Market Efficiency with Micro and Macro Information

Paul Glasserman       Harry Mamaysky*

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Abstract

We propose a tractable, multi-security model in which investors choose to acquire information about macro or micro fundamentals or remain uninformed. The model is solvable in closed form and yields a rich set of empirical predictions. Primary among these is an endogenous bias toward micro efficiency. A positive fraction of agents will always choose to be micro informed, but in some cases no agent will choose to be macro informed. Furthermore, for most reasonable choices of parameter values, prices will be more informative about micro than macro fundamentals. The key friction in our model is the assumption that uninformed investors cannot make costless inferences from individual stock prices. We explore the model’s implications for the cyclicality of investor information choices, for systematic and idiosyncratic return volatility, and for excess covariance and volatility.

Keywords: Asset Management; Investment; Information; Asset Pricing; Volatility; Attention

*Glasserman: Columbia Business School, pg20@columbia.edu. Mamaysky: Columbia Business School, hm2646@columbia.edu. We thank Patrick Bolton, Charles Calomiris, Larry Glosten, Bob Hodrick, Gur Huberman, Tomasz Piskorski, Dimitri Vayanos, and Laura Veldkamp, as well as seminar participants at Columbia University and the University of Amsterdam for valuable comments.
1 Introduction

Jung and Shiller (2006) gave the name “Samuelson’s dictum” to the hypothesis that the stock market is “micro efficient” but “macro inefficient.” More precisely, the dictum holds that the efficient markets hypothesis describes the pricing of individual stocks better than it describes the aggregate stock market. Jung and Shiller (2006) argued that this view is plausible if the market has access to more information about the fundamentals of individual companies than about fundamentals of the aggregate stock market; if the variation in information about individual companies is large relative to the variation in information about the aggregate market; and if the reasons behind changes in aggregate dividends are subtle and difficult for the investing public to understand.

We develop a tractable multi-security model with imperfect information to capture these and other features in order to investigate a potential wedge between micro and macro price efficiency. The general setting may be viewed as a multi-security generalization of the classical model of Grossman and Stiglitz (1980). Our market consists of a large number of individual stocks, each of which is exposed to a macro risk factor and an idiosyncratic risk. The macro risk factor is tradeable through an index fund that holds all the individual stocks and diversifies away their idiosyncratic risks. Throughout, we assume that no-arbitrage forces the price of the index fund to equal the sum of the prices of the underlying securities.

We model three types of agents: uninformed investors, investors informed about the macro risk factor, and investors informed about individual stocks. Informed agents have access to a learning technology that reduces their uncertainty about the payout of either the index fund or an individual stock. The learning technology specifies what portion of micro and macro risk is knowable, and we investigate the consequences of varying the precision of the two types of technology, as suggested by the argument of Jung and Shiller (2006).

Investors who are macro uninformed can make inferences costlessly from the price of the index fund about the macro risk factor. However, we assume that investors who are not micro informed must pay a small cost – which we interpret as the cost of effort or an expenditure of time – to make inferences from individual stock prices. We motivate this assumption by observing that people become aware of price fluctuations in the S&P500 in the course of their daily lives – on the evening news, for example, or on the front page of the newspaper after particularly large price moves. People then naturally make inferences from this information about the overall state of the market, without expending additional effort. However, making inferences from the price of an individual stock requires
dedicating attention to following the price and then expending mental effort in thinking about the implications of the stock price for the company’s prospects. We assume such activity, while allowable, entails a small cost. This cost represents the key asymmetry in our model.

To solve the model, we first take the fraction of uninformed, macro-informed, and micro-informed agents as given and solve for an explicit market equilibrium, assuming all agents have CARA preferences. Shares of individual stocks and the index fund are subject to exogenous supply shocks. Importantly, the exogenous supply shocks themselves exhibit a factor structure – there is a common component across all firms’ supply shocks, but each firm’s supply shock also has an idiosyncratic component which we interpret as noise trading. Supply shocks are not observable to investors, and therefore equilibrium prices are informative about, but not fully revealing of, the micro or macro information acquired by informed agents.

We show that, in response to even a small cost of conditioning on individual stock prices, non-micro informed investors will optimally choose not to condition on, and therefore not trade in, individual stocks. This endogenous non-participation of the micro uninformed allows all gains from accommodating idiosyncratic supply shocks to accrue to the micro informed, and this is a key driver of many of our results. We define explicit measures of micro and macro price informativeness for the index fund and for the individual stocks; these measures are a focus of much of our analysis.

We allow agents to choose between being micro informed and macro informed, and we characterize the equilibrium in which a marginal agent is indifferent between the two types of information. This analysis contrasts with the single-security setting of Grossman and Stiglitz (1980), where agents choose whether to become informed or remain uninformed, but the choice between micro and macro information is absent. In practice, developing the skills needed to acquire and apply investment information takes time — years of education and experience. In the near term, these requirements leave the total fraction of informed investors relatively fixed. By contrast, we suppose that informed investors can move comparatively quickly and costlessly between being macro informed or micro informed by shifting their focus of attention. Endogenizing this focus gives rise to an attention equilibrium centered on the choice between macro and micro information. Over a longer horizon, agents choose whether to gain the skills to become informed, as well as the type of information on which to focus. We therefore study an information equilibrium that endogenizes both decisions to determine equilibrium proportions of macro informed, micro informed, and uninformed investors.
A striking feature of our results is a recurring asymmetry between micro and macro information. For example, we show that the information equilibrium sometimes has no macro informed agents, but some fraction of agents will always choose to be micro informed. We show that increasing the precision of micro information makes micro informed agents worse off: we say that micro informed agents overtrade their information, driving down their compensation for liquidity provision. In contrast, macro informed agents may be better or worse off as a result of more precise macro information: they are better off when the fraction of macro informed agents or, equivalently, when the informativeness of the price of the index fund is sufficiently low. Similarly, the equilibrium fraction of macro informed agents always increases with the precision of micro information, but it can move in either direction with an increase in the precision of macro information.

A simple condition on the relative precision of micro and macro information determines whether the market is more micro efficient or more macro efficient. The conclusion is consistent with Jung and Shiller’s (2006) discussion of Samuelson’s dictum: if – in a sense we make precise – the knowable fraction of uncertainty is greater for individual stocks than for the index fund, then the market is more micro efficient than macro efficient. When we introduce a common cost for acquiring either micro or macro information, we show that among agents who choose to become informed, the fraction who choose to become macro informed declines as the cost increases. Like other implications of our model, this shows an endogenous bias toward micro over macro efficiency, particularly when information acquisition is very costly.

Our theoretical analysis leads to several testable empirical predictions: (i) Most importantly, we establish predictions about how the nature of the informational environment faced by investors affects the allocation of their focus to either micro or macro investments. Variation in the precision of micro and macro information, and in the cost of becoming informed, induce variation in assets invested in different actively managed strategies. (ii) As a direct consequence of these dynamics, idiosyncratic return volatility falls as more micro information becomes available or as the fraction of micro informed investors increases. (iii) As investors shift focus between micro and macro information, idiosyncratic volatility and systematic volatility move in opposite directions. (iii) Changes in the precision of micro information contribute to a common factor in idiosyncratic volatility. (iv) Low precision in macro information creates excess volatility and excess comovement in prices, compared with fundamentals. (vi) Declining costs for acquiring information shift a greater fraction of actively managed funds to macro focused strategies. (vii) Recessions characterized by a similar increase in macro and micro risk push informed investors to focus on micro
information, whereas recessions accompanied predominantly by increased macro risk and only a small increase in the price of risk push investors into macro information.

Our work is related to several strands of literature. Our model effectively nests Grossman and Stiglitz (1980) if we take the index fund as the single asset in their model. We also draw on the analysis of Hellwig (1980), Admati (1985) and Admati and Pfleiderer (1987) but address different questions (see Brunnermeier (2001) for a survey of related literature). In particular, Admati and Pfleiderer (1987) focus on understanding when signals are complements or substitutes; the issue of strategic complementarity in information acquisition is also investigated in Goldstein and Yang (2015) using a Grossman-Stiglitz type model. As in Kyle (1985), our noise traders are price insensitive and gains from trade against them accrue to the informed — thereby providing incentive to collect information. We shed light on the discussion in Black (1986) of the crucial role that “noise” plays in price formation by proposing a model in which the factor structure of noise trading plays an important role in determining the relative micro versus macro efficiency of markets.

Van Nieuwerburgh and Veldkamp (2009) analyze how investors’ choices to learn about the domestic or foreign market in the presence of asymmetric prior knowledge may explain the home bias puzzle. Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) develop a model of rational attention allocation in which fund managers choose whether to acquire macro or stock specific information before making investment decisions. Their model, like ours, has multiple assets subject to a common cash flow factor; but, in contrast to our setting, their agents ultimately all acquire the same information. Their focus is on explaining cyclical variation in attention allocation that is caused by exogenous changes in economic conditions. Kacperczyk et al. (2016) show that mutual fund managers change their focus from micro to macro fundamental information over the course of the business cycle. We discuss several mechanisms that give rise to such behavior, and contrast these with the explanations put forward in Kacperczyk et al. (2016).

Peng and Xiong (2006) also use a model of rational attention allocation to study portfolio choice. In their framework, investors allocate more attention to sector or market-wide information and less attention to firm-specific information. Their conclusion contrasts with ours (and with the Jung-Shiller discussion of Samuelson’s dictum and the Maćkowiak and Wiederholt 2009 model of sticky prices under rational intattention) primarily because in their setting a representative investor makes the information allocation decision; since macro uncertainty is common to all securities, while micro uncertainty is diversified away, the representative investor allocates more attention to macro and sector

\[^1\text{Van Nieuwerburgh and Veldkamp (2010) use related ideas to explain investor under-diversification.}\]
level information. Gårleanu and Pedersen (2016) extend the Grossman-Stiglitz model to link market efficiency and asset management through search costs incurred by investors in selecting fund managers, in a model with a single risky asset.

Bhattacharya and O’Hara (2016) study a Kyle-type model with an ETF and multiple underlying securities. Their model, like ours, contains macro informed and micro informed agents (their “informed speculators”), as well as supply shocks in the ETF and the underlying securities (their “liquidity traders”). They focus on the situation where the liquidity of the ETF is higher than that of the underlying hard-to-trade securities, where there is price impact from trade, where the no-arbitrage relationship between the ETF and underlying security prices can break down, and where the ETF conveys information about individual stock prices. They analyze whether ETFs can lead to short-term market instability in the underlying securities. Cong and Xu (2016) study optimal security design in a Kyle-type setting with two underlying securities. Glosten, Nallareddy, and Zou (2016) investigate (empirically and theoretically) the possibility that trading in an ETF can affect the informational efficiency of stocks (present in the ETF) whose primary markets have poor price discovery. Our focus is on how investors’ information choices play out over months or years, and therefore we largely abstract from these higher frequency microstructure issues by assuming agents trade with no price impact and by maintaining the no-arbitrage restriction that the index fund is equal to the price of the underlying basket of securities.

In discussing empirical implications of our model, we note a connection with the findings of Vuolteenaho (2002) that individual stock returns are driven more by information about cash flows than about discount rates, whereas Campbell and Ammer (1993) find the opposite for the aggregate stock market.

The rest of the paper is organized as follows. Section 2 presents the model, and Section 3 solves for the market clearing prices and demands. In Section 4 we analyze the attention equilibrium and derive the equilibrium fractions of macro and micro informed investors, holding fixed the total size of the informed population. In Section 5 we study the information equilibrium that endogenizes the decision to become informed along with the choice between macro and micro information. We also derive the threshold cost of making inferences from stock prices that will induce the micro uninformed to not trade in individual stocks. Section 6 presents testable implications of the model. Section 7 summarizes the paper and some of its empirical implications. Technical details and proofs are covered in an appendix.
The economy

Securities

We assume the existence of $N$ risky securities — called stocks — indexed by $i$. There is also an index fund, $F$, one share of which holds $1/N$ shares of each of the $N$ stocks. Finally, there is a riskless security with a gross return of $R$. All random variables in the model are normally distributed.

The time 2 dividend payouts of the stocks are given by

$$u_i = \beta_i M + S_i, \quad i = 1, \ldots, N,$$

where $M = m + \epsilon_M$ and $S_i = s_i + \epsilon_i$. We will discuss the covariance structure of the $s_i$'s and the $\epsilon_i$'s momentarily, but all other pairs of random variables associated with dividend payouts are independent. We set $E[m] = \bar{m}$ and $E[s_i] = E[\epsilon_i] = E[\epsilon_M] = 0$. The variance of $m$ is $\sigma^2_m > 0$, the variance of $s_i$ is $\sigma^2_s > 0$, and the variances of $\epsilon_i$ and $\epsilon_M$ are $\sigma^2_{\epsilon_s} > 0$ and $\sigma^2_{\epsilon_M} > 0$. As will be clear shortly, $m$ and $s_i$ will be observable (at a cost) to investors, and therefore represent the knowable part of dividend uncertainty. The ratio of the variance of this knowable part of the dividend to the total variance is given by

$$f_M = \frac{\sigma^2_m}{\sigma^2_M} \quad \text{and} \quad f_S = \frac{\sigma^2_s}{\sigma^2_S},$$

where $\sigma^2_M = \sigma^2_m + \sigma^2_{\epsilon_M}$ and $\sigma^2_S = \sigma^2_s + \sigma^2_{\epsilon_s}$.

The equilibrium in this economy is greatly simplified when the idiosyncratic cash flow shocks sum to zero over a finite number of stocks. We impose

$$\sum_{i=1}^{N} s_i = \sum_{i=1}^{N} \epsilon_i = 0$$

by assuming the covariances

$$\text{cov}(s_i, s_j) = -f_S \frac{\sigma^2_S}{N-1} \quad \text{and} \quad \text{cov}(\epsilon_i, \epsilon_j) = -(1 - f_S) \frac{\sigma^2_S}{N-1}. \quad (4)$$

In the absence of market frictions, instead of holding the index fund, investors can equivalently hold an identical number of shares of every stock. Representing such demand via an index fund is a notational convenience in our model.

This is a common assumption in the asset pricing literature when considering multi-security economies with a finite number of assets; see, for example, Ross (1978), Chen and Ingersoll (1983), and Kwon (1985). It ensures that idiosyncratic terms are fully diversifiable with $N$ finite.
which imply that the variances of the sums in (3) are zero.\footnote{This makes \((s_1, \ldots, s_N)\) exchangeable random variables, and similarly for \(\epsilon_i\). Each of the covariance matrices specified by (4) is diagonally dominant and therefore positive semidefinite.}

We assume that the average beta is 1 \((\sum \beta_i = N)\), so the index fund \(F\) pays

\[
u_F = M + \frac{1}{N} \sum_{i=1}^{N} S_i = M,
\]

where the second equality follows from our assumption in (3). Prices of individual stock are given by \(P_i\). The index fund price is \(P_F\), and precluding arbitrage requires that

\[
P_F = \frac{1}{N} \sum_{i=1}^{N} P_i.
\]

\section*{Agents and information sets}

Agents maximize expected utility over time 2 wealth, given by \(-E[\exp(-\gamma \tilde{W}_2)]\). For simplicity, we assume the same risk aversion parameter \(\gamma > 0\) for all agents. There are uninformed, macro \((M)\) informed and micro \((S)\) informed agents. Uninformed agents can choose to become either \(M\) or \(i\) informed at a cost \(c\). It is costless for agents to move from being \(M\) to \(S\) informed, and vice versa. The proportion of each group of agents is given by \(\lambda_U\), \(\lambda_M\), and \(\lambda_S\), respectively, with

\[
\lambda_U + \lambda_M + \lambda_S = 1.
\]

We analyze two types of informational equilibria in the paper. In the attention equilibrium, for a given \(\lambda_U\), the proportion of \(M\) and \(S\) informed will be chosen so as to make investors indifferent between the two information sets.\footnote{More precisely, this describes an interior equilibrium. We will also consider corner solutions.} In the information equilibrium, we also solve for the \(\lambda_U\) which makes the uninformed indifferent between staying uninformed or paying a cost \(c\) to become either macro or micro informed.

Macro informed agents observe \(m\) and in aggregate micro informed agents observe \(s_i\) for every \(i\). We think of \(f_M\) and \(f_S\) from (2) as exogenously specified technologies available to \(M\) and \(S\) informed investors.\footnote{In a dynamic setting, it is possible to think about \(f_M\) and \(f_S\) changing in response to the number of macro or micro informed investors from the prior period, as well as to technological and regulatory innovations (for example, Reg FD may have affected \(f_S\)). But here we take the \(f\)'s as given.} Say two investors expend the same amount of effort: one to study the prospects of an individual company, and the other to study the prospects of the aggregate stock market. We want to allow for the possibility that the first investor...
may learn more (or less) through this equal effort than the second – potentially because there is more (or less) knowable at the micro than the macro level. If more (less) micro information were knowable, we would have $f_S > f_M$ ($f_S < f_M$).

Agents who choose to be $S$ informed are randomly assigned to learn about security $i$ so that $\lambda_S/N$ agents are knowledgeable about $s_i$ for every $i$. We will also refer to $S$ informed investors who learn $s_i$ as $i$ informed investors. All agents who are not $M$ informed rationally – and costlessly – extract from $P_F$ the relevant information about $m$. In addition $i$ informed agents can condition on $P_i$ in making inferences about $m$ (though it will be shown that $P_i$ contains no useful information about $m$ once $P_F$ is known).

Importantly, we assume that non-$i$ informed agents must pay a “small” cost $\kappa_i$ to make inferences about $s_i$ from $P_i$ and then to trade in stock $i$. Though it is expressed in wealth terms, we interpret $\kappa_i$ to represent the time and effort required to analyze individual stock prices. This is a key assumption in our model, which we rationalize as follows: People are routinely exposed to information about the behavior of the overall stock market in the course of their daily activities. For example, when listening to the evening news, if people hear that the stock market has fallen 2% that day, they will costlessly make some inference about the state of the macroeconomy. However, agents do not, as a matter of course, observe an individual stock’s price moves, nor do they consider the implications of these for the idiosyncratic prospects of a given company without expending some minimal amount of effort. We will show in Section 5.2 that, in equilibrium, the amount of effort individuals would be willing to expend on this enterprise is very small. So for a plausibly small $\kappa_i$, non-$i$ informed investors will choose not to condition on $P_i$, will not trade stock $i$, and will only trade in the index fund. For now, we will analyze an economy where only the $i$ informed trade directly in stock $i$, and we show the optimality of this outcome in Section 5.2.

Supply shocks

The supply of the $i^{th}$ asset is given by

$$\frac{1}{N} (X_F + X_i - X_\beta).$$

We make the standard assumption that supply shocks are unobservable by the agents. $X_F$ is the common supply shock, normally distributed with mean $\bar{X}_F$ and variance $\sigma^2_{X_F}$. $X_i$ are normally distributed idiosyncratic shocks, each with mean 0 and variance $\sigma^2_X$. $X_\beta$ is an adjustment term that we discuss shortly. Supply shocks are independent of cash
flows, and $X_i$ is independent of $X_F$ for all $i$. Similarly to our restriction in (3) and (4) that the $s_i$’s and $\epsilon_i$’s sum to zero, we ensure that

$$\sum_{i=1}^{N} X_i = 0 \quad \text{by imposing the condition} \quad \text{cov}(X_i, X_j) = -\frac{\sigma_X^2}{N-1}. \quad (8)$$

We want to structure our supply shocks to make the total macro cash flow equal to $X_F \times M$, and to make each idiosyncratic cash flow equal to $X_i \times S_i/N$. Therefore aggregate cash flow in the economy should equal $X_F M + N^{-1}(X_1 S_1 + \cdots + X_N S_N)$. Given the supply shocks in (7), the aggregate cash flow is equal to $N^{-1} \sum_i (X_F + X_i - X_\beta)(\beta_i M + S_i)$ which becomes $X_F M + N^{-1} \sum_i X_i S_i + (N^{-1} \sum_i X_i \beta_i - X_\beta)M$, where we have used the fact that the betas are 1 on average, and that $\sum S_i = \sum X_i = 0$ from (3) and (8). Setting $X_\beta = \frac{1}{N} \sum_i X_i \beta_i$ will produce the desired aggregate cash flow in the economy.\footnote{None of our qualitative result change if this term is dropped, except that the index fund equilibrium would need to reflect the $M$ risk contribution of idiosyncratic supply shocks – which would be small for large $N$, and would only introduce notational complexity. Also by virtue of (8) if all the betas across stocks were constant, $X_\beta = 0$.}

The aggregate portion of supply shocks, $X_F$, is standard in the literature – as will become clear, it is analogous to the single security supply shock in Grossman and Stiglitz (1980). The idiosyncratic portion of the supply shock, $X_i$, proxies for price-insensitive noise trading in individual stocks. Some of this noise trading may be liquidity driven (for example, individuals needs to sell their employer’s stock to pay for unforeseen expenditures), but the majority is likely to come from either incorrect expectations or from other value-irrelevant triggers, such as an affinity for trading or fads. Our interpretation of noise traders follows Black (1986), who discusses how noise traders play a crucial role in price formation. Recent empirical studies either suggest (Brandt, Brav, Graham, and Kumar (2010)) or document (Foucault, Sraer, and Thesmar (2011)) a causal link from retail trading to idiosyncratic volatility of stock returns. For example, Foucault et al. (2011) “show that retail trading activity has a positive effect on [idiosyncratic] volatility of stock returns, which suggests that retail investors behave as noise traders.” Our model captures this exact phenomenon via $X_i$. In fact, as will be shown in Section 6 the volatility of $X_i$ directly enters into the idiosyncratic volatility of stock returns.
Market clearing

Let us write $q^U_i$, $q^M_i$, and $q^i$ for the demands of each investor group for security $i$, which can be one of the $N$ stocks or the index fund $F$. For any investor group, $q_i$ denotes that group’s direct demand for stock $i$. Note that each group’s $F$ demand, $q_F$, leads to an indirect demand of $q_F/N$ for every stock $i$.

Aggregate holdings of the index fund are given by

$$q_F \equiv \lambda_U q^U_F + \lambda_M q^M_F + \frac{\lambda_S}{N} \sum_{i=1}^{N} q^i_F.$$  \hspace{1cm} (9)

The market clearing condition for each stock $i$ can be written in its general form as

$$\frac{\lambda_S}{N} \sum_{j=1}^{N} q^j_i + \lambda_U q^U_i + \lambda_M q^M_i + \frac{q_F}{N} = \frac{1}{N} (X_F + X_i - X_\beta) \quad \forall i. \hspace{1cm} (10)$$

The first three terms on the left hand side are the direct demand for stock $i$ from the $i$ informed, uninformed, and $M$ informed, respectively. The fourth term is how much of stock $i$ is held in the index fund. The right hand side is the supply shock from (7). The direct and indirect demand (via $F$) of all agents for stock $i$ must equal its supply.

We now impose the restriction that non-$i$ informed agents do not own stock $i$, i.e. $q^U_i/M/j = 0$ for $j \neq i$. This simplifies (10) to

$$q_F = X_F - X_\beta - (\lambda_S q^i_i - X_i) \quad \forall i.$$  \hspace{1cm} (11)

As this must hold for all $i$, the quantity $\xi \equiv \lambda_S q^i_i - X_i$ cannot depend on $i$. We can therefore write $i$’s direct demand as

$$\lambda_S q^i_i = X_i + \xi,$$  \hspace{1cm} (11)

for some $\xi$ that does not depend on $i$, and we can therefore write $q_F$ as

$$q_F = X_F - X_\beta - \xi.$$  \hspace{1cm} (12)

While (11) and (12) are clearly sufficient for (10), we have therefore established that they are necessary as well.

We will show in Section 3.1 that in equilibrium $\xi$ must be zero, leading to two important implications. It will follow from (11) that the $i$ informed investors fully absorb the
idiosyncratic supply shock $X_i$, and it will follow from (12) that the index fund will hold the aggregate supply shock. We will interpret (11) as liquidity provision by the $i$ informed investors in the securities in which they specialize.

3 Market equilibrium

We construct an equilibrium in which the index fund price takes the form

$$P_F = a_F + b_F(m - \bar{m}) + c_F(X_F - \bar{X}_F),$$

and individual stock prices are given by

$$P_i = \beta_i P_F + b_S s_i + c_S (X_i + \xi).$$

Equation (13) makes the index fund price linear in the macro shock $m$ and the aggregate supply shock $X_F$. Equation (14) similarly makes the idiosyncratic part of the price of stock $i$, $P_i - \beta_i P_F$, linear in the micro shock $s_i$ and the idiosyncratic supply shock $X_i + \xi$.

3.1 Model solution

Recall from Section 2 that we assume that agents uninformed about security $i$ do not invest in that security. Therefore, the $M$ informed and uninformed agents will have demands only for the index fund, and $i$ informed agents will demand the index fund and security $i$. We assume that agents of type $U$, $M$, and $i$, respectively, set their demands by maximizing expected utility conditional on the information sets

$$\mathcal{I}_U = \{P_F\}, \quad \mathcal{I}_M = \{m, P_F\}, \quad \text{and} \quad \mathcal{I}_i = \{P_F, s_i\}.$$

In the equilibrium we construct, $P_F$ is a noisy version of $m$, so $M$-informed agents rationally ignore $P_F$ in evaluating conditional moments of $M$. Similarly, all agents rationally ignore the prices $P_1, \ldots, P_N$ in evaluating conditional moments of $M$, and $i$ informed agents rationally ignore these prices in evaluating conditional moments of $S_i$.

Standard arguments imply that the $M$ informed demand for the index fund is given by

$$q^M_F = \frac{1}{\gamma(1 - \hat{f}_M)\sigma^2_M} (m - RP_F),$$

as in equation (8) of Grossman and Stiglitz (1980), where $R$ is the risk-free gross return,
and uninformed demand for the index fund is given by

\[
q_U^F = \frac{1}{\gamma \text{var}(M|P_F)} (E[M|P_F] - RP_F).
\] (16)

If \( P_F \) takes the form in (13), then

\[
E[M|P_F] = K_F(P_F - a_F) + \bar{m},
\]
\[
\text{var}(M|P_F) = f_M\sigma_M^2(1 - K_F b_F) + (1 - f_M)\sigma_M^2,
\] (17)
\[
K_F = \frac{b_F f_M \sigma_M^2}{b_F f_M \sigma_M^2 + c_F^2 \sigma_X^2}.
\]

Demands of the \( i \) informed agents are given by the following proposition. See also Admati (1985).

**Proposition 1.** If the prices \( P_F \) and \( P_i \) take the form in (13) and (14), then the demands of \( i \) informed agents are given by

\[
q_i^F = \frac{R}{\gamma(1 - f_S)\sigma_S^2} (\beta_i P_F + s_i/R - P_i) \quad (18)
\]
\[
q_i^F = q_U^F - \beta_i q_i^i. \quad (19)
\]

Equation (19) shows that an \( i \) informed agent’s demand for the index fund consists of two components. The first component is the demand \( q_U^F \) of the uninformed agents: neither the \( i \) informed nor the uninformed have any information about \( M \) (recall \( u_F = M \)) beyond that contained in \( P_F \). The second term \(-\beta_i q_i^i\) offsets the exposure to \( M \) that the \( i \) informed agent takes on by holding stock \( i \). We interpret the second term as the \( i \) informed’s *hedging demand*: the \( i \) informed use the index fund to hedge out excess exposure to \( M \) that they get from speculating on their information about \( s_i \). The net result is that \( i \) informed and uninformed agents have the same exposure to \( M \).

Substituting (19) in (9) – which gives the aggregate index fund demand – and combining this with the index fund market clearing condition in (12) yields

\[
(\lambda_U + \lambda_S)q_U^F + \lambda_M q_M^F = X_F. \quad (20)
\]

This market clearing condition for the index fund aligns with the single security Grossman-Stiglitz equilibrium, with the percent of (macro) informed agents given by \( \lambda_M \) and the

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8Using (19) we see that \( N^{-1} \lambda_S \sum q_i^F \) from (9) is equal to \( \lambda_S q_U^F - N^{-1} \sum \beta_i \lambda_S q_i^i \). Using (11) this becomes \( \lambda_S q_U^F - N^{-1} \sum \beta_i (X_i + \xi) = \lambda_S q_U^F - X_\beta - \xi \), and combining this with (12) yields (20).
percent of (macro) uninformed agents given by \( \lambda_U + \lambda_S \).

With the demands (15)–(16) for the index fund and demands (18)–(19) for individual securities, market-clearing prices are given by the following proposition:

**Proposition 2.** The market clears at an index fund price of the form (13),

\[
P_F = a_F + b_F(m - \bar{m}) + c_F(X_F - \bar{X}_F), \quad \text{with} \quad \frac{c_F}{b_F} = -\frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M},
\]

and prices for individual stocks \( i \) of the form (14), given by

\[
P_i = \beta_i P_F + \frac{s_i}{R} - \frac{\gamma(1 - f_S)\sigma_S^2}{\lambda_S R} (X_i + \xi).
\]

The no-arbitrage condition (6) is satisfied if and only if \( \xi = 0 \).

The form of the index fund price \( P_F \) follows from Grossman and Stiglitz (1980); explicit expressions for the coefficients \( a_F, b_F, \) and \( c_F \), are derived in the appendix. Comparison of (14) and (22) shows that the ratio \( c_S/b_S \) in the price of stock \( i \) has exactly the same form as \( c_F/b_F \) in the price of the index fund in (21). In fact, if \( \lambda_M = 1 \), then \( b_F = 1/R \) and \( c_F \) has exactly the same form as \( c_S \). The stock \( i \) equilibrium is the direct analog of the index fund equilibrium with only \( M \)-informed agents.

As in Grossman and Stiglitz (1980) the equilibrium price of the index fund depends on the proportion of investors informed about the fund’s payout. Similarly, the prices of individual securities depend on the proportion of investors informed about these securities. Much of our analysis will center on the endogenous choice of these proportions.

A diversified fund holding \( X_i \) shares of each stock can’t exist in our model because supply shocks are assumed to be unobservable. Hence market clearing can only occur through a price channel, which explains the presence of the \( s_i \) and \( X_i \) terms in the stock price in (22). Despite this, we note that the unconditional CAPM holds because \( \text{E}[u_i - \text{RP}_i] = \beta_i \text{E}[M - \text{RP}_F] \).

### 3.2 Price efficiency

It will prove useful to measure the extent to which prices in our model are informative about fundamentals. For the case of the index fund, we define price efficiency, \( \rho_F^2 \), as the proportion of price variability that is due to variability in \( m \), the knowable portion of the aggregate dividend. This is the \( R^2 \) from regressing \( P_F \) on \( m \).
From the functional form of $P_F$ in (13), we see that

$$\rho_F^2 = \frac{b_F^2 f_F \sigma_M^2}{b_F^2 f_M \sigma_M^2 + c_F^2 \sigma_X^2}. \quad (23)$$

Note that this is equal to $b_F K_F$ from the $M$ uninformed’s inference problem, which implies that that for the $M$ uninformed agent, the variance of $m$ conditional on $P_F$ is given by

$$\text{var}(m|P_F) = f_M \sigma_M^2 (1 - \rho_F^2).$$

As the price efficiency goes to 1, $P_F$ becomes fully revealing about $m$.

Dividing both sides by $b_F^2 \sigma_M^2$ and using the expression for $c_F/b_F$ in (21), we can rewrite this as

$$\rho_F^2 = \frac{f_M}{f_M + \gamma^2 (1 - f_M)^2 \sigma_M^2 \sigma_X^2 / \lambda_M^2}. \quad (24)$$

For stock $i$ we define price efficiency as the proportion of the variability of the price that is driven by variability in $s_i$, the idiosyncratic dividend shock, once $P_F$ is known. From the functional form of $P_i$ in (14) and the fact that $\xi = 0$, this is given by

$$\rho_S^2 = \frac{b_S^2 f_S \sigma_S^2}{b_S^2 f_S \sigma_S^2 + c_S^2 \sigma_X^2}. \quad (25)$$

Using the expression for $c_S/b_S$ in (22) and simplifying as we did with $\rho_F^2$, we find that

$$\rho_S^2 = \frac{f_S}{f_S + \gamma^2 (1 - f_S)^2 \sigma_S^2 \sigma_X^2 / \lambda_S^2}. \quad (25)$$

As in the case of the index fund, as $\rho_S^2$ goes to 1, $P_i$ becomes fully revealing about $s_i$.

We note that the two efficiency measures have identical functional forms, with each using its respective set of moments and its $\lambda$. Furthermore, observe that our informative-ness measures are with regard to the knowable portion of the dividend payout, not the total dividend payout.

Differentiating (24) and (25) and straightforward algebra, yields the following result:

**Lemma 1** (When are prices more informative?).

(i) Micro (macro) prices are more efficient as either (a) the fraction of micro (macro) informed increases, or (b) as the micro (macro) learning technology improves. That is:

$$d\rho_F^2/d\lambda_F > 0 \quad \text{and} \quad d\rho_F^2/d\lambda_M > 0,$$

14
and

\[ d\rho^2_S/df > 0 \quad \text{and} \quad d\rho^2_F/df > 0. \]

(ii) Furthermore, when the fraction of micro (macro) informed is zero, or when the learning technology is non-informative, price efficiency is zero. In other words, \( \rho^2_F \to 0 \) as either \( \lambda_M \to 0 \) or \( f_M \to 0 \), and \( \rho^2_S \to 0 \) as either \( \lambda_S \to 0 \) or \( f_S \to 0 \).

(iii) When the learning technology is perfect prices become fully revealing. In other words, \( \rho^2_F \to 1 \) as \( f_M \to 1 \), and \( \rho^2_S \to 1 \) as \( f_S \to 1 \).

As the number of informed in a given market grows, prices in that market become more revealing. Similarly, as the information about future dividends that is known to informed investors becomes greater, these investors – facing less future cash flow risk – trade more aggressively (as if they had a smaller risk-aversion parameter \( \gamma \)) which incorporates more of their information into prices.

We will be able to say much more about both measures of price efficiency when we evaluate them at equilibrium proportions \( \lambda_M \) and \( \lambda_S \).

4 Attention equilibrium

The prior section analyzed the market equilibrium, taking the fraction of uninformed, \( M \) informed and \( i \) informed traders as given. In this section, we will endogenously determine these quantities.

As discussed in the introduction, we take the view that developing the skills needed to acquire and apply investment information takes time — perhaps seven to ten years of education and experience. In the near term, these requirements leave the total fraction of informed investors \( \lambda_M + \lambda_S \) fixed. Once investors have the skills needed to become informed, we suppose that they can move relatively quickly (over one or two years) and costlessly between macro and micro information by shifting the focus of their attention. We therefore distinguish a near-term attention equilibrium, in which \( \lambda_U \) is fixed and the split between \( \lambda_M \) and \( \lambda_S \) is endogeneous, from a longer-term information equilibrium, in which the decision to become informed is endogenized along with the choice of information on which to focus. We analyze the attention equilibrium in this section and address the information equilibrium in Section 5. The allocation of attention we study refers to the fraction of investors focused on each type of information, and not the allocation of attention by an individual agent.
4.1 Relative utility

Recall that we take an investor’s ex ante expected utility to be \( J \equiv E[-\exp(-\gamma \tilde{W}_2)] \), where the expectation is taken unconditionally over the time 2 wealth \( \tilde{W}_2 \).

Fixing the fraction of uninformed, the following lemma establishes the relative benefit of being \( M \) and \( S \) informed relative to being uninformed.

Lemma 2. If the cost of becoming informed is given by \( c \), then the benefit of being \( M \) informed relative to being uninformed is given by

\[
J_M/J_U = \exp(\gamma c) \left( 1 + \frac{f_M}{1 - f_M} (1 - \rho_F^2) \right)^{-\frac{1}{2}}.
\] (26)

The benefit of being \( i \) informed relative to being uninformed is given by

\[
J_i/J_U = \exp(\gamma c) \left( 1 + \frac{f_i}{1 - f_i} \left( \frac{1}{\rho_S^2} - 1 \right) \right)^{-\frac{1}{2}}.
\] (27)

Note that because utilities in our model are negative, a decrease in these ratios represents a gain in informed relative to uninformed utility.

Each of the ex ante utility ratios in the lemma is increasing in the corresponding measure of price efficiency – that is, informed investors become progressively worse off relative to uninformed as micro or macro prices become more efficient. But the dependence on \( \rho_S^2 \) in (27) differs from the dependence on \( \rho_F^2 \) in (26), a point we return to in Section 5.1.

Recalling from Lemma 1 that macro and micro price efficiency increase in \( \lambda_M \) and \( \lambda_S \), respectively, we immediately get that

Lemma 3 (Benefit of information decreases with number of informed). \( J_S/J_U \) strictly increases (making \( i \) informed worse off) in \( \lambda_S \). \( J_M/J_U \) strictly increases (making \( M \) informed worse off) in \( \lambda_M \).

Figure 1 illustrates the results of Lemma 2 and 1. The figure holds \( \lambda_U \) fixed, and the x-axis is indexed by \( \lambda_M \) (so \( \lambda_S = 0 \) is the rightmost point on the graph). As \( \lambda_M \) increases, \( J_M/J_U \) increases, indicating that the \( M \) informed are becoming worse off. Similarly, at the rightmost point of the graph, \( \lambda_S = 0 \), and as we move to the left, \( J_S/J_U \) increases, indicating that the \( i \) informed are becoming worse off as more of their type enter the economy.
Figure 1: The information equilibrium for a fixed number of uninformed investors. Relative utilities are shown assuming cost of becoming informed is $c = 0$. Parameter values are described in Section A.1.

4.2 Choice between macro and micro information

For the analysis of the attention equilibrium, we hold fixed the fraction of uninformed $\lambda_U$. We assume that once an agent chooses to pay cost $c$, the agent can decide to learn about either macro or micro information. At an interior equilibrium, the marginal investor must be indifferent between these two information sets, in which case equilibrium will be characterized by a $\lambda_M^*$ such that with that many macro informed investors and with $1 - \lambda_U - \lambda_M^*$ micro informed investors we will have $J_M = J_S$, which just sets (26) equal to (27). To cover the possibility of a corner solution, we define an equilibrium by a pair of proportions $\lambda_M \geq 0$ and $\lambda_S = 1 - \lambda_U - \lambda_M \geq 0$ satisfying

$$J_M < J_S \Rightarrow \lambda_M = 0 \quad \text{and} \quad J_S < J_M \Rightarrow \lambda_S = 0.$$  \hspace{1cm} (28)

The inequalities in this conditions are equivalent to $J_M/J_U > J_S/J_U$ and $J_S/J_U > J_M/J_U$, respectively, because $J_U < 0$.

Recall from Lemma 1 that when the fraction of $M$ or $i$ informed is zero, price efficiency
is also 0 (i.e. $\rho^2_F(\lambda_M = 0) = 0$ and $\rho^2_S(\lambda_S = 0) = 0$). From (26) and (27), we see that

$$J_M/J_U(\lambda_M = 0) = \sqrt{1 - f_M} \quad \text{and} \quad J_S/J_U(\lambda_S = 0) = 0;$$

we are taking $c = 0$ because the fraction of uninformed is fixed. From Lemma 3 we know that $J_M/J_U$ and $J_S/J_U$ both increase monotonically (i.e., make the informed worse off) with their respective $\lambda$'s. When $\lambda_M$ is zero, the $M$ informed achieve their maximal utility; when $\lambda_M = 1 - \lambda_U$, the $S$ informed achieve their maximal utility. As $\lambda_M$ increases from zero to $1 - \lambda_U$, $\lambda_S$ decreases, so the macro informed become progressively worse off and the micro informed become progressively better off. If at some $\lambda_M$ the two curves $J_M/J_U$ and $J_S/J_U$ intersect, we will have an interior equilibrium, and it must be unique because of the strict monotonicity in Lemma 3. This case is illustrated in Figure 1. If there is no interior equilibrium, then either macro or micro information is always preferred, and no investor will choose the other.

To make these observations precise, let us define

$$\tilde{\lambda}_M \equiv (1 - \lambda_U) \frac{1 - \sqrt{\varphi} \sqrt{1 + \frac{1 - \varphi}{\varphi} \frac{\sigma^2_M}{(1 - \lambda_U)^2}}}{1 - \varphi},$$

where

$$\varphi = \frac{(1 - f_S)\sigma^2_S\sigma^2_X}{(1 - f_M)\sigma^2_M\sigma^2_X} \quad \text{and} \quad \alpha = \frac{1 - f_M}{f_M} \frac{(1 - f_S)\sigma^2_S\sigma^2_X}{(1 - f_M)\sigma^2_M\sigma^2_X}.$$  

Note that $\varphi$ is the ratio of the total risk arising from the unknowable portion of idiosyncratic supply shocks (i.e. the variance of $\epsilon_i$ times the variance of $X_i$) to the total risk arising from macro supply shocks (i.e. the variance of $\epsilon_M$ times the variance of $X_F$). The larger $\varphi$ the more total unknowable risk comes from idiosyncratic rather than systematic sources.

The following proposition characterizes the equilibrium allocation of attention in the economy between macro information and micro information when the total fraction of informed investors $1 - \lambda_U$ is fixed.

**Proposition 3** (Attention equilibrium). Suppose $0 \leq \lambda_U < 1$, so some agents are informed.

(i) Interior equilibrium. If $\tilde{\lambda}_M \in [0, 1 - \lambda_U)$, then this point defines the unique equi-

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9 The expression on the right has a finite limit as $\varphi \to 1$, and we take that limit as the value of $\tilde{\lambda}_M$ at $\varphi = 1$.

10 We refer to (i) as the case of an interior equilibrium, even though it includes the possibility of a
librium: at $\lambda_M^* = \bar{\lambda}_M$, the marginal informed investor will be indifferent between becoming $M$ and $S$ informed.

(ii) If $\bar{\lambda}_M \notin [0, 1 - \lambda_U)$, the unique equilibrium is at the boundary $\lambda_M^* = 0$, where all informed agents are $i$ informed.

(iii) In equilibrium, we always have $\lambda_M^* < 1 - \lambda_U$. In other words, some informed agents will choose to be $i$ informed in equilibrium.

It bears emphasizing that our attention equilibrium – regardless of parameter values – precludes all informed agents from being $M$ informed. In contrast, it is possible for all informed agents to be $i$ informed. We therefore have, as a fundamental feature of the economy, a bias for micro over macro information. We will discuss this question further in Section 5.3.

To get some intuition into the drivers of the attention equilibrium, we consider two special cases in which $\bar{\lambda}_M$ simplifies: when $\gamma = 0$ and when $\varphi = 1$. If $\gamma = 0$, then

$$\bar{\lambda}_M = \frac{1 - \lambda_U}{1 + \sqrt{\varphi}}.$$  (32)

As noted before Proposition 3, $\varphi$ measures the relative magnitude of unknowable idiosyncratic and macro shocks. So, this expression suggests that, at least at low levels of risk aversion, agents favor information about the greater source of uncertainty, with $\bar{\lambda}_M$ increasing in macro uncertainty and decreasing in micro uncertainty. This feature is consistent with our discussion of Jung and Shiller (2006) in the introduction; in particular, greater micro uncertainty shifts greater focus to micro information.

At $\varphi = 1$,

$$\bar{\lambda}_M = \frac{(1 - \lambda_U)}{2} \left[ 1 - \frac{\gamma^2 \alpha}{(1 - \lambda_U)^2} \right].$$

Here it becomes evident that an increase in risk aversion moves investors toward micro information. The benefit to being macro informed comes from two sources, liquidity provision for the macro supply shocks $X_F$ and the ability to advantageously trade against the macro uninformed, whereas the benefit of being micro informed only comes from liquidity provision for the micro shocks $X_i$. When $\gamma$ increases, trade between the informed and uninformed falls, therefore diminishing the advantage of being $M$ informed. However,
the liquidity discount in micro and macro prices, i.e. $c_F$ and $c_S$ from (21) and (22), increases with risk aversion, making being micro informed relatively more attractive.

Similarly, an increase in micro (macro) volatility, as measured by $\sigma_S \sigma_X$ ($\sigma_M \sigma_{X_F}$), will increase the benefit of information to the micro (macro) informed, and will therefore decrease (increase) $\lambda^*_M$ when the economy is in an interior equilibrium.

These two results hold when the equilibrium is characterized by $\tilde{\lambda}_M$.

**Lemma 4** (Effects of risk aversion and risk on the attention equilibrium). We consider the case of an interior equilibrium with $\lambda^*_M > 0$.

(i) Risk aversion will push investors towards micro information:

$$\frac{d\lambda^*_M}{d\gamma} < 0.$$  

(ii) Increase in micro (macro) risk pushes investors towards micro (macro) information:

$$\frac{d\lambda^*_M}{d(\sigma_S \sigma_X)} < 0 \text{ and } \frac{d\lambda^*_M}{d(\sigma_M \sigma_{X_F})} > 0.$$  

4.3 Impacts of learning technology

**Effect of information precision on investor welfare**

Recall from (2) that $f_M$ ($f_S$) determines the amount of variation in $M$ ($S_i$) that is knowable from becoming informed. We refer to this as information precision.

We first show that, somewhat surprisingly, a better micro learning technology makes the $S$ informed investors worse off.

**Lemma 5** (The $S$ informed overtrade on their information). Better learning technology is worse for micro informed in the sense that

$$\frac{d(J_S/J_U)}{df_S} > 0 \quad (\text{micro informed are worse off}).$$

When investors become $i$ informed, the more they know about the ultimate idiosyncratic portion of the payout $S_i$ the less uncertainty they face from owning the stock. From (22) we see that the discount in the stock price due to idiosyncratic supply shocks $X_i$ will be zero when the learning technology is perfect, i.e. when $f_S = 1$. With no discount in the price, the compensation for liquidity provision goes to zero. Because atomic informed agents cannot act strategically and coordinate to limit their liquidity provision in
an optimal (for them) way, uncertainty about the dividend helps them by decreasing the sensitivity of their demand to price shocks, which in turn leads to a higher risk premium in prices.

In contrast to the informed, the informed may be better or worse off as their learning technology, $f_M$, improves.

**Lemma 6** (The informed can be better or worse off with more information). *Better learning technology is better for the informed if and only if*

$$\rho_F^2 < \frac{1}{1 + f_M},$$

(33)

*which is equivalent to*

$$\lambda_M < \gamma \sigma_M \sigma_{X_F} \frac{1 - f_M}{f_M}.$$  (34)

*In this case,*

$$\frac{d(J_M/J_U)}{df_M} < 0 \quad \text{(macro informed are better off).}$$

To gain intuition into this result recall that at $f_M = 0$ we would have $\rho_F^2 = 0$ (price reveals nothing when nothing about $M$ is knowable), and at $f_M = 1$ we would have $\rho_F^2 = 1$ (prices are fully revealing when $M$ is fully known). Furthermore, from Lemma 4 we know $\rho_F^2$ increases monotonically in $f_M$. So (33) implies that the informed benefit from an increase in the precision $f_M$ only when $f_M$ (hence also the price informativeness $\rho_F^2$) is low. Using (34) the condition can be reinterpreted as placing a limit on how many informed investors the economy can support before better macro precision begins to make the macro informed worse off.

The contrast between micro and macro information in Lemmas 5 and 6 can be understood as follows. In the market for the index fund, informed investors trade against uninformed investors as well as taking the other side of price insensitive liquidity shocks, introducing an effect that is absent in the market for individual stocks. With a poor learning technology, prices are not very informative, so a small improvement in the learning technology gives the informed an informational edge over the uninformed, allowing the informed to extract rents in trading. However, as the learning technology improves and price efficiency grows, the incremental ability to extract rents from trading against the uninformed diminishes, while the tendency to overtrade on information (as in the case of the market for individual stocks) grows. At some point, determined by $\rho_F^2 = 1/(1 + f_M)$, the overtrading tendency begins to dominate the rent-extraction effect.
Effect of information precision on attention equilibrium

Recall that for a fixed $\lambda_U$, the equilibrium $\lambda^*_M$ (proportion of macro informed) is determined by the condition that $J_M/J_U = J_S/J_U$, in the case of an interior equilibrium. We note that $J_M/J_U$ does not depend on $f_S$. Therefore as $f_S$ rises and $J_S/J_U$ increases, $J_M/J_U$ can only increase if $\lambda_M$ increases (from Lemma 3). It follows that an interior equilibrium $\lambda^*_M$ must increase with $f_S$. As the benefit of being micro informed diminishes due to more precise micro information, the fraction of informed investors that focus on macro information grows. Figure 2 demonstrates this equilibrium adjustment. For every $\lambda_M$, a higher $f_S$ makes the micro informed worse off, which pushes the equilibrium number of macro informed higher.

Figure 2: The effect of increasing micro precision $f_S$ on the attention equilibrium with fixed $\lambda_U$. Parameter values are described in Section A.1.

Similarly, since $J_S/J_U$ does not depend on $f_M$ but decreases in $\lambda_M$ (i.e. micro informed are better off as there are fewer micro informed), if $J_M/J_U$ decreases (increases) in $f_M$, then $\lambda^*_M$ must increase (decrease) in $f_M$. Figure 3 illustrates this phenomenon. In the figure, the equilibrium $\lambda^*_M$ is sufficiently small so that macro precision makes the macro informed better off. As macro precision $f_M$ increases, for a range of $\lambda_M$ that are sufficiently
small, the macro informed become better off, which increases \( \lambda^*_M \) (i.e. decreases the number of micro informed, thus making the remaining micro informed better off). Had the equilibrium \( \lambda_M \) been sufficiently high, the effect would have had the opposite sign, as can be seen by the fact the \( J_M/J_U \) increases with \( f_M \) for high \( \lambda_M \).

![Graph showing the ratio of \( J_M \) and \( J_S \) to \( J_U \) against \( \lambda_M \).](image)

Figure 3: The effect of increasing macro precision \( f_M \) on the attention equilibrium with fixed \( \lambda_U \). Parameter values are described in Section A.1.

The preceding arguments establish the following result:

**Lemma 7** (Effect of information precision on equilibrium). *In the case of an interior equilibrium with \( \lambda^*_M > 0 \), the number of macro informed increases as the micro learning technology becomes more precise:*

\[
\frac{d\lambda^*_M}{df_M} > 0.
\]

*Condition (33) (or equivalently (34)) is necessary and sufficient for the number of macro informed to increase as the macro learning technology becomes more precise:*

\[
\frac{d\lambda^*_M}{df_M} > 0 \quad \text{if and only if} \quad \rho_F^2 < \frac{1}{1 + f_M} \left( > \frac{1}{1 + f_M} \right).
\]
4.4 Equilibrium price informativeness

In equilibrium, the marginal investor is indifferent between becoming $M$ or $i$ informed. We define $\tau_M$ as

$$\tau_M \equiv \frac{f_M/(1-f_M)}{f_S/(1-f_S)},$$

which measures the precision of the macro learning technology relative to the micro learning technology. If, by expending an equal amount of effort, investors are able to learn a greater (lesser) proportion of $S$ than they can of $M$, then $\tau_M < 1$ ($\tau_M > 1$), and the micro learning technology is relatively more (less) precise. From Lemma 2, we see that an interior equilibrium, where $J_M/J_U = J_S/J_U$, micro price efficiency is related to macro price efficiency via $(1-\rho^2_S)/\rho^2_S = \tau_M(1-\rho^2_F)$, which yields

$$\rho^2_S = \frac{1}{1 + \tau_M(1-\rho^2_F)}. \quad (35)$$

From this we see that as markets become fully macro efficient, they must also become fully micro efficient, and vice versa. In other words,

$$\rho^2_S \rightarrow 1 \iff \rho^2_F \rightarrow 1.$$  

However, as macro price efficiency tends towards zero, micro price efficiency tends towards $1/(1 + \tau_M)$. Since both sides of (35) are decreasing as their respective $\rho$ falls, this also represents the lower bound for $\rho^2_S$ in equilibrium. This result is the direct consequence of part (iii) of Proposition 3 which shows that $\lambda_S^* > 0$, i.e. that in equilibrium there must be $i$ informed traders, which from (25) implies that $\rho^2_S$ cannot be zero. So we have

$$\rho^2_S \rightarrow \frac{1}{1 + \tau_M} \iff \rho^2_F \rightarrow 0.$$  

In the limits, we see therefore that markets are either micro efficient or both markets are fully revealing – the latter limit being unattainable, as has already been argued.

We can make a more general statement about the relative price efficiency of the two markets. The following result holds at any interior equilibrium.

**Proposition 4** (When are markets more micro or macro efficient?). If $\tau_M \leq 1$, then $\rho^2_S \geq \rho^2_F$ (markets are more micro efficient). If $\tau_M > 1$, then $\rho^2_S < \rho^2_F$ (markets are more macro efficient) for $\rho^2_F \in (1/\tau_M, 1]$ and otherwise $\rho^2_S \geq \rho^2_F$.

This result shows that if the micro learning technology is more precise than the macro
one (meaning $\tau_M < 1$) then markets will always be more micro efficient. This conclusion supports the comments of Jung and Shiller (2006) discussed in the introduction. If the macro technology is more precise then there will be a range of parameter values where markets can be either micro or macro efficient.

The difference in price efficiencies $\rho_S^2 - \rho_F^2$ is illustrated in Figure 4 for the two $\tau_M$ regimes. In the left panel, where micro information is more precise ($\tau_M < 1$), $\rho_S^2 > \rho_F^2$ and the difference is always positive. In the right panel, where macro information is more precise ($\tau_M > 1$), we see one region (shown as the filled-in area in the chart) where markets are more micro efficient and another region in which they are more macro efficient.

There is no obvious link between the precision of the learning technology and the incentive for agents to become $i$ or $M$ informed. In fact as Lemma 5 showed, $i$ informed are worse off as their learning technology improves (as $f_S$ increases). And Lemma 6 shows that $M$ informed can be better or worse off as their learning technology improves, depending on how macro efficient prices already are. Proposition 4 shows that despite this incentive structure, the relative precision of the micro and macro price technologies has an important impact on the relative informativeness of micro and macro prices, in equilibrium.

Proposition 4 suggests that markets tend towards micro efficiency. First, it is likely that $\tau_M < 1$, though this is an empirical question. But even if $\tau_M > 1$, there is still a range of parameter values where markets are more micro efficient.

### 5 Information equilibrium

In Section 4 we examined the choice among informed agents to become macro informed or micro informed, holding fixed the total fraction of informed agents. That analysis describes a medium-term equilibrium, over a time scale long enough for informed agents to shift their focus of attention, but not long enough for uninformed agents to acquire the skills needed to become informed.

In this section, we examine a longer-term equilibrium in which the uninformed can become informed by incurring a cost $c$. We endogenize not only the choice between micro and macro information, but also the decision to become informed. An equilibrium in this setting — which we refer to as an information equilibrium — is defined by nonnegative proportions ($\lambda_M, \lambda_S, \lambda_U = 1 - \lambda_M - \lambda_S$) such that no agent of a type in strictly positive proportion prefers switching to a different type. Extending (28), we require that, for any
Figure 4: These charts show the difference between micro and macro price efficiency, $\rho_S^2 - \rho_F^2$, as a function of macro price efficiency $\rho_F^2$. The shaded areas in the charts represents the region of micro efficiency. The two charts are labeled with their respective information precision ratios $\tau_M$. In the $\tau_M > 1$ regime, the function $\rho_S^2 - \rho_F^2$ has roots at $1/\tau_M$ and 1. Parameter values are described in Section A.1.

Recall that our utilities are negative, so the inequality on the left implies that type $i'$ is preferred to type $i$. The ratios $J_M/J_U$, $J_S/J_U$, and $J_S/J_M$ all have well-defined limits as some or all of $\lambda_M$, $\lambda_S$, and $\lambda_U$ approach zero. (This follows from the expressions for these ratios in (26) and (27) and the dependence of $\rho_F^2$ and $\rho_S^2$ on $\lambda_M$ and $\lambda_S$ in (24) and (25), respectively.) We may therefore evaluate and compare these ratios even in cases where one or more of the proportions $\lambda_i$ is zero.

### 5.1 Effect of information cost $c$ on the equilibrium

Figure 5 helps illustrate the general results that follow. The figure plots the equilibrium proportion of each type of agent as a function of the cost $c$ of information acquisition. The figure divides into three regions. At sufficiently low costs, all agents prefer to become...
informed, so $\lambda_U = 0$. At sufficiently high costs, no investors to choose to be macro informed, so $\lambda_M = 0$. At intermediate costs, we find agents of all three types. At all cost levels, some fraction of agents choose to be micro informed.

![Figure 5: Equilibrium proportions of macro informed, micro informed, and uninformed agents as functions of the cost of information acquisition $c$.](image)

To justify these assertions and to give an explicit characterization of the information equilibrium at each cost level $c > 0$, we first consider the possibility that all three types of agents are present in positive proportions. To be consistent with equilibrium, this outcome requires $J_M/J_U = J_S/J_U = 1$. Using the expressions for these ratios in (26) and (27), these equalities imply

$$
\rho^2_F = 1 - \frac{1 - f_M}{f_M} [e^{2\gamma c} - 1].
$$

(37)

and

$$
\rho^2_S = \left(1 + \frac{1 - f_S}{f_S} [e^{2\gamma c} - 1]\right)^{-1}.
$$

(38)

Setting these expressions equal to (24) and (25), respectively, we can solve for $\lambda_M$ and $\lambda_S$ to get

$$
\lambda_M(c) = \gamma(1 - f_M)\sigma_M\sigma_{X_F}\left(\frac{1}{(1 - f_M)(e^{2\gamma c} - 1)} - \frac{1}{f_M}\right)^{1/2}
$$

(39)
The expression for $\lambda_M(c)$ is valid for $c < \bar{c}$, with

$$\bar{c} = -\frac{1}{2\gamma} \log(1 - f_M).$$

(41)

If $\lambda_M(c) + \lambda_S(c) \leq 1$, then $(\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c))$ defines an information equilibrium with $J_M = J_S = J_U$.

Both $\lambda_M(c)$ and $\lambda_S(c)$ increase continuously and without bound as $c$ decreases toward zero, so the equation

$$\lambda_M(c) + \lambda_S(c) = 1,$$

defines the lowest cost at which we can meaningfully set $\lambda_U = 1 - \lambda_M(c) - \lambda_S(c)$. At lower cost levels, we need to consider the possibility of an equilibrium with $\lambda_U = 0$.

Once we fix a value for $\lambda_U$, the split between macro and micro informed agents is characterized by Proposition 3. Write $\lambda^*_M(0)$ for the value of $\lambda^*_M$ in Proposition 3 at $\lambda_U = 0$; this value is given either by the root $\tilde{\lambda}_M$ in (30) or zero. Set

$$\lambda_M, \lambda_S, \lambda_U = \begin{cases} 
\lambda^*_M(0), 1 - \lambda^*_M(0), 0, & 0 < c < \zeta; \\
\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c), & \zeta \leq c < \bar{c}; \\
0, \lambda_S(c), 1 - \lambda_S(c), & c \geq \bar{c},
\end{cases}$$

(42)

Then (42) makes explicit the equilibrium proportions illustrated in Figure 5: at large cost levels, $\lambda_M = 0$; at low cost levels, $\lambda_U = 0$ and $\lambda_M$ and $\lambda_S$ are constant; at intermediate cost levels, all three proportions are positive; at all cost levels, $\lambda_S > 0$. We always have $\zeta > 0$ and $\bar{c} < \infty$, so the low cost and high cost ranges are always present; but it is possible to have $\zeta = \bar{c}$, in which case the intermediate cost range is absent. This occurs when $\lambda^*_M(0) = 0$ (see, in particular, case (ii) of Proposition 3) and implies that no investor chooses to be macro informed at any cost level.

**Proposition 5** (Information equilibrium). *At each $c > 0$, the proportions in (42) define the unique information equilibrium.*

From this characterization, we can deduce several properties of the information equi-
librium. Let us define $\Pi_M$ as the fraction of informed who are macro informed, or

$$
\Pi_M \equiv \frac{\lambda_M}{\lambda_M + \lambda_S}.
$$

(43)

When we are at an interior attention equilibrium, i.e. $\lambda^*_M > 0$, we see that $\Pi_M$ is the coefficient of $(1 - \lambda_U)$ (since $1 - \lambda_U = \lambda_M + \lambda_S$) in (30). Differentiating with respect to $\lambda_U$ yields

$$
\frac{d\Pi_M}{d\lambda_U} < 0,
$$

(44)

when $\lambda_M > 0$. In other words, the more uninformed investors there are in the economy, the greater the fraction of informed investors who choose to be micro informed. The next result describes the dependence of $\Pi_M$ on $c$ and summarizes some features of the information equilibrium.

**Corollary 1** (Effect of information cost $c$ on information equilibrium). In equilibrium, with a cost of becoming informed given by $c$, the following will hold:

(i) As $c$ increases, the fraction $\Pi_M$ of informed investors who choose macro information falls; moreover, $\Pi_M$ is strictly decreasing in $c$ if $\lambda_M > 0$ and $\lambda_U > 0$.

(ii) There is a maximal cost given by (41) above which no agent will choose to be macro informed. If $\gamma^2 \alpha = 1$, with $\alpha$ as in (31), then no agent chooses to be macro informed at any cost level.

(iii) As $c$ grows the fraction of investors who are uninformed increases; moreover $\lambda_U$ is strictly increasing in $c$ wherever $\lambda_U > 0$.

(iv) Micro and macro price efficiency are decreasing in $c$.

As $c$ increases and the number of uninformed grows, if $\Pi_M$ doesn’t change, the already micro and macro informed (i.e. assuming they don’t have to pay $c$ to become informed) are relatively better off (this follows from Lemma 3). However, the micro informed gain disproportionately more than the macro informed – as will be discussed Section 5.3. In order to maintain the attention equilibrium at the new higher $c$, we therefore need more micro informed to equilibrate the relative benefits of micro vs macro information. Therefore, $\Pi_M$ must fall when $c$ increases.
5.2 No-trade condition for micro uninformed

Our analysis thus far has restricted non-\(i\) informed agents from trading in stock \(i\). Recall our assumption that non-\(i\) informed must pay a cost \(\kappa_i\) to condition on \(P_i\) and to trade in that stock. We now show that in an information equilibrium there will be some threshold cost, \(\kappa\), such that if \(\kappa_i > \kappa\) the \(i\)-uninformed are better off not trading in stock \(i\). Furthermore, we show that this cost is strictly less than \(c\), the cost of becoming informed, and that \(\kappa\) goes to zero as the learning technology for micro information improves.\(^{12}\)

To simplify the analysis, let \(P_{S_i} = P_i - \beta_i P_{F_i}\) denote the price of security that pays \(S_i\). Suppose \(0 < f_S < 1\), and define

\[
\kappa = \frac{1}{2\gamma} \log \left( \frac{\text{var}[S_i - R P_{S_i}]}{\text{var}[S_i | P_{S_i}]} \right), \tag{45}
\]

where

\[
\text{var}[S_i - R P_{S_i}] = (1 - f_S)\sigma_S^2 + R^2 \sigma_S^2 \sigma_X^2,
\]

and

\[
\text{var}[S_i | P_{S_i}] = (1 - f_S)\sigma_S^2 + f_S \sigma_S^2 (1 - \rho_S^2) = (1 - f_S \rho_S^2)\sigma_S^2. \tag{46}
\]

Suppose that the economy is in an information equilibrium. Suppose that agents uninformed about stock \(i\) (which includes any macro-informed agents, agents informed about stock \(j\), \(j \neq i\), and uninformed agents) may deviate from the equilibrium by investing in stock \(i\), making inferences based on its price and conditioning their demand on the price, but that they incur a cost \(\kappa_i\) in doing so. Recall that the total number of stocks is \(N \geq 2\).

**Proposition 6.** If \(\kappa_i > \kappa\), then no macro-informed or uninformed investors will deviate from equilibrium by investing in stock \(i\). No \(j\)-informed investors, \(j \neq i\), will deviate, for all sufficiently large \(N\).

When the number of stocks \(N\) is finite, condition (4) introduces a correlation of \(-1/(N - 1)\) between pairs of idiosyncratic terms. This correlation is negligible for even moderately large \(N\). The condition in the second sentence of the proposition is needed to ensure that stock-specific information about stock \(i\) available to a \(j\)-informed agent through the stock-specific information \(s_j\) is indeed negligible.

\(^{12}\)In an omitted result, we show that uninformed agents who do not condition on \(P_i\) would choose not to trade in individual stocks even if this were costless, as long as markets were sufficiently micro efficient (\(\rho_S^2\) large enough).
The next result shows that the threshold $\kappa$ for the conditioning cost to be consistent with the information equilibrium is smaller than the cost $c$ of becoming informed, and it becomes negligibly small as the micro informativeness $f_S$ increases. We restrict attention to information equilibria in which $\lambda_U > 0$ to ensure that the cost $c$ is binding on the decision to become informed. This condition rules out cases (such as $c = 0$) in which the cost is too low to be relevant to the equilibrium.

**Proposition 7.** In an information equilibrium with $\lambda_U > 0$, the following properties hold.

(a) For any $0 < f_S < 1$, we have $\kappa < c$.

(b) At $f_S = 0$, $\kappa = c$.

(c) As $f_S \to 1$, $\kappa \to 0$.

(d) $\kappa$ is a decreasing function of $f_S$.

Note that the threshold $\kappa$ in (45) is increasing in the precision $1/\text{var}[S_i|P_{S_i}]$ of the price signal about $S_i$, and also increasing in the unconditional return variance $\text{var}[S_i - RPS_i]$. As the learning technology $f_S$ improves, the price-signal precision improves, but so much of the learnable information is impounded into prices by the $i$ informed that the unconditional return variance drops even faster. This, in turn, reduces the incentive for the $i$-uninformed to learn about the security payout by conditioning on today’s price.

### 5.3 Are markets micro efficient?

A tendency for markets to be more micro than macro efficient has been a recurring theme of many of our results. For example:

- From Propositions [3] and [5], we note that the $\lambda^*_S > 0$ – there must always be some micro informed investors. It is, in fact, possible that there are only micro informed investors, so that $\lambda^*_M = 0$.

- From Lemma[4], we see that when investors in the economy become more risk averse, there will be more micro informed.

- From Proposition [4], when the micro learning technology is more precise than the macro we always have that $\rho^2_S > \rho^2_F$ – i.e. prices are more micro efficient than macro efficient. Even when the macro technology is more precise, for low enough macro efficiency, the market will be more micro efficient.
• Corollary 1 shows that when information is costlier, a larger fraction of the informed investors choose micro information.

• In fact, from Proposition 5 we know that at a sufficiently high cost of becoming informed, all informed investors choose to be micro informed.

The key driver of these results is our assumption that micro uninformed do not costlessly make inferences from individual stock prices. Furthermore we show that, when the micro learning technology is precise, the cost that agents would be willing to pay for this conditioning goes to zero. Therefore, the micro informed are the only investors who can collect surplus from accommodating idiosyncratic supply shocks. This creates a strong incentive for collecting micro information. Technically, we see from (29) that the benefit \( J_M/J_U \) to being the first macro informed investor is finite, whereas the benefit to being the first micro informed is infinite (\( J_S/J_U \) takes on its highest possible value – zero – at \( \lambda_S = 0 \)).

6 Applications

6.1 Evolution of investor attention

The most important insight generated by our analysis is the connection between the information environment faced by investors and their information choices. A key variable in our model is the ratio of knowable versus total risk, both at the macro and micro levels. Recall from Proposition 4 that if \( \tau_M \leq 1 \), markets are always more micro efficient (i.e. \( \rho_S^2 > \rho_F^2 \)). Determining \( \tau \) involves knowing \( f_M \) and \( f_S \), and is an important empirical challenge.

One approach to estimating these quantities is to run the following forecasting regression for return on assets at the individual name or industry (or economy-wide) level

\[
ROA_i(t + 1) = a + \Gamma'X_i(t) + \epsilon(t + 1),
\]

where \( X(t) \) is the set of predictive variables (potentially including lagged ROAs) for \( i \)'s earnings that are known prior to time \( t + 1 \). In the regression, \( i \) can be either a set of companies, a set of industries or a set of countries. The \( R^2 \) of this regression would be an estimate of a lower bound for either \( f_S \) (if run at the individual name level) or \( f_M \) (if run at some level of aggregation). The estimate is a lower bound because we cannot be sure that \( X \) includes all the relevant and knowable information for forecasting \( i \)'s earnings.\(^{13}\)

\(^{13}\)For example, Ball, Sadka and Sadka (2009) using US firms from 1950–2005 show that just under
Furthermore, it is likely that there is a relationship between today’s $\lambda^*_S$ and $\lambda^*_M$ and tomorrow’s $f_S$ and $f_M$ respectively – as more people become macro (micro) informed what is knowable about macro (micro) risk may increase. Using assets under management (AUM) in various institutional arrangements (such as macro hedge funds, or fundamentally driven mutual funds) as proxies for the model’s $\lambda$’s, one could analyze whether the explanatory power of the regression in (47) increases as AUM in the relevant set of funds increases.

If there is a relationship between future $f$’s and current $\lambda$’s this may induce cycles in the structure of the asset management industry. Investors may “overcommit” to a particular specialty thereby driving down the benefit of information, and then shift their focus to another specialty, at which point the cycle repeats. The key driver is the fact that the benefit to being macro informed can increase or decrease with $f_M$, as was shown in Lemma 6 which in turn causes $\lambda^*_M$ to be a hump-shaped function of $f_M$ (see Lemma 7). Figure 6 shows that assets under management devoted to different hedge fund styles exhibit large time variation, potentially in response to overcommitment by investors to a particular strategy, followed by the flight of investor assets to less “crowded” strategies. Note, in particular, the apparent negative correlation between AUM in macro and long-short equity (where macro risk is typically hedged out), and between distressed and fixed income. We interpret this as anecdotal evidence that information collection in one strategy type ultimately reveals so much information that speculators no longer find it worthwhile to focus on that strategy, and shift their attention to other opportunities.

**Time trends and cross-sectional implications**

We predict (as would Grossman and Stiglitz (1980)) that the amount of money being actively managed falls as information costs rise. This is a potential explanation for the increase in passively managed funds over the past few years\(^\text{14}\) As the cost of computational power and data has fallen, the amount of information that is in the public domain has increased, and the cost $c$ of learning information that is unknown to others has likely increased. Our model’s prediction that $d\Pi_M/dc < 0$ would imply that as money has been moving into passively managed products, the proportion of actively managed funds that are micro focused has increased.

60% of contemporaneous cross-sectional ROA variation is captured by the top 5 principal components of ROA. One could analyze the extent to which either the principal components or the firm residuals can be forecasted.

\(^{14}\)See, for example, “Active asset managers knocked by shift to passive strategies” from the *Financial Times*, April 11, 2016.
Related to the above point, it is likely that in less developed markets, less information is publically known, and the cost of learning information that is not widely known is likely lower than in developed markets. Therefore, the active money management industry in emerging markets should be more macro focused than in developed markets. We are not aware of any empirical work that has addressed these questions.

6.2 Systematic and idiosyncratic volatility

We define “excess returns”\footnote{Since prices can be negative in our model, the return concept is the dollar gain from an investment, rather than a percent return.} on stock $i$ in our model as $u_i - RP_i$. We decompose this into the systematic return component $\beta_i(M - RP_F)$ and an idiosyncratic return component given by

$$u_i - RP_i - \beta_i(M - RP_F) = \underbrace{\epsilon_i}_{\text{Unknowable portion of dividend}} + \underbrace{\gamma(1 - f_S)\sigma_S^2 X_i/\lambda_S}_{\text{Adjustment due to idiosyncratic supply shock}}.$$  \hspace{1cm} (48)
The idiosyncratic return variance can be written as

\[
Vol_{idio}^2 = \sigma_{\epsilon_s}^2 \left( 1 + \frac{\gamma^2 \sigma_{\epsilon_s}^2 \sigma_X^2}{\lambda_S^2} \right) = \sigma_{\epsilon_s}^2 \left( 1 + \frac{f_S}{1 - f_S} \left[ \frac{1}{\rho_S^2} - 1 \right] \right),
\]

(49)

where the second equation follows from (25). As can be seen from (48), idiosyncratic return volatility consists of two components. The first comes from the unknowable portion of the idiosyncratic dividend, \(\epsilon_i\), and the second comes from price adjustment in response to idiosyncratic supply shocks. Note that the knowable portion of the dividend payout \(s_i\) does not enter into the idiosyncratic return because of the \(s_i/R\) term in \(P_i\).

Interestingly, in an attention equilibrium, we can use (49) to rewrite \(J_S/J_U\) from (27) as

\[
J_S/J_U = \exp(\gamma c) \frac{\sigma_{\epsilon_s}}{Vol_{idio}}.
\]

(50)

This makes clear that the benefit of being \(i\) informed increases with the idiosyncratic return volatility (recall \(J_S/J_U\) falls when \(i\) informed are better off) but decreases in the unknowable part of idiosyncratic dividend volatility (\(\sigma_{\epsilon_s}\)).

With \(\beta_i = 1\), the systematic variance of stock returns (or equivalently, the variance of index fund returns) is given by

\[
Vol_{syst}^2 \equiv \text{var}(M - RPF) = \sigma_{\epsilon_M}^2 + (1 - Rb_F)^2 \sigma_m^2 + R^2 c_F^2 \sigma_X^2,
\]

(51)

which follows from (13). While we have a closed form solution for all quantities in this equation (see Proposition 2 and equation (A.2)), there unfortunately is no simple and general characterization for \(Vol_{syst}^2\).

We have a simple expression for systematic volatility in two special cases of the model. When \(f_M \to 0\),

\[
Vol_{syst}^2 \to \sigma_M^2 \left( 1 + \gamma^2 \sigma_M^2 \sigma_X^2 \right).
\]

(52)

Note that this is an exact relationship. Comparing this to the first equation in (49), we see that when no information is revealed about the common component of the dividend \(M\), the variance of the index fund return (i.e. \(M - RPF\)) consists of the variance of \(M\)

\[1\] This implies that \(\rho_F^2 = 0\) from (24) for any value of \(\lambda_M\). For a non-zero cost of becoming informed, we will also have that \(\lambda_M = 0\) since the macro signal will contain zero information. The relationship in (52) surprisingly holds even if \(\lambda_M > 0\).
and the variance in \( P_F \) that comes from accommodating the aggregate supply shock \( X_F \).

At the other extreme, when \( f_M \) approaches 1, and therefore when \( \rho_F^2 \) approaches 1, the systematic variance is given by

\[
Vol^2_{syst} = \sigma_{\epsilon M}^2 \left( 1 + \frac{\gamma^2 \sigma_{\epsilon M}^2 \sigma_{X_F}^2}{\lambda_M^2} \right) + o(\sigma_{\epsilon M}^4),
\]

which follows from Section A.2.1 and where \( \sigma_{\epsilon M}^2 = (1 - f_M) \sigma_{M}^2 \). Unlike (52) this is an approximate relationship because at \( f_M = \rho_F^2 = 1 \), the economy becomes degenerate. But (53) has exactly the same interpretation except that the dividend uncertainty is no longer about \( M \) but is about \( \epsilon_M \), the unknowable part of \( M \) (since the knowable part is fully revealed by the price). Note the similarity between the systematic return volatility decomposition in (53) and the idiosyncratic return volatility decomposition in (49).

Figure 7 shows a numerical example of how systematic volatility given by (51) behaves in the model as a function of \( \Pi_M \), the fraction of informed that are macro informed, for different levels of precision \( f_M \) of the learning technology. The blue circle in the upper left corner shows systematic volatility from (52) when macro prices reveal nothing about \( m \), and the colored diamonds on the right hand side of the figure show systematic volatility from (53) when the macro signal precision \( f_M \) approaches 1. The approximation improves, as can be expected, as \( f_M \) and \( \Pi_M \) grow (see Footnote 17).

**How volatility depends on attention allocation**

From (49) and the fact that \( \sigma_{\epsilon S}^2 = (1 - f_S) \sigma_{S}^2 \) we make two observations about idiosyncratic return volatility:

- \( Vol_{idio} \) falls as \( f_S \) increases (holding the \( \lambda \)'s fixed), and
- \( Vol_{idio} \) falls as \( \lambda_S \) (and therefore \( \rho_S^2 \)) increases.

As more of the idiosyncratic portion of the dividend \( S_i \) becomes knowable, idiosyncratic return volatility falls (holding all else equal). As there are more micro informed investors, prices become less sensitive to idiosyncratic supply shocks, and return volatility again falls.

We observe from Figure 7 that systematic volatility has the same qualitative dependence on \( f_M \) and \( \lambda_M \) as does idiosyncratic volatility on \( f_S \) and \( \lambda_S \) respectively. Finally, we note

\[
\text{Note that (53) would hold exactly when } \rho_F^2 = 1, \text{ which could happen for } f_M < 1 \text{ as } \gamma/\lambda_M \to 0. \text{ Therefore, for a given } f_M, \text{ the approximation in (53) is a better fit for higher values of } \lambda_M \text{ (and therefore of } \Pi_M).}
\]
Figure 7: This figure shows systematic volatility in the model for different sets of macro signal precisions $f_M$ as a function of the fraction of informed who are macro informed $\Pi_M$. The horizontal dashed line shows the volatility of the fundamental portion of the dividend $\sigma_M$. The shaded area above this line indicates the region of excess covariance. Parameter values are described in Section A.1. $\lambda_U$ is set to 0.4.

that holding all else equal, when $\lambda_M$ (or $\lambda_S$) changes $Vol_{idio}$ and $Vol_{syst}$ move in opposite directions.\footnote{Hong and Sraer (2015), Li (2015), and Gao et al. (2016) relate stock returns to levels of disagreement in analysts’ forecasts of either macro or micro fundamentals. Disagreement reflects the knowable fractions, $f_M$ and $f_S$, the fundamental volatilities, $\sigma_M$ and $\sigma_S$, and the fractions $\lambda_M$ and $\lambda_S$. Our model provides a possible framework for examining the implications of macro and micro disagreement simultaneously.}

**Behavior of idiosyncratic volatility**

The relationship in (49) sheds some light on the finding in Bekaert, Hodrick and Zhang (BHZ) (2012) (at the country level) and in Herskovic et al. (2016) (for US stocks) that idiosyncratic volatility has a strong common component. In a dynamic version of our economy $f_S$ acts as a state variable, whose value is affected either by (i) shocks (techno-
logical or regulatory) to the knowable portion of micro uncertainty or by (ii) past investor information choices (as discussed in Section 6.1). In an economy with a high $f_S$ more information about corporate cash flows will be reflected in prices, investors will demand less compensation for idiosyncratic supply shocks (since prices are more “accurate”) and idiosyncratic return volatility will fall. This will also happen when the cost of becoming informed falls, thereby increasing $\lambda_S$. Since these dynamics affect all stocks equally, variation in idiosyncratic volatility induced via this channel should have a strong common component. Note that this channel is quite different from that discussed in Herskovic et al. (2016) who show that idiosyncratic return volatility is related to the volatility of firm-level cash flows. Our explanation operates not through cash flows but through how much market participants know about cash flows.

BHZ (2012) find that an important determinant of idiosyncratic volatility is the market variance premium (MVP, defined as the difference between the square of the VIX index and a forecast of the future market variance). In an information equilibrium with a positive fraction of uninformed investors it is easy to verify that idiosyncratic volatility from (49) is given by $Vol^2_{idio} = \sigma^2_{\epsilon_S} \exp(2\gamma c)$. To the extent that the MVP increases due to increasing risk aversion $\gamma$, our model forecasts an equilibrium increase in idiosyncratic volatility. Furthermore, BHZ (2012) document that the correlation of country-level idiosyncratic volatility has increased over time. Our model would predict such an increase if variation in $f_S$ relative to other model variables has increased. Given recent changes in technological and regulatory factors, as well as the rapidly shifting structure of the asset management industry, an increase in the variation of $f_S$ is plausible.

### 6.3 Excess volatility and comovement

It has long been recognized that the majority of stock return variability cannot be explained by information about future dividends (LeRoy and Porter 1981 and Shiller 1981 are the classic papers in the area; see also Campbell 1991 and Campbell and Ammer 1993). Later empirical work has documented the fact that stocks also exhibit a higher covariance than is justified by their cash flows (for example, Pindyck and Rotemberg (1993) and Barberis, Shleifer and Wurgler (2005)). We show in this section that the excess volatility and excess comovement phenomena are, in fact, closely related, and can both be explained – at least partially – by the relative micro vs macro efficiency of markets.

In our model, assuming $N$ is large, the covariance between the dividend of stocks $i$
and \( j \) is given by
\[
\text{cov}(u_i, u_j) = \beta_i \beta_j \sigma^2_M.
\]
The covariance between excess returns of \( i \) and \( j \) is given by
\[
\text{cov}(u_i - R_{P_i}, u_j - R_{P_j}) = \beta_i \beta_j \text{Vol}^2_{\text{syst}}.
\]

If we define excess comovement as a higher covariance of returns than the covariance of dividends, then this reduces to
\[
\text{Vol}^2_{\text{syst}} > \sigma^2_M \iff \text{Excess comovement}.
\]

Return comovement exceeds earnings comovement when index fund returns are more volatile than index fund earnings. In other words, excess macro volatility leads to excess covariance.

The shaded region in Figure [7] shows when \( \text{Vol}_{\text{syst}} \) is higher than \( \sigma_M \) (indicated by the horizontal dashed line).\(^{20}\) We see that when macro signal precision \( f_M \) is high or when there are a large fraction of macro informed \( \Pi_M \), return volatility is lower than fundamental volatility – that is we have insufficient covariance. This happens because prices reveal so much information about \( M \) that \( u_i - R_{P_i} \) becomes relatively \( M \) insensitive.

Excess volatility and therefore excess covariance arise when the macro signal is very imprecise (i.e. \( f_M \) is low) or when there are relatively few macro informed investors (i.e. \( \Pi_M \) is small). The tendency of actual stocks to exhibit excess covariance and excess volatility, as empirical work suggests, lends more evidence to the micro efficiency (and macro inefficiency) of markets.

Peng and Xiong (2006) show that excess comovement can arise in a behavioral representative investor model when investors are sufficiently overconfident in the quality of their information. Veldkamp (2006) shows that excess comovement can be obtained in a fully rational framework when investors tend to learn the same information as other investors and then make inferences about security payoffs based on the prices of a small set of common securities. Our model identifies a new and very broad channel of excess comovement – the relative micro versus macro efficiency of the market.

\(^{20}\)Though the shaded excess covariance region is small in the figure to show the full range on systematic volatility behavior, our view is that this is the region most representative of actual markets.
Excess idiosyncratic volatility

From equation (49) we see that the square of the ratio of idiosyncratic return volatility to idiosyncratic dividend volatility is given by

$$\frac{Vol_{idio}^2}{\sigma_S^2} = 1 + f_s \left( \frac{1}{\rho_s^2} - 2 \right),$$

from which we see that there is excess idiosyncratic volatility (i.e. $Vol_{idio}^2 > \sigma_S^2$) if and only if

$$\rho_s^2 < \frac{1}{2}.$$  

Also excess idiosyncratic volatility falls when price efficiency increases. There is no equally clean characterization of the relationship between systematic return and dividend volatility, although Equations (52) and (53) yield comparable expressions for the limits $f_M \to 0$ and $f_M \to 1$ respectively.

Excess correlation

We should note that using Peng and Xiong’s (2006) definition of excess comovement as return correlation being higher than dividend correlation leads to the following characterization in our model

$$\frac{Vol_{syst}}{\sigma_M} > \frac{Vol_{idio}}{\sigma_S} \iff \text{Excess comovement},$$

for the case of two representative stocks with unit betas.$^{21}$ This characterization leads to the same intuition as in the covariance case. Relative micro efficiency (i.e. low $Vol_{idio}$ relative to $\sigma_S$ – which occurs when micro price efficiency is high, according to Equation (54)) is associated with excess comovement, as is relative macro inefficiency (i.e. high $Vol_{syst}$ relative to $\sigma_M$).

Equation (55) can be interpreted in terms of the finding in Vuolteenaho (2002) that firm level market adjusted returns (i.e. his analogue to $u_i - RP_i - \beta_i(M - RP_F)$) are driven predominantly by cash flow news (Table III in his paper) and that market excess returns (i.e. $M - RP_F$) are more driven by discount rate – and not cash flow – news (his Table VII). Since $\sigma_M$ and $\sigma_S$ proxy for cash flow news in our model, Vuolteenaho’s results suggest that the ratio on the left hand side of (55) is high, and the ratio on the right hand

$^{21}$With unit betas, return correlations are given by $Vol_{syst}^2/(Vol_{syst}^2 + Vol_{idio}^2)$ and cash flow correlations are given by $\sigma_M^2/($). Rearranging and taking square roots yields (55).
side is low. This is additional evidence of micro efficiency.

6.4 Recessions

Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) is a related model that studies how investors allocate limited attention to macro and micro uncertainty. In their paper, as in ours, the individual stock dividend process is given by beta times a macro shock plus an idiosyncratic shock. While our paper investigates the specialization choices of investors into micro and macro informed, their paper analyzes how informed investors, who all receive signals from the same distribution, would choose to allocate the precision of those signals between micro and macro information. The two papers analyze related problems, but from two different, and complementary, vantage points. Therefore contrasting the mechanisms at work in the two theories should be informative for our understanding of the functioning of actual markets.

The main theoretical results in Kacperczyk et al. (2016) follow from their Proposition 1, which states that the informed investors allocate more attention to securities with greater cash flow variance, and their Proposition 2, which states informed investors will allocate more attention to the macro component of cash flows ($M$ in our model) as risk aversion increases. A recession in their model is defined as a time of increased risk aversion and increased macro dividend volatility, $\sigma_M$, which allows them to conclude in Section 2 that

“...in recessions, the average amount of attention devoted to aggregate shocks should increase and the average amount of attention devoted to stock-specific shocks should decrease.”

The corresponding question in our model can be posed as follows: How does $\lambda^*_M$ depend on risk aversion $\gamma$ and dividend volatility (given by $\sigma_M$ and $\sigma_S$)? From Lemma 4 we know that (1) increasing risk aversion pushes investors towards micro information, (2) increasing macro dividend risk $\sigma_M$ pushes investors towards macro information, and (3) increasing idiosyncratic dividend risk $\sigma_S$ pushes investors to micro information. Whereas in Kacperczyk et al. (2016) the risk aversion and macro dividend risk effects reinforce each other, in our model they offset. Furthermore if idiosyncratic dividend risk $\sigma_S$ were to increase in a recession, we show that this would push investors further towards micro information.\(^{22}\) Our models only make the same prediction for recessions characterized by

\(^{22}\)Kacperczyk et al. (2016) document that during recessions systematic return volatility increases, while idiosyncratic return volatility increases by much less (and the latter increase is not statistically different
a small increase in $\gamma$, negligible increase in $\sigma_S$, and a large increase in $\sigma_M$.

A key difference in the two models is the fact that increasing risk aversion, or increasing aggregate risk, diminishes one benefit of being macro informed in our model (trade with the macro uninformed falls) but increases the benefit of liquidity provision to idiosyncratic noise trades by the micro informed. This mechanism isn’t present in the Kacperczyk et al. (2016) model, and may account for the different predictions.

Kacperczyk et al. (2016) show that funds’ portfolio deviations from the market co-vary more with future changes in industrial production during recessions, and that these porfolio deviations covary more with future firm-specific earnings shocks during booms. They interpret this as evidence that funds are more macro-focused during recessions, and more micro-focused during booms. An alternative explanation for this observation is that funds perceive the market to be more micro-efficient during recessions and therefore tailor their portfolios to take advantage of macro opportunities, and similarly during booms funds perceive the market to be more macro-efficient and therefore focus their portfolios on micro opportunities. This latter interpretation would explain the finding in Kacperczyk et al. (2014) that mutual funds seem to be better stock pickers during booms, and better market timers during recessions. Furthermore it is consistent with the implications of our model that investors gravitate towards micro information in recessions characterized by an increase in risk aversion and an increase in both macro and micro dividend risks.\(^{23}\)

![Image](image.png)

Which of these interpretations is a better fit for the combined results of Kacperczyk et al. (2014) and Kacperczyk et al. (2016) is an important area for future work.

7 Conclusion

Professional investors typically specialize along a particular dimension of knowledge. This choice of specialization has important effects on the behavior of market prices. We model an economy where investors can choose to be informed either about micro or macro fundamentals. An important driver of this choice is the ability for micro informed investors to accommodate, and thereby extract rents from, the idiosyncratic portion of security supply shocks. This opportunity arises because the micro uninformed choose not to trade individual stocks when there is even a small cost of making inferences from invididual

\(^{23}\)If $\sigma_M$ and $\sigma_S$ both increase in the same proportion, therefore leaving their ratio and $\varphi$ from (31) unchanged, then the same logic underlying the result in Lemma 4 that $d\lambda_M^*/d\gamma < 0$ implies that $\lambda_M^*$ would fall.
stock prices. Our model sheds light on a longstanding question in financial economics – whether markets are better at incorporating information at the micro or macro level.

In our setting, we show a general tendency of markets towards micro rather than macro efficiency. Furthermore, the choice by investors of specializing in micro or macro information has direct bearing on:

- The dynamics of AUM across different areas of specialization;
- The amount of information knowable about company-specific or aggregate earnings;
- The recent proliferation of passively managed money in the US;
- The relative micro–macro focus of emerging market versus developed market investors;
- The relationship between idiosyncratic and systematic volatility;
- The time series behavior of idiosyncratic volatility;
- The excess volatility of stock returns relative to earnings;
- The tendency of stocks to comove in excess of their cash flow covariance;
- The allocation of information choices in recessionary periods.

An empirical analysis of these (and other) questions raised by our model should yield valuable insights into the operation of financial markets.
A Appendix

A.1 Parameter values for numerical examples

The parameter values used in the paper's numerical examples are:

\[ \gamma = 1.5; [f_M, f_S] = [0.75, 0.75]; [\sigma^2_M, \sigma^2_S] = [0.25, 0.25]; R = 1.05; \bar{X}_F = 0; [\sigma^2_{X_F}, \sigma^2_X] = [0.5, 1]. \]

A.2 Solution of model

Proof of Proposition 1. The analysis is simplified if we allow informed agents to invest in the index fund and in a hedged security paying \( u_i - \beta_i u_F = S_i \), with price \( P_i - \beta_i P_F \).

If we let \( \tilde{q}_F \) and \( \tilde{q}_S \) denote the demands in this case, the demands in the original setting are given by \( q^i_F = \tilde{q}_S \) and \( q^i_F = \tilde{q}_F - \beta_i \tilde{q}_S \). By standard arguments, the modified demands are given by

\[
\begin{bmatrix}
\tilde{q}^i_F \\
\tilde{q}^i_S
\end{bmatrix} = \frac{1}{\gamma} \text{var} \left[ \frac{M}{S_i} | I_i \right]^{-1} \left( E \left[ \frac{M}{S_i} | I_i \right] - R \left[ \frac{P_F}{P_i - \beta_i P_F} \right] \right). 
\]

Now

\[
\text{var} \left[ \frac{M}{S_i} | I_i \right] = \left( \frac{\text{var}[M | I_i]}{\text{var}[S_i | I_i]} \right) = \left( \frac{\text{var}[M | P_F]}{(1 - f_S)\sigma^2_S} \right), \tag{A.1}
\]

and

\[
E \left[ \frac{M}{S_i} | I_i \right] = \left[ E[M | P_F] \right]_{s_i}.
\]

Thus, \( \tilde{q}^i_F = q^U_F \), with \( q^U_F \) as given in (16), and

\[
\tilde{q}^i_S = \frac{s_i - R(P_i - \beta_i P_F)}{\gamma(1 - f_S)\sigma^2_S}.
\]

As \( q^i_S = \tilde{q}^i_S \), equation (18) follows, and then \( q^i_F = \tilde{q}^i_F - \beta_i \tilde{q}^i_S = q^U_F - \beta_i q^i_S \) completes the proof.

Proof of Proposition 3. The price \( P_F \) can be derived from first principles, but we can simplify the derivation by reducing it to the setting of Grossman and Stiglitz (1980). The informed (15) and uninformed (16) demands for the index fund and the market clearing condition (20) reduce to the demands in equations (8) and (8') of Grossman and Stiglitz (1980) and their market clearing condition (9), once we take \( \lambda = \lambda_M \) and \( 1 - \lambda_M = \lambda_U + \lambda_S \).

The coefficients of the price \( P_F \) in (13) can therefore be deduced from the price in their equation (A10). Theorem 1 of Grossman-Stiglitz gives an expression for \( P_F \) in the form
\( \alpha_1 + \alpha_2 w_\lambda \), for constants \( \alpha_1 \) and \( \alpha_2 > 0 \), where, in our notation,

\[
w_\lambda = m - \frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M}(X_F - \bar{X}_F).
\]

Comparison with (13) now implies that

\[
c_F b_F = -\frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M}.
\]

Setting \( b_F \) equal to the coefficient of \( x \) (= \( X_F \)) in (A10) of Grossman-Stiglitz, we get

\[
b_F = \frac{1}{R} \frac{\lambda_M}{(1 - f_M)\sigma_M^2} + \frac{1 - \lambda_M}{\sigma_M^2} \frac{f_M\sigma_M^2}{\var{M|P_F}} + \frac{1 - \lambda_M}{\var{M|w_\lambda}}.
\]

Moreover,

\[
\var{w_\lambda} = (1 - f_M)\sigma_M^2 + \frac{\gamma^2(1 - f_M)^2\sigma_M^4}{\lambda_M^2} \sigma_{X_F}^2,
\]

and \( \var{M|w_\lambda} = \var{M|P_F} \). To evaluate \( \var{M|P_F} \), note that the only unknown term in (17) is \( K_F b_F \), which we can now evaluate using (21) to get

\[
K_F b_F = \frac{f_M\sigma_M^2}{f_M\sigma_M^2 + \frac{\gamma^2(1 - f_M)^2\sigma_M^4}{\lambda_M^2} \sigma_{X_F}^2}.
\]

This yields an explicit expression for \( \var{M|P_F} \) which in turn yields an explicit expression for \( b_F \) through (A.2). An expression for \( c_F \) then follows using (21). Finally, to evaluate the constant term \( a_F \), we can again match coefficients with the expression in (A10) of Grossman-Stiglitz. Alternatively, we can evaluate their (A10) at (using their notation) \( \theta = \mathbb{E}\theta^* \) and \( x = \mathbb{E}x^* \), which, in our notation yields

\[
a_F = \frac{\bar{m}}{R} - \frac{\bar{X}_F}{R} \left[ \frac{1 - \lambda_M}{\gamma \var{M|P_F}} + \frac{\lambda_M}{\gamma(1 - f_M)\sigma_M^2} \right]^{-1}.
\]

Equation (22) follows directly from (18) and market clearing. Summing over \( i \) we get

\[
\frac{1}{N} \sum_i P_i = P_F - \frac{(1 - f_S)\sigma_S^2}{\lambda_S R} \xi \text{ by virtue of the idiosyncratic shock conditions in (3) and (8) and the fact that betas are 1 on average. Therefore, condition (6) is satisfied if and only if } \xi = 0.
\]
A.2.1 Approximation of $P_F$ when $f_M$ is near 1

Let $\delta = (1 - f_M)\sigma_M^2$. Then

\[
\begin{align*}
    a_F &= \frac{m}{R} - \frac{X}{R} \gamma \delta + O(\delta^2) \\
    b_F &= \frac{1}{R} + O(\delta^2) \\
    c_F &= -\frac{\gamma}{R\lambda_M} \delta + O(\delta^2).
\end{align*}
\]

(A.4)

The key point is that there is no $O(\delta)$ term in $b_F$; in other words, $b_F' = 0$ at $\delta = 0$. It is intuitive that $b_F$ (the sensitivity of the index fund price to the macro fundamental $m$) is maximized in the fully revealing case, but it is surprising that its derivative is zero there. In this case the price of the index fund, $P_F$, is given by

\[
P_F = \frac{m}{R} - \gamma(1 - f_M)\sigma_M^2 \frac{\bar{X}}{R} - \frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M} (X_F - \bar{X}_F) + O((1 - f_M)^2).
\]

The expressions in (A.4) follow from differentiation of the coefficients derived in the proof of Proposition 2. We omit the details.

A.3 Attention equilibrium

Proof of Lemma 2. The factor $\exp(\gamma c)$ in (26) and (27) reflects the cost of information acquisition. Since we are holding $\lambda_U$ fixed and comparing $J_M/J_U$ with $J_S/J_U$, we may set $c = 0$ and omit this factor. We make repeated use of Proposition 3.1 of Admati and Pfleiderer (1987), which yields for $\iota \in \{M, U, S\}$,

\[
J_\iota = -\frac{|\text{var}[u_\iota - R P_\iota | \mathcal{I}_\iota]|^{1/2}}{|\text{var}[u_\iota]|^{1/2}} \exp \left( -\text{E}[u_\iota - R P_\iota]^{\top} \text{var}[u_\iota - R P_\iota]^{-1} \text{E}[u_\iota - R P_\iota] / 2 \right).
\]

Here, $u_\iota$ denotes the payoff of the asset(s) in which agents of type $\iota$ invest, which is simply $u_F$ for $\iota = M, U$, and $(u_F, u_i)^{\top}$ for $\iota = S$. The corresponding prices are recorded in $P_\iota$. The information sets are $\mathcal{I}_M = \{m, P_F\}$, $\mathcal{I}_U = \{P_F\}$, $\mathcal{I}_S = \{s_i, P_t, P_F\}$, and $| \cdot |$ indicates the determinant of a matrix. Thus,

\[
J_U = -\left( \frac{\text{var}[M | P_F]}{\text{var}[u_F - R P_F]} \right)^{1/2} \exp \left( -\text{E}[u_F - R P_F]^2 / 2\text{var}[u_F - R P_F] \right)
\]

and

\[
J_M = -\left( \frac{\text{var}[M | m]}{\text{var}[u_F - R P_F]} \right)^{1/2} \exp \left( -\text{E}[u_F - R P_F]^2 / 2\text{var}[u_F - R P_F] \right),
\]

so

\[
J_M/J_U = \left( \frac{\text{var}[M | P_F]}{(1 - f_M)\sigma_M^2} \right)^{-1/2}.
\]
Combining (17) and (23), we get
\[ \text{var}[M|PF] = f_M \sigma^2_M (1 - \rho^2_F) + (1 - f_M) \sigma^2_M, \]
from which (26) follows.

To evaluate \( J_S \), we can use the same transformation as in the proof of Proposition 1 and assume that \( i \) informed agents optimize over uncorrelated securities paying \( M \) and \( S_i \) rather than paying \( M \) and \( \beta_i M + S_i \). The achievable utility is unchanged by this linear transformation of payoffs. Let \( P_{S_i} = P_i - \beta_i P_F \) denote the price of the security paying \( S_i \).

Using \( \text{(A.1)} \) and the fact that \( \text{E}[S_i - RP_{S_i}] = 0 \), we get
\[ J_S = -\left( \frac{\text{var}[M|PF](1 - f_S)\sigma^2_S \gamma^2(1 - f_S)^2 \sigma^4_S \sigma^2_X}{\text{var}[u_F - RP_F]\text{var}[S_i - RP_{S_i}]} \right)^{1/2} \exp(-\text{E}[u_F - RP_F]^2/2\text{var}[u_F - RP_F]). \]

Thus,
\[ J_S/J_U = \left( \frac{\text{var}[S_i - RP_{S_i}]}{(1 - f_S)\sigma^2_S} \right)^{-1/2}. \]

Using first (22) and then (25), we get
\[ \text{var}[S_i - RP_{S_i}] = (1 - f_S)\sigma^2_S + \frac{\gamma^2(1 - f_S)^2 \sigma^4_S \sigma^2_X}{\lambda^2_S} \]
\[ = (1 - f_S)\sigma^2_S + f_S \sigma^2_S \left( \frac{1}{\rho^2_S} - 1 \right), \]
from which (27) follows.

Proof of Proposition 3. As noted in (29), \( J_S/J_U \) approaches zero as \( \lambda_S = 1 - \lambda_M - \lambda_U \) decreases to zero (and \( \lambda_M \) increases to 1 - \( \lambda_U \)). We know from (26) that \( J_M/J_U > 0 \) for all \( \lambda_M \); in fact, from (26) we know that \( J_M/J_U \geq \sqrt{1 - f_M} \). It follows from the strict monotonicity of \( J_M/J_U \) and \( J_S/J_U \) (Lemma 1) that either \( J_M/J_U > J_S/J_U \) for all \( \lambda_M \in [0, 1 - \lambda_U] \) or the two curves cross at exactly one \( \lambda_M \) in \([0, 1 - \lambda_U] \). In the first case, all informed agents prefer to be \( i \) informed than \( M \) informed, so the only equilibrium is \( \lambda^*_M = 0 \).

In the second case, the unique point of intersection defines the equilibrium proportion \( \lambda^*_M \), as explained in the discussion of Figure 1. We therefore examine at which \( \lambda_M \) (if any) we have \( J_M/J_U = J_S/J_U \). We can equate (26) and (27) by setting
\[ \frac{1 - f_M}{f_M} - \frac{1}{1 - \rho^2_F} = \frac{1 - f_S}{f_S} - \frac{\rho^2_S}{1 - \rho^2_S}. \]

Using the expressions for \( \rho^2_F \) and \( \rho^2_S \) in (24) and (25), this equation becomes
\[ \frac{1 - f_M}{f_M} + \frac{\lambda^2_M}{\gamma^2 \sigma^2_M \sigma^2_F} = \frac{(1 - \lambda_U - \lambda_M)^2}{\gamma^2(1 - f_S)\sigma^2_S \sigma^2_X}. \]
Thus, $\lambda_M$ satisfies a quadratic equation, which, with some algebraic simplification, can be put in the form $A\lambda_M^2 + B\lambda_M + C = 0$, where

$$
A = 1 - \varphi, \quad B = -2(1 - \lambda_U), \quad C = (1 - \lambda_U)^2 - \alpha \gamma^2,
$$

(A.5)

with $\varphi$ and $\alpha$ as defined in (31). One of the two roots of this equation is given by $\tilde{\lambda}_M$. Denote the other root by

$$
\eta = -\frac{B + \sqrt{B^2 - 4AC}}{2A}.
$$

We claim that $\eta \not\in [0, 1 - \lambda_U]$. We may assume $A \neq 0$, because $\eta \to \infty$ as $A \to 0$ because $B < 0$. If $A < 0$ then either $\eta$ is complex or $\eta < 0$, again because $B < 0$. If $A > 0$, then $A < 1$ because $\varphi > 0$. Then if $\eta$ is real, it satisfies $\eta \geq -B/2A > -B/2 = 1 - \lambda_U$.

Combining these observations, we conclude that either $\tilde{\lambda}_M \in [0, 1 - \lambda_U)$ and the information equilibrium has $\lambda^*_M = \tilde{\lambda}_M$, or else the equilibrium occurs at $\lambda^*_M = 0$.

Proof of Lemma 4. Differentiation of $\tilde{\lambda}_M$ with respect to $\gamma$ yields

$$
\frac{d\tilde{\lambda}_M}{d\gamma} = -\frac{1}{\alpha \sqrt{\varphi(1 - \lambda_U)}} \left[ 1 + \frac{\gamma^2 \alpha}{\varphi (1 - \lambda_U)^2} \right]^{-1/2}.
$$

At an interior equilibrium, $\tilde{\lambda}_M$ is real, so the expression on the right is real and therefore negative.

At an interior equilibrium, $\lambda^*_M$ is the solution to $A\lambda^2 + B\lambda + C = 0$, with the coefficients given by (A.5). Differentiating (with respect to some parameter, e.g. $\sigma_S\sigma_X$ or $\sigma_M\sigma_{X_F}$) yields $\dot{A}\lambda^2 + 2A\dot{\lambda} + B\dot{\lambda} + \dot{C} = 0$ (note $\dot{B} = 0$ because we assume $\lambda_U$ is fixed). Solving for $\dot{\lambda}$ yields

$$
\dot{\lambda} = -\frac{\dot{A}\lambda^2 + \dot{C}}{2A\lambda + B}.
$$

We note that $2A\lambda + B < 0$ can be rewritten $(1 - \varphi)\lambda < 1 - \lambda_U$ which is always true because $\varphi > 0$ and $\lambda_M < 1 - \lambda_U$ since $\lambda_S > 0$. Therefore, $\text{sgn}(\dot{\lambda}) = \text{sgn}(A\lambda^2 + C)$. Differentiating with respect to $\sigma_S\sigma_X$ yields $\dot{A} < 0$ and $\dot{C} < 0$, which implies $\dot{\lambda} < 0$; similarly, differentiating with respect to $\sigma_M\sigma_{X_F}$ yields $\dot{A} > 0$ and $\dot{C} = 0$, which implies $\dot{\lambda} > 0$.

Proof of Lemma 5. Using the expression for $\rho_S^2$ in (25), we get

$$
\frac{f_S}{1 - f_S} \left( 1 - \rho_S^2 \right) = \frac{\gamma^2(1 - f_S)\sigma_S^2\sigma_X^2}{\lambda_S^2}.
$$

The derivative of this expression with respect to $f_S$ is strictly negative, so the derivative of $J_S/J_U$ in (27) is strictly positive.

Proof of Lemma 6. We see from (26) that the derivative of $J_M/J_U$ is negative precisely if
Moreover, case (ii) in Proposition 3 entails \( \tau(24) \). By the monotonicity argument used in case (i), \( \tau(24) \), and therefore (36) holds.

Proof of Proposition 5. We first show that (42) defines an information equilibrium at each \( c > 0 \), then verify uniqueness. For all three cases in (42), the specified \( \lambda_M, \lambda_S, \) and \( \lambda_U \) are nonnegative and sum to 1, so it suffices to verify (36). For \( c \leq \bar{c} \), we have \( J_M = J_S = J_U \) by construction, so the condition holds. For \( c \geq \bar{c} \), we again have \( J_S/J_U = 1 \) by construction. With \( \lambda_M = 0 \), we have \( \rho_F^2 = 0 \), and \( J_M/J_U \) in (26) evaluates to \( \exp(\gamma c)\sqrt{1-J_M} \geq \exp(\gamma \bar{c})\sqrt{1-J_M} = 1 \), so \( J_U/J_M \leq 1 \). Combining the two ratios we get \( J_S/J_M \leq 1 \). Thus, (36) holds.

For \( c < \bar{c} \), we consider two cases. If suppose case (i) of Proposition 3 holds at \( \bar{c} \). By definition, \( 1 - \lambda_M(\bar{c}) - \lambda_S(\bar{c}) = 0 \) and \( J_M/J_U = J_S/J_U \) at \( (\lambda_M(\bar{c}), \lambda_S(\bar{c}), 0) \), so \( \lambda_M(\bar{c}) = \lambda_M^*(0) \) and \( \lambda_S(\bar{c}) = 1 - \lambda_M^*(0) \), by the definition of \( \lambda_M^* \). Because \( \lambda_M(c) \) and \( \lambda_S(c) \) are strictly decreasing in \( c \), they are strictly greater than \( \lambda_M^*(0) \) and \( 1 - \lambda_M^*(0) \). Decreasing \( \lambda_M \) decreases \( \rho_F^2 \), which decreases \( J_M/J_U \) in (26), and decreasing \( \lambda_S \) similarly decreases \( J_S/J_U \). By construction, \( J_M/J_U = J_S/J_U = 1 \) at \( (\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c)) \), even for \( c < \bar{c} \), so at \( (\lambda_M^*(0), 1 - \lambda_M^*(0), 0) \) we have \( J_M/J_U < 1 \), \( J_S/J_U < 1 \), and \( J_M/J_S = 1 \), confirming (36).

Now suppose case (ii) of Proposition 3 holds at \( \bar{c} \). Then \( \lambda_M^*(\bar{c}) = \lambda_M(\bar{c}) = 0 \), and (42) specifies \( \lambda_M = 0 \) for all \( c < \bar{c} \). By the monotonicity argument used in case (i), \( J_S/J_U < 1 \) at all \( c < \bar{c} \). Moreover, case (ii) in Proposition 3 entails \( J_S/J_M \leq 1 \), so this also holds for all \( c < \bar{c} \), and therefore (36) holds.

We now turn to uniqueness. At any \( c \), once we determine which proportions are strictly positive, the equilibrium is determined: if \( \lambda_U = 0 \), the other two proportions are determined by Proposition 3; if all three proportions are positive, they must satisfy \( J_M/J_U = J_S/J_U = 1 \) and must therefore be given by \( (\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c)) \); if \( \lambda_M = 0 \) and \( \lambda_U > 0 \), the proportions are determined by the requirement that \( J_S/J_U = 1 \). If we tried to set \( \lambda_S = 0 \), we would have \( \rho_F^2 = 0 \) and \( J_S/J_U = 0 \), in which case (36) would require \( \lambda_U = 0 \) and \( \lambda_M = 1 \); but this yields \( \rho_F^2 = 1 \) and \( J_M/J_U \geq 1 \) in (26), which would

A.4 Information equilibrium

Proof of Proposition 4. From (35) we see that \( \rho_F^2 \geq \rho_F^2 \) precisely when \( \rho_F^2 \leq 1/\tau_M \). If \( \tau_M \leq 1 \), then we necessarily have \( \rho_F^2 \leq 1/\tau_M \). But if \( \tau_M > 1 \), then markets are more macro efficient whenever \( \rho_F^2 > 1/\tau_M \).
implies \( J_S/J_M = 0 \), requiring \( \lambda_M = 0 \) and leading to a contradiction. The possibility that \( \lambda_S = 0 \) can therefore be excluded. For any \( c > 0 \), (27) rules out the possibility that \( \lambda_S = 1 \), because this would entail \( \rho_S^2 = 1 \) and \( J_S/J_U > 1 \).

It therefore suffices to show that at each \( c \), the set of agents with positive proportions is uniquely determined. Suppose we try to introduce uninformed agents into an equilibrium from which they are absent. The original equilibrium must have \( \lambda_M, \lambda_S > 0 \) and therefore \( J_M/J_U \leq 1 \) and \( J_S/J_U \leq 1 \). Increasing \( \lambda_U \) requires decreasing either \( \lambda_M \) or \( \lambda_S \) and therefore decreasing either \( J_M/J_U \) or \( J_S/J_U \), precluding \( \lambda_U > 0 \), in light of (36).

Suppose we try to introduce macro informed agents into an equilibrium with only micro informed and uninformed agents. The presence of uninformed agents requires \( J_M/J_U \geq 1 \); otherwise, the uninformed would prefer to become macro informed. Increasing \( \lambda_M \) would increase \( \rho_U^2 \) which increases \( J_M/J_U \), precluding \( \lambda_M > 0 \). \( \square \)

**Proof of Corollary A.4**  
(i) It suffices to consider the intermediate range \( \underline{c} \leq c \leq \bar{c} \) with \( \underline{c} < \bar{c} \), because \( \Pi_M \) is constant on \((0, \underline{c}] \) and identically zero on \([\bar{c}, \infty) \). It follows from (39) and (40) that
\[
\lambda_S^2(c) = \frac{\gamma^2(1 - f_S)^2 \sigma_S^2 \sigma_X^2}{f_S \tau_M} \left( \frac{\lambda_M^2(c) f_M}{\gamma^2(1 - f_M)^2 \sigma_M^2 \sigma_X^2} + 1 \right) \equiv a\lambda_M^2(c) + b, \quad a, b > 0.
\]
Because \( \lambda_M(c) \) is strictly decreasing in \( c \), dividing both sides by \( \lambda_M^2(c) \) shows that \( \lambda_S^2(c)/\lambda_M^2(c) \) is strictly increasing in \( c \), hence \( \lambda_M(c)/(\lambda_M(c) + \lambda_S(c)) \) is strictly decreasing in \( c \).

(ii) The first assertion follows from Proposition 5. For the second assertion, evaluate \( \lambda_S(c) \) in (40) at \( \bar{c} \) in (41) to get \( \lambda_S(\bar{c}) = \gamma^2 a \). If \( \lambda_S(\bar{c}) = 1 \), then \( \bar{c} = \underline{c} \), and \( \lambda_S = 1 \) at all \( c \leq \bar{c} \), so \( \lambda_M = 0 \) at all \( c \).

(iii) Follows from (42). (iv) We know from (24) and (25) that \( \rho_F^2 \) and \( \rho_S^2 \) are increasing in \( \lambda_M \) and \( \lambda_S \), respectively, so monotonicity of price efficiency follows from monotonicity in (42). \( \square \)

### A.5 No-Trade Results

**Proof of Proposition A.6**. Because all agents invest in the index fund, we may replace the decision to invest in a security having price \( P_i \) and paying \( S_i + M \) with a decision to invest in a security having price \( P_{S_i} \) and paying \( S_i \). In other words, we replace the original stocks with securities that hedge out the macro component of the individual stocks.

We have three types of agents to consider. If \( \lambda_U > 0 \), consider an uninformed agent. The squared expected utility for an uninformed agent that deviates from equilibrium by investing in stock \( i \) is
\[
J_i^2 = \frac{|\tilde{V}|}{|\tilde{\Psi}|} e^{-\tilde{\mu}^\top \tilde{\Psi}^{-1} \tilde{\mu}} \times e^{2\gamma \kappa_i}, \tag{A.6}
\]
where \( \tilde{\Psi} \) is the unconditional covariance matrix of \( u_F - RP, S_i - RP_{S_i}, \tilde{\mu} \) is the unconditional mean of \( u_F - RP_F, S_i - RP_{S_i} \), and \( \tilde{V} \) is the conditional covariance matrix of \( u_F, S_i \), given \( P_F, P_{S_i} \). This expression follow from Proposition 3.1 of Admati and Pfleiderer (1987); we have taken the agent’s initial wealth to be zero because it plays no role
in the comparison. We have

\[ \tilde{\Psi} = \begin{pmatrix} \text{var}[u_F - RP_F] & 0 \\ 0 & (1 - f_S)\sigma^2_S + R^2 c^2_S \sigma^2_X \end{pmatrix}, \quad \tilde{\mu}^\top = (E[u_F - RP_F], 0), \]

and

\[ \tilde{V} = \begin{pmatrix} \text{var}[u_F | P_F] & 0 \\ 0 & \text{var}[S_i | P_{si}] \end{pmatrix}. \]

The squared expected utility for an uninformed agent that does not deviate is given by

\[ J^2_U = \frac{\text{var}[u_F | P_F]}{\text{var}[u_F - RP_F]} \exp(-Q_F), \quad Q_F = \frac{E[u_F - RP_F]^2}{\text{var}[u_F - RP_F]}, \quad (A.7) \]

Because \( \tilde{\mu}^\top \tilde{\Psi}^{-1} \tilde{\mu} \) evaluates to \( Q_F \), the last factor is common to \( \tilde{J}^2 \) and \( J^2_U \). The agent prefers not to deviate so long as

\[ 1 < \frac{\tilde{J}^2}{J^2_U} = \frac{\text{var}[S_i | P_{si}]}{\text{var}[S_i - RP_{si}]} e^{2\gamma \kappa_i}; \]

that is, so long as \( \kappa_i > \kappa \).

Next, suppose \( \lambda_M > 0 \) and consider a macro-informed agent. The expression in (A.6) remains valid, except that now

\[ \tilde{V} = \begin{pmatrix} \text{var}[u_F | m] & 0 \\ 0 & \text{var}[S_i | P_{si}] \end{pmatrix}, \]

reflecting the agent’s information about \( M \), and \( \tilde{J}^2 \) should be multiplied by \( \exp(2\gamma c) \) to reflect the cost of becoming macro-informed. For a macro-informed agent that does not deviate,

\[ J^2_M = \frac{\text{var}[u_F | m]}{\text{var}[u_F - RP_F]} \exp(-Q_F) \exp(2\gamma c); \]

The ratio \( \tilde{J}^2 / J^2_M \) is the same as it was in the uninformed case, so the same condition on \( \kappa_i \) applies.

Finally, consider an agent informed about stock \( j \) who deviates and invests in stock \( i \neq j \). We now take \( \tilde{\mu} \) and \( \tilde{\Psi} \) to be the unconditional mean vector and covariance matrix of \( (u_F - RP_F, S_j - RP_{sj}, S_i - RP_{si}) \), and we take \( \tilde{V} \) to be conditional covariance matrix of \( (u_F, S_j, S_i) \), conditional on \( (P_F, s_j, P_{si}) \). We get

\[ \tilde{\Psi} = \begin{pmatrix} \text{var}[u_F - RP_F] & 0 & 0 \\ 0 & \text{var}[S_j - RP_{sj}] & \text{cov}[S_j - RP_{sj}, S_i - RP_{si}] \\ 0 & \text{cov}[S_j - RP_{sj}, S_i - RP_{si}] & \text{var}[S_i - RP_{si}] \end{pmatrix}, \]

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\( \tilde{\mu}^T = (E[u_F - RP_F], 0, 0) \), and

\[
\tilde{V} = \begin{pmatrix}
\text{var}[u_F|P_F] & 0 & 0 \\
0 & \text{var}[S_j|s_j, P_{S_i}] & \text{cov}[S_j, S_i|s_j, P_{S_i}] \\
0 & \text{cov}[S_j, S_i|s_j, P_{S_i}] & \text{var}[S_i|s_j, P_{S_i}]
\end{pmatrix}.
\]

As the number of stocks \( N \) grows without bound, the covariance terms in paper-equation (12) approach zero, so the off-diagonal entries of \( \tilde{\Psi} \) and \( \tilde{V} \) approach zero, and on the diagonal of \( \tilde{V} \), \( \text{var}[S_j|s_j, P_{S_i}] \to \text{var}[S_j|s_j] \) and \( \text{var}[S_i|s_j, P_{S_i}] \to \text{var}[S_i|P_{S_i}] \). Because the determinant of a matrix is a continuous function of its entries, it follows that by taking \( N \) sufficiently large, we can make \( \tilde{J}^2 \), the squared utility from deviating, arbitrarily close to

\[
\text{var}[u_F|P_F] \text{var}[S_j|s_j] \text{var}[S_i|P_{S_i}] \exp(-Q_F) \exp(2\gamma(c + \kappa_i)).
\]

For a \( j \)-informed investor who does not deviate, the squared expected utility is

\[
J^2_S = \frac{\text{var}[u_F|P_F]\text{var}[S_j|s_j]\text{var}[S_i|P_{S_i}]}{\text{var}[u_F - RP_F]\text{var}[S_j - RP_{S_i}]\text{var}[S_i - RP_{S_i}]} \exp(-Q_F) \exp(2\gamma c).
\]

The ratio \( \tilde{J}^2/J^2_S \) therefore approaches \((\text{var}[S_i|P_{S_i}]/\text{var}[S_i - RP_{S_i}])e^{2\gamma\kappa_i} \). If \( \kappa_i > \kappa \), then for all sufficiently large \( N \), \( \tilde{J}^2/J^2_S > 1 \), and the agent prefers not to deviate from the information equilibrium. □

This argument extends to any fixed number of stocks \( K \), for all sufficiently large \( N \). In Proposition 3, we let \( K \) increase with \( N \).

**Proof of Proposition 7.** In an information equilibrium with \( \lambda_U > 0 \), we have

\[
\frac{J^2_S}{J^2_U} = \frac{\text{var}[S_i|s_i]}{\text{var}[S_i - RP_{S_i}]}e^{2\gamma c} = 1;
\]

the first equality follows from (A.7) and (A.8) and the definition of \( c \) [or p.62 of the paper], and the second equality is the equilibrium condition for a marginal uninformed investor to be indifferent between remaining uninformed or paying the cost \( c \) to become informed and invest in stock \( i \). The definition of \( \kappa \) yields

\[
\frac{\text{var}[S_i|P_{S_i}]}{\text{var}[S_i - RP_{S_i}]}e^{2\gamma\kappa} = 1.
\]

Thus,

\[
e^{2\gamma(\kappa - c)} = \frac{\text{var}[S_i|s_i]}{\text{var}[S_i|P_{S_i}]} < 1,
\]

the inequality holding for any \( 0 < f_S < 1 \). Part (a) follows. For part (b), observe that at \( f_S = 0 \), neither \( s_i \) nor \( P_{S_i} \) is informative about \( S_i \); so we have \( \text{var}[S_i|s_i] = \text{var}[S_i] = \text{var}[S_i|P_{S_i}] \). Equality then holds in (A.9) and \( \kappa = c \).
Use (46) to write (A.9) as

\[ e^{2(\kappa - c)} = \frac{(1 - f_S)^2}{(1 - f_S)\sigma_S^2 + f_S\sigma_S^2(1 - \rho_S^2)} = \left(1 + f_S \frac{1 - \rho_S^2}{1 - f_S}\right)^{-1} \]

and then

\[ \kappa = c - \frac{1}{2\gamma} \log \left(1 + f_S \frac{1 - \rho_S^2}{1 - f_S}\right). \]  

Equation (43) in the paper yields

\[ 1 - \rho_S^2 = \frac{(1 - f_S)(e^{2\gamma c} - 1)}{f_S + (1 - f_S)(e^{2\gamma c} - 1)}, \]

so, as \( f_S \to 1, \)

\[ f_S \frac{1 - \rho_S^2}{1 - f_S} = \frac{f_S(e^{2\gamma c} - 1)}{f_S + (1 - f_S)(e^{2\gamma c} - 1)} \to e^{2\gamma c} - 1. \] (A.11)

Applying this limit in (A.10), we get \( \kappa \to 0, \) which is (c). The middle expression in (A.11) is an increasing function of \( f_S, \) so \( \kappa \) in (A.10) is a decreasing function of \( f_S. \) □

In Proposition 6, we considered deviations in which an agent invested in a single stock about which the investor was not informed. A similar result holds for deviations consisting of investments in \( K \) stocks. We assume that each time the agent makes inferences based on the price of a stock and then sets a demand for the stock conditional on the price, the agent incurs a cost \( \kappa_i. \) Thus, the cost of conditioning on \( K \) stock prices is \( K\kappa_i. \) The proof of the following result is lengthy and therefore omitted, but it is available from the authors.

**Proposition 8.** If \( \kappa_i > \kappa, \) then, no uninformed or macro-informed agent will deviate from equilibrium by investing in \( K \) stocks, \( 1 \leq K \leq N - 1. \) No micro-informed agent will deviate by investing in \( K - 1 \) stocks about which the agent is not informed, \( 2 \leq K \leq N - 2, \) for all sufficiently large \( N. \)

**References**


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