

Optimal Corporate Securities Values
in the Presence of Chapter 7 and Chapter 11
(Preliminary Draft)*

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Abstract

In a contingent claims framework, with a single issue of debt and full information, we show that the presence of a bankruptcy code with automatic stay, absolute priority rules, and potential debt forgiveness, can lead to significant conflicts of interest between the borrowers and lenders. In the first-best outcome, the code can add significant value to both parties by way of higher debt capacity, lower spreads, and improvement in the overall value of the firm. If the control is given to borrowers about ex-ante timing of entering into bankruptcy and the ex-post decision to liquidate once the firm goes into bankruptcy, most of the benefits of the code are appropriated by the borrowers at the expense of the creditors. We show that the lenders can restore the first-best outcome, in large measure, by seizing this control or by ex-post transfer of control rights which allows them to decide when to liquidate the firm that has been taken to the chapter 11 process by the borrowers. Irrespective of who is in control, our model implies, based on the term structure of probabilities of default and liquidation, that firms are more likely to default on average and are less likely to liquidate on average relative to the benchmark model of Leland (1994).

JEL classification:

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1 Introduction

The bankruptcy code, which includes a liquidation process (Chapter 7), and a reorganization process (Chapter 11), aims to resolve a number of important issues associated with a distressed firm. These issues can be classified either into the information asymmetry (e.g. quality of the firm, Heinkel and Zechner, 1993), agency problems (e.g. risk shifting, Jensen and Meckling, 1976), or coordination problems (e.g. debt of various maturities, Berglof and von Thadden, 1994). In this paper, we investigate if there is a place for a reorganization process in the absence of these frictions. We wish to characterize the states of a borrowing firm relative to its outstanding contractual debt obligations at different stages of financial distress assuming full information and a single issue of debt. We also determine the role played by the bankruptcy code in improving the welfare of borrowers and lenders and how its influence depends on the rights given to borrowers and lenders at various stages of financial distress.

More precisely, we build on the framework of structural modelling of debt (Merton, 1974; Black and Cox, 1976), which allows us to determine the overall firm value, along with the values of equity and debt. We assume, in line with this literature, that liquidation destroys part of the firm's value. We ask a basic and yet important question whether there is any value in the availability of Chapter 11 even in the absence of asymmetric information, agency costs, and coordination problems.¹ Put differently, we try to determine if a distressed firm would choose the reorganization option under nearly perfect market assumptions. Our model extends the work of Leland (1994) who characterizes exclusively the liquidation option (Chapter 7). We superimpose a sparse characterization of Chapter 11 aimed to preserve the critical characteristics of the process, such as automatic stay, grace period, absolute priority, and transfer of control rights from equity to debt in bad states.²

In deciding which features of the reorganization proceedings must be incorporated in our stylized model, we follow Hart (1999) who identifies the goals of an efficient bankruptcy procedure, which enjoy a broad consensus in the theoretical and empirical work on the subject. First, a good

¹Our paper will have nothing to say about the optimal choice between the different workout alternatives to bankruptcy proceedings. Nevertheless, our results will provide the benchmark for making this decision.

²The grace period is the time that creditors give the debtors to recover from distress before the firm is liquidated. The grace period could be thought of as a renegotiated exclusivity period – the time frame within which a reorganization plan must be filed.

bankruptcy procedure should deliver an ex-post efficient outcome. Intuitively, this translates into total firm value maximization, ex-post. Our model incorporates this feature by solving for the first best outcome and, subsequently, addresses the question of achieving this outcome ex-post by shifting control from equity to debt. Second, the bankruptcy code should penalize borrowers and managers to preserve the bonding role of debt. Absent this feature there will be typically no lending, ex-ante. Suspension of dividends and the imposition of absolute priority rules (APR) upon entering bankruptcy are very much in the spirit of this goal. We model the automatic stay provision – the key feature of Chapter 11 which penalizes borrowers by suspending dividends. We also enforce APR by giving all residual value to creditors upon liquidation. Third, while there should be strong case for absolute priority, some portion of value should be set aside for equity holders. This goal is modelled by allowing debt forgiveness once under chapter 11.

Our modelling approach allows us to draw a clear distinction between the notions of bankruptcy and liquidation. Absent the reorganization option, Black and Cox (1976) and Leland (1994) have shown that the firm should issue equity to finance its contractual debt obligations until the equity value is driven to zero. This rule implies that the firm is not bankrupt as long as its equity value is positive. This rule also endogenously determines the bankruptcy boundary which coincides with the liquidation boundary. There is no room for bankruptcy without liquidation because the firm meets its contractual obligations either from operating cash flows, or by issuing equity until it is liquidated. There is no state in which the equity values are positive, but the firm, nevertheless, can not fulfill its obligations. This intermediate state prior to liquidation is introduced in our model by the presence of reorganization under Chapter 11. Thus, we can define a firm as being bankrupt when it files for Chapter 11, i.e. its operating cash flows are below an additional (endogenously determined) boundary even though its equity value is positive.

The conclusions regarding the welfare improvement that emerge from our model are nuanced: at a first glance, one might expect that the addition of another outside option in the form of a reorganization besides the liquidation option should lead to an improvement of welfare for every one. That is certainly not the case, in general. Equity value maximization is no longer consistent with total firm value maximization (as opposed to Leland, 1994), when the agents are presented with the options to reorganize or to liquidate. This result reflects the fact that when equity holders (debtors) are given the right to decide when to file for Chapter 11, they tend to capture all the

rents associated with the additional option, such as debt forgiveness and suspension of contractual payments, by filing early. Debt forgiveness may also be in the interest of the lenders and the firm as a whole because costly liquidation could be avoided. Nonetheless total firm value maximization generally requires to file for Chapter 11 later (relative to debtors), because this extends the period of complete contractual payments by the debtors. The divergence between firm value maximization and equity value maximization brings up some important issues that are absent in the benchmark model of Leland (1994) and, more generally, in the corporate debt literature.³

We address the issue whether the “first best” (i.e., the one that maximizes the total firm value) outcome can be restored. We show that debt maximization leads to a strategy that is qualitatively very similar to the first best. Therefore, one way in which the first best can be restored is by “contingent transfer of control rights” once chapter 11 decision is made by the debtors: this feature has been stressed in the incomplete contracting literature (see Aghion and Bolton, 1992 and Dewatripont and Tirole, 1994, for example). We allow creditors to take control of the reorganization process by choosing the length of the grace period. This feature will induce debtors to file for Chapter 11 later because the length of the grace period introduces a trade-off between a credible liquidation threat (preferred by creditors) and the ability to recover from distress (preferred by debtors).

Irrespective of who has control over decision making process, the probabilities of default increase relative to the Leland benchmark. This is natural since bankruptcy in our model occurs no later than the bankruptcy in the Leland model. Therefore, separating different forms of distress allows us to resolve the shortcomings of the extant models, which tend to underestimate the default probabilities. One might argue that this result is built in mechanically by introducing a new boundary. However, the probability of filing for Chapter 11 is what everybody is concerned with in practice. This probability is closely linked to the popular expected default frequency (EDF) used by Moody’s|KMV (Crosbie and Bohn, 2002; Leland, 2002). Interestingly, once the firm value is so low that it files for Chapter 11 in our model, the probability of avoiding liquidation increases in many states of the world relative to the Leland model. Hence, though our model induces earlier bankruptcy, the bankruptcy mechanism can decrease the probability of liquidation.

The paper is organized into six sections. The next section presents the details of our model,

³To the best of our knowledge. Relevant references are greatly appreciated.

discusses valuation issues and contrasts the model with the related work. Section 3 discusses the model implications under the first-best scenario. Section 4 contrasts debt and equity maximization and establishes that debt maximization is very close to the first-best. Section 5 is concerned with transfer of control rights from equity to debt in order to achieve the first-best. The final section concludes.

2 Model of Default and Liquidation

2.1 The Setup

Following Goldstein, Ju, and Leland (2001), we choose our primitive variable to be the operating cash flows or Earnings before interest and taxes (EBIT). The EBIT are governed under the risk-neutral measure \mathbb{Q} by:

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dW_t(\mathbb{Q}) \quad (1)$$

This assumption implies that the unlevered value of the firm's asset (or, a claim on the entire payout) is equal to:

$$V_t = E_t^{\mathbb{Q}} \left(\int_t^{\infty} e^{-r(s-t)} \delta_s ds \right) = \frac{\delta_t}{r - \mu} \quad (2)$$

Note that $\mu < r$. Indeed, since μ is a constant (2) implies:

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t(\mathbb{Q}) \quad (3)$$

and, therefore,

$$\frac{dV_t + \delta_t dt}{V_t} = r dt + \sigma dW_t(\mathbb{Q}) \quad (4)$$

So, the total return on the unlevered value is equal to the risk-free rate under \mathbb{Q} , or, in other words, μ represents an EBIT-adjusted risk-free rate.

The firm is raising cash to finance its projects by issuing one consol bond. As a result, the coupon rate c determines the firm's capital structure. We assume that potential creditors, and the firm's management/equity holders, have full information about the EBIT characteristics μ , and σ . Also, they are able to observe the values of EBIT continuously. There is no meaningful role for

debt in this setting unless there are corporate taxes and costly bankruptcy (Modigliani and Miller, 1958, 1963; Kraus and Litzenberger, 1973). We assume that the coupon rate was selected optimally taking into account these frictions. However, since the tax advantages of debt are obvious for the firm's equity holders, we ignore taxes to emphasize the impact of bankruptcy. In section 3.3 we return to this issue and explicitly consider the optimal capital structure in the presence of taxes and bankruptcy costs.

We assume that the investment policy of the firm is fixed, and EBIT is used to pay off the debt. Any surplus $\delta_t - c$, if there is one, is distributed to the equity holders of the company as dividends. The first credit event in our model which triggers all the subsequent developments is when earnings become less than the promised (contractual) coupon obligations.

It is important to point out at this stage, that we are modelling financial rather than economical distress. Therefore, bankruptcy by itself is not going to cause poor performance.⁴ The firms that we consider are simply illiquid. We model this feature by keeping the same process for EBIT before and after bankruptcy. Figures 1 and 2 illustrate further events.

In the Leland (1994) model the firm issues additional equity to meet the coupon payments. It can do so until the equity value becomes equal to zero. In our model, the firm may choose to default (Chapter 11) prior to completely destroying the equity value. This decision may still lead to liquidation (Chapter 7), or it may result in recovery from default. We model these possibilities by introducing two (potentially endogenous) barriers for V_t . The first barrier, V_B , determines the Chapter 11 filing, the second barrier, V_L , corresponds to liquidation. This is the point of qualitative departure from Leland (1994), who allows for only the liquidation boundary, V_L .

In practice, prior to default, the firm has an option to restructure its debt either through a private workout in case of the bank debt, or an exchange offer in case of public debt.⁵ We focus on the value of the bankruptcy option. Therefore, while our paper will have nothing to say about the optimal choice between the different workout alternatives, our results will nevertheless provide the benchmark for making this decision.

Once the company defaults, the automatic stay provision takes effect for the duration the

⁴Although, once under bankruptcy there are some real costs that are borne by the firm.

⁵Hackbarth, Hennessy, and Leland (2003) consider an optimal mixture of bank and public debt. Fan and Sundaresan (2000) study the exchange offers. Chen (2003) determines the optimal choice of workout vs bankruptcy in the presence of asymmetric information.

exclusivity period of 120 days, which is often extended. In particular, this means that interest payments stop on all unsecured debt effectively extending the maturity of all firm's debt obligations. We model the automatic stay by accumulating the unpaid coupons in arrears, A_t . The firm may return to the liquid state at some future point T and pay θA_T , $0 \leq \theta \leq 1$. Effectively, the creditors will forgive a fraction $1 - \theta$ of the arrears. This modelling strategy reflects the desired goal that equity holders also get something in the chapter 11 process. In our model with consol bond, the parameter θ will effectively control by how much the creditors are willing to extend the debt's maturity.⁶

While in bankruptcy, the firm will be exposed to the continuously accruing proportional distress cost ω . This cost can represent legal fees, lost business and valuable employees. The cost insures that the firm would not want to drag out the bankruptcy forever. This also implies that bankruptcy results in some economic distress as well.

In part, the feasibility of resurfacing from default will be determined by the ability of the firm to repay the accumulated arrears. In our model, the firm does not pay dividends to the shareholders while it is in default. Instead the entire EBIT is accumulated in a separate account, S_t . If the firm emerges from the default at time T , the entire amount in the account S_T is applied towards the arrears θA_T . A leftover, if there is any, is distributed to the shareholders. If S_T does not contain an amount sufficient to repay the arrears, the equity is diluted to raise the remaining amount.

Finally, if the company spends too much time (more than the grace period d) in default, or the value of unlevered assets reaches V_L , the firm liquidates with proportional cost α .⁷ The first condition is equivalent to accumulating too much debt in arrears, $A_T = cd$. This feature matches the real-life practice of the bankruptcy judge removing the automatic stay provision after the current exclusivity period has ended and creditors are not willing to extend it. The second condition is equivalent to the equity value falling to zero as in the Leland (1994) model.

⁶Emanuel (1983) models the arrears on the preferred stock dividend in a similar fashion. However, since his focus is on the valuation of preferred stock, there is no role for debt in his model. Bartolini and Dixit (1991) value sovereign debt, and hence do not allow for default. They model the arrears by letting the sovereign borrower to capitalize the unpaid interest.

⁷On average, Chapter 11 cases last 2.5 years (see Helwege, 1999 and references therein).

2.2 The Equity, Debt, and Firm Values

Given our setup, we can value the firm, the equity and the debt. We follow Bielecki and Rutkowski (2002) in computing these values via the risk-neutral valuation, or martingale, approach. In order to succinctly describe the details of our valuation, we need to describe the evolution of the objects of interest mathematically and to introduce additional notations. Table 1 contains the summary of all notations.

Denote the fraction of arrears to be paid out by $a : a_t = \theta A_t$. Then a will evolve according to:

$$da_t = \begin{cases} ra_t dt + \theta c dt & \text{if } V_L \leq V_t < V_B \\ -a_t dt & \text{if } V_t \geq V_B \end{cases} \quad (5)$$

In addition, we model the accumulation of earnings in the default region via the process S :

$$dS_t = \begin{cases} rS_t dt + \delta_t dt & \text{if } V_L \leq V_t < V_B \\ -S_t dt & \text{if } V_t \geq V_B \end{cases} \quad (6)$$

Note that in the second line S resets back to zero. This is happening because accumulated earnings are being distributed between the arrears and equity upon the emergence from default.

Let

$$\tau_t^{V_B} = \sup\{s \leq t : V_s = V_B, a_s = 0\} \quad (7)$$

be the most recent time of default before time t . One of the reasons the firm can liquidate is that it spent too much time (longer than d) in default. In this case, the time of liquidation can be computed as:

$$\tau_t^d = \inf\{s \geq t : s - \tau_s^{V_B} \geq d\} \quad (8)$$

The other reason the firm liquidates is that the firm value simply reaches the liquidation value, or, equivalently, if equity value falls to zero. Therefore, liquidation can occur at time:

$$\tau_t^{V_L} = \inf\{s \geq t : V_s = V_L\} = \inf\{s \geq t : E(\delta_s) = 0\} \quad (9)$$

Therefore, the liquidation time is the smallest of the two:

$$\mathcal{T}_t = \tau_t^d \wedge \tau_t^{V_L} \wedge \infty \quad (10)$$

Now, given the definition of the liquidation time, we can determine the total equity value as:

$$E(\delta_t) = (1 - \tau) \times \left(E_t^{\mathbb{Q}} \left\{ \int_t^{\mathcal{T}_t} e^{-r(s-t)} [(\delta_s - c) \mathbb{I}_{\{V_s \geq V_B, a_s = 0\}} - \omega V_s \mathbb{I}_{\{V_L < V_s < V_B\}} + (S_s - a_s) \mathbb{I}_{\{V_s \geq V_B, a_s > 0\}}] ds \right\} \right) \quad (11)$$

where τ is the effective tax rate.⁸ As we mentioned, we assume that $\tau = 0$, and revisit the tax effects in the section 3.3. Note that the first term in the expression represents either the normal operation of the company in the healthy state, or equity dilution prior to filing for Chapter 11. The next term is associated with the bankruptcy costs. The third term represents clearing of the arrears when the unlevered firm value reaches V_B from below. In this case, if accumulated earnings exceed the arrears, the difference $S_t - a_t$ is positive and leftover goes to the shareholders increasing the equity value. On the contrary, if the difference is negative, the firm has to issue additional equity to clear the arrears.

The value of debt is equal to:

$$D(\delta_t) = E_t^{\mathbb{Q}} \left\{ \int_t^{\mathcal{T}_t} e^{-r(s-t)} [c \mathbb{I}_{\{V_s \geq V_B, a_s = 0\}} + a_s \mathbb{I}_{\{V_s \geq V_B, a_s > 0\}}] ds \right\} + (1 - \alpha) E_t^{\mathbb{Q}} \left\{ e^{-r(\mathcal{T}_t - t)} (V_{\mathcal{T}_t} + S_{\mathcal{T}_t}) \right\} \quad (12)$$

Note that creditors recover a fraction of earnings accumulated during the automatic stay at liquidation. Finally, the total value of the firm's assets is equal to:

$$v(\delta_t) = E_t^{\mathbb{Q}} \left\{ \int_t^{\mathcal{T}_t} e^{-r(s-t)} [\delta_s \mathbb{I}_{\{V_s \geq V_B, a_s = 0\}} - \omega V_s \mathbb{I}_{\{V_L < V_s < V_B\}} + S_s \mathbb{I}_{\{V_s \geq V_B, a_s > 0\}}] ds \right\} + (1 - \alpha) E_t^{\mathbb{Q}} \left\{ e^{-r(\mathcal{T}_t - t)} (V_{\mathcal{T}_t} + S_{\mathcal{T}_t}) \right\} \quad (13)$$

The fact that the EBIT accumulated during default stays in the firm is reflected in the first component of the expression.

We can not evaluate the above expressions analytically. We use binomial tree based methodology developed by Kaya (2004). The details of the procedure are available in the Appendix A.

⁸Following Goldstein, Ju, and Leland (2001), we apply corporate taxes to earnings adjusted for the interest payout, as opposed to reducing the interest payout in proportion to the tax rate. In particular, this removes counterintuitive property that increase in taxes leads to the increase in the equity value.

2.3 Optimal Securities Values

Up until now we have treated the values of V_B as fixed. Intuitively, this boundary should be located between the distress ($\delta_t = c$) and liquidation (V_L) boundaries. However, it is not clear whether the firm should default immediately upon running out of earnings to pay the coupon, or should issue equity and default at or close to the boundary discussed in the literature. In order to answer this question, we first solve for V_B , which maximizes the total firm value, v (first-best). Then we compare the obtained bankruptcy barrier with the ones corresponding to the strategies which maximize the equity value or the debt value. As we will see in the subsequent section, the optimal default boundary leads to the conflict of interest between the debtors and creditors – a feature absent in the extant models. In order to address the tension between the objectives of equity and debt holders, we investigate in the section 5 how control can be shifted to achieve the first-best outcome.

Also, as we have already mentioned, we find V_L by setting the equity value to zero. This solution is consistent with both firm and equity values maximization (Leland, 1994). Overall, the liquidation region also depends on the length of the grace period d , and on earnings accumulated in default S . In this respect our setting is different from Leland’s where the liquidation region is constant.

2.4 The Model and Bankruptcy Proceedings

Our model is able to capture a number of salient features of the bankruptcy proceedings. We explicitly incorporate the automatic stay provision by stopping all the payments to creditors and shareholders, and keeping track of the accumulated interest and earnings. We also parsimoniously replicate the spirit behind the debtor-in-possession (DIP) financing. The grace period d does not exactly represent the exclusivity period, it is rather a length of time that firm is eventually allowed to spend in default, potentially after multiple renegotiations of the exclusivity period.⁹ We allow for senior borrowing, which is typical for DIP financing, via the parameter θ . This parameter serves a dual role. On the one hand it works as one of the bankruptcy costs when it is not equal to zero. On the other hand, θ ’s proximity to zero shows how much arrears are going to be forgiven, which can be interpreted as additional (senior) borrowing.

⁹In section 5 we explicitly address negotiation over d .

An important part of the bankruptcy proceedings is who retains control at which stage. For example, Aghion and Bolton (1992) and Dewatripont and Tirole (1994) emphasize the importance of switching control from equity to debt in bad states. Moreover, Skeel (2003) points out that the recent trend is that in practice creditors are able to affect a lot of decisions in bankruptcy in contradiction to the letter of the bankruptcy law. This happens because creditors use DIP financing and executive compensation as a lever to introduce new officers on the companies' boards and to provide the incentive to the executives to complete reorganization in a fast and efficient fashion. We are able to incorporate this features of the model by choosing both the bankruptcy level V_B , and the grace period d endogenously.

2.5 Related Structural Models

Before we proceed with the discussion of the model properties and implications, we briefly discuss earlier papers which rely on structural models of default. In general, the development of this literature is driven by the desire to explain empirically observed credit spreads. The conventional wisdom is that the structural models tend to underestimate the spreads. This motivates more recent papers to introduce additional realistic features which would lead to the increase in spreads.¹⁰ Simultaneously, researchers attempt to preserve analytical valuation of debt and equity values. A model combining the two affects – analytical expressions and increased spreads – is considered to be successful. However, Leland (2002) disputes whether this is an appropriate metric.

With this background in mind we proceed discussing related papers. Four recent papers that are close in spirit to the issues that we study here are by Francois and Morellec (2004), Galai, Raviv and Wiener (2003), Moraux (2002), and Paseka (2003). These papers make a distinction between default and liquidation. Moreover they attempt to capture some dimensions of the bankruptcy code.

The paper by Francois and Morellec (2004) keeps track of the cumulative time spent by the firm contiguously under the default barrier. When this cumulative time just exceeds the grace period d , the firm gets liquidated. In their model the firm can not get liquidated if the unlevered value of the assets is too low: $\mathcal{T}_t = \tau_t^d \wedge \infty$.¹¹ As a result, limited liability could be violated.

¹⁰However, recent empirical evidence from Eom, Helwege, and Huang (2004) shows that more recent models indeed underestimate spreads for high quality bonds, but overestimate spreads for low quality bonds.

¹¹In other words, while Leland (1994) allows only for V_L , Francois and Morellec (2004) allow only for V_B .

Absent limited liability, incentives of the creditors are much more closely aligned with those of debtors. Therefore, it is not surprising that the authors limit themselves to finding the optimal values of V_B by maximizing the equity value only. As we will show, considering the first-best and debt maximization scenarios leads to different implications for optimal bankruptcy decisions and securities values. In the Francois-Morellec model excursions below the default boundary are always associated with implicit forgiving of contractual obligations, $\theta = 0$. Therefore, it is not surprising that the authors find the probability of liquidation to decrease with d . Moreover, as we will see below, such an assumption can encourage the equity to default even when $\delta > c$. Finally, the authors obtain closed-form solutions for the firm, debt, and equity values only prior to default.

Moraux (2002) attempts to penalize the firms for default by keeping track of the cumulative time spent by the firm under the default barrier, which makes a difference for multiple bankruptcies. Apart from this feature, his setup is similar to that of Francois and Morellec. In particular, θ is still equal to zero. He is able to find the Laplace transform of the values of interest. Hence, the values themselves require numerical computation.

Galai, Raviv and Wiener (2003) recognize that Moraux's improvement ignores the severity of default, i.e. how far the unlevered firm value travels below V_B . In their model V_B is time-dependent and exogenous as in Black and Cox (1976). The firm liquidates when a new state variable, effectively computed as weighted average of the distances from V_t to V_B throughout the entire history of the firm's defaults, exceeds a certain value. Arrears in our model ($\theta \neq 0$) capture the same effect in a more natural way that is consistent with actual bankruptcy practices. Galai, Raviv and Wiener derive partial differential equations and solve them numerically to obtain debt and equity values.

Paseka (2003), like our model, explicitly recognizes the presence of both default and liquidation boundaries. However, he specifies V_L as an exogenously determined fraction of V_B . This may lead to negative values of equity similar to the above papers. Paseka focuses on the outcome of the reorganization plan instead of the cash flows during the automatic stay period. The reorganization plan is proposed by equity as soon as V_t reaches a new boundary $V_R > V_B$. A dynamic bargaining game takes place. In equilibrium the plan is accepted, and the firm exits from bankruptcy debt free. Such a setup effectively gives all the control to the equity. As in the other papers, the first-best and debt maximization scenarios are not explored. Finally, the author computes analytical solution for debt and equity as a function of debt value upon default, $D(V_B)$. However, $D(V_B)$ and the optimal

value of V_B itself require non-trivial numerical computations.

In summary, these papers make a contribution by thinking about the time spent in chapter 11, the role of exclusivity period and the nature of excursions while in chapter 11. These are very useful insights. However, none of the papers (with the exception of Paseka, 2003) considers two separate barriers for default and liquidation, and none of the papers considers the optimal choice of these two boundaries driven by different objectives. We believe that these issues are at the core of financial distress, because they allow to assess how valuable the bankruptcy option is. Endogenous V_B and V_L will tell us how early would a firm liquidate in the Leland (1994) world without bankruptcy as opposed to our model. This, in turn, will have implications for the values of debt, equity, and the firm overall.

3 First-Best Outcome

In this section we characterize the first-best outcomes: assuming that the total firm value is maximized ex-ante upon filing for chapter 11. This is clearly a useful theoretical benchmark. We then ask in a later section, how the first-best outcomes can be enforced by lenders. Throughout, we will use Leland (1994) as the benchmark case corresponding to the case where only liquidation option is available to the lenders and borrowers. This way, we can get an appreciation of the incremental value added by the bankruptcy code in alleviating liquidity problems. We will report all the quantities of interest, such the optimal default boundary V_B , or the firm value $v(\delta_t)$ as percentage of the appropriate values implied by the Leland model.

In the discussions that follow, we have set the deep parameters of the model at these levels: $\mu = 1\%$, $\sigma = 20\%$, $r = 5\%$, and $c = 3\%$. We vary the costs of being under chapter 11, denoted by ω , from 0% to 2% and initially set the liquidation costs $\alpha = 50\%$ (later we also consider extreme values of 10% and 90%). The debt forgiveness parameter θ is set at either 100% (no forgiveness) or at 50% (half of the arrears forgiven). We vary the length of the grace period d that the firm is allowed to spend under chapter 11 from zero to ten years.

3.1 Bankruptcy Boundary

The first panel of the figure 3 reports the bankruptcy boundary V_B as a percentage of the liquidation boundary V_L in the Leland (1994) model. The rationale is simple. In the Leland world V_B is equal to V_L . We would like to see if the availability of the Chapter 11 induces the firm to stop diluting equity earlier. One of the key implications from the plot is that the equity issuance region is always smaller once chapter 11 option is available to the agents.

Note that, from the perspective of the firm value maximization, the illiquidity problem is sorted out sooner than in Leland (1994) by filing for chapter 11. The particular choice of the bankruptcy level V_B will depend on the configurations of the debt forgiveness θ and the bankruptcy cost ω . Obviously, when there is no debt reduction ($\theta = 100\%$), and no bankruptcy costs ($\omega = 0\%$) the firm is willing to default early especially if the grace period d is sufficiently large to minimize the probability of liquidation.

The interesting outcome of the models is that if there are bankruptcy costs ($\omega = 2\%$) then firm chooses to default earlier if some of the debt is forgiven. This happens, because the liquidation cost α forces the firm to delay the default, i.e. get closer to the Leland model, as in the case $\theta = 100\%$, $\omega = 2\%$. However, the ability to forgive part of the debt reduces the probability of the firm being liquidated, i.e. reduces the probability to incur the liquidation costs, and therefore induces the firm to default earlier.

Since in our model, the firm defaults earlier than in the Leland model under many circumstances, this implies that the probability of default is higher than in Leland (probability of default is determined by the default boundary, which is different across the models, and the properties of the EBIT generating process (1), which is identical for the different models). These probabilities are a big focus in research because many practitioners use Moody's|KMV EDF (expected default frequency), which is a closely related measure. Hence qualitative difference between our and extant models is potentially important. We will explore this issue in greater detail in the section 4.2.

3.2 The Equity, Debt, and Firm Values

The first natural question is what happens to the firm value once the optimal, or first-best, bankruptcy boundary is selected. To address this question, we plot in figure 3 the total value of the firm as a percentage of the respective value in the Leland model. We see that the highest

firm value is achieved with zero bankruptcy cost and partial debt forgiveness. This is natural because zero bankruptcy cost increases overall firm value (see (13)), and debt forgiveness increases the probability of avoiding the costly liquidation. The firm values for the cases with either no forgiveness (but with bankruptcy cost), or no bankruptcy cost (but with forgiveness) are smaller because one of the advantageous components is missing. Finally, the case with no debt forgiveness and with bankruptcy cost is naturally the worst, and basically coincides with the Leland benchmark.

As we have discovered, it makes sense for the firm as a whole to invoke chapter 11 to get debt relief before liquidation if the chapter 11 process is less expensive. Is it necessarily in the best interests of the lenders? We plot in figure 3 the values of equity and debt. We see that both equity (debtors) and debt (creditors) benefit from the improvement in the firm value.

Note, however, that almost all of the savings from Chapter 11 are passed along to the lenders. This results in significant decline in the spreads. Eom, Helwege, and Huang (2004) report that low quality firms tend to have excessively high spreads in the Leland-type models. Our model could serve as a potential resolution of this drawback of the structural models.

Since in much of the scenarios, the benefits of the code appears to pass to the lenders under the first-best, we must consider the two incentives implied by this result. First, lenders will have a strong incentive to move to the first-best. Second, unless some concessions are given to the equity holders (in the form of debt forgiveness or reduced bankruptcy cost), they will not be interested in the first-best outcome. Since the total value increases in the presence of the code, lenders will have the right incentives to make such concessions. These observations suggest that the presence of chapter 11 as an additional outside option may lead to very interesting debt renegotiations. This is clearly an interesting topic for future research.¹²

We have experimented with other, more extreme, values of the liquidation cost. Qualitatively, the same results hold. Interestingly, when the cost is small ($\alpha = 10\%$), liquidation is not a big threat to the firm. As a result, all the values converge to the one in Leland, i.e. the bankruptcy boundary is very close to the liquidation boundary despite the benefits of the Chapter 11. When the liquidation cost is high ($\alpha = 90\%$), all the improvements observed in the base case of $\alpha = 50\%$ are even stronger: Chapter 11 is beneficial for a firm with potentially costly liquidation. Last, but

¹²For example, see Anderson and Sundaresan (1996), and Mella-Barral and Perraudin (1997) for models of debt renegotiation.

not the least, in the presence of costly liquidation, long grace period d enhances the individual values of debt and equity, and the value of the firm as a whole. This happens precisely because it delays the liquidation.

3.3 Debt Capacity and Optimal Capital Structure

The ability of the firm to repay its debt affects the amounts it can borrow, and, therefore, affects the optimal capital structure. In our context, the option of Chapter 11 should affect the ability of the firm to borrow, as pointed out in Acharya, Sundaram, and John (2004). The debt contract as modelled in our paper is not optimal: it exposes the firm to dead-weight losses but does not confer either tax benefits nor performs the role of a commitment tool to reduce agency costs.

It is easy to introduce tax advantages and show that debt contract is optimal for some tax configurations. We start by characterizing debt capacity. We plot on figure 4 the value of the debt for each level of promised coupon for the fixed grace period, $d = 2$ years. Note that the Chapter 11 option increases the debt capacity because the firm is able to avoid premature liquidations. Addition of tax advantage from debt will lead to an optimal capital structure to which we turn next.

The introduction of tax advantage allows us to compute the debt level which maximizes the overall firm value. This is illustrated by the plot in figure 5. We work with a corporate tax rate of 35% throughout this discussion. Two interesting points are worth emphasizing. First, consistently with our previous observations, debt relief or small bankruptcy costs lead to the greater use of debt. Second, equity values are declining in the use of debt and basically coincide with the Leland values despite the different configurations of parameters controlling debt/bankruptcy relief.¹³

These observations suggest a tension between the level of debt that equity holders would want to use and the level implied by the first-best. This trade-off suggests that some “side-payments” can be designed between the creditors and the borrowers whereby some of the tax advantages associated with the use of debt can be credibly promised to borrowers to induce them to use the optimal level of debt.

¹³Because of such similarity with the Leland results, we do not provide the respective plots to conserve space. The results are available upon request, or could be deduced from the equation (17) in Leland (1994).

4 Equity/Debt Maximization

4.1 The Securities Values

We have analyzed the first-best outcome so far. This is a natural theoretical benchmark to assess how the outcomes might differ if borrowers or lenders have the ability to choose when to file for chapter 11 or when to liquidate the firm. In the subsequent sections, we will provide a discussion of these issues. We continue with the same parameter configurations that were used in the previous section.

As noted earlier, equity holders have a strong incentive not to follow the first-best as the outcomes under such a scenario mostly favors lenders. Intuitively, in maximizing total value, we select the default boundary V_B lower than what equity holders might prefer: they would like to file for Chapter 11 sooner to get debt relief, especially when the grace period d is long. In fact, if the debt relief is high and the process of Chapter 11 is not too costly, they might file for Chapter 11 even when the EBIT is higher than promised coupon obligations, i.e. the firm is liquid. These observations are confirmed by figure 6. Note that when the equity value is maximized, the debt value declines (the spreads increase) relative to that in Leland (1994). It means that the equity holders have appropriated all the rents associated with the Chapter 11 option.

Clearly, the incentive of debtors to deviate from the first-best can be mitigated by restricting the debt relief, as we can see in the figure 6 (the case $\theta = 100\%$, $\omega = 2\%$). This may, however, not be total value maximizing, as noted earlier. Hence, as an alternative, sufficiently small values of d can keep the deviations from first-best to be small. However, these remedies make the outside option of filing for Chapter 11 less valuable. A more productive approach is to let the lender take an active role in either deciding when the firm should file for chapter 11 or taking the reins of the firm once the borrowers decide when to file for chapter 11. Section 5 is dedicated to the latter option. We consider the former option next.

The debt value maximization is presented on figure 7. Comparing this to figure 3, we immediately see that giving creditors the full control leads to outcomes that are very similar to the first-best. When debt is in control, the debt values is slightly higher, and the equity value is slightly smaller. Despite being a useful alternative to equity maximization, the debt maximization can not be implemented in practice: typically equity holders make decisions about filing for Chapter 11.

However, as Skeel (2003) points out, in practice creditors get to control decision making in distress by introducing their officers into the firm's board and by using executive compensation contracts to align the managers incentives with theirs. In the next section, we introduce a stylized way to transfer control in bad states.

4.2 Term Structure of Default Probabilities

As we observed in the section 3, the presence of Chapter 11 increases the probability of default because the new default boundary is typically above that of Leland (1994). Indeed the probability that the firm defaults in t years, \mathbb{P}_B , can be computed as:

$$\begin{aligned} \mathbb{P}_B(t, \delta_0) &= \mathbb{P} \left(\sup_{0 \leq s \leq t} \delta_s \leq \delta_B | \delta_0 \right) \\ &= \Phi \left(\frac{-\log(\delta_0/\delta_B) - (\mu + \lambda - 0.5\sigma^2)t}{\sigma\sqrt{t}} \right) \\ &+ \exp \left(-\frac{2\log(\delta_0/\delta_B)(\mu + \lambda - 0.5\sigma^2)}{\sigma^2} \right) \Phi \left(\frac{-\log(\delta_0/\delta_B) + (\mu + \lambda - 0.5\sigma^2)t}{\sigma\sqrt{t}} \right) \end{aligned} \quad (14)$$

where \mathbb{P} is the actual (as opposed to risk-neutral) probability measure, λ is the equity risk premium (associated with the probability measure switch from \mathbb{P} to \mathbb{Q}), and $\Phi(\cdot)$ is the standard normal cumulative distribution function (see Harrison, 1990 for details). In practice, one is interested in the actual, and not risk-neutral, probability of default, therefore we introduce the risk premium λ , which we take equal to 4%. In our model, $\delta_B = V_B(r - \mu)$ (see (2)). In the case of Leland, default and liquidation boundaries coincide, and, therefore, $\delta_B = V_L(r - \mu)$.

Leland (2002) argues that \mathbb{P}_B is a more informative measure than credit spreads for distinguishing different models. In practice spreads reflect not only the probability and severity of a credit event, but liquidity effects as well.¹⁴ The extant structural models do not explicitly model liquidity effects, hence they are expected to fail in matching spreads. Also, our model can generate only one (infinite maturity) spread. Default probability \mathbb{P}_B resolves both problems, because default is not related to the ease of trading in the particular security, and, since the probability value depends on the horizon, we can generate a whole term structure of \mathbb{P}_B 's.

While formal calibration of the model is beyond the scope of this paper (see Leland, 2002, for a relevant exercise), we would like to contrast the probabilities term structures implied by our and

¹⁴Here the notion of liquidity is different from the one we use throughout the paper. In the present context, liquidity refers to the ease of trading in the particular bond.

Leland's models. In particular, we want to contrast the equity and debt maximization cases in our model. We use the firm's distress boundary as a starting point ($\delta_0 = c$), and compute the term structures of \mathbb{P}_B for the usual parameter configurations. Figures 8 and 9 show the equity and debt maximization cases respectively.

We see that, perhaps not surprisingly, the differences between the Leland and our models is most dramatic when equity is in control. The only case with similar default term structures occurs when equity gets the harshest penalties for default (no debt forgiveness and presence of bankruptcy costs). Generally, debt maximization aligns the default probabilities with Leland (1994) more closely. The exception is, naturally, when equity has to pay out the arrears in full, and there are no bankruptcy costs, which causes debt to default earlier.

Most importantly, irrespective of particular quantitative differences, the \mathbb{P}_B 's deviate from zero for short maturities. The extant models are not capable of generating this effect (Leland, 2002).¹⁵ It appears that our model has sufficient flexibility to generate rich patterns of default probabilities. This property is a natural outcome of our attempt to distinguish default from liquidation.

4.3 Default vs Liquidation

Our framework enables us to construct more precise measures of the differences between default and liquidation. For instance, one natural question to ask is whether availability of Chapter 11 affects the chances of eventual liquidation. We can follow the strategy of the previous section and compute the probabilities of liquidation, \mathbb{P}_L , by replacing δ_B by δ_L in (14).

Figures 10 and 11 plot \mathbb{P}_L for a 10-year and 1-month horizon respectively. Probability of liquidation depends on the time spent in default, $t - \tau_t^{V_B}$ (see (7)), and the amount of earnings accumulated in the account S_t (see (6)). Therefore, we have to restrict ourselves to the most interesting scenarios. Based on the earlier discussions we select $d = 2$, $\theta = 50\%$, $\omega = 2\%$, and $\alpha = 50\%$. A natural starting point δ_0 is the level of EBIT after which scenarios with and without Chapter 11 diverge. This the point where the firm files for Chapter 11, i.e. δ_B . Hence we are

¹⁵There are two exceptions which rely on alternative mechanisms to generate such deviations from zero at short maturities. Zhou (2001) adds a jump component to the unlevered firm value V . Duffie and Lando (2001) introduce default uncertainty at short horizons via additional noise term (representing imprecise accounting information) in the dynamics of V .

computing probabilities of liquidation starting at the default boundary.¹⁶ \mathbb{P}_L from the Leland model is reported on top of each chart.¹⁷

Equity-value maximization leads to states of the world in which there are more chances that firm gets liquidated earlier than in the Leland model. In fact, this happens about half of the time when the firm spends a sufficiently long time in default. When we evaluate the one-month probability of liquidation, it becomes greater than zero only when the grace period d is about to expire.

Debt-value maximization leads to smaller chances of liquidation in 1after Chapter 11 in many states of the world. When the firm spends a lot of time in default, and there is not much accumulated in the account S , \mathbb{P}_L flattens out at its highest possible value. The probability flattens out because zero equity value dominates other reasons for liquidation in those states of the world.

When t is equal to 10 years the largest probability value is not much higher than the liquidation probability from Leland (1994).¹⁸ The wedge between the two probabilities is much larger when the liquidation horizon is short (one month). The Leland model produces very small probability, because one can not generate a large change in the EBIT in a short period of time via a diffusive movement. In our model, the firm might liquidate if it overstays its time in default, or it does not have enough cash to clear the arrears. Hence \mathbb{P}_L increases sharply in these cases.

Overall, we see that in many cases the probability of liquidation in our model is below that of Leland despite the fact that probability of default is always higher. Hence, in many states of the world, the presence of Chapter 11 helps firms to avoid unnecessary liquidation. Longer grace period d would be helpful in reducing \mathbb{P}_L 's even further.

5 Transfer of control rights

The intuition for ex-post transfer of control has been stressed in the incomplete contracting literature, as noted earlier in our paper. An alternative to lenders taking over the control of the decision to file for chapter 11 is to give lenders ex-post transfer of control once the decision is made by the borrowers to file for chapter 11. While transfer of control can include many things, we consider a

¹⁶This level has no particular meaning in the Leland (1994) model.

¹⁷As we mentioned earlier, equity and debt and maximization are aligned in the Leland model, and, therefore, V_L should be the same across the two. However, since V_B is changing, the probability of liquidation will be different even in the Leland world.

¹⁸In fact, when the liquidation cost is small, $\alpha = 10\%$, the highest \mathbb{P}_L in our model coincides with that of Leland.

restricted version of transfer in which the lenders decide on the length of grace period d once the firm is under chapter 11. The choice of d plays off the potential liquidation threat (if d is chosen to be too small) with the ability to sort out the illiquidity problem through automatic stay and debt relief.¹⁹

The borrowers know that if they choose V_B to be too high, then the lenders can choose a d too low ex-post and get a very high liquidation value leaving the borrowers with nothing. This will induce the borrowers to choose a lower value of V_B than they would have chosen otherwise. The figure 12, which plots the bankruptcy boundary V_B and the total firm value v as functions of θ conveys this intuition.

In addition to the usual parameter configurations, we explicitly report the results for three values of the liquidation cost α : 10%, 50%, and 90%. As before, when the liquidation cost is low, all values collapse to those in Leland (1994). When the liquidation cost is high, it is impossible to reach even the Leland levels. The threat of liquidation is so high for creditors, that they are willing to give a long grace period to the debtors in the hope that they will clear the bankruptcy eventually. Long grace period removes the liquidation threat from the debtors, and they act as in the regular equity maximization case.

The most interesting interaction occurs for the intermediate, and the most relevant empirically, value of α . In this case, if the debt forgiveness is substantial, $\theta \leq 40\%$, and the bankruptcy cost is small, $\omega \leq 1\%$, than total firm value increases relative to the Leland case. Thus, even if the first-best is not achieved, there is a movement in the right direction.

In order to understand what happens when the values of θ and ω are small, it is useful to revisit equity and debt maximization presented in figures 6 and 7. As we can see, creditors generally prefer a low bankruptcy boundary V_B : for our parameter configurations it is at most 120% of the Leland liquidation boundary. Equity selects similar values of V_B only for small d . Therefore, debt has to choose small d when equity files for Chapter 11 (ex-post) to induce equity to select a small V_B ex-ante. Short d is effective when there is a lot of subsidy to equity, i.e. θ and ω are small. This explains our results.

¹⁹Empirical evidence in Helwege (1999) emphasizes the link between the length of default and bargaining.

6 Conclusion

We have presented a simple model of a firm which has risky debt outstanding in its capital structure. The lender and the borrower have the options of filing for Chapter 11 and liquidating (Chapter 7). Chapter 11 in our model takes into an account automatic stay, grace period and debt relief. The first-best outcome is shown to be different from equity-value maximizing outcome. This is in sharp contrast to Leland (1994) who shows that, with the liquidation option only, the first-best outcome coincides with equity value maximizing outcome.

An important result of our paper is that the first-best outcome could be restored in large measure by giving creditors either the control to declare Chapter 11 or the right to liquidate the firm once it is taken to Chapter 11 by the equity holders. The intuition for this result is that equity holders will have an incentive to file for Chapter 11 too soon in the absence of the liquidation threat from the creditors.

Irrespective of who is in control, our model generates probabilities of default that are larger than those in the Leland model. In particular, the probabilities are above zero at very short maturities – a feature that was previously generated only by modifying the dynamics of the state variable via a jump or additional noise component. Interestingly, our model often generates lower probabilities of liquidation as compared to Leland. This implies that Chapter 11 facilitates recovery of firms from financial distress.

A Computational methodology

TO BE COMPLETED

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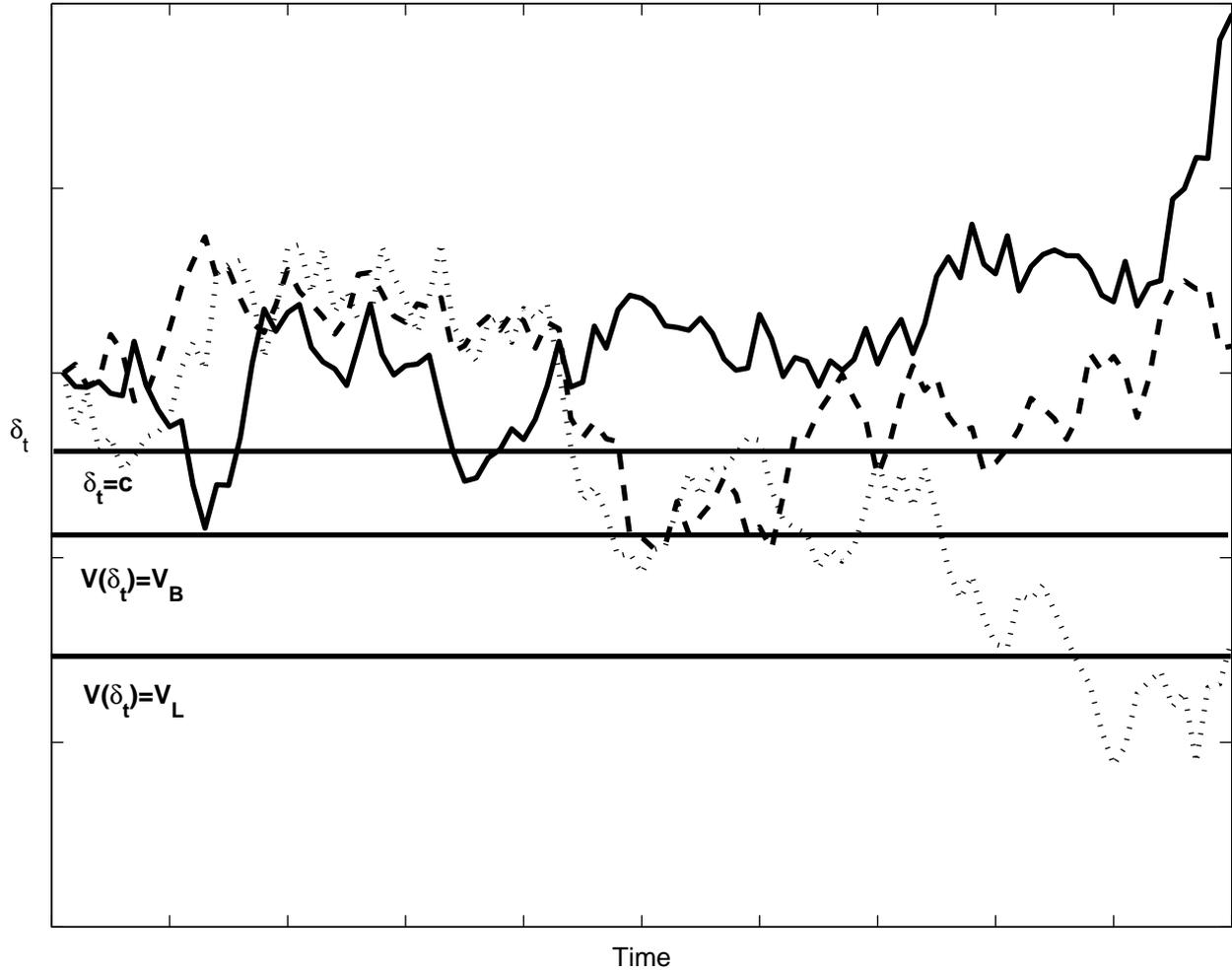
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Table 1 : Notations

Notation	Explanation
Value of unlevered assets	
δ_t	EBIT (payout flow)
μ	Drift of EBIT under \mathbb{Q}
σ	Volatility of EBIT
r	Risk-free interest rate
c	Coupon rate
V_t	Present value of EBIT (Value of unlevered assets)
Default	
V_B	Default boundary
d	The grace period – maximal amount of time the firm is allowed to stay in default
$\tau_t^{V_B}$	the last time healthy firm's value hit V_B prior to t
τ_t^d	time of liquidation due to long time spent in default ($t - \tau_t^{V_B} \geq d$)
$\tau_t^{V_L}$	time of liquidation due to limited liability violation
A_t	Arrears
θ	fraction of the arrears to be paid out
a_t	θA_t
S_t	the amount of earnings accumulated in the default
ω	Distress costs
$\mathbb{P}_B(t, \delta_0)$	Probability of default in t years starting at δ_0
Liquidation	
V_L	Liquidation boundary
α	Liquidation costs
$\mathbb{P}_L(t, \delta_0)$	Probability of liquidation in t years starting at δ_0

Figure 1. Time Series of Events

On this figure we plot possible paths of the EBIT process (δ_t) and the key boundaries which determine the state of the firm: $\delta_t = c$ - distress; $V_t = V_B$ - Chapter 11; and $V_t = V_L$ - liquidation (Chapter 7).



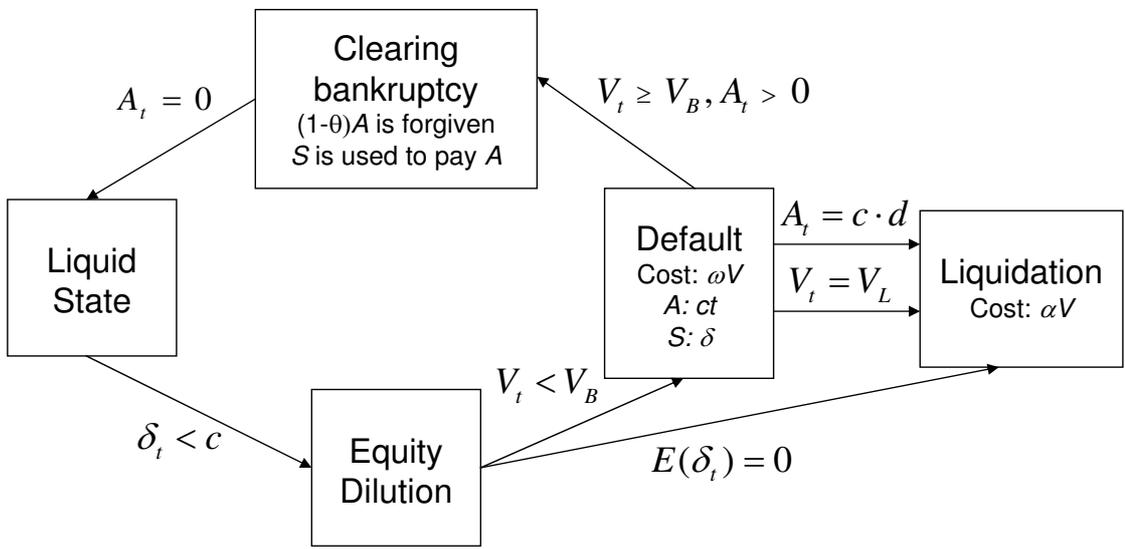


Figure 3. Firm Value Maximization (First-Best)

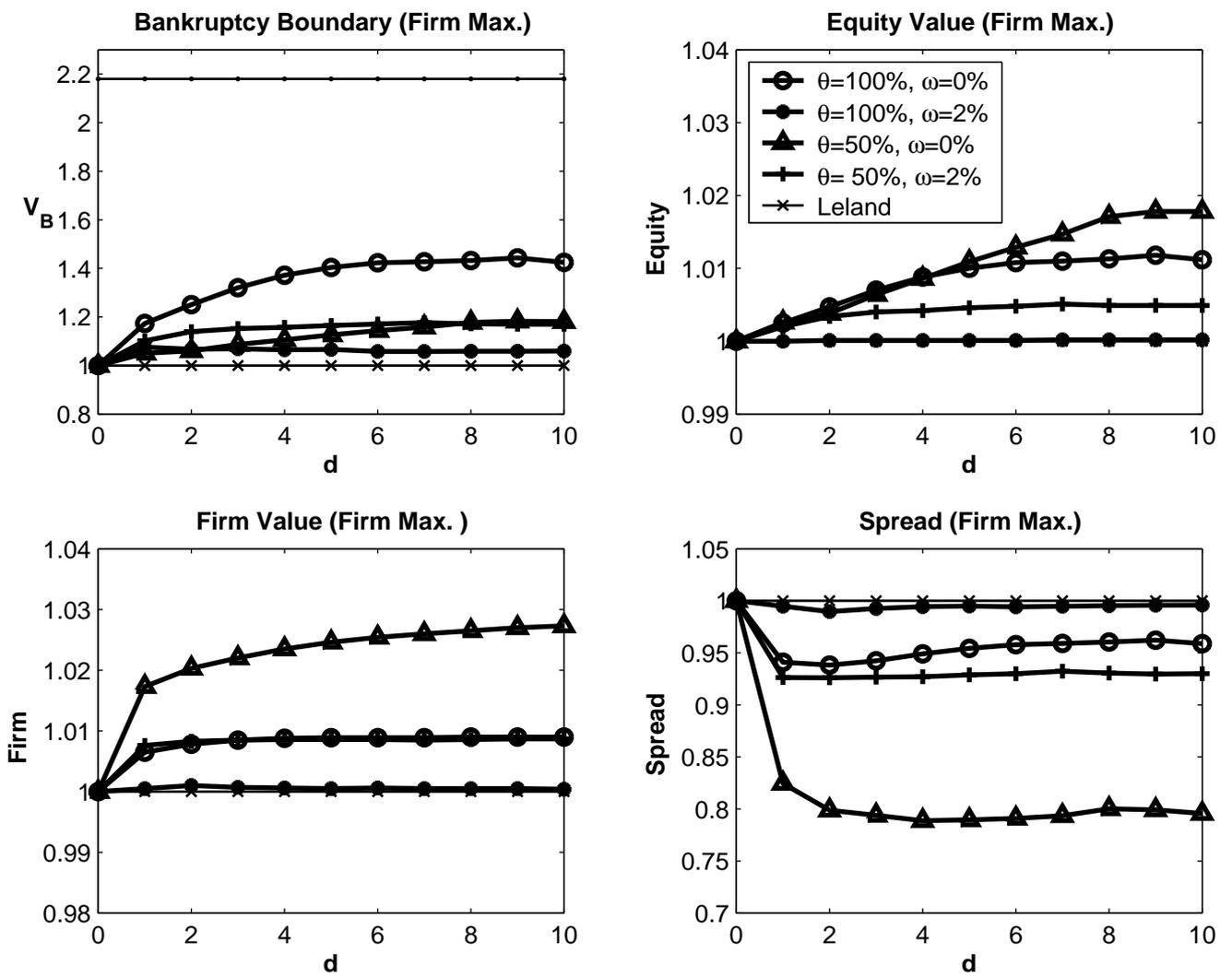


Figure 4. Debt Capacity

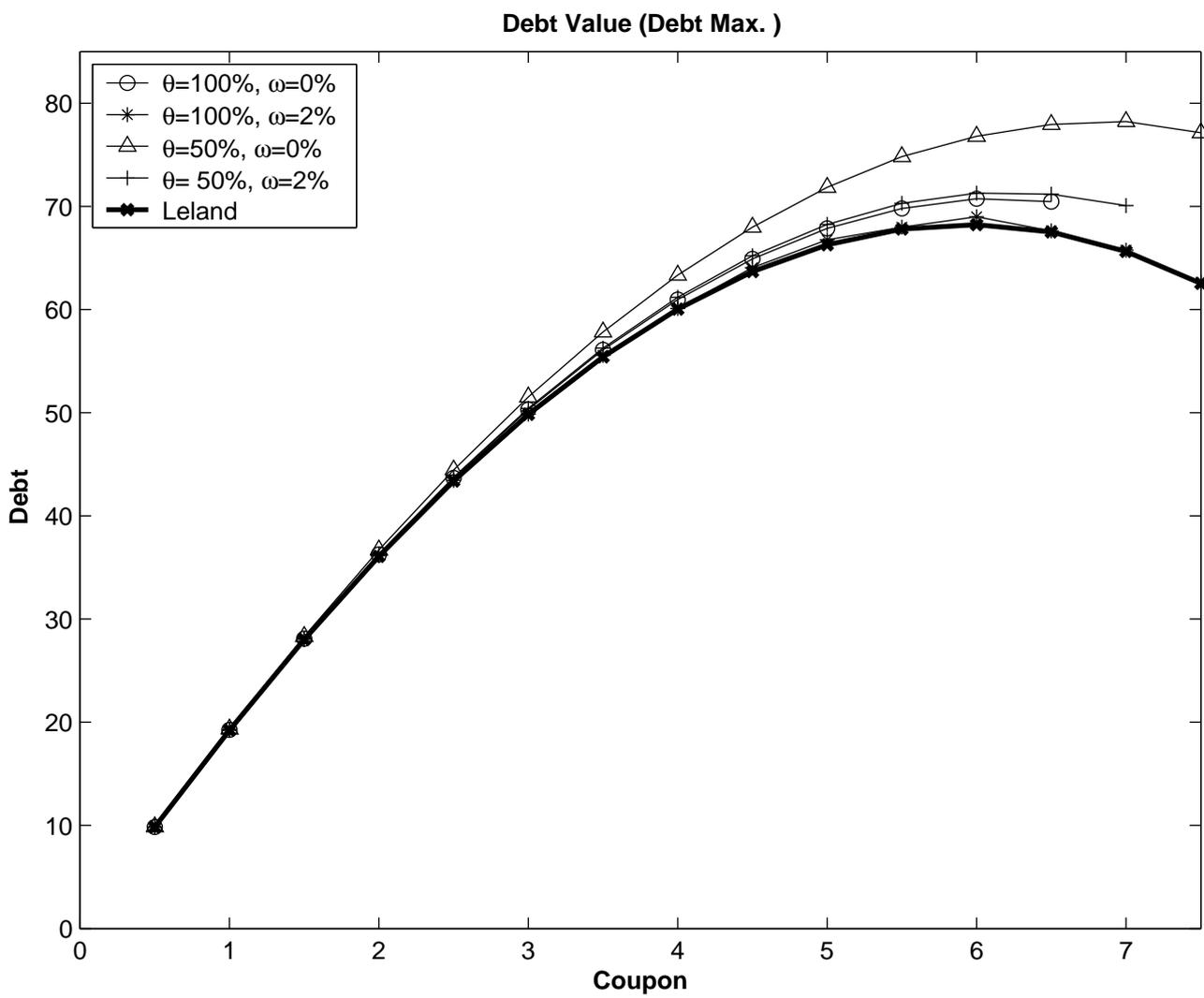


Figure 5. Optimal Capital Structure

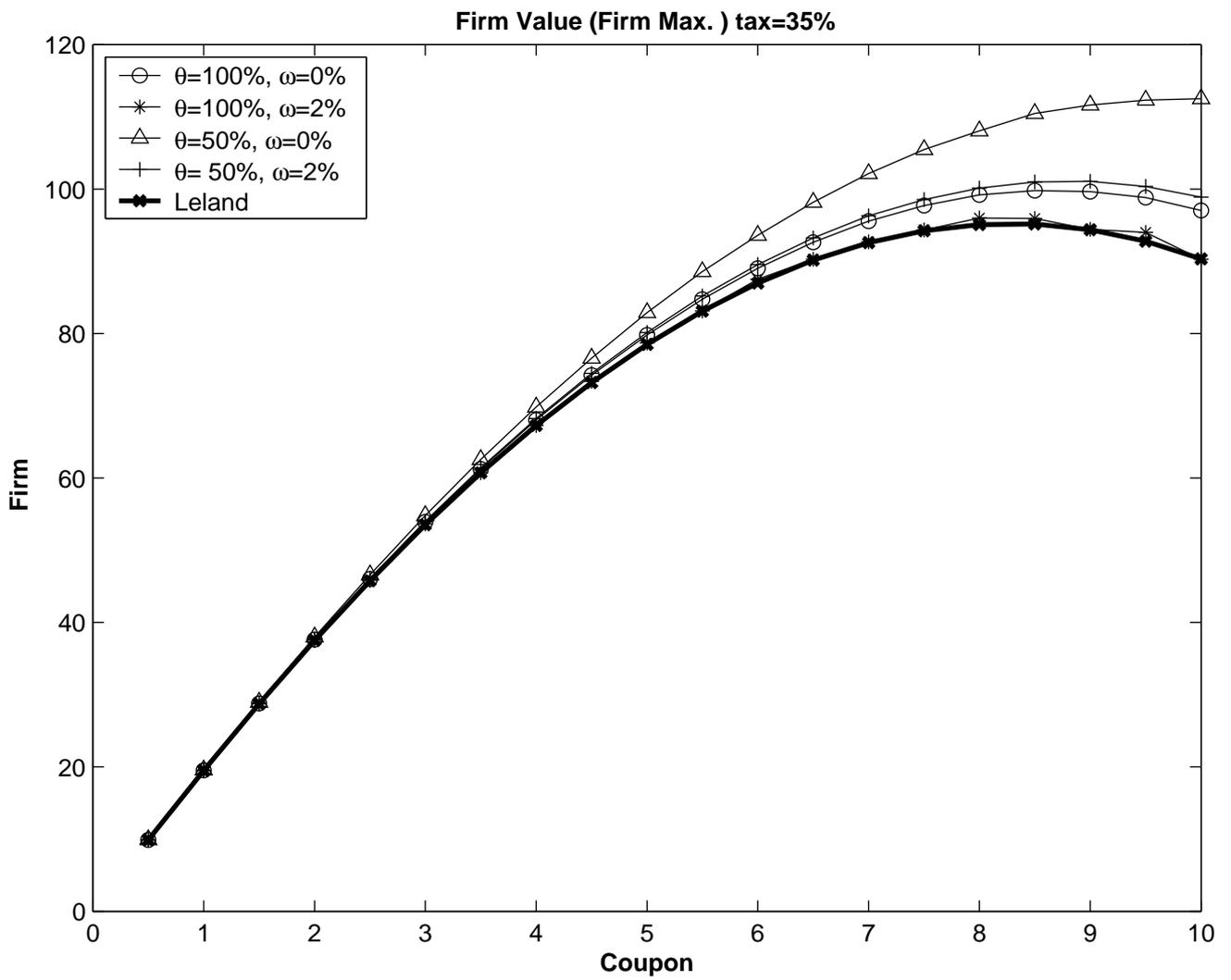


Figure 6. Equity Value Maximization

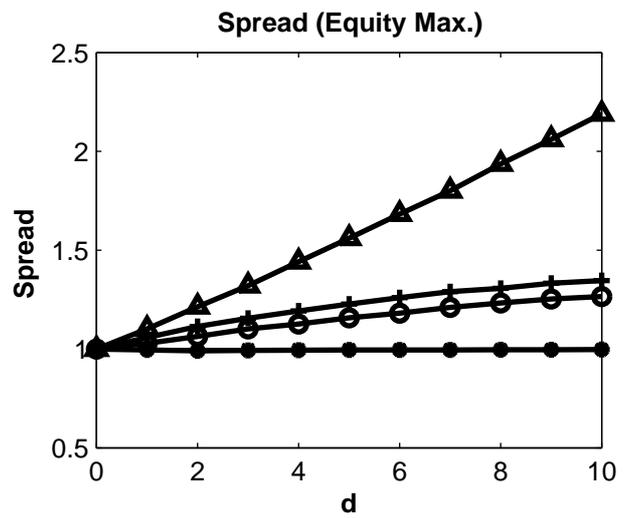
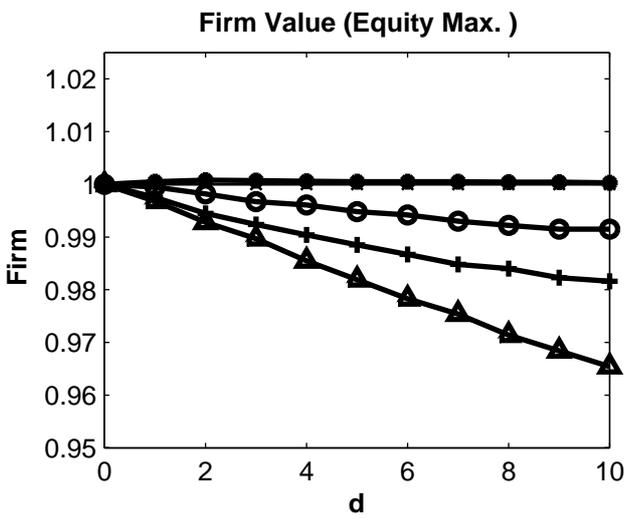
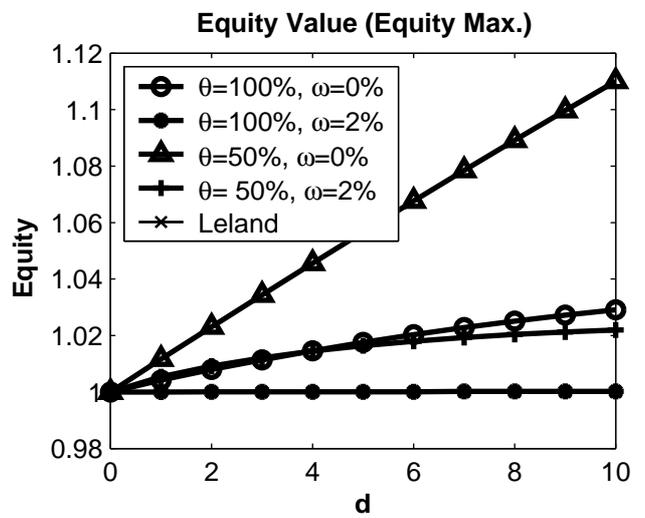
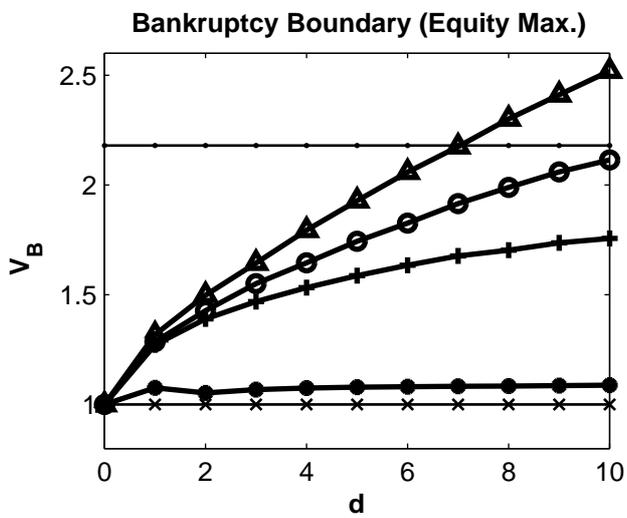


Figure 7. Debt Value Maximization

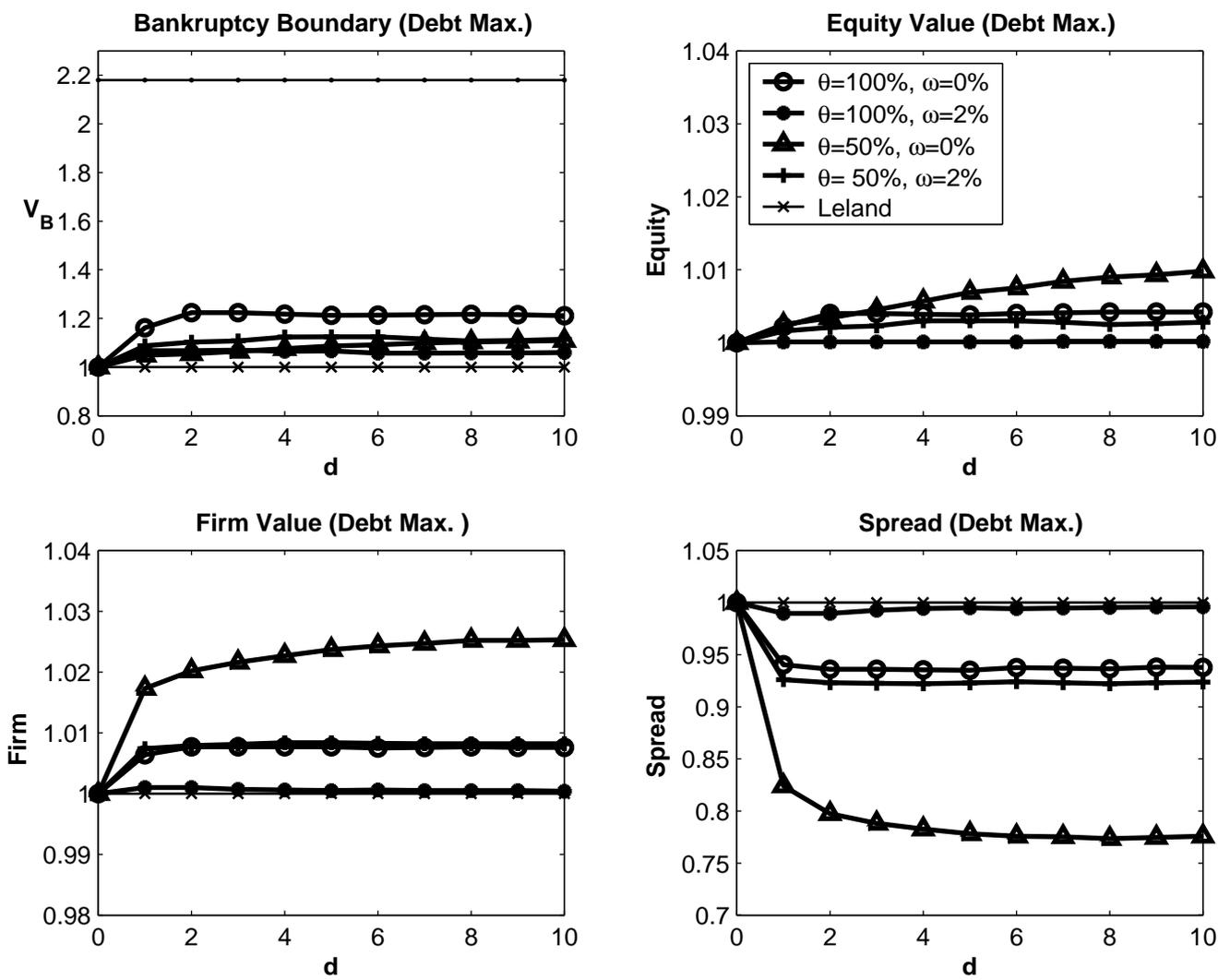


Figure 8. Term Structure of Default Probabilities (Equity Maximization)

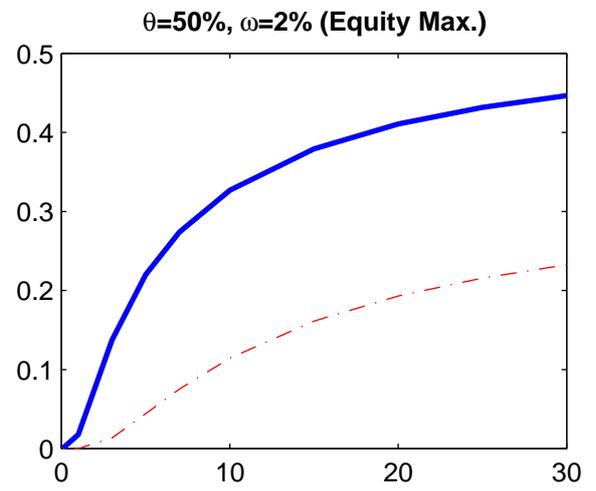
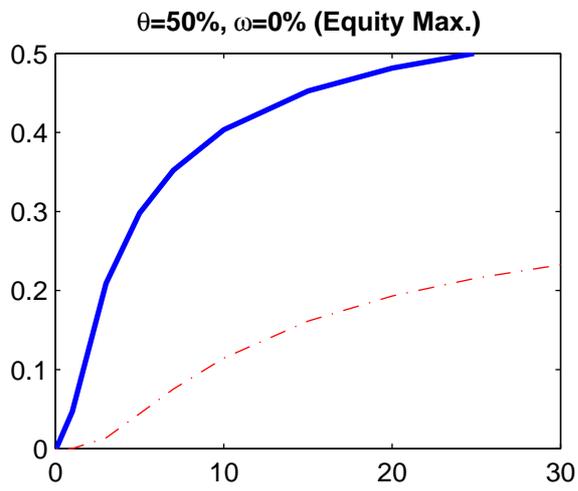
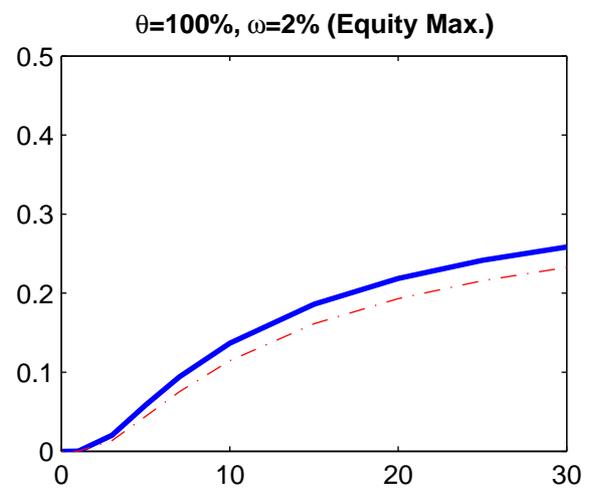
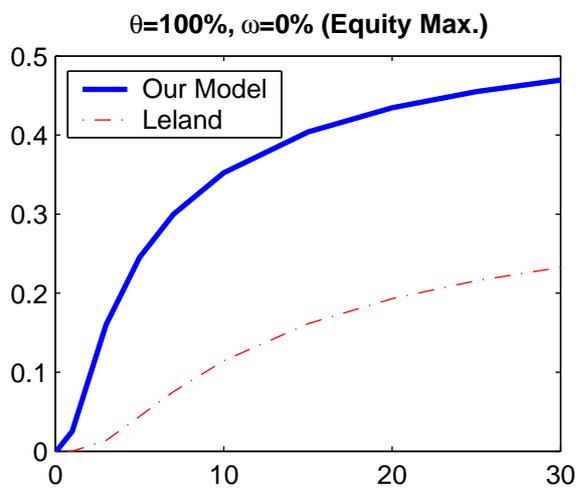


Figure 9. Term Structure of Default Probabilities (Debt Maximization)

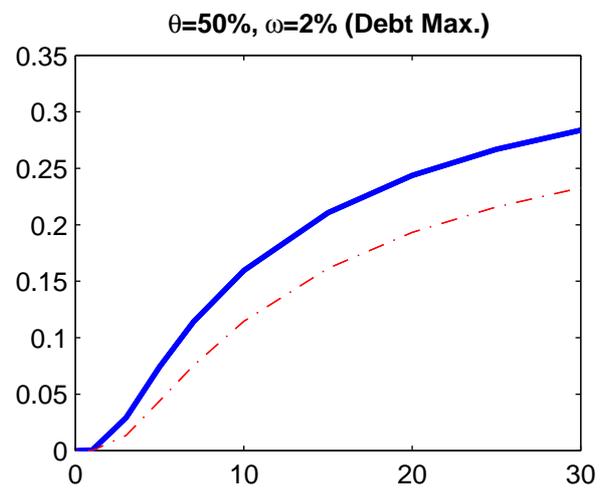
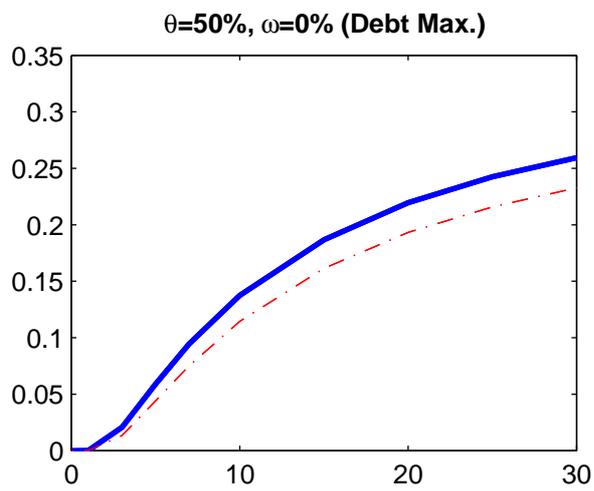
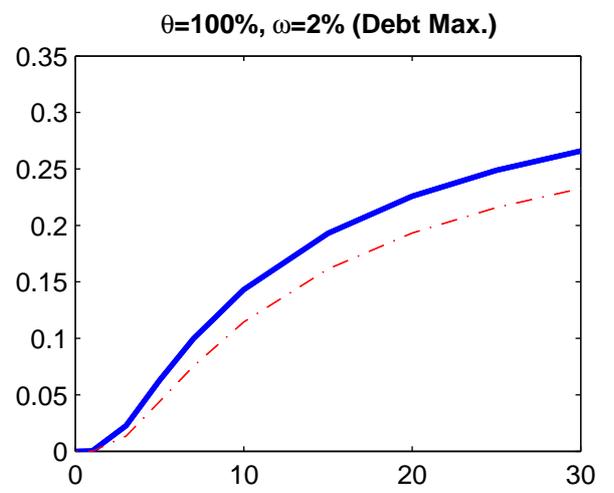
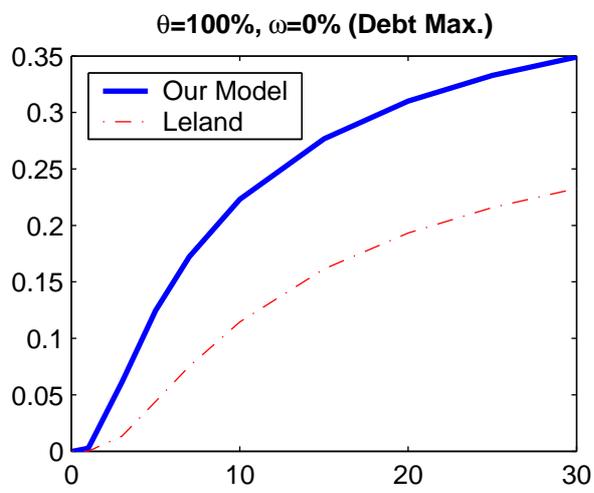


Figure 10. Probability of liquidation in 10 years

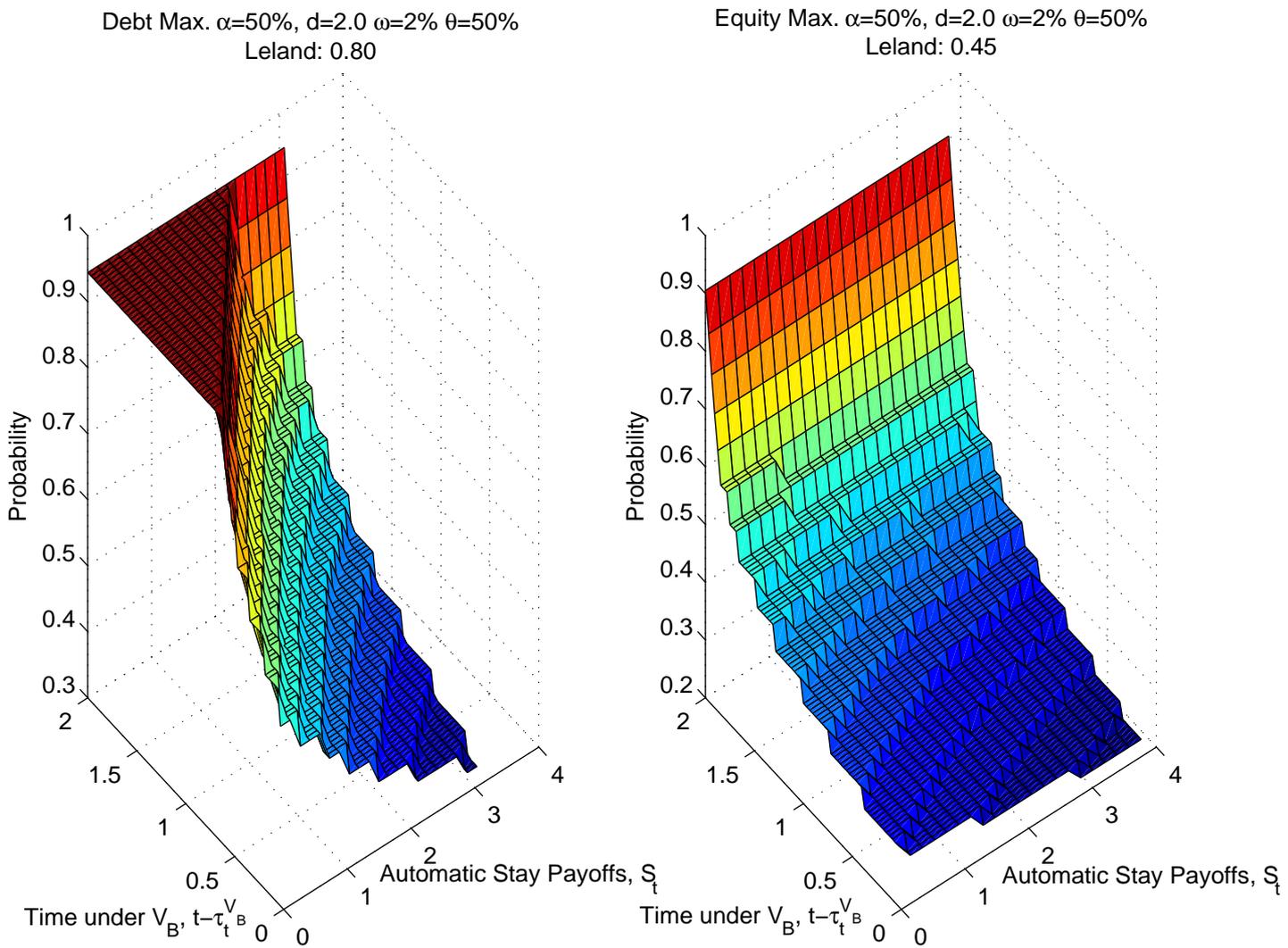


Figure 11. Probability of liquidation in 1 month

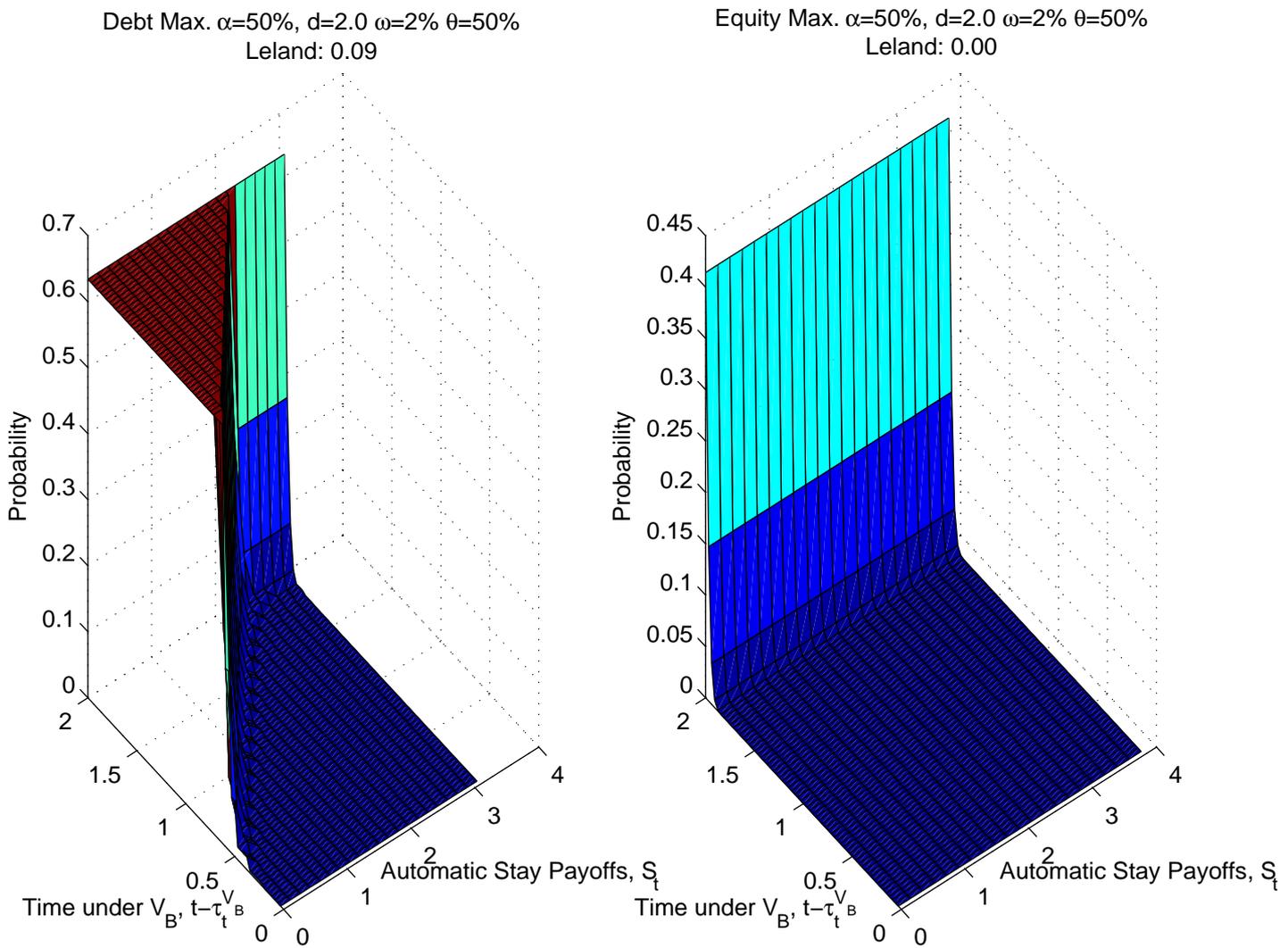


Figure 12. Transfer of Control Rights

