Ownership, incentives, and the hold-up problem

Forthcoming in *The Rand Journal of Economics*

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Vertical integration is often proposed as a way to resolve hold-up problems, ignoring the empirical fact that division managers tend to maximize divisional (not firmwide) profit when investing. This paper develops a model with asymmetric information at the bargaining stage and investment returns taking the form of cash and "empire benefits." Owners of a vertically integrated firm then will provide division managers with low-powered incentives so as to induce them to bargain "more cooperatively," resulting in higher investments and overall profit as compared with non-integration. Thus, vertical integration mitigates hold-up problems even without profit sharing.

1 Introduction

This paper examines the contracting relationship between owners and managers of vertically related units. The units trade an intermediate good for which there does not exist an external market, e.g., a specialized component. If the specifics of the transaction are non-contractible, then it is well understood that such bilateral monopoly situations are prone to underinvestment problems (Williamson, 1985). The early incomplete contracting literature has proposed vertical integration as a solution to such "hold-up" problems. [1] Yet, most vertically integrated firms are run in a decentralized fashion with decision-makers compensated mostly based on divisional profit so that hold-up problems are bound to

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I would like to thank Rick Antle, Raymond Deneckere (the Editor), Mike Kirschenheiter, Brian Mittendorf, Stefan Reichelstein, Mike Riordan, Phil Stocken, two anonymous referees, workshop participants at Columbia University, Humboldt Universität Berlin, University of California at Berkeley, Yale University, the EIASM Workshop on Accounting and Economics (Frankfurt), and especially Stan Baiman and Bjorn Jorgensen for helpful discussions.

[1] Klein, Crawford and Alchian (1978), Williamson (1985), Grossman and Hart (1986). A separate strand of literature, Rogerson (1984), Chung (1991) and Edlin and Reichelstein (1996), has shown that the hold-up problem can be solved with complex bargaining mechanisms involving non-Nash bargaining and fixed-price upfront contracts that are subsequently renegotiated.

resurface, as is supported by the evidence in Eccles (1985). The main purpose of this paper therefore is to investigate whether vertical integration nevertheless matters in that it can be value-enhancing even in the absence of firm-wide profit sharing.

A separate aspect of the vertical integration debate is concerned with how incentives differ across organizations. Williamson (1985) argues that integrated firms have to resort to lower-powered incentives—as compared with non-integrated firms—because of central management's inability to commit not to intervene in ongoing operations. This argument, too, is undermined by the practice of decentralization as non-integrated firms are seldom owner-managed and hence susceptible to similar commitment problems. Consequently, this paper assumes delegated decision-making under either ownership structure. Nonetheless, it predicts incentives to be lower-powered in integrated firms because of a subtle link between incentives and managers' bargaining behavior: In equilibrium, managers in this model negotiate more "cooperatively" under muted incentives, thereby alleviating hold-up problems and enhancing overall efficiency.

This paper departs from the vast body of incomplete contracting studies in two important ways. First, it follows the recent "empire benefits" literature: Managers are assumed to consume private benefits of control from the assets under their purview provided these assets are used productively (here: provided the transfer takes place). [2] Second, information at the bargaining stage is assumed to be asymmetric, whereas most prior studies have assumed unverifiable, yet symmetric information. [3] That approach has been taken mainly for tractability reasons as it allowed modelers to use the Nash bargaining solution. However, there is general consensus that asymmetric information is a pervasive phenomenon between, but also within firms. In this model, division managers privately observe their valuations for the intermediate good after investing; the subsequent negotiations take the form of a sealed-bid mechanism (Chatterjee and Samuelson, 1983).

[2]Empire benefits are discussed anecdotally in Donaldson (1984) and Burrough and Helyar (1991), and formally modelled in Harris and Raviv (1996), Lambert (2001), Hart and Holmstrom (2002) and Baldenius (2003). Hennessy and Levy (2002) find strong empirical evidence for empire benefits to affect investment decisions. Murphy (1986) and Baker, Jensen, and Murphy (1988) examine, theoretically and empirically, the incentive role of career concerns which are important determinants of empire benefits.

[3]Exceptions are Tirole (1986), Riordan (1989) and Baiman and Rajan (2002). Vaysman (1998) considers negotiated transfer pricing under asymmetric information in an adverse selection framework without relationship-specific investments.

Combining these model ingredients establishes a link between effort incentives and bargaining behavior. Bargaining under asymmetric information results in ex post inefficient transfers as both managers "shade" their bids for the intermediate product: The seller bids above his reservation price while the buyer bids below his reservation price. When submitting a bid, each manager trades off the probability that trade occurs with his resulting payoff—compensation and empire benefits. The incentive weights then become design instruments for the shareholders to fine-tune the managers' tradeoff between compensation (increasing in the probability of trade and in the amount of bid shading) and empire benefits (increasing only in the probability of trade).

Facing muted incentives, a manager assigns relatively more weight to empire benefits and thus bids more cautiously—that is, closer to his reservation price—to ensure the transfer takes place. In equilibrium, the other manager takes advantage of his counterpart's softened bargaining behavior by bidding more aggressively. As this paper shows, however, ultimately the efficiency of ex post trade—and hence also of ex ante investments—is enhanced by low-powered incentives. This is in stark contrast to the symmetric information Nash bargaining solution that always ensures efficient trade, ex post.

The paper proceeds by applying these insights to the question of optimal incentive provision under alternative ownership structures. To isolate the effect of incentives on specific investments (and to rule out a trivial case for vertical integration), the information structure and bargaining mechanism are held constant across regimes. The model predicts that the owners of vertically related but non-integrated firms provide their managers with high-powered incentives so as to commit them to bargain aggressively at the trading stage. As a result, the firms find themselves in a prisoners' dilemma characterized by severe trading inefficiencies and underinvestment. In a vertically integrated firm, on the other hand, one principal contracts with both managers and is residual claimant for the aggregate surplus net of managerial compensation. In the optimal solution, this principal provides the managers with low-powered incentives, resulting in more cooperative bargaining, improved trading and investment efficiency, and consequently higher profits.

The key insight of this paper therefore is that ownership matters even if investment decisions are delegated to managers who aim to maximize divisional profit. This holds because ownership determines optimal incentive provision which in turn affects the managers' bargaining behavior. The prediction of muted incentives under vertical integration is consistent with Williamson (1985), albeit with a different flavor: Rather than reflecting commitment problems, low-powered effort incentives under integration may actually enhance the efficiency of intrafirm trade. [4] The paper also discusses the implications for how the managers' expected compensation should vary across organizational forms.

Prior studies have taken different routes to identifying benefits of vertical integration in incomplete contracting settings with delegated decision-making. Holmstrom and Tirole (1991) show that higher effort incentives induce managers to acquire more (personally costly) skills; vertical integration then is a way to internalize the attendant externality on other divisions. In Anctil and Dutta (1999), firm-wide profit sharing alleviates hold-up problems at the cost of higher risk premia. [5] Baldenius, Reichelstein, and Sahay (1999) show that negotiations at times are dominated by alternative transfer pricing mechanisms (e.g., based on self-reported costs) that are difficult to enforce between independent firms. However, none of these earlier studies suggests vertical integration to be of any value in the important setting where: (i) decision-makers are compensated solely based on divisional performance; (ii) specific investments are in physical capital (not skills); and (iii) prices are negotiated both between and within firms. The present paper aims to close this gap.

My paper has several features in common with existing studies. Hart and Holmstrom (2002) also incorporate empire benefits into an incomplete contracting model. However, they assume non-verifiability not just of ex ante investments but also of the ex post transfer decision. Baldenius (2000) compares the performance (in terms of ex post trade and ex ante investment efficiency) of the equal-split sealed-bid mechanism with a "lopsided" bargaining protocol where one division can make a take-it-or-leave-it offer. Matouschek

[4]These results are sensitive to the nature of specific investments. A shown in a model extension in Section 6, if managers also invest in personally costly skill-acquisition and capital investments are negligible, then a vertically integrated firm may actually adopt high-powered incentives. This will elicit higher levels of skill-acquisition, which has a positive externality on the respective other division. Thus, there is a need to distinguish between investments in capital versus managerial skills in future empirical work on the link between incentives, organizational form, and investments.

[5]The free-rider problem associated with profit sharing is discussed in Baker, Jensen, and Murphy (1988). Empirically, profit sharing only plays a minor role for division managers' compensation (Merchant, 1989). Bushman, Indjejikian, and Smith (1995) and Keating (1997) show that the weight on firmwide performance measures increases with the degree of interdependence between divisions. Yet, divisional metrics remain the key drivers of performance evaluation except for top-level executives.

(2004) introduces information asymmetry into a property rights model without specific investments. Analyzing sealed-bid and general revelation mechanisms, he shows that asset ownership affects ex post trade efficiency by determining the aggregate disagreement payoffs during bargaining. As a consequence, joint asset ownership may be optimal, contrary to earlier results in the property rights literature. While the present paper shares the focus of Baldenius (2000) and Matouschek (2004) on the centrality of ex post inefficiencies for determining optimal governance structures, its innovation lies in addressing managerial compensation to study the link between incentives and bargaining.

Lastly, the results in this paper are in line with the literature on incentives and cooperation in organizations. For instance, Holmstrom (1999) develops a multi-task moral hazard setting with manipulable performance measures and argues that firms use low-powered incentives to encourage employee cooperation in situations where strong market incentives would undermine incentives for cooperation. Closely related, Argyres (1995) compares the governance structures of GM and IBM and finds that low-powered incentives contributed to IBM's successful effort at interdivisional coordination, whereas high-powered incentives at GM failed along this dimension. While Argyres only considers vertically integrated firms, his conclusion is consistent with the results presented here that high-powered incentives adversely affect cooperation. I am not aware of any theoretical study prior to mine that has established such a link in a bargaining setting.

The paper is organized as follows: Section 2 describes the model. Section 3 lays out the bargaining process and the managers' effort and investment incentives for given contracts. Section 4 derives optimal compensation contracts under alternative ownership structures for one-sided investments. Sections 5 and 6 extend the model by allowing for bilateral investments and managerial skill acquisition. Section 7 concludes.

2 The model

The model entails two vertically related units. Unit 1 manufactures an indivisible intermediate good and transfers it to Unit 2 which further processes it and sells a final good. For now the generic term "unit" is used; Section 4 will then distinguish between non-integration where both units are independent firms, and vertical integration where they are divisions within the same firm. The intermediate good may be thought of as a specialized component for which there does not exist an outside market. Let $q \in \{0, 1\}$ indicate whether the transfer takes place. If it does ($q = 1$), then Unit 2 pays a (transfer) price t to Unit 1. Each Unit i can undertake relationship-specific investments, denoted $I_i \in [0, \bar{I}_i]$, to enhance the gains from trade.

Unit i is run by Manager i, who exerts unobservable effort, $a_i \in \mathbb{R}_+$, with marginal productivity normalized to one. The respective income measures are

$$\pi_i \equiv a_i + M_i(\theta_i, I_i, t)q - C_i(I_i), \quad i = 1, 2, \tag{1}$$

where $C_i(I_i)$ is Unit i's fixed cost associated with its investment,

$$\begin{aligned} M_1(\theta_1, I_1, t) &\equiv t - (\theta_1 - I_1), \\ M_2(\theta_2, I_2, t) &\equiv \theta_2 + I_2 - t, \end{aligned}$$

are the units' respective contribution margins resulting from the transfer, and θ_i is Unit i's cost or revenue type. That is, the seller's investment I_1 lowers its variable production cost, while the buyer's investment I_2 raises its net-revenue from the transaction. To simplify the derivation of the managers' bargaining strategies, types θ_i are commonly known to be uniformly distributed over the intervals $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ for $i = 1, 2$, with $\overline{\theta}_2 \geq \overline{\theta}_1$ and $\underline{\theta}_2 \geq \underline{\theta}_1$, according to densities $f_i = (\overline{\theta}_i - \underline{\theta}_i)^{-1}$ with corresponding distributions $F_i(\theta_i)$. Let $\theta = (\theta_1, \theta_2)$, $I = (I_1, I_2)$ and $a = (a_1, a_2)$. In contrast to most incomplete contracting studies, I assume that type realizations are *privately* observed—that is, only Manager 1 observes θ_1, and only Manager 2 observes θ_2. This appears to be descriptive of many exchange situations, both across and within firms. [6]

As argued in the Introduction, I assume unit managers are compensated based on their respective unit's income only. That is, I ignore the possibility of firm-wide profit sharing under vertical integration. Restricting attention to linear contracts, let

$$s_i = \alpha_i + \beta_i \pi_i \tag{2}$$

[6] As noted in Tirole (1986), asymmetric information about costs often arises from cost allocations if the supplier produces multiple products. The buyer's valuation may contain subjective elements or it may depend on further processing costs, in which case cost allocation issues arise again.

denote Manager i's compensation. While linear contracts will generally not be optimal, they allow for a simple parametrization of incentive weights in terms of the bonus coefficients β_i. Assuming risk neutrality, Manager i's payoff equals [7]

$$s_i - v_i(a_i) + b_i I_i q, \tag{3}$$

where v_i is his (increasing and convex) disutility of effort function and the parameter $b_i \geq 0$ captures private ("empire") benefits of control from the assets under his purview. For convenience, let $v_i'(0) = 0,\ v_i'(\infty) = \infty,\ C_i(0) = C_i'(0) = 0,\ C_i''(I_i) \geq 0$ and $C_i'(\bar{I}_i) = \infty$.

Apart from the usual perks-related interpretation, one may think of empire benefits as a reduced form of an unmodeled additional effort variable: If the transaction takes place, then the managers can carry it out with less personally costly effort, the more equipment or staff is in place. Alternatively, empire benefits can arise from career concerns (Baker, Jensen, and Murphy, 1988): Managers overseeing large divisions are more likely than those in charge of smaller units to be promoted unless the assets in place prove to be unprofitable, ex post. All of these interpretations suggest that managers consume empire benefits only if the investment pays off (i.e., only if the transaction is completed, $q = 1$) and that these benefits are increasing in I_i. Qualitatively similar insights could be established if the managers only cared about compensation (i.e., no empire benefits), but the compensation contracts entailed an additional discrete bonus paid in case the transfer takes place. [8]

In line with the incomplete contracting literature, investments are made at an early stage before the managers observe their respective types. This seems descriptive as managers often have to commit to a certain technology, install non-redeployable equipment or hire staff before the exact nature of the product is known. [9] Neither the investments nor the resulting fixed costs are verifiable to the principal(s). It is important to note that I

[7] Due to risk neutrality, the moral hazard problem associated with the managers' choice of a_i, by itself, is not very interesting. It is included in the model because it creates a need for incentive provision, and it provides a natural anchor in that $\beta_i = 1$ would always be optimal if incentives did not affect investments.

[8] I would like to thank a reviewer for suggesting this "transfer bonus" interpretation. For the question studied in this paper, the empire benefits interpretation has certain advantages. First, the term "bonus coefficient" would be ambiguous otherwise, as it would refer to β_i as well as to the transfer bonus. Second, one would expect the transfer bonus to be chosen endogenously, which is beyond the scope of this paper.

[9] If the managers could wait with their investment until after they observe their respective types, investments would effectively signal the managers' private information (ignoring randomized investment strategies). The bargaining game would then revert back to a symmetric information formulation.

assume lack of verifiability and asymmetric information even under vertical integration—i.e., integration by itself does not make contracts "more complete" or information more symmetric. [10] If a division is engaged in many different transactions, then investments can generally not be linked to certain products and hence are not contractible.

The sequence of events entails four dates and is outlined in Figure 1. At date 1, compensation contracts (α, β) are offered to the managers with $\alpha = (\alpha_1, \alpha_2) \in \mathbb{R}^2$ and $\beta = (\beta_1, \beta_2) \in \mathbb{R}^2$. These contract offers are publicly observable and the principal(s) can credibly commit to them. [11] I assume that the managers can commit to stay on the job for any realization of the types θ (ex ante individual rationality) and that their reservation utilities are sufficiently low so they always accept the contracts in equilibrium. At date 2, the managers choose their efforts and investments (a, I); these choices are observable to the respective other manager. At date 3, Manager i privately observes his type θ_i. At date 4, the transfer decision is made in accordance with the bargaining mechanism specified below and the managers are compensated.

—Insert Figure 1 about here—

Before solving for the bargaining outcome, it is helpful to clarify the scope of the analysis: The decentralized firm structure with linear compensation schemes and sealed-bid bargaining is taken as given in this model. [12] In particular, I ignore direct revelation mechanisms (Myerson and Satterthwaite, 1983), whereby managers report their private information to the principal(s). The mechanism design approach characterizes implementable allocations and total surplus, while individual payoffs determining ex ante investment incentives remain indeterminate. As Myerson and Satterthwaite (1983) have shown, however, the sealed-bid mechanism achieves second-best efficiency if the relevant cost and revenue supports coincide. While this happens only in knife-edge cases in this model, Assumption 1 below ensures that the relevant type supports do not vary too much,

[10] This is in line with Grossman and Hart (1986) and Matouschek (2004). On the other hand, the information-enhancing role of vertical integration has been emphasized by Arrow (1975), Riordan (1989), and Baker and Hubbard (2003).

[11] See Tirole (1986) and Bagwell (1995) on the issues of observability and commitment.

[12] Moreover, all participants are assumed to be risk neutral. Introducing risk aversion would lower the optimal bonus coefficients in absolute value, but the relative ranking of effort and investment incentives across integration and non-integration—the focus of this paper—would be unaffected.

so by continuity the sealed-bid mechanism remains "fairly efficient." [13]

As a useful benchmark, I begin by characterizing the first-best, or fully efficient, solution. The first-best trading rule for any investment and state θ at date 4 is given by:

$$q^*(I,\theta) = 1 \ \text{ if, and only if, } \ \theta_2 - \theta_1 + \sum_{i=1}^{2}(1+b_i)I_i \geq 0. \tag{4}$$

First-best expected total surplus as a function of effort and investment then equals (with $E_\theta[\cdot] = \int_{\Theta_1}\int_{\Theta_2}[\cdot]\ dF_2(\theta_2)dF_1(\theta_1)$ as the expectation operator):

$$\Pi^*(a,I) \equiv \sum_{i=1}^{2}[a_i - v_i(a_i) - C_i(I_i)] + E_\theta\left[\left(\theta_2 - \theta_1 + \sum_{i=1}^{2}(1+b_i)I_i\right)q^*(I,\theta)\right].$$

At date 2, the *first-best* program reads:

$$\mathcal{P}^{\mathcal{FB}}: \qquad \max_{a,I}\ \Pi^*(a,I).$$

Let (a^*, I^*) denote the solution to program $\mathcal{P}^{\mathcal{FB}}$. Efficient effort choices are given by $v_i'(a_i^*) = 1$. Assuming a unique interior solution and using the envelope theorem, efficient investments are characterized by the first-order conditions:

$$(1+b_i)Q^*(I_1^*, I_2^*) - C_i'(I_i^*) = 0, \quad i = 1,2, \tag{5}$$

where $Q^*(I) \equiv Pr[q^*(I,\theta) = 1]$. Intuitively, the higher the probability of trade $Q^*(I)$, the greater the marginal expected investment return, both in terms of incremental contribution margin and empire benefits. The next two sections develop the main insights of the paper for the special case where only the seller invests—that is, $I_2 = 0$. By symmetry of the setup, all results carry over to the complementary case where only the buyer invests.

3 The link between incentives, bargaining, and investment

This section analyzes the unit managers' effort, investment, and bargaining behavior for given compensation contracts, assuming $I_2 = 0$. Note that, for given contracts, the managers' decisions are the same under non-integration and vertical integration. The sequential game can be solved by backward induction, beginning with the bargaining procedure

[13] An alternative approach to model negotiations is via alternating offers. However, such bargaining protocols are plagued by serious technical problems (Ausubel, Cramton, and Deneckere, 2002).

which takes the form of an equal-split sealed-bid mechanism (Chatterjee and Samuelson, 1983). At date 4, both managers are asked to simultaneously and non-cooperatively submit bids, $\sigma = (\sigma_1, \sigma_2)$, regarding their valuations for the good. If the buyer's bid σ_2 exceeds the seller's bid σ_1, then $q(\sigma) = 1$ and $t(\sigma) = \frac{1}{2}(\sigma_1 + \sigma_2)$. If $\sigma_2 < \sigma_1$, then $q(\sigma) = t(\sigma) = 0$.

To better understand the bargaining outcome, it is important to note that Manager 1, the investing manager, benefits from his investment in two ways if trade occurs: The relevant cost of producing the intermediate good is lower which increases his compensation and he consumes empire benefits. This can be seen from Manager 1's incremental payoff if $q = 1$ which, according to (1)–(3), equals

$$\beta_1(t - \theta_1 + I_1) + b_1 I_1 \;\equiv\; \beta_1[t - \theta_1 + \nu_1(\beta_1)I_1].$$

The relevant cost of producing the good as perceived by Manager 1 equals $\theta_1 - \nu_1(\beta_1)I_1$, where $\nu_1(\beta_1) \equiv 1 + b_1/\beta_1$ reflects the fact that his tradeoff between compensation and empire benefits depends on his bonus coefficient β_1.

At date 4, Manager i's information set is $\{\omega, \theta_i\}$, where $\omega = \{\beta, I_1\}$ summarizes the publicly known information (recall that for now $I_2 = 0$ is assumed to hold). [14] Manager i's bidding strategy constitutes a mapping $\sigma_i : \mathbb{R}^2 \times [0, \bar{I}_1] \times \Theta_i \to \mathbb{R}$. These strategies form a Bayesian-Nash equilibrium if the following holds:

$$\sigma_1(\theta_1 \mid \omega) \;\in\; \arg\max_{\sigma_1} \; E_{\theta_2}\left[\beta_1\left(\frac{\sigma_1 + \sigma_2(\theta_2 \mid \omega)}{2} - \theta_1 + \nu_1(\beta_1)I_1\right) q(\omega, \theta)\right], \tag{6}$$

$$\sigma_2(\theta_2 \mid \omega) \;\in\; \arg\max_{\sigma_2} \; E_{\theta_1}\left[\beta_2\left(\theta_2 - \frac{\sigma_1(\theta_1 \mid \omega) + \sigma_2}{2}\right) q(\omega, \theta)\right]. \tag{7}$$

The trading rule is given by $q(\omega, \theta) \equiv q(\sigma_1(\omega, \theta_1), \sigma_2(\omega, \theta_2)) = 1$ if, and only if, $\sigma_2(\omega, \theta_2) \geq \sigma_1(\omega, \theta_1)$, in which case the resulting transfer price equals

$$t(\omega, \theta) \equiv t(\sigma_1(\theta_1 \mid \omega), \sigma_2(\theta_2 \mid \omega)) = \frac{\sigma_1(\theta_1 \mid \omega) + \sigma_2(\theta_2 \mid \omega)}{2}. \tag{8}$$

Following Chatterjee and Samuelson (1983), I only consider bidding strategies that are (piecewise) linear in each manager's private information parameter, θ_i. [15] Moreover,

[14] While the fixed salaries α and the effort levels a are also public information, they do not affect the managers' bargaining behavior and hence can be excluded from ω for brevity. Section 6 considers managerial effort choices that pay off only if trade occurs, in which case effort affects bargaining.

[15] Leininger, Linhart and Radner (1989) show there exist infinitely many equilibria apart from the linear one. However, Radner and Schotter (1989) and Daniel, Searle, and Rapoport (1998) found the bidding strategies employed by agents in experiments to be broadly consistent with the linear equilibrium.

the relevant cost and revenue supports are assumed to overlap sufficiently to avoid trivial solutions. A complication now arises in that the relevant valuation supports are endogenous: Manager 1's valuation support depends on his investment I_1 which in equilibrium (as shown in Proposition 2 below) will be decreasing in his bonus coefficient β_1. Hence, there exists a lower bound, β_1^o, for which he chooses his maximum investment—that is, $I_1(\beta_1^o) = \bar{I}_1$. Setting $\beta_1 < \beta_1^o$ would only serve to reduce Manager 1's effort input without improving investments. This lower bound is used for the following technical assumption which implicitly imposes upper bounds on investments and empire benefits: [16]

Assumption 1. Given $\beta_1 = \beta_1^o$ and $I_1 = \bar{I}_1$, the conditional probability of trade is strictly less than one for any type $\theta_i,\ i = 1, 2$.

The following Lemma characterizes the linear equilibrium bidding strategies.

Lemma 1. Suppose only the seller invests and Assumption 1 holds. Then, for any public information ω and any type profile (θ_1, θ_2), the equal-split sealed-bid mechanism yields the following linear equilibrium bidding strategies:

$$\sigma_1(\theta_1 \mid \omega) = \begin{cases} \tilde{\sigma}_1(\theta_1 \mid \omega) \equiv \frac{3\bar{\theta}_2 + \underline{\theta}_1 - 9\nu_1(\beta_1)I_1 + 8\theta_1}{12}, & \text{if } \tilde{\sigma}_1(\theta_1 \mid \omega) \leq \tilde{\sigma}_2(\bar{\theta}_2 \mid \omega), \\ \theta_1 - \nu_1(\beta_1)I_1, & \text{if } \tilde{\sigma}_1(\theta_1 \mid \omega) > \tilde{\sigma}_2(\bar{\theta}_2 \mid \omega), \end{cases}$$

$$\sigma_2(\theta_2 \mid \omega) = \begin{cases} \tilde{\sigma}_2(\theta_2 \mid \omega) \equiv \frac{\bar{\theta}_2 + 3\underline{\theta}_1 - 3\nu_1(\beta_1)I_1 + 8\theta_2}{12}, & \text{if } \tilde{\sigma}_2(\theta_2 \mid \omega) \geq \tilde{\sigma}_1(\underline{\theta}_1 \mid \omega), \\ \theta_2, & \text{if } \tilde{\sigma}_2(\theta_2 \mid \omega) < \tilde{\sigma}_1(\underline{\theta}_1 \mid \omega). \end{cases}$$

Proof: See the Appendix for all proofs.

A key insight obtained from Lemma 1 is that only Manager 1's bonus coefficient affects the bargaining outcome provided $b_1 > 0$, which from here on is assumed to hold. On the other hand, β_2 does not affect bargaining as Manager 2 has no investment opportunity;

[16] Assumption 1 postulates that $\overline{\theta}_1 \geq \frac{3}{4}\overline{\theta}_2 + \frac{1}{4}\underline{\theta}_1 + \frac{3}{4}\nu_1(\beta_1^o)\bar{I}_1$ and $\underline{\theta}_2 \leq \frac{1}{4}\overline{\theta}_2 + \frac{3}{4}\underline{\theta}_1 - \frac{3}{4}\nu_1(\beta_1^o)\bar{I}_1$, where $\nu_1(\beta_1) \equiv 1 + b_1/\beta_1$. If this assumption is violated (e.g., investments can take on very high values), then it is possible that $\tilde{\sigma}_2(\theta_2 \mid \omega) > \tilde{\sigma}_1(\overline{\theta}_1 \mid \omega)$ for some θ_2, ω. This would give rise to a third branch in the bidding strategies in Lemma 1, cluttering notation without adding much insight. Similar arguments would apply to very low cost types θ_1. See Chatterjee and Samuelson (1983) for a discussion of these boundary cases.

hence $\omega = \{\beta_1, I_1\}$. As for Manager 1's bonus coefficient β_1, recall that the optimal seller bid in (6) trades off the probability that trade occurs with the realized payoff consisting of empire benefits and compensation. Holding constant I_1, a variation in β_1 affects the relative weights Manager 1 assigns to compensation (increasing in σ_1, conditional on $q = 1$) and empire benefits (independent of σ_1, conditional on q). The lower β_1, the more weight Manager 1 places on empire benefits and hence on the probability that trade occurs. As a consequence, his perceived relevant cost (scaled by β_1) of $\theta_1 - \nu_1(\beta_1)I_1$ decreases and he will submit a lower bid. Put differently, Manager 1 will bargain *less aggressively* ($\partial\sigma_1/\partial\beta_1 \geq 0$). In equilibrium, Manager 2 will react by bidding *more aggressively*, that is, demand a higher discount for lower β_1 (as $\partial\sigma_2/\partial\beta_1 \geq 0$).

—Insert Figure 2 about here—

This discussion is summarized in Figure 2. There, the managers' bidding strategies are depicted as bold lines and, with a slight abuse of notation, $\sigma_i^o(\theta_i) \equiv \sigma_i(\theta_i \mid \beta_1^o, I_1)$ and $\sigma_i^{oo}(\theta_i) \equiv \sigma_i(\theta_i \mid \beta_1^{oo}, I_1)$ for $\beta_1^{oo} > \beta_1^o$. As this figure demonstrates, Manager 1's relevant cost of $\theta_1 - \nu_1(\beta_1)I_1$ shifts downward as β_1 becomes smaller. As a consequence, he will lower his bid pointwise: $\sigma_1^o(\theta_1) < \sigma_1^{oo}(\theta_1)$ for all θ_1. At the same time, Manager 2's valuation θ_2 (as given by the 45 degree line) is unaffected by a change in β_1, but his bid in equilibrium will deviate further from this valuation as β_1 decreases, i.e., $\sigma_2^o(\theta_2) < \sigma_2^{oo}(\theta_2) < \theta_2$.

In light of these countervailing effects on the managers' bidding strategies, how does Manager 1's incentive weight β_1 affect the efficiency of trade for given investments? The next result evaluates this tradeoff and shows that the resulting change in Manager 2's bargaining strategy is dominated by the more direct effect on Manager 1's bargaining strategy. To this end, let

$$\begin{aligned} Q(\omega) &\equiv Pr[q(\omega, \theta) = 1], \\ S(\omega) &\equiv E_\theta[(\theta_2 - \theta_1 + (1 + b_1)I_1)q(\omega, \theta)] \end{aligned}$$

denote, respectively, the transfer probability and the expected total surplus (contribution margin cum empire benefits) if trade occurs.

Lemma 2. Suppose only the seller invests and A1 holds. Then, holding constant the seller's investment I_1, both $Q(\omega)$ and $S(\omega)$ are decreasing in β_1.

Lowering the effort incentives for the investing party—here: decreasing β_1—improves trading efficiency in terms of both transfer probability and expected gains from trade, at the expense of decreased effort input by that party. This insight will be useful for the subsequent analysis which addresses the managers' investment incentives.

At date 2, before observing their respective types, the managers make their effort and investment choices (a, I) so as to maximize their respective expected utilities,

$$U_i(a, I \mid \alpha, \beta) = \alpha_i + \beta_i E_\theta[\pi_i(\cdot)] - v_i(a_i) + b_i I_i Q(\omega). \tag{9}$$

Given $I_2 = 0$, it is immediate to see that the managers' choices of (a_1, a_2, I_1) are independent of one another. Manager i's effort incentive constraint is

$$a_i(\beta_i) \in \arg\max_{a_i} \{\beta_i a_i - v_i(a_i)\}, \quad i = 1, 2, \tag{10}$$

so that $a_i(\beta_i)$ is solely determined by β_i, according to the (necessary and sufficient) first-order condition $v_i'(a_i(\beta_i)) = \beta_i$. Manager 1's investment incentive constraint reads

$$I_1(\beta_1) \in \arg\max_{I_1} \; \beta_1[E_\theta\{[t(\beta_1, I_1, \theta) - \theta_1 + \nu_1(\beta_1)I_1]q(\beta_1, I_1, \theta)\} - C_1(I_1)], \tag{11}$$

subject to the anticipated outcome of the bidding game in Lemma 1. The investing manager's objective function in (11) is assumed concave in I_1 for all $\beta_1 \geq \beta_1^o$ with an interior maximum. [17] Then, as shown in the Appendix, Manager 1's investment decision is given by the first-order condition

$$\frac{3}{4}\nu_1(\beta_1)Q(\beta_1, I_1(\beta_1)) = C_1'(I_1(\beta_1)). \tag{12}$$

It is instructive to consider a benchmark case where both managers are residual claimants for their respective units; put differently, the firms are owner-managed:

[17] Concavity of the objective in (11) is ensured if $C_1(I_1)$ is sufficiently convex so that the second-order condition $\frac{27}{64}(\nu_1(\beta_1))^2(\overline{\theta}_2 - \underline{\theta}_1 + \nu_1(\beta_1)I_1(\beta_1)) \leq C_1''(I_1(\beta_1))$ holds for all $\beta_1 \geq \beta_1^o$. One can show that concavity of the first-best program $\mathcal{P}^{\mathcal{FB}}$ implies concavity of each individual unit's investment problem, unless β_1 is "small".

Proposition 1. Suppose only the seller invests and Assumption 1 holds. If $\beta = (1, 1)$, then both managers exert first-best effort levels but Manager 1 underinvests, $I_1(1) < I_1^*$.

Being residual claimants, clearly both managers will exert efficient effort. However, an underinvestment problem arises for two reasons. First, the ex post probability that trade occurs—and that the investment pays off—is suboptimal: $Q(\beta_1 = 1, I_1) < Q^*(I_1)$ for any I_1. This follows from the fact that both managers bid strategically. Second, Manager 1 anticipates that Manager 2 will bid more aggressively upon observing a higher investment ($\partial\sigma_2/\partial I_1 \leq 0$, by Lemma 1), because the induced distribution over seller bids is shifted to the left as I_1 goes up. This reflects the familiar hold-up problem.

—Insert Figure 3 about here—

Ex ante investment incentives in incomplete-contracting models depend on the anticipated outcome of the ex post bargaining game. The fact that effort incentives affect the bargaining outcome, therefore, establishes a link between incentive intensity and investments. Specifically, I now ask how $I_1(\beta_1)$ varies in β_1. By Lemma 2, lowering β_1 results in a higher probability of trade and therefore enhances Manager 1's investment incentives, all else equal. On the other hand, Lemma 1 implies that lowering β_1 will exacerbate the hold-up problem. This is demonstrated in Figure 3 which depicts Manager 2's bidding strategy for high- and low-powered seller incentives ($\beta_1^{oo} > \beta_1^o$), and for high and low seller investments ($I_1^{oo} > I_1^o$). For low-powered seller incentives, Manager 2 not just includes a higher discount into his bid ($\sigma_2(\theta_2 \mid \beta_1^o, I_1) < \sigma_2(\theta_2 \mid \beta_1^{oo}, I_1)$ for any I_1), but also reacts more sharply to an increase in the seller's investment ($\sigma_2(\theta_2 \mid \beta_1^o, I_1^o) - \sigma_2(\theta_2 \mid \beta_1^o, I_1^{oo}) > \sigma_2(\theta_2 \mid \beta_1^{oo}, I_1^o) - \sigma_2(\theta_2 \mid \beta_1^{oo}, I_1^{oo})$). Evaluating this tradeoff, the next result shows that the net effect of lowering β_1 on $I_1(\beta_1)$ is nonetheless positive.

Proposition 2. If only the seller invests and Assumption 1 holds, then $I_1'(\beta_1) \leq 0$.

As shown in the Appendix, the impact of a decrease in β_1 on the seller's marginal investment returns in equilibrium is ultimately determined by two first-order effects: First, the transfer probability increases, as $\partial Q/\partial\beta_1 < 0$ by Lemma 2. Second, $\nu_1(\beta_1)$ increases as Manager 1's tradeoff between empire benefits and compensation is tilted in favor of empire benefits which, in contrast to compensation, are unaffected by the fixed cost from investing. This further improves his investment incentives at the margin. [18]

Proposition 2 is of key importance to the subsequent analysis as it identifies a potential role for compensation to alleviate underinvestment problems: Lowering the investing manager's effort incentives induces him to invest more despite the exacerbated hold-up problem. The next section uses the above results to investigate the optimal contract design choices for the principal(s) under alternative ownership structures.

4 Ownership structure and incentives

Under non-integrated ownership each Unit i is a stand-alone firm owned by Principal i. Principal i designs a compensation contract for Manager i, anticipating the ensuing subgame described in the preceding section. Since managers are risk-neutral and able to commit to stay on the job for any type realization, each principal can extract the expected surplus (including empire benefits) from her respective manager. That is, α_i is set so that $U_i = 0$ in (9). As a consequence, the remaining choice variable is β_i, and Principal i's objective as a function of β_i becomes

$$\Pi_i^{NI}(\beta) \equiv a_i(\beta_i) - v_i(a_i(\beta_i)) - C_i(I_i(\beta)) + E_\theta\{[M_i(\omega,\theta) + b_i I_i(\beta)]q(\omega,\theta)\},$$

with $M_i(\omega,\theta) \equiv M_i(\theta_i, I_i(\beta), t(\beta, I(\beta), \theta))$ and similarly for $q(\omega,\theta)$. Thus, the principals face the following contract design problem under non-integration (NI): [19]

$$\mathcal{P}^{\mathcal{NI}}: \qquad \max_{\beta_i} \; \Pi_i^{NI}(\beta), \quad i = 1, 2,$$
$$\text{subject to: (6), (7), (10), (11).}$$

Let β_i^{NI} denote the solution to program $\mathcal{P}^{\mathcal{NI}}$ for Firm i.

Consider Firm 2 first. Since only Firm 1 is assumed to invest, Lemma 1 implies that β_2 only affects Manager 2's effort input. Thus, Principal 2 optimally sets $\beta_2 = 1$ so as to elicit first-best effort from her manager. Now consider Firm 1. At $\beta_1 = 1$, Manager 1 is

[18] I am grateful to an anonymous reviewer for suggesting this interpretation.

[19] In general, the principals' contract offers should be viewed as a Nash equilibrium. In this one-sided investment scenario, however, the contract offers are independent. This will no longer hold in Section 5.

residual claimant for Firm 1's profit and hence fully internalizes Principal 1's objective. While this might suggest that both firms should set $\beta_i = 1$, as in Proposition 1, this intuition is incomplete as the following result shows.

Proposition 3. If the firms are non-integrated, only the seller (Firm 1) invests and Assumption 1 holds, then $\beta_2^{NI} = 1$ and $\beta_1^{NI} > 1$.

To gain some intuition for why $\beta_1^{NI} > 1$ holds, suppose Principal 1 were to set $\beta_1 = 1$. Manager 1 then would choose the first-best effort a_1^* and investment $I_1(1)$, maximizing the expected Firm-1 surplus (cum empire benefits) holding constant the distribution over Manager 2's bids. Now consider a variation in that β_1 is raised slightly. This only has a second-order effect on Firm 1's profit via changes in Manager 1's effort, investment and bidding choices. At the same time, there is a first-order gain for Firm 1 arising from the fact that the buyer will bid more cautiously. Distorting Manager 1's incentives away from "naive" profit maximization thus serves as a commitment device for Firm 1 to bargain more aggressively, thereby imposing a negative externality on Firm 2.

This commitment argument is reminiscent of Fershtman and Judd's (1987) results on strategic incentive contracts in oligopoly. These authors have shown that by compensating managers based on sales, in addition to income, firms can commit to high-output strategies under Cournot competition. In contrast, the interaction studied here is between vertically related firms negotiating the terms of trade.

Under vertical integration, both units are divisions of the same firm. Division i is again run by Manager i whose effort and investment decisions are as described by (10) and (11), above. However, now there is only one Principal who simultaneously contracts with both managers. While this ensures that a vertically integrated firm always weakly dominates a non-integrated structure in this model, I seek to characterize the contractual means (short of firm-wide profit sharing) by which integration can *strictly* outperform non-integration.

The managers' fixed salaries α can again be adjusted so as to extract the entire expected surplus including empire benefits. The Principal's objective function thus becomes

$$\Pi^{INT}(\beta) \equiv \sum_{i=1}^{2}[a_i(\beta_i) - v_i(a_i(\beta_i))] - C_1(I_1(\beta_1)) + E_\theta\{[\theta_2 - \theta_1 + (1 + b_1)I_1(\beta_1)]q(\omega, \theta)\},$$

and the contract design problem under vertical integration (INT) reads:

$$\mathcal{P}^{\mathcal{INT}}: \quad \max_{\beta} \ \Pi^{INT}(\beta), \\ \text{subject to: (6), (7), (10), (11).}$$

Let β^{INT} denote the solution to program $\mathcal{P}^{\mathcal{INT}}$. As under non-integration, the principal will set $\beta_2 = 1$ so as to induce first-best effort input from Manager 2. However, the Principal now internalizes the cost that Manager i's strategic bidding behavior imposes on Division j. As a consequence, the investing division's incentives should optimally be muted within a vertically integrated firm.

Proposition 4. If the units are vertically integrated, only the seller (Division 1) invests and Assumption 1 holds, then $\beta_2^{INT} = 1$ and $\beta_1^{INT} < 1$.

By lowering β_1, the Principal induces Manager 1 to bid more cautiously at the trading stage, which by Proposition 2 yields an improvement in $I_1(\beta_1)$. In equilibrium—that is, for $I_1(\beta_1)$ and the resulting bids $\{\sigma_i(\theta_i \mid \beta_1, I_1(\beta_1))\}_{i=1}^{2}$—this raises the probability that trade occurs and it improves firm-wide gains from trade (Lemma 2). Loosely speaking, incentives in an integrated firm are muted so as to induce more "cooperative" bargaining behavior from divisional managers.

Propositions 3 and 4 illustrate the central message of this paper: Ownership matters even under divisional performance evaluation because it determines optimal incentive provision, which in turn affects the managers' bargaining behavior. Since this is a key departure from the incomplete contracting literature, it should be emphasized that both modelling innovations—asymmetric information and empire benefits—are necessary for this incentives/bargaining link to obtain. In the absence of empire benefits ($b_i = 0$), bonus coefficients have no bearing on bargaining and investments; the same holds if information is symmetric, in which case Nash bargaining ensures ex post efficiency.

As noted in the Introduction, Williamson (1985) argues that integrated firms have to resort to muted incentives because of commitment problems. Propositions 3 and 4 provide a complementary but more positive explanation for this phenomenon: High-powered incentives under vertical integration are not *optimal* even if they may be *feasible*. Low-powered incentives serve to moderate the managers' bargaining behavior and hence to alleviate the externality problem in connection with relationship-specific investments. This is consistent with the empirical evidence reported in Argyres (1995) on the link between incentives and interdivisional coordination at IBM and GM.

The preceding analysis has made a number of simplifying assumptions. Most notably, only one of the units was assumed to invest specifically, and this investment opportunity was assumed to be in physical assets only. The remaining sections generalize the model by allowing for bilateral investments and investments in managerial skill-acquisition.

5 Bilateral investments

Suppose now that both managers simultaneously choose capital investments, $I = (I_1, I_2)$, after accepting their contracts, but before observing their respective types. For any I, the buyer's bidding strategy at date 4 then amounts to choosing

$$\sigma_2(\theta_2 \mid \omega) \ \in \ \arg\max_{\sigma_2} \ E_{\theta_1} \left[\beta_2 \left(\theta_2 - \frac{\sigma_1(\theta_1 \mid \omega) + \sigma_2}{2} + \nu_2(\beta_2) I_2 \right) q(\omega, \theta) \right], \qquad (13)$$

where $\nu_2(\beta_2) \equiv 1 + b_2/\beta_2$. The seller's bidding strategy remains as given in (6). Each manager's bid now depends on both bonus coefficients and investments (hence $\omega = \{\beta, I\}$), and investments are chosen in form of a pure-strategy Nash equilibrium. As will become clear momentarily, this holds because (i) both bonus coefficients now affect the bargaining outcome and (ii) investments are strategic complements.

Our earlier finding that investment is decreasing in effort incentives can be shown to carry over to the bilateral investment case; see Lemma A1 in the Appendix. However, Assumption 1 needs to be adapted for the case of bilateral investments. To this end, let $\mathcal{B}^o = \{\beta \mid I_i(\beta) = \bar{I}_i, \ i = 1, 2\}$ denote the set of bonus coefficients for which both managers choose their respective maximum investments. By the same logic as in connection with Assumption 1, attention can be confined to the set $\mathcal{B}$ that contains all β's for which $I(\beta) \leq (\bar{I}_1, \bar{I}_2)$. [20]

[20] I adopt the vector convention that $(x_1, ..., x_k) \geq (y_1, ..., y_k)$ means $x_i \geq y_i, \forall i$, and $(x_1, ..., x_k) > (y_1, ..., y_k)$ means $x_i \geq y_i, \forall i$, with at least one inequality strict. Formally then, $\mathcal{B} = \{\beta \mid \nexists \beta^o \in \mathcal{B}^o$ such that $\beta^o > \beta\}$. That is, $\mathcal{B}$ is the upper contour set with respect to the one-dimensional manifold $\mathcal{B}^o$. For any $\beta \notin \mathcal{B}$ there exists another $\beta' \in \mathcal{B}$ that induces the same investments but higher effort levels and is hence preferred. Assumption $1'$ then amounts to $\overline{\theta}_1 \geq \frac{3}{4}\overline{\theta}_2 + \frac{1}{4}\underline{\theta}_1 + \frac{3}{4}\Phi$ and $\underline{\theta}_2 \leq \frac{1}{4}\overline{\theta}_2 + \frac{3}{4}\underline{\theta}_1 - \frac{3}{4}\Phi$,

Assumption 1′. For any $\beta \in \mathcal{B}^o$ and $I = (\bar{I}_1, \bar{I}_2)$, the conditional probability of trade is strictly less than one for any type $\theta_i,\ i = 1, 2$.

The managers' equilibrium bidding strategies can be found along similar lines as in Lemma 1; see the proof of Proposition 5 in the Appendix for details. For given contracts, the managers' investments $I(\beta)$ constitute a pure-strategy Nash equilibrium whenever

$$I_i(\beta) \in \arg\max_{I_i}\ U_i(a_i, I_i, I_j(\beta) \mid \alpha, \beta), \quad i, j = 1, 2,\ i \neq j,\ \beta \in \mathcal{B}, \tag{14}$$

subject to the ensuing bargaining subgame. The following assumption facilitates the comparison of the equilibrium contract offers under the alternative ownership structures.

Assumption 2. For any $\beta \in \mathcal{B}$, there exists a unique interior pure-strategy Nash equilibrium in investments, $I(\beta)$, that solves (14).

The Appendix contains a technical condition that ensures Assumption 2 holds.

At the contract design stage (date 1), the objective functions of the principal(s) are the same as in programs $\mathcal{P}^{\mathcal{NI}}$ and $\mathcal{P}^{\mathcal{INT}}$ above, except that the choices (β_1, β_2) now are interdependent. In a Nash equilibrium in contracts offered under non-integration, for any conjectured bonus coefficient β_j, Principal i chooses her optimal response $\beta_i^{NI}(\beta_j)$ so as to maximize Π_i^{NI} in program $\mathcal{P}^{\mathcal{NI}}$, subject to the constraints in (6) and (13) regarding the managers' bidding strategies; and (10) and (14) regarding effort and investment choices. Under vertical integration, on the other hand, the Principal maximizes Π^{INT} in program $\mathcal{P}^{\mathcal{INT}}$ over both (β_1, β_2), subject to the same constraints. I assume there exists a unique pure-strategy Nash equilibrium β^{NI} under non-integration, as well as a unique global maximum β^{INT} for the vertically integrated firm. [21] Then the following result obtains:

Proposition 5. If both units can invest and Assumptions 1′ and 2 hold, then $\beta^{NI} > \beta^{INT}$ and the resulting equilibrium investment amounts are greater under integration than under non-integration—that is, $I^{INT} \equiv I(\beta^{INT}) > I^{NI} \equiv I(\beta^{NI})$.

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for all $\beta \in \mathcal{B}^o$, where $\Phi \equiv \sum_{i=1}^{2} \nu_i(\beta_1)\bar{I}_i$ and $\nu_i(\beta_i) \equiv (1 + b_i/\beta_i)$.

[21] These uniqueness assumptions will hold for C_i sufficiently convex. If there exist multiple equilibria/maxima with $\underline{\beta}^k$ and $\overline{\beta}^k$ as the respective smallest and highest equilibrium/maximum for $k \in \{NI, INT\}$, then Proposition 5 would have to be restated as: $\underline{\beta}^{NI} > \underline{\beta}^{INT}$ and $\overline{\beta}^{NI} > \overline{\beta}^{INT}$.

Under non-integration, the firms find themselves in a prisoners' dilemma with each principal inducing her respective manager to bargain aggressively. In equilibrium this leads to less efficient trade, lower investments, and lower profits than under vertical integration where the Principal pays out $\beta^{INT} < (1, 1)$. [22]

Proposition 5 has implications for the managers' expected compensation payments across the two organizational modes. Since $\beta_i^{INT} < \beta_i^{NI}$, each manager will exert less effort and negotiate more cautiously within a vertically integrated firm. Both these effects result in lower expected compensation because both managers (i) incur lower disutility of effort and (ii) consume higher expected empire benefits due to higher investments and a higher transfer probability as compared with non-integration. This discussion is summarized in the following corollary, which is presented without formal proof.

Corollary. Each manager's expected compensation is lower under vertical integration than under non-integration.

This corollary is in stark contrast to Hart and Holmstrom (2002) who predict compensation to be lower under non-integration. [23] The reason for these diverging predictions is that in their model decisions are assumed non-contractible not just ex ante, as in my model, but also ex post. Compensation then is not a viable instrument to affect workers' incentives and the "boss" cannot expropriate the workers' private benefits via reduced pay. Moreover, their model suppresses personally costly effort. Hence, of the two effects that drive the corollary, the disutility of effort-effect is absent from Hart and Holmstrom while the empire benefits-effect is reversed: In their model employees of integrated firms have to be compensated for the fact that their boss ignores their private benefits when making decisions. [24]

[22] Note that, unlike Proposition 3 (where $I_2 = 0$), Proposition 5 does not claim that $\beta_i^{NI} > 1$ holds for $i = 1, 2$. The reason is that raising β_i now results in a decrease in I_j which indirectly harms firm i. This "feedback" effect was absent in the one-sided investment setting.

[23] The corollary does not conflict with the empirical regularity that larger firms tend to pay higher wages on average (e.g., Brown and Medoff, 1989), because vertically integrated firms are not necessarily larger than stand-alone firms.

[24] While ex post contractibility of divisional income and of transfer decisions appears to be a reasonable assumption per se, a caveat is in order. In practice the shareholders may be imperfectly informed about how their managers value private benefits (recall that b_i is assumed to be commonly known in my model). Hence, they may not be able to perfectly extract these benefits by adjusting compensation.

6 Skill-acquisition effort

Transfers of intermediate products often require non-contractible, relationship-specific investments not just in capital but also in effort; e.g., engineering effort at the product design stage (Holmstrom and Tirole, 1991; Baiman and Rajan, 1998). Suppose that Manager i can select an additional input variable to the production process, denoted by $e_i \in [0, \bar{e}_i]$, and let $e = (e_1, e_2)$. Unlike capital investments that are paid from divisional accounts, such "skill-acquisition" effort is personally costly to the manager. The attendant disutility is denoted by $D_i(e_i)$; it is assumed to be increasing, convex, and separable from the disutility from general-purpose effort, $v_i(a_i)$. [25] Manager i's expected payoff then equals

$$\widehat{U}_i(a_i, e, I \mid \alpha, \beta) = \alpha_i + \beta_i E_\theta[\widehat{\pi}_i(\cdot)] - v_i(a_i) - D_i(e_i) + b_i I_i Q(\cdot).$$

The probability of trade, Q, will now depend also on e. The modified profit measures are $\widehat{\pi}_i \equiv a_i + \widehat{M}_i(\theta_i, I_i, e_i, t)q - C_i(I_i)$, with $\widehat{M}_1(\cdot) \equiv t - \theta_1 + I_1 + e_1$ and $\widehat{M}_2(\cdot) \equiv \theta_2 + I_2 + e_2 - t$ as the transfer-related contribution margins. If the transfer does not materialize, each manager is left with his sunk costs, $\beta_i C_i(I_i)$ from the capital investment and $D_i(e_i)$ from the skill-acquisition effort.

Now consider the managers' incentives in this modified setting. The Bayesian-Nash equilibrium in bidding strategies for any $\omega = \{\beta, I, e\}$ and θ now is given by [26]

$$\widehat{\sigma}_i(\theta_i \mid \omega) \in \arg\max_{\sigma_i} E_{\theta_j}\{[\beta_i \widehat{M}_i(\theta_i, I_i, e_i, t) + b_i I_i]\widehat{q}(\omega, \theta)\}, \quad i = 1, 2, \ i \neq j,$$

where the notation mirrors that from preceding subsections. The following assumption once more generalizes Assumption 1 by imposing an upper bound on total relationship-specific investments, now in terms of capital and skill-acquisition. In keeping with the discussion of Assumption $1'$, above, let $\widehat{\mathcal{B}}^o$ denote the set of β's for which $e_i(\beta) = \bar{e}_i$ and $I_i(\beta) = \bar{I}_i$, $i = 1, 2$, and $\widehat{\mathcal{B}}$ the set of β's for which $e_i(\beta) \leq \bar{e}_i$ and $I_i(\beta) \leq \bar{I}_i$. [27]

[25] The implicit assumption here is that managers operate below full workload capacity. If instead disutility were non-separable across (a_i, e_i), this would affect the absolute magnitude of the equilibrium incentive provisions under each organizational form, but I conjecture their relative ranking will be unaffected.

[26] The first-best solution and the derivation of the equilibrium effort, investment, and bidding strategies under decentralization are characterized in the Appendix, proof of Proposition 6.

[27] As in footnote 20, Assumption $1''$ amounts to assuming that $\overline{\theta}_1 \geq \frac{3}{4}\overline{\theta}_2 + \frac{1}{4}\underline{\theta}_1 + \frac{3}{4}\widehat{\Phi}$ and $\underline{\theta}_2 \leq \frac{1}{4}\overline{\theta}_2 + \frac{3}{4}\underline{\theta}_1 - \frac{3}{4}\widehat{\Phi}$, for all $\beta \in \widehat{\mathcal{B}}^o$, where $\widehat{\Phi} \equiv \sum_{i=1}^{2}[\nu_i(\beta_i)\bar{I}_i + \bar{e}_i]$.

Assumption 1''. For any $\beta \in \widehat{\mathcal{B}}^o$ and $(I, e) = (\bar{I}_1, \bar{I}_2, \bar{e}_1, \bar{e}_2)$, the conditional probability of trade is strictly less than one for any type $\theta_i,\ i = 1, 2$.

The link between effort incentives and the managers' bargaining behavior now becomes more complex. In previous sections, muted incentives have resulted, unambiguously, in increased capital investments due to more cooperative bargaining and an attendant improvement in equilibrium trading efficiency. Now there is a countervailing effect: For instance, lowering β_1 decreases Manager 1's input of skill-acquisition effort e_1, which in turn reduces the probability that trade occurs. Since all investment variables (I_1, e_1, I_2, e_2) are strategic complements, the net effect on ex post trade efficiency and ex ante investments will generally depend on the "elasticities" of these respective input variables. To evaluate the ensuing tradeoffs most transparently, it is therefore useful to return to the scenario where only the seller invests in capital assets; i.e., $I_2 = 0$. A pure-strategy Nash equilibrium in investments obtains if the following two conditions hold simultaneously:

$$\begin{aligned}(I_1(\beta), e_1(\beta)) &\in \arg\max_{I_1, e_1} \widehat{U}_1(a_1, e_1, e_2(\beta), I_1 \mid \alpha, \beta),\\ e_2(\beta) &\in \arg\max_{e_2} \widehat{U}_2(a_2, e_2, e_1(\beta), I_1(\beta) \mid \alpha, \beta).\end{aligned}$$

I now adapt Assumption 2 by allowing for relationship-specific effort.

Assumption 2'. Given $I_2 = 0$, for any $\beta \in \widehat{\mathcal{B}}$ there exists a unique interior pure-strategy Nash equilibrium, $(I_1(\beta), e_1(\beta), e_2(\beta))$.

It is straightforward to show that all previous findings for non-integrated firms carry over to this setting. The next result therefore only addresses vertically integrated firms.

Proposition 6. Consider a vertically integrated firm where only the selling division can invest in physical assets ($I_2 = 0$), but both managers can invest in personally costly skill-acquisition effort ($e_i \geq 0,\ i = 1, 2$). Given Assumptions $1''$ and $2'$:

(i) $\beta_2^{INT} > 1$;

(ii) If Manager 1 can only invest in skill acquisition ($C_1'(0) \to \infty$), then $\beta_1^{INT} > 1$; whereas if Manager 1 can only invest in capital assets ($D_1'(0) \to \infty$), then $\beta_1^{INT} < 1$.

If the buyer cannot invest in physical capital, then β_2 will affect the bargaining outcome only via its effect on e_2. The higher β_2, the greater e_2, and hence the higher the probability of trade. This in turn increases the seller's incentives to invest in I_1 and e_1, as in Holmstrom and Tirole (1991). The same holds for the seller in the corresponding case where $C_1'(0) \to \infty$. If instead $D_1'(0) \to \infty$, then there is no scope for seller investments in skill acquisition and the logic from previous sections carries over: The Principal will provide Manager 1 with weaker effort incentives (low β_1) so as to induce more cooperative bargaining.

7 Conclusion

This paper has addressed two related questions: (i) Does the widespread practice of divisional performance evaluation undermine any coordination advantages associated with vertical integration? (ii) How should incentive strengths compare between vertically integrated firms and stand-alone firms? The answer to question (i) is yes: Vertical integration mitigates hold-up problems even if the integrated firm is subsequently run in a decentralized fashion. The claims in the early incomplete contracting literature (Klein, Crawford, and Alchian, 1978; Williamson, 1985) therefore hold up in the presence of divisional performance evaluation, if perhaps a little less forcefully. The channel through which vertical integration improves value is related to question (ii), above. Even if high-powered incentives are feasible within integrated firms, optimally incentives should be muted so as to moderate managers' bargaining behavior. This in turn will improve investment incentives and overall profit. That is, this model supports the claims in the management literature that high-powered effort incentives hamper inter-divisional cooperation.

It is instructive to contrast the approach taken here with the property rights theory literature, e.g., Grossman and Hart (1986), Hart and Moore (1988), Hart and Holmstrom (2002), Matouschek (2004). For the most part, this literature has sidestepped the issue of separation of ownership and control and instead has stipulated a direct link between ownership and investment efficiency by assuming that asset ownership determines the disagreement payoffs in the bargaining game. In my model, this ownership/investments link is less direct: Ownership determines optimal incentive provision which in turn affects the managers' bargaining behavior and investment incentives. [28] By allowing for delegated decision-making, richer models can be developed that may generate empirically testable hypotheses regarding relative incentive strengths across firms. However, a limitation of the approach taken here is that it can only highlight the benefits from vertical integration, while the property rights theory illustrates both benefits and costs. [29]

As noted above, there are alternative modelling approaches to establishing a link between effort incentives and bargaining behavior. For instance, suppose empire benefits are negligible but the compensation contract contains a third entry (in addition to fixed salary and a bonus proportional to divisional profit): a discrete bonus paid out whenever the transaction takes place. Note that the transaction *per se* is a verifiable and hence contractible event. Such a "transfer bonus" essentially becomes an additional control instrument at the principal's disposal. Endogenizing this additional control is an interesting avenue for future research.

Progress in analyzing hold-up problems under asymmetric information has been stymied by the problematic task of modelling the bargaining process (Ausubel, Cramton, and Deneckere, 2002). While the sealed-bid mechanism used in this paper requires some structure to be placed on the underlying distributions of the private information variables and achieves second-best efficiency only in knife-edge cases, it is an otherwise flexible modeling tool that may prove useful for future research in applied contract theory. A particularly appealing feature of this mechanism is that, under reasonable assumptions, managers' bargaining strategies become smooth functions of their bonus coefficients with higher coefficients translating into more aggressive bargaining. Such plausible predictions are difficult to obtain within cooperative bargaining models based on the Nash solution, as noted by Holmstrom and Tirole (1991, p.211).

[28] Moreover, the present paper borrows from Williamson's (1975) transaction cost economics the notion of ex post governance costs—here: trade distortions that are affected by compensation.

[29] Other factors that work against vertical integration and that are ignored here are, among others, limited span of control (Ziv, 2000) or influence costs (Bagwell and Zechner, 1993).

Appendix: Proofs

Proof of Lemma 1: I follow Chatterjee and Samuelson (1983) in deriving the Bayesian-Nash equilibrium in linear bidding strategies. The proof allows for bilateral investments, $I_i \geq 0,\ i = 1, 2$, with Lemma 1 as a special case where $I_2 = 0$.

Dividing the bidding strategies in (6)–(7) by β_i yields

$$\begin{aligned}
\sigma_1(\theta_1 \mid \omega) &\in \arg\max_{\sigma_1} E_{\theta_2}\left[\left(\frac{\sigma_1 + \sigma_2(\theta_2 \mid \omega)}{2} - \theta_1 + \nu_1(\beta_1)I_1\right) q(\omega, \theta)\right], \\
\sigma_2(\theta_2 \mid \omega) &\in \arg\max_{\sigma_2} E_{\theta_1}\left[\left(\theta_2 + \nu_2(\beta_2)I_2 - \frac{\sigma_1(\theta_1 \mid \omega) + \sigma_2}{2}\right) q(\omega, \theta)\right],
\end{aligned}$$

where $\nu_i(\beta_i) \equiv 1 + b_i/\beta_i$. Let $\tau_1 \equiv \theta_1 - \nu_1(\beta_1)I_1$, $\tau_2 \equiv \theta_2 + \nu_2(\beta_2)I_2$ denote the managers' relevant valuations for the intermediate good, adjusted for empire benefits. This induces (uniform) distributions $\widehat{F}_i(\tau_i)$ over $[\underline{\tau}_i, \bar{\tau}_i]$, where $\underline{\tau}_1 \equiv \underline{\theta}_1 - \nu_1(\beta_1)I_1$, $\bar{\tau}_1 \equiv \bar{\theta}_1 - \nu_1(\beta_1)I_1$, $\underline{\tau}_2 \equiv \underline{\theta}_2 + \nu_2(\beta_2)I_2$, and $\bar{\tau}_2 \equiv \bar{\theta}_2 + \nu_2(\beta_2)I_2$. Restate the managers' optimization problems:

$$\begin{aligned}
\widehat{\sigma}_1(\tau_1 \mid \omega) &\in \arg\max_{\sigma_1} \int_{\sigma_1}^{\widehat{\sigma}_2(\bar{\tau}_2|\omega)} \left(\frac{\sigma_1 + \widehat{\sigma}_2(\tau_2 \mid \omega)}{2} - \tau_1\right) dG_2(\widehat{\sigma}_2 \mid \omega), \\
\widehat{\sigma}_2(\tau_2 \mid \omega) &\in \arg\max_{\sigma_2} \int_{\widehat{\sigma}_1(\underline{\tau}_1|\omega)}^{\sigma_2} \left(\tau_2 - \frac{\widehat{\sigma}_1(\tau_1 \mid \omega) + \sigma_2}{2}\right) dG_1(\widehat{\sigma}_1 \mid \omega).
\end{aligned}$$

The bid distribution functions G_i are defined by the underlying type distributions $\widehat{F}_i(\tau_i)$ and the bidding strategies $\widehat{\sigma}_i$, where $G_i(\cdot \mid \omega) \equiv \widehat{F}_i(\widehat{\sigma}_i^{-1}(\cdot \mid \omega))$. The first-order condition for the seller is

$$\frac{1 - G_2(\widehat{\sigma}_1(\tau_1 \mid \omega) \mid \omega)}{2} - [\widehat{\sigma}_1(\tau_1 \mid \omega) - \tau_1] g_2(\widehat{\sigma}_1(\tau_1 \mid \omega) \mid \omega) = 0,$$

with g_i denoting the density function to G_i. Defining $x_2 \equiv \widehat{\sigma}_2^{-1}(\widehat{\sigma}_1 \mid \omega)$ results in $g_2(\widehat{\sigma}_1 \mid \omega) = f_2(x_2)/\widehat{\sigma}_2'(x_2 \mid \omega)$, $\tau_1 = \widehat{\sigma}_1^{-1}(\widehat{\sigma}_2(x_2 \mid \omega) \mid \omega)$, and $G_2(\widehat{\sigma}_1 \mid \omega) = F_2(x_2)$, where F_i is the distribution function over Manager i's types θ_i. Hence, the first-order condition can be restated as

$$\widehat{\sigma}_1^{-1}(\widehat{\sigma}_2(x_2 \mid \omega) \mid \omega) = \widehat{\sigma}_2(x_2 \mid \omega) - \frac{1}{2}\widehat{\sigma}_2'(x_2 \mid \omega)\frac{1 - \widehat{F}_2(x_2)}{\widehat{f}_2(x_2)}.$$

Proceeding in a similar fashion for the buyer yields

$$\widehat{\sigma}_2^{-1}(\widehat{\sigma}_1(x_1 \mid \omega) \mid \omega) = \widehat{\sigma}_1(x_1 \mid \omega) + \frac{1}{2}\widehat{\sigma}_1'(x_1 \mid \omega)\frac{\widehat{F}_1(x_1)}{\widehat{f}_1(x_1)}, \quad \text{for } x_1 \equiv \widehat{\sigma}_1^{-1}(\widehat{\sigma}_2 \mid \omega).$$

A Bayesian-Nash equilibrium now is a solution to these two linked differential equations. Restricting attention to linear bidding strategies of the form $\widehat{\sigma}_i(\tau_i \mid \omega) = y_i(\omega) + z_i(\omega)\tau_i$, where the coefficients (y_i, z_i) are allowed to vary in ω, gives

$$\begin{aligned}
\widehat{\sigma}_1^{-1}(\widehat{\sigma}_2(\tau_2 \mid \omega) \mid \omega) &= \widehat{\sigma}_2(\tau_2 \mid \omega) - \frac{z_2(\omega)}{2}(\bar{\tau}_2 - \tau_2), \\
\widehat{\sigma}_2^{-1}(\widehat{\sigma}_1(\tau_1 \mid \omega) \mid \omega) &= \widehat{\sigma}_1(\tau_1 \mid \omega) + \frac{z_1(\omega)}{2}(\tau_1 - \underline{\tau}_1).
\end{aligned}$$

By differentiation, we can determine the slopes of the strategies, which turn out to be independent of ω: $z_1(\omega) \equiv z_2(\omega) \equiv \frac{2}{3}$. It follows that the intercept terms are also independent of ω: $y_1(\omega) \equiv \frac{1}{4}\bar{\tau}_2 + \frac{1}{12}\underline{\tau}_1$ and $y_2(\omega) \equiv \frac{1}{12}\bar{\tau}_2 + \frac{1}{4}\underline{\tau}_1$. Rescaling both managers' valuations in terms of $\theta_i = \tau_i \pm \nu_i(\beta_i)I_i$, yields the linear strategies

$$\begin{aligned}
\sigma_1(\theta_1 \mid \omega) &= \frac{3\bar{\theta}_2 + \underline{\theta}_1 - 9\nu_1(\beta_1)I_1 + 3\nu_2(\beta_2)I_2 + 8\theta_1}{12}, \\
\sigma_2(\theta_2 \mid \omega) &= \frac{\bar{\theta}_2 + 3\underline{\theta}_1 - 3\nu_1(\beta_1)I_1 + 9\nu_2(\beta_2)I_2 + 8\theta_2}{12},
\end{aligned}$$

given the first-order conditions are necessary and sufficient. For a detailed discussion of the boundary conditions, the reader is referred to Chatterjee and Samuelson (1983). *Q.E.D.*

Proof of Lemma 2: The probability of trade for given $\omega = \{\beta_1, I_1\}$—recall that $I_2 = 0$, by assumption, i.e. β_2 does not affect bargaining (by Lemma 1)—equals

$$Q(\omega) \equiv Pr[q(\omega, \theta) = 1] = \int_{\underline{\theta}_1}^{\theta_1^o(\bar{\theta}_2|\omega)} \int_{\theta_2^o(\theta_1|\omega)}^{\bar{\theta}_2} f_1 f_2 d\theta_2 d\theta_1.$$

The integral boundaries for the case where $I_2 = 0$ are

$$\theta_1^o(\bar{\theta}_2 \mid \omega) \equiv \sigma_1^{-1}(\sigma_2(\bar{\theta}_2 \mid \omega) \mid \omega) = \frac{3\bar{\theta}_2 + \underline{\theta}_1 + 3\nu_1(\beta_1)I_1}{4}, \tag{A1}$$

$$\theta_2^o(\theta_1 \mid \omega) \equiv \sigma_2^{-1}(\sigma_1(\theta_1 \mid \omega) \mid \omega) = \theta_1 + \frac{\bar{\theta}_2 - \underline{\theta}_1 - 3\nu_1(\beta_1)I_1}{4}, \tag{A2}$$

with (A1) as the maximum seller cost realization for which the conditionally expected probability of trade is strictly positive, and (A2) as the buyer revenue realization for which $\sigma_2(\cdot) = \sigma_1(\cdot)$. Using these gives

$$Q(\omega) = \int_{\underline{\theta}_1}^{\theta_1^o(\bar{\theta}_2|\omega)} \frac{3\bar{\theta}_2 + \underline{\theta}_1 + 3\nu_1(\beta_1)I_1 - 4\theta_1}{4} f_1 f_2 d\theta_1. \tag{A3}$$

Straightforward partial differentiation, holding constant I_1, yields

$$\left.\frac{\partial Q}{\partial \beta_1}\right|_{I_1} = -\frac{3b_1}{4\beta_1^2} I_1[\theta_1^o(\overline{\theta}_2 \mid \omega) - \underline{\theta}_1] f_1 f_2 < 0. \tag{A4}$$

The expected total trading surplus (including empire benefits) equals

$$S(\omega) = \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \int_{\theta_2^o(\theta_1|\omega)}^{\overline{\theta}_2} [\theta_2 - \theta_1 + (1+b_1)I_1] f_1 f_2 d\theta_2 d\theta_1.$$

Differentiating this term, again holding constant I_1:

$$\left.\frac{\partial S}{\partial \beta_1}\right|_{I_1} = \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \left(-\left.\frac{\partial \theta_2^o(\theta_1 \mid \omega)}{\partial \beta_1}\right|_{I_1} \right) [\theta_2^o(\theta_1 \mid \omega) - \theta_1 + (1+b_1)I_1] f_1 f_2 d\theta_2.$$

This last expression is strictly negative as $\partial\theta_2^o/\partial\beta_1 = 3b_1 I_1/(4\beta_1^2) > 0$ and the term in square brackets is positive. *Q.E.D.*

Proof of Proposition 1: Each unit Manager i aims at maximizing his payoff as given by the expected utility expression in (9). The managers' effort choices are determined by the first-order conditions $v_i'(a_i(\beta_i)) = \beta_i$; hence $a_i(1) = a_i^*$. Manager 1 also chooses an investment I_1. Ignoring fixed salaries, denote manager 1's expected payoff by the function $\phi_1(a_1, I_1 \mid \beta_1)$:

$$\begin{aligned}\phi_1(a_1, I_1 \mid \beta_1) \;\equiv\; & \beta_1 \left\{ a_1 + \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \int_{\theta_2^o(\theta_1|\omega)}^{\overline{\theta}_2} [t(\omega,\theta) - \theta_1 + \nu_1(\beta_1)I_1] f_1 f_2 d\theta_2 d\theta_1 \right\} \\ & -\beta_1 C_1(I_1) - v_1(a_1), \end{aligned} \tag{A5}$$

where $t(\omega,\theta) = \frac{1}{2}[\sigma_1(\theta_1 \mid \omega) + \sigma_2(\theta_2 \mid \omega)]$. The integral boundaries are given by (A1) and (A2). Taking the first-order derivative with respect to I_1 yields:

$$\begin{aligned}\frac{\partial \phi_1}{\partial I_1} &= \beta_1 \left\{ \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \int_{\theta_2^o(\theta_1|\omega)}^{\overline{\theta}_2} \left[\frac{\partial t(\omega,\theta)}{\partial I_1} + \nu_1(\beta_1) \right] f_1 f_2 d\theta_2 d\theta_1 \right. \\ & \qquad \left. - \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \frac{\partial \theta_2^o(\theta_1 \mid \omega)}{\partial I_1} [\sigma_1(\omega,\theta_1) - \theta_1 + \nu_1(\beta_1)I_1] f_1 f_2 d\theta_1 - C_1'(I_1) \right\} \\ &= \beta_1 \left[\frac{\nu_1(\beta_1)}{2} Q(\beta_1, I_1) \right. \\ & \qquad \left. + \nu_1(\beta_1) \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \frac{3\overline{\theta}_2 + \underline{\theta}_1 + 3\nu_1(\beta_1)I_1 - 4\theta_1}{16} f_1 f_2 d\theta_1 - C_1'(I_1) \right] \\ &= \frac{3}{4}\beta_1 \nu_1(\beta_1) Q(\beta_1, I_1) - \beta_1 C_1'(I_1). \end{aligned} \tag{A6}$$

Setting this term equal to zero gives the necessary first-order condition for $I_1(\beta_1)$ in (12). Given the technical assumption in footnote 17, the first-order condition is also sufficient as

$$\frac{27}{64}(\nu_1(\beta_1))^2\left[\overline{\theta}_2-\underline{\theta}_1+\nu_1(\beta_1)I_1(\beta_1)\right]\leq C_1''(I_1(\beta_1)), \tag{A7}$$

for all $\beta_1 \geq \beta_1^o$, ensures concavity of Manager 1's investment problem.

To prove the underinvestment result, I only need to show that the seller's marginal returns to investing in I_1 are strictly less than first-best. By standard revealed preference arguments, then $I_1(1) < I_1^*$ will hold. The marginal return to I_1 in the first-best solution equals $(1+b_1)Q^*(I_1)$, by (5). Under delegation, the seller's marginal investment return in (A6), divided by β_1 (recall that Manager 1 also bears only a fraction β_1 of the incremental fixed costs $C_1'(I_1)$), equals $\frac{3}{4}\nu_1(\beta_1)Q(\beta_1, I_1)$. At $\beta_1 = 1$,

$$\frac{3}{4}\nu_1(1)Q(1,I_1)=\frac{3}{4}(1+b_1)Q(1,I_1)<(1+b_1)Q^*(I_1),$$

because $\nu_1(1) = 1 + b_1$ and $Q(1, I_1) < Q^*(I_1)$. Manager 1's marginal investment return is strictly less than first-best and he will therefore underinvest. *Q.E.D.*

Proof of Proposition 2: The proof proceeds by applying the implicit function theorem to Manager 1's first-order investment condition [30]

$$\frac{\partial U_1}{\partial I_1}=\beta_1\left[\frac{3}{4}\nu_1(\beta_1)Q(\beta_1,I_1(\beta_1))-C_1'(I_1(\beta_1))\right]=0.$$

Provided the concavity condition in (A7) is satisfied, we have

$$sign[I_1'(\beta_1)]=sign\left[\left.\frac{\partial^2 U_1}{\partial I_1\partial\beta_1}\right|_{I_1(\beta_1)}=\frac{3}{4}\beta_1\left(\frac{\partial\nu_1}{\partial\beta_1}Q(\beta_1,I_1(\beta_1))+\nu_1(\beta_1)\left.\frac{\partial Q}{\partial\beta_1}\right|_{I_1(\beta_1)}\right)\right].$$

Now, $\partial\nu_1/\partial\beta_1 = -b_1/\beta_1^2$ and, by (A4), $\partial Q/\partial\beta_1 \leq 0$. It follows that $I_1'(\beta_1) \leq 0$. *Q.E.D.*

Proof of Proposition 3: First, as apparent from Lemma 1, β_2 does not affect the parties' equilibrium bidding strategies, $(\sigma_1(\cdot), \sigma_2(\cdot))$, given that only the seller invests. Instead, β_2 only affects the effort chosen by Manager 2; hence Principal 2 will optimally set $\beta_2 = 1$ so as to induce first-best effort.

[30] Note that Manager 1's objective function U_i in (9) does not have increasing differences in I_1 and β_1 everywhere; thus the simpler methods of monotone comparative statics do not apply here.

Now consider Firm 1. For any bonus coefficient, β_1, Manager 1 will choose his effort and investment $(a_1(\beta_1), I_1(\beta_1))$ so as to maximize U_1 in (9). The first-order conditions describing the seller's choices are given by $v_1'(a_1(\beta_1)) = \beta_1$ and by (A6), above. Under non-integration (NI), Principal 1 sets β_1 so as to maximize

$$\Pi_1^{NI}(\beta) \equiv a_1(\beta_1) - v_1(a_1(\beta_1)) + \psi_1(\cdot),$$

where the new function

$$\begin{aligned}
\psi_1(\cdot) &\equiv E_\theta\{[M_1(\cdot) + b_1 I_1(\beta_1)]q(\omega,\theta)\} - C_1(I_1(\beta_1)) \\
&= \int\limits_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \int\limits_{\theta_2^o(\theta_1|\omega)}^{\overline{\theta}_2} \left[\frac{\sigma_1(\theta_1 \mid \omega) + \sigma_2(\theta_2 \mid \omega)}{2} - \theta_1 + (1+b_1)I_1(\beta_1)\right] f_1 f_2 d\theta_2 d\theta_1 - C_1(I_1(\beta_1))
\end{aligned}$$

captures the expected total investment return to Unit 1, subject to the constraint that Manager 1 chooses effort and investment so as to maximize (A5). Note that the function $\psi_1(\cdot)$ is differentiable almost everywhere in both managers' bids, $\sigma_1(\theta_1 \mid \omega)$ and $\sigma_2(\theta_2 \mid \omega)$, and each bid function, $\sigma_i(\theta_i \mid \omega)$, is differentiable a.e. in β_i.

Note in particular that $\Pi_1^{NI}(1) = \phi_1(a_1(1), I_1(1) \mid 1)$, that is, the objectives of Manager 1 and Principal 1 coincide for $\beta_1 = 1$, for any given bid submitted by the buyer, $\sigma_2(\cdot)$. By the envelope theorem, therefore,

$$\left.\frac{d\Pi_1^{NI}}{d\beta_1}\right|_{\beta_1=1} = \left.\frac{\partial \psi_1}{\partial \sigma_2}\frac{\partial \sigma_2}{\partial \beta_1}\right|_{\beta_1=1}.$$

A change in σ_2, as a consequence of a change in β_1, affects ψ_1 in two ways: via its impact on $t(\omega,\theta)$ and on the cutoff $\theta_2^o(\theta_1 \mid \omega)$ because the seller's payoff from transferring is strictly positive at this cutoff. The change in that cutoff, holding Manager 1's choices constant, is found by applying the implicit function theorem to the right-hand side of the following equation which implicitly defines $\theta_2^o(\theta_1 \mid \omega)$:

$$\begin{aligned}
\sigma_1(\theta_1 \mid \omega) &\equiv \sigma_2(\theta_2^o(\theta_1 \mid \omega) \mid \omega) = \frac{\overline{\theta}_2 + 3\underline{\theta}_1 - 3\nu_1(\beta_1)I_1 + 8\theta_2^o(\theta_1 \mid \omega)}{12} \\
\Rightarrow \left.\frac{\partial \theta_2^o}{\partial \beta_1}\right|_{I_1,\sigma_1} &= -\frac{3b_1 I_1}{8(\beta_1)^2}.
\end{aligned}$$

Using this yields:

$$\left.\frac{d\Pi_1^{NI}}{d\beta_1}\right|_{\beta_1=1} = \frac{1}{2}\frac{\partial \sigma_2}{\partial \beta_1}Q(1, I_1(1)) + \frac{3b_1 I_1(1)}{8} \int\limits_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} [\sigma_1(\theta_1 \mid \omega) - \theta_1 + (1+b_1)I_1(1)] f_1 f_2 d\theta_1$$

$$= \quad \frac{b_1 I_1(1)}{8} Q(1, I_1(1)) + \frac{3b_1 I_1(1)}{8} \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \frac{3\overline{\theta}_2 + \underline{\theta}_1 + 3(1+b_1)I_1(1) - 4\theta_1}{12} f_1 f_2 d\theta_1$$

$$= \quad \frac{1}{4} b_1 I_1(1) Q(1, I_1(1)) \; > \; 0,$$

where the last equality uses (A3). Hence, Principal 1 optimally sets $\beta_1^{NI} > 1$. *Q.E.D.*

Proof of Proposition 4: Under integration (Program $\mathcal{P}^{\mathcal{INT}}$), the divisional managers' bidding strategies for publicly known information, ω, and any privately observed type, θ_i, are again independent of β_2. Moreover, Manager 1's investment and effort problem is given by (A5) and hence coincides with that under non-integration. However, under integration the Principal's objective function now becomes (ignoring Manager 2's effort and disutility):

$$\Pi^{INT}(\beta_1) \;\; \equiv \;\; a_1(\beta_1) - v_1(a_1(\beta_1)) - C_1(I_1(\beta_1)) \qquad \text{(A8)}$$
$$+ \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \int_{\theta_2^o(\theta_1|\omega)}^{\overline{\theta}_2} \left[\theta_2 - \theta_1 + (1+b_1)I_1(\beta_1)\right] f_1 f_2 d\theta_2 d\theta_1,$$

subject to (A5).

To evaluate the first-order derivative of Π^{INT} with respect to β_1 at $\beta_1 = 1$, use Leibnitz' rule together with the fact that $\theta_2^o(\theta_1^o(\overline{\theta}_2 \mid \omega) \mid \omega) \equiv \overline{\theta}_2$:

$$\left.\frac{d\Pi^{INT}}{d\beta_1}\right|_{\beta_1=1} = \left[(1+b_1)Q(\beta_1, I_1(\beta_1)) - C_1'(I_1(\beta_1))\right] I_1'(\beta_1)$$
$$-\frac{d\theta_2^o(\theta_1 \mid \omega)}{d\beta_1} \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \left[\theta_2^o(\theta_1 \mid \omega) - \theta_1 + (1+b_1)I_1(\beta_1)\right] f_1 f_2 d\theta_1$$
$$< \; -\frac{d\theta_2^o(\theta_1 \mid \omega)}{d\beta_1} \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \left[\theta_2^o(\theta_1 \mid \omega) - \theta_1 + (1+b_1)I_1(\beta_1)\right] f_1 f_2 d\theta_1$$
$$< \; 0.$$

The first inequality makes use of the necessary first-order investment condition (obtained by setting the term in (A6) equal to zero) together with $I_1'(\beta_1) < 0$ (see proof of Proposition 2, above). The second inequality holds because the integrand is positive and, by (A2):

$$\frac{d\theta_2^o(\theta_1 \mid \omega)}{d\beta_1} = -\frac{3}{4}\left[-\frac{b_1}{(\beta_1)^2} I_1(\beta_1) + \nu_1(\beta_1) I_1'(\beta_1)\right] > 0,$$

given that $I_1'(\beta_1) < 0$. This establishes that the firm-wide profit function $\Pi^{INT}(\beta_1)$ under integration is decreasing in β_1 at $\beta_1 = 1$, which completes the proof. *Q.E.D.*

Proof of Proposition 5: The proof proceeds by backward induction. For any public information $\omega = \{\beta_1, \beta_2, I_1, I_2\}$ (ignoring the effort choices a as they do not affect investment incentives) and any privately observed types θ_i, the managers' optimal bidding strategies solve (6) and (13), respectively. Performing a simple change in variables in the proof of Lemma 1, the linear Bayesian-Nash equilibrium can be restated as:

$$\sigma_1(\theta_1 \mid \omega) = \begin{cases} \tilde{\sigma}_1(\theta_1 \mid \omega) \equiv \frac{3\bar{\theta}_2 + \underline{\theta}_1 - 9\nu_1(\beta_1)I_1 + 3\nu_2(\beta_2)I_2 + 8\theta_1}{12}, & \text{if } \tilde{\sigma}_1(\theta_1 \mid \omega) \leq \tilde{\sigma}_2(\bar{\theta}_2 \mid \omega), \\ \theta_1 - \nu_1(\beta_1)I_1, & \text{if } \tilde{\sigma}_1(\theta_1 \mid \omega) > \tilde{\sigma}_2(\bar{\theta}_2 \mid \omega), \end{cases}$$

$$\sigma_2(\theta_2 \mid \omega) = \begin{cases} \tilde{\sigma}_2(\theta_2 \mid \omega) \equiv \frac{\bar{\theta}_2 + 3\underline{\theta}_1 - 3\nu_1(\beta_1)I_1 + 9\nu_2(\beta_2)I_2 + 8\theta_2}{12}, & \text{if } \tilde{\sigma}_2(\theta_2 \mid \omega) \geq \tilde{\sigma}_1(\underline{\theta}_1 \mid \omega), \\ \theta_2 + \nu_2(\beta_2)I_2, & \text{if } \tilde{\sigma}_2(\theta_2 \mid \omega) < \tilde{\sigma}_1(\underline{\theta}_1 \mid \omega). \end{cases}$$

where $\nu_i(\beta_i) \equiv (1 + b_i/\beta_i)$, $i = 1, 2$. As in the one-sided investment setting, $q(\omega, \theta) = 1$ and $t(\omega, \theta) = \frac{1}{2}[\sigma_1(\cdot) + \sigma_2(\cdot)]$, if $\sigma_2(\theta_2 \mid \omega) \geq \sigma_1(\theta_1 \mid \omega)$; otherwise $q(\omega, \theta) = t(\omega, \theta) = 0$.

For any incentive weights β, the managers, in equilibrium, choose their efforts and investments so as to maximize their respective payoffs. That is, $(a_i(\beta_i), I_i(\beta))$ maximize (ignoring fixed salaries):

$$U_i(a_i, I \mid \beta) = \beta_i \left[a_i + \int_{\underline{\theta}_1}^{\theta_1^o(\bar{\theta}_2|\omega)} \int_{\theta_2^o(\theta_1|\omega)}^{\bar{\theta}_2} \left(M_i(\cdot) + \frac{b_i}{\beta_i} I_i \right) f_1 f_2 d\theta_2 d\theta_1 \right] - \beta_i C_i(I_i) - v_i(a_i),$$

for $i = 1, 2$, where:

$$\begin{aligned} \theta_1^o(\bar{\theta}_2 \mid \omega) &= \frac{1}{4}[3\bar{\theta}_2 + \underline{\theta}_1 + 3\left(\nu_1(\beta_1)I_1 + \nu_2(\beta_2)I_2\right)], \\ \theta_2^o(\theta_1 \mid \omega) &= \theta_1 + \frac{1}{4}[\bar{\theta}_2 - \underline{\theta}_1 - 3\left(\nu_1(\beta_1)I_1 + \nu_2(\beta_2)I_2\right)], \\ M_1(\cdot) &= t(\beta, I(\beta), \theta) - \theta_1 + I_1, \\ M_2(\cdot) &= \theta_2 + I_2 - t(\beta, I(\beta), \theta). \end{aligned}$$

The resulting necessary first-order conditions are: $v_i'(a_i(\beta_i)) = \beta_i$, and

$$\frac{\partial U_i}{\partial I_i} = \frac{3}{4}\nu_i(\beta_i) Q(\beta, I_i(I_j \mid \beta), I_j) - C_i'(I_i(I_j \mid \beta)) = 0, \tag{A9}$$

for any conjectured investment I_j by the respective other manager.

I now present a sufficient condition for existence of a unique Nash equilibrium in investments:

Lemma A1. For any $\beta \in \mathcal{B}$, a unique Nash equilibrium $I(\beta)$ exists, if the following holds for $X \equiv \frac{27}{64}\left[\overline{\theta}_2 - \underline{\theta}_1 + \sum_{k=1}^{2} \nu_k(\beta_k) I_k(\beta)\right] f_1 f_2$:

$$\nu_i(\beta_i)[\nu_1(\beta_1) + \nu_2(\beta_2)]X \leq C_i''(I_i(\beta)), \; i = 1, 2. \tag{A10}$$

Proof: First, note that (A10) implies concavity of Manager i's investment problem $I_i(I_j \mid \beta)$ for any given I_j. To see this, differentiate the first-order condition in (A9) once more with respect to I_i:

$$\frac{27}{64}[\nu_i(\beta_i)]^2 \left(\overline{\theta}_2 - \underline{\theta}_1 + \sum_{k=1}^{2} \nu_k(\beta_k) I_k(\beta)\right) f_1 f_2 - C_i''(I_i(\beta)) = [\nu_i(\beta_i)]^2 X - C_i''(I_i(\beta)) \leq 0,$$

which is indeed negative by (A10). To prove Lemma A1 it suffices to show that the Hessian

$$H = \begin{bmatrix} \frac{\partial^2 U_1}{\partial I_1^2} & \frac{\partial^2 U_1}{\partial I_1 \partial I_2} \\ \frac{\partial^2 U_2}{\partial I_1 \partial I_2} & \frac{\partial^2 U_2}{\partial I_2^2} \end{bmatrix}$$

has a dominant diagonal—that is, $|\partial^2 U_i / \partial I_i^2| \geq |\partial^2 U_i / \partial I_1 \partial I_2|$, for $i = 1, 2$. This inequality can be rewritten as

$$C_i''(I_i(\beta)) - [\nu_i(\beta_i)]^2 X \geq \nu_1(\beta_1)\nu_2(\beta_2) X,$$

which coincides with the condition in (A10). *Q.E.D.*

Next, I show that the Nash equilibrium in investments is decreasing in each divisional manager's effort incentives:

Lemma A2. For any $\beta_i, \beta_j, \tilde{\beta}_j \in \mathcal{B}$ with $\tilde{\beta}_j > \beta_j$, $i, j \in \{1, 2\}$: $I(\beta_i, \beta_j) > I(\beta_i, \tilde{\beta}_j)$.

Proof: First, note that the date-2 investment game in which for given β each Manager i chooses I_i so as to maximize $U_i(a_i, I \mid \alpha, \beta)$ is not supermodular as the payoff functions fail to exhibit everywhere increasing differences in (I_i, β_i). Therefore, conventional "local" comparative statics methods have to be employed.

The following preliminaries will prove useful below:

$$\left.\frac{\partial^2 U_i}{\partial I_i \partial I_j}\right|_{I(\beta)} = \frac{3}{4}\beta_i \nu_i(\beta_i) \frac{\partial Q}{\partial I_j} > 0 \tag{A11}$$

$$\left.\frac{\partial^2 U_i}{\partial I_i \partial \beta_j}\right|_{I(\beta)} = \frac{3}{4}\beta_i \nu_i(\beta_i)\frac{\partial Q}{\partial \beta_j} < 0 \tag{A12}$$

$$\begin{aligned}\left.\frac{\partial^2 U_i}{\partial I_i \partial \beta_i}\right|_{I(\beta)} &= \frac{3}{4}\nu_i(\beta_i)Q(\omega) - C_i'(I_i(\beta)) + \frac{3}{4}\left(\frac{\partial \nu_i(\beta_i)}{\partial \beta_i}Q(\omega) + \nu_i(\beta_i)\frac{\partial Q}{\partial \beta_i}\right)\\ &= \frac{3}{4}\left(\frac{\partial \nu_i(\beta_i)}{\partial \beta_i}Q(\omega) + \nu_i(\beta_i)\frac{\partial Q}{\partial \beta_i}\right) < 0, \end{aligned}\tag{A13}$$

where the last equality makes use of the first-order condition (A9).

First consider the comparative statics with respect to a change in β_1. Totally differentiating the first-order conditions in (A9) yields the following system:

$$\begin{bmatrix} \frac{\partial^2 U_1}{\partial I_1^2} & \frac{\partial^2 U_1}{\partial I_1 \partial I_2} \\ \frac{\partial^2 U_2}{\partial I_1 \partial I_2} & \frac{\partial^2 U_2}{\partial I_2^2} \end{bmatrix} \begin{bmatrix} \frac{\partial I_1}{\partial \beta_1} \\ \frac{\partial I_2}{\partial \beta_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 U_1}{\partial I_1 \partial \beta_1} \\ -\frac{\partial^2 U_2}{\partial I_2 \partial \beta_1} \end{bmatrix},$$

where all derivatives are evaluated at $I(\beta)$. Applying Cramer's rule:

$$\frac{\partial I_1}{\partial \beta_1} = \frac{1}{|H|}\left(\frac{\partial^2 U_2}{\partial I_2 \partial \beta_1}\frac{\partial^2 U_1}{\partial I_1 \partial I_2} - \frac{\partial^2 U_1}{\partial I_1 \partial \beta_1}\frac{\partial^2 U_2}{\partial I_2^2}\right).$$

By concavity, $|H| > 0$. Together with (A11)-(A13) this yields $\partial I_1/\partial \beta_1 < 0$. Following similar logic, $\partial I_i/\partial \beta_j$ can be shown to be negative for all $i, j = 1, 2$. *Q.E.D.*

Now consider the date-1 contract offer game between the two principals under non-integration. For any conjectured bonus coefficient β_j, Principal i's optimal response $\beta_i^{NI}(\beta_j)$ maximizes her objective function $\Pi_i^{NI}(\beta)$ in program $\mathcal{P}^{\mathcal{NI}}$ in the main text, subject to the constraints in (6), (10), (13) and (14). The Principal of the vertically integrated firm, in contrast, chooses $(\beta_1^{INT}, \beta_2^{INT})$ jointly so as to maximize Π^{INT} in program $\mathcal{P}^{\mathcal{INT}}$, subject to the same constraints.

Ignoring effort-related variables, let

$$\begin{aligned}\psi_1(\beta) &\equiv E_\theta\left\{\left[\frac{\sigma_1(\theta_1 \mid \omega) + \sigma_2(\theta_2 \mid \omega)}{2} - \theta_1 + (1 + b_1)I_1(\beta)\right]q(\omega, \theta)\right\} - C_1(I_1(\beta)),\\ \psi_2(\beta) &\equiv E_\theta\left\{\left[\theta_2 + (1 + b_2)I_2(\beta) - \frac{\sigma_1(\theta_1 \mid \omega) + \sigma_2(\theta_2 \mid \omega)}{2}\right]q(\omega, \theta)\right\} - C_2(I_2(\beta)),\end{aligned}$$

respectively, denote the surplus generated by each unit from investing and transferring the good. Now, denote by $\beta_i^{NI}(\beta_j)$ the solution to the problem:

$$\max_{\beta_i}\ \psi_i(\beta),\ \text{subject to: (6), (10), (13), (14).}$$

Similarly, denote by $\beta_i^{INT}(\beta_j)$ the solution to:

$$\max_{\beta_i} \{\psi^{INT} \equiv \psi_1(\beta) + \psi_2(\beta)\}, \text{ subject to: (6), (10), (13), (14).}$$

By revealed preference, for $\beta^{INT} < \beta^{NI}$ to hold as claimed in Proposition 5, I therefore only need to show that $\partial\psi_j/\partial\beta_i \leq 0,\ i \neq j$. To see that this is true, consider any equilibrium investments $I(\beta)$ and subsequent bargaining strategies $\{\sigma_i(\theta_i \mid \omega)\}_i$. Suppose now that Manager i's bonus coefficient is lowered to $\beta_i - \varepsilon,\ \varepsilon > 0$. Holding fixed $I_j(\beta)$ and $\sigma_j(\theta_j \mid \beta, I(\beta))$, Manager j's payoff will increase because (i) I_i increases (Lemma A2) and (ii) Manager i bids less aggressively at the trading stage; therefore, $Q(\omega)$ will increase and the transfer price, if $q = 1$, will be more favorable to Manager j. By revealed preference, Manager j (and therefore Principal j) can do even better by readjusting his investment and bidding choices in an optimal fashion.

Thus, for any β_j, $\beta_i^{NI}(\beta_j) > \beta_i^{INT}(\beta_j)$. By the assumed uniqueness of the Nash equilibrium/profit maximum, $\beta^{NI} > \beta^{INT}$ holds. Finally, applying Lemma A2 yields $I^{NI} \equiv I(\beta^{NI}) < I^{INT} \equiv I(\beta^{INT})$, completing the proof. *Q.E.D.*

Proof of Proposition 6: I begin by deriving the first-best benchmark solution for the general case where opportunities exists for investing in physical assets, I_i, as well as in personally costly skill-acquisition, e_i. In the absence of informational asymmetries, the Principal of a vertically integrated firm chooses $(\widehat{a}^*, \widehat{e}^*, \widehat{I}^*)$ as the solution to:

$$\max_{a,e,I} \left\{ E_\theta \left[\left(\theta_2 - \theta_1 + \sum_{i=1}^{2} [e_i + (1+b_i)I_i] \right) \widehat{q}^*(\theta, I, e) \right] + \sum_{i=1}^{2} [a_i - v_i(a_i) - D_i(e_i) - C_i(I_i)] \right\},$$

subject to:

$$\widehat{q}^*(\theta, I, e) = 1, \quad \text{if, and only if,} \quad \theta_2 - \theta_1 + \sum_{i=1}^{2} [e_i + (1+b_i)I_i] \geq 0.$$

Assuming a unique interior optimum, the corresponding necessary and sufficient first-order conditions are

$$D_i'(\widehat{e}_i^*) = \frac{C_i'(\widehat{I}_i^*)}{1+b_i} = \widehat{Q}^*(\widehat{I}^*, \widehat{e}^*),$$

where $\widehat{Q}^*(I, e) \equiv Pr[\widehat{q}^*(\theta, I, e) = 1]$.

Now return to the case where only Manager i observes θ_i after choosing (a_i, I_i, e_i). Performing a change in variables in the proof of Lemma 1, given Assumption 1$''$ yields the following linear Bayesian-Nash equilibrium for any previously determined (α, β, a, I, e):

$$\widehat{\sigma}_1(\theta_1 \mid \omega) = \begin{cases} \widehat{\sigma}_1^o(\theta_1 \mid \omega) \equiv \frac{3\bar{\theta}_2 + \underline{\theta}_1 - 9\gamma_1 + 3\gamma_2 + 8\theta_1}{12}, & \text{if } \widehat{\sigma}_1^o(\theta_1 \mid \omega) \leq \widehat{\sigma}_2^o(\bar{\theta}_2 \mid \omega), \\ \theta_1 - \nu_1(\beta_1) I_1, & \text{if } \widehat{\sigma}_1^o(\theta_1 \mid \omega) > \widehat{\sigma}_2^o(\bar{\theta}_2 \mid \omega), \end{cases}$$

$$\widehat{\sigma}_2(\theta_2 \mid \omega) = \begin{cases} \widehat{\sigma}_2^o(\theta_2 \mid \omega) \equiv \frac{\bar{\theta}_2 + 3\underline{\theta}_1 - 3\gamma_1 + 9\gamma_2 + 8\theta_2}{12}, & \text{if } \widehat{\sigma}_2^o(\theta_2 \mid \omega) \geq \widehat{\sigma}_1^o(\underline{\theta}_1 \mid \omega), \\ \theta_2 + \nu_2(\beta_2) I_2, & \text{if } \widehat{\sigma}_2^o(\theta_2 \mid \omega) < \widehat{\sigma}_1^o(\underline{\theta}_1 \mid \omega), \end{cases}$$

where $\gamma_i \equiv \nu_i(\beta_i) I_i + e_i$ and $\nu_i(\beta_i) \equiv (1 + b_i/\beta_i)$, $i = 1, 2$. The units' respective contribution margins if trade occurs are $\widehat{M}_1(\cdot) \equiv t - \theta_1 + I_1 + e_1$ and $\widehat{M}_2(\cdot) \equiv \theta_2 + I_2 + e_2 - t$. Generalizing the cutoffs in (A1) and (A2), we have $\theta_1^o(\overline{\theta}_2 \mid \omega) = \frac{1}{4}[3\overline{\theta}_2 + \underline{\theta}_1 + 3(\gamma_1 + \gamma_2)]$, and $\theta_2^o(\theta_1 \mid \omega) = \theta_1 + \frac{1}{4}[\overline{\theta}_2 - \underline{\theta}_1 - 3(\gamma_1 + \gamma_2)]$. The expected surplus (cum empire benefits) generated by Unit i equals

$$\widehat{\psi}_i(\beta) \equiv E_\theta \left\{ \left[\widehat{M}_i(\theta, I(\beta), e(\beta), t(\omega, \theta)) + b_i I_i(\beta) \right] \widehat{q}(\omega, \theta) \right\} - C_i(I_i(\beta)) - D_i(e_i(\beta)).$$

Recall that, by assumption, only the seller can invest in capital assets in Proposition 6 (i.e., $I_2 = 0$).

Case 1: $C_1'(0) \to \infty$. In this case, both managers only invest in skill acquisition effort, e_i, while $I_1 = I_2 = 0$. Each manager then solves (suppressing general-purpose effort, a_i, and fixed salaries, α_i):

$$e_i(\beta) \in \arg\max_{e_i} \left\{ \widehat{U}_i(e \mid \beta) = \beta_i \int_{\underline{\theta}_1}^{\theta_1^o(\overline{\theta}_2|\omega)} \int_{\theta_2^o(\theta_1|\omega)}^{\overline{\theta}_2} \widehat{M}_i(\cdot) f_1 f_2 d\theta_2 d\theta_1 - D_i(e_i) \right\}.$$

Now I can apply the methods of monotone comparative statics for the date-2 game of skill-acquisition choices, because $\widehat{U}_i$ is supermodular in (e_1, e_2) and has increasing differences in (e_i, β_i, β_j). For $\widehat{X} \equiv \frac{27}{64}[\overline{\theta}_2 - \underline{\theta}_1 + e_1 + e_2] f_1 f_2$, we have:

$$\frac{\partial^2 \widehat{U}_i}{\partial e_i \partial e_j} = \beta_i \widehat{X} > 0; \quad \frac{\partial^2 \widehat{U}_i}{\partial e_i \partial \beta_i} = \widehat{X} > 0; \quad \frac{\partial^2 \widehat{U}_i}{\partial e_i \partial \beta_j} = 0.$$

Hence, the (assumed unique) Nash equilibrium $e(\beta)$ is monotonically non-decreasing in β_i, $i = 1, 2$.

It only remains to be shown that the Principal will indeed set $\beta_i > 1$. This immediately follows from similar arguments as in the proof of Proposition 4: At $\beta_i = 1$, Manager i who maximizes $\widehat{U}_i$ at the same time maximizes $\widehat{\psi}_i$. Thus, by the envelope theorem, $\partial\Pi^{INT}/\partial\beta_i|_{\beta_i=1} = \partial\widehat{\psi}_j/\partial\beta_i|_{\beta_i=1}$. The latter term is clearly positive due to the positive externality the attendant increase in $e_i(\beta)$ has on $\widehat{\psi}_j$.

Case 2: $D_1'(0) \to \infty$. I only sketch this proof as it closely resembles that of Proposition 5. In this case, $I_2 = e_1 = 0$. Since $\widehat{U}_1$ again fails to exhibit increasing differences everywhere in (I_1, β_1), the relevant cross partials have to be evaluated at the Nash equilibrium $(I_1(\beta), e_2(\beta))$:

$$\frac{\partial^2\widehat{U}_1}{\partial I_1\partial e_2} > 0; \quad \frac{\partial^2\widehat{U}_1}{\partial I_1\partial \beta_2} > 0; \quad \frac{\partial^2\widehat{U}_1}{\partial I_1\partial \beta_1} < 0; \quad \frac{\partial^2\widehat{U}_2}{\partial I_1\partial e_2} > 0; \quad \frac{\partial^2\widehat{U}_2}{\partial e_2\partial \beta_1} < 0; \quad \frac{\partial^2\widehat{U}_2}{\partial e_2\partial \beta_2} > 0.$$

Applying Cramer's rule then shows that this (assumed unique) Nash equilibrium is non-increasing in β_1 and non-decreasing in β_2. The remainder of the proof is identical to that of Proposition 5. *Q.E.D.*

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FIGURE 1
TIMELINE

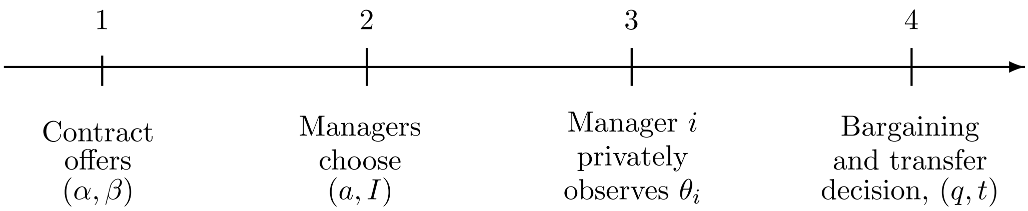

FIGURE 2
BIDDING STRATEGIES FOR GIVEN INVESTMENT AND $\beta_1^{oo} > \beta_1^{o}$

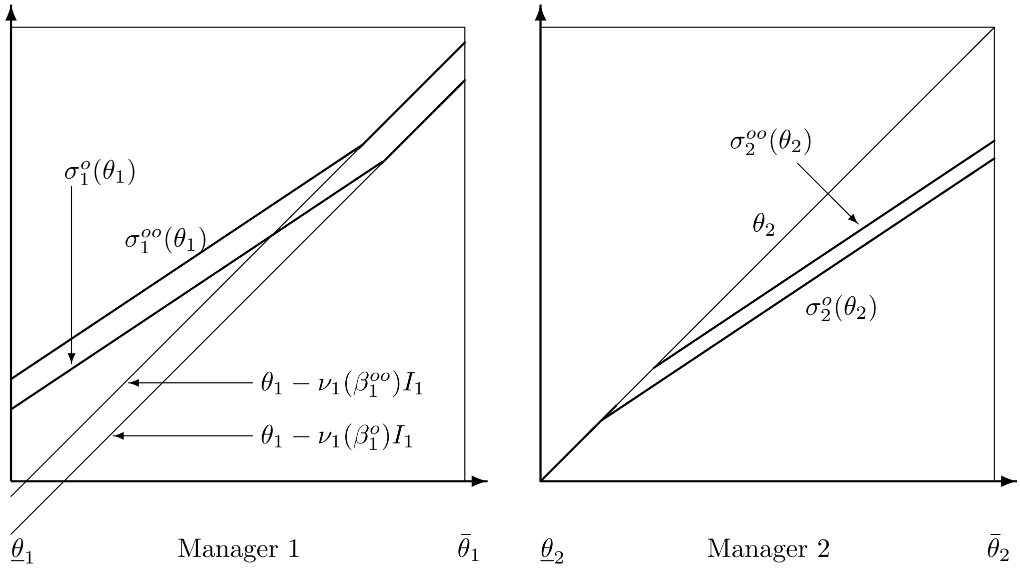

FIGURE 3
MANAGER 2's BIDDING STRATEGY: THE HOLD-UP PROBLEM