Savings Gluts and Financial Fragility*

Patrick Bolton
Columbia University and NBER

Tano Santos
Columbia University and NBER

Jose A. Scheinkman
Columbia University, Princeton University and NBER

May 22, 2017

Abstract

We investigate the effects of an increase in liquidity (a “savings glut”) on the incentives to originate high quality assets, and on the fragility of the financial sector. Originators incur private costs when originating high quality assets. Assets are subsequently distributed in two markets: A private market where informed intermediaries operate, and an exchange where uninformed investors trade. Uninformed investors pay the same price irrespective of the quality of the assets, which discourages good origination. Informed investors identify and pick the best assets by offering a premium over the uninformed price, which encourages originators to supply good assets. We show that there is a positive origination effect of an increase in savings of uninformed investors when the overall level of savings is low, but the opposite is true when liquidity is abundant, so that an increase in savings has a non-monotone effect on origination incentives. Leverage increases monotonically with savings and is highest precisely when incentives for good asset origination are at their lowest. Thus plentiful liquidity leads to fragile balance sheets: On the asset side non-performing assets accumulate and on the liability side more of those assets are funded with debt. We relate our findings to some of the stylized facts observed in financial markets in the lead up to the Great Recession.

*We thank Hyun Shin and Chris Sims for insightful comments on an earlier version of the paper. We are grateful to Tobias Adrian and Tomasz Piskorski for the data they have provided to construct Figures 2 and 3.
“Large quantities of liquid capital sloshing around the world should raise the possibility that they will overflow the container.” Robert M. Solow vii in Foreword of Manias, Panics, and Crashes: A History of Financial Crises by Charles P. Kindleberger and Robert Aliber (2005)

1 Introduction

Large changes in capital flows have long been linked to financial crises (Kindleberger and Aliber, 2005). The typical narrative is that capital inflows (‘hot money’) boost asset prices and set in motion a lending and real estate boom. Eventually, when capital flows stop and real estate values decline, a debt crisis ensues (see e.g. Aliber, 2011, and Calvo, 2012). But, as compelling as the historical evidence is, the microeconomic mechanisms that bring about financial fragility as a result of large capital inflows are still poorly understood.

The focus of our paper is the idea that easy financial conditions and risk-spread compression caused by a savings glut undermine origination incentives. This can lead to a deterioration of the quality of originated assets and to a greater fragility of financial intermediaries. Three broad stylized facts are linked together by the mechanism explored in our paper.

The first is the spread compression that has resulted from the savings glut, as illustrated in Figure 1. The risk-spread variable we have selected is the U.S. high yield option-adjusted spread, which is a good measure of the pricing of risk in credit markets. The savings variable we center on is cash holdings or savings by U.S. corporations, which is the largest component of aggregate U.S. savings.\(^1\) As can be seen in Figure 1, US corporate savings grew dramatically in the years prior to the crisis, and this growth is negatively correlated with the high yield option-adjusted spread, suggesting that increases in US corporate savings contributed to yield compression prior to the crisis.\(^2\)

The other two stylized facts relate to the increase in the fragility of financial intermediaries prior to the crisis, both on the asset and liability side of their balance sheets. The second stylized fact displayed in Figure 2 is a phenomenon first noted by Adrian and Shin (2010),

\(^1\)As Karabarbounis and Neiman (2012) emphasize 80% of US savings is by corporations.
\(^2\)Indeed, financial analysts have noted and explored this precise link. See Loeys, Mackie, Meggesye and Panigirtzoglou (2005).
namely that US brokers-dealers became more fragile in the years leading up to the crisis in part because the dramatic expansion of their balance sheets was financed primarily through an increase in leverage. The third stylized fact is the growing fragility of financial intermediaries on the asset side of their balance sheets, as a result of the gradual erosion of their underwriting standards. Figure 3 Panel A shows how much the percentage of private label mortgage loans with poor or no documentation rose in the years prior to the Great Recession.\(^3\) Panel B of Figure 3 in turn shows the average cumulative default paths of non-agency securitized loan vintages from 2000 to 2007. Although default rates at first declined from 2000 to 2003, they became worse and worse as more and more low documentation loans were underwritten. All in all, leverage of financial institutions started rising just when the quality of originated assets was deteriorating.

The model we develop in this paper is to our knowledge the first to link savings gluts, whatever their source, to greater financial intermediary fragility both on the liability side, through greater leverage, and on the asset side, through the deterioration of origination incentives. In the model it takes costly effort for an originator to produce an asset of high quality. Originated assets are distributed to investors in two different markets: An organized securities market, where the asset can be sold to uninformed investors, and an over-the-counter (OTC) market, where the asset may be bought by an informed intermediary, who has superior, but noisy, information about the quality of originated assets.

The central mechanism we study is the incentives financial markets provide originators and how these market incentives are affected by changes in savings or capital flows. The organized exchange cannot provide any incentives, since investors in this market are unable to distinguish between high and low quality assets. Market incentives for origination of good assets come from informed investors who are willing to pay more in the OTC market for assets they identify as high quality. A central result of our analysis is that origination incentives at first improve with capital inflows but eventually the savings glut narrows spreads so much that origination incentives are undermined. We focus on the situation where capital in the hands of informed investors is relatively scarce and thus in equilibrium informed traders obtain a higher

\(^3\)See also Keys, Piskorski, Seru and Vig (2013) Figure 4.4. We thank Tomasz Piskorski for providing us the quarterly data used to construct Figure 3 Panels A and B and Tobias Adrian for the data for Figure 2.
rate of return and leverage their superior information by borrowing from uninformed traders. In this situation informed traders act as financial intermediaries. We show that intermediary leverage increases monotonically as more savings pour into capital markets and is highest when origination incentives are at their lowest. In other words, the savings glut gradually increases the fragility of financial intermediaries: on the liability side, they are increasingly leveraged, and on the asset side, their balance sheets increasingly contain low quality ‘non-performing’ assets.

The rising financial fragility prior to the crisis is often attributed to changes in investors’ psychology (Minsky 1992). According to Minsky financial fragility is simply due to the growing risk appetite of investors. But he establishes no clear link between greater risk taking and increasing origination of non-performing assets. Whether it is greater risk appetite or rising savings that are associated with the lending boom is immaterial as far as the effect on yield compression is concerned. The critical variable is the aggregate risk-weighted capital flowing into asset markets. Thus, our analysis can be also be seen as a microfoundation of a Minsky-type explanation of the lending boom.

The heart of our analysis is a rich modeling of the financial sector, which features three markets. First, there is a private OTC market where informed investors purchase high quality assets from originators. Second, there is a public market where the remaining assets are distributed to uninformed investors. Third, there is a collateralized debt, or repo market, where informed financial intermediaries can borrow from investors. We do much of our analysis under the convenient assumption that there is cash-in-the-market pricing of assets in equilibrium (Allen and Gale, 1999). In other words, asset prices are determined by the ratio of aggregate savings and the volume of distributed assets. In this way, a savings glut directly feeds through to higher asset prices.

The aggregate savings that investors deploy in the market are exogenously given and we compare equilibria for different levels of aggregate savings, keeping constant the distribution of capital across informed and uninformed investors. A key feature of our analysis is that asset prices in the OTC market rise less than proportionately with aggregate savings, so that the overall effect of the rise in aggregate savings is to compress the spread between the price of
high quality assets traded in the OTC market and the asset price on the exchange. In addition, the balance sheet of financial intermediaries grows with aggregate savings. Moreover, this growth in the balance sheet is mostly financed with debt though there is also some growth in book equity. The end result is that leverage amongst financial intermediaries grows as aggregate liquidity increases. The reason is that, as in Kiyotaki and Moore (1997), the rise in asset prices relaxes leverage constraints, thereby increasing the debt capacity of financial intermediaries.

Since financial intermediaries have information on the quality of originated assets and are willing to pay more for high quality assets, the increase of their balance sheets ought to result in better incentives for originators. However, there is a countervailing effect through the narrowing of spreads, which undoes origination incentives. We show that when the aggregate capital of informed investors is small, the effect of savings on origination incentives has an inverted U-shape. Initially increases in savings improve incentives but eventually they lead to a deterioration of origination standards, as is reflected in Panel B of Figure 3.

The deterioration of the quality of the assets on intermediaries’ balance sheets is due to the fact that, although they are informed about asset quality, they on occasion mistake a non-performing asset for a high quality asset. When origination incentives deteriorate intermediaries are more exposed to making such a mistake because the deal flow they see is of lesser quality. All in all, we show that even if the quality of the intermediaries’ information does not change, they end up accumulating a greater fraction of non-performing assets in a savings glut. In sum, our model predicts several of the main stylized patterns observed during the years leading up to the crisis of 2007-09 without relying on extrapolation biases in asset prices or any other behavioral bias.

These biases most likely did play a role in the run up to the crisis and we explore the effects of the addition of an overconfidence bias of informed investors in section 5.2, by allowing for the possibility that these agents exaggerate the precision of the signals they receive about the quality of the assets up for sale, as in Scheinkman and Xiong (2003). When intermediaries overestimate the precision of their signals, they bid more aggressively for assets when they receive a positive signal about asset quality. Obviously, their overconfidence could lead
them to overbid and thereby stimulate an asset price bubble. That effect is well understood. But a less obvious and novel insight from our model is that the book leverage of overconfident intermediaries (the ratio of debt to book value of assets) is lower than the book leverage of unbiased intermediaries. What is more, in our model an overconfident intermediary *reporting a lower book leverage is also a more fragile intermediary*. This simple but striking observation has implications for the prudential regulation of banks: it illustrates the potentially misleading nature of book leverage as a measure of financial fragility. What appears to be a financially healthy bank by its low reported leverage, could in fact be a bank that has unwittingly taken too much risk.

Finally, we also consider the effect of changes in the distribution of capital across informed and uninformed investors and show that as the fraction of capital in the hands of the uninformed increases, the price in the exchange increases, origination standards worsen and leverage increases. If one believes that in the run up to the crisis total capital increased but with a bias towards uninformed investors, then these results would help explain the increase in prices (narrowing of the spreads), the deterioration of origination standards and the increase in leverage.

Our results use a cash-in-the-market pricing device. A more conventional, but somewhat more involved, way of delivering the same effect would be to have a model with risk-averse investors and to let their aversion to risk be decreasing in their accumulated savings. This is consistent with the findings of the empirical asset pricing literature, which found that compensation for risk is countercyclical: high during troughs and low at the peak of the cycle. Under this alternative model asset prices on the exchange would be affected by both the fundamental quality of assets and the risk attitudes of uninformed investors. The key mechanism would then be that a reduction in the discount rate would lead to a reduction in the spread between the price for high quality assets in the OTC market and the uninformed price on the exchange, and thereby weakening origination incentives. The Allen and Gale notion of cash-in-the-market pricing should therefore be thought of more generally as simply a more tractable way of modeling the effects of decreasing risk aversion that may result from a savings glut.

---

4The literature on stock return predictability is large. See Cochrane (2008) for a survey and summary.
Related literature. We provide microfoundations for one of the leading hypotheses on the origins of the crisis of 2007-09: a savings glut combined with balance-sheet expansion and greater leverage of financial intermediaries. Other commentators, most notably Bernanke (2005), Shin (2012), Gourinchas (2012), and Borio and Disyatat (2011) have argued that the rise in global liquidity more than global imbalances was the major cause of the crisis. Bernanke (2005) famously argued that a ‘global savings glut’ was the main cause of low long-term interest rates before the crisis of 2007-09. With low interest rates households could afford bigger mortgages, which, in turn, fuelled real-estate price inflation. Bernanke’s thesis is sometimes narrowly interpreted as meaning that the rise of Chinese savings caused the fall in US interest rates, but this narrow interpretation is not completely borne out in the data.\(^5\)

The savings glut hypothesis, however, is not dependent on a particular source of savings. Any surge in savings, whatever the sector, yields similar implications.

Shin (2011, 2012) points to the role of rising gross capital flows across countries, channelled through an increasingly globalized banking industry, as a major source of financial fragility. He attributes this increase in cross-border flows to the transition to Basel II capital regulation, which allowed banks to lever up, and to the advent of the Euro, which eliminated currency risk within the Eurozone.\(^6\) With higher leverage and greater reliance on fickle foreign capital inflows, Shin argues that banks became fundamentally more fragile. It would not take much to put their viability at risk with a liquidity crunch. Borio and Disyatat (2011) also emphasize the critical role played by the financial sector in the crisis. They argue that the main cause of the crisis was the dynamic expansion of the balance sheets of large, complex, financial institutions in response to the savings glut. The ‘excess elasticity’ of bank balance sheets, the term they employ, was the main cause of the crisis in their analysis. In effect, banks acted as a multiplier of the savings glut, pouring vast quantities of new fuel on the fire.

More systematic evidence that financial crises are preceded by above trend credit growth was found by Gourinchas, Valdés and Landerretche (2001) for the period of the 1960s to 2000. More recently Jordà, Schularick and Taylor (2011) have looked at a much longer time-series

---


\(^6\) See Santos (2017) for an account of flows in the Spanish banking sector in the years leading up to the Eurozone crisis.
(140 years) and have found that abnormally high credit growth and low interest rates precede
global financial crises. Moreover, they find that credit growth is a better predictor of financial
crises than global imbalances, the latter playing an additional role only before WWII.

Several other theories linking savings gluts to financial instability have been proposed. An early model by Caballero, Fahri and Gourinchas (2008) links rising global imbalances to low interest rates, through a limited global supply of safe assets. However, they do not explore the effects of these imbalances on origination standards, leverage, and financial fragility. In independent related work, Martinez-Miera and Repullo (2016) propose a model of intermediation similar to Holmstrom and Tirole (1997), where banks’ incentives to monitor are affected by a savings glut. In their model, safe projects are financed by non-monitoring (distributing) banks, while riskier projects are held on the balance sheet of traditional monitoring banks. Their theory is built on a different economic mechanism than ours and makes somewhat different predictions. In particular, any effect of the savings glut on intermediary leverage is absent from their model. A related theory by Neuhann (2016) points to the additional fragility caused by excessive ‘risk transfer’ from originating banks to ‘financiers’. This risk transfer gradually undermined origination incentives of commercial banks as more and more risk was transferred to a rising non-bank sector. Boissay, Collard and Smets (2016) consider a dynamic model of the interbank market, where banks borrow from other lenders. Borrowing is limited by adverse selection problems: Lenders don’t know whether they are lending to banks with good or bad investment opportunities. As interbank rates rise only the banks with the best investment opportunities borrow in the interbank market, so that the volume of interbank loans increases and the pool of borrowing banks improves. In other words, there is a positive correlation between interbank rates and leverage in their model. A crisis occurs when there is a sudden increase in savings, which causes interbank rates to drop, and consequently leads to a deterioration of the pool of borrowing banks, together with a collapse in interbank lending volume. In contrast, in our model intermediaries borrow in a repo market and leverage and repo rates are negatively related. This seems to better match the stylized facts in the years leading up to the Great Recession, when investment banks and broker-dealers greatly increased the size of their balance sheets (and their leverage), increasingly relying on repo financing, while
repo rates kept falling.

An alternative theory of financial fragility that does not depend on savings gluts is developed by Gennaioli, Shleifer and Vishny (2012 and 2013). In their model some agents underestimate or neglect tail risk, perhaps because they extrapolate from recent history. Financial intermediaries that identify this misperception can exploit it through securitizations to build up more risk. In addition, to control their residual risk, financial intermediaries purchase claims against other intermediaries which generates excessive financial instability. We consider the possibility of misperceptions by financial intermediaries instead of by final investors in Section 5.2 and show how it can lead to additional financial fragility by further undermining origination standards.\(^7\)

\section{The Model}

The model we develop focuses on the financial market mechanism linking the pricing of assets in financial markets and incentives of originators to supply high quality assets. In particular, our analysis centers on the question of how this mechanism is affected by changes in aggregate liquidity flowing into financial markets. Accordingly, our model must comprise at least two classes of agents, asset originators and investors, interacting over two periods.

\subsection{Agents}

We assume that each class of agents is of fixed size (we normalize the measure of each class to 1), and that both originators and investors have risk-neutral preferences.

\textbf{Originators.} In period 1 each originator can generate one asset that produces payoffs in period 2, which are either \(x_h > 0\) or \(x_l = 0\). An asset can be interpreted to mean a business or consumer loan, a mortgage, or other assets. The quality of an originated asset depends on the amount of effort \(e \in [e, 1)\) exerted by the originator, where we assume that \(e > 0\). Without loss of generality we set the probability that an asset yields a high payoff \(x_h\) equal to the effort

\(^7\)For evidence that financial intermediaries misperceived risk in the run up to the financial crisis see Gerardi, Lehnert, Sherlund and Willen (2008) or Cheng, Raina and Xiong (2014).
Asset payoffs are only revealed in period $2$, so that the only private information originators have in period $1$ is their choice of effort. Originators only value consumption in period $1$ and they incur a disutility cost of effort $e$, so that their utility function takes the form:

$$u(e, c_1) = -\psi(e - e) + c_1,$$

where $c_1$ stands for consumption in period $1$. We assume that the disutility of effort function $\psi(z)$ satisfies the following properties: (i) $\psi(0) = \psi'(0) = 0$; (ii) $\psi'(z) > 0$ if $z > 0$; (iii) $\psi'(1 - e) > x_h$ and (iv) $\psi''(z) >> 0$. Given that $\psi(0) = 0$, originators always (weakly) prefer to originate an asset as long as they can sell this asset at a non-negative price in period $1$.

**Investors.** Each investor has an initial endowment of $K$ units of capital in period $1$ and a utility function

$$U(c_1, c_2) = c_1 + c_2,$$

where again $c_\tau \geq 0$ denotes consumption at time $\tau = 1, 2$. Since investors are indifferent between consumption in period $1$ and $2$, they are natural buyers of the assets that originators would like to sell in period $1$. Our model captures in a simple way changes in aggregate savings by varying $K$.

There are two types of investors. A first group, which we refer to as **uninformed investors**, are unable to identify the quality of an asset for sale. We denote by $M$ the fraction of uninformed investors and assume that $0 < M \leq 1$. The second group, which we label as **informed investors** are better able to determine the quality of assets and can identify those assets that are more likely to yield a high payoff $x_h$. We let $N = 1 - M$ denote the fraction of informed intermediaries.

### 2.2 Financial Markets

There are three different financial markets in which agents can trade: 1) an opaque, over-the-counter (OTC) market where originators can trade assets with informed investors; 2) An organized, competitive, transparent and regulated exchange where originators can sell their asset to uninformed investors; and 3) A secured debt market, where informed investors can
borrow from uninformed investors. The dual market structure for assets builds on Bolton, Santos and Scheinkman (2012 and 2016). A key distinction between these two types of markets is how buyers and sellers meet and how prices are determined. In the organized exchange all price quotes are disclosed, so that effectively asset trades occur at competitively set prices. In the private market there is no price disclosure and all transactions are negotiated on a bilateral basis between one buyer and one seller.

2.2.1 OTC Market

Originators are willing to trade with informed investors in the OTC market despite the lack of competition among intermediaries and the lack of transparency in the hope that their asset will be identified as a high quality asset. Informed intermediaries observe a signal that is correlated with the quality of the asset and are willing to pay a higher price for high quality assets than the price at which a generic asset is sold on the organized exchange. Even though informed traders are free to buy in any market, we show that when informed capital is scarce and uninformed capital is not too scarce, they only operate in the OTC market in equilibrium, and they only purchase assets that they judge to be high quality. As we detail below, although informed traders can borrow from uninformed investors, they have a limited borrowing capacity, due to the collateral requirements. Therefore, even after exhausting their borrowing capacity, informed traders may not have sufficient capital to deploy to purchase all available high quality assets. In this case, any asset that informed traders are not able to purchase will be sold on the exchange.

Each informed trader observes a signal $\sigma \in \{\sigma_h, \sigma_l\}$ on the quality of any asset offered for sale such that

$$\text{prob}(\sigma_h|x_h) = 1 \quad \text{and} \quad \text{prob}(\sigma_h|x_l) = \alpha \in [0, 1).$$

The parameter $\alpha$ captures possible valuation mistakes of informed trader. Conditional on observing $\sigma_h$ the probability that the asset yields a payoff $x_h$ is:

$$g := \text{prob}(x_h|\sigma_h) = \frac{e}{e + \alpha(1 - e)}.$$
Note that when $\alpha = 0$ we have $g = 1$; in other words, $\sigma_h$ is a perfectly informative signal for the payoff $x_h$. The higher is $\alpha$ the less informative is the signal $\sigma_h$, and when $\alpha = 1$ there is no additional information conveyed by $\sigma_h$.

Conditional on observing $\sigma_l$ the investor knows that the asset will pay 0 in period 2. In this case, the investor would offer to pay no more than zero for the asset, so that the originator prefers to sell the asset on the anonymous exchange. In sum, all OTC trades with informed investors involve high quality assets with signal $\sigma_h$.

We denote by $p^d$ the price at which an asset with a signal $\sigma_h$ trades in the OTC market, and by $p$ the competitive price for a generic asset on the exchange. As in Bolton, Santos and Scheinkman (2012 and 2016) we assume that a bilateral trade on the OTC market takes place at negotiated terms via Nash bargaining and that

$$p^d = \kappa g x_h + (1 - \kappa) p,$$

(4)

where $\kappa > 0$ denotes the bargaining strength of the originator. Note that once an informed investor offers any positive price, the originator learns that the signal associated with her asset is $\sigma_h$. The bargaining parties are then symmetrically informed. The pricing formula (4) is the Nash bargaining solution of a bargaining game between an informed investor and an originator for the purchase of an asset with signal $\sigma_h$, where the bargaining weights of the originator and investor are respectively $\kappa$ and $(1 - \kappa)$, and where the disagreement point is given by a sale of the asset on the exchange at price $p$. This OTC bargain only takes place if $g x_h \geq p$, which is a condition satisfied by the equilibrium price on the exchange $p$, as we show below.\(^8\)

Let $q^i$ denote the number of assets acquired by each informed trader on the OTC market. The probability that an originator with an asset with signal $\sigma_h$ sells her asset to an informed trader in a symmetric equilibrium is then given by the ratio of the total number of assets purchased by informed intermediaries to the total number of assets with signal $\sigma_h$ (provided that this ratio does not exceed 1):

$$m := \min \left\{ \frac{N q^i}{e + (1 - e) \alpha}; 1 \right\}.$$  

(5)

\(^8\)See Bolton, Santos and Scheinkman (2016) for a detailed derivation of the Nash bargaining solution.
2.2.2 Organized Exchange

All originated assets that are not cream-skimmed in the OTC market end up being distributed on the organized exchange. Therefore, the volume of high-quality assets traded on the exchange in a symmetric equilibrium is equal to $e(1 - m)$. To see this, observe first that originators produce assets with payoff $x_h$ with probability $e$. The fraction of assets $x_h$ in a symmetric equilibrium is then $e$, of which a fraction $m$ is bought by financial intermediaries. Second, the volume of low-quality assets sold on the exchange is $(1 - e) - (1 - e)\alpha m$, so that the total volume of assets distributed on the exchange is $1 - em - (1 - e)\alpha m$. The expected value of an asset traded on the exchange is then:

$$p^f = \frac{e(1 - m)x_h}{1 - em - (1 - e)\alpha m},$$

where the superscript $f$ stands for “fair value”. Expression (6) highlights that the fair value of assets traded on the exchange depends on the fraction of assets that are cream-skimmed by informed investors.

In Bolton, Santos and Scheinkman (2016) we assume that risk-neutral uninformed investors are always able to pay the fair value $p^f$ and focus on the implications of cream-skimming of high quality assets in the OTC market. Here, we generalize the model to allow asset prices to respond to changes in aggregate liquidity. Specifically, if the total stock of liquidity brought by investors to the exchange is $T$, then we assume that asset prices on the exchange must satisfy:

$$p \leq p^{\text{cim}} := \frac{T}{1 - em - (1 - e)\alpha m},$$

so that:

$$p = \min\{p^f, p^{\text{cim}}\} = \min\left\{\frac{e(1 - m)x_h}{1 - em - (1 - e)\alpha m}; \frac{T}{1 - em - (1 - e)\alpha m}\right\},$$

since $p \leq p^f, p \leq gx_h$ or $p \leq p^d$.

The superscript “cim” stands for cash-in-the-market pricing, which obtains whenever the total pool of liquidity is less than $e(1 - m)x_h$.

It is helpful to introduce two further pieces of notation: We denote by $R$ the return from buying an asset with signal $\sigma_h$ on the OTC market, and by $r^x$ the return from investing on the
exchange, when \( p > 0 \). That is:

\[
R = \frac{g x_h}{p^d} \quad \text{and} \quad r^x = \frac{e(1 - m)x_h}{p \left(1 - em - (1 - e)\alpha m\right)}.
\] (9)

### 2.2.3 The Secured Credit Market

Investors can borrow from other investors in the form of risk free collateralized loans akin to *repo* contracts. Under a typical loan an investor borrows at the rate \( r \) against the assets it acquires. Since informed traders have access to all trades available to uninformed traders we may assume without loss of generality that in equilibrium only informed traders act as borrowers. Formally, we assume that an intermediary can borrow against purchased assets at the exchange market value \( p \) of these assets with a haircut \( \eta > 0 \). More precisely, if an intermediary purchases \( y \) units it can borrow any amount \( D \) that satisfies the constraint

\[
D \leq (1 - \eta)py.
\] (10)

Note that the valuation of the collateral in constraint (10) is determined using the exchange price \( p \). In section 5.3 we justify using the exchange price \( p \) as the reference price, by assuming an intermediary period between periods 1 and 2, when lenders hit by a liquidity shock have the option not to rollover a loan. If the loan is not rolled over, the lender ends up as the owner of an asset that he can only sell in the exchange.\(^9\) Constraint (10) also features a haircut \( \eta \), which, in practice, reflects lenders’ beliefs about the risk with respect to selling the asset in the secondary market.

### 2.3 Expected Payoffs

Having described how originated assets are distributed and priced in period 1, we are now in a position to specify originator and investor payoffs. Originators sell assets to investors in

\(^9\)Kiyotaki and Moore (1997, equation (3), page 218), take the price of the asset at time \( t + 1 \) to value the collateral in their borrowing constraint. Our specification reflects the market practice for repo transactions, in which valuation of the collateral is based on the current market price. Nevertheless, constraint (10) also incorporates the main feedback effect of Kiyotaki and Moore (1997), whereby increases in asset prices can relax collateral constraints for secured loans.
period 1, who hold them until maturity. Replacing \( c_1 \) in (1) with the expected price of the originated asset we obtain the following expression for an originator’s expected payoff:

\[
-\psi(e - \xi) + e\left(\mp^d + (1 - m)p\right) + (1 - e)\left[\alpha\left(\mp^d + (1 - m)p\right) + (1 - \alpha)p\right].
\]  

(11)

An originator who chooses origination effort \( e \) is able to generate an asset with a payoff \( x_h \) with probability \( e \). In this case he is matched with an informed investor with probability \( m \) and obtains a price \( p^d \) in the OTC market, whereas if he is not matched with an informed investor he sells the asset on exchange for a price \( p \). If he originates an asset with a low payoff, he is still able, with probability \( \alpha \), to obtain a price \( p^d \) if matched to an informed investor who obtains a good signal \( \sigma_h \).

Uninformed investors do not have access to the OTC market. They can acquire assets on the exchange or lend to informed intermediaries. Let \( q^u \geq 0 \) be the quantity of assets bought on the exchange and \( D^u = q^u p - K \) the amount borrowed by uninformed investors (if \( K > q^u p \) uninformed investors are net lenders in the repo market). Then, an uninformed investor’s expected payoff is given by:

\[
V^u(q^u) := q^u p r x - D^u r.
\]  

(12)

Feasible choices for \( D^u \) satisfy the leverage constraint:

\[
\eta q^u \leq K.
\]  

(13)

Informed investors choose an amount \( q^i \geq 0 \) to purchase in the OTC market and an amount \( y - q^i \geq 0 \) to purchase on the exchange. Informed investors borrow the amount \( D^i = p^d q^i + p(y - q^i) - K \) (or lend if this amount is negative). An informed investor’s expected payoff is thus given by:

\[
V^i(q^i, y) = q^i p^d R + (y - q^i) p r^x - D^i r
\]

\[
= q^i p^d (R - r) + p(y - q^i)(r^x - r) + K r
\]  

(14)

Again feasible choices for \( D^i \) satisfy the leverage constraint:

\[
\eta y \leq K - (p^d - p) q^i.
\]  

(15)
We will be especially interested in situations where \( R > r^x = r \). In such situations informed intermediaries only trade in OTC markets, so that \( q^i = y \) and the leverage constraint takes the form:

\[
D^i = p^d q^i - K \leq (1 - \eta) pq^i. \tag{16}
\]

Note that the constraint (16) features two margins. The first is the standard haircut in secured transactions as captured by the parameter \( \eta \). The second is slightly more subtle and arises because informed investors acquire assets in private transactions at a price \( p^d \), but the collateral value of those assets is \( p < p^d \). In effect, informed equity capital is required to acquire assets in OTC markets.

### 2.4 Cash-in-the-Market (CIM) Equilibrium

We parameterize our economy by \((K, N, \alpha)\) and determine under what conditions a unique equilibrium exists. We then characterize how the equilibrium varies with \((K, N, \alpha)\). Given \((K, N, \alpha)\), a vector of equilibrium prices \((p, p^d, r, r^x, R)\) and quantities \((e, g, q^u, D^u, q^i, y, D^i, m)\) is such that:

1. \( g, p^d \) and \( m \) satisfy equations (2)-(4),
2. \( p \) satisfies (8) when \( T = p(q^u + y - q^i) \),
3. \((e, q^u, D^u, q^i, y, D^i)\) solve the maximization problems of respectively originators, uninformed investors, and informed intermediaries and furthermore the following market clearing conditions hold:
   \[
   M q^u + N y = 1 \quad \text{and} \quad M D^u + N D^i = 0.
   \]
4. \( r D \leq q^i g x_h \), that is, collateralized loans are indeed risk free.

A first immediate observation is that there is no equilibrium where \( r > r^x \). The reason is that if \( r > r^x \) uninformed investors strictly prefer to lend all their savings to informed intermediaries and the latter also prefer to lend rather than purchase assets on the exchange.
It follows that we must then have \( p = 0 \). But then informed investors cannot borrow, as the leverage constraint always binds, so that \( MD^u + ND^i < 0 \).

A second immediate observation is that if \( p > 0 \) and \( R > r \), then it is optimal for informed investors to borrow as much as possible and earn the spread \((R - r)\) on every dollar they borrow, so that their leverage constraint binds. Moreover, when \( R > r \) an equilibrium can exist only if \( r = r^x \). To be sure, if \( r < r^x \) then both informed and uninformed investors want to borrow, so that \( D^i \) and \( D^u \) are positive and \( MD^u + ND^i > 0 \). In sum, in any (strict) maximal leverage equilibrium we must have \( r = r^x \).

**Definition 1:** A *fair value equilibrium* is such that \( r^x = 1 \).

**Definition 2:** A *CIM equilibrium* is such that \( r^x > 1 \), and a *strict CIM equilibrium* is a CIM equilibrium such that \( R > r > 1 \) and \( p > 0 \).

Notice that to establish that a candidate configuration is indeed a strict CIM equilibrium it suffices to verify 1-3 above. Condition 4 follows because

\[
 rp = r^x p = \frac{e(1 - m)x_h}{1 - em - (1 - m)\alpha m} < ex_h < gx_h
\]

and thus

\[
 rD \leq \frac{D}{p} gx_h \leq (1 - \eta) qgx_h < qgx_h,
\]

where we have used (10) to obtain the second inequality.

Our analysis focuses mostly on strict CIM equilibria. In a strict CIM equilibrium there is market segmentation, with informed investors trading only in the OTC market (that is \( q^i = y \)) and uninformed investors trading on the exchange. In addition, informed investors act as intermediaries and borrow from uninformed investors, as \( R > r \) and \( r^x = r \). Since \( R > r \), informed investors exhaust their leverage constraint. We illustrate the financial flows that obtain in a strict CIM equilibrium in Figure 4.

Strict CIM equilibria are of special interest because changes in aggregate liquidity affect asset prices in both markets in equilibrium. That is, in a strict CIM equilibrium assets sell below their fair value on the exchange, where \( r^x > 1 \), and the capital that informed intermediaries can garner is not sufficient to purchase all high quality assets. Intermediaries are constrained by their borrowing capacity, so that \( m < 1 \) and \( p > 0 \).
In what follows we show that CIM equilibrium exist for certain configurations of parameters \((K, N, \alpha)\). We then derive the main comparative statics results with respect to \(K\) and use these to interpret the main stylized facts on asset origination and leverage of financial intermediaries that preceded the financial crisis of 2007-09. We also derive necessary and sufficient conditions for an equilibrium such that \(R > r\), which we report in Appendix A.1.

3 Strict CIM Equilibrium

In Appendix A.1 we show that we can completely determine all prices and quantities in a strict CIM equilibrium, once we know the vector \((p; e)\). For this reason we often refer to a strict CIM equilibrium \((p; e)\) in what follows. We can characterize the necessary conditions for existence of a strict CIM equilibrium as the solution to a single pair of equations \(f(p, e) = 0\) given in Appendix A.1. We show that the converse result also holds: Starting from a solution \((p, e)\) to \(f(p, e) = 0\), we can construct a unique candidate equilibrium with \(r = r^*\) that is a strict CIM equilibrium provided that \(R > r^* > 1\). Moreover, the system of equations \(f(p, e) = 0\) also allows us to characterize the main properties of strict CIM equilibria.

3.1 Existence and Uniqueness of a strict CIM equilibrium

We begin the analysis by establishing the existence of a strict CIM equilibrium in an economy where the measure \(N\) of informed intermediaries is small. When this measure is small enough, the aggregate capital available in the OTC market will be too small to absorb all originated high quality assets. To save on notation we write for each \(\tilde{\alpha} \geq 0\), the minimum probability that a high quality asset (with signal \(\sigma_h\)) yields the payoff \(x_h\) as:

\[
\tilde{g} := g(\tilde{\alpha}, e) = \frac{e}{e + \tilde{\alpha}(1 - e)}.
\]

Our main existence and uniqueness result is then as follows.

**Proposition 1** Fix \(\tilde{\alpha} > 0\) and suppose that

\[
\frac{e\tilde{g}Kx_h}{\tilde{g} - \tilde{\alpha} + e\tilde{g}K} < \tilde{K} < e x_h.
\]

(17)
Then there exists a neighborhood $\mathcal{N}$ of $(\tilde{K}, 0, \tilde{\alpha})$ such that for $(K, N, \alpha) \in \mathcal{N}$, $N > 0$ and $\alpha \geq 0$ there is a unique equilibrium, which is a strict CIM equilibrium.

**Proof:** This follows from Propositions A.8 and A.9 in the Appendix.

The inequality (17) puts an upper and lower bound on the amount of capital in the economy for a strict CIM equilibrium to exist. First, $K$ cannot be too low for otherwise the expected rate of return $r^x$ on the exchange would be so large that informed investors prefer to deploy their capital on the exchange. Second, $K$ cannot be too high for then there could be so much liquidity in the economy that assets trade at their fair value $p^f$.

In a strict CIM equilibrium informed intermediaries earn a strictly positive spread on any dollar they borrow, so that they borrow up to their debt capacity ((16) is met with equality). Uninformed investors lend to informed intermediaries and earn the same expected return on the collateralized debt claims they hold as on the assets they purchase on the exchange, so that: $D^i > 0$, $D^u < 0$, and $R > r^x = r > 1$. Accordingly, in a strict CIM equilibrium capital flows across markets as illustrated in Figure 4.

4 Comparative Statics with respect to $K$

We are now in a position to study the central question of our analysis, namely how the strict CIM equilibrium is affected by changes in aggregate savings $K$. To this end we derive key comparative statics properties of CIM equilibria as a function of $K$. In particular, we characterize how asset prices in the exchange, $p$, origination incentives, $e$, and leverage of financial intermediaries, $D^i$, vary with $K$.

4.1 Asset Prices

Consider first the CIM prices in the exchange, given by (7). It is not immediately obvious that an increase in $K$ results in higher prices $p$, as there is a direct and an indirect effect. The direct effect of an increase in $K$ is that investor savings $MK$ increase, which should result in higher asset prices, except that intermediaries also increase their borrowing $ND^i$, thereby reducing
the savings that investors channel to the exchange. Still, the next proposition shows that the net effect of an increase in $K$ is an increase in the price $p$.

**Proposition 2.** There exists an $ar{\epsilon} > 0$ such that if $NK_2 < \bar{\epsilon}$ and if there are continuous functions $p(K)$ and $e(K)$ defined in an interval $(K_1, K_2)$ such that $(p(K), e(K))$ is a strict CIM equilibrium for parameters $(K, N, \alpha)$, then the price in the exchange $p(K)$ is an increasing function of $K$.

**Proof:** This follows immediately from Lemma A.5. □

Note that a stricter bound than in Proposition 1 is required to guarantee that prices in the exchange are an increasing function of $K$, specifically that the total amount of capital in the hands of informed intermediaries, $NK$, is sufficiently small. Intuitively, if capital in the hands of informed intermediaries is small, so will be the share of incremental savings of uninformed investors that flows to informed intermediaries through the secured debt market. Most of the increase in uninformed capital is then invested in the exchange, thereby pushing up asset prices on the exchange.

Propositions 1 and 2 are illustrated in Figure 5. As can be seen in Panel A, when $K = 1$ the fair value of assets on the exchange is $p^f \approx 2.7$ while the CIM price is much lower at $p \approx 0.8$. As $K$ increases more money flows into asset markets, which pushes up the price $p$ but also results in more cream-skimming by informed intermediaries in the OTC market. There is a direct effect and an indirect effect from this greater cream-skimming, as we explain in greater detail below when we look at the comparative statics of $e$ with respect to $K$. The direct effect is that, other things equal, a worse quality pool of assets is sold on the exchange as a result of the greater cream-skimming. The indirect effect, which at first dominates, is that the greater cream-skimming improves origination incentives, so much so that the average quality of assets for sale on the exchange net of cream-skimming is also improved. As $K$ rises further, the direct effect dominates at some point, so that the greater cream-skimming results in a deterioration of expected asset quality on the exchange and therefore a reduction in $p^f$. Eventually, as $K$ is increased even further, $p = p^f$, at which point there is no more cash in the market pricing. When $p = p^f$ additional increases in $K$ only affect asset prices to the extent that they change the average quality of assets for sale on the exchange.
Consider next Panel B, which plots the expected rate of return of informed intermediaries, $R$, and the expected rate of return of uninformed investors, $r^x$ (which equals the equilibrium interest rate $r$), as a function of aggregate savings $K$. At the smallest value of $K$ the two returns are equal, $r^x = R$, at which point a strict CIM equilibrium ceases to exist.\footnote{Recall that as $p$ is increases with $K$, asset prices in the OTC market $p^d$ mechanically increase as well (see (4)).} As $K$ rises beyond this low value informed intermediaries returns $R$ grow larger and larger relative to uninformed investors’ returns $r^x$. But, note that both returns decline as more savings get channeled into asset markets. In sum, Figure 5 plots the strict CIM equilibrium for the entire admissible range of $K$. At the lowest value for $K$ we have $r^x = R$, and at the highest value for $K$ we have $p = p^f$.

Proposition 2 is an admittedly intuitive, yet fundamental, result for our analysis. It establishes under what conditions increases in aggregate savings result in a glut that has the effect of increasing asset prices. As we show next, changes in asset prices also affect origination incentives and the expected quality of the assets traded on the exchange.

### 4.2 Origination Incentives

As we have already hinted at above, the asset price changes induced by changes in $K$, $dp/dK$, in turn affect origination incentives. We will show that at first an increase in aggregate savings improves origination incentives, but eventually, as a savings glut emerges, origination incentives are impaired. More precisely, since: i) $p \leq gx_h$; ii) $\psi(.)$ is convex, with $\psi(0) = \psi'(0) = 0$; and, iii) $\psi'(1 - \varepsilon) > x_h > \kappa gx_h$, the uniquely optimal origination effort $\varepsilon < 1$ satisfies the first-order condition:

$$
\psi'(e - \varepsilon) = (1 - \alpha) m \left( p^d - p \right).
$$

This condition makes clear that there are three determinants of origination incentives:

1. The precision of intermediaries’ information about asset quality as captured by the term $(1 - \alpha)$. The higher the precision $(1 - \alpha)$ with which an asset yielding $x_h$ is identified the higher are origination incentives.
2. Originators’ market incentives are given by the prospect of selling a high quality asset at price $p^d$ to an informed intermediary rather than at price $p$. But to be able to sell an asset at price $p^d$ it is not sufficient to originate a high quality asset. Conditional on producing such an asset, the originator must also get an offer from an informed intermediary. This occurs with probability $m$, so that the higher is the matching probability $m$ the stronger are origination incentives.

3. Finally, the size of the spread $(p^d - p) = \kappa (gx_h - p)$ naturally affects origination incentives. Intuitively, if $p$ is very close to $p^d$, there is little point in exerting costly effort to produce good assets, given that the price paid by intermediaries is very close to the price paid by an uninformed investor.

Given that asset prices $p$ increase with aggregate savings $K$ in a strict CIM equilibrium, the spread $(p^d - p)$ is decreasing with aggregate incentives. This is the main reason why savings gluts undermine origination incentives. Savings gluts are associated with spread compression, which reduces incentives to bring higher quality assets to the market.

But there is an important countervailing effect through the matching probability $m$. As we show in Figure 6 Panel B, the matching probability $m$ may well be increasing in $K$. In the example plotted in Figure 6 we assume that $\psi(e) = \theta e^2/2$, with $\theta = 0.25$; $\kappa = 0.15$; $\eta = 0.5$; $M = 0.75$, and $x_h = 5$. The underlying economic reason why $m$ may be increasing in $K$ is that the capital that informed intermediaries can deploy increases with $K$ both directly—as intermediaries’ own capital $NK$ is increasing—and indirectly, as intermediaries’ borrowing capacity increases, and their borrowing costs decrease with $p$. As a result, intermediaries may be able to purchase more high quality assets, thereby increasing $m$ and origination incentives.

When $m$ increases with $K$, as in the example, the net effect of an increase in savings $K$ on origination incentives $e$ is such that the effect through $m$ dominates at low levels of $K$, and the effect through the spread $(p^d - p) = \kappa (gx_h - p)$ dominates at high levels of $K$, so that on net $e$ at first rises with $K$ and then decreases with $K$, as shown in Panel A of Figure 6. In other words, $e(K)$ is a non-monotonic function of $K$. This non-monotonicity of $e(K)$

\footnote{There is a second order effect through the dependence of the spread on $g$, as defined in (3). The lower is $\alpha$ the smaller is this second order effect.}
is a robust outcome of the model, and we have identified sufficient conditions under which it holds in the proposition below. Essentially, as long as informed intermediaries’ capital is not too large and the haircut on the collateralized debt is sufficiently small, increases in savings $K$ at first improve origination incentives, but eventually reduce them when savings have reached a critical high level.

**Proposition 3** (Single peakedness of effort) Let $K_1 < K_2$ and suppose that there are continuous functions $p(K)$ and $e(K)$ defined in $(K_1, K_2)$ such that $(p(K); e(K))$ is a strict CIM equilibrium for parameters $(K, N, \alpha)$ with $K_2N < \bar{\epsilon}$. Then if $\eta < \kappa$ then (a) If $K_1$ is small enough, $e'(K_1) > 0$; (b) If $K_2$ is large enough $e'(K_2) < 0$; and (c) the function $e(K)$ is either monotone or has a single global maximum.

**Proof:** This is a consequence of Lemma A.5. □

It is intuitive that $\kappa$ must be sufficiently large for $e(K)$ to be non-monotonic. For, suppose that $\kappa = 0$; this would imply that $(p^d - p) = 0$, so that there would be no origination incentives at all. Observe also that when the haircut $\eta$ is larger, financial intermediaries can borrow less, so that $m$ is lower other things equal. To make up for the lower $m$ the spread $(p^d - p)$ must be larger to preserve incentives, which explains why $\kappa$ must also be larger when $\eta$ is larger.

An additional economic effect that complicates the analysis is that when $K$ increases, so do asset prices. This means that, although intermediaries’ debt capacity always increases with $K$, $m$ may not necessarily increase, as asset prices could rise more than intermediaries’ debt capacity. Still, the robust result is that under the sufficient condition in Proposition 3 $e(K)$ is non-monotonic.

Proposition 3 is a central result of our analysis. It provides a powerful explanation based on economic fundamentals for why late in a lending boom origination incentives deteriorate. As Figure 3 shows, the proportion of incomplete or no documentation private label mortgage loans increased consistently in the years up to the Great Recession (Panel A), yet the delinquency rates first improved for the vintages or 2001 to 2003, and only started to deteriorate in the final phase of the run-up to the crisis (Panel B). Our model can explain this
non-monotonicity in the quality of originated mortgages because there are fundamentally two countervailing forces affecting market origination-incentives.\(^\text{12}\)

The deterioration of origination standards is a phenomenon commonly observed in episodes shortly preceding the onset of a financial crisis, and it has puzzled economic historians (see Kindleberger and Aliber, 2005). Minsky (1992), most notably, offers a popular explanation of this phenomenon rooted in investor psychology. It emphasizes the growing risk appetite, to the point of recklessness, of investors over the lending boom. It is easy to find examples of investor irresponsibility in boom periods, which lend credibility to this explanation. Yet, economists have generally been wary of explanations based on changing investor psychology because of their fundamental circularity: booms occur because of growing investor irresponsibility, and investors become more reckless because of the boom. Hence, one advantage of our theory is that we do not need to appeal to changing psychological factors to explain the decline in asset quality origination. Of course, to the extent that such behavior is prevalent it would reinforce our underlying economic mechanism. Moreover, our mechanism based on economic fundamentals is closely intertwined with two other phenomena also observed before financial crises, the rise in asset demand and the rise in intermediary leverage.

The non-monotonicity of \(e(K)\), in turn, explains the non-monotonicity of \(p^I(K)\) (see Panel A of Figure 5). There are two effects influencing the average quality of assets traded in the exchange: Origination effort and cream skimming.\(^\text{13}\) The latter effect always reduces the quality of assets traded in the exchange. But when \(e(K)\) increases with \(K\) the overall increase in asset quality at first outweighs the effects of cream-skimming, so that the non-monotonicity of \(e(K)\) also translates into a non-monotonicity of \(p^I(K)\). Under cash-in-the-market pricing, however, the expected payoff of assets traded on the exchange is delinked from asset pricing: The price of the average asset traded in the exchange \(p(K)\) rises with \(K\) even though the expected payoff \(p^I(K)\) decreases. As a result, the deterioration of origination standards is masked by the monotone narrowing of spreads. This prediction of our model is consistent

\(^{12}\)There were, of course, other forces in play around the crisis of 2007-2009, in particular house price dynamics, which led to lower rates of non-performing mortgages as long as prices were rising and to a sharp increase, when house prices collapsed during the crisis.

\(^{13}\)We studied the effect of cream skimming in Bolton, Santos and Scheinkman (2016).
with the evidence of Krishnamurthy and Muir (2015).

We show next that savings gluts are also accompanied by increasing financial fragility.

### 4.3 Intermediary Financial Fragility

In the strict CIM equilibrium informed intermediaries exhaust their borrowing capacity, so that (16) is met with equality. When aggregate savings \( K \) increase so do asset prices \( p(K) \), which relaxes the constraint (16), thereby increasing intermediary leverage. More formally, we define leverage as follows:

\[
\mathcal{L} = \frac{\text{Total assets}}{\text{Book equity}} = \frac{K + D_i}{K} = 1 + \ell^i \quad \text{with} \quad \ell^i := \frac{D_i}{K},
\]

Straightforward computations show that

\[
\ell^i = \frac{1 - \eta}{\frac{\rho^i}{\eta} - (1 - \eta)},
\]

and thus changes in leverage are determined by changes in the ratio \( p/p^d \). Holding effort (and hence \( g \)) constant, an increase in \( p \) always results in a increase in \( p/p^d \) and thus an increase in leverage \( \ell^i \). The next proposition shows that changes in effort do not overturn this result.

**Proposition 4** (Leverage) There exists an \( \hat{N} \) such that if \((\bar{p}, \bar{e})\) is a strict CIM equilibrium for parameter values \( (\bar{K}, N, \alpha) \), with \( N \) sufficiently small, then \( \mathcal{L} \) is an increasing function of \( K \).

**Proof:** The result follows directly from Proposition A.7 in the Appendix. \( \square \)

Since intermediary borrowing is constrained by the market value of its collateral it is obvious that borrowing increases with \( K \). But, the proposition states a much stronger result: As aggregate savings increase, leverage, that is the amount of debt per unit of capital, also increases. In other words, intermediary borrowing increases more than proportionally with \( K \). This implication is tested in Adrian and Shin (2010), who stress that there is: “a strongly positive relationship between changes in leverage and changes in the balance sheet size,” [Adrian and Shin, page 419, 2010, and as shown in Figure 2].

\[\text{Note that this definition takes the marks of informed intermediaries, } p^dq^i, \text{ to value their assets rather than } pq^i. \text{ The reason is that otherwise intermediaries would have to immediately mark down any assets they acquire, which is counterfactual.}\]
What is more, to the greater fragility on the liability side, induced by the higher leverage, also corresponds a greater fragility on the asset side of intermediaries’ balance sheets. This is due to the deterioration in origination standards, which also affects the quality of assets distributed to informed intermediaries. This effect is far from obvious. After all, intermediaries are able to identify high quality assets through their informational advantage. Yet, the fraction \( g = \Pr (x_h | \sigma_h) \) of assets paying off \( x_h \) acquired by intermediaries as given in (3), is an increasing function of \( e \). In other words, as origination standards deteriorate, the proportion of non-performing assets on the balance sheet of intermediaries also increases, since the fraction of non-performing assets with a signal \( \sigma_h \) distributed in the OTC market increases. This is shown in Figure 7. Panel A illustrates how the asset side of intermediaries’ balance sheet mirrors the non-monotonicity of origination effort in \( K \). Panel B illustrates Proposition 4: As aggregate savings \( K \) increase so does intermediaries’ leverage. Thus, for a while there is a positive correlation between leverage and asset quality on intermediaries’ balance sheets, but once aggregate savings reach such a high point that there is a glut compressing spreads and eroding origination incentives, the correlation turns negative. In sum, our model can account for the double source of fragility in the balance sheet of financial intermediaries documented in the years leading up to the great Recession (see Figures 1 and 3).

Moreover, intermediary leverage is higher the less precise is their information about asset quality (the higher is \( \alpha \)). Other things equal, an increase in \( \alpha \) lowers the expected payoff of acquired assets, and therefore the price intermediaries are willing to pay, \( p^d \). This, in turn, results in a narrower spread, \( p^d - p \), which economizes the equity capital that intermediaries need to hold, thereby increasing their leverage, \( \ell^i = D^i / K \).

### 4.4 Implications

The savings glut pinpointed by Bernanke (2005) and commonly proposed as a fundamental cause of the financial crisis conjures the image of a financial system that is not equipped to absorb vast pools of new savings. But beyond this image there is limited appreciation of the precise mechanisms that lead an economy awash with liquidity to a financial crisis. Our model and formal analysis is an attempt to uncover these mechanism and thereby gain sharper policy
implications.

The central mechanism in our model is the effect of the savings glut in compressing spreads, and thereby undermining origination incentives. The growing demand for assets is met with greater supply of lower quality assets. Our key observation is that cash-in-the-market pricing masks the effect of spread compression and the deterioration of origination incentives. Uninformed investors are affected by changes in the cash-in-the-market price \( p \), which does not reflect the deteriorating fair-value price of the assets on the exchange \( p' \). In other words, as a result of the savings glut, asset prices on the exchange rise even when the fair value of assets falls.

A second, reinforcing, mechanism is through the rise of leverage of financial intermediaries caused by the savings glut and the higher collateral values it induces. This expansion of intermediaries balance sheets through increased leverage occurs just as underwriting standards of originated assets deteriorate, resulting in greater financial fragility of the financial intermediary sector.

These main predictions of our model are broadly in line with developments in the financial sector in the run-up to the crisis of 2007-09: Inflows of greater emerging market savings into global asset markets produced the rise in asset prices, a compression of yields, an expansion of repo markets and bank wholesale funding along with a deterioration of mortgage origination standards and greater fragility of financial intermediaries.

Of course, some financial institutions, such as Lehman Brothers, Merrill Lynch and Bear Stearns, were more aggressive in expanding their balance sheets, to the point where they ultimately failed. One intriguing possibility in terms of our model is that the these institutions had underestimated their own \( \alpha \). They were overly relying on information on their past returns on the assets they purchased to assess their own ability to control risks. If they were unaware of the deterioration in origination incentives, as seems plausible, they may have unwittingly added a larger proportion of non-performing assets to their balance sheets than they could handle (see Foote, Gerardi and Willen, 2012). When larger losses than predicted by their own risk models materialized and these financial institutions realized that the proportion of good assets on their balance sheet was lower than estimated it was too late. To capture this behavior,
we could extend the model to allow for the possibility of an endogenous $\alpha$. We could add to the model that informed intermediaries are engaged in costly endogenous screening of assets and that they determine their screening effort based on the past history of non-performing assets they acquired. Then, as origination standards improve ($e(K)$ increases) intermediaries would respond by cutting their screening effort, which, in turn, could amplify their financial fragility at the peak of the savings glut and lead to their insolvency.

### 4.5 Other Comparative Statics

Our exclusive focus so far has been on comparative statics with respect to $K$. But our model yields two other important comparative statics results with respect to $N$ and $\eta$, which we characterize below.

#### 4.5.1 Distribution of capital and knowledge

How are asset prices and origination incentives affected when the increase in liquidity is concentrated within the financial intermediary sector? We can address this question by looking at the comparative statics with respect to $N$, the measure of informed traders. Indeed, by increasing $N$ we increase the relative liquidity of the intermediary sector, in equilibrium.

**Proposition 5** Let $N_2$ and $K$ be such that $N_2 K < \tau$ and $N_1 < N_2$. Suppose there exists continuous functions $p(N)$ and $e(N)$ for $N \in [N_1, N_2]$ such that $(p(N); e(N))$ is a strict CIM equilibrium for parameters $(K, N, \alpha)$. Then $p(N)$ is decreasing, $e(N)$ is increasing in $N$ and $\mathcal{L}(N)$ is decreasing in $N$.

**Proof:** Follows from Lemma A.2.

In words, asset prices on the exchange $p$ are a decreasing function of the proportion of capital held by intermediaries (and therefore, since $N = 1 - M$, an increasing function of the proportion of savings held by “dumb” investors). Origination incentives, $e$, are an increasing function of the proportion of capital held by intermediaries, $N$. Intuitively, an increase in intermediary capital results in a higher probability $m$ of selling a high quality asset to an intermediary and also an increase in the spread $(p^d - p)$, so that origination incentives
are improved. Finally, leverage decreases with $N$. This follows because as $N$ increases, effort increases and thus $g$ increases. As a result $p^d$ goes down by less than $p$, and thus $p/p^d$ decreases and so does $\ell^i$ (see equation (19)).

### 4.5.2 Haircuts and incentives for good origination

Could excessively low repo haircuts have been a contributing factor in worsening the fragility of financial intermediaries before the crisis? To address this question we need to characterize the comparative statics with respect to $\eta$. As the proposition below establishes, a lower haircut allows informed intermediaries to borrow more, resulting in lower asset prices on the exchange and higher origination incentives.

**Proposition 6** Let $N$ and $K$ be such that $NK < \varepsilon$, and suppose that there exist continuous functions $p(\eta)$ and $e(\eta)$ that correspond to strict CIM equilibria for parameter values $(K, N, \alpha, \eta)$, $\eta \in [\eta_1, \eta_2]$. Then $p(\eta)$ is increasing and $e(\eta)$ is decreasing in $\eta$.

**Proof:** Follows directly from Lemma A.3. □

In other words, an increase in the haircut $\eta$ limits the amount of liquidity flowing to informed intermediaries through the repo market. More liquidity gets channelled to the exchange, resulting in higher asset prices $p(\eta)$. Thus, an unintended consequence of a policy seeking to strengthen financial stability by imposing higher haircuts $\eta$ for secured loans is to undermine origination incentives and thereby to adversely affect the quality of assets distributed in financial markets.

### 5 Robustness

#### 5.1 Endogenous origination volume and distribution

A simplifying assumption in our analysis so far has been that the total volume of originated assets is fixed and completely price inelastic. But, when a savings glut pushes up asset prices, isn’t it natural to expect a supply response and an increase asset origination? A striking example of such a response was the large increase in the float of dot.com IPOs following the
expiration of lock up provisions, which contributed to the bursting of the dot.com bubble (Ofek and Richardson, 2003). Similarly, the rise in real estate prices up to 2007 gave rise to a construction and securitization boom. As this increased real origination volume was not sufficient to quell the market’s thirst for new mortgage-backed securities, it was further augmented at the peak of the cycle by a larger and larger volume of synthetically created assets.

Our model can be extended to allow for a price-elastic origination volume and our results are robust as long as the supply of assets is not too price elastic. Indeed, if origination volume were perfectly price elastic there could not be a savings glut. A particularly interesting way of introducing a price-elastic origination volume is to let originators choose whether to distribute or retain the asset they originated until maturity and to allow for different demand for liquidity across originators. Then, in equilibrium, for any given $K$, a fraction $\lambda(p)$ of originators would prefer to distribute their asset, and the fraction $(1 - \lambda(p))$ to hold on to their asset until maturity. The fraction $\lambda(p)$ would be increasing in $p$, thus giving rise to a price-elastic volume of distributed assets. In this more general formulation of the model, a savings glut would doubly undermine origination incentives. Not only would spread compression reduce market incentives of originators who intend to distribute their assets, but also a smaller fraction of originators would have skin in the game by retaining their assets to maturity. This more general version of the model could thus also account for the deterioration of origination standards in the run-up to the crisis of 2007-09 that was caused by lower skin-in-the-game incentives.\footnote{See on this issue, for instance, Bord and Santos (2012), Keys, Mukherjee, Seru and Vig (2010) and Purnanandam (2011). There are of course additional issues associated with distribution such as active misrepresentation by originators as in Piskorski, Seru and Witkin (2015).}

5.2 Overconfidence and ‘bad apples’

Our model can also be extended to introduce overconfident investors. Consider, for example, the equilibrium situation where informed investors have a screening technology with $\alpha = .2$, but a single, atomistic, informed investor believes that her $\alpha = 0$. That is, this optimistic investor believes that the risk management systems in place guarantee that only good assets
enter the balance sheet. The optimistic investor takes as given the equilibrium price on the exchange $p^*$ and interest rate $r^*$. The quantity of assets bought and leverage of the optimistic investor are then:

$$
\tilde{q} = \frac{K}{\kappa (x_h - p^*) + \eta p^*} \quad \text{and} \quad \tilde{D} = \frac{(1 - \eta) p^* K}{\kappa (x_h - p^*) + \eta p^*},
$$

which should be compared with (A.14) and (A.15).

Note first that $q^* > \tilde{q}$, as the optimistic investor bids more for assets in the OTC market than the other informed intermediaries who have the correct assessment of $\alpha$. Indeed the optimistic investor bids

$$
\tilde{p}^d = \kappa x_h + (1 - \kappa) p^*,
$$

which is higher than the price offered by the other intermediaries (see (4)). As a result $D^* > \tilde{D}$, and thus $L^* > \tilde{L}$, as the optimistic investor would have less collateral to post in the repo market. In other words, leverage of the optimistic intermediary is lower than that of intermediaries who have an accurate estimate of the precision of the signal.

The optimistic intermediary believes his expected equity in period 2 is

$$
E^2_o = \tilde{q} x_h - r^* \tilde{D},
$$

whereas the true level of capital is

$$
E^2_{true} = \tilde{q} g^* x_h - r^* \tilde{D} < E^2_o.
$$

Second, note that the optimistic investor’s true average level of capital is lower than the average level of capital of investors with an accurate assessment of $\alpha$, $E^2$, which is

$$
E^2 = q^* g^* x_h - r^* D^* = \left( \frac{\kappa (x_h - p^*) + \eta p^*}{\kappa (g^* x_h - p^*) + \eta p^*} \right) E^2_{true} > E^2_{true}.
$$

In sum, optimistic investors have smaller balance sheets and take on less leverage, but have less equity capital at $\tau = 2$ than investors with an accurate estimate of $\alpha$. This simple example remarkably illustrates how a bank supervisor focusing on book leverage would be misled to believe that the optimistic intermediary is the safer one. This is illustrated in Panel A of Figure 8 where the top line represents the equity capital under the wrong beliefs and
the bottom line represents the true equity capital. Leverage, simply put, does not equal risk exposure.

There is evidence that during the credit bubble many financial intermediaries, although fully cognizant of the link between home price appreciation (HPA) and MBS values, did not consider possible the sharp nationwide negative scenarios of HPA that actually came to pass. Analysts most extreme scenarios at a major investment bank did not go beyond a 5% correction in housing prices, far less than was experienced (see Gerardi, Lehnert, Sherlund and Willen, 2008). We could also model a situation of aggregate optimism by assuming that all informed intermediaries believe that \( \alpha_o = 0 \) even though \( \alpha > 0 \). Because all intermediaries are excessively optimistic about their signal, they bid aggressively for “good” assets beyond what their expected payoff should warrant. As above, book leverage would be relatively low in this situation and the average period 2 equity capital would be lower than anticipated. Panel B of Figure 8 shows the level of capital under the wrong beliefs (\( \alpha_o = 0 \)) and the actual level of capital under the true value of \( \alpha \), for \( \alpha = .4 \).

5.3 Motivating the Collateral Constraint

Our analysis builds on the assumption that intermediary borrowing is limited by constraint (10). One way of motivating this constraint is to introduce the possibility that the lender may change her mind in period 1 and refuse to roll over the loan in period 1. That is, suppose that period 1 is broken into two sub-periods, \( 1_− \) and \( 1_+ \). In sub-period \( 1_− \) assets are originated, borrowing and lending occurs, and the OTC market is operating. In period \( 1_+ \) only the exchange is open for trade and all other markets (OTC and Repo) are closed. However, any loan agreed to in period \( 1_− \) is callable in period \( 1_+ \), or in other words, the repo loan extended in period \( 1_− \) is open to a roll-over decision in period \( 1_+ \). Suppose, in addition, that a small set of uninformed agents learns in period \( 1_+ \) that they must consume immediately in this period and for this reason need to call their loans. The intermediaries who borrowed from them must then surrender their collateral to the lender, who subsequently pays an “iceberg cost” \( \eta \) before selling the collateral in the only market which is open - the exchange. If lenders understand that they may face this liquidity risk in period \( 1_+ \), and want to make sure that they can consume
at least their endowment in period $1_+$, then they will impose constraint (10) on the borrower. In other words, the collateral constraint (10) is determined by the rollover risk lenders face in period $1_+$ and is not driven by concerns about the creditworthiness of the borrower.

Alternatively, the collateral requirement could be driven by credit risk and the concern that the borrower does not engage in a strategic default. More precisely, consider the situation where the borrower could at any time abscond with a fraction $\eta$ of the assets after the loan has been agreed, but before date 2 cash flows are realized. In this case the borrower expects to obtain an expected payoff of $\eta q g x_h$. To prevent such a move the lender must make sure that the borrower is better off realizing the returns on his portfolio in period 2 and repaying his total debt obligation $D$. In other words, the following incentive constraint must hold:

$$q g x_h - D \geq \eta q g x_h$$

or,

$$D \leq (1 - \eta) q g x_h.$$  \hspace{1cm} (20)

Moreover, to obtain a finite $D$ it must be the case that in equilibrium, $p^d > (1 - \eta) g x_h$.

Preliminary analysis shows that the main qualitative results of our model are robust to the replacement of leverage constraint (10) with (20).

There is no obvious way to derive a constraint analogous to (10) but with $p^d$ replacing $p$. Often accounting conventions define haircut as a function of the book-price of an asset. This, however, does not insure that modeling the haircut as a constant proportion of book-price is adequate. Each of the two economic motives for a haircut that we presented would imply a haircut as a proportion of $p^d$, but this proportion would vary with market conditions.

### 5.4 Variation in risk premia or cash-in-the-market pricing?

We have modeled the savings glut and its effect on asset prices, spreads and origination incentives as an aggregate liquidity asset pricing phenomenon captured in simple terms by cash-in-the-market pricing. But, an alternative account of the effects of an increased pool of savings is possible based on the more classical notions of discount rates and “risk adjusted capital.” Indeed, there is ample evidence coming from the asset pricing literature that discount rates are
countercyclical, high during troughs and low at the peak of the cycle. Under this alternative interpretation, asset prices on the exchange are affected by both the fundamental quality of assets and the risk appetite of uninformed investors. Indeed, Shin (2012) develops a model in which bondholders’ risk appetite is pro-cyclical, inducing banks to lever up more in booms. In our model, when the discount rate of uninformed investors moves counter-cyclically, so will the excess return \( r^x - 1 \). A reduction in discount rates then leads to higher asset prices on the exchange, smaller price spreads \( p^d - p \) and consequently weaker origination incentives. It also results in higher leverage of financial intermediaries, yielding the pro-cyclical leverage patterns identified by Adrian and Shin (2010). This alternative account also matches the evidence that leverage is highest, and the worst assets are originated at the peak of the cycle, which leads to maximal financial fragility just when the economy is booming.

### 5.5 Double Margin

Our key single-peakedness of effort result (Proposition 3) is driven by the interplay of two countervailing forces. A first force is that when savings \( K \) increase the spread \( p^d - p \) narrows, undermining incentives to originate good assets. The second force is that when savings \( K \) increase the price \( p \) increases and the return \( r^x \) decreases, relaxing the borrowing constraint of informed intermediaries. As a result, more informed capital can be deployed to purchase good assets, increasing origination incentives. The basic economic logic behind Proposition 3 is that at low levels of savings \( K \) the latter effect dominates, but at higher levels of savings the former effect dominates, giving rise to our single-peakedness result.

It turns out that this result depends on the presence of a double margin, the \( p^d - p \) margin and the collateral haircut \( \eta \). When there is no haircut, \( \eta = 0 \), it turns out that

\[
\frac{\partial e}{\partial K} = \frac{N}{\psi_e + e\psi_{ee}} > 0.
\]

This can be immediately observed by setting \( \eta = 0 \) in equations A.17 to A.19, and computing \( \frac{\partial e}{\partial K} \) by totally differentiating with respect to \( e \) and \( K \) in equation A.17. The fact that \( \frac{\partial e}{\partial K} \) always has a positive sign when \( \eta = 0 \) is not entirely surprising.\(^{16}\) In effect, a positive haircut

\(^{16}\)Thus to explain the deterioration of standards while maintaining the assumption that \( \eta = 0 \) one would need
acts like a speed-bump is reducing the flow of capital from the uninformed exchange to the informed dealer market. This speed-bump reduces the strength of the countervailing force on incentives of a greater share of informed capital being deployed. It is only when the speed-bump is entirely removed that this countervailing force is always dominant. Paradoxically, our analysis suggests that financial fragility can by lessened by lowering the haircut on repo loans. A reduction in $\eta$ increases the flow of savings to informed intermediaries and results in a lower uninformed price $p$, thereby improving origination incentives.

6 Conclusion

We have developed an extremely simple model in which asset prices, spreads, origination incentives and leverage are driven by aggregate liquidity conditions. The notion of cash-in-the-market pricing, first introduced by Allen and Gale (1998), is central for tractability and a clear analysis of potentially complex effects. The other central building block is the dual representation of financial markets as in Bolton, Santos and Scheinkman (2016), with an organized exchange where uninformed investors trade, and an OTC market where informed traders cream skim the best assets. The third essential element is a repo market where agents can borrow against collateral.

Asset originators distribute assets across the two markets. A key economic mechanism in our analysis centers on origination incentives, which arise from the ability of informed investors to identify the better assets, and their offer of a price improvement relative to the exchange for those higher quality assets. There are two effects of aggregate liquidity on origination incentives. First, the equilibrium price improvement for higher quality assets narrows as liquidity surges, which weakens origination incentives. Second, as aggregate liquidity increases some of it will find its way to the balance sheets of informed investors, who then can buy more high quality assets, which is good for origination incentives. A central result in our model is that the latter effect dominates when the level of aggregate liquidity is low, whereas the former effect dominates when it is high. An important implication of this result is that to assume that the capital increase is biased towards uniformed investors (see Proposition 5 above).
origination incentives eventually deteriorate with increasing aggregate liquidity.

Another basic result is that the balance sheet of financial intermediaries becomes more fragile as liquidity increases. There are two reasons why financial fragility increases. First, unless the screening abilities of informed investors are perfect, the fraction of good assets in intermediaries’ balance sheets is an increasing function of the fraction of good assets originated. Thus, as origination standards deteriorate and the fraction of originated high-quality assets falls, the balance sheets of informed intermediaries necessarily absorb an increasing fraction of non-performing assets. Second, a further fragility is induced because more of the balance sheet is financed with leverage. Indeed, we show that leverage is increasing in aggregate liquidity conditions. Thus as liquidity becomes abundant, there are worse assets on the intermediaries balance sheets with more leverage. Our model thus offers a particularly simple account, based on a straightforward economic mechanism, of the years leading up to the Great Recession. In essence, we argue that the savings glut emphasized by Bernanke (2005) in his account of low interest rates in the run-up to the crisis was amplified by the financial sector, as suggested by Borio and Disyatat (2011), and that this resulted in weaker origination standards and a fragile financial system.
References


Aliber, Robert (2011) “Financial turbulence and international investment”, *BIS working paper No 58*.


Borio, Claudio and Piti Disyatat (2011) “Global imbalances and the financial crisis: Link or no link?,” BIS Working Papers, no. 346, May.


Loeys, Jan, David Mackie, Paul Meggyesi and Nikolaos Panigirtzoglou (2005) “Corporates are driving the global savings glut,” Global Issues, June 24th, JPMorgan.

Martinez-Miera, David and Rafael Repullo (2015) “Search for Yield,” manuscript, CEMFI.


Figure 1: Corporate savings in billions of US$ (left axis) and BofA Merrill Lynch US High Yield Option-Adjusted Spread in % (right axis). Quarterly:1997Q1-2017Q1. Not seasonally adjusted. Data Source: Federal Reserve Bank of St. Louis
Figure 2: Broker dealers: Total assets in billions of dollars and leverage (total assets to book equity). Quarterly: 2000Q1-2007Q2 Data source: Federal Reserve Board
Figure 3: Panel A: Fraction of loans that were originated with no/low/reduced documentation among non-agency mortgages with known documentation status in the Loan Performance (LP) database. Panel B: Average cumulative default paths for the non-agency securitized loans by year of their origination. The cumulative delinquency rates are of all privately securitized residential mortgages calculated from loan level, monthly Corelogic-Blackbox data. A loan is defined as delinquent if it ever becomes 60 days past due and is considered delinquent thereafter. The graph is separated by year of mortgage origination, the y-axis plots cumulative delinquency rates in each quarter following origination which is depicted on the x-axis. Data source: Corelogic
Figure 4: Flows and stocks in a strict CIM equilibrium

\[
p = \min \left\{ \frac{e(1-m)x_h}{1-em-(1-e)am}, \frac{MK - ND^i}{1-em-(1-e)am} \right\}
\]

Originators: 1

Financial intermediaries: \( N \)

Assets | Liabilities
--- | ---
\( q^i p^d \) | \( D^i \)
\( K \) | \( K \)

Investors: \( M \)

Uninformed capital

\( MK - ND^i \)

Incentives: \( e \)

\( p^d \)

\( g x_h \)
Figure 5: Panel A: Prices in the exchange $p$ and expected payoff (dashed line) as a function of capital, $K$. Panel B: Expected rate of return in the opaque market, $R$, expected rate of return in the exchange $r^x(=r)$; the horizontal line is set at one. In this example, $\psi(e) = \theta \frac{e^2}{2}$ with $\theta = .25$. In addition, $\kappa = .15$, $\eta = .5$, $M = .75$ and $x_h = 5$. 

---

**Panel A**

- $p^f(K)$, dashed green line
- $p(K)$, solid blue line

**Panel B**

- $r^x(K)$, dashed green line
- $R(K)$, solid blue line
- $r = r^x$, solid red line
Figure 6: Panel A: Origination effort $e$, as a function of capital, $K$, for the cases $\alpha = 0$ (dashed line, in green) and $\alpha = .2$ (continuous line, in blue). Panel B: $x_h p$, as a function of capital $K$ (continuous line, in blue, left axis) and matching probability $m$ as a function of capital, $K$, (dashed line, in green, right axis). In this example, $\psi(e) = \theta \frac{e^2}{2}$ with $\theta = .25$. In addition, $\kappa = .15, \eta = .5, M = .75$ and $x_h = 5$. 

Panel A

Panel B
Figure 7: Fragility. Panel A: Fraction of high payoff assets in the balance sheet of financial intermediaries, \( g \), as a function of capital, \( K \). Panel B: Leverage, \( \ell := D/K \), as a function of capital \( K \) for the cases \( \alpha = 0 \) (dashed line, in green) and \( \alpha = .2 \) (continuous line, in blue). In this example, \( \psi(e) = \theta e^2 / 2 \) with \( \theta = .25 \). In addition, \( \kappa = .15, \eta = .5, M = .75 \) and \( x_h = 5 \).
Figure 8: Panel A: Equity capital under the wrong beliefs, $E_o^2$ (dashed line) and the true equity capital at date 2, $E_{true}^2$, of the single optimistic intermediary who believes $\alpha = 0$ when $\alpha = .2$.

Panel B: Equity capital when the entire intermediary sector believes that $\alpha_o = 0$ when the true value of $\alpha$ is $\alpha = .4$ (dashed line) and true capital of financial intermediaries (continuous line). In this example, $\psi(e) = \theta e^2 / 2$ with $\theta = .25$. In addition, $\kappa = .15$, $\eta = .5$, $M = .75$ and $x_h = 5$. 

Panel A

Panel B
A Appendix

A.1 Equations defining equilibria

In this section we write down necessary and sufficient conditions for a vector describing prices \((p, p^d, r, R, r^x)\) and quantities \((e, g, q^u, D^u, q^i, y, D^i, m)\) in which \(R > r > 1\) to be an equilibrium. Recall that in this case we necessarily have \(r^x = r\), and \(y = q^i\). In fact, we can parametrize the equilibrium which much fewer variables. Given \((p, r)\) and candidate choices \((e, q^u, q^i)\), we may define

\[
p^d := \kappa g x_h + (1 - \kappa) p \tag{A.1}
\]
\[
m := \min \left\{ \frac{Nq^i}{e + \alpha (1 - e)}, 1 \right\} \tag{A.2}
\]
\[
g := \frac{e}{e + \alpha (1 - e)} \tag{A.3}
\]
\[
r^x := \frac{e(1 - m)x_h}{p(1 - m(e + \alpha (1 - e)))}\quad\text{and}\quad R := \frac{gx_h}{p^d} \tag{A.4}
\]
\[
D^u := pq^u - K \tag{A.5}
\]
\[
D^i := p^d q^i - K \tag{A.6}
\]

\((p, r)\) and candidate choices \((e, q^u, q^i)\) form an equilibrium with \(R > r\) if and only if,

\[
D^u \leq 0 \tag{A.7}
\]
\[
D^i = (1 - \eta) pq^i \tag{A.8}
\]
\[
p = \min \left\{ \frac{MK - ND^i}{1 - m (e + \alpha (1 - e))}, \frac{e(1 - m)x_h}{1 - m (e + \alpha (1 - e))} \right\} \tag{A.9}
\]
\[
\psi' (e - e) = (1 - \alpha) m \kappa (gx_h - p) \tag{A.10}
\]
\[
r = r^x < R \tag{A.11}
\]
\[
Mq^u + Nq^i = 1 \tag{A.12}
\]
\[
MD^u + ND^i = 0 \tag{A.13}
\]
Equation (A.8) is the leverage constraint and (A.6) is the budget constraint of informed investors. Equation (A.9) is the price in the exchange, which is the minimum of the cash-in-the-market price and the fair value one. (A.10) is the first order condition of originators. Equations (A.12) and (A.13) are the market clearing conditions.

It follows from (A.6) and (A.8) that in an equilibrium with \( r > R \) then
\[
q^i = \frac{K}{\kappa (gx_h - p) + \eta p}, \quad (A.14)
\]
\[
D^i = \frac{(1 - \eta)pK}{\kappa (gx_h - p) + \eta p}. \quad (A.15)
\]
Furthermore since \( D^i > 0, D^u \leq 0 \). Thus (A.7) can be ignored.

It is easy to verify that in a strict CIM equilibrium, the pair \((p, e)\) completely determines all the prices and quantities in equilibrium. Given \( p \), equations (A.1), (A.5), (A.6), (A.12) and (A.13) uniquely determine \((q^u, D^u, q^i, D^i)\). Thus given \((p, e)\) the RHS of equations (A.1)-(A.6) are uniquely defined. As a consequence, we may parameterize the set of equilibria by the pair \((p, e)\).

### A.2 A system of equations in \( p \) and \( e \)

We will consider the model as parameterized by \((K, N, \alpha)\) and to save notation we will often set \( \theta = (N, \alpha) \). The system of equations (A.9)-(A.15) can be simplified to yield a tractable system in two equations with two unknowns, the price in the exchange \( p \) and the originators’ effort \( e \). Indeed straightforward algebra shows that in a strict CIM equilibrium with \( R > r \) and \( r^x > 1 \) then
\[
f^1 (p, e, K, \theta) := p - K (M + N \gamma) = 0 \quad (A.16)
\]
\[
f^2 (p, e, K, \theta) := (e + \alpha (1 - e)) \psi'(e - \varepsilon) - (1 - \alpha) NK \beta = 0 \quad (A.17)
\]
where
\[
\gamma := \frac{\eta p}{\kappa (gx_h - p) + \eta p} \quad (A.18)
\]
\[
\beta := \frac{\kappa (gx_h - p)}{\kappa (gx_h - p) + \eta p} = 1 - \gamma, \quad (A.19)
\]
Notice that

\[-\beta_p = \gamma_p = \frac{\eta \kappa gx_h}{(\kappa (gx_h - p) + \eta p)^2}\]  \hfill (A.20)

\[-\beta_e = \gamma_e = -\frac{\eta \kappa x_h g_e}{(\kappa (gx_h - p) + \eta p)^2}\]

where \[g_e = \frac{\alpha}{(e + \alpha (1 - e))^2} > 0\]  \hfill (A.21)

The function \(f\) is (mathematically) well defined for \(0 \leq N \leq 1\), for \(0 \leq \alpha \leq 1\) and for \(0 \leq p \leq x_h\), \(e \leq e \leq \bar{e}\) and \(K > 0\).

A “converse” also holds: Under additional conditions, the zeros of the system (A.16)-(A.17) correspond to CIM equilibria with \(R > r = r^x > 1\). To show this let

\[f(p, e, K, \theta) := \begin{pmatrix} f^1(p, e, K, \theta) \\ f^2(p, e, K, \theta) \end{pmatrix}.\]  \hfill (A.22)

Suppose that \((p, e)\) with \(e \geq e\) solves \(f(p, e, K, \theta) = 0\), with \(N \geq 0\) and \(0 \leq \alpha \leq 1\). Use equation (A.3) to calculate the implied \(g\) and then equation (A.14) to calculate the implied \(q^i\). Then equations (A.2), (A.1) and (A.4) can be used to yield the implied \(m, p^d, r^x\) and \(R\). If \(R > r^x\), choose \(r = r^x\), set \(D^i\) using (A.15) and \(D^u\) using (A.13). It is then easy to check that (A.10) is satisfied. Set \(q^u\) using (A.6). Walras Law insures that the goods market is in equilibrium, that is (A.12) holds. Thus to obtain a strict CIM equilibrium from a zero of \(f\) it suffices to apply this algorithm and verify that \(R > r := r^x > 1\). This algorithm also shows that there is a unique strict CIM equilibrium that would generate the same \((p, e)\). For this reason, with a slight abuse of language, we call \((p, e)\) a strict CIM equilibrium. The algorithm also shows that the implied prices \((p^d, r^x, r, R)\) and quantities \((g, q^u, D^u, q^i, y, D^i, m)\) vary continuously with \((p, e, K, \theta)\).

We make use of the implicit function theorem below, both to show the existence of an equilibrium where \(R > r = r^x > 1\) and to characterize its basic properties. We will use as a starting point \(\alpha = N = 0\), \(e = e\) and thus, to guarantee that this starting point is interior, we need to extend the domain of \(f\). Notice first that \(f\) is mathematically defined for \(N < 0\), and \(\alpha > \frac{-e}{1-\bar{e}}\). We extend the definition of \(f\) for \(e \in (e - \epsilon, 1), \epsilon\) small. We do this by defining \(\psi(z)\) for \(z > -\epsilon\) in a \(C^2\) manner, while preserving convexity. Notice that the convexity of the extended \(\psi\) guarantees that \(\psi'(z) \leq 0\) for \(z < 0\).
The calculation of the matrix of the partial derivatives of \( f \) at \((p, e, K, N, \alpha)\) is straightforward.

\[
\begin{align*}
    f^1_p & := \frac{\partial f}{\partial p} = 1 - NK\gamma_p \quad (A.23) \\
    f^1_e & := \frac{\partial f}{\partial e} = -NK\gamma_e \quad (A.24) \\
    f^2_p & := \frac{\partial f}{\partial p} = (1 - \alpha)NK\gamma_p \quad (A.25) \\
    f^2_e & := \frac{\partial f}{\partial e} = (1 - \alpha)\psi' + (e + \alpha(1 - e))\psi'' + (1 - \alpha)NK\gamma_e \quad (A.26)
\end{align*}
\]

Write

\[
\partial_{p,e} f(p, e, K, \theta) = \begin{pmatrix} f^1_p & f^1_e \\ f^2_p & f^2_e \end{pmatrix} \quad \text{with} \quad |\partial_{p,e} f| = f^1_p f^2_e - f^2_p f^1_e, \quad (A.27)
\]

For later use, notice that

\[
\begin{align*}
    f^1_K & := \frac{\partial f}{\partial K} = -(M + N\gamma) < 0 \quad (A.28) \\
    f^2_K & := \frac{\partial f}{\partial K} = -(1 - \alpha)N\beta \leq 0 \quad (A.29)
\end{align*}
\]

**Lemma A.1** There exists \( \bar{\epsilon} > 0 \) such that if \( N < \bar{N}(K) := \min\{1, \frac{5}{K}\} \) and \((p, e)\) is a strict CIM equilibrium for parameter values \((K, N, \alpha)\) and if \(e\) is the associated effort in this equilibrium then:

\[
\begin{align*}
    f(p, e, K, N, \alpha) &= 0 \\
    f^2_e(p, e, K, N, \alpha) &= 0 \quad (A.30) \\
    |\partial_{p,e} f(p, e, K, N, \alpha)| &= 0. \quad (A.31)
\end{align*}
\]

**Proof:** We have already shown that in a strict CIM equilibrium for parameter values \((K, N, \alpha)\), \(f(p, e, K, N, \alpha) = 0\). To prove the remaining claims, first notice that (A.9) guarantees that \(gx_h < p\). Thus \(\gamma < 1\) and \(\beta > 0\). Furthermore

\[
NK\gamma_p = NK \frac{\eta\kappa gx_h}{(\kappa (gx_h - p) + \eta p)^2} = Nq \frac{\eta\kappa gx_h}{\kappa (gx_h - p) + \eta p} = (e + \alpha(1 - e)) m \frac{\eta\kappa gx_h}{\kappa (gx_h - p) + \eta p}.
\]
Here we used (A.20) for the first equality, (A.14) for the second equality and (A.2) for the third equality. If \( \eta \geq \kappa \) then,

\[
\kappa < \frac{\eta \kappa g x_h}{\kappa (g x_h - p) + \eta p} \leq \eta, \tag{A.32}
\]

and if \( \kappa > \eta \)

\[
\eta < \frac{\eta \kappa g x_h}{\kappa (g x_h - p) + \eta p} < \frac{\eta \kappa g x_h}{\eta (g x_h - p) + \eta p} \leq \kappa.
\]

Since \( e < (e + \alpha (1 - e)m) < 1 \), for any \( \alpha \),

\[
f_p^1 \geq 1 - \max \{\eta, \kappa\} > 0, \tag{A.33}
\]

\[
f_p^2 \leq (1 - \alpha) \max \{\eta, \kappa\}. \tag{A.34}
\]

Notice that if \( \eta \leq \kappa \),

\[
\frac{p}{(k (g x_h - p) + \eta p)^2} \leq \frac{p}{(\eta g x_h)^2} \leq \frac{1}{\eta^2 g x_h}
\]

and if \( \kappa > \eta \),

\[
\frac{p}{(k (g x_h - p) + \eta p)^2} \leq \frac{1}{\kappa^2 g x_h}
\]

Thus

\[
(1 - \alpha)|\gamma_e| \leq \frac{(1 - \alpha) g_e}{g \min \{\eta, \kappa\}} \leq \frac{(1 - \alpha) \alpha}{\epsilon^2 \min \{\eta, \kappa\}} \leq \frac{1}{4 \epsilon^2 \min \{\eta, \kappa\}}.
\]

Since \( \psi'' >> 0 \), there exists \( \epsilon_1 \) such that if \( NK < \epsilon_1 \) then \( f_e^2 \geq \frac{\epsilon \inf \psi''}{2} >> 0. \) Hence (A.30) holds. In addition, there exists \( \bar{\epsilon} \leq \epsilon_1 \) such that if \( NK < \bar{\epsilon} \) then

\[
(1 - \max \{\eta, \kappa\}) \frac{\epsilon \inf \psi''}{2} - \frac{\max \{\eta, \kappa\} NK}{4 \epsilon^2 \min \{\eta, \kappa\}} > 0,
\]

which insures (A.31).

\[\Box\]

**Lemma A.2** If \( N < \tilde{N}(K) := \min \{1, \frac{\epsilon}{K} \} \) and \((p, e)\) is a strict CIM equilibrium for parameter values \((K, N, \alpha)\) then \( p_N < 0, e_N > 0 \) and \( \mathcal{L}_N < 0 \).

**Proof:** This follows from the previous Lemma, since

\[
f_N^1 = K \beta > 0 \quad \text{and} \quad f_N^2 = -(1 - \alpha) K \beta < 0.
\]

In addition, since \( e_N > 0 \) and \( g_e > 0 \), \( p/p^d \) decreases with \( N \) and thus \( \mathcal{L}_N < 0 \). \( \Box \)
Lemma A.3 If \( N < \bar{N}(K) := \min\{1, \frac{\bar{r}}{K}\} \) and \((p, e)\) is a strict CIM equilibrium for parameter values \((K, N, \alpha, \eta)\) then \( p_\eta > 0 \) and \( e_\eta < 0 \).

**Proof:** This follows since \( \gamma_\eta > 0, f_\eta^1 = -KN\gamma_\eta < 0 > 0 \) and \( f_\eta^2 = (1 - \alpha)KN\gamma_\eta > 0 \). \( \square \)

The following Lemma gives bounds for the price in the exchange \( p \).

**Lemma A.4** In a strict CIM equilibrium prices in the exchange satisfy \( K > p > \frac{MK}{2 - \eta} > 0 \).

**Proof:** Equations (A.5), (A.6), (A.12), (A.13) and \( p^d \geq p \) imply \( p \leq K \). Furthermore, in a strict CIM equilibrium

\[
\begin{align*}
p &= \frac{MK - ND^i}{1 - m(e + \alpha(1 - e))} \\
&> MK - ND^i \\
&= MK - N(1 - \eta)pq^i \\
&> MK - (1 - \eta)p, \quad \text{since} \quad Nq^i < 1.
\end{align*}
\]

Lemma A.5 Suppose \((\tilde{p}, \tilde{e})\) is a strict CIM equilibrium for parameter values \((\tilde{K}, \tilde{N}, \tilde{\alpha})\), with \( \tilde{N}\tilde{K} < \tilde{e} \). Then (i) There exists a \( \delta > 0 \) and \( \epsilon > 0 \) such that for \(|K - \tilde{K}| < \delta \) there exists a unique \((p(K), e(K))\) that is a strict CIM equilibrium for parameter values \((K, \tilde{N}, \tilde{\alpha})\) and within \( \epsilon \) from \((\tilde{p}, \tilde{e})\). Furthermore (ii) \( p'(\tilde{K}) > 0 \), and (iii) When \( \eta < \kappa \), then (a) If \( \tilde{K} < \frac{\tilde{\alpha}q}{\kappa + \eta} \) then \( e'(\tilde{K}) > 0 \); (b) if \( \tilde{K} > \frac{(2 - \eta)\tilde{\alpha}q}{2 - \eta + \tilde{M}} \) then \( e'(\tilde{K}) < 0 \); and (c) if \( e'(\tilde{K}) = 0 \) then \( \tilde{K} \) is a local maximum of \( e \).

**Proof:** To simplify notation we omit the arguments \( \tilde{N} \) and \( \alpha \) in what follows. Since \((\tilde{p}, \tilde{e})\) is a strict CIM equilibrium, by Lemma A.1 there exists \( \delta > 0 \) and \( \delta' \) such for \(|K - \tilde{K}| < \delta \) there exists unique \((p(K), e(K))\) with \(|(p(K), e(K)) - (\tilde{p}, \tilde{e})| < \delta' \) and \( f(p(K), e(K), K) = 0 \). Furthermore the functions \( p \) and \( e \) are at least \( C^2 \). The continuity of the candidate equilibrium constructed from a zero of \( f(p, e, K) \) guarantees that \((p(K), e(K))\) is indeed a strict CIM equilibrium and the uniqueness of the zeros of \( f(p, e, K) \) in a neighborhood of \((\tilde{p}, \tilde{e})\) establishes (i).
(ii) is immediate since $f^1_K < 0$ and $f^2_K \leq 0$.

Proof of (iii): For $|K - \tilde{K}| < \delta$ let

$$
\mathcal{A}(K) := \frac{f^2_p(p(K), e(K))f^1_K(p(K), e(K)) - f^1_p(p(K), e(K))f^2_K(p(K), e(K))}{1 - \alpha}
$$

and thus

$$
e'(K) = \frac{\mathcal{A}(K)(1 - \alpha)}{|\partial_{p,e,f}(p(K), e(p(K)))|}
$$

where, omitting the argument $(p(K), e(K))$ to simplify the expressions,

$$
\mathcal{A}(K) = NK\beta_p(M + N\gamma) + (1 - NK\gamma_p)) N\beta
$$

$$
= N\beta + NMK\beta_p + N^2K \left[ \gamma\beta_p - \gamma_p\beta \right]
$$

$$
= N\beta + NMK\beta_p + N^2K \left[ \gamma\beta_p + \beta_p\beta \right]
$$

$$
= N\beta + \beta_p NK \quad \text{using (A.19) and the fact that } M + N = 1.
$$

Thus, writing again the argument $(p(K), e(K))$

$$
\mathcal{A}(K) = N\beta(p(K), e(K)) + \beta_p(p(K), e(K))NK.
$$

If $\eta < \kappa$ then

$$
(gx_h - p) < \frac{\beta}{|\beta_p|} < \frac{\kappa}{\eta}(gx_h - p). \quad (A.35)
$$

If $K < \frac{\beta}{|\beta_p|}$ then $\mathcal{A}(K) > 0$. Since $g(\tilde{\alpha}e) = \frac{e}{e + \alpha(1 - e)}$ is monotone increasing in $e$, $g \geq \tilde{g}$. The upper bound on $p$ established in Lemma A.4 implies (a). Furthermore, if $K > \frac{(2 - \eta)x_h}{2 - \eta + M}$ then the lower bound in (A.35) and the upper bound on $p$ established in Lemma A.4 imply (b). Suppose now that that $e'(\tilde{K}) = 0$, and hence $\mathcal{A}(\tilde{K}) = 0$. Since $\eta < \kappa, \beta_pp_p(p(\tilde{K}), e(\tilde{K})) < 0$, and thus

$$
\mathcal{A}'(\tilde{K}) = N \left[ \beta_p(p(\tilde{K}), e(\tilde{K}))p'(\tilde{K}) + \beta_p(p(\tilde{K}), e(\tilde{K}))p'(\tilde{K})\tilde{K} + \beta_p(p(\tilde{K}), e(\tilde{K})) \right] < 0.
$$

This proves (c). \qed

**Remark A.6** Suppose the assumptions of Lemma A.5 hold at $(\tilde{K}, \tilde{N}, \tilde{\alpha})$. Then while

$$
|\partial_{p,e,f}(p, e, K, \tilde{N}, \alpha)| > 0
$$
one can prolong the domain of the functions $p(K)$ and $e(K)$. The Jacobian stays positive at least while $N(K) \geq N$. Since the function $N$ is non-increasing, a decrease in $K$ is always possible, but an increase in $K$ may lead to a violation of the bound on $N$. Furthermore, except for a $K$ close enough to $\tilde{K}$, there is no guarantee that the solution $(p(K), e(K))$ would lead to a candidate equilibrium with $R(K) > r(K) := r^z(K) > 1$. However if for $K \in [K_1, K_2]$ $(p(K), e(K))$ leads to a CIM equilibrium with $R(K) > r(K) := r^z(K) > 1$ and $N \leq \tilde{N}(K_2)$ we can use Lemma A.5 to compare the equilibria $(p(K), e(K))$. In particular the exchange price $p$ increases with $K$. If $K_1$ is small enough and $K_2$ large enough, the level of effort has a single global maximum - it increases if $K < \tilde{K}$ and decreases for $K > \tilde{K}$ for some $K_1 \leq \tilde{K} \leq K_2$.

**Proposition A.7** (i) If $(\tilde{p}, \tilde{e})$ is a strict CIM equilibrium for parameter values $(\tilde{K}, N, \alpha)$ with $0 < N < \tilde{N}(\tilde{K})$ and $e'(\tilde{K}) \leq 0$ then $\mathcal{L}' (\tilde{K}) > 0$. (ii) There exists a $\hat{N}$ such that if $(\tilde{p}, \tilde{e})$ is a strict CIM equilibrium for parameter values $(\tilde{K}, N, \alpha)$, with $N < \min\{\hat{N}, \tilde{N}(\tilde{K})\}$ then $\mathcal{L}' (\tilde{K}) > 0$

**Proof.** In a strict CIM equilibrium

$$D^i = \frac{(1-\eta)pK}{\kappa (gx_h - p) + \eta p}.$$  

Omitting the argument $\tilde{K}$ to lighten up notation:

$$(D^i)' = \left[ (p + p' \tilde{K}) (\kappa(gx_h - p) + \eta) - p \tilde{K} (\eta - \kappa) p' - \tilde{p} \kappa g x_h e' x_h \right]$$

$$\times \frac{1-\eta}{(\kappa (gx_h - p) + \eta p)^2}$$

$$= \left[ p (\kappa(gx_h - p) + \eta p) + \tilde{K} \kappa x_h (p' - pg e') \right] \times \frac{1-\eta}{(\kappa (gx_h - p) + \eta p)^2}$$

Thus,

$$\mathcal{L}' (\tilde{K}) = \frac{(D^i)' \tilde{K} - D^i}{K^2} = \frac{(1-\eta) \kappa x_h (p' - pg e')}{\tilde{K} (\kappa (gx_h - p) + \eta p)^2}.$$
Thus (i) follows from Lemma A.5. Furthermore if $N < \bar{N}(K)$,

$$
e' < \frac{1}{|\partial_{p,e,f}(p,e,K,N,\alpha)|} f_p^1(1 - \alpha)N\beta$$

$$\le \frac{(1 - \max\{\eta, \kappa\})N}{|\partial_{p,e,f}(p,e,K,N,\alpha)|}$$

$$\le \frac{N}{|\partial_{p,e,f}(p,e,K,N,\alpha)|}$$

and if $N < \bar{N}(K)$

$$p' > \frac{1}{|\partial_{p,e,f}(p,e,K,N,\alpha)|} \inf \psi'' \left( M + N\gamma \right)$$

$$\ge \frac{\epsilon}{|\partial_{p,e,f}(p,e,K,N,\alpha)|} \inf \psi'' \frac{\gamma}{2}$$

In the last equation we used $M + N\gamma = 1 - N + N\gamma > \gamma$. Furthermore,

$$\gamma \ge \frac{\min\{\kappa, \eta\}p}{\max\{\kappa, \eta\}x_h}$$

Thus by choosing

$$\hat{N} = \epsilon \frac{\inf \psi''}{2} \frac{\min\{\kappa, \eta\}}{\max\{\kappa, \eta\}x_h},$$

we have that for $N < \min\{\hat{N}, \bar{N}(\tilde{K})\}$, since $g_e \le 1$

$$p' - pg_e e' \ge \frac{p}{|\partial_{p,e,f}(p,e,K,N,\alpha)|} \left[ \frac{\inf \psi''}{2} \frac{\min\{\kappa, \eta\}}{\max\{\kappa, \eta\}x_h} - N \right] > 0 \quad (A.36)$$

establishing the second claim.

Until now, we have assumed the existence of a CIM equilibrium. The next Lemma shows that for a set of parameter values there exists a CIM equilibrium with $R > r^x$. Recall that we defined for each $\tilde{\alpha} \ge 0$,

$$\tilde{g} = g(\tilde{\alpha}, \epsilon) = \frac{\epsilon}{\epsilon + \tilde{\alpha}(1 - \epsilon)}$$

**Proposition A.8 (Existence)** Suppose that

$$\frac{\epsilon\tilde{g}\kappa x_h}{\epsilon - \epsilon + \epsilon\kappa} < \hat{K} < \epsilon x_h.$$  \quad (A.37)
Then (i) there exists a neighborhood \( N \) of \((\tilde{K}, 0, \tilde{\alpha})\) and \( \epsilon > 0 \), such that for every \((K, N, \alpha) \in N\) there is a unique \( p(K, N, \alpha) > 0 \) and \( e(K, N, \alpha) > 0 \) that is solution of \( f(p, e, K, N, \alpha) = 0 \) with \( |(p(K, N, \alpha), e(K, N, \alpha)) - (\tilde{K}, \tilde{\epsilon})| < \epsilon \). The functions \( p(K, N, \alpha) \) and \( e(K, N, \alpha) \) are \( C^2 \). (ii) If \( N > 0 \) and \( \alpha \geq 0 \) then \( e > \tilde{\epsilon} \). (iii) One may choose \( N \) such that \( (p(K, N, \alpha), e(K, N, \alpha)) \) is a strict CIM equilibrium for the parameter values \((K, N, \alpha) \in N\).

**Proof:** It is easy to check that \( f(\bar{K}, \bar{e}, \bar{K}, 0, \tilde{\alpha}) = 0 \). Since \( |\partial_{p,e}f(p, e, \bar{K}, 0, \tilde{\alpha})| > 0 \), the implicit function theorem guarantees that there exist a neighborhood \( N \) of \((\bar{K}, 0, N, \tilde{\alpha})\) and \( \epsilon > 0 \) such that for each \((K, N, \alpha) \in N\) there exists a unique \( (p, e) > 0 \) with \( |(p, e) - (\bar{K}, \bar{e})| < \epsilon \) such that \( f(p, e, K, N, \alpha) = 0 \), and that the functions \( p(K, N, \alpha) \) and \( e(K, N, \alpha) \) are \( C^2 \). Thus (i) holds. To establish (ii) note that if \( N > 0 \) and \( \alpha \geq 0 \) then from equation (A.17). \( \psi_e(e(K, N, \alpha) - \bar{e}) > 0 \) and hence \( e(K, N, \alpha) > \bar{e} \).

To show (iii), we first show that \((\bar{K}, \bar{e})\) is a strict CIM equilibrium for parameters \((\bar{K}, 0, \alpha)\). In fact by setting \( g = \bar{g} \) and using equation (A.14) to calculate the implied \( q^i \), it is easy to check that \( m = 0 \) solves (A.2), and \( p^d, r^x \) and \( R \) can be calculated using equations (A.1) and (A.4). The first inequality in (A.37) guarantees that \( R > r^x \) an thus we may choose \( r = r^x \), set \( D^i \) using (A.15) and \( D^u = 0 \) to satisfy (A.13). The second inequality in (A.37) insures that \( r^x > 1 \), and thus we may set \( q^u = 1 \). The remaining of the proof is as in the proof of Lemma A.5. Using the continuity of the implied strict CIM equilibrium prices and quantities with respect to the solution of \( f(p, e, K, N, \alpha) = 0 \) we may choose \( N \) such that the conditions \( m < 1 \), \( R > r := r^x > 1 \) are always satisfied whenever \( N > 0 \) and \( \alpha \geq 0 \). Hence \((p(K, N, \alpha), e(K, N, \alpha))\) is a strict CIM equilibrium when the parameters are given by \((K, N, \alpha) \in N\) and \( N > 0 \), \( \alpha \geq 0 \).

**Proposition A.9** In addition, one can choose the neighborhood \( N \) such that there are no other equilibria other than the (unique) strict CIM equilibrium.

**Proof:** Suppose there is a sequence of equilibria \((p_n, e_n)\) associated with the sequence of parameter values \((K_n, N_n, \alpha_n) \to (\tilde{K}, 0, \tilde{\alpha})\). Using equations (A.8) and (A.6), since \( g_n < 1 \), we obtain:

\[
(\kappa g_n x_n + (\eta - \kappa) p_n) q^i_n = K_n
\]

58
Since \( p_n < g_n x_h \)

\[
\eta n q_n^i < K_n.
\]

Since \( M_n \to 1 \) and \( K_n \to \bar{K} \), Lemma A.4 implies that \( q_n^i \) is bounded, and thus \( m_n \to 0 \), and \( e_n \to e \). Since \( p_n q_n \) is bounded, the leverage constraint implies \( D_n^i \) bounded and thus, in equilibrium, \( D_n^u \to 0 \). Given any \( \delta > 0 \) (A.9) implies that for \( n \) large, \( p_n < K_n + \delta \). If \( \delta \) is small enough, \( p_n \) cannot correspond to a Fair Value equilibrium if \( n \) is large, since \( K_n < e x_h \). If the equilibrium associated with \( N_n, \alpha_n \) is a CIM equilibrium, then necessarily \( p_n \to \bar{K} \) and since \( e_n \to e \) and (A.37) holds, for \( n \) large \( R_n > r_n^x \geq r_n \) and \( (p_n, e_n) \) is a strict CIM equilibrium. Thus \( f(p_n, e_n, K_n, N_n, \alpha_n) = 0 \).

and since for \( n \) large, \( |(p_n, e_n) - (\bar{K}, e)| < \epsilon \), by the previous Proposition, \( (p_n, e_n) = (p(K_n, N_n, \alpha_n), e(K_n, N_n, \alpha_n)) \). \( \square \)