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Operational risk and the Solvency II capital aggregation formula: implications of the hidden correlation assumptions

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ABSTRACT

We analyze the Solvency II standard formula (SF) for capital risk aggregation in relation to the treatment of operational risk (OR) capital. We show that the SF implicitly assumes that the correlation between OR and the other risks is very high: a situation that seems to be at odds with both the empirical evidence and the view of most industry participants. We also show that this formula, which somehow obscures the correlation assumptions, gives different insurance companies different benefits for diversification effects in relation to OR. Unfortunately, these benefits are based on the relative weights of the six basic capital components and not on any risk-related metric. Hence, contrary to what has been claimed, the SF does give diversification benefits (although minor ones) in relation to OR. Further, since the SF does not treat the correlation between OR and the other risks explicitly, it provides no incentive to gather data regarding this effect. Given all these considerations, for the time being, we recommend the adoption of the well-known linear aggregation formula, using low-to-moderate correlation assumptions between OR and the other risks.

Keywords: operational risk (OR); correlation; Solvency II (S2) standard formula (SF); capital risk aggregation; diversification benefits.

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1 INTRODUCTION

Solvency II (S2), the new European insurance regulatory framework, came into effect on January 1, 2016. One of its main purposes is to make sure that insurance companies will have enough capital to withstand stressful scenarios.

S2 recognizes six broad types of risk: (1) market risk, (2) default (credit) risk, (3) life underwriting risk, (4) health underwriting risk, (5) nonlife underwriting risk and (6) operational risk (European Insurance and Occupational Pensions Authority 2014a, Section SCR.1). For the purpose of estimating the necessary capital that an insurance company must have (the Solvency Capital Requirement; or, in S2 parlance, the SCR), S2 employs a two-step process. First, it estimates the SCR for each one of the six individual risks mentioned. Second, it aggregates these six figures to arrive at a combined overall SCR.

The goal of this paper is to look at the implications of the aggregation formula proposed by S2 (the second step) in relation to operational risk (OR) and the other risks. That is, we aim to unmask the correlation assumptions between OR and the other risks at the root of the S2 aggregation scheme. Hence, we take as a given the value of the SCR for each of the risks already identified.

2 BACKGROUND

In more precise terms, the aggregation problem described above consists of combining information from six random variables (the SCRs associated with each of the risks) in order to arrive at a single figure of merit: the overall SCR. In principle, this task can be accomplished in a number of ways.

The easiest approach is to resort to the usual linear aggregation expression based on the variance–covariance matrix (Mittnik *et al* 2013; Li *et al* 2015). The simplicity of this approach (its main advantage) is counterbalanced by the fact that oftentimes linear correlation coefficients are insufficient to fully capture the heavy-tail dependences that are critical in loss estimation analyses. Giacometti *et al* (2007) provide a good discussion on the importance of considering heavy-tailed distributions in the context of OR.

At the other extreme, we have the copula-based methods, in which one combines several one-dimensional probability distributions to construct a multidimensional distribution (Embrechts *et al* 2003; Brechmann *et al* 2013, 2014). These methods offer great flexibility at the expense of computational complexity. However, it is not always clear which copula is the best choice, as there are many alternatives, all offering different relative advantages. The use of copula-based aggregation models for operational risk, and, more broadly, some key mathematical issues relevant in general risk aggregation problems, has been addressed in detail by Giacometti *et al* (2008). Of course, regardless of the approach one takes, it is necessary to have an estimate of the correlation that links the random variables.

Our concern is the correlation between OR and the other five risks, or, more formally, between the SCR associated with OR and the other SCRs. Estimating this correlation is challenging for a number of reasons. First, as operational losses are typically sparse, the result has been a scarcity of data (Cruz 2012). A second complication that magnifies the difficulties associated with estimating these correlations, also mentioned by Cruz (2012), is that often operational-driven losses manifest themselves with an important time lag in reference to market events (litigation-related losses are a typical example). A third obstacle is simply the very nature of OR, or rather, the high variety of risks covered by this umbrella. Take for example the malicious destruction of equipment by a disgruntled employee. Clearly, one could make the case that the correlation between this type of event and, say, market risk is zero. On the other hand, at least conceptually, we could argue that market risk (think volatility) and execution risk (another type of OR) could exhibit some nonnegligible correlation. And fourth, there is the frequency and severity issue. The real danger in OR is the single event that can bring down an institution (Barings) or cause significant losses (the London Whale), rather than a sequence of small losses due to software glitches or employees' mistakes. Aggregating losses based on frequency-based correlations is mathematically manageable; also, there is some data regarding this type of correlation. Assembling data regarding severity-based correlations is more challenging. Moreover, severity-based correlations are very difficult to tackle from a modeling viewpoint (Frachot *et al* 2004).

In any event, assumptions must be made. So, what do we know about the correlation between OR and other risks?

The insurance industry seems to believe that this correlation is low. For example, Long and Whitworth (2004, Slide 14) presented a diagram advocating this view, but they did not commit to a specific numerical estimate. Chief Risk Officer Forum (2005, p. 18), an industry group representing fifteen global insurance companies, expressed a similar view in a technical report related to S2.

Larsson (2009, Table 12) used data related to the banking sector in Denmark, Finland, Norway and Sweden, from 1983 to 2008, to conclude that the correlation between OR and market or life risk should be treated as zero. She suggested a value of 49% for between OR and credit risk. Li *et al* (2012, Table 5), based on data from Austrian banks, employed in their study a correlation of 30% between OR and market or credit risk. In their OR handbook, Chernobai *et al* (2007, Tables 13.8 and 13.10) cite studies that deal with financial conglomerates that recommend correlation values in the 20–40% range between OR and market or credit risk.

It might be argued that the data related to banks is not applicable to the case of insurance companies. However, although banks and insurance companies differ in many ways, it is reasonable to think that in the case of banks, OR exposures are more highly correlated with other risks than in the case of the insurance sector. The reason is that banks, unlike insurers, are more sensitive to liquidity and trading losses. They are also exposed to bank runs. These are all phenomenons that tend to exacerbate correlation. Thus, whatever the correct value is for the correlation between OR and market or credit risk in a bank, in an insurance company this value should be lower.

Dexter *et al* (2007) also identified the lack of data as a big challenge, especially in reference to situations where loss events are infrequent. Nevertheless, they suggest starting with a zero correlation assumption between OR and other risks when there is no obvious common driver. They also present results from a survey of practitioners in which the estimates for such correlations fluctuate between low (10–30%) and medium (30–70%). Finally, Frachot *et al* (2004), in their paper aimed at discussing OR in the context of Basel III, conclude that there is a strong argument in favor of low correlation levels between aggregate losses. Towers Perrin & OpRisk Advisory (2010) also share this view, but they emphasize the importance of modeling OR by means of heavy-tail distributions.

In summary, and notwithstanding the many caveats embedded in the preceding observations, one thing is certain: the case for assuming a high (linear or Pearson) correlation between OR and the other five risks is at most very weak. We turn now to explore the aggregation formula employed by S2.

3 RISK AGGREGATION UNDER SOLVENCY II

Our concern is the S2 aggregation scheme known as the standard formula (SF). We suspect that this approach will be adopted, at least initially, by a large number of insurers, since it does not require approval from the national regulator. (Insurers intending to use an internal model instead of the SF need first to gain regulatory approval.)

3.1 The standard risk aggregation formula

Let $\mathrm{SCR}_1, \dots, \mathrm{SCR}_6$ denote the capital requirements associated with each of the risks mentioned, and let $\mathrm{SCR}_{\mathrm{TOT}}$ be the total capital required. S2 states that

$$\mathrm{SCR}_{\mathrm{TOT}} = \mathrm{SCR}_{\mathrm{BASIC}} + \mathrm{SCR}_6, \tag{3.1}$$

where $\mathrm{SCR}_{\mathrm{BASIC}}$ refers to the capital required based on the first five risks, and SCR_6 is the capital due to OR (European Insurance and Occupational Pensions Authority 2014a, Section SCR.1).

According to S2, $\mathrm{SCR}_{\mathrm{BASIC}}$ should be calculated following the standard linear aggregation formula.

Thus,

$$\mathrm{SCR}_{\mathrm{BASIC}} = \sqrt{\sum_{i=1}^{5}\sum_{j=1}^{5} \rho_{ij} \times \mathrm{SCR}_i \times \mathrm{SCR}_j}, \tag{3.2}$$

where the off-diagonal terms in the 5×5 correlation matrix are assumed to be equal to 0.25, except that $\rho_{52} = \rho_{25} = 0.5$ (European Insurance and Occupational Pensions Authority 2014a, Section SCR.1). For simplicity, and since it is immaterial to this discussion, we are not explicitly considering intangible assets risk and adjustments due to technical provisions or deferred taxes.

This aggregation scheme can be described as "mixed" since it relies on a linear aggregation expression for the first five risks; however, the capital for the sixth risk (OR) is simply added to the combined value of the previous five capital risks instead of incorporated under the linear aggregation formula. This approach has led some practitioners to state (erroneously, as we will see) that there are no diversification benefits between OR and other risks (Herzog 2011, p. 4; Internal Model Industry Forum 2015, p. 9). More surprisingly, this mistaken opinion has also been expressed by both the Basel Committee on Banking Supervision: Joint Forum (2010, p. 11) and the European Insurance and Occupational Pensions Authority (EIOPA; 2014b, Section 6). Incidentally, it should be mentioned that Basel III, under the advanced approach formula, also adds the capital due to OR directly to those figures arising from market and credit risk.

3.2 Operational risk and the implicit correlation assumptions with others risks

It certainly strikes one as curious that the capital due to OR (SCR_6) has been left outside the linear aggregation expression, where it could have been naturally included simply by expanding the correlation matrix to a 6×6 matrix. Obviously, this would have required estimating the correlation between OR and the other five risk components.

One disadvantage of estimating the capital required by invoking (3.1) and (3.2) is that somehow this obscures the correlation that is implicitly assumed between OR and the other risks. This, however, can be made clear very easily by assuming that the correlation between OR and the other risks is the same; consequently, let $\rho_{i6} = \rho_{6i} = \rho^*$ for $i = 1, \dots, 6$. This assumption allows us to expand the previously defined 5×5 correlation matrix to a 6×6 matrix and employ the linear aggregation formula to combine all six risks. Let us define SCR^* the capital obtained with this expression:

$$\mathrm{SCR}^* = \sqrt{\sum_{i=1}^{6}\sum_{j=1}^{6} \rho_{ij} \times \mathrm{SCR}_i \times \mathrm{SCR}_j}. \tag{3.3}$$

Then, by solving

$$\mathrm{SCR}^{*} = \mathrm{SCR}^{*}(\rho^{*}) = \mathrm{SCR}_{\mathrm{TOT}} \tag{3.4}$$

for ρ^{*} (the implicit correlation embedded in the SF), we can get a sense for the level of correlation hidden behind (3.1) between the OR and the other five risks. In short, by solving (3.4) for several representative cases, we can uncover the correlation assumption behind the S2 approach to handle OR. We now turn to this task.

4 EXAMPLE

We consider ten hypothetical but realistic insurers, whose individual SCRs are shown in Table 1 (Panel A). Panel B displays the basic capital ($\mathrm{SCR}_{\mathrm{BASIC}}$ from (3.2)) and the total capital ($\mathrm{SCR}_{\mathrm{TOT}}$) based on the S2 aggregation equation (3.1). This panel also shows, for each case, the implicit correlation assumption ρ^{*} for OR behind the S2 approach (from solving (3.4)). Panel C simply shows what the total capital would have been if we had employed the linear aggregation expression (3.3) with different assumptions for the correlation between OR and the other risks. Panel D displays the diversification benefit (in relation to OR) provided by the SF; it is simply the ratio between $[\mathrm{SCR}_{\mathrm{TOTAL}} - \mathrm{SCR}^{*}(\rho^{*} = 100\%)]$ and $\mathrm{SCR}_{\mathrm{TOTAL}}$. Note that $\mathrm{SCR}^{*}(\rho^{*} = 100\%)$ is the SCR obtained by invoking (3.3), assuming $\rho^{*} = 100\%$.

5 DISCUSSION

In reference to the results shown in Table 1, we can make the following observations.

(1) The implicit correlation imposed by (3.1) between OR and the remaining five risks is, on average, 79%, which is a high correlation number in absolute terms (based on the data shown in Panel B, bottom line). It is also higher than any of the correlation values assumed by EIOPA in reference to the other five risks. Additionally, the 79% value seems to be at odds with the industry and academic views discussed earlier in this paper.

(2) These results also show that asserting that the SF does not provide diversification benefits, as we mentioned before, is plain wrong. In fact, there are diversification benefits; however, they are very limited. Panel C, bottom line, shows what the SCR would have been in case we had assumed a 100% correlation between OR and each of the other risks ($\rho^{*} = 100\%$). Clearly, this capital value is higher (although not by much) than the total SCR figure shown in Panel B, second line. Finally, a measure of the diversification benefits is shown in Panel D. In essence, adding the SCR_6 to the aggregated SCR based on the other five risks is equivalent to assuming that the correlation between the OR and the combined capital resulting from the other risks ($\mathrm{SCR}_{\mathrm{BASIC}}$) is 1, but – and this is key – the

TABLE 1 SCRs for ten hypothetical insurance companies.

		Firm									
		A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Panel A	Market risk	100	30	40	40	35	10	40	10	10	12
	Default risk	10	5	80	55	10	2	10	10	3	5
	Life risk	500	70	0	0	7	0	20	55	30	0
	Health risk	10	20	0	3	0	50	5	10	10	55
	Nonlife risk	0	0	20	99	50	5	80	10	15	0
	OR	80	20	10	12	80	10	35	40	10	11
Panel B	SCR_{BASIC}	539.7	92.6	109.6	152.5	78.9	55.7	113.1	69.8	46.7	60.8
	SCR_{TOT}	619.7	112.6	119.6	164.5	155.9	65.7	148.1	109.8	56.7	71.8
	Implicit ρ^*	0.87	0.75	0.79	0.77	0.74	0.93	0.73	0.74	0.77	0.84
Panel C	$\rho^* = 0$	545.6	94.7	110	152.9	110.2	56.6	118.4	80.5	47.7	61.7
	$\rho^* = 25\%$	567.8	101.1	113.1	156.8	127.4	59.4	129.4	91.5	51.2	64.9
	$\rho^* = 50\%$	589.3	107.1	116.2	160.5	142.5	62.2	139.4	101.4	54.4	67.9
	$\rho^* = 100\%$	630	118.2	122.1	167.7	168.7	67.4	157.7	118.6	60.3	73.5
Panel D	Diversification benefits	1.6%	4.7%	2.0%	1.9%	7.6%	2.5%	6.1%	7.4%	6.0%	2.3%

Panel A shows the values of the six SCRs for ten insurance firms; Panel B shows the basic and total SCRs calculated with the SF as well as the implicit correlation (ρ^*) between OR and the other risks based on the SF aggregation formula; Panel C displays the SCR using different correlation coefficients for OR; and Panel D shows the diversification benefits (in reference to OR) implicit in each case according to the SF.

correlation between OR and each of the five risks, taken individually, is not 1 (this is the source of the mistaken view that there are no diversification benefits in terms of OR). It should be noted, nevertheless, that the diversification benefits vary a great deal as a function of the relative value of the six SCRs. In summary: adding the OR capital is simply equivalent to assuming very high (but less than 1) varying correlation values between OR and the other five risks.

(3) From (2), it follows that the SF is fundamentally unfair: it assumes different correlation values between OR and the other risks depending on the relative size of the six basic risk capital components. These variations can be significant, between 0.73 and 0.93 in our example. This situation is undesirable, for it creates the possibility of regulatory arbitrage. By simply manipulating the relative value of $\mathrm{SCR}_1, \dots, \mathrm{SCR}_6$ an insurer can achieve a reduction in the overall SCR without actually decreasing its risk.

(4) It is obvious from the structure of the SF that the regulator wanted to impose a high correlation between OR and the other risks. Leaving aside the merits of this decision, it also obvious that it would have been more transparent (and to some extent more intellectually honest) to simply include the OR within the linear aggregation expression using high (but fixed) correlation coefficients. There is no clear explanation for why this option was not adopted, since at least it would have had one clear merit: the correlation assumptions would have been openly displayed for all to see.

(5) Since the SF makes no attempt to incorporate explicitly in the formula the correct correlation between OR and the other risks (granted, an elusive figure), it also acts as a disincentive to gather data to explore this correlation issue more fully.

(6) The differences between the $\mathrm{SCR}_{\mathrm{TOTAL}}$ (Panel B) and what that value could have been if S2 had relied on a linear aggregation formula using a lower correlation assumption for OR (Panel C) is very telling. For example, a value of $\rho^* = 50\%$ – a lower value than the figure implied by the SF but under no conditions unreasonable – renders SCR values that can be as much as 10% lower. Hence, the effects of the SF structure and assumptions are not trivial.

6 CONCLUSION

The standard capital aggregation expression used by S2 is odd. It assumes that the correlation between OR and the other risks is very high, and it relies on a “mixed” formula that somehow obscures the correlation assumptions. It also gives different insurance companies different benefits for diversification effects in relation to OR, and these benefits are not really based on any sound diversification metric; rather, they are based on the relative weights of the six capital components. Finally, it acts as a disincentive to study in more detail the true correlation between OR and the other risks.

Moreover, this expression will probably result in overly conservative capital charges. Considering that in general OR capital charges are in the 3–15% range in reference to the total capital charges, it is clear that this excess capital is not negligible. Obviously, it should be of no concern to the regulator whether additional capital charges will affect the return to the insurance company stockholders, or whether these charges will be passed on to clients in the form of higher insurance premiums. However, there is an area in which capital charges (that is, excessive capital charges) might have an effect that is more serious and far reaching: annuities. The challenges that the European insurance sector, and, indeed, society as a whole, will soon face in relation to pensions is troubling. There is the combined effect of an increasingly older population, who will be retiring in a very unfavorable interest-rate environment. It is here that an overly conservative SCR formula could do more damage than good.

OR management is a young and vibrant discipline, where many problems are still not satisfactorily solved, much less understood. Just in the narrow arena of risk aggregation there are many pending issues. Can we rely on linear correlation coefficients to describe heavy-tail dependences well? What is the best copula to combine OR losses? Should aggregation schemes incorporate the distribution of the correlation coefficients (instead of using the average value of such coefficients)? Should we give up on traditional capital aggregation schemes and adopt, as Li *et al* (2014) suggest, a mutual information-based dependence? All these questions are open and important.

EIOPA's decision to adopt such a *sui generis* expression to aggregate risks is unfortunate, not only because of the consequences already explained, but also because it comes at an unwelcome time. The industry and the academic community, instead of using their resources to investigate the above-mentioned issues, will have to spend some time trying to convince the regulators that the SF needs some urgent re-engineering and propose a better choice. This is vital if we consider that EIOPA has recently admitted that it lacks the resources to review the SF, let alone investigate alternative models (Internal Model Industry Forum 2015, p. 18). It is therefore incumbent upon the industry and the academic community to embrace this task. A good start would be the adoption of the linear aggregation formula, but with lower (and fixed) OR correlation assumptions, until other alternatives are more fully explored. At least this will eliminate the distorting effects brought by the SF.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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