Research Note

Some Empirical Regularities in Market Shares

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We present some empirical regularities in the market shares of brands. Our cross-sectional data on market shares consists of 1,171 brands in 91 product categories of foods and sporting goods sold in the United States. One of our results is that the pattern of market shares for each of the categories (many of which are fundamentally dissimilar, such as breakfast cereals and rifles) is represented well by the power law. The power law also does better than an alternative model—namely, the exponential form—which has previously been studied in the literature but without having been compared to any alternative. These two models have sharply different implications; for example, the power law predicts that the ratio of market shares for two successively ranked brands becomes smaller as one progresses from higher-ranked to lower-ranked brands, whereas the exponential form predicts that this ratio is a constant. Our findings have several managerial and research implications, which we summarize.

Key words: marketing; product policy; competitive strategy

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1. Introduction

This report presents some empirical findings concerning patterns of market shares of brands and summarizes their implications. Our data consist of 506 brands in 48 product categories of foods and 665 brands in 43 product categories of sporting goods sold in the United States. In the spirit of Bass (1995, p. G7), Ehrenberg (1982, 1995), and Simon (1968), our analysis is descriptive rather than causal. The power law is a central organizing concept of our analysis. It says that if brand \( j \) has the \( r \)th highest market share \( s_j \), then \( s_j = A(a + r_j)^{-b} \), where \( A \), \( a \), and \( b \) are constants. A testable alternative to this model is the exponential form, \( s_j = Ge^{-gr_j} \), where \( G \) and \( g \) are constants, which Buzzell (1981) tested in an important study but without comparing it to any alternative specification. These two models have sharply different implications. For example, the power law predicts that the ratio of market shares for two successively ranked brands becomes smaller as one progresses from higher-ranked to lower-ranked brands, whereas the exponential form predicts that this ratio is a constant.

Our main empirical findings, when product categories are considered one at a time, are as follows: (1) The power law holds very well in an absolute sense; the \( R^2 \) values are consistently greater than 0.90 and often greater than 0.95. (2) The power law describes the data significantly better than the exponential form does, taking into account the fact that the former has one more parameter than the latter. (3) The relative superiority of the power law over the exponential form is greater for product categories that have lower values of \( b \). (4) The exponential form fits better for lower market shares than for higher market shares.

Contributions

By definition, the market share and its rank are monotonically related; a higher-ranked brand has a larger market share. However, within a given product category, this relationship can a priori take any one of an infinite number of shapes: no functional form at all or any of an infinite number of possible functional forms, including linear, concave, convex, or a combination of these. As the drivers of demand can be vastly different in different product categories, the above monotonic relationship can, a priori, differ across product categories. Despite these possibilities, we show that the same relationship is repeated across product categories, and it is well approximated by the power law, which, as mentioned earlier, has specific
implications. The exponential form is a natural benchmark because it has the desired monotonicity, it has been studied in the literature in the same context, and it has different implications.

To our knowledge, this report is the first to examine the power law for a large number of product categories, rather than for just one or another product category. It is also the first to compare, for one or more product categories, the power law with any alternative model, which, in the present report, is the exponential form. There are potential disadvantages of examining the power law while limiting oneself to just one product category, for example, because the presence of the power law may then be attributed to the special characteristics of that particular category, without perhaps recognizing its widespread prevalence. Examples of the application of the power law, or its special cases, to individual product categories are Chung and Cox (1994) on the number of hit albums produced by a music group, Adamic and Huberman (2000) on the number of links to a website, and Kalyanaram et al. (1995) on a relationship between market share and the order of entry of firms producing prescription anti-ulcer drugs and certain packaged consumer goods.

In addition to the above results, in which each product category is considered one at a time, we also find some previously unreported patterns in the market shares across product categories. The latter results are quite counterintuitive, because, for example, on the face of it, categories such as soccer shoes and handguns are highly dissimilar. Because of space constraints, these across-categories results are not reported here but in Kohli and Sah (2006), which also contains an exploratory theoretical framework to help understand the observed patterns, both within and across product categories. The foundation of this model, adapted from Hill (1974) and related work, is that the brand choices are made by consumers according to special cases of the Dirichlet-multinomial paradigm. These special cases include but are not limited to Bose-Einstein statistic, which implies that each possible vector of \( N \) purchases of \( n \) brands has the same probability of occurrence. The Bose-Einstein statistic is also obtained from the Hendry System, when there are no partitions (Rubinson et al. 1980). Hill’s model simultaneously considers a large number of product categories. It assumes that that the number of within-category brands is a random variable, that purchases are independent across categories, and that the ratio \( n/N \) is independent across categories. Ijiri and Simon (1975) provide an alternative derivation of a power law starting with a Bose-Einstein characterization of purchases and allowing the entry of new brands in a product category.

2. Data, Methods, and Empirical Results

Preliminaries

Consider one product category with \( n \) brands. Assign the index \( j = 1, 2, 3, \ldots, n \), to brands in nonincreasing order of market shares. Let \( s_j \) denote the market share of brand \( j \) and \( r_j \) its market-share rank. Thus, in the absence of ties in market shares, \( r_j = j \) for all \( j \), and the index value \( j = 1 \) is assigned to the highest ranked brand, which has the largest market share. The power law and the exponential form are, respectively:

\[
s_j = A(a + r_j)^{-b} \tag{1}
\]

\[
s_j = Ge^{-b r_j} \tag{2}
\]

Define the “share ratio” of two successively ranked brands as \( f_j \equiv s_j/s_{j+1} \). We assume that \( A > 0, a > -1, \) and \( b > 0 \) for the power law, and that \( G > 0 \) and \( g > 0 \) for the exponential form. These inequalities ensure that \( s_j > 0 \) and \( f_j > 1 \). The share ratio is \( f_j = [1 + (1/(a + r_j))]^b \) for the power law and \( f_j = e^{b r_j} \) for the exponential form. It follows that \( f_j > f_{j+1} \) for the power law and \( f_j = f_{j+1} \) for the exponential form. As noted earlier, this is a crucial difference between the empirical contents of (1) and (2).

Data

We examine two sets of data. The first, made available by Nielsen Market Research, reports the market shares of 506 brands in 48 product categories of foods. These market shares are for a large urban market in the southwestern United States, aggregated over the 120 weeks from January 1993 to May 1995. The number of brands in each of these food categories is displayed in Table 1, in descending order of the number of brands. The names of the product categories are withheld on the advice of those providing the data. The second data set is published by the Sporting Goods Association of America. It contains the market shares in the United States for 665 brands in 43 product categories of sporting goods, aggregated over the 1999 calendar year.

The two data sets reflect the motivations and constraints of those who created them. In brief, the following aspects are noteworthy: (a) The data on foods, collected at the store level for a smaller geographical area, are perhaps more accurate than those on sporting goods. The former data exclude certain types of

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1 Sometimes the power law is expressed as \( s_j = A'(a' + h r_j)^{-b} \), where \( A', a', h, \) and \( b \) are constants. This yields (1) by setting \( A \equiv A'h^{-b} \) and \( a \equiv a'/h \). Expression (1) or its special cases are often referred to, without consistency, as the Pareto Law, the discrete Pareto distribution, and Zipf’s Law. The special case with \( a = 0 \) and \( b = 1 \) has been used extensively in the study of city sizes; see Gabaix (1999). Kalyanaram et al. (1995) use the special case with \( a = 0 \) and \( b = 1/2 \).
results and conclusions are likely to be robust in spite of the unique characteristics or limitations of the data sets. For a discussion on differentiated replication, see Lindsey and Ehrenberg (1993) and Uncles and Wright (2004).  

Estimation Methods

We use two methods to obtain parameter estimates. First, we use a hierarchical Bayes approach to estimate the parameters in a random-coefficients model (RCM), pooling the data across product categories. Second, we obtain parameter estimates for each product category, using the data for only that particular category. The latter, simpler method minimizes the

stores from the Nielsen audits. (b) We do not have a random selection of product categories of foods and sporting goods. We have used all of the data available to us. (c) All market shares are in equivalent (quantity) units, without distinguishing brand variations and stock-keeping units. (d) The data exclude brands with market shares smaller than 1%, for which sampling error is likely to be a problem. (e) As seen in Table 1, the number of brands is small for several product categories. (f) The data do not contain details of individual purchase histories or sales over shorter time spans.

Our two data sets represent a broad range of products in their respective markets. These two markets are quite unrelated, including regarding consumers’ reasons for buying or not buying particular goods or brands and producers’ methods of selling their products. The two data sets differ in the length of time over which the data have been collected: One year for foods and 18 months for sporting goods. The data sets also differ in their geographical coverage: regional for foods and national for sporting goods. Moreover, these two data sets have been constructed by two different organizations under different procedures and with different objectives, without any coordination with each other.

Notwithstanding these key differences, our empirical findings from the two sets of data are very similar. This could be viewed as a partial indication that our results and conclusions are likely to be robust in spite of the unique characteristics or limitations of the data sets. For a discussion on differentiated replication, see Lindsey and Ehrenberg (1993) and Uncles and Wright (2004).  

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**Table 1** Estimates for Each Product Category: Foods

<table>
<thead>
<tr>
<th>No. of brands</th>
<th>Exponential form</th>
<th>Power law</th>
<th>No. of brands</th>
<th>Exponential form</th>
<th>Power law</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g</td>
<td>$R^2$</td>
<td>a</td>
<td>b</td>
<td>$R^2$</td>
</tr>
<tr>
<td>27</td>
<td>0.06</td>
<td>0.84</td>
<td>-0.17</td>
<td>0.59</td>
<td>0.97</td>
</tr>
<tr>
<td>22</td>
<td>0.11</td>
<td>0.91</td>
<td>1.61</td>
<td>1.30</td>
<td>0.97</td>
</tr>
<tr>
<td>21</td>
<td>0.12</td>
<td>0.94</td>
<td>-0.65</td>
<td>0.75</td>
<td>0.91</td>
</tr>
<tr>
<td>20</td>
<td>0.12</td>
<td>0.97</td>
<td>16.92</td>
<td>3.35</td>
<td>0.98</td>
</tr>
<tr>
<td>20</td>
<td>0.15</td>
<td>0.91</td>
<td>1.22</td>
<td>1.47</td>
<td>0.99</td>
</tr>
<tr>
<td>20</td>
<td>0.15</td>
<td>0.95</td>
<td>4.78</td>
<td>2.03</td>
<td>0.99</td>
</tr>
<tr>
<td>20</td>
<td>0.12</td>
<td>0.92</td>
<td>2.35</td>
<td>1.31</td>
<td>0.95</td>
</tr>
<tr>
<td>19</td>
<td>0.13</td>
<td>0.89</td>
<td>0.40</td>
<td>1.06</td>
<td>0.99</td>
</tr>
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<td>0.18</td>
<td>0.96</td>
<td>3.66</td>
<td>2.00</td>
<td>0.99</td>
</tr>
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<td>0.98</td>
<td>17.04</td>
<td>4.52</td>
<td>0.99</td>
</tr>
<tr>
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<td>0.97</td>
<td>9.56</td>
<td>3.41</td>
<td>0.98</td>
</tr>
<tr>
<td>13</td>
<td>0.25</td>
<td>0.90</td>
<td>0.87</td>
<td>1.63</td>
<td>0.98</td>
</tr>
<tr>
<td>13</td>
<td>0.23</td>
<td>0.96</td>
<td>4.40</td>
<td>2.38</td>
<td>0.98</td>
</tr>
<tr>
<td>12</td>
<td>0.20</td>
<td>0.90</td>
<td>0.75</td>
<td>1.23</td>
<td>0.98</td>
</tr>
<tr>
<td>12</td>
<td>0.24</td>
<td>0.90</td>
<td>0.31</td>
<td>1.33</td>
<td>0.98</td>
</tr>
<tr>
<td>12</td>
<td>0.29</td>
<td>0.99</td>
<td>28.58</td>
<td>10.00</td>
<td>0.99</td>
</tr>
<tr>
<td>11</td>
<td>0.30</td>
<td>0.94</td>
<td>28.08</td>
<td>10.00</td>
<td>0.94</td>
</tr>
<tr>
<td>11</td>
<td>0.30</td>
<td>0.99</td>
<td>27.33</td>
<td>10.00</td>
<td>0.99</td>
</tr>
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<td>1.98</td>
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<td>0.98</td>
</tr>
<tr>
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<td>0.90</td>
<td>0.89</td>
<td>1.62</td>
<td>0.94</td>
</tr>
<tr>
<td>10</td>
<td>0.38</td>
<td>0.83</td>
<td>-0.61</td>
<td>1.28</td>
<td>0.97</td>
</tr>
<tr>
<td>10</td>
<td>0.40</td>
<td>0.97</td>
<td>19.95</td>
<td>10.00</td>
<td>0.96</td>
</tr>
<tr>
<td>10</td>
<td>0.41</td>
<td>0.97</td>
<td>19.29</td>
<td>10.00</td>
<td>0.97</td>
</tr>
<tr>
<td>9</td>
<td>0.35</td>
<td>0.93</td>
<td>4.57</td>
<td>3.20</td>
<td>0.93</td>
</tr>
</tbody>
</table>
sums of squared errors (MSSE) and provides unbiased parameter estimates. Unlike the RCM, it does not require the assumption that the category-specific parameters are random draws from a relevant population distribution. However, it can give larger standard errors of estimates than the RCM, especially for those product categories that have very few brands. Notably, the results from both empirical methods suggest that the power law is a substantially better fit to the data than is the exponential form. For example, the values of the Bayes factors, which are commonly used for comparing such nonnested models (and here take into account the fact that the power law has one more parameter than the exponential form), are $e^{263}$ for the product categories of foods and $e^{425}$ for those of sporting goods, showing an overwhelming superiority of the power law over the exponential form. We describe here the results of the MSSE estimation for the foods. Details of the MSSE estimates for sporting goods, as well as of the RCM estimates for foods and sporting goods, are in Kohli and Sah (2006).

We use a nonlinear procedure to estimate the parameters of the power law by rewriting (1) as $\ln s_j = \ln A - b \ln(a + r_j)$. For the exponential form, we linearly estimate the parameters by rewriting (2) as $\ln s_j = \ln G - g r_j$. In certain cases, the market shares in a product category are identical up to two decimal places. In such cases, we assign the same average rank to these tied data points.

The natural constraints on market shares are that each of them be nonnegative and that they satisfy the adding-up condition that their sum is at most unity. The results presented here are those without building in these constraints into the estimations; among the reasons to do so are the following: Without the constraints, the predicted market share of each brand is positive and less than unity. Also, without the constraints, each of an overwhelming proportion of our product categories (84 of 91) satisfies the adding-up condition.$^3$

For some product categories, the $R^2$ values for the power law estimates increase monotonically with the value of $b$, and this increase is nearly imperceptible for values of $b$ larger than 10. For example, an increase in the value of $b$ from 10 to 50 typically increases the $R^2$ value by less than 0.01, on base values of $R^2$ generally in excess of 0.9. These product categories typically have two distinguishing features: They have fewer brands and they yield approximately the same $R^2$ values for the power law and the exponential form. The latter suggests an interpretation in view of Mandelbrot’s (1963) prediction that a power law with sufficiently large values of $b$ approximates an exponential form. The interpretation is that, within the limitations of the data, the power law and the exponential form are roughly equally good descriptions for these product categories. Keeping this in mind, for these product categories, we have used a value of 10 for $b$ in the parameter estimates presented in this report; estimates with larger values of $b$ are available on request from the authors.

**Results**

Table 1 displays the estimated parameters and the corresponding values of $R^2$ for the power law and the exponential form, separately for each of the 48 food categories. These results suggest that the power law holds well in an absolute sense. For example, the value of $R^2$ for the power law is greater than or equal to 0.95 for 37 out of a total of 48 product categories, and it is larger than 0.9 for 44 product categories.$^4$ A value of 1 is displayed for the $R^2$ in some cases in this report because we have rounded off these values to two places after the decimal. Parameter estimates that are statistically significant at the 95% confidence level are shown in boldface in Table 1. The overall picture in this regard is that if a parameter estimate is not significant, then it is typically but not always the case that the corresponding product category has a small number of brands; in these cases, the statistical significance of the parameter estimates is based on only a few degrees of freedom. Error estimates are not applicable for $b$ if its displayed value is 10, given the restriction mentioned earlier.

Note in Table 1 that the estimated parameters are different for different product categories. This is what we would expect. Suppose, to the contrary, that the parameters were the same for two product categories that have the same number of brands. Then the brands with the same rank will have identical market shares across these two categories. This is almost entirely contrary to what we know about market shares.

Figure 1 shows how the $R^2$ values for the power law and the exponential form vary across values of $b$.$^5$

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$^3$ A reason for the seven exceptions is as follows. The estimated market shares, and therefore their sum, are random variables. The sum of the predicted market shares is therefore more likely to exceed unity for a product category for which the sum of the actual market shares is closer to unity. Among the exceptions, the sum of the actual market shares ranges from 0.968 to 0.997. Further, for these exceptions, the quantitative conclusions concerning the power law are unchanged if the adding-up condition is built into the estimations; these estimates are available from the authors. See Deaton and Muellbauer (1980, Chapter 3), in a different context, for literature on whether to constrain the estimation or to obtain unconstrained estimates and then verify the required restrictions. This literature has evolved in favor of the latter approach.

$^4$ For sporting goods, the value of $R^2$ for the power law is greater than or equal to 0.95 for 41 of a total of 43 product categories, and it is larger than 0.9 for all 43 product categories. The number of brands varies from 4 to 37 across the sporting goods categories.

$^5$ The $R^2$ value here refers to the proportion of explained variance in the logarithm of the values of market shares for each product.
The values of $b$ are taken from Table 1, and the product categories are reordered in ascending values of $b$. The numbers displayed on the horizontal axis of this panel are the labels of the product categories, after this reordering. These numbers are in themselves not relevant to what this figure shows. The vertical axis displays the corresponding values of $R^2$ for the power law as well as for the exponential form. This figure shows that, for lower values of $b$, the values of $R^2$ for the power law are substantially larger than those for the exponential form and that the values of $R^2$ from the two models are less distinguishable at higher values of $b$. These findings are consistent with the aforementioned theoretical prediction of Mandelbrot (1963).

Finally, in Figures 2 and 3, we illustrate our earlier observation that the exponential form fits better for lower-ranking market shares than for higher-ranking market shares. We plot in Figure 2 (Figure 3) the actual market shares, and the estimated power law and exponential form for the first (17th) product category in the left-hand (right-hand) panel of Table 1. Note that the product category in Figure 2 has 27 brands, and the product category in Figure 3 has 5 brands. In each case, the fit obtained by the exponential form is markedly better for brands with low market shares, but there is no such visual asymmetry in the fit provided by the power law.

3. Some Implications

Managerial Implications

One view in marketing is that managers place too much emphasis on market share rather than on other fundamentals of brand performance (Jacobson and Aaker 1985). Our results suggest that such an emphasis may be justifiable for high-share brands, where there are large, discrete gaps in the values of market share. This contrasts with the view that market shares change in incremental steps in response to small changes in advertising, promotions, pricing, or other activities. If the latter were true, one would expect to find no consistent pattern of the kind we obtain, except that, by definition, a higher-ranked brand will have a larger market share. One could say nothing about the relationship between the market shares of successively ranked brands. In contrast, we find a recurrent pattern across product categories: Jumps in market shares are predicted by a power law to a remarkable degree of accuracy. This suggests that, in mature markets, each market has a set of “stable” market-share levels. If this is true, then incremental changes in efforts can produce only short-term changes in market shares. Over time, these will average out and a stable pattern of market shares will emerge. In stable markets these patterns will persist until one firm or another invests large resources to procure the large gains in share that can take it to a higher stable share level. Accordingly, managers seeking sustained gains in market share in stable markets may not be successful by increasing efforts and resources devoted to a brand in small increments.

A related implication concerns the setting of market-share goals, a common task for brand managers. Our results suggest that the smallest target for an increase is the market share that the power law associates with the next-highest rank. Higher possible targets are those associated with higher ranks. These target levels can be useful for allocating resources to a brand. If the next-highest market share is much larger than the current market share for a brand, a firm may need to commit resources for several years, assuming that the power law remains stable over time.

The present results can be useful for assessing the performance of brands and for relating this assessment to marketing actions. Comparing a brand’s current market share against the level predicted for its rank by the power law for the product category can indicate whether it is performing below or above par. If a brand has a higher market share than that predicted by its rank, it might be ready to move up a rank. If it has a lower market share than that predicted by its rank, it might need to be better defended against competitors. This analysis can extend to cross-category comparisons of brands. Thus, a brand with 15% market share in one product category might be a better performer than another brand with 25% market share in another product category if the first is performing above par for its rank and the second is not. Managers can use such information when allocating resources across brands in different product categories. Financial analysts can use it to evaluate the performance of brands in different product categories.
The above discussion is only appropriate for stable markets. In such cases, our results can also be useful for assessing potential instabilities in markets. For example, if two brands in a product category have the same market shares, then the market is likely to be unstable—one or another brand will gain or lose market share until a power law pattern reappears. This information can be useful for identifying opportunities and potential threats in an unstable market.

The present results are consistent with the logic of the NBD-Dirichlet multinomial framework (e.g., Ehrenberg et al. 2004). This framework, like the present research, describes stationary markets, albeit with the difference that they use longitudinal rather than cross-sectional data.

Implications for Research
Our focus in this report has been on establishing the existence of an empirical regularity in market shares and establishing the suitability of a power law for representing the regularity. Besides testing the boundaries of the power law, which we discuss earlier in the report, future research might examine if there are systematic patterns across product categories in the parameter values of the power law; if these patterns differ for stable and growing markets; and how the parameter values for a product category, or a group of categories, change over time.

A separate implication of our results is that it may be useful to construct models in which marketing efforts and activities are related to the discrete levels of market shares predicted by a power law. The power law parameters themselves would then be related to such category-level variables as the total advertising expenditures and average prices across brands in the category.

Finally, the marketing literature on pioneering advantage suggests that earlier entrants sustain larger market shares than later entrants. Kalyanaram and coauthors (1995) find a power law relation between
order of entry and market shares. For prescription anti-ulcer drugs and certain packaged consumer goods, they estimate the special case of (1) with \( a = 0 \) and \( b = 1/2 \), where \( r \) is the order of market entry. We find that the power law pattern holds for a vastly larger number of product categories without any reference to the order of entry. This suggests the need to separate the effects of pioneering advantage from the rank-share relations we report in this paper.

**Acknowledgments**

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