constraint based upon the law of large numbers.

**Unconstrained Optimum**

In the unconstrained optimum, the government controls $e$ (so there is no moral hazard problem) and can perfectly smooth consumption (there is, in effect, perfect insurance.) Maximizing the expected utility function $V$ subject to constraint (2), with respect to consumption in each date and state \( \{C_1, \hat{C}_{n,t}, \hat{C}_{u,t}\} \) and with respect to the threshold search cost $\hat{e}$, we have

\[
\hat{C}_1 = \hat{C}_{n,t} = \hat{C}_{u,t} = \hat{C} = \frac{M + 2 - \hat{q}}{M + 2 + N} w,
\]

for all $t (> 1)$—there is complete consumption smoothing - and

\[
\hat{e} = U'(\hat{C}) w,
\]

where $\hat{q} \equiv q(1 - F(\hat{e}))$. In our model, search always yields a job\(^{16}\), which generates a wage of $w$ and a marginal utility of $U'w$; it pays to search so long as the cost of search (in utility terms) is less than the marginal utility of the income generated.

The expected utility $\hat{V}$ of an individual under the unconstrained optimum would then be

\[
\hat{V} = (M + 2 + N)U(\hat{C}) - \int_0^{\hat{e}} edF.
\]

\(^{16}\text{In general, however, search may not yield a job. This is both because of the stochastic nature of search and a shortage of jobs. With a given number of jobs available to individuals, search generates an externality: when one person finds a job, it makes it less likely that others do. In this respect the optimal threshold search cost $\hat{e}$ in this model, where the externality of search is not taken into account, should be distinguished from the first-best one. The first best socially optimal search intensity would be somewhat less than that indicated above.}\)
Constrained optimum

In actuality, however, the government cannot directly control individual search behavior. If the government guaranteed consumption irrespective of search, no individual would have an incentive to search. The threshold search cost $e'$ chosen by an individual worker will be the one that maximizes his expected utility $V$ without taking into account its effect upon constraint (2). Thus, we have

$$e' = \sum_{t=2}^{M+N+2} U(C_{n,t}) - \sum_{t=2}^{M+N+2} U(C_{u,t}).$$

The optimal outcome that the government can achieve given its inability to control individual search behavior, otherwise called the constrained optimum, can be characterized by maximizing the expected utility $V(.)$ subject to the constraint (2) and the individual search behavior constraint (6). A detailed analysis of the constrained optimum will be given later in this section. But one can easily see that the additional constraint (6) implies that the set of consumptions $\{C_{1}^{*}, C_{n,t}^{*}, C_{u,t}^{*}\}$ and threshold search cost $e^{*}$ under the constrained optimum satisfies

$$C_{n,t}^{*} = C_{u}^{*},$$
$$C_{u,t}^{*} = C_{u}^{*}$$

for all $t = 2, \ldots, M+N+2$, and

$$C_{n}^{*} > C_{u}^{*}$$

$$e^{*} = (M + N + 1)\{U(C_{n}^{*}) - U(C_{u}^{*})\}.$$
The condition (7) says that, from period 2 on, the consumption in each period should be equal for each state N or U, which is also required for the unconstrained optimum. The condition (8), however, states that per period consumption for those in state U should be less than that for those in state N: There is incomplete insurance. Individuals have to be provided with incentives to search, and this requires that there be a difference in well-being depending on whether the individual is or is not unemployed.  

II.2. Characterizing the Constrained Optimum

In this section we will characterize the constrained optimum set of savings/dissavings and unemployment, presuming that the capital market is perfect so that an individual can borrow against his future income. In our model the UI tax $T$ is paid only in period 1. The income available for consumption when the individual is unemployed in period 2 consists of the benefit $r$ funded by UI tax $T$ and the benefit $R$ funded by lifetime savings. $R$ represents the self-insurance of an unemployed worker, and can be financed either by his pre-unemployment savings or by borrowings from his future pension savings. (Later, the distinction between these two will turn out to be important; with perfect capital markets, $R$ represents simply a subtraction from what can be spent in other periods over his lifetime.).
Let $s_1$, $s_u$, and $s_n$ be the savings rate in period 1, the savings rates for those with state U and N, respectively. Then, we have

$$\begin{align*}
C_1 &= w(1 - s_1 - T) \\
C_{n,t} &= w(1 - s_n), \quad t = 2, \ldots, M + 2 \\
C_{n,t} &= w\frac{(M + 1)s_n + s_1}{N}, \quad t = M + 3, \ldots, M + N + 2 \\
C_{u,t} &= w(r + R) \\
C_{u,t} &= w(1 - s_n), \quad t = 3, \ldots, M + 2 \\
C_{u,t} &= w\frac{Ms_u + s_1 - R}{N}, \quad t = M + 3, \ldots, M + N + 2
\end{align*}$$

(10)

Let $\{s_1^*, s_n^*, s_u^*, r^*, R^*, T^*\}$ be a set of the optimal variables that can achieve the constrained optimum. Then, by substituting (10) into the expected utility function $V(\cdot)$ in (1), we can solve for these variables by maximizing $V(.)$ subject to the individual search behavior constraint (6) and the government budget constraint. More specifically, the variables $\{s_1^*, s_n^*, s_u^*, r^*, R^*, T^*\}$ solve the following optimization problem:

$$\begin{align*}
\text{Max} \quad V(\cdot) \\
\text{subject to} \\
(12) \quad e^* &= I(M, N, s_1) - J(r, M, N, s_1) \quad \text{(Individual Search Constraint)} \\
(13) \quad T &= \bar{q}r. \quad \text{(Government Budget Constraint)} \\
(14) \quad r &\geq 0 \quad \text{(Non-negativity of UI benefit)}^{22}
\end{align*}$$

$^{21}$ Equations 10 simply represent a recharacterization of consumption, making use of the previously established result that, conditional on the individual being in a state where he was employed (unemployed) in period 2, consumption in subsequent periods is perfectly smoothed.
where

\[ \bar{q} \equiv q(1 - F(e^\ast)) \]

\[(15) \quad I(M, N, s_1) = \max_{s_1} (M + 1)U((1 - s_w)w) + NU\left\{ \frac{(M + 1)s_n + s_1}{N} w \right\} \]

\[(16) \quad J(r, M, N, s_1) = \max_{s_1, r} U((r + R)w) + MU((1 - s_u)w) + NU\left\{ \frac{M_s + s_1 - R}{N} w \right\} . \]

The valuation function I(.) or J(.) measure the payoff as of period 2 for state N or U, respectively. For the purpose of simplicity, we will hereafter normalize the wage at unity.

The maximization problems (15) and (16) imply perfect intertemporal consumption smoothing for those in a given state N or U (obviously, from period 2 on, when the individual knows which state he is in). That is, for given savings \( s_1 \) and tax-funded benefit \( r \), we have

\[(17) \quad 1 - s_n^\ast \equiv C_n^\ast = \frac{M + 1 + s_1}{M + N + 1}, \]

\[(18) \quad 1 - s_u^\ast \equiv C_u^\ast = \frac{M + r + s_1}{M + N + 1} = C_n^\ast - X_{u,n}^\ast, \text{ where } X_{u,n}^\ast = \frac{1 - r}{N + M + 1}, \]

\[(19) \quad R^\ast = C_u^\ast - r. \]

Conditions (17) and (18) characterize the optimal consumption per period for a worker in state N and U, respectively. \( X_{u,n}^\ast \) is reduction in consumption per period facing a worker who is unemployed in period 2. Although the government provides a UI benefit \( r^\ast \), it will be less than the wage he would otherwise have earned; and this is necessary to provide search incentives. Given the limitation of the UI benefit \( r^\ast \), an unemployed worker may rely on a certain amount \( R^\ast \) of self-insurance that supplements \( r^\ast \), but will not bring consumption up to the level that it would have been had he not been

\[ \text{We ignore in this paper the possibility that } r < 0, \text{ i.e., that an unemployed individual is taxed.} \]
The optimal savings $s_1^*$ in period 1 and the optimal UI benefit $r^*$ (and its tax $T^*$) are determined not only by consumption smoothing across periods and states but also by their effects upon search incentives. Differentiation of (12) with respect to $r$ and $s_1$ will provide us with the following results on the search incentives:

\[
\frac{\partial e^*}{\partial r} = -U'\left(\frac{M + r + s_1}{N + M + 1}\right) < 0
\]

\[
\frac{\partial e^*}{\partial s_1} = \left\{U'\left(\frac{M + 1 + s_1}{N + M + 1}\right) - U'\left(\frac{M + r + s_1}{N + M + 1}\right)\right\} < 0.
\]

(20) shows that an individual worker’s search decision is adversely affected by the UI benefit $r$. This is the source of well-known welfare cost associated with the UI system. (21) shows that the savings in period 1 also negatively affects search effort decision, because a worker with greater savings does not take as seriously the prospect of a reduction in lifetime income as a result of a period of unemployment. Treating such savings as “self-insurance,” self-insurance adversely affects search, though not as much as ordinary insurance does.\(^{24}\) This is a standard externality-like effect that arises in insurance markets.\(^{25}\)

In characterizing the constrained optimum we will consider the two cases: Case I in which the constrained optimum involves positive amount of unemployment insurance and Case II in which the constrained optimum entails no unemployment insurance, i.e., there is complete reliance on self-insurance.

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\(^{23}\) Theoretically $R^*$ could be negative as $r^*$ becomes greater than one’s optimal consumption level in period 2, in which case a part of the tax-funded benefit has to be left for retirement consumption. Although this can be perceived as another type of integration between UI and pension, we will not pursue this possibility in this paper.

\(^{24}\) Several authors, including Costain (1999), Kocherlakota (2002) and Shimmer and Werning (2003) have analyzed the impact of savings on the intertemporal structure of UI benefits when there is not a single period of unemployment (as here.) We focus on a somewhat different question, the effect of savings on the extent to which there should be reliance on UI benefits.

\(^{25}\) See Arnott-Stiglitz, [1985].
II.2.A. Case I: constrained optimum entails a positive amount of unemployment insurance

Technically this is the case in which the solution for the above problem is an interior one. Taking into account incentive effects and using the envelope theorem, we can write down the necessary conditions for an interior solution \((s_1^*, r^*)\) as follows:

\[
(22) \quad -(U_{1^*} - U_{n^*}) + \eta(U_u^* - U_{n^*})(1 - \eta \frac{U_u^{1^*}}{U_u}) = 0, \\
(23) \quad -U_{1^*}U_u^{1^*}(1 - \eta \frac{U_u^{1^*}}{U_u}) = 0.
\]

where \(U_i \equiv U'(C_i^*), \ i = 1, n, u\) and
\[
\eta \equiv \frac{\partial q}{\partial r} = \frac{f'(e^*)}{1 - F(e^*)}U'(C_u^*)r, \ 	ext{indicating the elasticity of reemployment (for an unemployed worker) with respect to UI benefit. The equations (22) and (23) will jointly determine } s_1^* \text{ and } r^*. 26
\]

One important point about the constrained optimum is that the optimal pre-unemployment savings \(s_1^*\) will not be the same as the one an individual would choose, given UI benefit \(r^*.\) In fact, \(s_1^*\) is lower than the individually optimal savings \(\hat{s}_1,\) because at \(s_1 = \hat{s}_1\) we have

\[
(24) \quad -(U_{1^*} - U_{n^*}) + \eta(U_u^* - U_{n^*})(1 - \eta \frac{U_u^{1^*}}{U_u}) < 0,
\]

since \(- (U_{1^*} - U_{n^*}) + \eta(U_u^* - U_{n^*}) = 0\) at \(s_1 = \hat{s}_1.\) This is due to the incentive effect of \(s_1,\) reflected in the term \(1 - \eta \frac{U_u^{1^*}}{U_u}\) in (22). Greater savings leads to less search, because the costs of unemployment are lower. As in standard moral hazard problems, individual’s in determining their search behavior do not take into account the societal costs of the higher probability of the insured-

\[26\] With the assumption of constant \(\eta\) and absolute risk-aversion \(\delta,\) it is proved in the Appendix that the second-order condition is satisfied under plausible circumstances.
against event occurring. It can be shown, however, that the difference between $\hat{s}_1$ and $s_1^*$ is fairly small for plausible values of the relevant parameters.\(^{27}\) Notice also that the government can use tax on savings to get individual choose the optimal one $s_1^*$ instead of $\hat{s}_1$.\(^{28}\)

We can rewrite (22) and (23) as follows

\[
\begin{align*}
(22-A) & \quad -D_{1,n} + qD_{u,n} = 0 \\
(23-A) & \quad (1-q)D_{u,n} - \eta = 0,
\end{align*}
\]

where $D_{i,j} = \frac{U_i - U_j}{U_i}$ for $i, j = 1, n, u$.

This rewriting of the first order conditions allows up to make two other important points. Condition (22-A) suggests that the pre-unemployment savings $s_1^*$ involves *lifetime* precautionary savings against unemployment, which is defined as the amount by which consumption in period 1 would have to be reduced if the loss of unemployment is spread over an individual’s life.\(^{29}\) The *lifetime* precautionary savings just defined should be distinguished from what we refer to as the *precautionary savings against an unemployment shock*, money which is to be used next period for consumption by an unemployed worker. Without the government provision of pension-funded borrowing—or equivalently here, the ability to borrow against future wages—and in the absence of full unemployment insurance, individuals would have to rely on savings $s_1$ for their consumption.

\(^{27}\) Since individual choices $\hat{s}_1$ are determined as $-D_{1,n} + qD_{u,n} = 0$, where $D_{i,j} = \frac{U_i - U_j}{U_i}$, the size of the difference between $\hat{s}_1$ and $s_1^*$ will be set by the size of the difference between $D_{i,j}$ and $D_{i,j}^*$, which is of the order of $\left(\frac{1}{M + N + 1}\right)^2$, and is very small.

\(^{28}\) This would be consistent with the more general argument that in response to this type of externality taxing activities that are substitutes for search and first period consumption and subsidizing activities that are complements are necessary. (See Greenwald and Stiglitz [1986], and Arnott and Stiglitz [1990])

\(^{29}\) Note that lifetime precautionary savings provides only partial insurance so that $C_1 > C_u$. In particular, if $U''=0$ or if $(1/(M+N+1))^2$ is very small, the precautionary lifetime savings would be just equal to the actuarial value of the amount by which consumption each period has to be reduced, i.e., $\frac{q}{(M + N + 1)}$. 
during a period of unemployment. This is the case for the conventional UI system in the absence of integration with imperfect capital markets, which will be discussed later.

Finally, condition (23-A) suggests that the optimal UI benefit $r^*$ is set so as to balance consumption smoothing between the two states $N$ and $U$ (i.e., the provision of insurance) with concerns of incentives for insurance. While the second term of (23-A) captures the incentive cost that the increase in UI benefit $r$ causes, the first term $D_{u,n}$ reflects the marginal insurance benefit provided by $r$, which is positively related to $X_{u,n}^{30}$, the amount of consumption reduction per period for a worker in state $U$. This implies that the risk burden that an individual has to bear due to the limitation of $r$ depends upon the amount of consumption reduction per period, $X_{u,n}$, not upon the total amount of consumption reduction, $(1-r)$. This is because a perfect capital market allows an individual to spread out the reduction in lifetime income over the working and retirement periods and thus to reduce the risk burden associated with the incomplete provision of insurance against unemployment.31 This is how the capital market perfection improves the trade-off between insurance and incentive, thereby enhancing welfare. This also implies that an individual may rely less on the UI benefit and more on self-insurance in the constrained optimum than he would under the conventional (unintegrated) UI when he cannot borrow against his pension account. Indeed, it is even possible that the constrained optimum involves no UI benefit under integration, i.e., that $r^* = 0$, the case which we will turn to in the next subsection.

II.2.B. Case 2: No unemployment insurance is required.

Case 2 will be true when the non-negativity constraint (14) is binding. The condition for Case 2, therefore, will be that, at $r^* = 0$,

30 Note that $D_{u,n} \approx \delta X_{u,n} - \frac{1}{2} (\delta X_{u,n})^2$ since $\delta$ is constant.

31 Also, from the macroeconomic point of view, an Integrated UI system has a strong automatic stabilizing effect on the economy as it allows individuals to consume more than what they earn during the bad times through borrowing against their future incomes. The stabilization effect would be stronger than in a system relying on UI alone because it offers unemployed individuals to consume more than they otherwise would; it also maintains stronger search incentives, but in a serious economic downturn, it is lack of jobs, not lack of search which limits employment.
\[ (22-B) \quad -(U'_1-U'_u) + \bar{q}(U'_u-U'_u)(1-\eta \frac{U'_1}{U'_u}) = 0, \]

and

\[ (23-B) \quad -U'_1 + U'_u (1-\eta \frac{U'_1}{U'_u}) \leq 0 \]

or

\[ (23-B') \quad (1-\bar{q})D_{u,n} - \eta \leq 0. \]

**II.3. Two Main Issues**

Having characterized the constrained optimum, we are now able to deal with the two important issues: (a) identifying circumstances under which the constrained optimum would entail no unemployment insurance (or only self-insurance), (b) identifying the circumstances in which integration is welfare enhancing.

In the Section IV we will identify more in detail the parameter space (in terms of risk-aversion \( \delta \), elasticity of search \( \eta \) and relative size of unemployment shock indicated by M and N) in which no UI is necessary, and the set of parameters in which integration is relevant. Additionally, we will examine how the optimal mix of unemployment insurance and self-insurance (in Case 1) varies with the parameters.

Before analyzing these issues, we will give a more detailed explanation of the relevance of integration.

**III. Relevance of Integration**

In general, we can think of two circumstances under which we need government intervention to achieve the constrained optimum: first, the constrained optimum may not be decentralizable through individual choices in competitive markets; and second, the constrained optimum may not be realizable if capital markets are imperfect, as it requires individual borrowing against future income.
Let us first look at the decentralization issue, assuming for the moment that the capital market is perfect. It is easy to see that, given \( r \) and \( s_1 \), the optimal savings and dissavings \( \{s_h^*, s_u^*, R^*\} \) coincides with individual choices, so that they may be decentralizable with individual decision making. As for the pre-unemployment savings \( s_1^* \), it is not decentralizable by individual choices without government intervention,\(^{32}\) as evidenced by (24). As previously mentioned, however, \( s_1^* \) is decentralizable by employing a tax on pre-unemployment savings. One can also see that the optimal UI benefit \( r^* \) can be supported as an equilibrium in a competitive insurance market. Since the government budget condition (13) is equivalent to the zero profit condition for an individual insurance firm, the maximization problem for \( r^* \) will be identical to the one for an equilibrium amount of insurance in a competitive market.\(^{33}\)

With the constrained optimum being decentralizable by a (appropriately set) tax on pre-unemployment savings, the government would not need to intervene further if capital markets were perfect, so that individuals could borrow against future income.\(^{34}\) But because capital markets are not perfect, integrating UI with a pension program is required to attain the constrained optimum,\(^{35}\) whenever an individual has to borrow to sustain his optimal consumption.\(^{36}\) To see if the government has to intervene with the integrated UI program, i.e., if the integration is relevant (or welfare-enhancing), we need to check whether or not an unemployed individual will have sufficient savings available to finance the desired consumption \( C_u^* \) given the UI benefit \( r^* \).

\(^{32}\) Note, however, that the difference between \( s_1^* \) and the individual choice of \( s_1 \) is very small.

\(^{33}\) Note that all the individuals are of the same risk type in this model, so that insurance firms are not subject to adverse selection problems caused by privately-informed individual heterogeneity. Once we allow for individual heterogeneity, then the optimal UI benefit \( r^* \) for all types of individuals would not be supported as a competitive equilibrium.

\(^{34}\) There are, of course, other reasons that government intervention might be desirable. We focus here on problems of moral hazard. All insurance markets also face problems of adverse selection. In general, such markets are not constrained Pareto efficient. See Greenwald and Stiglitz [1986] and Arnott, Greenwald, and Stiglitz [1994].

\(^{35}\) Integrated UI is a way to effectively collateralize pension savings and future income in an imperfect capital market. In most countries, there are strong limits on garnishing wages, an alternative mechanism for ensuring repayment; and the rules governing garnishment can have large effects on labor supply. Especially in many developing countries, it is in any case hard to enforce credit contracts without physical collateral.

\(^{36}\) Note that some of retirement savings with which UI is integrated needs to be mandatory to serve as a collateral of the borrowings. If the mandatory rate of savings for retirement is set equal to the optimal savings rate, in our model the constrained optimum would be realizable by the integrated UI.
III.1. Conditions for Relevance of Integration

The government often introduces mandatory retirement savings program for some reasons exogenous to the model.\(^{37}\) The amount an unemployed individual would have to borrow to sustain the (constrained) optimal level of consumption in the absence of integration would then be closely related to the mandatory savings in period 1 under a public pension program, because such savings cannot be withdrawn until retirement. In other words, the level of mandatory retirement savings set by the government is one of the important parameters affecting the relevance of integration.

Let \(\alpha(>0)\) be the mandatory savings rate imposed by the government, and assume for simplicity that \(\alpha\) does not affect \(s_1^*\).\(^{38}\) The optimal amount of borrowing for an unemployed worker will then be \((C_w^*-(r^*+s_1^*-\alpha))\). Thus, integration will be needed to support the constrained optimum if the amount of borrowing required to sustain the optimum is positive, i.e., if

\[
(25) \quad r^* + s_1^* - \alpha < \frac{M}{M + N}
\]

by (18) and (19). Integration of UI with the public pension program will be welfare enhancing under the condition (25), because without integration an individual would need more UI benefit \(r\) and/or pre-unemployment savings \(s_1\) (net of mandatory savings) than the (constrained) optimal levels. It can also be seen from (25) that the increase in the mandatory savings rate would make integration more likely to be relevant. More specifically, as \(\alpha\) approaches \(s_1^*\) (all of first period savings are mandatory) then integration is \textit{always} desirable so long as in the constrained optimum, there is some pension-

\(^{37}\) These include excessive myopia—individuals when they are young do not adequately take into account the consequences of a lack of income when they are old--and the moral hazard problem described earlier--individuals expect the government to bail them out when they end up with no savings at the time of retirement may not save for retirement or may put their money into excessively risky investments.

\(^{38}\) Note that \(s_1^*\) is the optimal savings rate \textit{in the absence of mandatory} savings. When the government implements public mandatory pension program mainly because of moral hazard on the part of individuals (as mentioned in footnote 31), \(\alpha\) would not affect \(s_1^*\) (recall that \(s_1\) is defined as the \textit{total} savings, include the mandatory and the non-mandatory savings.). Also, this assumption would not affect our arguments on the relevance of integration. If \(\alpha\) changes the optimal savings rate in period 1 to \(s_1'\), for example, the LHS of (25') will be \(r^* + s_1' - \alpha\).
funded self-insurance, which there will be so long as individuals are not very risk averse and/or there is a moral hazard problem associated with the search.\(^{39}\) This suggests that an economy with a strong public pension program should integrate unemployment insurance with the pension program.

In improving welfare relative to the conventional (unintegrated) UI, there might seem to be an alternative to integrating the public pension program with the unemployment insurance scheme: reducing \(\alpha\) to zero or eliminating the public pension program. This is not the occasion to provide a full rationale for the existence of such programs.\(^{40}\) Our analysis is simply predicated on the observation that governments have chosen to provide such programs; if they do so, then our analysis has demonstrated that the unemployment insurance scheme should be integrated with it.\(^{41}\)

But note that there are a number of factors which may make such schemes more or less attractive. For instance, societies in which there is more concern that elderly individuals do not fall below a certain threshold level—but in which at the same time there are many individuals willing to take advantage of society’s compassion—will find it desirable to have at least a minimal mandatory program.\(^{42}\) The fact that mandatory pension savings cannot be withdrawn until retirement, however, means that there is a considerable welfare burden placed upon individuals who may experience several shocks other than unemployment during their lifetime. These concerns may have placed some limits on the extent of desirable mandatory savings. Integration may, accordingly, lead to still further welfare benefits (beyond those formally modeled in this paper): integration allows for an increase in

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\(^{39}\) Obviously, the result depends on our assumption that unemployment occurs early in life. If we had formulated a continuous time model, and assumed that there was some probability of unemployment in every period after the beginning of work, then obviously, some integration (if only limited to borrowing against future pension savings for early periods of an individual’s life) would be desirable so long as there is not complete unemployment insurance, i.e. some reliance on private savings.

\(^{40}\) See, for instance, Stiglitz [1992].

\(^{41}\) We can treat the public pension program as endogenous in the model and characterize the optimal mandatory pension savings and ascertain the benefits of integration, taking into account the rationale for the program. This will not be analyzed in this paper, but an important feature of the comprehensive model will be sketched in Section V. As we comment below, there are conceptual difficulties in conducting a welfare analysis, including formulating what is meant by ‘optimal mandatory savings’ when the rationale for mandatory savings is myopia. When reason for mandatory savings is ‘moral hazard,’ then the analysis is fairly straightforward, and the conclusions of this paper concerning the desirability of integration remain valid.

\(^{42}\) For a more extensive discussion of the role of government in risk bearing, see Stiglitz [1992].
the level of mandatory savings, and thus an improvement in the quality of the safety net provided to the elderly.\textsuperscript{43}

In this respect integration will be especially desirable for those with low income, for whom the incidence of unemployment is particularly high and for whom the mandatory savings constitute a large fraction of their total savings. In the United States, for instance, those at the bottom of the income distribution have little savings to which they can get access, and at the same time face higher risks of unemployment. Similarly, integration is likely to be of particular value in developing countries, where the poor both face high risks of unemployment and have low savings.\textsuperscript{44}

\textbf{III.2. Welfare Advantage of Integration}

We have stressed that the welfare advantage of an integrated UI program over the usual UI system arises from capital market imperfections. Suppose for simplicity that $\alpha = 0$ in period 1, i.e., that there is no mandatory savings. Without integration, an unemployed worker would have to use period 1 savings to maintain the optimal consumption level. When (25) holds, however, the sum of UI benefit and pre-unemployment savings under the constrained optimum is less than the desired consumption level under unemployment. In the absence of integration, therefore, an unemployed individual may not only need more UI benefits but also undertake some additional (precautionary) savings in period 1 to supplement the UI benefit $r$.

Just as the UI benefit entails a welfare cost associated with adverse search incentives, precautionary savings often imposes a welfare burden both in terms of its distortionary effects both on search and intertemporal consumption smoothing. In the absence of integration, the “third best” levels of first period savings and the unemployment benefit will be higher than they would be in the constrained optimum with integration. We now show that under this circumstance a small degree of integration, i.e., allowing for even a small amount of borrowing is welfare enhancing. In comparing the unintegrated UI with an integrated UI, we will suppose that an unemployed individual in the

\textsuperscript{43} That is, the optimal safety net depends on the extent to which there is a moral hazard problem; strong mandatory programs may reduce the extent to which moral hazard problems arise.

\textsuperscript{44} In the formal analysis of this paper, we have assumed all individuals are identical. A richer model would recognize individual heterogeneity, but that governments have to have relatively simple rules and programs that apply to all individuals.
unintegrated system can borrow zero, while in the integrated system he is faced with a borrowing constraint, \( R \), imposed by the public pension program.

Let \( \hat{r} \) and \( \hat{s} \) be the optimal UI benefit and savings in period 1 in the absence of integration (i.e., when \( R = 0 \)), respectively. Let also \( V^*(R) \) be the maximized expected utility of an individual who solves the problem (11) subject to the constraints (12)-(16) and to the additional constraint of borrowing \( R \). Then we can prove the following results.

Proposition 1.45 46 Suppose (25) holds. Then,

(i) \( r^* + s^* + \hat{r} + \hat{s} < C_u^* \),

(ii) \( \frac{\partial V^*(R)}{\partial R} > 0 \) at \( R = 0 \).

The first part of the Proposition 1 confirms the earlier intuition: Although UI benefit and precautionary savings can be increased in the absence of integration or borrowing possibility to ameliorate the consequences of unemployment shock, they would not be sufficient enough to secure the desired consumption for an unemployed individual; these are constrained due to the efficiency costs of UI benefit and precautionary savings. Under these circumstances, as the second part of Proposition 1 indicates, it pays to integrate UI with a pension by relaxing the borrowing constraint for individuals.47

IV. Identifying Circumstances For Constrained Optimum With No UI and For Welfare-Enhancing Integration

45 It should be clear that proposition 1 provides sufficient, not necessary, conditions for integration to be welfare increasing.

46 The proof of each of the propositions is contained in the NBER working paper No w9199, Stiglitz and Yun [2002 ], available at http://papers.nber.org/papers/w9199, or in the most recently revised version available at http://home.ewha.ac.kr/~jyyun/.

47 Section V provides a convincing argument that this still holds even when the public pension program is treated as endogenous.
With the conditions for the (constrained) optimum with no unemployment insurance (Case 2) and for the relevance of integration being set, in this section we will identify in greater detail the parameter space in which self-insurance is all that is desired to support the constrained optimum and the parameter space in which integration is relevant. Before conducting this analysis in the general context, we will first consider an interesting special case: when M and N are very large or when the length of unemployment spell is vanishingly small. The analysis of these two issues in the context of the limiting case is important for the two reasons. First, this limiting case may be realistic as it has been empirically confirmed by Altman-Feldstein [1998] that the size of the typical unemployment shock is fairly small compared to one’s lifetime income.\textsuperscript{48} Second, this limiting case provides us with insights not only about the desirable pattern of integration but also about its welfare benefit; in the limited case, the entire welfare advantage of integration stems from its ability to ease the risk burden caused by the insufficiency of (optimal) UI benefits (plus first period savings) by spreading out the income loss over one’s lifetime.

\textit{IV.1. Limiting Case : } $M, N \to \infty$

We will suppose that $a = \frac{M + 1}{N}$, the ratio of the working period to retirement period is kept constant. We can establish the following Proposition.

Proposition 2. As $M$ (or $N$) goes to $\infty$ with $a$ being kept constant, $r^* \to 0$, $s^*_i \to \frac{1}{1 + a}$, i.e., the constrained optimum does not entail any unemployment insurance but entails self-insurance only. When $a \geq 1$, integration is desirable. Finally, the integration approximates the unconstrained optimum.

Proposition 2 highlights one of the important aspects of an integrated UI system. When the unemployment spell is vanishingly minute relative to the individual’s working lifetime, there is no

\textsuperscript{48} This will be discussed again later in Section V.
need for an unemployment insurance scheme, i.e., full self-insurance is optimal. Also, when the working period for an individual is longer than his retirement period (which is in fact the case), integration is desirable. Importantly, integration provides perfect insurance with no attenuation of incentives, that is, integration allows for the achievement of the unconstrained optimum.\(^{49}\)

As previously noted, what we have called the unconstrained optimum may differ from the “first-best” optimum because of the externality associated with individual search.\(^{50}\) The question one can raise will then be whether integration that can support the first-best optimum. We will now show that in the limiting case, not only can integration attain complete insurance and complete intertemporal consumption smoothing, but we can also induce any desired level of search intensity, in particular that associated with the first best optimum.

To see this more clearly, we will first take UI benefit \(r\) as given. We can then rewrite (12) as follows:

\[
e(r, M, N) = (M + 1 + N) \left\{ U \left( \frac{M + 1 + s_1(r, M, N)}{M + N + 1} \right) - U \left( \frac{M + r + s_1(r, M, N)}{M + N + 1} \right) \right\},
\]

and we can prove the following Proposition.

**Proposition 3.** For any small \(\varepsilon > 0\) such that \(\varepsilon < -\sum_{i,j=1,\mu,n} C_i - C_j\), there exists \(K < \infty\) such that for \(M(\text{or } N) > K\), the integration achieves the outcome of \(\{C_1, C_\mu, C_n\}\) with the threshold search cost being \(e(r, M, N)\).

Proposition 3 demonstrates that integration can make the risk burden arbitrarily small, taking as

\(^{49}\) If \(a < 1\), however, the relevance of the integrated UI would depend upon the mandatory savings rate \(\alpha\).

\(^{50}\) We have in this paper thus identified several different “constrained” optima: (a) that in which there is an externality associated with search, but the government does (can do) nothing about it; (b) that in which there is no externality associated with search, but there is a moral hazard problem associated with search, and the government cannot perfectly offset it; but capital markets are perfect; (c) that in which there is no externality associated with search, there is a moral hazard problem associated with search, and individuals cannot borrow against future income.
given the level of search incentive determined by UI benefit $r$. Thus, the outcome of full insurance with any desired level of search incentive on the part of individuals can be approximated by the integrated UI as the unemployment spell gets shorter relative to their lifetime period. Alternatively, Proposition 3 implies that when M and N are very large an integrated UI program can be designed to achieve both full consumption-smoothing across periods and states and any desired level of search, including matching the first-best one.

IV.2. General Case

We now establish, for general case, the following Proposition on the possibility of full self-insurance and the desirability of integration:

Proposition 4. For given M and N, there exists a set of critical parameter values $(\delta^o, \eta^o)$, such that for $(\delta, \eta) \in \Omega$, where $\Omega = \{\delta, \eta \mid \delta \leq \delta^o \text{ and } \eta \geq \eta^o \}$, the constrained optimum does not entail any amount of unemployment insurance. Also, integration is relevant for $(\delta, \eta) \in \Omega$.

Proposition 4 demonstrates that a constrained optimum entails self-insurance only, when individual risk-aversion $\delta$ (or elasticity $\eta$ of reemployment with respect to search) is low enough (or high enough). It also shows that the constrained optimum can be realized by government intervention through integration. Furthermore Proposition 4 implies that a constrained optimum involves a combination of unemployment insurance and self-insurance for a certain set of parameter values.

For more detailed and graphical analysis, we will set utility function of an individual to be a following type of a constant risk aversion.

$$U(C) = -\exp(-\delta C)$$

where $\delta$ represents the constant risk-aversion of an individual. Focusing on Case 1 (i.e., on the case for interior solution of $r^*$), we can then present the following comparative statics results on the
Proposition 5.

(i) \( \frac{\partial r^*}{\partial \eta} < 0 \)

(ii) \( \frac{\partial r^*}{\partial \delta} > 0 \)

Proposition five shows that as the elasticity of reemployment (indicated by \( \eta \)) increases, the potential incentive cost of unemployment insurance grows, making it more desirable to rely on self-insurance. Greater risk-aversion of a worker (a higher \( \delta \)) implies a greater need for insurance against the risk of unemployment and thus a greater reliance on UI benefits. It is intuitively obvious that as workers are more seriously subject to moral hazard associated with search incentive or as workers less more risk-averse, the benefit of integration (or of UI) becomes greater (or less), leading to less UI and more of self-insurance.

Based upon the comparative static results of Proposition 5, we can identify more specifically (than we did in Proposition 4) the parameter space in which the constrained optimum involves self-insurance only or the parameter space in which integration of UI with retirement insurance is relevant.

Proposition 6. The constrained optimum does not entail any UI (i.e., \( r^* = 0 \)) if

\[
\eta \geq (1 - \bar{q}^o)(1 - \exp(-\frac{\delta}{M + N + 1})) = H(\delta; q, M, N),
\]

where \( \frac{\partial H}{\partial \delta} > 0 \), \( \frac{\partial H}{\partial M} \bigg|_{q} < 0 \), and \( \bar{q}^o \) is the probability of getting unemployed in period 2 when no UI is offered. Also, integration is relevant when
\[ \eta \geq G(\delta; q, M, N), \]

where \( G(\delta; q, M, N) < H(\delta; q, M, N) \) for all \( \delta \) given \( (q, M, N) \) and \( \frac{\partial G}{\partial \delta} > 0 \).

Note that \( H(.) \) or \( G(.) \) refer to the lowest value of search elasticity of reemployment that is necessary for the constrained optimum to entail no UI or for the integration to be desirable, given \( \delta \) and \( M(\text{or } N) \), respectively. Proposition 6 shows that the larger the elasticity \( \eta \) of reemployment or the smaller the risk aversion \( \delta \), and the shorter the unemployment spell (or the larger \( M \) and \( N \)), the more likely the constrained optimum is to involve no UI benefit. This makes sense because the incentive concern associated with UI benefit increases with \( \eta \) while the need for insurance against unemployment varies positively with \( \delta \) and negatively with \( M \) (or \( N \)).

In particular, the parameter space \( (\delta, \eta) \) over which \( r^* = 0 \) is depicted in Figure II; the constrained optimum never entails any UI benefit regardless of risk-aversion if the elasticity of reemployment is greater than \( (1 - \bar{q}^o) \), the probability of not being unemployed under no UI. It also indicates that, to the extent that \( M \) or \( N \) is fairly large, or that total unemployment spell for a typical individual is fairly short compared to his lifetime, \( r^* \) would be zero unless \( \eta \) is far below \( (1 - \bar{q}^o) \).\(^{51}\)

As for the relevance of integration, integration is more likely to be welfare-improving when the optimal UI benefit \( r^* \) gets smaller, because the optimal pre-unemployment savings \( s_1^* \) would not make up for the insufficiency of UI benefit.\(^{52}\) By Proposition 5, therefore, the lowest value \( G(\delta; q, M, N) \) of search elasticity \( \eta \) that warrant integration should be increasing in \( \delta \). Note that the region of the two parameters \( (\eta, \delta) \) for which integration is relevant, as indicated by the shaded area in Figure III, includes the parameter space \( (\eta, \delta) \) for which there is zero UI. This suggests that whenever the constrained optimum involves a fairly small level of UI benefit it always pays to integrate it with a pension program.

\(^{51}\) Many empirical studies have shown that the elasticity of reemployment (for an unemployed worker) with respect to UI benefit is not that small although it is mostly less than 1.

\(^{52}\) This is indicated by the slope of \( s_1(r) \) (derived from (22)) that is less than 1.
Figure III shows that the integrated UI is more likely to be relevant (i) as individuals are less risk-averse or subject to a more serious incentive problem and (ii) as the unemployment spell is shorter relative to one’s lifetime. Conversely, when the unemployment benefit \( r \) is large—as when the replacement rate approaches unity—then the individual can rely totally on first period savings to smooth consumption (consistent with the constrained optimum).

V. Remarks

V.1. Discounting and Wage Variability

We have thus far assumed away discounting in the model. Discounting may influence the main results of the model mainly through its effects on pre-unemployment savings. The relative size of interest rate compared to discount rate affects savings.\(^{53}\) When the interest rate is lower than the discount rate, for instance, the cost of precautionary savings is higher, so that an individual will undertake less savings in period 1 than he does in the current model with zero interest and discount rates. Thus, the low interest rate relative to discount rate makes it more likely that there are welfare gains associated with integration.

The variability of wage over one’s lifetime is also an important factor for the relevance of integration. Typically, wages increase with age; with a desire for intertemporal consumption smoothing, individuals will save less earlier in his career than he would were wages constant. Thus, an increasing wage profile for an individual worker increases the likelihood that integration will be welfare enhancing.\(^{54}\)

V.2. Multiple Unemployment Shocks

Up to now, we have dealt with a simple case where an individual experiences only one unemployment shock in his career. In fact, however, people may experience several unemployment


\(^{54}\) The wage variability among the heterogenous workers would also affect the relevance of integration. The integration will be more relevant for low-wage workers who save little, especially in the presence of a mandatory pension program.
shocks, which are to be covered by an integrated UI program. We will discuss briefly the effectiveness and benefit structure of the integrated UI system when there are multiple unemployment shocks.

Formally, one can view the risk of unemployment in each period as a separate risk, to be covered by a separate (unemployment) insurance policy. Each period, we can ask what the optimal mix of self-insurance and formal insurance is. Replacing some of the UI benefits by self-insurance (including borrowing against retirement funds) would still alleviate the attenuation of incentives associated with UI benefit while reducing risk burden by enabling consumption smoothing each period. Clearly, the (constrained) optimum consumption both before and after each potential episode of unemployment will be history dependent, i.e. depend on whether the individual has experienced unemployment in previous periods. We can ask whether the constrained optimum level of savings in prior years in combination with the unemployment insurance benefit suffices to sustain consumption during unemployment at the constrained optimum level. Later in life, if there is no mandatory savings (so that individuals have access to all of the funds set aside for retirement) there will be little benefit from integration. The benefit of the integration would be especially marked when the unemployment shocks occur earlier (rather than later) in one’s career. Proposition 2 argued that if the unemployment shock was small relative to lifetime income (and if the unemployment shock came early in life) then integration was likely to be of particular value. Conversely, with multiple, highly correlated shocks, there is a risk that, in effect, each unemployment shock is, in effect, large relative to lifetime, so that there is a real need for unemployment insurance and the gains from integration may be small.

Note that much of the earlier literature has focused on a rather different problem, where an individual who is unemployed in period t will remain unemployed in period t + 1 unless search is successful at time t. In terms of our model, it is as if an individual who experiences an unemployment shock at time t has a probability distribution of losses in income. (In our model, there are, in effect, only two outcomes.) Here, we worry about the fact that once the individual becomes re-employed, he still faces the risk of another episode of unemployment.

One of the questions which we do not answer in this paper is how the mix (and terms) of UI and self-insurance should vary with the state of the economy. It should be clear from our analysis, however, that the optimal system as described here depends on certain key parameters which are likely to change over the cycle. In addition, however, optimal social insurance might entail redistributions from those lucky enough to have lived a large fraction of their life during periods in which the unemployment rate is low to those unlucky enough to have spent more of their working life in a period of high unemployment. An analysis of these intergenerational redistributions would take us beyond the scope of this paper.
The question then is whether or not the total duration of unemployment for an individual is short in comparison to his lifetime, which can be explored empirically. Feldstein and Altman [1998] demonstrated in a simulation study based on PSID that if an individual deposits a modest amount out of his income to his savings account to finance his unemployment benefit (at the level under the current UI system), the terminal balance of an individual savings account is positive for most individuals. Similar studies by Folster [2002] and Yun [2002], focusing on countries such as Sweden and Korea, respectively, show that for most individuals the amount of hypothetical lifetime unemployment benefit for an individual is only a small portion of lifetime savings. These results seem to support empirically the view that there would be significant welfare gains from integration.

With multiple unemployment shocks the optimal amount of borrowing (from one’s pension) at a certain point in time is positively related to the expected level of his pension at that time. Thus, the optimal integrated UI system will involve a different mix of formal unemployment insurance and self-insurance for the unemployed depending upon their past employment histories. For an unemployed worker who has been unemployed for a relatively long period and expects to have a low level of pension savings in the future, for example, the amount of the desired borrowing out of the pension program will be small. These arguments suggest that the amount of borrowing offered to the unemployed by the integrated UI would decrease—and the UI benefit would increase-- as the total duration of previous unemployment increases.

V.3. Myopic Behavior

The integrated UI system enables an individual to self-insure himself against unemployment by using his pension and thereby maintaining his search incentive. The favorable incentive effect of self-

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57 Feldstein and Altman showed that if workers contribute 4% of wage incomes to their individual savings accounts up to a very restrictive ceiling and withdraw from the accounts the same amount of money as provided by the current UI system, only 5-6% of them may experience negative balance in their accounts at the end of their careers. Considering that the amount of contribution simulated in Feldstein-Altman is much smaller than individual pension, we can easily see that the total amount of UI benefit for an individual will be very small in general compared to his pension fund.

58 In fact, our model already incorporates this type of mandatory savings: an individual is mandated to save the optimal level $s_{k}$ after experiencing unemployment, which is greater than the optimal savings level $s_{n}$ for those with no unemployment experience in period 2.
insurance is thus based upon the presumption that an individual recognizes and responds to the future costs—reduced pension benefits, increased contributions to the pension fund to make up for the short fall—if he fails to search intensively. Some may argue, however, that an individual worker is so myopic that he may not respond fully. In particular, especially for the young unemployed workers, the repayment of borrowing or reduced pension benefits may be too remote to affect their search incentives; for such individuals, the impact on search of benefits financed through self-insurance is no different from that of government provided insurance. For such individuals, search can only be affected by differences in income experienced in the short run; that is, only limitations on the total “benefit” \((r + R)\) will induce search.

Earlier, we noted that if an individual is not myopic, he will reduce his consumption (or increase his savings) after being reemployed in order to spread out the burden of borrowing, as indicated by the savings \(s_n^*\) and \(s_u^*\) characterized in (17) and (18). This implies that a rational individual in fact would start to pay for his borrowings just after a period of unemployment. With myopia, individuals may not do this.

The government can mandate a myopic individual to save more for his retirement after experiencing unemployment. Even if individuals do not fully integrate changes in consumption thirty years from now into their behavior, they are likely to respond to policies which have a more immediate effect on consumption. Thus our model suggests that the government can use differential mandatory savings policy (effectively, basing contributions in the integrated unemployment and pension scheme on experience-rating) to help induce search. With myopic behavior, the government might need to impose a greater burden for repayment earlier in an individual’s life in order to induce the second best level of search. At the same time, it is still true that integration—allowing the individual to borrow against future pension benefits—enables greater income smoothing with less of an attenuation of incentives.

The formal welfare analysis of economies with myopic individuals is complicated by the difficulty of ascertaining the appropriate valuation function. Traditional economic analysis is concerned with individual’s expected utility over their entire life. But with myopic individuals, there is no loss to the
individual’s expected utility (viewed at the beginning) to payments made out of retirement benefits to finance unemployment benefits.

V.4. Distributional Issue

Once we introduce into the model various types of workers differing in wages, unemployment probabilities, etc. from each other, we may have to consider distributional effects in characterizing the optimal mix between UI benefit and pension-borrowings. It has often been argued that the integrated UI system with individual accounts aggravates inequality among individuals. High-wage workers, who tend to get unemployed less frequently than low-wage workers, are able to accumulate more savings than under the tax-funded UI system. By the same token, low-wage individuals are subsidized by high-wage individuals under UI because the former group tends to have high unemployment probability than the latter one. Thus, if the social welfare function puts greater weight on the payoff to the low-wage individuals than to the high-wage individuals (often the case in designing a social safety net), the optimal mix would involve more of UI benefit and less of self-insurance than characterized in this paper.

On the other hand, we would like to emphasize two other important points about the distributional aspects of integration. First, there is an element of the integrated UI system that has a favorable implication for distribution. A key point of the integrated UI system is that it allows a worker to borrow against his future savings to finance a part of his unemployment benefit. This would be particularly beneficial to the low-wage workers who do not have much (precautionary) savings. Many studies have shown that the wealth distribution among individuals (in particular, wealth that is liquid, i.e. not tied up in housing) at a particular point in time is more unequal than the distribution of annual

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61 In fact, in experience rated systems described before, the extent of redistribution through the UI system is limited.

62 For an economy where the wage is equal to the average wage. Another interesting issue may arise when individuals are privately informed about their characteristics, such as search proclivity. The optimum may involve a separating equilibrium in which different types of individuals have different degrees of integration. In this case the optimum degree of integration for an individual with strong search proclivity, for example, may be smaller than in the case of perfect information due to the self-selection constraint. If individuals differ only in their wages (w), which are observable, then the optimal system may involve different replacement rates (different degrees of UI insurance and self-insurance) for individuals at different wages; such an optimum scheme may involve redistributions from those with high w to those with low w, i.e. the budget constraint applies only to the entire system, not to each wage group.
incomes, which in turn is more unequal that the distribution of lifetime incomes. A young low-wage worker, who has a higher probability of becoming unemployed, also has a high chance of moving up within the income strata. Thus it would be more beneficial to such a worker than to, say, a high-wage worker to use his future income to finance current consumption when he is unemployed.

Second, the government can provide an explicit subsidy to compensate for the implicit subsidy under the unintegrated system. Appropriately designed explicit subsidies can mimic the distributional impacts of the implicit subsidies, and would entail no incremental adverse incentive effects.

V.5. Endogenous Mandatory Pension Program

Our analysis has so far assumed that if there is a public, mandatory pension program, there are advantages to integrating the unemployment insurance program with the pension program, enabling individuals to borrow against future pension income. Indeed, the fact that individuals cannot borrow against future income/savings provides a rationale for a mandatory pension program, in addition to the usual arguments noted earlier, myopia and moral hazard. Presumably, however, if these other rationale are important, then we should simultaneously solve for the extent of the mandatory pension program and the degree of integration. If the public pension program is to be treated endogenously in the model, the optimal mandatory savings for retirement would be determined by balancing its marginal benefit (in the light of any rationale for it) against its marginal welfare cost. While a more

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63 See Bjorklund [1993] and OECD [1996].

64 It is important to emphasize that an actuarial fair payment to a worker who is unemployed is not a subsidy; there is an implicit subsidy only when the actuarial value of the benefits paid to a worker of a particular type (e.g., a worker with a particular wage in a particular industry with a particular job history) exceeds the value of his unemployment insurance contributions.

65 An easy way to provide a subsidy to low-income individuals under the integrated UI would be to make public mandatory pension program more progressive. That is, as we introduce integration, we could redesign the public pension system such that low-income individuals contribute less while getting more at the time of retirement. This would entail some efficiency costs, of course, as the pension system gets less actuarially fair. The optimal integrated UI should also take into account the benefits and efficiency costs of introducing a greater degree of redistribution within the UI system. This remains a future research topic.

66 Note that several Latin American countries have established a kind of unemployment insurance that emphasizes self-insurance (instead of tax-funded UI) by setting up individual savings accounts for unemployment benefits, and that some of those countries have adopted redistributitional measures to counteract the adverse distributional effects of the programs. Chile, for example, has recently set up a so-called Solidarity Fund or Common Fund financed mainly by the government to guarantee a minimum amount of unemployment insurance for low-income workers who have not accumulated enough resources in their individual accounts.
complete analysis of optimal integration remains a topic for the future research, to check roughly the optimality of integration in this comprehensive framework, however, we can consider a small degree of integration of UI with a pension in the unintegrated system. The envelope theorem and Proposition 1 (ii) suggest that it always pays to integrate UI with a pension (if UI is not integrated) even when pension program is endogenously determined in the model.

Suppose, on the other hand, that the reason for the public pension program is that there is a social norm that dictates a minimum standard of living, and that this is guaranteed by the public pension. This is a case where one might have thought that there might be no welfare gains from integration, since integration would require an increase in the mandatory level of pension savings, to avoid individuals who face unemployment and who borrow against future pension income falling below the minimum norm. To see that even in this extreme case some degree of integration is welfare-enhancing, assume (as the worst case) that there is no incremental (social) benefit attached to any extra pension savings above the level set in the original situation (before integration), But, so long as the mandatory savings is less than that which the individual would do on their own, the welfare cost of the incremental mandatory savings would be zero because one can borrow against it whenever necessary at the time of unemployment, while the integration is welfare-increasing by Proposition 1 (ii).

VI. Toward a Joint Integration of Multiple Social Insurance Programs with Pension: Integrated Lifetime Insurance

We have argued that allowing a worker to withdraw from his pension to finance his consumption when faced with an unemployment shock can yield some welfare improvement. The idea of integration can be applied to other forms of social insurance than UI, such as health and disability insurance, which cover various shocks an individual may face in his life. We can then think of a comprehensive integration system that jointly integrates several social insurance programs with a pension program through an individual account. We refer to this as an integrated lifetime insurance system. If an individual experiences only a few shocks, so that the amount of corresponding loss is small compared
to his retirement savings, the previous argument for integration should still apply. Suppose, however, that an individual experiences so many of those shocks that were there heavy reliance on borrowing against future pension savings, his retirement account would be so drained that retirement consumption would fall below the socially acceptable level; there would have to be a government bailout. Since early in his life, the individual does not know what shocks await him in the future, there would seem to be a risk associated with integration. Given this possibility associated with multi-risk case, we will in this subsection briefly examine the welfare implications of joint integration of multiple social insurance programs with a pension program.

Before providing the more detailed arguments, we note that qualitatively there is an obvious answer to these concerns. The degree of integration—of allowed borrowing against future pension saving—can be state dependent; and the amount of tax funded insurance provided against risks later in an individual’s life can depend on his cumulative experience. An individual with no experience of unemployment early in life will self-insure against small health and disability risks later in life.

In general, the integrated lifetime insurance system consists of several social insurance programs integrated with the public pension program, providing an individual facing a particular shock with a mixture of a tax-funded insurance benefit and pension-funded borrowing. As in the integrated UI, the benefit mix for a particular shock would depend not only upon the risk-aversion and incentive ‘sensitivity’ of an individual but also upon his history (his experience with various shocks) and the stochastic processes describing future shocks (conditional on past shocks). An important factor determining the benefit mix (the magnitude of the pension-funded borrowing) of an integrated social insurance is the amount of pension savings that is expected to be available at the time of retirement. If an individual facing an adverse shock (unemployment or health) expects to have a relatively large amount of pension savings at the time of retirement, he would be offered a benefit mix consisting of large borrowings and a small tax-funded insurance benefit.

There are a couple of reasons for the positive relationship between pension-funded borrowing offered by an integrated social insurance for an individual and his total future pension savings expected at the time of a shock. First, the greater amount of retirement savings for an individual (with a given expected length of retirement) implies that more borrowing is necessary for intertemporal
consumption smoothing in the event of a shock. Second, lower expected pension savings for an individual implies a higher probability that the government bails him out to sustain a certain minimum level of retirement consumption. If the government bails out low pension savings individuals, the pension-funded borrowing to finance consumption in the event of a shock would in fact have adverse disincentive effects. The individual would know that his borrowing to sustain his consumption might not be repaid if he experiences more shocks in the future. The reason for reliance on pension-funded borrowing is to eliminate the adverse incentive effects of insurance; the possibility of a government bailout means that there is still some implicit insurance, and therefore there is still some adverse incentive effect. Note, however, that this adverse disincentive effect of borrowing is still less severe than a tax-funded benefit, so long as there is some probability that the borrowing is repaid.

The expected pension savings available to an individual at the time of retirement depends upon the size of the loss caused by the shocks he experiences and the amount of borrowing under the integrated social insurance program. Hence, the desired amount of pension-funded borrowing offered by an integrated social insurance for an individual will also be determined by two other factors. First, the amount of borrowing for an individual facing a “shock” would be negatively related to the number and the sizes of other shocks that he has experienced in the past and is expected to go through in the future. If he has already experienced several shocks and borrowed substantially against his future pension his expected pension savings at retirement will be lower, implying that should he experience another shock, he should not rely much on pension-funded borrowing. If shocks are highly positively correlated with each other, the benefit mix of an integrated social insurance involves less self-insurance and more tax-funded benefits. Second, the level of integration for a particular social insurance program— the amount of pension-funded borrowing offered by an integrated social insurance against that particular risk—is constrained by the amount of pension-funded borrowing offered under other integrated social insurance programs. This is because the given amount of pension savings has to be shared for self-insurance against the shocks dealt with under several integrated social insurance programs.

67 This implies that the benefit mix offered by an integrated social insurance for an individual would also depend upon the individual’s employment history as well as the nature of the other shocks he suffers from.
This suggests that as more social insurance programs are integrated with the pension program, the level of integration for each will be lowered, raising the question: Would it be relevant to integrate all the social insurance programs with the public pension scheme or to exclude some of them from the integration? The answer is that all the social insurances should be jointly integrated unless one risk is perfectly, positively correlated with another. With the risks being imperfectly correlated to each other, there is always some positive probability that an individual suffering from a shock later in his life has not experienced any other shock before, in which case some borrowing against his pension may be offered to him, and this will attenuate the adverse incentive effect of the tax-funded insurance program.

The basic idea behind the integrated lifetime insurance system is thus that there should be no constraint on the fund that confines its use to a specific set of shocks only. Such a constraint unambiguously lowers welfare. The joint integration allows us to have a common pool of pension savings that we can draw upon in funding benefit payments under different shocks. For those who have not experienced any shock and thus have not had to borrow against their future pension savings, for example, the system allows a relatively large amount of their pension savings to be used for the upcoming shocks (and retirement). Also, more pension savings can be used to finance the borrowing against an early shock if other subsequent shocks are expected to occur with a low probability before retirement.

The extent of welfare improvement from an integrated lifetime insurance would depend upon the correlation among the risks. If all the risks were perfectly, negatively correlated with each other, the joint integration would be able to give us the maximum welfare benefit by setting the highest possible level of integration for each social insurance program. As the risks are more positively correlated to each other, the level of integration for each social insurance program that can be set under joint integration would become lower. Unless the risks are perfectly positively correlated, however, the integrated lifetime insurance system will always bring some welfare gain.68

These arguments for the integrated lifetime insurance system lend support to the Provident Fund in

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68 There are some studies which indirectly suggest that shocks are not highly correlated for an individual during his lifetime: Bjorklund [1993] showed based upon Swedish data that lifetime income is more equally distributed among individuals than annual income. OECD [1996] also reported that the upward income mobility for low income individuals is higher than for high income individuals.
Singapore, Malaysia, and recently in Hong Kong.\textsuperscript{69} The Provident Fund, to which individuals contribute a portion of their wage earnings, covers several risks, such as disability, medical, and retirement risks. It should be noted, however, that there is an important difference between the Provident Fund and the integrated lifetime insurance introduced here: while the integrated lifetime insurance allows an individual to withdraw more than what he has contributed, the Provident Fund limits the amount of individual withdrawal to what he has accumulated in his account. Thus the levels of insurance and intertemporal consumption smoothing it can provide to individuals are considerably limited, compared to the integrated lifetime insurance system that allows them to access savings out of future income.

\section*{VII. Conclusion}

The failure of markets to provide adequate insurance against certain risks has long been recognized.\textsuperscript{70} This, combined with the fact that social norms do not allow individuals in their old age to suffer from insufficient income, whether their misfortune arises partly because they have chosen to save insufficiently (either because of excessive myopia or because of rational exploitation of these social norms), or because they have undertaken excessive risks provides a rationale for a compulsory public pension program. This paper has developed a further advantage to such a program; it allows for the collateralization of future wage income in a way which is not easily possible otherwise, thus allowing individuals in effect to self insure against a variety of risks.

This paper has addressed two related issues. The possibility of pension-funded self-insurance does not eliminate the desirability of some tax-funded unemployment insurance. We have identified the factors on which the optimal degree of pension-funded self-insurance depends. Although our analysis suggests a heavy reliance on pension-funded self-insurance and argues for the relevance of integration,

\textsuperscript{69} For detailed information on the system, see Asher [1994].

\textsuperscript{70} Just as this paper has just touched on why governments all over the world provide mandatory retirement insurance, this paper has not explained why markets do not provide adequate unemployment insurance, and virtually everywhere, there is heavily reliance on government unemployment insurance programs. (As usual, problems of moral hazard and adverse selection arising from asymmetries of information are at the core of the explanation.)
there are some limiting circumstances under which integration is not welfare enhancing. This paper also identifies the conditions under which this is the case.

Regardless of the motivation for public pension programs, they have become a part of the basic policy frameworks of all advanced industrial countries, and increasingly of countries in the developing world. The political processes which shape those programs are complex, and there is no reason to believe that the resulting social security programs are necessarily optimal, in any sense. This paper can be looked at in another way: it suggests that a simple reform, allowing for the limited borrowing against public pension savings, may be welfare enhancing. 71

When there are multiple risks (including the risk of multiple bouts of unemployment), again some reliance on pension-funded self-insurance is in general desirable. The integrated lifetime insurance system can always generate a welfare gain from allowing a common pool of pension savings to be shared to finance the benefits for those facing various shocks. The general principle naturally leads to the suggestion of a fully integrated lifetime insurance system through a joint individual account, an extension of the Provident Fund of Singapore and Malaysia, where major risks including disability and health are integrated with the public pensions program.

71 And the argument put forward earlier, that compulsory savings programs allows for borrowing against future savings, effectively collateralizing part of future wage income, suggests that welfare may be higher than it would be with a reduced public pension program.
<Figures>

**Period 1** Unemp. shock  |  **Period 2**  |  **Periods (3- (M+2))**  |  **Periods((M+2)- (M+2+N))**
Employment  |  Search Effort  |  Emp./Unemp.  |  Employment  |  Retirement

\[U(w(1-S_n)) \rightarrow U(w(1-S_n)) \rightarrow U(w(S_n(M+1)+S_1)/N)\]

\[(1-q)\]

\[U(w(1-T-S_1))\]

\[q\]

\[0 \rightarrow U(w(1-S_n)) \rightarrow U(w(1-S_n)) \rightarrow U(w(S_n(M+1)+S_1)/N)\]

(Search)

\[0 \rightarrow U(w(r+R)) \rightarrow U(w(1-S_u)) \rightarrow U(w(S_uM+S_1-R)/N)\]

(No Search)

*Figure 1*  Model

\[\eta\]

H(δ;q,M,N)

1- \(\eta\)

0  \(\delta/(M+N+1)\)

*Figure 2*  Region for constrained optimum with no UI
<Figure 3> Region for integration to be relevant

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