Market Efficiency with Micro and Macro Information

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Abstract

We propose a tractable, multi-security model in which investors choose to acquire information about macro or micro fundamentals or remain uninformed. The model is solvable in closed form and yields a rich set of empirical predictions. Primary among these is an endogenous bias toward micro efficiency. A positive fraction of agents will always choose to be micro informed, but in some cases no agent will choose to be macro informed. Furthermore, for most reasonable choices of parameter values, prices will be more informative about micro than macro fundamentals. A key driver of our results is that only micro informed investors take the other side of idiosyncratic noise trading in individual stocks. We explore the model’s implications for systematic and idiosyncratic return volatility and trading volume, for excess covariance and volatility, and for the cyclicality of investor information choices.

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1 Introduction

Jung and Shiller (2006) give the name “Samuelson’s dictum” to the hypothesis that the stock market is “micro efficient” but “macro inefficient.” More precisely, the dictum holds that the efficient markets hypothesis describes the pricing of individual stocks better than it describes the aggregate stock market. Jung and Shiller (2006) argue that this view is plausible under the following circumstances:

- The market has access to more information about the fundamentals of individual companies than about fundamentals of the aggregate stock market;
- The variation in information about individual companies is large relative to the variation in information about the aggregate market;
- Changes in aggregate dividends are less dramatic than changes in dividends for individual firms, and the reasons behind these aggregate changes are subtle and difficult for the investing public to understand.

We develop a tractable multi-security model with imperfect information to capture these and other features in order to investigate a potential wedge between micro and macro price efficiency. The general setting may be viewed as a multi-security generalization of the classical model of Grossman and Stiglitz (1980). Our market consists of a large number of individual stocks, each of which is exposed to a macro risk factor and an idiosyncratic risk. The macro risk factor is tradeable through an index fund that holds all the individual stocks and diversifies away their idiosyncratic risks.

We model three types of agents: uninformed investors, investors informed about the macro risk factor, and investors informed about individual stocks. Informed agents have access to an information technology that reduces their uncertainty about the payout of either the index fund or an individual stock. The information technology specifies what portion of micro and macro risk is knowable, and we investigate the consequences of varying the precision of the two types of technology.

We first take the fraction of uninformed, macro-informed, and micro-informed agents as given and solve for an explicit market equilibrium, assuming all agents have CARA preferences. Shares of individual stocks and the index fund are subject to exogenous supply shocks. Importantly, the exogenous supply shocks themselves exhibit a factor structure – there is a common component across all firms’ supply shocks, but each firm’s supply shock also has an idiosyncratic component which we interpret as noise trading. Supply shocks are not observable to investors, and therefore equilibrium prices are informative about
but not fully revealing of the micro or macro information acquired by informed agents. We define explicit measures of micro and macro price informativeness for the index fund and for the individual stocks; these measures are a focus of much of our analysis.

We then allow agents to choose between being micro informed and macro informed, and we characterize the equilibrium in which a marginal agent is indifferent between the two types of information. This analysis contrasts with the single-security setting of Grossman and Stiglitz (1980), where agents choose whether to become informed or remain uninformed, but the choice between micro and macro information is absent. In practice, developing the skills needed to acquire and apply investment information takes time — years of education and experience. In the near term, these requirements leave the total fraction of informed investors relatively fixed. By contrast, we suppose that informed investors can move comparatively quickly and costlessly between being macro informed or micro informed by shifting their focus of attention. Endogenizing this focus gives rise to an attention equilibrium centered on the choice between macro and micro information. Over a longer horizon, agents choose whether to gain the skills to become informed, as well as the type of information on which to focus. We therefore study an information equilibrium that endogenizes both decisions to determine equilibrium proportions of macro informed, micro informed, and uninformed investors.

A striking feature of our results is a recurring asymmetry between micro and macro information. For example, we show that the information equilibrium sometimes has no macro informed agents, but some fraction of agents will always choose to be micro informed. We show that increasing the precision of micro information makes micro informed agents worse off: we say that micro informed agents overtrade their information, driving down their compensation for liquidity provision. In contrast, macro informed agents may be better or worse off as a result of more precise macro information: they are better off when the fraction of macro informed agents is low or, equivalently, when the informativeness of the price of the index fund is low relative to the knowable fraction of macro uncertainty. Similarly, the equilibrium fraction of macro informed agents always increases with the precision of micro information, but it can move in either direction with an increase in the precision of macro information.

A simple condition on the relative precision of micro and macro information determines whether the market is more micro efficient or more macro efficient. The conclusion is consistent with Jung and Shiller’s (2006) discussion of Samuelson’s dictum: if – in a sense we make precise – the knowable fraction of uncertainty is greater for individual stocks than for the index fund, then the market is more micro efficient than macro efficient. When we
introduce a common cost for acquiring either micro or macro information, we show that among agents who choose to become informed, the fraction who choose to become macro informed declines as the cost increases. Like other implications of our model, this shows an endogenous bias toward micro over macro efficiency, particularly when information acquisition is very costly.

For most of our results, we constrain the investment opportunities for agents based on their information: uninformed and macro informed agents invest only in the index fund; each micro informed agent may invest in the index fund and in the stock about which that agent is informed. In other words, agents invest based on their information or they diversify. These restrictions are a priori sensible; we endogenize them under additional conditions ensuring that agents would invest negligibly small amounts in other assets if unconstrained. The fact that only the micro informed trade in individual stocks means that they are the only group of investors able to take the other side of idiosyncratic noise trading (or equivalently, of the idiosyncratic portion of stock supply shocks) – this is a key driver of many of the results in the paper.

It is possible that as more investors shift their attention to macro information, over time a greater fraction of macro uncertainty becomes knowable – with more people looking for information, more information may be found. And similarly as investors shift attention towards micro information, more of micro information may be knowable to market participants. Once enough of a certain type of information is known, the relative value of that type of information diminishes. This leads to endogenous attention cycles, in which investors shift from macro to micro attention and back again. Importantly, these attention cycles are not responses to exogenously changing economic conditions, but are an endogenous outcome of attention allocation – and ultimately misallocation – by investors in the model.

Our theoretical analysis leads to several testable empirical predictions: (i) Idiosyncratic volatility falls as more micro information becomes available or as the fraction of micro informed investors increases. (ii) As investors shift focus between micro and macro information, idiosyncratic volatility and systematic volatility move in opposite directions. (iii) Changes in the precision of micro information contribute to a common factor in idiosyncratic volatility. (iv) Low precision in macro information creates excess volatility and excess comovement in prices, compared with fundamentals. (v) The variation in the precision of micro and macro information, and the variation in investor attention to the two types of information, help explain variation in assets invested in different hedge fund strategies. (vi) Declining costs for acquiring information shift a greater fraction of
actively managed funds to macro focused strategies. (vii) Recessions characterized by a similar increase in macro and micro risk push informed investors to focus on micro information, whereas recessions accompanied predominantly by increased macro risk and only a small increase in the price of risk push investors into macro information. (viii) An increase in macro (micro) trading volume resulting from an increase in supply volatility increases the the fraction of macro (micro) informed agents, but it does not change price informativeness.

Several of the assumptions underlying the model lend themselves to empirical analysis. For example, the relative precision of knowable micro and macro information can be estimated by regressing company-specific and aggregate earnings on predictive variables. Furthermore, how the precision of micro (macro) information responds to the presence of micro (macro) informed investors can be analyzed by studying explanatory power of the above-mentioned regressions and its sensitivity to changing information sets amongst investors. This would allow us to test, for example, whether the precision of macro information is more sensitive to the fraction of macro informed investors in emerging markets than in developed markets.

Our work is related to several strands of literature. Our model effectively nests Grossman and Stiglitz (1980) if we take the index fund as the single asset in their model. We also draw on the analysis of Hellwig (1980), Admati (1985) and Admati and Pfleiderer (1987) but address different questions (see Brunnermeier (2001) for a survey of related literature). In particular, Admati and Pfleiderer (1987) focus on understanding when signals are complements or substitutes; the issue of strategic complementarity in information acquisition is also investigated in Goldstein and Yang (2015) using a Grossman-Stiglitz type model. As in Kyle (1985), our noise traders are price insensitive and gains from trade against them accrue to the informed – thereby providing incentive to collect information. We shed light on the discussion in Black (1986) of the crucial role that “noise” plays in price formation by proposing a model in which the factor structure of noise trading plays an important role in determining the relative micro versus macro efficiency of markets.

Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) develop a model of rational attention allocation in which fund managers choose what information to acquire in making investments. Their model, like ours, has multiple assets subject to a common cash flow factor; but, in contrast to our setting, their agents ultimately all acquire the same information. Their focus is on explaining cyclical variation in attention allocation that is caused by exogenous changes in economic conditions. Kacperczyk et al. (2016) show that mutual fund managers change their focus from micro to macro fundamental information.
over the course of the business cycle. We discuss several mechanism that give rise to such behavior, and contrast these with the explanations put forward in Kacperczyk et al. (2016).

Garleanu and Pedersen (2016) extend the Grossman-Stiglitz model to link market efficiency and asset management through search costs incurred by investors in selecting fund managers, in a model with a single risky asset. Portfolio choice and information acquisition is also studied in Van Nieuwerburgh and Veldkamp (2010). Peng and Xiong (2006) also use a model of rational attention allocation to study portfolio choice. In their framework, investors allocate more attention to sector or marketwide information and less attention to firm-specific information. Their conclusion contrasts with ours (and with the Jung-Shiller discussion of Samuelson’s dictum and the Mackowiak and Wiederholt (2009) model of sticky prices under rational intattention) primarily because in their setting a representative investor makes the information allocation decision; since macro uncertainty is common to all securities, while micro uncertainty is diversified away, the representative investor allocates more attention to macro and sector level information. Our model also relates to the question reviewed in Merton (1987) of how differences in information lead investors to hold different sets of securities. But in Merton’s (1987) formulation knowing about a security means knowing the parameters of its return distribution, whereas in our setting informed investors know something about the realization of returns. In discussing empirical implications of our model, we note a connection with the findings of Vuolteenaho (2002) that individual stock returns are driven more by information about cash flows than about discount rates, whereas Campbell and Ammer (1993) find the opposite for the aggregate stock market.

The rest of the paper is organized as follows. Section 2 presents the model, and Section 3 solves for the market clearing prices and demands. In Section 4, we analyze the attention equilibrium and derive the equilibrium fractions of macro and micro informed investors, holding fixed the total size of the informed population. In Section 5, we study the information equilibrium that endogenizes the decision to become informed along with the choice between macro and micro information. Section 6 discusses the dynamic evolution of an economy where the level of investor attention affects the precision of the information technology — in particular, as more investors become macro informed, more macro information becomes knowable. Section 7 presents testable implications of the model. Section 8 summarizes the paper and some of its empirical implications. Technical details and proofs are covered in an appendix.
2 The economy

Securities

We assume the existence $N$ risky securities – called stocks – indexed by $i$. There is also an index fund, $F$, one share of which holds $1/N$ shares of each of the $N$ stocks. Finally, there is a riskless security with a rate of return of $R - 1$. The time 2 dividend payouts of the stocks are given by

$$u_i = \beta_i M + S_i, \quad i = 1, \ldots, N, \quad (1)$$

where $M = m + \epsilon_M$ and $S_i = s_i + \epsilon_i$. We do not assume that $s_i$ is independent of $s_j$, nor that $\epsilon_i$ is independent of $\epsilon_j$. All other pairs of random variables associated with dividend payouts are independent. We set $E[m] = \bar{m}$ and $E[s_i] = E[\epsilon_i] = E[\epsilon_M] = 0$. The variance of $m$ is $\sigma_m^2 > 0$, the variance of $s_i$ is $\sigma_s^2 > 0$, and the variances of $\epsilon_i$ and $\epsilon_M$ are $\sigma_{\epsilon_s}^2 > 0$ and $\sigma_{\epsilon_M}^2 > 0$. As will be clear shortly, $m$ and $s_i$ will be observable (at a cost) to investors, and therefore represent the knowable part of dividend uncertainty. The ratio of the variance of this knowable part of the dividend to the total variance is given by

$$f_M = \frac{\sigma_m^2}{\sigma_M^2} \quad \text{and} \quad f_S = \frac{\sigma_s^2}{\sigma_S^2}, \quad (2)$$

where $\sigma_M^2 = \sigma_m^2 + \sigma_{\epsilon_M}^2$ and $\sigma_M^2 = \sigma_s^2 + \sigma_{\epsilon_S}^2$.

All of these random variables are normally distributed. We assume that the average beta is 1 ($\sum_i \beta_i = N$), so the payout of $F$ is

$$u_F = M + \frac{1}{N} \sum_{i=1}^{N} S_i. \quad (3)$$

Prices of individual stock are given by $P_i$. The index fund price is $P_F$, and no arbitrage requires that

$$P_F = \frac{1}{N} \sum_{i=1}^{N} P_i. \quad (4)$$

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1 The nature of their dependence is discussed in Section 3.
Agents and information sets

Agents maximize expected utility over time 2 wealth, given by $-\mathbb{E}[\exp(-\gamma \hat{W}_2)]$. For simplicity, we assume the same risk aversion parameter $\gamma > 0$ for all agents. There are uninformed, macro ($M$) informed and micro ($S$) informed agents. Uninformed agents can choose to become either $M$ or $S$ informed at a cost $c$. It is costless for agents to move from being $M$ to $S$ informed, and vice versa. The proportion of each group of agents is given by $\lambda_U$, $\lambda_M$, and $\lambda_S$, respectively, with

$$\lambda_U + \lambda_M + \lambda_S = 1.$$  

We analyze two types of informational equilibria in the paper. In the attention equilibrium, for a given $\lambda_U$, the proportion of $M$ and $S$ informed will be chosen so as to make investors indifferent between the two information sets.\footnote{More precisely, this describes an interior equilibrium. We will also consider corner solutions.} In the information equilibrium, we also solve for the $\lambda_U$ which makes the uninformed indifferent between staying uninformed or paying a cost $c$ to become either macro or micro informed.

Macro informed agents observe $m$ and in aggregate micro informed agents observe $s_i$ for every $i$. For the time being we think of $f_M$ and $f_S$ from (2) as exogenously specified information technologies available to $M$ and $S$ informed investors.\footnote{Section 6 will discuss the effects on equilibrium when the information technology depends on the number of informed agents.} Say two investors expend the same amount of effort: one to study the prospects of an individual company, and the other to study the prospects of the aggregate stock market. We want to allow for the possibility that the first investor may learn more (or less) through this equal effort than the second – potentially because there is more (or less) knowable at the micro than the macro level. If more (less) micro information were knowable, we would have $f_S > f_M$ ($f_S < f_M$).

Agents who choose to be $S$ informed are randomly assigned to learn about security $i$ so that $\lambda_S/N$ agents are knowledgeable about $s_i$ for every $i$. We will often refer to $S$ informed investors who learn $s_i$ as $i$ informed investors. All agents who are not $M$ informed rationally extract from $P_F$ the relevant information about $m$. In addition $i$ informed agents can condition on $P_i$ in making inferences about $m$ (though it will be shown that $P_i$ contains no useful information about $m$ once $P_F$ is known).

In initially solving the model, we will assume that non-$i$ informed agents are not allowed to trade in stock $i$. Intuitively, macro uninformed investors may have legitimate reasons for trading stock market indexes, but investors who have not studied the fun-
damentals of a particular company should not trade in that company’s stock. We will show in Section 3.4 that, under certain plausible assumptions about investor behavior, this restriction can be endogenized. We defer this analysis to keep development of the model as simple as possible.

Supply and market clearing

The supply of the $i^{th}$ asset is given by

$$\frac{1}{N} \left( X_F - \frac{1}{N} \sum_{j=1}^{N} \beta_j X_j + X_i \right),$$

(5)

where $X_F$ is the common supply shock, normally distributed with mean $\bar{X}_F$ and variance $\sigma_{X_F}^2$, and $X_i$ are normally distributed idiosyncratic shocks, each with mean 0 and variance $\sigma_X^2$. Supply shocks are independent of cashflows, and $X_i$ is independent of $X_F$ for all $i$, though we do not assume that the $X_i$’s are wholly uncorrelated with one another (see Section 3). We make the standard assumption that supply shocks are unobservable by the agents. The summation term in (5) ensures that in aggregate the idiosyncratic shocks do not add any $M$ risk to the economy.

The aggregate portion of supply shocks, $X_F$, is standard in the literature – as will become clear, it is analogous to the single security supply shock from Grossman and Stiglitz (1980). The idiosyncratic portion of the supply shock, $X_i$, proxies for price insensitive noise trading in individual stocks. Perhaps some of this noise trading is liquidity driven (for example, individuals needs to sell their employer’s stock to pay for unforeseen expenditures), but the majority is likely to come from either incorrect expectations or from other value-irrelevant triggers, such as an affinity for trading. Our interpretation of noise traders follows Black (1986), who discusses how noise traders play a crucial role in price formation. Recent empirical studies either suggest (Brandt, Brav, Graham, and Kumar (2010)) or document (Foucault, Sraer, and Thesmar (2011)) a causal link from retail trading to idiosyncratic volatility of stock returns. For example, Foucault et al. (2011) “show that retail trading activity has a positive effect on [idiosyncractic] volatility

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4None of our qualitative result change if this term is dropped, except that the index fund equilibrium would need to reflect the $M$ risk contribution of idiosyncratic supply shocks – which would (1) be small for large $N$, and (2) would only clutter the analysis.
of stock returns, which suggests that retail investors behave as noise traders.” Our model captures this exact phenomenon via $X_i$. In fact, as will be shown in Section 7, the volatility of $X_i$ directly enters into the idiosyncratic volatility of stock returns.

Let us write $q_U^i$, $q_M^i$, and $q_i^i$ for the demands of each investor group for security $i$, which can be one of the $N$ stocks or the index fund $F$. For any investor group, $q_i$ denotes that group’s direct demand for stock $i$. Note that each group’s $F$ demand, $q_F$, leads to an indirect demand of $q_F/N$ for every stock $i$.

Let us define the aggregate holdings of the index fund as follows

$$q_F \equiv \lambda_U q_U^F + \lambda_M q_M^F + \frac{\lambda_S}{N} \sum_{i=1}^{N} q_i^F.$$ (6)

The market clearing condition for each stock $i$ can be written in its general form as

$$\frac{\lambda_S}{N} \sum_{j=1}^{N} q_i^j + \lambda_U q_U^i + \lambda_M q_M^i + \frac{q_F}{N} = \frac{1}{N} (X_F - \sum_{j=1}^{N} \beta_j X_j/N + X_i) \quad \forall i. $$ (7)

The first three terms on the left hand side are the direct demand for stock $i$ from the $S$ informed, uninformed, and $M$ informed, respectively. The fourth term is how much of stock $i$ is held in the index fund. The right hand side is the supply shock from (5). The direct and indirect demand (via $F$) of all agents for stock $i$ must equal its supply.

We now impose the restriction that non-$i$ informed agents do not own stock $i$, i.e. $q_{i}^{U/M/j} = 0$ for $j \neq i$. This simplifies the above condition to

$$\lambda_S q_i^i + q_F = X_F - \frac{1}{N} \sum_j \beta_j X_j + X_i \quad \forall i. $$ (8)

Rewriting (8) as

$$q_F = X_F - \frac{1}{N} \sum_j \beta_j X_j - (\lambda_S q_i^i - X_i) \quad \forall i,$$

and observing that this must hold for all $i$, implies that $\xi \equiv \lambda_S q_i^i - X_i$ cannot depend on $i$. We can therefore write $i$’s direct demand as

$$\lambda_S q_i^i = X_i + \xi,$$ (9)

5We also disallow the presence of a fund which owns $X_i/N$ shares of each stock. If every investor owned one share of this fund, all idiosyncratic supply shocks could be held in a way that fully diversifies away all idiosyncratic risk for large $N$. However, because supply shocks are not observable, such a fund is precluded from existing in the model.
for some $\xi$ that does not depend on $i$. The market clearing condition in (8) implies that

$$q_F = X_F - \frac{1}{N} \sum_j \beta_j X_j - \xi.$$  \hspace{1cm} (10)

While (9) and (10) are clearly sufficient for (8), we have therefore established that they are necessary as well.

A nonzero $\xi$ would be a common component in the demands of agents informed about individual stocks that would be offset by holdings in the index fund. We will see in Section 3.1 that the choice of $\xi$ does not affect the market clearing condition for the index fund because investors use the index fund to hedge out the $M$ component of any additive term in their individual security demands. However, $\xi$ does affect market clearing for individual stocks. We will show in Section 3.2 that this, together with the no-arbitrage condition (4) restricts $\xi$ to be zero. A non-zero $\xi$ implies that the aggregate supply shock is held in a non-diversified form – via direct demand for individual stocks – thus requiring compensation for stocks’ idiosyncratic risk, which is not required when the aggregate supply shock is held in diversified form via the index fund. Therefore, in equilibrium we will have $q_F = X_F - \sum_j \beta_j X_j/N$ – the index fund will hold the aggregate supply shock.\footnote{In the absence of market frictions, instead of holding the index fund, investors can equivalently hold an identical number of shares of every stock. Representing such demand via an index fund is a notational convenience in our model.}

This result is related to Kwon’s (1985) proof that risk-averse investors will hold the market portfolio in equilibrium, regardless of quadratic preferences or normality of returns, as long as idiosyncratic returns are mean zero conditional on the market return.
3 Market equilibrium

The equilibrium in this economy is greatly simplified when the idiosyncratic cashflow and supply shocks sum to zero over a finite number of stocks. We will impose

$$\sum_{i=1}^{N} s_{i} = \sum_{i=1}^{N} \epsilon_{i} = \sum_{i=1}^{N} X_{i} = 0 \quad (11)$$

by assuming the covariances

\[
\begin{align*}
\text{cov}(X_{i}, X_{j}) &= -\frac{\sigma_{X}^{2}}{N - 1}, \\
\text{cov}(s_{i}, s_{j}) &= -f_{S} \frac{\sigma_{S}^{2}}{N - 1}, \\
\text{cov}(\epsilon_{i}, \epsilon_{j}) &= -(1 - f_{S}) \frac{\sigma_{S}^{2}}{N - 1},
\end{align*}
\]

which imply that the variances of the sums in (11) are zero. Given these assumptions, it follows from (3) that \( u_{F} = M \), i.e. the payout of the index fund is just equal to the common component of the dividends.

In this case the equilibrium prices become particularly simple. We conjecture that the index fund price takes the form

$$P_{F} = a_{F} + b_{F} (m - \bar{m}) + c_{F} (X_{F} - \bar{X}_{F}) \quad (13)$$

and that individual stock prices are given by

$$P_{i} = \beta_{i} P_{F} + b_{S} s_{i} + c_{S} (X_{i} + \xi) \quad (14)$$

We construct an equilibrium in which these conjectured prices hold. Note that the price of the index fund and the individual security prices must satisfy the no arbitrage condition.

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7 This is a common assumption in the asset pricing literature when considering multi-security economies with a finite number of assets. See, for example, Ross (1978), Chen and Ingersoll (1983), and Kwon (1985). At the cost of additional complexity, we can solve our model in the case of finite \( N \) with uncorrelated idiosyncratic cashflow and supply shocks. The price equations in (13) and (14) will contain loadings on two additional terms, \( \bar{s} \) (the average \( s_{i} \) across all stocks) and \( \bar{X} \) (the average \( X_{i} \)), which reflect risk sharing among agents when idiosyncratic shocks don’t cancel in aggregate. We can then show that the equilibrium we analyze in the paper is the limit of these finite \( N \), uncorrelated shock economies as \( N \) becomes large. This result is available from the authors upon request, but is not included in the paper to conserve space.

8 This makes \((X_{1}, \ldots, X_{N})\) exchangeable random variables, and similarly for the \( s_{i} \) and \( \epsilon_{i} \). Each of the covariance matrices specified by (12) is diagonal dominant and therefore positive semidefinite.
in (4) that the mean stock price is equal to the price of the index fund.

### 3.1 Market clearing

In equilibrium, the market clearing conditions in (7) and (10) have a very intuitive interpretation. To develop this, we need to first preview a result: $i$ informed demand for the index fund $F$ will be given by

$$q_i^F = q_U^i - \beta_i q_i^i. \tag{15}$$

An $i$ informed agent’s demand for the index fund consists of two components: (1) the demand of the uninformed agents, since neither has any information about $M$ (recall $u_F = M$) beyond that contained in $P_F$, minus (2) the exposure to $M$ that the $i$ informed agent already has through stock $i$. This will result in the $i$ informed and the uninformed agents maintaining the same risk exposure to $M$. We refer to the last term in (15) as the $i$ informed’s **hedging demand** – the $i$ informed use the index fund to hedge out undesired exposure to $M$ that they get from speculating on their information about $s_i$.

From (9) we know that $\lambda_S q_i^1 = X_i + \xi$. The $i$ informed absorb the entire idiosyncratic portion of stock $i$’s supply shock (up to an additive term that does not depend on $i$ or any of the $X_i$). Indeed it is this liquidity provision service – the fact that all non-index-fund absorbed supply shocks in stock $i$ need to be absorbed by the $i$ informed agents – which creates incentive for agents to become $i$ informed in the first place.

Now consider the index fund demand identity from (6). Given the $i$ informed index fund demand in (15), together with $\lambda_S q_i^1$ from (9), we can rewrite fund demand as

$$q_F = (\lambda_U + \lambda_S) q_U^F + \lambda_M q_M^F - \left(\xi + \frac{1}{N} \sum_i \beta_i X_i\right), \tag{16}$$

where we have used the fact that average betas are $1^{9}$.

We now impose the index fund market clearing condition $q_F = X_F - \sum_j \beta_j X_j/N - \xi$, note that the hedging demand cancels from both sides of the equation, and observe that $X_F$ will be held via investors’ demands for the index fund:

$$X_F = (\lambda_U + \lambda_S) q_U^F + \lambda_M q_M^F. \tag{17}$$

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9Note that $\lambda_S/N \sum_i q_i^F = \lambda_S q_U^F - 1/N \sum_i \beta_i \lambda_s q_i^i$ from (15). Using (9) this becomes $\lambda_S q_U^F - 1/N \sum_i \beta_i (X_i + \xi)$, from which (16) follows.
As already mentioned, \( \xi \) does not enter into the index fund market clearing condition – and therefore into \( P_F \) – because demand for it via the index fund is exactly offset by agents’ hedging demands.

Surprisingly, the index fund market clearing condition in (17) describes exactly the single security Grossman-Stiglitz equilibrium, with the percent of \( M \) informed agents given by \( \lambda_M \) and the percent of macro uninformed agents given by \( \lambda_U + \lambda_S \). In the attention equilibrium portion of our analysis, we will endogenously determine the fraction of agents choosing to be \( S \) versus \( M \) informed by requiring that the marginal investor be indifferent between the two choices. Our model is therefore a natural generalization of Grossman and Stiglitz (1980) into a multi-security framework. Our index fund market clearing condition is exactly the one from the Grossman and Stiglitz (1980), and our idiosyncratic supply shock condition in (9) characterizes speculator liquidity provision in the securities in which they specialize.

### 3.2 Model solution

Recall from Section 2 that we assume that agents uninformed about security \( i \) do not invest in that security. Therefore, the \( M \) informed and uninformed agents will have demands only for the index fund, and \( i \)-informed agents will demand the index fund and security \( i \). We assume that agents of type \( U \), \( M \), and \( i \), respectively, set their demands by maximizing expected utility conditional on the information sets

\[
\mathcal{I}_U = \{P_F\}, \quad \mathcal{I}_M = \{m, P_F\}, \quad \text{and} \quad \mathcal{I}_i = \{P_F, P_i, s_i\}.
\]

In the equilibrium we construct, \( P_F \) is a noisy version of \( m \), so \( M \)-informed agents rationally ignore \( P_F \) in evaluating conditional moments of \( M \). Similarly, all agents rationally ignore the prices \( P_1, \ldots, P_N \) in evaluating conditional moments of \( M \). Because of the covariance structure in (12), the price \( P_j \) will include some information about \( S_i, i \neq j \), but this information is negligible for large \( N \), so we assume that \( i \)-informed agents ignore it computing conditional moments of \( u_i \). We examine this point in greater detail in Section 3.4.

Standard arguments imply that the \( M \) informed demand for the index fund is given by

\[
q^M_F = \frac{1}{\gamma(1 - f_M)\sigma^2_M}(m - RP_F), \quad (18)
\]

as in equation (8) of Grossman and Stiglitz (1980), where \( R \) is the risk-free gross return,
and uninformed demand for the index fund is given by

\[ q^U_F = \frac{1}{\gamma \text{var}(M|P_F)}(E[M|P_F] - RP_F). \]  

(19)

If \( P_F \) takes the form in (13), then

\[ \text{E}[M|P_F] = K_F(P_F - a_F) + \bar{m}, \]
\[ \text{var}(M|P_F) = f_M\sigma^2_M(1 - K_Fb_F) + (1 - f_M)\sigma^2_M, \]  

(20)

\[ K_F = \frac{b_Ff_M\sigma^2_M}{b_F^2f_M\sigma^2_M + c_F^2\sigma^2_X}. \]

Demands of the \( i \) informed agents are given by the following proposition. See also Admati (1985).

**Proposition 1.** If the prices \( P_F \) and \( P_i \) take the form in (13) and (14), then the demands of \( i \) informed agents are given by

\[ q^i_i = R \frac{\gamma(1 - f_S)(\beta_i P_F + s_i/R - P_i)}{\lambda S}, \]  

(21)

\[ q^i_F = q^U_F - \beta q^i_i. \]  

(22)

We restate here the results from (9) and Section 3.1 that market clearing requires that

\[ (\lambda_U + \lambda_S)q^U_F + \lambda_Mq^M_F = X_F, \]  

(23)

\[ \lambda_S q^i_i = X_i + \xi. \]  

(24)

With the demands (18)–(19) for the index fund and demands (21)–(22) for individual securities, market-clearing prices are given by the following proposition:

**Proposition 2.** The market clears at an index fund price of the form (13),

\[ P_F = a_F + b_F(m - \bar{m}) + c_F(X_F - \bar{X}_F), \quad \text{with} \quad \frac{c_F}{b_F} = -\frac{\gamma(1 - f_M)\sigma^2_M}{\lambda M}, \]  

(25)

and prices for individual stocks \( i \) of the form (14), given by

\[ P_i = \beta_i P_F + \frac{s_i}{R} - \frac{\gamma(1 - f_S)\sigma^2_S}{\lambda SR}(X_i + \xi). \]  

(26)

The no-arbitrage condition (4) is satisfied if and only if \( \xi = 0 \).
The form of the index fund price $P_F$ follows from Grossman and Stiglitz (1980); explicit expressions for the coefficients $a_F$, $b_F$, and $c_F$, are derived in the appendix. Comparison of (14) and (26) shows that the ratio $c_S/b_S$ in the price of stock $i$ has exactly the same form as $c_F/b_F$ in the price of the index fund in (25). In fact, if $\lambda_M = 1$, then $b_F = 1/R$ and $c_F$ has exactly the same form as $c_S$. The stock $i$ equilibrium is the direct analog of the index fund equilibrium with only $M$ informed agents.

As in Grossman and Stiglitz (1980) the equilibrium price of the index fund depends on the proportion of investors informed about the fund’s payout. Similarly, the prices of individual securities depend on the proportion of investors informed about these securities. Much of our analysis will center on the endogenous choice of these proportions.

### 3.3 Price efficiency

It will prove useful to measure the extent to which prices in our model are informative about fundamentals. For the case of the index fund, we define price efficiency, $\rho^2_F$, as the proportion of price variability that is due to variability in $m$, the knowable portion of the aggregate dividend. This is the $R^2$ from regressing $P_F$ on $m$.

From the functional form of $P_F$ in (13), we see that

$$
\rho^2_F = \frac{b^2_Ff_M\sigma^2_M}{b^2_Ff_M\sigma^2_M + c^2_F\sigma^2_X}. \tag{27}
$$

Note that this is equal to $b_FK_F$ from the $M$ uninformed’s inference problem, which implies that that for the $M$ uninformed agent, the variance of $m$ conditional on $P_F$ is given by

$$
\text{var}(m|P_F) = f_M\sigma^2_M(1 - \rho^2_F). 
$$

As the price efficiency goes to 1, $P_F$ becomes fully revealing about $m$.

Dividing both sides by $b^2_F\sigma^2_M$ and using the expression for $c_F/b_F$ in (25), we can rewrite this as

$$
\rho^2_F = \frac{f_M}{f_M + \gamma^2(1 - f_M)^2\sigma^2_M\sigma^2_{X_F}/\lambda^2_M}. \tag{28}
$$

For stock $i$ we define price efficiency as the proportion of the variability of the price that is driven by variability in $s_i$, the idiosyncratic dividend shock, once $P_F$ is known. From the functional form of $P_i$ in (14) and the fact that $\xi = 0$, this is given by

$$
\rho^2_S = \frac{b^2_Ss_F\sigma^2_S}{b^2_Ss_F\sigma^2_S + c^2_S\sigma^2_X}. 
$$

16
Using the expression for $c_S/b_S$ in (26) and doing the same simplification as for $\rho_F^2$, we find that

$$\rho_S^2 = \frac{f_S}{f_S + \gamma^2(1 - f_S)^2\sigma_S^2\sigma_X^2/\lambda_S^2}. \quad (29)$$

As in the case of the index fund, as $\rho_S^2$ goes to 1, $P_i$ becomes fully revealing about $s_i$.

We note that the two efficiency measures have identical functional forms, with each using its respective set of moments and its $\lambda$. Furthermore, observe that our informativeness measures are with regard to the knowable portion of the dividend payout, not the total dividend payout.

Differentiating (28) and (29) and straightforward algebra, yields the following results:

**Lemma 1** (When are prices more informative?).

(i) Micro (macro) prices are more efficient as either (a) the fraction of micro (macro) informed increases, or (b) as the micro (macro) information technology improves. That is:

$$d\rho_S^2/d\lambda_S > 0 \quad \text{and} \quad d\rho_F^2/d\lambda_M > 0,$$

and

$$d\rho_S^2/df_S > 0 \quad \text{and} \quad d\rho_F^2/df_M > 0.$$

(ii) Furthermore, when the fraction of micro (macro) informed is zero, or when the information technology is non-informative, price efficiency is zero. In other words, $\rho_F^2 \to 0$ as either $\lambda_M \to 0$ or $f_M \to 0$, and $\rho_S^2 \to 0$ as either $\lambda_S \to 0$ or $f_S \to 0$.

(iii) When the information technology is perfect prices become fully revealing. In other words, $\rho_F^2 \to 1$ as $f_M \to 1$, and $\rho_S^2 \to 1$ as $f_S \to 1$.

As the number of informed in a given market grows, prices in that market become more revealing. Similarly, as the information about future dividends that is known to informed investors becomes greater, these investors—facing less future cashflow risk—trade more aggressively (as if they had a smaller risk-aversion parameter $\gamma$) which incorporates more of their information into prices.

We will be able to say much more about both measures of price efficiency when we evaluate them at equilibrium proportions $\lambda_M$ and $\lambda_S$.

### 3.4 No-trade conditions

Our analysis thus far has restricted non-$i$ informed agents from trading in stock $i$. This restriction can be endogenized in one of two ways.
Assumption 1. Agents who are not informed about the payout of security i \( s_i \) cannot condition their demands on the idiosyncratic portion of the price \( P_i \) of stock i.

If an agent is informed about the payout of stock 1 only, we assume it is too time consuming for this agent to submit \( N - 1 \) separate demands for other stocks as a function of their prices. Alternatively we can assume that

Assumption 2. Agents who are not informed about the payout of security i \( s_i \) do not update their beliefs about \( s_i \) based on \( P_i \), but are allowed to submit demands as a function of \( P_i \). Furthermore, the market is sufficiently micro efficient in the sense that \( \frac{\rho_s^2}{(1 - \rho_s^2)} > \frac{1}{1 - f_s} \).

The first statement in this assumption corresponds to the competitive (Walrasian) equilibrium concept in Lang et al. (1992). We assume it is too costly for agents informed about stock 1 to make separate inferences about the other \( N - 1 \) idiosyncratic stock payouts. Agents are allowed to condition their demands on prices, as long as the market is sufficiently micro efficient.

In the appendix, we formulate these conditions precisely and show that given either Assumption 1 or 2 for large \( N \), any agent who is not i informed will optimally choose not to trade stock i. Either of these assumptions thus endogenizes the portfolio restrictions that we imposed on agents at the outset.

Under Assumption 1 non i informed investors can submit demands for all stocks, but their demands cannot depend on those stocks’ prices – though they are allowed to depend on \( P_F \) (and \( P_j \) for the j informed). This is analogous to an investor submitting a basket sell order to a broker to “work” over the course of several hours – the amount traded for each stock will not depend on the average price realized over the course of the order – with the condition that the entire order can be cancelled if the stock market were to have a large move during the duration of order execution. The payoff to an investor from trading stock i hedged with the index fund is

\[ q_i(u_i - R(P_i - \beta_i P_F)). \]

When \( P_i - \beta_i P_F \) is known (i.e. \( q_i \) conditions on \( P_i \)), the investor faces uncertainty only from \( u_i \). When \( P_i - \beta_i P_F \) is not known (i.e. \( q_i \) cannot condition on \( P_i \)) the investor faces additional uncertainty from \( s_i \) and \( X_i \), both of which are in \( P_i \) (see equation (26)). However, in expectation, \( P_i \) only compensates the investor for i’s systematic risk loading and does not compensate the investor for bearing additional risk from \( s_i \) and \( X_i \) (i.e. \( E[P_i] = \beta_i E[P_F] \)), which makes trading in i unattractive.
To appreciate why Assumption 2 leads to a no-trade result as well, recall from (26) that the idiosyncratic portion of $P_i$ is given by

$$P_i - \beta_i P_F = \frac{s_i}{R} - \frac{\gamma(1 - f_S)\sigma^2_S}{\lambda_S R} X_i.$$ 

An $i$ uninformed investor who does not update beliefs about $s_i$ based on its price attributes all variation in $P_i - \beta_i P_F$ to liquidity demand and none of it to potential changes in $s_i$. The agent therefore overtrades the stock. For example, when $P_i - \beta_i P_F$ is low, the $i$ uninformed agent, by not updating properly about $s_i$, believes that $s_i$ is equal to its unconditional expectation of 0 and that the price must be low due to a high level of liquidity selling. The agent therefore buys a suboptimally large number of shares if part of the move in $P_i$ was caused by a realization of $s_i$ below 0. This investment will leave the agent worse off if the market is sufficiently micro efficient, in the sense of Assumption 2.

### 3.4.1 No-trade condition for the index fund

An analogous result to the no-trade conditions for individual stocks obtains in the case of the index fund when the macro information technology approaches infinite precision, i.e. when $f_M \to 1$. We show in Section A.2.1 that in this case the price of the index fund, $P_F$, is given by

$$P_F = \frac{m}{R} - \frac{\gamma(1 - f_M)\sigma^2_M}{\lambda_M} \bar{X}_F - \frac{\gamma(1 - f_M)\sigma^2_M}{\lambda_M} (X_F - \bar{X}_F) + O((1 - f_M)^2). \tag{30}$$

Comparing this to $P_i$ in (26) (and setting $\bar{X}_F = 0$ to be comparable to the zero mean idiosyncratic supply shock), we see that the idiosyncratic component of stock $i$’s price ($P_i - \beta_i P_F$) is exactly analogous to the index fund price when the information technology is fully precise. Except, in the single stock case, the idiosyncratic portion of the price maintains this functional form for all $f_S$.

The $X_F - \bar{X}_F$ term in the (30) has only $\lambda_M$ in the denominator – the proportion of $M$ uninformed doesn’t matter. As the index fund price approaches being fully revealing about $m$, only the $M$ informed absorb the stochastic portion $X_F - \bar{X}_F$ of the aggregate supply shock. The discount in the price due to the aggregate supply shock $X_F$ falls so quickly that the $M$ uninformed choose to completely stay out of the market – their informational disadvantage relative to the $M$ informed looms very large when the price $P_F$ doesn’t provide enough compensation for supply noise risk.
4 Attention equilibrium

The prior section analyzed the market equilibrium, taking the fraction of uninformed, $M$ informed and $S$ informed traders as given. In this section, we will endogenously determine these quantities.

As discussed in the introduction, we take the view that developing the skills needed to acquire and apply investment information takes time — perhaps seven to ten years of education and experience. In the near term, these requirements leave the total fraction of informed investors $\lambda_M + \lambda_S$ fixed. Once investors have the skills needed to become informed, we suppose that they can move relatively quickly (over one or two years) and costlessly between macro and micro information by shifting the focus of their attention. We therefore distinguish a near-term attention equilibrium, in which $\lambda_U$ is fixed and the split between $\lambda_M$ and $\lambda_S$ is endogeneous, from a longer-term information equilibrium, in which the decision to become informed is endogenized along with the choice of information on which to focus. We analyze the attention equilibrium in this section and address the information equilibrium in Section 5. The allocation of attention we study refers to the fraction of investors focused on each type of information, and not the allocation of attention by an individual agent.

4.1 Relative utility

Recall that we take an investor’s ex ante expected utility to be $J \equiv \mathbb{E}[-\exp(-\gamma \tilde{W}_2)]$, where the expectation is taken unconditionally over the time 2 wealth $\tilde{W}_2$.

Fixing the fraction of uninformed, the following lemma establishes the relative benefit of being $M$ and $S$ informed relative to being uninformed.

**Lemma 2.** If the cost of becoming informed is given by $c$, then the benefit of being $M$ informed relative to being uninformed is given by

$$
J_M/J_U = \exp(\gamma c) \left( 1 + \frac{f_M}{1 - f_M} (1 - \rho_F^2) \right)^{-\frac{1}{2}}.
$$

(31)

The benefit of being $S$ informed relative to being uninformed is given by

$$
J_S/J_U = \exp(\gamma c) \left( 1 + \frac{f_S}{1 - f_S} \left( \frac{1}{\rho_S^2} - 1 \right) \right)^{-\frac{1}{2}}.
$$

(32)

Note that because utilities in our model are negative, a decrease in the above ratios...
represents a gain in informed relative to uninformed utility.

Each of the ex ante utility ratios in the lemma is increasing in the corresponding measure of price efficiency – that is, informed investors become progressively worse off relative to uninformed as micro or macro prices become more efficient. But the dependence on $\rho_S^2$ in (32) differs from the dependence on $\rho_F^2$ in (31), a point we return to in Section 5.1. Recalling from Lemma 1 that macro and micro price efficiency increase in $\lambda_M$ and $\lambda_S$, respectively, we immediately get that

**Lemma 3** (Benefit of information decreases with number of informed). $J_S/J_U$ strictly increases (making $S$ informed worse off) in $\lambda_S$. $J_M/J_U$ strictly increases (making $M$ informed worse off) in $\lambda_M$.

Figure 1 illustrates the results of Lemma 2 and 1. The figure holds $\lambda_U$ fixed, and the x-axis is indexed by $\lambda_M$ (so $\lambda_S = 0$ is the rightmost point on the graph). As $\lambda_M$ increases, $J_M/J_U$ increases, indicating that the $M$ informed are becoming worse off. Similarly, at the rightmost point of the graph, $\lambda_S = 0$, and as we move to the left, $J_S/J_U$ increases, indicating that the $S$ informed are becoming worse off as more of their type enter the economy.

4.1.1 On the impossibility of informationally efficient markets

Recall that $\rho_F^2$ and $\rho_S^2$ are each zero when the respective fraction of $M$ and $S$ informed is zero. At this point the benefit of becoming informed is greatest. Micro and macro price efficiency increase with their respective $\lambda$’s and with the risk tolerance $1/\gamma$ of the informed investors. As $\lambda/\gamma$ (the mass of informed scaled by their risk tolerance) grows, price efficiency approaches 1. At this point, as can be seen from Lemma 2 the relative utility of informed and uninformed are identical when the cost of becoming informed is zero. If the cost to becoming informed is non-zero, no one will choose to do so, $\lambda/\gamma$ will fall, and price efficiency will fall away from 1. Therefore, it is not possible for prices to be fully revealing when information is costly. This is exactly the Grossman-Stiglitz result, but we have now established it as well for micro information.

As long as prices aren’t fully revealing, informed agents will be better off than the uninformed.

4.2 Choice between macro and micro information

For the analysis of the attention equilibrium, we hold fixed the fraction of uninformed $\lambda_U$. We assume that once an agent chooses to pay cost $c$, the agent can decide to learn about
Figure 1: The information equilibrium for a fixed number of uninformed investors. Relative utilities are shown assuming cost of becoming informed is $c = 0$. Parameter values are described in Section A.1.

either macro or micro information. At an interior equilibrium, the marginal investor must be indifferent between these two information sets, in which case equilibrium will be characterized by a $\lambda_M^*$ such that with that many macro informed investors and with $1 - \lambda_U - \lambda_M^*$ micro informed investors we will have $J_M = J_S$, which just sets (31) equal to (32). To cover the possibility of a corner solution, we define an equilibrium by a pair of proportions $\lambda_M \geq 0$ and $\lambda_S = 1 - \lambda_U - \lambda_M \geq 0$ satisfying

$$J_M < J_S \Rightarrow \lambda_M = 0 \quad \text{and} \quad J_S < J_M \Rightarrow \lambda_S = 0.$$  

(33)

The inequalities in this conditions are equivalent to $J_M / J_U > J_S / J_U$ and $J_S / J_U > J_M / J_U$, respectively, because $J_U < 0$.

Recall from Lemma [1] that when the fraction of $M$ or $S$ informed is zero, price efficiency
is also 0 (i.e. \( \rho_F^2(\lambda_M = 0) = 0 \) and \( \rho_S^2(\lambda_S = 0) = 0 \)). From (31) and (32), we see that

\[
J_M/J_U(\lambda_M = 0) = \sqrt{1 - f_M} \quad \text{and} \quad J_S/J_U(\lambda_S = 0) = 0;
\]

we are taking \( c = 0 \) because the fraction of uninformed is fixed. From Lemma 3 we know that \( J_M/J_U \) and \( J_S/J_U \) both increase monotonically (i.e. make the informed worse off) with their respective \( \lambda \)'s. When \( \lambda_M \) is zero, the \( M \) informed achieve their maximal utility; when \( \lambda_M = 1 - \lambda_U \), the \( S \) informed achieve their maximal utility. As \( \lambda_M \) increases from zero to \( 1 - \lambda_U \), \( \lambda_S \) decreases, so the macro informed become progressively worse off and the micro informed become progressively better off. If at some \( \lambda_M \) the two curves \( J_M/J_U \) and \( J_S/J_U \) intersect, we will have an interior equilibrium, and it must be unique because of the strict monotonicity in Lemma 3. This case is illustrated in Figure 1. If there is no interior equilibrium, then either macro or micro information is always preferred, an no investor will choose the other.

To make these observations precise, let us define

\[
\tilde{\lambda}_M \equiv (1 - \lambda_U) \frac{1 - \sqrt{\varphi} \sqrt{1 + \frac{1 - \varphi}{\varphi} \frac{\gamma^2 \alpha}{(1 - \lambda_U)^2}}}{1 - \varphi},
\]

where

\[
\varphi = \frac{(1 - f_S) \sigma_S^2 \sigma_X^2}{(1 - f_M) \sigma_M^2 \sigma_{X_F}^2} \quad \text{and} \quad \alpha = \frac{1 - f_M}{f_M} \frac{(1 - f_S) \sigma_S^2 \sigma_X^2}{(1 - f_M) \sigma_M^2 \sigma_{X_F}^2}.
\]

Note that \( \varphi \) is the ratio of the total risk arising from the unknowable portion of idiosyncratic supply shocks (i.e. the variance of \( \epsilon_i \) times the variance of \( X_i \)) to the total risk arising from macro supply shocks (i.e. the variance of \( \epsilon_M \) times the variance of \( X_F \)). The larger \( \varphi \) the more total unknowable risk comes from idiosyncratic rather than systematic sources.

The following proposition characterizes the equilibrium allocation of attention in the economy between macro information and micro information when the total fraction of informed investors \( 1 - \lambda_U \) is fixed.

**Proposition 3** (Attention equilibrium). Suppose \( 0 \leq \lambda_U < 1 \), so some agents are informed.

(i) Interior equilibrium\(^{11}\) If \( \tilde{\lambda}_M \in [0, 1 - \lambda_U) \), then this point defines the unique equi-

\(^{10}\)The expression on the right has a finite limit as \( \varphi \to 1 \), and we take that limit as the value of \( \tilde{\lambda}_M \) at \( \varphi = 1 \).

\(^{11}\)We refer to (i) as the case of an interior equilibrium, even though it includes the possibility of a
librium: at $\lambda_M^* = \tilde{\lambda}_M$, the marginal informed investor will be indifferent between becoming $M$ and $S$ informed.

(ii) If $\tilde{\lambda}_M \not\in [0, 1 - \lambda_U)$, the unique equilibrium is at the boundary $\lambda_M^* = 0$, where all informed agents are $S$ informed.

(iii) In equilibrium, we always have $\lambda_M^* < 1 - \lambda_U$. In other words, some informed agents will choose to be $S$ informed in equilibrium.

It bears emphasizing that our attention equilibrium – regardless of parameter values – precludes all informed agents from being $M$ informed. In contrast, it is possible for all informed agents to be $S$ informed. We therefore have, as a fundamental feature of the economy, a bias for micro over macro information. We will discuss this question further in Section 5.2.

To get some intuition into the drivers of the attention equilibrium, we consider two special cases in which $\tilde{\lambda}_M$ simplifies: when $\gamma = 0$ and when $\varphi = 1$. If $\gamma = 0$, then

$$
\tilde{\lambda}_M = \frac{1 - \lambda_U}{1 + \sqrt{\varphi}}. \tag{37}
$$

As noted before Proposition 3, $\varphi$ measures the relative magnitude of unknowable idiosyncratic and macro shocks. So, this expression suggests that, at least at low levels of risk aversion, agents favor information about the greater source of uncertainty, with $\tilde{\lambda}_M$ increasing in macro uncertainty and decreasing in micro uncertainty. This feature is consistent with our discussion of Jung and Shiller (2006) in the introduction; in particular, greater micro uncertainty shifts greater focus to micro information.

At $\varphi = 1$,

$$
\tilde{\lambda}_M = \frac{(1 - \lambda_U)}{2} \left[ 1 - \frac{\gamma^2 \alpha}{(1 - \lambda_U)^2} \right].
$$

Here it becomes evident that an increase in risk aversion moves investors toward micro information. The benefit to being macro informed comes from two sources, liquidity provision for the macro supply shocks $X_F$ and the ability to advantageously trade against the macro uninformed, whereas the benefit of being micro informed only comes from liquidity provision for the micro shocks $X_i$. When $\gamma$ increases, trade between the informed and uninformed falls, therefore diminishing the advantage of being $M$ informed. However,
the liquidity discount in micro and macro prices, i.e. $c_F$ and $c_S$ from (25) and (26), increases with risk aversion, making being micro informed relatively more attractive.

Similarly, an increase in micro (macro) volatility, as measured by $\sigma_S \sigma_X (\sigma_M \sigma_X F)$, will increase the benefit of information to the micro (macro) informed, and will therefore decrease (increase) $\lambda^*_M$ when the economy is in an interior equilibrium.

These two results hold when the economy is characterized by $\tilde{\lambda}_M$.

**Lemma 4** (Effects of risk aversion and risk on the attention equilibrium). We consider the case of an interior equilibrium with $\lambda^*_M > 0$.

(i) Risk aversion will push investors towards micro information:

$$\frac{d\lambda^*_M}{d\gamma} < 0.$$  

(ii) Increase in micro (macro) risk pushes investors towards micro (macro) information:

$$\frac{d\lambda^*_M}{d(\sigma_S \sigma_X)} < 0 \quad \text{and} \quad \frac{d\lambda^*_M}{d(\sigma_M \sigma_X F)} > 0.$$  

### 4.3 Impacts of information technology

**Effect of information precision on investor welfare**

Recall from (2) that $f_M$ ($f_S$) determines the amount of variation in $M$ ($S_i$) that is knowable from becoming informed. We refer to this as information precision.

We first show that, somewhat surprisingly, a better micro information technology makes the $S$ informed investors worse off.

**Lemma 5** (The $S$ informed overtrade on their information). Better information technology is worse for micro informed in the sense that

$$\frac{d(J_S / J_U)}{df_S} > 0 \quad \text{(micro informed are worse off)}.$$  

When investors become $S$ informed, the more they know about the ultimate idiosyncratic portion of the payout $S_i$ the less uncertainty they face from owning the stock. From (26) we see that the discount in the stock price due to idiosyncratic supply shocks $X_i$ will be zero when the information technology is perfect, i.e. when $f_S = 1$. With no discount in the price, the compensation for liquidity provision goes to zero. Because atomic informed agents cannot act strategically and coordinate to limit their liquidity provision in
an optimal (for them) way, uncertainty about the dividend helps them by decreasing the sensitivity of their demand to price shocks, which in turn leads to a higher risk premium in prices.

In contrast to the $S$ informed, the $M$ informed may be better or worse off as their information technology, $f_M$, improves.

**Lemma 6** (The $M$ informed can be better or worse off with more information). Better information technology is better for the $M$ informed if and only if

$$\rho_F^2 < \frac{1}{1 + f_M},$$  \hspace{1cm} (38)

which is equivalent to

$$\lambda_M < \gamma \sigma_M \sigma_X \frac{1 - f_M}{f_M}.$$  \hspace{1cm} (39)

In this case,

$$\frac{d(J_M/J_U)}{df_M} < 0 \quad \text{ (macro informed are better off).}$$

To gain intuition into this result recall that at $f_M = 0$ we would have $\rho_F^2 = 0$ (price reveals nothing when nothing about $M$ is knowable), and at $f_M = 1$ we would have $\rho_F^2 = 1$ (prices are fully revealing when $M$ is fully known). Furthermore, from Lemma 1 we know $\rho_F^2$ increases monotonically in $f_M$. So (38) implies that the $M$ informed benefit from an increase in the precision $f_M$ only when $f_M$ (hence also the price informativeness $\rho_F^2$) is low. Using (39) the condition can be reinterpreted as placing a limit on how many $M$ informed investors the economy can support before better macro precision begins to make the macro informed worse off.

The contrast between micro and macro information in Lemmas 5 and 6 can be understood as follows. In the market for the index fund, informed investors trade against uninformed investors as well meeting price insensitive liquidity shocks, introducing an effect that is absent in the market for individual stocks. With a poor information technology, prices are not very informative, so a small improvement in the information technology gives the $M$ informed an informational edge over the uninformed, allowing the informed to extract rents in trading. However, as the information technology improves and price efficiency grows, the incremental ability to extract rents from trading against the uninformed diminishes, while the tendency to overtrade on information (as in the case of the market for individual stocks) grows. At some point, determined by $\rho_F^2 = 1/(1 + f_M)$, the overtrading tendency begins to dominate the rent-extraction effect.
Effect of information precision on attention equilibrium

Recall that for a fixed $\lambda_U$, the equilibrium $\lambda_M^*$ (proportion of macro informed) is determined by the condition that $J_M/J_U = J_S/J_U$, in the case of an interior equilibrium. We note that $J_M/J_U$ does not depend on $f_S$. Therefore as $f_S$ rises and $J_S/J_U$ increases, $J_M/J_U$ can only increase if $\lambda_M$ increases (from Lemma 3). It follows that an interior equilibrium $\lambda_M^*$ must increase with $f_S$. As the benefit of being micro informed diminishes due to more precise micro information, the fraction of informed investors that focus on macro information grows. Figure 2 demonstrates this equilibrium adjustment. For every $\lambda_M$, a higher $f_S$ makes the micro informed worse off, which pushes the equilibrium number of macro informed higher.

Figure 2: The effect of increasing micro precision $f_S$ on the attention equilibrium with fixed $\lambda_U$. Parameter values are described in Section A.1.

Similarly, since $J_S/J_U$ does not depend on $f_M$ but decreases in $\lambda_M$ (i.e. micro informed are better off as there are fewer micro informed), if $J_M/J_U$ decreases (increases) in $f_M$, then $\lambda_M^*$ must increase (decrease) in $f_M$. Figure 3 illustrates this phenomenon. In the
figure, the equilibrium $\lambda_M^*$ is sufficiently small so that macro precision makes the macro informed better off. As macro precision $f_M$ increases, for a range of $\lambda_M$ that are sufficiently small, the macro informed become better off, which increases $\lambda_M^*$ (i.e. decreases the number of micro informed, thus making the remaining micro informed better off). Had the equilibrium $\lambda_M$ been sufficiently high, the effect would have had the opposite sign, as can be seen by the fact the $J_M/J_U$ increases with $f_M$ for high $\lambda_M$.

Figure 3: The effect of increasing macro precision $f_M$ on the attention equilibrium with fixed $\lambda_U$. Parameter values are described in Section A.1.

The preceding arguments establish the following result:

**Lemma 7** (Effect of information precision on equilibrium). In the case of an interior equilibrium with $\lambda_M^* > 0$, the number of macro informed increases as the micro information technology becomes more precise:

$$\frac{d\lambda_M^*}{df_M} > 0.$$  

Condition (38) (or equivalently (39)) is necessary and sufficient for the number of macro
informed to increase as the macro information technology becomes more precise:

\[
\frac{d\lambda^*_M}{df_M} > 0 \quad (0) \quad \text{if and only if} \quad \rho^2_F < \frac{1}{1 + f_M} \left( > \frac{1}{1 + f_M} \right).
\]

### 4.4 Equilibrium price informativeness

In equilibrium, the marginal investor is indifferent between becoming \( M \) or \( S \) informed. We define \( \tau_M \) as

\[
\tau_M \equiv \frac{f_M}{1 - f_M} / \frac{f_S}{1 - f_S},
\]

which measures the precision of the macro information technology relative to the micro information technology. If, by expending an equal amount of effort, investors are able to learn a greater (lesser) proportion of \( S \) than they can of \( M \), then \( \tau_M < 1 \) (\( \tau_M > 1 \)), and the micro information technology is relatively more (less) precise. From Lemma 2 we see that an interior equilibrium, where \( J_M/J_U = J_S/J_U \), micro price efficiency is related to macro price efficiency via \( (1 - \rho^2_S)/\rho^2_S = \tau_M(1 - \rho^2_F) \), which yields

\[
\rho^2_S = \frac{1}{1 + \tau_M(1 - \rho^2_F)}.
\]  

From this we see that as markets become fully macro efficient, they must also become fully micro efficient, and vice versa. In other words,

\[
\rho^2_S \to 1 \iff \rho^2_F \to 1.
\]

However, as macro price efficiency tends towards zero, micro price efficiency tends towards \( 1/(1 + \tau_M) \). Since both sides of (40) are decreasing as their respective \( \rho \) falls, this also represents the lower bound for \( \rho^2_S \) in equilibrium. This result is the direct consequence of part (iii) of Proposition 3 which shows that \( \lambda^*_S > 0 \), i.e. that in equilibrium there must be \( S \) informed traders, which from (29) implies that \( \rho^2_S \) cannot be zero. So we have

\[
\rho^2_S \to \frac{1}{1 + \tau_M} \iff \rho^2_F \to 0.
\]

In the limits, we see therefore that markets are either micro efficient or both markets are fully revealing – the latter limit being unattainable, as has already been argued.

We can make a more general statement about the relative price efficiency of the two markets. The following result holds at any interior equilibrium.
Proposition 4 (When are markets more micro or macro efficient?). If $\tau_M \leq 1$, then $\rho^2_S \geq \rho^2_F$ (markets are more micro efficient). If $\tau_M > 1$, then $\rho^2_S < \rho^2_F$ (markets are more macro efficient) for $\rho^2_F \in (1/\tau_M, 1]$ and otherwise $\rho^2_S \geq \rho^2_F$.

This result shows that if the micro information technology is more precise than the macro one (meaning $\tau_M < 1$) then markets will always be more micro efficient. However, if the macro technology is more precise then there will be a range of parameter values where markets can be either micro or macro efficient.

The difference in price efficiencies $\rho^2_S - \rho^2_F$ is illustrated in Figure 4 for the two $\tau_M$ regimes. In the left panel, where micro information is more precise ($\tau_M < 1$), $\rho^2_S > \rho^2_F$ and the difference is always positive. In the right panel, where macro information is more precise ($\tau_M > 1$), we see one region (shown as the filled-in area in the chart) where markets are more micro efficient and another region in which they are more macro efficient.

There is no obvious link between the precision of the information technology and the incentive for agents to become $S$ or $M$ informed. In fact as Lemma 5 showed, $S$ informed are worse off as their information technology improves (as $f_S$ increases). And Lemma 6 shows that $M$ informed can be better or worse off as their information technology improves, depending on how macro efficient prices already are. Proposition 4 shows that despite this incentive structure, the relative precision of the micro and macro price technologies has an important impact on the relative informativeness of micro and macro prices, in equilibrium.

Proposition 4 suggests that markets tend towards micro efficiency. First, it is likely that $\tau_M < 1$ though this is an empirical question. But even if $\tau_M > 1$, there is still a range of parameter values where markets are more micro efficient.

## 5 Information equilibrium

In Section 4 we examined the choice among informed agents to become macro informed or micro informed, holding fixed the total fraction of informed agents. That analysis describes a medium-term equilibrium, over a time scale long enough for informed agents to shift their focus of attention, but not long enough for uninformed agents to acquire the skills needed to become informed.

In this section, we examine a longer-term equilibrium in which the uninformed can become informed by incurring a cost $c$. We endogenize not only the choice between micro and macro information, but also the decision to become informed. An equilibrium in this setting — which we refer to as an information equilibrium — is defined by nonnegative
proportions \((\lambda_M, \lambda_S, \lambda_U = 1 - \lambda_M - \lambda_S)\) such that no agent of a type in strictly positive proportion prefers switching to a different type. Extending (33), we require that, for any \(\iota, \iota' \in \{M, S, U\}\),

\[
J_{\iota} / J_{\iota'} > 1 \Rightarrow \lambda_i = 0.
\] (41)

Recall that our utilities are negative, so the inequality on the left implies that type \(\iota'\) is preferred to type \(\iota\). The ratios \(J_M/J_U, J_S/J_U, \) and \(J_S/J_M\) all have well-defined limits as some or all of \(\lambda_M, \lambda_S, \) and \(\lambda_U\) approach zero. (This follows from the expressions for these ratios in (31) and (32) and the dependence of \(\rho_F^2\) and \(\rho_S^2\) on \(\lambda_M\) and \(\lambda_S\) in (28) and (29), respectively.) We may therefore evaluate and compare these ratios even in cases where one or more of the proportions \(\lambda_i\) is zero.
5.1 Effect of information cost $c$ on the equilibrium

Figure 5 helps illustrate the general results that follow. The figure plots the equilibrium proportion of each type of agent as a function of the cost $c$ of information acquisition. The figure divides into three regions. At sufficiently low costs, all agents prefer to become informed, so $\lambda_U = 0$. At sufficiently high costs, no investors choose to be macro informed, so $\lambda_M = 0$. At intermediate costs, we find agents of all three types. At all cost levels, some fraction of agents choose to be micro informed.

![Figure 5: Equilibrium proportions of macro informed, micro informed, and uninformed agents as functions of the cost of information acquisition $c$.](image)

To justify these assertions and to give an explicit characterization of the information equilibrium at each cost level $c > 0$, we first consider the possibility that all three types of agents are present in positive proportions. To be consistent with equilibrium, this outcome requires $J_M/J_U = J_S/J_U = 1$. Using the expressions for these ratios in (31) and (32), these equalities imply

$$\rho_F^2 = 1 - \frac{1 - f_M}{f_M} \left[ e^{2\gamma c} - 1 \right].$$  \hspace{1cm} (42)
\[ \rho_s^2 = \left( 1 + \frac{1 - f_s}{f_s} [e^{2\gamma c} - 1] \right)^{-1}. \]  

(43)

Setting these expressions equal to (28) and (29), respectively, we can solve for \( \lambda_M \) and \( \lambda_S \) to get

\[ \lambda_M(c) = \gamma (1 - f_M) \sigma_M \sigma_x \left( \frac{1}{(1 - f_M)(e^{2\gamma c} - 1)} - \frac{1}{f_M} \right)^{1/2} \]  

(44)

and

\[ \lambda_S(c) = \gamma (1 - f_S) \sigma_S \sigma_x \left( \frac{1}{(1 - f_S)(e^{2\gamma c} - 1)} \right)^{1/2}. \]  

(45)

The expression for \( \lambda_M(c) \) is valid for \( c < \bar{c} \), with

\[ \bar{c} = -\frac{1}{2\gamma} \log(1 - f_M). \]  

(46)

If \( \lambda_M(c) + \lambda_S(c) \leq 1 \), then \((\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c))\) defines an information equilibrium with \( J_M = J_S = J_U \).

Both \( \lambda_M(c) \) and \( \lambda_S(c) \) increase continuously and without bound as \( c \) decreases toward zero, so the equation

\[ \lambda_M(\bar{c}) + \lambda_S(\bar{c}) = 1, \]

defines the lowest cost at which we can meaningfully set \( \lambda_U = 1 - \lambda_M(c) - \lambda_S(c) \). At lower cost levels, we need to consider the possibility of an equilibrium with \( \lambda_U = 0 \).

Once we fix a value for \( \lambda_U \), the split between macro and micro informed agents is characterized by Proposition 3. Write \( \lambda^*_M(0) \) for the value of \( \lambda^*_M \) in Proposition 3 at \( \lambda_U = 0 \); this value is given either by the root \( \bar{\lambda}_M \) in (35) or zero. Set

\[ \lambda_M, \lambda_S, \lambda_U = \begin{cases} 
\lambda^*_M(0), 1 - \lambda^*_M(0), 0, & 0 < c < \xi; \\
\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c), & \xi \leq c < \bar{c}; \\
0, \lambda_S(c), 1 - \lambda_S(c), & c \geq \bar{c},
\end{cases} \]  

(47)

Then (47) makes explicit the equilibrium proportions illustrated in Figure 5: at large cost levels, \( \lambda_M = 0 \); at low cost levels, \( \lambda_U = 0 \) and \( \lambda_M \) and \( \lambda_S \) are constant; at intermediate cost levels, all three proportions are positive; at all cost levels, \( \lambda_S > 0 \). We always have \( \xi > 0 \) and \( \bar{c} < \infty \), so the low cost and high cost ranges are always present; but it is

\footnote{Curiously, the condition for \( \lambda_M(c) \) to satisfy (39) is \( c > -(1/2\gamma) \log(1 - f_M) \), suggesting that an increase in the precision of macro information benefits the macro informed when the cost of information is high.}
possible to have \( \zeta = \bar{c} \), in which case the intermediate cost range is absent. This occurs when \( \lambda_*^M(0) = 0 \) (see, in particular, case (ii) of Proposition 3) and implies that no investor chooses to be macro informed at any cost level.

**Proposition 5** (Information equilibrium). *At each \( c > 0 \), the proportions in (47) define the unique information equilibrium.*

From this characterization, we can deduce several properties of the information equilibrium. Let us define \( \Pi_M \) as the fraction of informed who are macro informed, or

\[
\Pi_M \equiv \frac{\lambda_M}{\lambda_M + \lambda_S}.
\]

When we are at an interior attention equilibrium, i.e. \( \lambda^*_M > 0 \), we see that \( \Pi_M \) is the coefficient of \( 1 - \lambda_U \) (since \( 1 - \lambda_U = \lambda_M + \lambda_S \)) in (35). Differentiating with respect to \( \lambda_U \) yields

\[
\frac{d\Pi_M}{d\lambda_U} < 0,
\]

when \( \lambda_M > 0 \). In other words, the more uninformed investors there are in the economy, the greater the fraction of informed investors who choose to be micro informed. The next result describes the dependence of \( \Pi_M \) on \( c \) and summarizes some features of the information equilibrium.

**Corollary 1** (Effect of information cost \( c \) on information equilibrium). *In equilibrium, with a cost of becoming informed given by \( c \), the following will hold:*

(i) *As \( c \) increases, the fraction \( \Pi_M \) of informed investors who choose macro information falls; moreover, \( \Pi_M \) is strictly decreasing in \( c \) if \( \lambda_M > 0 \) and \( \lambda_U > 0 \).

(ii) *There is a maximal cost given by (46) above which no agent will choose to be macro informed. If \( \gamma^2\alpha = 1 \), with \( \alpha \) as in (36), then no agent chooses to be macro informed at any cost level.

(iii) *As \( c \) grows the fraction of investors who are uninformed increases; moreover \( \lambda_U \) is strictly increasing in \( c \) wherever \( \lambda_U > 0 \).

(iv) *Micro and macro price efficiency are decreasing in \( c \).*

As \( c \) increases and the number of uninformed grows, if \( \Pi_M \) doesn’t change, the already micro and macro informed (i.e. assuming they don’t have to pay \( c \) to become informed) are relatively better off (this follows from Lemma 3). However, the micro informed gain
disproportionately more than the macro informed – as will be discussed Section 5.2. In order to maintain the attention equilibrium at the new higher $c$, we therefore need more micro informed to equilibrate the relative benefits of micro vs macro information. Therefore, $\Pi_M$ must fall when $c$ increases.

5.2 Are markets micro efficient?

A tendency for markets to be more micro than macro efficient has been a recurring theme of many of our results. For example, we have noted that:

- From Propositions 3 and 5, we note that the $\lambda^*_S > 0$ – there must always be some micro informed investors. It is, in fact, possible that there are only micro informed investors, so that $\lambda^*_M = 0$.

- From Lemma 4, we see that when investors in the economy become more risk averse, there will be more micro informed.

- From Proposition 4 when the micro information technology is more precise than the macro we always have that $\rho^2_S > \rho^2_F$ – i.e. prices are more micro efficient than macro efficient. Even when the macro technology is more precise, for low enough macro efficiency, the market will be more micro efficient.

- Corollary 1 shows that when information is costlier, a larger fraction of the informed investors choose micro information.

- In fact, from Proposition 5 we know that at a sufficiently high cost of becoming informed, all informed investors choose to be micro informed.

The direct cause for these results comes from Lemma 2 and equation (34). At $\rho^2_F = 0$ (and consequently when $\lambda_M = 0$ from (28)), $J_M/J_U = e^{\gamma c} \sqrt{1 - \lambda_M}$ and $d(J_M/J_U)/d\rho^2_S$ is finite. However, at $\rho^2_S = 0$ (and consequently when $\lambda_S = 0$ from (29)), $J_S/J_U = 0$ and $d(J_S/J_U)/d\rho^2_S = \infty$. The desire to become micro informed when micro price efficiency is zero is infinitely large – the rate of decrease in a micro investor’s utility as new micro informed enter is infinite\(^{13}\) – whereas when $\lambda_M = \rho^2_F = 0$ the desire to become macro informed is large but bounded. This larger marginal propensity towards micro information leads the model towards an information allocation favoring micro price efficiency.

The fundamental driver of this asymmetry in the model comes from the fact that macro supply shocks $X_F$ can be accommodated by macro informed and macro uninformed

\(^{13}\)Recall $J_S/J_U$ increasing means the $S$ informed are worse off.
investors. However, micro informed investors are the only ones capable of accommodating idiosyncratic supply shocks \(X_i\). And the benefit of being the sole liquidity provider to the first marginal micro investor is infinitely large. This is a fundamental feature of our model because of the no-trade results that we establish in Section 3.4. It isn’t simply that we assume that the non-\(i\)-informed do not participate in liquidity provision for idiosyncratic supply shocks \(X_i\), but in fact as long as the \(i\)-uninformed are unable to properly condition on \(P_i\), this result obtains endogenously.

The key assumption therefore is the inability of non-\(i\)-informed to make proper inferences about that company’s fundamentals from the price of stock \(i\). Given that non-\(i\)-informed know nothing about \(i\)’s fundamental prospects to begin with, this seems to be a reasonable assumption. Most people, whether they follow the stock market or not, form some opinion about the overall state of the economy from the level of the Dow Jones Industrial Average. However, people who are completely uninformed about the fundamentals of IBM are not likely to make any inferences about IBM’s fundamental prospects from observing its price – and therefore should (and in our model do) choose to stay out of the market for IBM’s stock.

6 Dynamics of Micro and Macro Attention

We now embed our model in a dynamic setting. We let the precision \(f_M\) of the macro information technology depend on \(\lambda^*_M\), the fraction of agents that choose to be macro informed, and then do the same for \(f_S\). We assume that \(f_M\) is increasing in \(\lambda^*_M\), so that as more agents focus on acquiring macro information, more macro information becomes available. The change in \(f_M\) in turn influences \(\lambda^*_M\), setting off an iterative process. The resulting process can generate cycles of high and low macro and micro informativeness. In effect, investors shift their focus and investments as opportunities to acquire information change, and then reverse their moves once the opportunities are exploited.

Our interest is in how changes in the precision of the information technology interact with the allocation of attention in the economy between macro and micro information, rather than on the decision to become informed. Throughout this section, we therefore hold \(\lambda_U\) fixed. As discussed in Section 4, the transitioning from being uninformed to an informed market participant may take seven to ten years.

\(^{14}\)Given the change in \(f_M\), we assume that the economy reaches the new equilibrium attention allocation \(\lambda^*_M\). An individual agent cannot affect \(f_M\), and each agent responds optimally to the change in \(f_M\). We assume that agents cannot coordinate. If they could, then they might collectively prefer a different allocation of attention in anticipation of the impact on \(f_M\).
Our basic setup is illustrated in the left panel of Figure 6. The curved line shows $\lambda^*_M$ as a function of $f_M$, based on Proposition 3; the curve has a flat segment in the lower left where $\lambda^*_M = 0$, as in part (ii) of the proposition. Consistent with Lemma 7, the relationship is not monotonic. The diagonal line in the figure shows a possible mapping from $\lambda^*_M$ to $f_M$. The key point is that the knowable fraction $f_M$ of macro uncertainty increases with the fraction of agents that become macro informed, and the simplest such relationship is linear. The value of $f_M$ at which the line reaches the horizontal access measures the knowable fraction of macro uncertainty when the first agent becomes macro informed.

The point at which the curve and the diagonal line cross is a fixed point of the iteration that updates each of $f_M$ and $\lambda^*_M$ in response to a change in the other. As illustrated in the figure, this equilibrium is approached from any initial point with $\lambda^*_M > 0$. At $\lambda^*_M = 0$, the value of $f_M$ is sufficiently large to attract investors, making $\lambda^*_M$ positive and then driving the iteration toward the equilibrium.

The configuration in the right panel of Figure 6 has three fixed points. In this example, the knowable fraction $f_M$ at $\lambda^*_M = 0$ is too small to attract investors, so we have an equilibrium with no macro informed agents in the lower left. The second intersection of the two lines is an unstable equilibrium: from initial conditions with a slightly larger value of $\lambda^*_M$, the increase in $f_M$ attracts more macro informed agents and drives $f_M$ toward the equilibrium in the upper right. Starting from a slightly lower value of $\lambda^*_M$, the low value of $f_M$ eventually drives out all macro informed agents.

In these examples, changes to $f_M$ and $\lambda^*_M$ occur only out of equilibrium. The examples of Figure 7 show that more complex dynamics are possible. The configuration in the left panel of Figure 7 is similar to that of the left panel of Figure 6 but the sensitivity of $f_M$ to $\lambda^*_M$ is now greater. In this case, an equilibrium cycle emerges: starting from either of the circled points in the figure, the economy oscillates between those two points. Numerical experiments suggest that the system is drawn toward the equilibrium cycle from any initial condition, as in the case illustrated in the figure.15 The right panel of Figure 7 shows an equilibrium cycle that oscillates through four combinations of $f_M$ and $\lambda^*_M$.

In the examples of Figure 7, fluctuation in the fraction of macro informed agents becomes an equilibrium phenomenon, within the limitations of our dynamic formulation. When $f_M$ is at its lower value in the left panel, agents see an opportunity and more of them choose to become macro informed. These agents act myopically, not anticipating the effect

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15 The point of intersection of the diagonal line and the curve is still a fixed point, but it is now an unstable equilibrium, with points starting nearby drawn toward the equilibrium cycle.
that their decision to become macro informed will have. They drive up $f_M$ which, beyond a certain point, makes macro information less valuable, prompting a decrease in $\lambda^*_M$. The economy thus oscillates between periods when macro information is underexploited (high $f_M$, low $\lambda^*_M$), and periods in which it is overexploited (low $f_M$, high $\lambda^*_M$). We think of these cycles as occurring over several years, which supports the assumption that agents do not anticipate the consequences that their attention choices have on the future level of $f_M$.

It is not accidental that the length of the equilibrium cycle lengths as the diagonal line becomes less steep, as we move from the left panel of Figure 6 to the two panels of Figure 7. This is the pattern we would expect based on results for similar mappings in May (1976) and Grandmont (1985). The discussion in May (1976) suggests the possibility of cycles of arbitrary length and even “chaotic” dynamics as the diagonal line flattens. A flatter diagonal line implies a greater sensitivity of $f_M$ to $\lambda^*_M$. We would expect a high sensitivity of $f_M$ to $\lambda^*_M$ in an emerging market — where macro-relevant information may be easier to obtain — and much less sensitivity in a developed economy. This interpretation then predicts that emerging markets would experience greater variability in $f_M$ and $\lambda^*_M$.

In our final example, we allow $f_S$, the precision of the micro information technology, to vary as well. Each of the curves in the right panel of Figure 8 shows a mapping from $f_S$ to $\lambda^*_S = 1 - \lambda_U - \lambda^*_M$, the fraction of micro informed agents. Recall from Lemma 7 that $\lambda^*_M$ increases with $f_S$, so $\lambda^*_S$ decreases with $f_S$. The two curves in the right panel

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16More precisely, what matters is the steepness of the line relative to the slope of the curve at the point of intersection.
Figure 7: The left panel shows an equilibrium cycle that oscillates between the two circled points. The right panel shows an equilibrium cycle of length four.

illustrate this relationship at two different levels of $f_M$. The diagonal line in the figure shows the response of $f_S$ to $\lambda^*_S$: as more agents become micro informed, the knowable fraction of micro uncertainty increases.

The two panels of Figure 8 together illustrate equilibrium dynamics that can emerge when both $f_M$ and $f_S$ change. The economy cycles between two states, indicated by the circles in the figure. When $\lambda^*_M$ is at its higher value in the left panel, $\lambda^*_S$ is at its lower value in the right panel. These proportions cause $f_M$ to increase and $f_S$ to decrease. With both of these parameters changing, the mappings in the two figures shift from the solid to the dashed curves: the $\lambda^*_M$ curve on the left shifts down, and the $\lambda^*_S$ curve on the right shifts up, moving the economy to a state with a higher fraction of micro informed agents and a lower fraction of macro informed agents.

This pattern is consistent with a cycle observed in practice. In some periods, the market rewards stock selection; in others, macro risk dominates and the main decision facing investors is not which stocks to buy but at what level to participate in the market. In our setting, these cycles arise without a change in the relative magnitudes of macro and micro risk, as measured by $\sigma_M$ and $\sigma_S$. Instead, they are driven by the interaction between the knowable fraction of macro and micro uncertainty and agents’ choices to become macro or micro informed.

17In this example, $\lambda_U = 0.4$, so $\lambda^*_M + \lambda^*_S = 0.6$. 
Figure 8: An equilibrium cycle in which both \( f_M \) and \( f_S \) oscillate. The circled points in the left panel show the equilibrium pairs \((f_M, \lambda^*_M)\), and the circled points in the right panel show the equilibrium pairs \((f_S, \lambda^*_S)\). The solid (dashed) curve in the left panel is the mapping \( f_M \mapsto \lambda^*_M \) when \( f_S \) is at the higher (lower) of the two circled values in the right panel. The solid (dashed) curve in the right panel is the mapping \( f_S \mapsto \lambda^*_S \) when \( f_M \) is at the lower (higher) of the two circled values in the left panel.

7 Applications

7.1 Systematic and idiosyncratic volatility

We define excess returns on stock \( i \) in our model as \( u_i - R_P \). We decompose this into the systematic return component \( \beta_i(M - R_P) \) and an idiosyncratic return component

\[
\begin{align*}
  u_i - R_P - \beta_i(M - R_P) &= \epsilon_i + \gamma(1 - f_S)\sigma^2_S X_i / \lambda_S .
\end{align*}
\]

The idiosyncratic return variance can be written as

\[
\begin{align*}
  Vol^2_{idio} &= \sigma^2_{\epsilon_S} \left( 1 + \frac{\gamma^2 \epsilon^2_{\epsilon_S} \sigma^2_X}{\lambda^2_S} \right) \\
  &= \sigma^2_{\epsilon_S} \left( 1 + \frac{f_S}{1 - f_S} \left[ \frac{1}{\rho^2_S} - 1 \right] \right) .
\end{align*}
\]
where the second equation follows from (29). Interestingly, we can use this relationship to rewrite $J_S/J_U$ from (32) as

$$J_S/J_U = \exp(\gamma c) \frac{\sigma_{\epsilon S}}{Vol_{\text{idio}}}.$$  

This makes clear that the benefit of being $S$ informed increases with the idiosyncratic return volatility (recall $J_S/J_U$ falls when $S$ informed are better off) but decreases in the unknowable part of idiosyncratic dividend volatility.

As can be seen from (50), idiosyncratic return volatility consists of two components. The first comes from the unknowable portion of the idiosyncratic dividend, $\epsilon_i$, and the second comes from price adjustment in response to idiosyncratic supply shocks. Note that the knowable portion of the dividend payout $s_i$ does not enter into the idiosyncratic return because of the $s_i/R$ term in $P_i$.

The systematic variance of stock returns (or equivalently, the variance of index fund returns) is given by

$$Vol^2_{syst} \equiv \text{var}(M - RP_F) = \sigma^2_{\epsilon M} + (1 - Rb_F)^2 \sigma^2_M + R^2 c_F^2 \sigma^2_{X_F},$$  

which follows from (13). While we have a closed form solution for all quantities in this equation (see Proposition 2 and equation (A.2)), there unfortunately is no simple and general characterization for $Vol^2_{syst}$.

We have a simple expression for systematic volatility in two special cases of the model. When $f_M \to 0$\footnote{This implies that $\rho^2_F = 0$ from (28) for any value of $\lambda_M$. For a non-zero cost of becoming informed, we will also have that $\lambda_M = 0$ since the macro signal will contain zero information. The relationship in (53) surprisingly holds even if $\lambda_M > 0$.}, we will have

$$Vol^2_{syst} \to \sigma^2_M \left(1 + \gamma^2 \sigma^2_M \sigma^2_{X_F}\right)$$  

where we have assumed $\beta_i = 1$. Note that this is an exact relationship. Comparing this to the first equation in (51), we see that when no information is revealed about the common component of the dividend $M$, the variance of the index fund return (i.e. $M - RP_F$) consists of the variance of $M$ and the variance in $P_F$ that comes from accommodating the aggregate supply shock $X_F$.

At the other extreme, when $f_M$ approaches 1, and therefore when $\rho^2_F$ approaches 1,
the systematic variance is given by

\[ Vol_{syst}^2 = \sigma^2_{\epsilon_M} \left( 1 + \frac{\gamma^2 \sigma^2_{\epsilon_M} \sigma^2_{X_F}}{\lambda_M^2} \right) + o(\sigma^4_{\epsilon_M}), \tag{54} \]

which follows from (A.4) and (30), and where \( \sigma^2_{\epsilon_M} = (1 - f_M)\sigma^2_M \). Unlike (53) this is an approximate relationship because at \( f_M = \rho^2_F = 1 \), the economy becomes degenerate. But (54) has exactly the same interpretation except that the dividend uncertainty is no longer about \( M \) but is about \( \epsilon_M \), the unknowable part of \( M \) (since the knowable part is fully revealed by the price). Note the similarity between the systematic return volatility decomposition in (54) and the idiosyncratic return volatility decomposition in (51). Indeed, in the case of macro signal precision approaching 1, the \( M \) informed play the same role in the index fund market and the \( S \) informed play in the market for the idiosyncratic portion of stock \( i \)'s supply shock.²⁰

Figure 9 shows a numerical example of how systematic volatility given by (52) behaves in the model as a function of \( \Pi_M \), the fraction of informed that are macro informed, for different levels of precision \( f_M \) of the information technology. The blue circle in the upper left corner shows systematic volatility from (53) when macro prices reveal nothing about \( m \), and the colored diamonds on the right hand side of the figure show systematic volatility from (54) when the macro signal precision \( f_M \) approaches 1. The approximation improves, as can be expected, as \( f_M \) and \( \Pi_M \) grow (see Footnote 19).

7.1.1 How volatility depends on attention allocation

From (51) and the fact that \( \sigma^2_{\epsilon_S} = (1 - f_S)\sigma^2_S \), we make two observations about idiosyncratic return volatility:

- \( Vol_{idio} \) falls as \( f_S \) increases, and
- \( Vol_{idio} \) falls as \( \lambda_S \) (and therefore \( \rho^2_S \)) increases.

As more of the idiosyncratic portion of the dividend \( S_i \) becomes knowable, idiosyncratic return volatility falls. As there are more micro informed investors, prices become less sensitive to idiosyncratic supply shocks, and return volatility again falls.

We observe from Figure 7 that systematic volatility has the same qualitative dependence on \( f_M \) and \( \lambda_M \) as does idiosyncratic volatility on \( f_S \) and \( \lambda_S \) respectively.

²⁰See discussion in Section 3.4.1

²⁹Note that (54) would hold exactly when \( \rho^2_F = 1 \), which could happen for \( f_M < 1 \) as \( \gamma/\lambda_M \to 0 \). Therefore, for a given \( f_M \), the approximation in (54) is a better fit for higher values of \( \lambda_M \) (and therefore of \( \Pi_M \)).
Given this discussion, we make one more observation about the joint behavior of idiosyncratic and systematic volatilities:

- **Holding all else equal, when $\lambda_M$ (or $\lambda_S$) changes $Vol_{idio}$ and $Vol_{syst}$ move in opposite directions.**

Empirical tests of these implications would require the identification of plausibly exogenous variation in $f_M$, $f_S$, $\lambda_M$, and/or $\lambda_S$. Events such as the closure of a research department at a brokerage firm for reasons having nothing to do with the characteristics of the stocks the department covered may provide a setting where this effect can be identified.\(^\text{21}\)

\(^{21}\)Hong and Sraer (2015), Li (2015), and Gao et al. (2016) relate stock returns to levels of disagreement in analysts’ forecasts of either macro or micro fundamentals. Disagreement reflects the knowable fractions,
7.1.2 Common factor in idiosyncratic volatility

The relationship in (51) sheds some light on the finding in Herskovic, Kelly, and Lustig (2014) that idiosyncratic volatility has a strong common component. As Section 6 makes clear, in the dynamic version of our economy \( f_S \) becomes a state variable as shifts of investors from one information set to the other affect the productivity of the information technology. Idiosyncratic return volatility has two components. The first, \( \sigma^2_{\epsilon_S} \), is the variance of the unknowable portion of idiosyncratic dividend payouts. The second component in (51) is a common component across all stocks, and has to do with the model’s information equilibrium. Since \( f_S \) is a state variable (which also enters into \( \rho^2_S \)) this second component induces a strong factor structure into idiosyncratic volatility in our economy.

To differentiate this information acquisition hypothesis from the household income risk explanation proposed in Herskovic, Kelly, and Lustig (2014) is an interesting area for future empirical work.

7.2 Excess volatility and comovement

It has long been recognized that stock returns exhibit more volatility than can be justified by a present value relationship between prices and future earnings or dividends (LeRoy and Porter (1981) and Shiller (1981) are the classic papers in the area). Later empirical work has documented the fact that stocks also exhibit a higher covariance than is justified by their cashflows (for example, Pindyck and Rotemberg (1993) and Barberis, Shleifer and Wurgler (2005)). We show in this section that the excess volatility and excess comovement phenomena are, in fact, closely related, and are both driven by the relative micro vs macro efficiency of markets.

In our model the covariance between the dividend of stocks \( i \) and \( j \) is given by

\[
\text{cov}(u_i, u_j) = \beta_i \beta_j \sigma^2_M.
\]

The covariance between excess returns of \( i \) and \( j \) is given by

\[
\text{cov}(u_i - RP_i, u_j - RP_j) = \beta_i \beta_j \text{Vol}^2_{syst}.
\]

If we define excess comovement as a higher covariance of returns than the covariance of \( f_M \) and \( f_S \), the fundamental volatilities, \( \sigma_M \) and \( \sigma_S \), and the fractions \( \lambda_M \) and \( \lambda_S \). Our model provides a possible framework for examining the implications of macro and micro disagreement simultaneously.
dividends, then this reduces to

\[ Vol_{syst}^2 > \sigma_M^2 \iff \text{Excess comovement.} \]

Return comovement exceeds earnings comovement when index fund returns are more volatile than index fund earnings. In other words, excess macro volatility leads to excess covariance.

The shaded region in Figure 9 shows when \( Vol_{syst} \) is higher than \( \sigma_M \) (indicated by the horizontal dashed line).\(^{22}\) We see that when macro signal precision \( f_M \) is high or when there are a large fraction of macro informed \( \Pi_M \), return volatility is lower than fundamental volatility – that is we have insufficient covariance. This happens because prices reveal so much information about \( M \) that \( u_i - RP_i \) becomes relatively \( M \) insensitive.

Excess volatility and therefore excess covariance arise when the macro signal is very imprecise (i.e. \( f_M \) is low) or when there are relatively few macro informed investors (i.e. \( \Pi_M \) is small). The tendency of actual stocks to exhibit excess covariance and excess volatility, as empirical work suggests, lends more evidence to the micro efficiency (and macro inefficiency) of markets.

Peng and Xiong (2006) show that excess comovement can arise in a behavioral representative investor model when investors are sufficiently overconfident in the quality of their information. Veldkamp (2006) shows that excess comovement can be obtained in a fully rational framework when investors tend to learn the same information as other investors and then make inferences about security payoffs based on the prices of a small set of common securities. Our model identifies a new and very broad channel of excess comovement – the relative micro versus macro efficiency of the market.

### 7.2.1 Excess idiosyncratic volatility

From equation (51), we see that the square of the ratio of idiosyncratic return volatility to idiosyncratic dividend volatility is given by

\[
\frac{Vol_{idio}^2}{\sigma_S^2} = 1 + f_s \left[ \frac{1}{\rho_S^2} - 2 \right],
\]

\(^{22}\)Though the shaded excess covariance region is small in the figure to show the full range on systematic volatility behavior, our view is that this is the region most representative of actual markets.
from which we see that there is excess idiosyncratic volatility (i.e. $Vol_{idio}^2 > \sigma_S^2$) if and only if

$$\rho_S^2 < \frac{1}{2}.$$ 

Therefore excess idiosyncratic volatility falls when price efficiency increases. There is no equally clean characterization of the relationship between systematic return and dividend volatility, although Equations (53) and (54) yield comparable expressions for the limits $f_M \to 0$ and $f_M \to 1$ respectively.

### 7.2.2 Excess correlation

We should note that using Peng and Xiong’s (2006) definition of excess comovement as return correlation being higher than dividend correlation leads to the following characterization in our model

$$\frac{Vol_{syst}}{\sigma_M} > \frac{Vol_{idio}}{\sigma_S} \iff \text{Excess comovement},$$

for the case of two representative stocks with unit betas. This characterization leads to the same intuition as in the covariance case. Relative micro efficiency (i.e. low $Vol_{idio}$ relative to $\sigma_S$) (which happens when micro price efficiency is high, according to Equation (55)) is associated with excess comovement, as is relative macro inefficiency (i.e. high $Vol_{syst}$ relative to $\sigma_M$).

Equation (56) can be interpreted in terms of the finding in Vuolteenaho (2002) that firm level market adjusted returns (i.e. his analogue to $u_i - RP_i - \beta_i(M - RP_F)$) are driven predominantly by cashflow news (Table III in his paper) and that market excess returns (i.e. $M - RP_F$) are more driven by discount rate – and not cashflow – news (their Table VII). Since $\sigma_M$ and $\sigma_S$ proxy for cashflow news in our model, Vuolteenaho’s results therefore suggest that the ratio on the left hand side of (56) is high, and the ratio on the right hand side is low. This is additional evidence of micro efficiency.

This discussion suggests that further empirical work is needed to connect excess comovement to measures of micro efficiency, as well as to the information choices (i.e. micro or macro) made by investors.

### 7.3 Empirical tests of $f_M$ versus $f_S$

One of the key drivers of results in our model has been the ratio of knowable versus total risk, both at the macro and micro levels. Recall from Proposition 4 if $\tau_M \leq 1$, markets
are always micro efficient (i.e. $\rho_S^2 > \rho_F^2$). Determining $\tau$ involves knowing $f_M$ and $f_S$. One approach to estimating these quantities is to run the following earnings forecasting regression at the individual name or industry (or economy-wide) level

$$\Delta EPS_i(t + 1) = a + \gamma' X_i(t) + \epsilon(t + 1),$$

where $X(t)$ is the set of predictive variables (potentially including lagged $EPS_i$) for $i$’s earnings that are known prior to time $t + 1$. In the regression, $i$ can be either a set of companies, a set of industries or a set of countries. The $R^2$ of this regression would be an estimate of a lower bound for either $f_S$ (if run at the individual name level) or $f_M$ (if run at some level of aggregation). The estimate is a lower bound because we cannot be sure that $X$ includes all the relevant and knowable information for forecasting $i$’s earnings. Such a study would shed light on an important relationship for modeling micro versus macro efficiency.

Furthermore, the analysis in Section 6 posits the existence of a relationship between $f_S$ and $f_M$ and $\lambda^*_S$ and $\lambda^*_M$ respectively. If we use assets under management (AUM) in various institutional arrangements (such as macro hedge funds, or fundamentally driven mutual funds) as proxies for the model’s $\lambda$’s, then we can analyze whether the explanatory power of (57) increases as AUM in the relevant set of funds increases, as posited by the theoretical analysis.

### 7.4 Evolution of active money management

Our model has profound implications for the evolution of the active money management sector. As Section 6 argued, if investors are unable to anticipate what effect their specialization today will have on investment performance in the next several years, they overcommit to a particular specialty thereby driving down the benefit of information, and then shift their focus to another specialty, at which point the cycle repeats. The key driver of this effect is the fact that the benefit to being macro informed can increase or decrease with $f_M$, as was shown in Lemma 6, which in turn causes $\lambda^*_M$ to be a hump-shaped function of $f_M$ (see Lemma 7).

Figure 10 shows that assets under management devoted to different hedge fund styles exhibit large time variation, presumably in response to overcommitment by investors to a particular strategy, followed by the flight of investor assets to less “crowded” strategies. Note, in particular, the apparent negative correlation between AUM in macro and long-short equity (where macro risk is hedged), and between distressed and fixed income. We
interpret this as anecdotal evidence that saturation of opportunities in one asset class is followed by capital flows into related asset classes that are perceived by investors to offer better opportunities.

![Hedge fund strategy AUM as percent of total](image)

Figure 10: Percent of aggregate hedge fund assets under management devoted to different styles. Data is from BarclayHedge at [http://www.barclayhedge.com/research/indices/ghs/mum/HF_Money_Under_Management.html](http://www.barclayhedge.com/research/indices/ghs/mum/HF_Money_Under_Management.html).

The variation in $f_S$, $f_M$, $\lambda_S$ and $\lambda_M$ implied by the discussion in Section 6 has far reaching implications for markets. As investors move into micro focused styles, we should expect: micro volatility to fall, micro volatility to comove, macro volatility to increase, and for prices to exhibit excess comovement and volatility relative to fundamentals. Furthermore, regression of the type in (57), should exhibit higher $R^2$'s for single names as opposed to industry or macro level aggregation. As micro focused styles become saturated, all these effects should diminish or reverse. Such variation should therefore be correlated with, and indeed caused by, fluctuations in investor specialization choices.

### 7.4.1 Effects of changes in the cost of information

**Time trends**

We predict (as would Grossman and Stiglitz (1980)) that the amount of money being actively managed falls as information costs rise. This is a potential explanation for the
increase in passively managed funds over the past few years

As the cost of computational power and data has fallen, the amount of information that is in the public domain has increased, and the cost $c$ of learning information that is unknown to others has increased. Our model’s prediction that $d\Pi_M/dc < 0$ would imply that as money has been moving into passively managed products, the proportion of actively managed funds that are micro-focused has increased. We are not aware of any work that has addressed whether this has or has not taken place.

**Cross-sectional implications**

Related to the above point, it is likely that in less developed markets, less information is publically known, and the cost of learning information that is not widely known is likely lower than in developed markets. Therefore, the active money management industry in emerging markets should be more macro-focused than in developed markets. Furthermore, using the methodology proposed in Section 7.3, we can investigate the conjecture from our discussion of equilibrium dynamics from Section 6 that the slope of the $f_M$ curve (as a function of $\lambda^*_M$) is steeper in emerging relative to developed markets. Again, we are not aware of formal empirical analysis of these questions.

### 7.5 Recessions

Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) is a related model that studies how investors allocate limited attention to macro and micro uncertainty. In their paper, as in ours, the individual stock dividend process is given by beta times a macro shock plus an idiosyncratic shock. While our paper investigates the specialization choices of investors into micro and macro informed, their paper analyzes how informed investors, who all receive signals from the same distribution, would choose to allocate the precision of those signals between micro and macro information. The two papers analyze related problems, but from two different, and complementary, vantage points. Therefore contrasting the mechanisms at work in the two theories should be informative for our understanding of the functioning of actual markets.

The main theoretical results in Kacperczyk et al. (2016) follow from their Proposition 1, which states that the informed investors allocate more attention to securities with greater cashflow variance, and their Proposition 2, which states informed investors will allocate more attention to the macro component of cashflows ($M$ in our model) as risk.

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23 See, for example, “Active asset managers knocked by shift to passive strategies” from the *Financial Times*, April 11, 2016.
aversion increases. A recession in their model is defined as a time of increased risk aversion and increased macro dividend volatility, $\sigma_M$, which allows them to conclude in Section 2 that

“...in recessions, the average amount of attention devoted to aggregate shocks should increase and the average amount of attention devoted to stock-specific shocks should decrease.”

The corresponding question in our model can be posed as follows: How does $\lambda^*_M$ depend on risk aversion $\gamma$ and dividend volatility (given by $\sigma_M$ and $\sigma_S$)? From Lemma 4 we know that (1) increasing risk aversion pushes investors towards micro information, (2) increasing macro dividend risk $\sigma_M$ pushes investors towards macro information, and (3) increasing idiosyncratic dividend risk $\sigma_S$ pushes investors to micro information. Whereas in Kacperczyk et al. (2016) the risk aversion and macro dividend risk effects reinforce each other, in our model they offset. Furthermore if idiosyncratic dividend risk $\sigma_S$ were to increase in a recession, we show that this would push investors further towards micro information. Our models only make the same prediction for recessions characterized by a small increase in $\gamma$, negligible increase in $\sigma_S$, and a large increase in $\sigma_M$.

A key difference in the two models is the fact that increasing risk aversion, or increasing aggregate risk, diminishes one benefit of being macro informed in our model (trade with the macro uninformed falls) but increases the benefit of liquidity provision to idiosyncratic noise trades by the micro informed. This mechanism isn’t present in the Kacperczyk et al. (2016) model, and may account for the different predictions.

Kacperczyk et al. (2016) show that funds’ portfolio deviations from the market co-vary more with future changes in industrial production during recessions, and that these portfolio deviations co-vary more with future firm-specific earnings shocks during booms. They interpret this as evidence that funds are more macro-focused during recessions, and more micro-focused during booms. An alternative explanation for this observation is that funds perceive the market to be more micro-efficient during recessions and therefore tailor their portfolios to take advantage of macro opportunities, and similarly during booms funds perceive the market to be more macro-efficient and therefore focus their portfolios on micro opportunities. This latter interpretation would explain the finding in Kacperczyk et al. (2014) that mutual funds seem to be better stock pickers during booms, and better

\[24\] Kacperczyk et al. (2016) document that during recessions systematic return volatility increases, while idiosyncratic return volatility increases by much less (and the latter increase is not statistically different from zero). However, because $\sigma_M$ and $\sigma_S$ measure dividend rather than return uncertainty, their behavior in recessions may be different.
market timers during recessions. Furthermore it is consistent with the implications of our model that investors gravitate towards micro information in recessions characterized by an increase in risk aversion and an increase in both macro and micro dividend risks.

A better understanding of which of these interpretations is a better fit for the combined results of Kacperczyk et al. (2014) and Kacperczyk et al. (2016) is an important area for future work.

7.6 Trading volume

Lo and Wang (2000) document rich, but slow moving (see their Figure 1), time series variation in aggregate and idiosyncratic turnover (shares traded over shares outstanding) in US stocks over the period 1962–1996. Such low frequency variation in trading volume has direct implications in our model for the incentive to become micro or macro informed, as well as for the choice between being uninformed and informed.

We note from (28) and (29) that \( \rho_S^2 \) and \( \rho_F^2 \) are: (a) decreasing in the variance of supply shocks \( \sigma_{X_F}^2 \) and \( \sigma_X^2 \) respectively, and (b) increasing in the proportion of macro and micro informed (\( \lambda_M \) and \( \lambda_S \)) respectively. At an interior equilibrium, \( J_U = J_M = J_S \), so for a fixed cost \( c \) we see from (31) and (32) that as supply variation increases, the proportion of informed must increase as well in order to maintain all investor groups at their indifference points. Indeed, this is exactly what we see from Equations (44) and (45) which show that the information equilibrium shares of macro and micro informed, \( \lambda_M(c) \) and \( \lambda_S(c) \), are increasing in \( \sigma_{X_F}^2 \) and \( \sigma_X^2 \) respectively. Furthermore, this must occur in a way so as to keep \( \rho_S^2 \) and \( \rho_F^2 \) exactly unchanged – as we see from Equations (42) and (43) information equilibrium price efficiency does not depend on \( \sigma_X \) or \( \sigma_{X_F} \). This result is a generalization of Theorem 4 from Grossman and Stiglitz (1980) to the case of systematic and idiosyncratic risk.

The following Lemma summarizes this relationship:

Lemma 8 (Effect of supply volatility on proportion of informed.). Assuming a fixed cost \( c \) of becoming informed and an equilibrium with all three types of investors in positive proportions, as the variation of micro \( \sigma_X^2 \) (macro \( \sigma_{X_F}^2 \)) supply shocks increases, the fraction of micro \( \lambda_S \) (macro \( \lambda_M \)) investors in the economy increases, the fraction of macro (micro) investors stays the same, and the fraction of uninformed \( \lambda_U \) falls. Furthermore, \( \rho_S^2 \) (\( \rho_F^2 \)) stays exactly the same.

\(^{25}\text{If } \sigma_M \text{ and } \sigma_S \text{ both increase in the same proportion, therefore leaving their ratio and } \varphi \text{ from (36) unchanged, then the same logic underlying the result in Lemma 4 that } d\lambda_M'/d\gamma < 0 \text{ implies that } \lambda_M' \text{ would fall.}\)
Since trade in our model is caused by supply shocks, higher turnover at the macro and micro level is associated with higher $\sigma^2_{X_F}$ and $\sigma^2_X$ respectively. As aggregate turnover increases, we should expect the number of uninformed investors to fall, and the number of macro informed investors to increase. As the idiosyncratic portion of turnover increases, we expect the number of uninformed investors to again fall, and for the number of micro informed investors to increase. However, surprisingly, in both cases we do not expect price informativeness to change, as a higher number of either micro or macro informed enter the market to exactly offset the effects of increased turnover. This connection between turnover and the number of informed investors has not been investigated to our knowledge.

Foucault et al. (2011) study a natural experiment – the introduction of a new settlement procedure – in the French stock market that has the effect of raising the cost of speculative trading by individual investors. They show that this new settlement procedure lowers retail trading volume and also lowers the volatility of stock returns, a result which the authors interpret to mean that “retail investors behave as noise trades.” If we think of supply shocks as proxying for all price insensitive trading, whether from liquidity shocks or irrational beliefs, then the natural experiment of Foucault et al. maps directly into a reduction in $\sigma_X$ in our model, which then reduces idiosyncratic return volatility as shown in (51). Our model therefore predicts that following this event, once the market settled into its new information equilibrium, the number of micro informed investors in the French stock market should have fallen. Whether this happened or not is an interesting area for future empirical work.

8 Conclusion

Professional investors typically specialize along a particular dimension of knowledge. This choice of specialization has important effects on the behavior of market prices. We model an economy where investors can choose to be informed either about micro or macro fundamentals. An important driver of this choice is the ability for micro informed investors to accommodate, and thereby extract rents from, the idiosyncratic portion of security supply shocks. Our model sheds light on a longstanding question in financial economics – whether markets are better at incorporating information at the micro or macro level.

In our setting, we show a general tendency of markets towards micro rather than macro efficiency. Furthermore, the choice by investors of specializing in micro or macro information has direct bearing on:

- The dynamics of AUM across different areas of specialization;
• The relationship between idiosyncratic and systematic volatility;
• The excess volatility of stock returns relative to earnings;
• The tendency of stocks to comove in excess of their cashflow covariance;
• The amount of information knowable about company-specific or aggregate earnings;
• The recent proliferation of passively managed money in the US;
• The relative micro–macro focus of emerging market versus developed market investors;
• The allocation of information choices in recessionary periods;
• The impact of trading volume on information choices.

An empirical analysis of these (and other) questions raised by our model should yield valuable insights into the operation of financial markets.

A Appendix

A.1 Parameter values for numerical examples

The parameter values used in the paper’s numerical examples are:

\[ \gamma = 1.5; [f_M, f_S] = [0.75, 0.75]; [\sigma^2_M, \sigma^2_S] = [0.25, 0.25]; R = 1.05; \bar{X}_F = 0; [\sigma^2_{X_F}, \sigma^2_X] = [0.5, 1]. \]

A.2 Solution of model

Proof of Proposition 1. The analysis is simplified if we allow \( i \)-informed agents to invest in the index fund and in a hedged security paying \( u_i - \beta_i u_F = S_i \), with price \( P_i - \beta_i P_F \). If we let \( \bar{q}_F \) and \( \bar{q}_S_i \) denote the demands in this case, the demands in the original setting are given by \( q^i = \bar{q}_S_i \) and \( q^i_F = \bar{q}_F - \beta_i \bar{q}_S_i \). By standard arguments, the modified demands are given by

\[
\begin{bmatrix}
\bar{q}^i_F \\
\bar{q}^i_S_i
\end{bmatrix} = \frac{1}{\gamma} \text{var} \left[ \begin{bmatrix} M \\ S_i \end{bmatrix} | I_i \right]^{-1} \left( \text{E} \left[ \begin{bmatrix} M \\ S_i \end{bmatrix} | I_i \right] - R \left[ \begin{bmatrix} P_F \\ P_i - \beta_i P_F \end{bmatrix} \right] \right).
\]

Now

\[
\text{var} \left[ \begin{bmatrix} M \\ S_i \end{bmatrix} | I_i \right] = \begin{bmatrix} \text{var}[M | I_i] & \text{var}[S_i | I_i] \\ \text{var}[S_i | I_i] & \text{var}[S_i | I_i] \end{bmatrix} = \begin{bmatrix} \text{var}[M | P_F] & \text{var}[M | P_F] \\ (1 - f_S) \sigma^2_S \end{bmatrix},
\]

53
and

\[ E \left[ \frac{M}{S_i} \mid I_i \right] = \frac{E[M|P_F]}{s_i}. \]

Thus, \( \bar{q}^i_F = q^U_F \), with \( q^U_F \) as given in (19), and

\[ \bar{q}^i_{S_i} = \frac{s_i - R(P_i - \beta_i P_F)}{\gamma(1 - f_S)\sigma_S^2}. \]

As \( q^i_i = \bar{q}^i_{S_i} \), equation (21) follows, and then \( q^i_F = \bar{q}^i_F - \beta_i \bar{q}^i_{S_i} = q^U_F - \beta_i q^i_i \) completes the proof.

\[ \square \]

Proof of Proposition 2. The price \( P_F \) can be derived from first principles, but we can simplify the derivation by reducing it to the setting of Grossman and Stiglitz (1980). The informed (18) and uninformed (19) demands for the index fund and the market clearing condition (23) reduce to the demands in equations (8) and (8') of Grossman and Stiglitz (1980) and their market clearing condition (9), once we take \( \lambda = \lambda_M \) and \( 1 - \lambda_M = \lambda_U + \lambda_S \). The coefficients of the price \( P_F \) in (13) can therefore be deduced from the price in their equation (A10). Theorem 1 of Grossman-Stiglitz gives an expression for \( P_F \) in the form

\[ \alpha_1 + \alpha_2 w \lambda, \]

for constants \( \alpha_1 \) and \( \alpha_2 > 0 \), where, in our notation,

\[ w \lambda = m - \frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M}(X_F - \bar{X}_F). \]

Comparison with (13) now implies that

\[ \frac{c_F}{b_F} = -\frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M}. \]

Setting \( b_F \) equal to the coefficient of \( x (= X_F) \) in (A10) of Grossman-Stiglitz, we get

\[ b_F = \frac{\lambda_M}{\lambda_M (1 - f_M)\sigma_M^2} \frac{1 - \lambda_M}{\text{var}[M|w]}, \]

and

\[ \frac{f_M \sigma_M^2}{\text{var}[M|w]} + \frac{1 - \lambda_M}{\text{var}[M|w]} \]

Moreover,\n
\[ \text{var}[w \lambda] = (1 - f_M)\sigma_M^2 + \frac{\gamma^2(1 - f_M)^2\sigma_M^4}{\lambda_M^2}\sigma_X^2, \]

and \( \text{var}[M|w \lambda] = \text{var}[M|P_F] \). To evaluate \( \text{var}[M|P_F] \), note that the only unknown term in (20) is \( K_F b_F \), which we can now evaluate using (25) to get

\[ K_F b_F = \frac{b_F^2 f_M \sigma_M^2}{b_F^2 f_M \sigma_M^2 + c_F^2 \sigma_X^2} \]

\[ = \frac{f_M \sigma_M^2}{f_M \sigma_M^2 + \frac{\gamma^2(1 - f_M)^2\sigma_M^4}{\lambda_M^2}\sigma_X^2}. \]

54
This yields an explicit expression for \( \text{var}[M|P_F] \) which in turn yields an explicit expression for \( b_F \) through (A.2). An expression for \( c_F \) then follows using (25). Finally, to evaluate the constant term \( a_F \), we can again match coefficients with the expression in (A10) of Grossman-Stiglitz. Alternatively, we can evaluate their (A10) at (using their notation) \( \theta = E\theta^* \) and \( x = Ex^* \), which, in our notation yields

\[
a_F = \frac{\bar{m}}{R} - \frac{\bar{X}}{R} \left[ \frac{1 - \lambda_M}{\gamma \text{var}(M|P_F)} + \frac{\lambda_M}{\gamma(1 - f_M)\sigma^2_M} \right]^{-1}. \tag{A.3}
\]

Equation (26) follows directly from (21) and market clearing. Summing over \( i \) we get

\[
\frac{1}{N} \sum_i P_i = P_F - \frac{\gamma (1-f_S)\sigma^2_S}{\lambda_S R} \xi \quad \text{by virtue of the idiosyncratic shock conditions in (11) and the fact that betas are 1 on average. Therefore, condition (4) is satisfied if and only if } \xi = 0. \]

A.2.1 Approximation of \( P_F \) when \( f_M \) is near 1

Let \( \delta = (1-f_M)\sigma^2_M \). Then

\[
a_F = \frac{\bar{m}}{R} - \frac{\bar{X}}{R} \gamma \delta + O(\delta^2) \]
\[
b_F = \frac{1}{R} + O(\delta^2) \quad \tag{A.4}
\]
\[
c_F = -\frac{\gamma R}{\lambda_M} \delta + O(\delta^2).
\]

The key point is that there is no \( O(\delta) \) term in \( b_F \); in other words, \( b_F = 0 \) at \( \delta = 0 \). It is intuitive that \( b_F \) (the sensitivity of the index fund price to the macro fundamental \( m \)) is maximized in the fully revealing case, but it is surprising that its derivative is zero there. The expressions in (A.4) follow from differentiation of the coefficients derived in the proof of Proposition 2. We omit the details.

A.3 No-Trade Results

Analysis under Assumption 1

Without loss of generality, we consider the perspective of an agent informed about stock 1. The agent may trade in the index fund \( F \) and in stocks \( 1, \ldots, K \), with \( K \leq N-1 \); the \( N \)th stock is a redundant asset to an agent that may invest in the fund and all other stocks because \( \sum_i S_i = \sum_i X_i = 0 \).

The agent chooses investment levels \( q^1_F, q^1_1, \ldots, q^1_K \) by solving

\[
\max_{q_F, q_1, \ldots, q_K} E \left[ -\exp \left( -\gamma [q_F(u_F - R P_F) + \sum_{k=1}^K q_k(u_k - R P_k)] \right) \right] \left| s_1, P_1, P_F \right] ,
\]

where we have dropped the superscripts on the quantities \( q_j \) for simplicity. In this formu-
lation, we do not allow the agent to condition on the prices \( P_j, j = 2, \ldots, K \); in particular, then, the agent cannot condition the demand \( q_j \) on the price \( P_j \). Because the payoffs to the securities are conditionally Gaussian, the agent’s optimal portfolio is given by

\[
\begin{pmatrix}
q_F \\
q_1 \\
\vdots \\
q_K
\end{pmatrix} = \frac{1}{\gamma \text{var}} \begin{pmatrix}
\mathbb{E} \left[ u_F - RP_F \right] \\
\mathbb{E} \left[ u_1 - RP_1 \right] \\
\vdots \\
\mathbb{E} \left[ u_K - RP_K \right]
\end{pmatrix}^{-1} \begin{pmatrix}
\mathbb{E} \left[ S \right] - \mathbb{E} \left[ RP_S \right] \\
\mathbb{E} \left[ S \right] - \mathbb{E} \left[ RP_S \right]
\end{pmatrix}.
\] (A.5)

If the idiosyncratic payouts \( S_k, k = 1, \ldots, K \), and supply shocks \( X_k, i = 1, \ldots, K \), were independent, then \( s_1, P_1, \) and \( P_F \) would carry no information about the idiosyncratic portion of \( u_j - RP_j, j \neq 1 \), and the agent would have no reason to take on the corresponding idiosyncratic risk — in other words, we would have \( q_j^1 = 0, j \neq 1 \). (This conclusion will be confirmed in the proof below.) Even with the small amount of dependence we have introduced through the covariances in (12), these investments remain negligibly small:

**Proposition 6.** The agent’s demands satisfy \(|q_j^1| = O(K/N^2), j = 2, \ldots, K\). In particular, if the agent is restricted to a fixed number \( K \) of stocks, these demands are \( O(1/N^2) \), and if \( K = N - 1 \), then the demands are \( O(1/N) \).

Here and in the proof that follows, an expression like \( O(1/N^2) \) means a quantity whose absolute value is bounded by \( c/N^2 \), for all sufficiently large \( N \), for some constant \( c \) not depending on \( N \).

**Proof.** The agent’s portfolio optimization problem is equivalent to choosing a position in the index fund and in hedged stocks with payoffs \( u_k - \beta_k M = S_k \) and prices \( P_{S_k} = P_k - \beta_k P_F, k = 1, \ldots, K \). With \( K \leq N - 1 \), the agent’s demand for the \( k \)th hedged stock in this formulation coincides with the agent’s demand for the \( k \)th stock in the original formulation because

\[
S_k - RP_{S_k} = (u_k - RP_k) - \beta_k(u_F - RP_F), \quad k = 1, \ldots, K.
\]

In terms of hedged stocks, the agent’s vector of optimal demands becomes

\[
\begin{pmatrix}
q_F \\
q_1 \\
\vdots \\
q_K
\end{pmatrix} = \frac{1}{\gamma \text{var}} \begin{pmatrix}
\text{var}[M|s_1, P_1, P_F] & 0 \\
0 & \text{var}[S - RP_S|s_1, P_1, P_F]
\end{pmatrix}^{-1} \begin{pmatrix}
\mathbb{E} \left[ u_F - RP_F \right] \\
\mathbb{E} \left[ S - RP_S \right]
\end{pmatrix} \left( s_1, P_1, P_F \right),
\]

where we have written \( S \) for \((S_1, \ldots, S_K)^\top\) and \( P_S \) for the corresponding vector of prices. In light of the block diagonal structure of the conditional covariance matrix, it suffices to evaluate

\[
\text{var}[S - RP_S|s_1, P_1, P_F]^{-1} \mathbb{E}[S - RP_S|s_1, P_1, P_F].
\] (A.6)

From (14) and (25), we have

\[
RP_{S_k} = s_k + cX_k \quad \text{and} \quad S_k - RP_{S_k} = \epsilon_k - cX_k, \quad k = 1, \ldots, K,
\]
with $c = -\gamma(1 - f_s)\sigma_s^2/\lambda_s$. We may therefore write (A.6) as

$$\text{var}[S - RP_S | X_1]^{-1}E[S - RP_S | X_1].$$

Let $a = \sigma_s^2$ and $b = c^2\sigma_s^2$. For all distinct $j$ and $k$,

$$\text{var}[S_k - RP_{S_k}] = a + b, \quad \text{cov}[S_j - RP_{S_j}, S_k - RP_{S_k}] = -\frac{a + b}{N - 1}.$$

Also,

$$\text{var}[S_1 - RP_{S_1} | X_1] = a, \quad \text{var}[S_j - RP_{S_j} | X_1] = a + b - \frac{b}{(N - 1)^2}, \quad j = 2, \ldots, K,$n

and, for $j \neq k$,

$$\text{cov}[S_j - RP_{S_j}, S_k - RP_{S_k} | X_1] = \begin{cases} -\frac{a}{(N - 1)}, & j = 1; \\ -\frac{(a + b)}{(N - 1)} - \frac{b}{(N - 1)^2}, & j \neq 1. \end{cases}$$

Set $\delta = 1/N$. These expressions then yield

$$\text{var}[S - RP_S | X_1] = \begin{pmatrix} a & -a\delta & -a\delta & \cdots & -a\delta \\ -a\delta & a + b(1 - \delta^2) & -(a + b)\delta - b\delta^2 & \cdots & -(a + b)\delta - b\delta^2 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ -a\delta & -(a + b)\delta - b\delta^2 & -(a + b)\delta - b\delta^2 & \cdots & a + b(1 - \delta^2) \end{pmatrix}.$$  (A.7)

So

$$\text{var}[S - RP_S | X_1] = \begin{pmatrix} \sqrt{a} & \sqrt{a + b} & \cdots & \sqrt{a + b} \\ \sqrt{a + b} & \cdots & \sqrt{a + b} \\ \vdots & \ddots & \ddots & \sqrt{a + b} \\ \sqrt{a + b} & \cdots & \sqrt{a + b} \end{pmatrix} (I - \delta B) \begin{pmatrix} \sqrt{a} & \sqrt{a + b} & \cdots & \sqrt{a + b} \\ \sqrt{a + b} & \cdots & \sqrt{a + b} \\ \vdots & \ddots & \ddots & \sqrt{a + b} \\ \sqrt{a + b} & \cdots & \sqrt{a + b} \end{pmatrix},$$

where

$$B = \begin{pmatrix} 0 & \sqrt{a}/\sqrt{a + b} & \sqrt{a}/\sqrt{a + b} & \cdots & \sqrt{a}/\sqrt{a + b} \\ \sqrt{a}/\sqrt{a + b} & b\delta/(a + b) & 1 + b\delta/(a + b) & \cdots & 1 + b\delta/(a + b) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \sqrt{a}/\sqrt{a + b} & 1 + b\delta/(a + b) & 1 + b\delta/(a + b) & \cdots & b\delta/(a + b) \end{pmatrix}.$$  

The sum of the entries in the first row of $\delta B$ is

$$(K - 1)\delta \frac{\sqrt{a}}{\sqrt{a + b}} < \frac{\sqrt{a}}{\sqrt{a + b}} < 1,$$
and in any other row the entries sum to 
\[ \delta \frac{\sqrt{a}}{\sqrt{a} + b} + (K - 2)\delta + (K - 1)\delta^2 \frac{b}{a + b} < (K - 1)\delta + (K - 1)\delta^2 < 1. \]

It follows that
\[ (I - \delta B)^{-1} = I + \delta B + \delta^2 B^2 + \cdots = I + \delta B + O(\delta^2 K), \]
and then \( \text{var}[S - RP_S|X_1]^{-1} \) is given by
\[
\begin{pmatrix}
\sqrt{a} \\
\sqrt{a} + b \\
\vdots \\
\sqrt{a} + b
\end{pmatrix}^{-1}
(I + \delta B)
\begin{pmatrix}
\sqrt{a} \\
\sqrt{a} + b \\
\vdots \\
\sqrt{a} + b
\end{pmatrix}^{-1} + O(\delta^2 K),
\]
where the error term denotes a matrix whose entries are \( O(\delta^2 K) \). This expression simplifies to
\[
\text{var}[S - RP_S|X_1]^{-1} = \begin{pmatrix}
1/a & \delta/(a + b) & \cdots & \delta/(a + b) \\
\delta/(a + b) & 1/(a + b) & \cdots & \delta/(a + b) \\
\vdots & \vdots & \ddots & \vdots \\
\delta/(a + b) & \cdots & 1/(a + b)
\end{pmatrix} + O(\delta^2 K).
\]

For the conditional means we have
\[ E[S_1 - RP_S|X_1] = -cX_1 \quad \text{and} \quad E[S_j - RP_S|X_1] = \frac{cX_1}{N - 1} = cX_1 \delta, \quad j = 2, \ldots, K. \]

The agent’s demand for the hedged stocks is then given by
\[
\begin{pmatrix}
1/a & \delta/(a + b) & \cdots & \delta/(a + b) \\
\delta/(a + b) & 1/(a + b) & \cdots & \delta/(a + b) \\
\vdots & \vdots & \ddots & \vdots \\
\delta/(a + b) & \cdots & 1/(a + b)
\end{pmatrix}
\begin{pmatrix}
-cX_1 \\
cX_1 \delta \\
\vdots \\
cX_1 \delta
\end{pmatrix} + O(\delta^2 K) = \begin{pmatrix}
-cX_1 / \gamma a + O(K \delta^2) \\
O(K \delta^2) \\
O(K \delta^2) \\
O(K \delta^2)
\end{pmatrix}.
\]

In other words, an investor informed about stock 1 takes a negligibly small position in the other stocks. \( \square \)

The demand \( q_1 = -cX_1 / \gamma a \) simplifies to \( X_1 / \lambda_S \), which is the market clearing condition when only 1-informed agents can invest in stock 1. If there were no correlation across stocks (in payoffs or supply shocks), we would have \( \delta = 0 \) in \([A.7]\), and the conclusion of the proposition would then be that \( q_j = 0, \ j = 2, \ldots, K. \)
Analysis under Assumption 2

In contrast to the setting of Proposition 6, we now allow agents to set their demands based on market prices, but we assume that agents do not otherwise make inferences or update their beliefs based on the prices of the expanded set of assets.

In more detail, each agent makes inferences based on information $I$. For uninformed agents, this is just $P_F$; for $M$-informed agents, this is $\{s_1, P_1, P_F\}$; and for 1-informed agents this is $\{s_1, P_1, P_F\}$. We consider an agent of any of these types who may invest in the index fund and in stocks $1, \ldots, K$. An agent who makes inferences based on all prices $P = (P_F, P_1, \ldots, P_K)\top$, would choose investment levels $q = (q_F, q_1, \ldots, q_K)\top$ by maximizing

$$
E \left[ -e^{-\gamma q\top(u-\text{RP})} | I, P \right] = -e^{-\gamma q\top(E[u|I,P]-\text{RP})+(\gamma^2/2)q\top\text{var}[u|I,P]q},
$$

with $u = (u_F, u_1, \ldots, u_K)\top$. Instead, we suppose the agent maximizes

$$
-e^{-\gamma q\top(E[u|I]-\text{RP})+(\gamma^2/2)q\top\text{var}[u|I]q}.
$$

(A.8)

In this sense, the agent uses all prices in choosing the quantities $q$, but the agent makes inferences based only on $I$. The agent does not learn about fundamentals from prices not contained in $I$. This coincides with the competitive (Walrasian) equilibrium concept of Lang et al. (1992), p.322.

Let $\tilde{J}_{K,N}$ denote the agent’s expected utility when choosing a portfolio by maximizing (A.8). This expected utility depends on $K$ through the set of stocks in which the agent is allowed to invest, and it depends on $N$ through the covariances in (12). Let $J$ denote the agent’s expected utility under the earlier restrictions ($M$-informed and uninformed agents investing only in the index fund, 1-informed agents investing in the fund and stock 1). This expected utility does not depend on $K$ or $N$; in particular, the joint distribution of $(M, S_1)$ does not depend on $K$ or $N$. The agent benefits from the expanded set of investments if $\tilde{J}_{K,N}/J < 1$.

Maximizing (A.8) yields

$$
q = \frac{1}{\gamma} \text{var}[u|I]^{-1}(E[u|I] - \text{RP}),
$$

with expected utility

$$
\tilde{J}_{K,N} = E \left[ -e^{-(E[u|I]-\text{RP})\text{var}[u|I]^{-1}(u-\text{RP})} \right].
$$

(A.9)

For fixed $K$, the covariance terms in (12) approach zero as the total size of the market increases. We suppose that for any $K \geq 2$ there is some $\gamma' > 1$ such that, for fixed $K$,

$$
\sup_{N \geq K} E \left[ e^{-\gamma'(E[u|I]-\text{RP})\text{var}[u|I]^{-1}(u-\text{RP})} \right] < \infty.
$$

Only the covariances in (12) change as $N$ varies in this expression. This condition is
sufficient to ensure that $\tilde{J}_K = \lim_{N \to \infty} \tilde{J}_{K,N}$ exists and equals the expected utility in (A.9) evaluated with $\text{cov}[s_j, s_k] = \text{cov}[\epsilon_j, \epsilon_k] = \text{cov}[X_j, X_k] = 0$, for all distinct $j, k = 1, \ldots, K$.

The following result shows that the agent is better off not investing in the expanded set of assets when the prices of those assets are sufficiently informative.

**Proposition 7.** If $\rho_S^2/(1 - \rho_S^2) < 1/(1 - f_s)$, then $\tilde{J}_{K,N}/J < 1$, for all sufficiently large $K$ and $N$. If $\rho_S^2/(1 - \rho_S^2) > 1/(1 - f_s)$, then $\tilde{J}_{K,N}/J > 1$, for all sufficiently large $K$ and $N$. In particular, the second case holds in any information equilibrium with $\lambda_M > 0$ and $f_M \leq f_s$.

The last condition indicates that under typical circumstances the agent is better off not investing in the expanded set of assets.

**Proof.** We evaluate the limit $\tilde{J}_K$ by evaluating the expectation (A.9) with $\text{cov}[s_j, s_k] = \text{cov}[\epsilon_j, \epsilon_k] = \text{cov}[X_j, X_k] = 0$, for all distinct $j, k = 1, \ldots, K$. As in the proof of Proposition 6, we may replace the original securities with hedged securities. Thus, we may take $u = (M, S_1, \ldots, S_K)^\top$, with prices $P = (P_F, P_{S_1}, \ldots, P_{S_K})^\top$, and $R P_{S_k} = s_k + cX_k$, $k = 1, \ldots, K$, where $c = -\gamma(1 - f_s)\sigma_S^2/\lambda_S$. In the case of a 1-informed investor, we have $\mathcal{I} = \{s_1, P_{S_1}, P_F\}$.

$$V = \text{var}[u|\mathcal{I}] = \begin{pmatrix} \text{var}[M|P_F] & \sigma_{\epsilon S}^2 & \sigma_{\epsilon S}^2 \\ \sigma_{\epsilon S}^2 & \sigma_S^2 & \sigma_S^2 \\ \vdots & \sigma_S^2 & \sigma_S^2 \end{pmatrix},$$

and

$$E[u|\mathcal{I}] - R P_S = (E[M|P_F] - R P_F, -cX_1, -s_2 - cX_2, \ldots, -s_K - cX_K)^\top.$$

Thus,

$$\tilde{J}_K = -E \left[ e^{-\frac{(E[M|P_F] - R P_F)(M - R P_F)}{\text{var}[M|P_F]}} \right] \left[ e^{-\frac{cX_1(\epsilon_1 - cX_1)}{\sigma_S^2}} \prod_{k=2}^K \left[ e^{-\frac{(s_k + cX_k)(\epsilon_k - cX_k)}{\sigma_S^2}} \right] \right].$$

The behavior of $\tilde{J}_K$ for large $K$ is therefore determined by the factors inside the product over $k$, all of which are identical. To characterize these factors, we use the following lemma:

**Lemma 9.** Let $(Y, Z)$ have a bivariate normal distribution with mean zero, $\text{var}[Y] = \text{var}[Z] = 1$ and $-1 < \text{cov}[Y, Z] = r < 0$. Let $G(\theta) = E[\exp(\theta Y Z)]$, for $-1/(1 - r) < \theta < 1/(1 + r)$. Then $G(\theta) < 1$ for $\theta \in (0, -2r/(1 - r^2))$ and $G(\theta) > 1$ for $\theta$ outside the interval $[0, -2r/(1 - r^2)]$. 

60
Proof. From equation (10) of Craig (1936), we get

\[ G(\theta) = \frac{1}{\sqrt{(1 - (1 + r)\theta)[1 + (1 - r)\theta]}}. \]

The function \( G \) is strictly convex (because it is a moment generating function) and \( G(\theta) \uparrow \infty \) as \( \theta \downarrow -1/(1-r) \) or \( \theta \uparrow 1/(1+r) \). Moreover, \( G(0) = G(-2r/(1-r^2)) = 1 \) and \( G'(0) < 0 \). Thus, \( G(\theta) < 1 \) between these two roots and \( G(\theta) > 1 \) outside the closed interval defined by the two roots.

To apply this result, set

\[ Y = \frac{s_k + cX_k}{\sqrt{f_S\sigma_S^2 + c^2\sigma_X^2}}, \quad Z = \frac{\epsilon_k - cX_k}{\sqrt{(1-f_S)\sigma_S^2 + c^2\sigma_X^2}}, \]

with correlation

\[ r = \frac{-c^2\sigma_X^2}{\sqrt{f_S\sigma_S^2 + c^2\sigma_X^2}\sqrt{(1-f_S)\sigma_S^2 + c^2\sigma_X^2}}. \]

Then

\[ a \equiv E \left[ e^{(s_k+cX_k)(\epsilon_k-cX_k)\theta}\sigma_S^2 \right] = E[e^{\theta Y Z}] \]

with

\[ \theta = \frac{\sqrt{f_S\sigma_S^2 + c^2\sigma_X^2}\sqrt{(1-f_S)\sigma_S^2 + c^2\sigma_X^2}}{\sigma_S^2}. \]

Thus, we have \( a < 1 \) if \( \theta < -2r/(1-r^2) \). This condition simplifies to \( c^2\sigma_X^2 > f_S(1-f_S)\sigma_S^2 \). At the same time, we have \( \rho_S^2 = f_S\sigma_S^2/(f_S\sigma_S^2 + c^2\sigma_X^2) \). Combining these conditions, we conclude that \( a < 1 \) under the first condition in the proposition, and \( a > 1 \) under the second condition. It follows that, as \( K \) increases, \( \tilde{J}_K/J \to 0 \) under the first condition and \( \tilde{J}_K/J \to \infty \) under the second condition, from which the proposition follows.

We have detailed the case of a 1-informed agent. In the case of an \( M \)-informed or uninformed agent, \( \tilde{J}_K \) is proportional to \( a^K \), so the same conclusion holds.

For the last assertion in the proposition, we can rewrite the condition \( \rho_S^2/(1-\rho_S^2) > 1/(1-f_S) \) as

\[ \frac{\lambda_S^2}{1-f_S} > \alpha \gamma^2 \frac{f_M}{1-f_M}, \]

with \( \alpha \) as defined in (36). Evaluating \( \lambda_S(\hat{c}) \) in (45) at \( \hat{c} \) in (46) yields \( \alpha \gamma^2 = \lambda_S(\hat{c}) \), so this condition is equivalent to

\[ \frac{\lambda_S^2}{\lambda_S^2(\hat{c})} > \tau_M. \]

If \( f_M \leq f_S \), then \( \tau_M \leq 1 \), so this condition holds if \( \lambda_S > \lambda_S(\hat{c}) \). It follows from (47) and Proposition 5 that this condition is satisfied in any information equilibrium with \( \lambda_M > 0 \). 

\[ \square \]
A.4 Attention equilibrium

Proof of Lemma 2. The factor \( \exp(\gamma c) \) in (31) and (32) reflects the cost of information acquisition. Since we are holding \( \lambda_U \) fixed and comparing \( J_M/J_U \) with \( J_S/J_U \), we may set \( c = 0 \) and omit this factor. We make repeated use of Proposition 3.1 of Admati and Pfleiderer (1987), which yields for \( \iota \in \{M, U, S\} \),

\[
J_{\iota} = -\frac{\text{var}[u_{\iota} - RP_{\iota}|I_{\iota}]}{\text{var}[u_{\iota}]}^{1/2} \exp \left( E[u_{\iota} - RP_{\iota}]^\top \text{var}[u_{\iota} - RP_{\iota}]^{-1}E[u_{\iota} - RP_{\iota}]/2 \right).
\]

Here, \( u_{\iota} \) denotes the payoff of the asset(s) in which agents of type \( \iota \) invest, which is simply \( u_F \) for \( \iota = M, U \), and \( (u_F, u_i)^\top \) for \( \iota = S \). The corresponding prices are recorded in \( P_{\iota} \). The information sets are \( I_M = \{m, P_F\} \), \( I_U = \{P_F\} \), \( I_S = \{s_i, P_i, P_F\} \), and \( |\cdot| \) indicates the determinant of a matrix. Thus,

\[
J_U = -\left( \frac{\text{var}[M|P_F]}{\text{var}[u_F - RP_F]} \right)^{1/2} \exp(-E[u_F - RP_F]^2/2\text{var}[u_F - RP_F])
\]

and

\[
J_M = -\left( \frac{\text{var}[M|m]}{\text{var}[u_F - RP_F]} \right)^{1/2} \exp(-E[u_F - RP_F]^2/2\text{var}[u_F - RP_F]),
\]

so

\[
J_M/J_U = \left( \frac{\text{var}[M|P_F]}{(1 - f_M)^2 \sigma_M^2} \right)^{-1/2}.
\]

Combining (20) and (27), we get

\[
\text{var}[M|P_F] = f_M \sigma_M^2(1 - \rho^2_F) + (1 - f_M) \sigma_M^2,
\]

from which (31) follows.

To evaluate \( J_S \), we can use the same transformation as in the proof of Proposition 1 and assume that \( i \)-informed agents optimize over uncorrelated securities paying \( M \) and \( S_i \) rather than paying \( M \) and \( \beta_i M + S_i \). The achievable utility is unchanged by this linear transformation of payoffs. Let \( P_{S_i} = P_i - \beta_i P_F \) denote the price of the security paying \( S_i \). Using (A.1) and the fact that \( E[S_i - RP_{S_i}] = 0 \), we get

\[
J_S = -\left( \frac{\text{var}[M|P_F]}{\text{var}[u_F - RP_F]\text{var}[S_i - RP_{S_i}]} \right)^{1/2} \exp(-E[u_F - RP_F]^2/2\text{var}[u_F - RP_F]).
\]

Thus,

\[
J_S/J_U = \left( \frac{\text{var}[S_i - RP_{S_i}]}{(1 - f_S)^2 \sigma_S^2} \right)^{-1/2}.
\]
Using first (26) and then (29), we get

\[
\text{var}[S_i - RP_{S_i}] = (1 - f_S)\sigma_{S_i}^2 + \frac{\gamma^2(1 - f_S)^2\sigma_X^4}{\lambda_S^2}\sigma_X^2,
\]

\[
= (1 - f_S)\sigma_{S_i}^2 + f_S\sigma_{S_i}^2\left(\frac{1}{\rho_{S_i}^2} - 1\right),
\]

from which (32) follows.

\[\square\]

Proof of Proposition 3. As noted in (34), \(J_{S_i}/J_{U}\) approaches zero as \(\lambda_S = 1 - \lambda_M - \lambda_U\) decreases to zero (and \(\lambda_M\) increases to \(1 - \lambda_U\)). We know from (31) that \(J_{M}/J_{U} > 0\) for all \(\lambda_M\); in fact, from (34) we know that \(J_{M}/J_{U} \geq \sqrt{1 - f_M}\). It follows from the strict monotonicity of \(J_{M}/J_{U}\) and \(J_{S_i}/J_{U}\) (Lemma 1) that either \(J_{M}/J_{U} > J_{S_i}/J_{U}\) for all \(\lambda_M \in [0, 1 - \lambda_U]\) or the two curves cross at exactly one \(\lambda_M\) in \([0, 1 - \lambda_U]\). In the first case, all informed agents prefer to be \(S\) informed than \(M\) informed, so the only equilibrium is \(\lambda^*_M = 0\).

In the second case, the unique point of intersection defines the equilibrium proportion \(\lambda^*_M\), as explained in the discussion of Figure 1. We therefore examine at which \(\lambda_M\) (if any) we have \(J_{M}/J_{U} = J_{S_i}/J_{U}\). We can equate (31) and (32) by setting

\[
\frac{1 - f_M}{f_M} = \frac{1}{1 - \rho_{F}^2} = \frac{1 - f_S}{f_S} = \frac{\rho_{S}^2}{1 - \rho_{S}^2}.
\]

Using the expressions for \(\rho_{F}^2\) and \(\rho_{S}^2\) in (28) and (29), this equation becomes

\[
\frac{1 - f_M}{f_M} + \frac{\lambda^2_M}{\gamma^2\sigma_M^2\sigma_{X_F}^2} = \frac{(1 - \lambda_U - \lambda_M)^2}{\gamma^2(1 - f_S)\sigma_X^2\sigma_{X_F}^2}.
\]

Thus, \(\lambda_M\) satisfies a quadratic equation, which, with some algebraic simplification, can be put in the form \(A\lambda^2_M + B\lambda_M + C = 0\), where

\[
A = 1 - \varphi, \quad B = -2(1 - \lambda_U), \quad C = (1 - \lambda_U)^2 - \alpha\gamma^2, \quad (A.10)
\]

with \(\varphi\) and \(\alpha\) as defined in (36). One of the two roots of this equation is given by \(\tilde{\lambda}_M\). Denote the other root by

\[
\eta = \frac{-B + \sqrt{B^2 - 4AC}}{2A}.
\]

We claim that \(\eta \not\in [0, 1 - \lambda_U]\). We may assume \(A \neq 0\), because \(\eta \to \infty\) as \(A \to 0\) because \(B < 0\). If \(A < 0\) then either \(\eta\) is complex or \(\eta < 0\), again because \(B < 0\). If \(A > 0\), then \(A < 1\) because \(\varphi > 0\). Then if \(\eta\) is real, it satisfies \(\eta \geq -B/2A > -B/2 = 1 - \lambda_U\).

Combining these observations, we conclude that either \(\tilde{\lambda}_M \in [0, 1 - \lambda_U]\) and the information equilibrium has \(\lambda^*_M = \tilde{\lambda}_M\), or else the equilibrium occurs at \(\lambda^*_M = 0\). \[\square\]
**Proof of Lemma 4.** Differentiation of \( \tilde{\lambda}_M \) with respect to \( \gamma \) yields

\[
\frac{d\tilde{\lambda}_M}{d\gamma} = -\frac{1}{\alpha\sqrt{\phi(1-\lambda_U)}} \left[ 1 + \frac{1-\varphi}{\varphi} \frac{\gamma^2\alpha}{(1-\lambda_U)^2} \right]^{-1/2}.
\]

At an interior equilibrium, \( \tilde{\lambda}_M \) is real, so the expression on the right is real and therefore negative.

At an interior equilibrium, \( \lambda^*_M \) is the solution to \( A\lambda^2 + B\lambda + C = 0 \), with the coefficients given by (A.10). Differentiating (with respect to some parameter, e.g. \( \sigma_S\sigma_X \) or \( \sigma_M\sigma_{X_F} \)) yields \( \dot{A}\lambda^2 + 2A\dot{\lambda} + B + \dot{C} = 0 \) (note \( \dot{B} = 0 \) because we assume \( \lambda_U \) is fixed). Solving for \( \dot{\lambda} \) yields

\[
\dot{\lambda} = -\frac{\dot{A}\lambda^2 + \dot{C}}{2A\lambda + B}.
\]

We note that \( 2A\lambda + B < 0 \) can be rewritten \((1-\varphi)\lambda < 1 - \lambda_U \) which is always true because \( \varphi > 0 \) and \( \lambda_M < 1 - \lambda_U \) since \( \lambda_S > 0 \). Therefore, \( \text{sgn}(\dot{\lambda}) = \text{sgn}(\dot{A}\lambda^2 + \dot{C}) \). Differentiating with respect to \( \sigma_S\sigma_X \) yields \( \dot{A} < 0 \) and \( \dot{C} < 0 \), which implies \( \dot{\lambda} < 0 \); similarly, differentiating with respect to \( \sigma_M\sigma_{X_F} \) yields \( \dot{A} > 0 \) and \( \dot{C} = 0 \), which implies \( \dot{\lambda} > 0 \).

**Proof of Lemma 5.** Using the expression for \( \rho^2_S \) in (29), we get

\[
\frac{f_S}{1-f_S} \left( \frac{1}{\rho^2_S} - 1 \right) = \frac{\gamma^2(1-f_S)\sigma^2_S\sigma^2_X}{\lambda^2_S}.
\]

The derivative of this expression with respect to \( f_S \) is strictly negative, so the derivative of \( J_S/J_U \) in (32) is strictly positive.

**Proof of Lemma 6.** We see from (31) that the derivative of \( J_M/J_U \) is negative precisely if the derivative of

\[
\frac{1-f_M}{f_M} \frac{1}{1-\rho^2_F} = \frac{1}{\gamma^2(1-f_M)\sigma^2_M\sigma^2_{X_F}/\lambda^2_F} + \frac{1}{f_M} - 1
\]

is negative, using the expression for \( \rho^2_F \) in (28). Differentiation yields

\[
\frac{1}{\gamma^2(1-f_M)^2\sigma^2_M\sigma^2_{X_F}/\lambda^2_F} - \frac{1}{f_M^2} = \frac{1}{f_M} \left( \frac{1}{1-\rho^2_F} - 1 \right) - \frac{1}{f_M^2},
\]

which is negative precisely if (38) holds. The equivalence of (38) and (39) follows from (28).

**Proof of Proposition 4.** From (40) we see that \( \rho^2_S \geq \rho^2_F \) precisely when \( \rho^2_F \leq 1/\tau_M \). If \( \tau_M \leq 1 \), then we necessarily have \( \rho^2_F \leq 1/\tau_M \). But if \( \tau_M > 1 \), then markets are more macro efficient whenever \( \rho^2_F > 1/\tau_M \).
A.5 Information equilibrium

Proof of Proposition 3. We first show that (47) defines an information equilibrium at each $c > 0$, then verify uniqueness. For all three cases in (47), the specified $\lambda_M$, $\lambda_S$, and $\lambda_U$ are nonnegative and sum to 1, so it suffices to verify (41). For $\underline{c} \leq c < \bar{c}$, we have $J_M = J_S = J_U$ by construction, so the condition holds. For $c \geq \bar{c}$, we again have $J_S/J_U = 1$ by construction. With $\lambda_M = 0$, we have $\rho_F^2 = 0$, and $J_M/J_U$ in (31) evaluates to $\exp(\gamma c)\sqrt{T - J_M} \geq \exp(\gamma c)\sqrt{T - J_S} = 1$, so $J_U/J_M \leq 1$. Combining the two ratios we get $J_S/J_M \leq 1$. Thus, (41) holds.

For $c < \underline{c}$, we consider two cases. First suppose case (i) of Proposition 3 holds at $\underline{c}$. By definition, $1 - \lambda_M(\underline{c}) - \lambda_S(\underline{c}) = 0$ and $J_M/J_U = J_S/J_U$ at $(\lambda_M(\underline{c}), \lambda_S(\underline{c}), 0)$, so $\lambda_M(\underline{c}) = \lambda_M^*(0)$ and $\lambda_S(\underline{c}) = 1 - \lambda_M^*(0)$, by the definition of $\lambda_M^*$. Because $\lambda_M(c)$ and $\lambda_S(c)$ are strictly decreasing in $c$, they are strictly greater than $\lambda_M^*(0)$ and $1 - \lambda_M^*(0)$. Decreasing $\lambda_M$ decreases $\rho_F^2$, which decreases $J_M/J_U$ in (31), and decreasing $\lambda_S$ similarly decreases $J_S/J_U$. By construction, $J_M/J_U = J_S/J_U = 1$ at $(\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c))$, even for $c < \underline{c}$, so at $(\lambda_M^*(0), 1 - \lambda_M^*(0), 0)$ we have $J_M/J_U < 1$, $J_S/J_U < 1$, and $J_M/J_S = 1$, confirming (41).

Now suppose case (ii) of Proposition 3 holds at $\underline{c}$. Then $\lambda_M^*(0) = \lambda_M(\underline{c}) = 0$, and (47) specifies $\lambda_M = 0$ for all $c < \underline{c}$. By the monotonicity argument used in case (i), $J_S/J_U < 1$ at all $c < \underline{c}$. Moreover, case (ii) in Proposition 3 entails $J_S/J_M \leq 1$, so this also holds for all $c < \underline{c}$, and therefore (41) holds.

We now turn to uniqueness. At any $c$, once we determine which proportions are strictly positive, the equilibrium is determined: if $\lambda_U = 0$, the other two proportions are determined by Proposition 3 if all three proportions are positive, they must satisfy $J_M/J_U = J_S/J_U = 1$ and must therefore be given by $(\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c))$; if $\lambda_M = 0$ and $\lambda_U > 0$, the proportions are determined by the requirement that $J_S/J_U = 1$. If we tried to set $\lambda_S = 0$, we would have $\rho_S^2 = 0$ and $J_S/J_U = 0$, in which case (41) would require $\lambda_U = 0$ and $\lambda_M = 1$; but this yields $\rho_F^2 = 1$ and $J_M/J_U \geq 1$ in (31), which would imply $J_S/J_M = 0$, requiring $\lambda_M = 0$ and leading to a contradiction. The possibility that $\lambda_S = 0$ can therefore be excluded. For any $c > 0$, (32) rules out the possibility that $\lambda_S = 1$, because this would entail $\rho_S^2 = 1$ and $J_S/J_U > 1$.

It therefore suffices to show that at each $c$, the set of agents with positive proportions is uniquely determined. Suppose we try to introduce uninformed agents into an equilibrium from which they are absent. The original equilibrium must have $\lambda_M, \lambda_S > 0$ and therefore $J_M/J_U \leq 1$ and $J_S/J_U \leq 1$. Increasing $\lambda_U$ requires decreasing either $\lambda_M$ or $\lambda_S$ and therefore decreasing either $J_M/J_U$ or $J_S/J_U$, precluding $\lambda_U > 0$, in light of (41). Suppose we try to introduce macro informed agents into an equilibrium with only micro informed and uninformed agents. The presence of uninformed agents requires $J_M/J_U \geq 1$; otherwise, the uninformed would prefer to become macro informed. Increasing $\lambda_M$ would increase $\rho_F^2$ which increases $J_M/J_U$, precluding $\lambda_M > 0$. □

Proof of Corollary 4. (i) It suffices to consider the intermediate range $\underline{c} \leq c \leq \bar{c}$ with $\underline{c} < \bar{c}$, because $\Pi_M$ is constant on $(0, \underline{c}]$ and identically zero on $[\bar{c}, \infty)$. It follows from (41).
and (45) that
\[
\lambda_S^2(c) = \frac{\gamma^2(1 - f_S)^2\sigma_S^2\sigma_X^2}{f_S\tau_M} \left( \frac{\lambda_M^2(c)f_M}{\gamma^2(1 - f_M)^2\sigma_M^2\sigma_X^2} + 1 \right) \equiv a\lambda_M^2(c) + b, \quad a, b > 0.
\]

Because \(\lambda_M(c)\) is strictly decreasing in \(c\), dividing both sides by \(\lambda_M^2(c)\) shows that \(\lambda_S^2(c)/\lambda_M^2(c)\) is strictly increasing in \(c\), hence \(\lambda_M(c)/(\lambda_M(c) + \lambda_S(c))\) is strictly decreasing in \(c\).

(ii) The first assertion follows from Proposition 5. For the second assertion, evaluate \(\lambda_S(c)\) in (45) at \(\bar{c}\) in (46) to get \(\lambda_S(\bar{c}) = \gamma^2\alpha\). If \(\lambda_S(\bar{c}) = 1\), then \(\bar{c} = \bar{c}\), and \(\lambda_S = 1\) at all \(c \leq \bar{c}\), so \(\lambda_M = 0\) at all \(c\).

(iii) Follows from (47). (iv) We know from (28) and (29) that \(\rho_F^2\) and \(\rho_S^2\) are increasing in \(\lambda_M\) and \(\lambda_S\), respectively, so monotonicity of price efficiency follows from monotonicity in (47).

\[\square\]

Proof of Lemma 8. The assertions about \(\lambda_M\), \(\lambda_S\), \(\rho_F^2\), and \(\rho_S^2\) can be read from (42)–(45). With either \(\lambda_M\) or \(\lambda_S\) increasing and the other remaining fixed, \(\lambda_U\) must decrease. \[\square\]
References


