Investor Information Choice with Macro and Micro Information

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Current version: January 2018

Abstract

We study information and portfolio choices in a market of securities whose dividends depend on an aggregate (macro) risk factor and idiosyncratic (micro) shocks. Investors can acquire information about dividends at a cost. We first establish a general result showing that investors endogenously choose to specialize in either macro or micro information. We then develop a specific model with this specialization and study the equilibrium mix of macro-informed and micro-informed investors and the informativeness of macro and micro prices. We discuss empirical implications for excess volatility, excess covariance, and security prices in recessions. Our results favor Samuelson’s dictum, that markets are more micro efficient than macro efficient.

Keywords: Information choice; asset pricing; price efficiency; attention

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1 Introduction

Samuelson’s dictum, as discussed in Shiller (2000), is the hypothesis that the stock market is “micro efficient” but “macro inefficient.” More precisely, the dictum holds that the efficient markets hypothesis describes the pricing of individual stocks better than it describes the aggregate stock market. Jung and Shiller (2005) review and add to empirical evidence that supports the dictum, including evidence of macro inefficiency in Campbell and Shiller (1988) and evidence for somewhat greater micro efficiency in Vuolteenaho (2002) and Cohen et al. (2003).

We develop a model of investor information choice to study a potential wedge between micro and macro price efficiency. Our setting may be viewed as a multi-security generalization of the classical model of Grossman and Stiglitz (1980). Our market consists of a large number of individual stocks, each of which is exposed to a macro risk factor and an idiosyncratic risk. The macro risk factor is tradeable through an index fund that holds all the individual stocks and diversifies away their idiosyncratic risks.

We begin with a very general formulation in which investors may choose to acquire information processing capacity at a cost. This capacity allows an investor to observe and make inferences from signals about fundamentals. Subject to their capacity constraint, informed investors may choose to learn about the macro risk factor, about the micro (idiosyncratic) risks of individual stocks, or any combination of the two. The capacity constraint limits the fraction of uncertainty about dividends an informed investor can remove from a collection of securities.

In formulating this capacity constraint, we differentiate the index fund from individual stocks. We posit that the capacity consumed in making inferences from the price of the index fund is fixed, irrespective of the informativeness of the price. This assumption is based on the view that the implications of the overall level of the stock market are widely discussed and accessible in a way that does not apply to individual stocks. In conditioning demand for the index fund on its price, an investor allocates a fixed capacity to paying attention to this information.

Our first main result shows that investors endogeneously choose to specialize in either macro or micro information. Our investors are ex ante identical, and once they incur the cost of becoming informed they are free to choose general combinations of signals, yet in equilibrium they concentrate in two groups, macro-informed and micro-informed investors. The macro-informed use all their capacity to learn about the macro factor and invest only in the index fund; a micro-informed investor acquires a signal about a single stock and invests in that stock and the index fund; some investors choose to remain
uninformed. This outcome — heterogeneous information choices among ex ante identical investors — contrasts with the related literature, as we explain later.

Having demonstrated that specialization in macro or micro information is a general phenomenon, we then construct a specific model by imposing this specialization as a constraint. In other words, our general result shows that specialization is a necessary property in equilibrium, and the constrained model demonstrates that such an equilibrium is in fact feasible.

The constrained model has three types of investors: uninformed, macro-informed, and micro-informed, as required by our general result. To solve the model, we first take the fractions of each type as given and solve for an explicit market equilibrium, assuming all agents have CARA preferences. Shares of individual stocks and the index fund are subject to exogenous supply shocks. The exogenous supply shocks themselves exhibit a factor structure. A common component, reflecting the aggregate level of supply, affects the supply of shares for all firms. In addition, noise trading in individual stocks contributes an idiosyncratic component to the supply of each stock. Supply shocks are not observable to investors; as a consequence, equilibrium prices are informative about, but not fully revealing of, the micro or macro information acquired by informed agents.

We then allow informed investors to choose between being micro-informed and macro-informed, and we characterize the equilibrium in which a marginal agent is indifferent between the two types of information. In practice, developing the skills needed to acquire and apply investment information takes time — years of education and experience. In the near term, these requirements leave the total fraction of informed investors relatively fixed. By contrast, we suppose that informed investors can move comparatively quickly and costlessly between being macro-informed or micro-informed by shifting their focus of attention. Endogenizing this focus gives rise to an attention equilibrium centered on the choice between macro and micro information.

Over a longer horizon, agents choose whether to gain the skills to become informed, as well as the type of information to acquire. We therefore study an information equilibrium that endogenizes both decisions to determine equilibrium proportions of macro-informed, micro-informed, and uninformed investors. An information equilibrium in the constrained model delivers an explicit case of the necessary specialization established in our general formulation: the investors in this model would not prefer to deviate from their specialization and select other combinations of signals that consume the same capacity.

Working with the constrained model, we find a recurring asymmetry between micro and macro information. For example, we show that the information equilibrium sometimes
has no macro-informed agents, but some fraction of agents will always choose to be micro-informed. We show that increasing the precision of micro information makes micro-informed investors worse off — we say that the micro-informed overtrade their information, driving down their compensation for liquidity provision. In contrast, macro-informed investors may be better or worse off as a result of more precise macro information: they are better off when the fraction of macro-informed agents or, equivalently, the informativeness of the price of the index fund is sufficiently low. Similarly, the equilibrium fraction of macro-informed agents always increases with the precision of micro information, but it can move in either direction with an increase in the precision of macro information. A simple condition on the relative precision of micro and macro information determines whether the market is more micro efficient or more macro efficient.

As applications of our theoretical analysis, we discuss some empirical predictions. Our analysis predicts that idiosyncratic return volatility falls as more micro information becomes available or as the fraction of micro-informed investors increases. As investors shift focus between micro and macro information, idiosyncratic volatility and systematic volatility move in opposite directions. Changes in the precision of micro information contribute to a common factor in idiosyncratic volatility. Low precision in macro information creates excess volatility and excess comovement in prices, compared with fundamentals. Recessions characterized by similar increases in macro and micro risk push informed investors to focus on micro information, whereas recessions accompanied predominantly by increased macro risk and only a small increase in the price of risk push investors into macro information.

Our work is related to several strands of literature. Our model effectively nests Grossman and Stiglitz (1980) if we take the index fund as the single asset in their model. We also draw on the analysis of Hellwig (1980), Admati (1985) and Admati and Pfleiderer (1987) but address different questions; see the books by Brunnermeier (2001) and Veldkamp (2011) for a survey of related literature. Admati and Pfleiderer (1987) and Goldstein and Yang (2015) focus on understanding when signals are complements or substitutes; our specialization result — that an informed investor will choose either macro or micro information, but not both — makes macro and micro information strategic substitutes. Schneemeier’s (2015) model predicts greater micro than macro efficiency when managers use market prices in their investment decisions. As in Kyle (1985), our noise traders are price insensitive, and gains from trade against them accrue to the informed, which provides an incentive to collect information. We shed light on the discussion in Black (1986) of the crucial role that “noise” plays in price formation by proposing a model in which
the factor structure of noise trading plays an important role in determining the relative micro versus macro efficiency of markets.

Van Nieuwerburgh and Veldkamp (2009) analyze how investors’ choices to learn about the domestic or foreign market in the presence of asymmetric prior knowledge may explain the home bias puzzle, and Van Nieuwerburgh and Veldkamp (2010) use related ideas to explain investor under-diversification. Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) develop a model of rational attention allocation in which fund managers choose whether to acquire macro or stock specific information before making investment decisions. Their model, like ours, has multiple assets subject to a common cash flow factor; but, in contrast to our setting, their agents ultimately all acquire the same information, a point we return to later. Their focus is on explaining cyclical variation in attention allocation that is caused by exogenous changes in economic conditions. Kacperczyk et al. (2016) show that mutual fund managers change their focus from micro to macro fundamental information over the course of the business cycle. We discuss alternative mechanisms that could give rise to such behavior and contrast these with the explanations in Kacperczyk et al. (2016).

Peng and Xiong (2006) also use a model of rational attention allocation to study portfolio choice. In their framework, investors allocate more attention to sector or marketwide information and less attention to firm-specific information. Their conclusion contrasts with ours (and with the Jung-Shiller discussion of Samuelson’s dictum and the Maćkowiak and Wiederholt 2009 model of sticky prices under rational intattention) primarily because in their setting a representative investor makes the information allocation decision; since macro uncertainty is common to all securities, while micro uncertainty is diversified away, the representative investor allocates more attention to macro and sector level information. Gărleanu and Pedersen (2016) extend the Grossman-Stiglitz model to link market efficiency and asset management through search costs incurred by investors in selecting fund managers, in a model with a single risky asset.

Bhattacharya and O’Hara (2016) study a Kyle-type model with an ETF and multiple underlying securities. Their model, like ours, contains macro-informed and micro-informed agents (their “informed speculators”), as well as supply shocks in the ETF and the underlying securities (their “liquidity traders”). They focus on the situation where the liquidity of the ETF is higher than that of the underlying hard-to-trade securities, where there is price impact from trade, where the no-arbitrage relationship between the ETF and underlying security prices can break down, and where the ETF conveys information about individual stock prices. They analyze whether ETFs can lead to short-term
market instability in the underlying securities. Glosten, Nallareddy, and Zou (2016) investigate (empirically and theoretically) the possibility that trading in an ETF can affect the informational efficiency of stocks held by the ETF whose primary markets have poor price discovery. Our focus is on how investors’ information choices play out over longer horizons, so we abstract from higher frequency microstructure issues by assuming agents trade with no price impact and by maintaining the no-arbitrage restriction that the index fund is equal to the price of the underlying basket of securities.

Section 2 describes our securities and the information choices available to investors, and it then presents our general result showing that investors endogenously specialize in macro or micro information. Section 3 introduces the constrained model and adds additional features (supply shocks and market clearing) that lead to explicit expressions for prices and price efficiency in the market equilibrium of Section 4. Sections 5 and 6 investigate the attention equilibrium and information equilibrium, respectively, in the constrained model. Section 7 discusses applications. Proofs are deferred to an appendix.

2 The economy

Securities

We assume the existence $N$ risky securities — called stocks — indexed by $i$. There is also an index fund, $F$, one share of which holds $1/N$ shares of each of the $N$ stocks. There is a riskless security with a gross return of $R$.

The time 2 dividend payouts of the stocks are given by

$$u_i = M + S_i, \quad i = 1, \ldots, N.$$  

(1)

We interpret $M$ as a macro factor and the $S_i$ as idiosyncratic contributions to the dividends. The random variables $M, S_1, \ldots, S_N$ are jointly normal, with $E[M] = \bar{m}$, $E[S_i] = 0$, $\text{var}[M] = \sigma^2_M$, $\text{var}[S_i] = \sigma^2_S$, and $E[MS_i] = 0$, $i = 1, \ldots, N$. To make the idiosyncratic shocks fully diversifiable with a finite number of stocks, we assume that

$$\text{corr}(S_i, S_j) = -\frac{1}{N-1}, \quad i \neq j,$$  

(2)

1Our results hold with minor modifications if $M$ is replaced with $\beta_i M$, provided the $\beta_i$s average to 1.

2This condition makes $(S_1, \ldots, S_N)$ exchangeable random variables, meaning that their joint distribution is invariant under permutations of the variables. The correlation matrix specified by (2) is diagonally dominant and therefore positive semidefinite.
which implies that $\sum_{i=1}^{N} S_i = 0^{[3]}$. As a consequence, the index fund $F$ pays

$$u_F = \frac{1}{N} \sum_{i=1}^{N} u_i = M + \frac{1}{N} \sum_{i=1}^{N} S_i = M; \quad (3)$$

the last equality is the benefit of imposing (2).

Prices of individual stocks are given by $P_i$. The index fund price is $P_F$, and precluding arbitrage requires that

$$P_F = \frac{1}{N} \sum_{i=1}^{N} P_i. \quad (4)$$

We also define the price $P_{S_i} = P_i - P_F$, $i = 1, \ldots, N$, of a security paying $u_i - M = S_i$, the idiosyncratic portion of the dividend of stock $i$.

**Agents and information sets**

A unit mass of agents maximize expected utility, $-E[\exp(-\gamma \tilde{W}_2)]$, over time time-2 wealth

$$\tilde{W}_2 = W_1 R + q_F(u_F - R P_F) + \sum_{i=1}^{N} q_i(u_i - R P_i),$$

where $q_F$ and $q_i$ are the shares invested in the index fund and stock $i$. The initial wealth $W_1$ does not affect an investor’s decisions. The risk aversion parameter $\gamma > 0$ is common to all investors.

Agents can choose to acquire information capacity $\kappa$, $0 < \kappa < 1$, by incurring a cost $c$. This capacity allows an agent to select signals $m'$ about $M$ and signals $s'_i$ about the $S_i$. We measure the informativeness of signals $m'$ and $s'_i$ through the variance reduction ratios $(\text{var}[M] - \text{var}[M|m'])/\text{var}[M]$ and $(\text{var}[S_i] - \text{var}[S_i|s'_i])/\text{var}[S_i]$. Informativeness will be constrained by $\kappa$, and the available signals will allow full use of $\kappa$.

In more detail, for any level of informativeness $f \in [0, 1]$, there is a signal $s_i(f)$, with $s_i(0) = 0$ and $s_i(1) = S_i$. Each $s_i(f)$ has mean zero and variance $f \sigma^2_S$, with $\text{var}[S_i|s_i(f)] = (1-f)\sigma^2_S$. Similarly, the signal $m(f)$ has $E[m(f)] = \bar{m}$, $\text{var}[m(f)] = f \sigma^2_M$, and $\text{var}[M|m(f)] = (1-f)\sigma^2_M$. All macro signals $m(f)$ are independent of signals $s_i(f')$ about idiosyncratic payouts, and all signals and payouts are jointly normal. We henceforth

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3This condition ensures that idiosyncratic terms are fully diversifiable with $N$ finite. This assumption is common in the asset pricing literature when considering multi-security economies with a finite number of assets; see, for example, Ross (1978), Chen and Ingersoll (1983), and Kwon (1985). We think of $N$ as large, so the correlation required by (2) is small. Correlations of this form, which we will encounter in several places, may be interpreted as “independence for large $N$.”

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omit the argument $f$ from the signals unless needed for clarity.

An informed investor selects a set of securities about which to acquire signals and in which to invest. The consideration set of securities contains $K$ stocks, $i_1, \ldots, i_K$, for some $0 \leq K \leq N - 1$, and may contain the index fund, in which case $K \leq N - 2$. We assume that prices are freely available, so once an investor chooses to become informed about a security, the investor knows at least the price of the security.

Together with a set of securities, an investor chooses a corresponding information set

$\mathcal{I}_K^{(0)} = \{(s'_{i_1}, P_{Si_1}), \ldots, (s'_{i_K}, P_{Si_K})\},$

$\mathcal{I}_K^{(1)} = \{P_F, (s'_{i_1}, P_{Si_1}), \ldots, (s'_{i_K}, P_{Si_K})\},$

or

$\mathcal{I}_K^{(2)} = \{(m', P_F), (s'_{i_1}, P_{Si_1}), \ldots, (s'_{i_K}, P_{Si_K})\},$

depending on whether the index fund is in the consideration set and, if it is, whether the investor learns more than the fund’s price.

With additional structure (which we introduce later), prices will reflect investors’ information choices. For now, we keep the discussion general and just assume that prices and signals are jointly normal. We also assume that the fund price $P_F$ is uncorrelated with the prices $P_{Si}$ of the idiosyncratic payouts.

Write $\Sigma^{(\iota,K)}$ for the unconditional covariance matrix of the payouts $M, S_{i_1}, \ldots, S_{i_K}$ or $S_{i_1}, \ldots, S_{i_K}$ in the consideration set, with $\iota \in \{1, 2\}$ if the index fund is in the set, and $\iota = 0$ if it is not. The off-diagonal elements of $\Sigma^{(0,K)}$ are determined by (2), and we have

$$\Sigma^{(2,K)} = \Sigma^{(1,K)} = \begin{pmatrix} \text{var}[M] & 0 \\ 0 & \Sigma^{(0,K)} \end{pmatrix}.$$

After observing signals, the investor evaluates the posterior distribution of the security payoffs and evaluates the conditional covariance matrix for the payoffs in the consideration set, which we denote by $\hat{\Sigma}^{(\iota,K)}$. Because every macro signal $m'$ is independent of every micro signal $s'_i$, we assume that $P_F$ is independent of $s'_i$ and $P_{Si}$ is independent of $m'$. The
conditional covariance matrices therefore have the form:

\[
\hat{\Sigma}^{(1,K)} = \begin{pmatrix}
\text{var}[M|P_F] & 0 \\
0 & \hat{\Sigma}^{(0,K)}
\end{pmatrix},
\hat{\Sigma}^{(2,K)} = \begin{pmatrix}
\text{var}[M|m', P_F] & 0 \\
0 & \hat{\Sigma}^{(0,K)}
\end{pmatrix}.
\]

Investors are constrained in how much information they can acquire, and we model this constraint through a bound on signal precision. Using \(|\cdot|\) to indicate the determinant of a matrix, for \(\iota = 0\) or \(\iota = 2\), we impose the constraint:

\[
\frac{|\hat{\Sigma}^{(\iota,K)}|}{|\Sigma^{(\iota,K)}|} \geq \kappa,
\]

where \(0 < \kappa < 1\) measures the information capacity an investor attains at the cost \(c\). Smaller \(\kappa\) corresponds to greater variance reduction and thus greater capacity. For the case of \(\iota = 1\), meaning signal set \(\mathcal{T}^{(1)}_K\), we impose the constraint:

\[
\delta_F \frac{|\hat{\Sigma}^{(0,K)}|}{|\Sigma^{(0,K)}|} \geq \kappa,
\]

with \(\delta_F < 1\); in other words, we have replaced \(\text{var}[M|P_F]/\text{var}[M]\) in the determinant ratio with a fixed quantity \(\delta_F\).

This modification treats the index fund price differently from other types of information. We are particularly interested in the case

\[
\frac{\text{var}[M|P_F]}{\text{var}[M]} < \delta_F.
\]

When this inequality holds, making inferences from the price of the index fund consumes less capacity than would be expected from the variance reduction achieved. The idea is that the implications of the overall state of the market, as measured by the index fund, are widely discussed and publicly disseminated; \(\delta_F\) is the capacity consumed by paying attention to this ambient information. If \(7\) holds, then making inferences from the price of the index fund is at least slightly easier than making inferences from other information,

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4The posterior distribution preserves the independence of macro and idiosyncratic sources of risk, but we do not assume that \(\hat{\Sigma}^{(0,K)}\) has the same dependence structure as \(\Sigma^{(0,K)}\), a point emphasized in Sims (2011), p.167.

5The determinant ratio is a multivariate generalization of a variance ratio, and it generalizes one minus a regression \(R^2\). The constraint in \(5\) is very similar to the entropy constraint used by Sims (2003), Mondria (2010), Van Nieuwerburgh and Veldkamp (2010), Hellwig et al. (2012) and others. With no information, \(\Sigma = \hat{\Sigma}\) and the determinant ratio is 1, indicating that no capacity is consumed, whereas the entropy measure includes a term depending on the number of assets \(K\). When the number of assets is fixed, the measures are equivalent. Take the determinant of empty matrices to be one, so \(|\Sigma^{(0,K)}| = |\hat{\Sigma}^{(0,K)}| = 1\) if \(K = 0\).
holding fixed the level of variance reduction. We do not assume (7); we show that it follows from more basic assumptions.

Because we condition on prices as well as nonpublic signals in (5) and (6), our formulation implies that making inferences from prices consumes some of the capacity $\kappa$. This point merits emphasis. The capacity $\kappa$ accounts for two types of effort: the effort required to acquire nonpublic signals $m'$ or $s'_i$, and also the effort required to make inferences from these signals and from publicly available prices. Price information is freely available, but regularly following the prices of hundreds of stocks and extracting investing implications from these prices consumes attention and effort.

Uninformed investors — those who do not incur the cost $c$ to acquire the capacity $\kappa$ — observe market prices, but they cannot observe signals $m(f)$ or $s_i(f)$, $f > 0$. Because making inferences from prices requires some information processing capacity, we endow uninformed investors with capacity $\delta_F$. This allows the uninformed to invest in the index fund and condition their demand on the price of the index. They may also reallocate this capacity to make inferences from the prices of individual stocks.

**Equilibrium**

Once investors choose their information sets, their optimal portfolios (chosen to maximize expected utility) are determined by the price system. An equilibrium consists of a collection of information choices and a joint distribution (assumed normal) for prices, dividends, and signals, under which investors do not want to deviate from their choices.

We consider equilibria with the following features:

(e1) The joint distribution of the pairs $(S_i, P_{S_i})$, $i = 1, \ldots, N$, is invariant under permutation of the indices, and every $(m(f), P_F)$ is independent of every $(s_i(f'), P_{S_i})$.

(e2) If no investors choose a macro signal $m(f)$, $f > 0$, then $\text{var}[M|P_F] = \text{var}[M]$, and if no investors choose any micro signal $s_i(f)$, $f > 0$, $i = 1, \ldots, N$, then $\text{var}[S_j|P_{S_j}] = \text{var}[S_j]$.  

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6Van Nieuwerburgh and Veldkamp (2010), p.796, propose a capacity constraint in which different variance ratios are raised to different powers. Our constraint can be viewed as raising $\text{var}[M|P_F]/\text{var}[M]$ to a power of zero and scaling it by a constant.

7In Section 4.1 and Appendix A.5, $\delta_F$ has an alternative interpretation as the capacity consumed when an investor in a stock trades the index fund to hedge part of the macro risk in the stock. Trading to hedge rather than invest is less dependent on the information in the price.

8Our formulation thus addresses a point in Sims (2011), p.177, that processing of price information, like processing of other signals, should be subject to a capacity constraint. Kacperczyk et al. (2016) also analyze a variant of their main model with costly learning from prices.

9More precisely, no information set that is selected by a positive fraction of agents is strictly dominated by another information set.
var\{S_j\}, for all \(j = 1, \ldots, N\).

(e3) The information cost and capacity parameters satisfy \(e^{2\gamma c} \kappa < \delta_F\).

(e4) A positive fraction of investors choose to remain uninformed, and a positive fraction of these invest in the index fund.

Condition (e1) restricts attention to equilibria that are symmetric in the individual stocks, which is a reasonable restriction given that their dividends are ex ante identically distributed. This restriction works against finding equilibria in which investors make heterogeneous information choices. The second part of (e1) is consistent with the interpretation of the \(S_i\) as idiosyncratic components. Condition (e2) ensures that prices do not contain exogenous information about dividends — only information acquired by investors; the condition leaves open the possibility of a spillover of information from some \(s_i(f)\) to \(P_{S_j}\), \(j \neq i\). The last two conditions limit us to “interior” equilibria: we will see that (e3) ensures that there is a benefit to becoming informed, whereas (e4) ensures that not all investors become informed.

To state the main result of this section, we highlight two types of information choices. Call informed investors who choose the information set \(\mathcal{I}_0^{(2)} = \{m, P_F\}\) macro-informed, and call informed investors who choose any information set \(\mathcal{I}_1^{(1)} = \{P_F, (s_i, P_{S_i})\}, i = 1, \ldots, N\), micro-informed. Here, \(m\) and \(s_i\) are the maximally informative macro and micro signals that can be achieved in these information sets with capacity \(\kappa\).

**Theorem 2.1.** In any equilibrium satisfying (e1)–(e4), all informed investors choose to be either macro-informed or micro-informed, and both types of investors are present in positive proportions.

Under the conditions in the theorem, all informed investors choose one of two types of information. In particular, we obtain heterogeneous information choices by ex ante identical investors. This result stands in marked contrast to most of the related literature. In a partial equilibrium setting with exogenous prices, Van Nieuwerburgh and Veldkamp (2010) show that investors with exponential utility and a variance-ratio information constraint are indifferent across all feasible information choices: their investors have no preference to cluster in specific information sets.\[^{10}\] Mondria (2010) finds cases of asymmetric equilibria numerically, but these are outside the scope of his theoretical analysis, which focuses on identical signal choices by investors. In Kacperczyk et al. (2016) all informed investors choose the same information, up to minor differences in how investors

\[^{10}\]See Appendix A.5 for more on this scenario.
break ties. Moreover, in their framework, there is no difference between having positive proportions of investors choosing two different information sets and having all investors split their attention between two information sets in the same proportions. In our setting, specialized micro- and macro-informed investors cannot be replaced with identical investors who divide their attention between micro and macro information.

In Goldstein and Yang (2015), the dividend of a single stock depends on two types of fundamentals. Their interpretation is different, but one could think of the two fundamentals as macro and micro sources of uncertainty. In their equilibrium, investors choose to learn about both fundamentals unless a cost penalty is introduced that makes the cost of acquiring both types of information greater than the sum of the costs of acquiring each type of information separately. Their outcome therefore differs from ours, in which investors choose to focus on one source of uncertainty. Investors in Goldstein and Yang (2015) have just one security through which to trade on two types of dividend information, so information about one signal can be inferred from the other; the two types of information can be substitutes or complements, depending on the strength of the interaction effect. Our setting has as many securities as sources of dividend information, which removes the interaction effect; because investors specialize, macro and micro signals are strategic substitutes. Investors in Van Nieuwerburgh and Veldkamp (2009) also specialize, but their specialization depends on differences in prior information.

The proof of Theorem 2.1 requires several steps, as detailed in the appendix. Here we provide some brief intuition. We show that conditions (e1)–(e4) imply (7), which means that, in equilibrium, making inferences from the price of the index fund consumes at least slightly less capacity than would be expected from the variance reduction achieved. More surprisingly, in equilibrium, learning about individual stocks effectively introduces a fixed cost (in expected utility) to following each additional a stock, in addition to the variable cost associated with increased signal precision. This effect discourages informed investors from spreading their capacity across multiple stocks.

Going forward, we will denote by \( m = m(f_M) \) the maximally informative macro signal chosen by a macro-informed investor, \( \text{var}[m] = f_M \text{var}[M] \), and we will represent \( M \) as

\[
M = m + \epsilon_M, \tag{8}
\]

where \( m \) and \( \epsilon_M \) are uncorrelated. Similarly, we will write

\[
S_i = s_i + \epsilon_i, \quad i = 1, \ldots, N, \tag{9}
\]
where $s_i$ and $\epsilon_i$ are uncorrelated with each other, and where $s_i = s_i(f_S)$, with $f_S = \text{var}[s_i]/\text{var}[S_i]$, is the maximally informative micro signal chosen by a micro-informed investor, recalling that the micro-informed also observe the index fund price $P_F$. The information choices $\{m, P_F\}$ and $\{P_F, (s_i, P_{S_i})\}$, consume the investor’s full capacity, so we have

$$\kappa = \frac{\text{var}[M|m, P_F]}{\text{var}[M]} = \delta_F \frac{\text{var}[S_i|s_i, P_{S_i}]}{\text{var}[S_i]}.$$  \hspace{1cm} (10)

### 3 The constrained model

Theorem 2.1 shows that a necessary condition for an equilibrium in our setting is that all informed investors are either macro-informed or micro-informed. We will now show that such an equilibrium does in fact exist. We do so by imposing the necessary condition as a constraint from the outset. In other words, we now consider a market with just three types of investors: uninformed, macro-informed, and micro-informed, with respective fractions $\lambda_U$, $\lambda_M$, and $\lambda_S = 1 - \lambda_U - \lambda_M$. The macro-informed select the signal $m$ in (8), and a micro-informed investor selects $P_F$ and a signal $s_i$ from (9); no other signals are chosen by any investors. We assume that the mass $\lambda_S$ of micro-informed investors is evenly divided among the $N$ stocks, so $\lambda_S/N$ investors observe each signal $s_i$, $i = 1, \ldots, N$, and only these investors invest directly in stock $i$. We limit the uninformed to investing in the index fund. We extend the correlation condition in (2) to the $s_i$ and $\epsilon_i$, so that

$$\sum_{i=1}^{N} s_i = \sum_{i=1}^{N} \epsilon_i = 0.$$  \hspace{1cm} (11)

#### Supply shocks

Investor demands for the securities will follow from their utility maximizing decisions. We now detail the supply of the securities. We suppose that the supply has a factor structure similar to that of the dividends in (1), with the supply of the $i^{th}$ stock given by

$$\frac{1}{N} (X_F + X_i).$$  \hspace{1cm} (12)

Here, $X_F$ is the common supply shock, normally distributed with mean $\bar{X}_F$ and variance $\sigma_{X_F}^2$. The $X_i$ are normally distributed idiosyncratic shocks, each with mean 0 and variance $\sigma_{X_i}^2$. Supply shocks are independent of cash flows, and $X_i$ is independent of $X_F$ for all $i$. The $X_i$ have the same correlation structure as the $S_i$ in (2), so the idiosyncratic shocks
diversify, in the sense that
\[ \sum_{i=1}^{N} X_i = 0. \] (13)
We make the standard assumption that supply shocks are unobservable by the agents.

The aggregate portion of supply shocks, \( X_F \), is standard in the literature — as will become clear, it is analogous to the single security supply shock in Grossman and Stiglitz (1980). The idiosyncratic portion of the supply shock, \( X_i \), proxies for price-insensitive noise trading in individual stocks. Some of this noise trading may be liquidity driven (for example, individuals needs to sell their employer’s stock to pay for unforeseen expenditures), but the majority is likely to come from either incorrect expectations or from other value-irrelevant triggers, such as an affinity for trading or fads. Our interpretation of noise traders follows Black (1986), who discusses how noise traders play a crucial role in price formation. Recent empirical studies (Brandt, Brav, Graham, and Kumar 2010 and Foucault, Sraer, and Thesmar 2011) document a link from retail trading to idiosyncratic volatility of stock returns. For example, Foucault et al. (2011) “show that retail trading activity has a positive effect on [idiosyncratic] volatility of stock returns, which suggests that retail investors behave as noise traders.” Our model captures this exact phenomenon via \( X_i \). In fact, as will be shown in Section 7, the volatility of \( X_i \) directly enters into the idiosyncratic volatility of stock returns.

**Market clearing**

Let us write \( q^U_i \), \( q^M_i \), and \( q^i_i \) for the demands of each investor group for security \( i \), which can be one of the \( N \) stocks or the index fund \( F \). For each stock \( i \), \( q^i_i \) denotes the *direct demand* for stock \( i \) by investors informed about stock \( i \). Each group’s \( F \) demand, \( q_F \), leads to an *indirect demand* of \( q_F / N \) for every stock \( i \).

Aggregate holdings of the index fund are given by
\[ q_F = \lambda_U q^U_F + \lambda_M q^M_F + \frac{\lambda_S}{N} \sum_{i=1}^{N} q^i_F. \] (14)

The market clearing condition for each stock \( i \) is given by
\[ \frac{\lambda_S}{N} q^i_i + \frac{q_F}{N} = \frac{1}{N} (X_F + X_i), \quad i = 1, \ldots, N. \] (15)

The first term on the left is the direct demand for stock \( i \) from investors informed about that stock; these are the only investors who invest directly in the stock. The second term
is the amount of stock $i$ held in the index fund. The right side is the supply shock from (12). The direct and indirect demand for stock $i$ must equal its supply.

As (15) must hold for all $i$, the quantity $\xi \equiv \lambda_S q_i^d - X_i$ cannot depend on $i$. We can therefore write the direct demand for stock $i$ and the total demand for the index fund as

$$\lambda_S q_i^d = X_i + \xi, \quad q_F = X_F - \xi,$$

for some $\xi$ that does not depend on $i$.

We will show in Section 4.1 that in equilibrium $\xi$ must be zero, leading to two important implications. It will follow from (16) that micro-informed investors fully absorb the idiosyncratic supply shock $X_i$, and that the index fund holds the aggregate supply shock. We will interpret the first equation in (16) as liquidity provision by the micro-informed investors in the securities in which they specialize.

## 4 Market equilibrium in the constrained model

We construct an equilibrium in which the index fund price takes the form

$$P_F = a_F + b_F(m - \bar{m}) + c_F(X_F - \bar{X}_F),$$

and individual stock prices are given by

$$P_i = P_F + b_S s_i + c_S(X_i + \xi), \quad i = 1, \ldots, N.$$

Here, $m$ and $s_i$ are the macro and micro signals in (8) and (9). Equation (17) makes the index fund price linear in the macro shock $m$ and the aggregate supply shock $X_F$. Equation (18) makes the idiosyncratic part of the price of stock $i$, $P_i - P_F$, linear in the micro shock $s_i$ and the idiosyncratic supply shock $X_i + \xi$. These prices satisfy (e1) and, consistent with (e2), the only information they contain about dividends comes from the selected signals $m$ and $s_i$.

### 4.1 Model solution

Macro-informed and uninformed investors have demands only for the index fund, and a micro-informed investor demands the index fund and one security $i$. Investors set their demands by maximizing expected utility conditional on their information sets. These sets
are \( \{P_F\} \) for the uninformed, \( \{m, P_F\} \) for the macro-informed, and \( \{P_F, P_S, s_i\} \) for the micro-informed.

By standard arguments, the macro-informed demand for the index fund is given by

\[
q^M_F = \frac{1}{\gamma(1 - f_M)\sigma^2_M}(m - RP_F),
\]

as in equation (8) of Grossman and Stiglitz (1980), where \( R \) is the risk-free gross return, and uninformed demand for the index fund is given by

\[
q^U_F = \frac{1}{\gamma \text{var}[M|P_F]}(E[M|P_F] - RP_F).
\]

If \( P_F \) takes the form in (17), then

\[
E[M|P_F] = K_F(P_F - a_F) + \bar{m},
\]

\[
\text{var}[M|P_F] = \text{var}[m|P_F] + \text{var}[\epsilon_M] = f_M\sigma^2_M(1 - K_F b_F) + (1 - f_M)\sigma^2_M,
\]

\[
K_F = \frac{b_F f_M \sigma^2_M}{b^2_F f_M \sigma^2_M + c^2_F \sigma^2_X_F}.
\]

Demands of the micro-informed agents are given by the following proposition.

**Proposition 4.1.** If the prices \( P_F \) and \( P_i \) take the form in (17) and (18), then the demands of \( i \) informed agents are given by

\[
q^i = \frac{R}{\gamma(1 - f_S)\sigma^2_S}(P_F + s_i/R - P_i) \tag{22}
\]

\[
q^i_F = q^U_F - q^i. \tag{23}
\]

Equation (23) shows that a micro-informed agent’s demand for the index fund consists of two components. The first component is the demand \( q^U_F \) of the uninformed agents: neither the micro-informed nor the uninformed have any information about \( M \) beyond that contained in \( P_F \). The second term \(-q^i_i\) offsets the exposure to \( M \) that the micro-informed agent takes on by holding stock \( i \). We interpret the second term as the micro-informed’s *hedging demand*: the micro-informed use the index fund to hedge out excess exposure to \( M \) that they get from speculating on their signal \( s_i \). The net result is that micro-informed and uninformed agents have the same exposure to \( M \).

Substituting (23) in (14) — which gives the aggregate index fund demand — and
combining this with the index fund market clearing condition in (16) yields

\[(\lambda_U + \lambda_S)q_F^U + \lambda_M q_F^M = X_F.\]  
(24)

This market clearing condition for the index fund aligns with the single security Grossman-Stiglitz equilibrium, with the fraction of macro-informed agents given by \(\lambda_M\) and the fraction of macro-uninformed agents given by \(\lambda_U + \lambda_S\).

With the demands (19)–(20) for the index fund and demands (22)–(23) for individual securities, market-clearing prices are given by the following proposition:

**Proposition 4.2.** The market clears at an index fund price of the form (17),

\[P_F = a_F + b_F (m - \bar{m}) + c_F (X_F - \bar{X}_F), \quad \text{with} \quad \frac{c_F}{b_F} = -\frac{\gamma(1 - f_M)\sigma^2_M}{\lambda_M},\]  
(25)

and prices for individual stocks \(i\) of the form (18), given by

\[P_i = P_F + \frac{s_i}{R} - \frac{\gamma(1 - f_S)\sigma^2_S}{\lambda_S R}(X_i + \xi).\]  
(26)

The no-arbitrage condition (4) is satisfied if and only if \(\xi = 0\).

The form of the index fund price \(P_F\) follows from Grossman and Stiglitz (1980); explicit expressions for the coefficients \(a_F\), \(b_F\), and \(c_F\), are derived in the appendix. Comparison of (18) and (26) shows that the ratio \(c_S/b_S\) in the price of stock \(i\) has exactly the same form as \(c_F/b_F\) in the price of the index fund in (25). In fact, if \(\lambda_M = 1\), then \(b_F = 1/R\) and \(c_F\) has exactly the same form as \(c_S\). The stock \(i\) equilibrium is the direct analog of the index fund equilibrium with only macro-informed agents.

As in Grossman and Stiglitz (1980) the equilibrium price of the index fund depends on the proportion of investors informed about the fund’s payout. Similarly, the prices of individual securities depend on the proportion of investors informed about these securities. We will investigate endogenous choices of these proportions.

When the proportions \(\lambda_U\), \(\lambda_M\), and \(\lambda_S\) are all endogenously positive and \(f_M > f_S\), the constrained model solved by Proposition 4.2 realizes the equilibrium conditions of Theorem 2.1. The constrained model is more general in the sense that it does not impose a relationship between the information ratios \(f_M\) and \(f_S\). With prices as in Proposition 4.2,

\[\text{Using (23) we see that } N^{-1}\lambda_S \sum_i q_F^i \text{ in (14) equals } \lambda_S q_F^U - N^{-1} \lambda_S q_F^i. \text{ Using the first equation in (16) this becomes } \lambda_S q_F^U - N^{-1} \sum_i (X_i + \xi) = \lambda_S q_F^U - \xi, \text{ and the second equation in (16) then yields (24).} \]
we can drop $P_F$ and $P_{S_i}$ from the conditioning in (10) and write (10) as

$$\kappa = 1 - f_M = \delta_F(1 - f_S).$$  \hspace{1cm} (27)

Here we need $f_M > f_S$: the informativeness of the macro signal $m$ is greater than that of the micro signal $s_i$. We will see in Section 5.2 that (e3) leads to an interior equilibrium in the constrained model through (27).

The equality $\kappa = \delta_F(1 - f_S)$ suggests an alternative interpretation of $\delta_F$. To trade on their signal $s_i$, micro-informed investors trade stock $i$, which changes their exposure to macro risk, compared with an uninformed investor. We can interpret $\delta_F$ as the capacity consumed by hedging this extra macro risk, leaving informativeness $f_S$ for $s_i$. A fixed $\delta_F$ then means that hedging capacity does not depend on the informativeness of prices.

### 4.2 Price efficiency

We will investigate the extent to which prices reflect available information, and to do so we need a measure of price efficiency. For the case of the index fund, we define price efficiency, $\rho^2_F$, as the proportion of price variability that is due to variability in $m$, the knowable portion of the aggregate dividend. This is the $R^2$ from regressing $P_F$ on $m$.

The squared correlation between $P_F$ in (17) and $m$ is given by

$$\rho^2_F = \frac{b^2_F f_M \sigma^2_M}{b^2_F f_M \sigma^2_M + c_F \sigma^2_{X_F}}. \hspace{1cm} (28)$$

This equals $b_F K_F$ in (21), so we can use (21) to write

$$\text{var}[m|P_F] = f_M \sigma^2_M (1 - \rho^2_F).$$

As the price efficiency goes to 1, $P_F$ becomes fully revealing about $m$. Dividing both sides of (28) by $b^2_F \sigma^2_M$ and using the expression for $c_F/b_F$ in (25), we get

$$\rho^2_F = \frac{f_M}{f_M + \gamma^2 (1 - f_M)^2 \sigma^2_M \sigma^2_{X_F} / \lambda^2_M}. \hspace{1cm} (29)$$

For stock $i$ we define price efficiency as the proportion of the variability of the price that is driven by variability in $s_i$, the idiosyncratic dividend shock, once $P_F$ is known.
From the functional form of $P_i$ in (18) and the fact that $\xi = 0$, this is given by

$$\rho_S^2 = \frac{b_f^2 f_s\sigma_S^2}{b_f^2 f_s\sigma_S^2 + c_S^2 \sigma_X^2}.$$  

Using the expression for $c_S/b_S$ in (26) and simplifying as we did with $\rho_F^2$, we find that

$$\rho_S^2 = \frac{f_s}{f_s + \gamma^2 (1 - f_s)^2 \sigma_S^2 \sigma_X^2 / \lambda_S^2}.$$  

(30)

As in the case of the index fund, as $\rho_S^2$ goes to 1, $P_i$ becomes fully revealing about $s_i$. The two efficiency measures have identical functional forms, with each using its respective set of moments and its $\lambda$. Each measures price informativeness with respect to the *knowable* portion of the dividend payout, as given by $f_M$ and $f_S$, not the total dividend payout.

Differentiating (29) and (30) and straightforward algebra, yields the following result:

**Proposition 4.3** (When are prices more informative?).

(i) Micro (macro) prices are more efficient as either (a) the fraction of micro (macro) informed increases, or (b) as the micro (macro) signal informativeness improves:

$$d\rho_S^2/d\lambda_S > 0 \quad \text{and} \quad d\rho_F^2/d\lambda_M > 0,$$

and

$$d\rho_S^2/df_S > 0 \quad \text{and} \quad d\rho_F^2/df_M > 0.$$  

(ii) Furthermore, when the fraction of micro (macro) informed is zero, or when the signals are non-informative, price efficiency is zero. In other words, $\rho_F^2 \to 0$ as either $\lambda_M \to 0$ or $f_M \to 0$, and $\rho_S^2 \to 0$ as either $\lambda_S \to 0$ or $f_S \to 0$.

(iii) When the signals are perfectly informative, prices become fully revealing. In other words, $\rho_F^2 \to 1$ as $f_M \to 1$ if $\lambda_M > 0$, and $\rho_S^2 \to 1$ as $f_S \to 1$ if $\lambda_S > 0$.

As the number of informed in a given market grows, prices in that market become more revealing. As the information about future dividends that is known to informed investors grows, these investors – facing less future cash flow risk – trade more aggressively, which incorporates more of their information into prices. We will have more to say about the price efficiency measures when we evaluate them at equilibrium proportions $\lambda_M$ and $\lambda_S$.  

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5 Attention equilibrium in the constrained model

The prior section analyzed the market equilibrium, taking the proportions of uninformed, macro-informed and micro-informed investors as given. In this section and then next, we will endogenously determine these proportions.

Recall that in Section 2, we allowed investors to acquire information processing capacity at a cost and then to allocate this capacity. In this section, we focus on the allocation decision, taking the decision to acquire capacity or remain uninformed as given. In other words, we hold \( \lambda_U \) fixed and investigate the equilibrium mix of \( \lambda_M \) and \( \lambda_S \).

As context for this investigation, we take the view that part of the cost of becoming informed lies in developing the skills needed to acquire and apply investment information, and that this process takes time — perhaps seven to ten years of education and experience. In the near term, these requirements leave the total fraction of informed investors \( \lambda_M + \lambda_S \) fixed. Once investors have the skills needed to become informed, we suppose that they can move relatively quickly (over one or two years) and costlessly between macro and micro information by shifting the focus of their attention. We therefore distinguish a near-term attention equilibrium, in which \( \lambda_U \) is fixed and the split between \( \lambda_M \) and \( \lambda_S \) is endogeneous, from a longer-term information equilibrium, in which the decision to become informed is endogenized along with the choice of information on which to focus. We analyze the attention equilibrium in this section and address the information equilibrium in Section 6.

The allocation of attention we study refers to the fraction of investors focused on each type of information, and not the allocation of attention by an individual agent.

5.1 Relative utility

Recall that an investor’s ex ante expected utility is given by 
\[
J \equiv E[- \exp(-\gamma \bar{W}_2)],
\]
where the expectation is taken unconditionally over time 2 wealth. Write \( J_M \), \( J_S \), and \( J_U \) for expected utility of macro-informed, micro-informed, and uninformed investors, respectively.

Fixing the fraction of uninformed, the following proposition establishes the relative benefit of being macro- or micro-informed relative to being uninformed.

Proposition 5.1. If the cost of becoming informed is given by \( c \), then the benefit of being macro-informed relative to being uninformed is given by

\[
\frac{J_M}{J_U} = \exp(\gamma c) \left(1 + \frac{f_M}{1 - f_M} (1 - \rho_F^2)\right)^{-\frac{1}{2}}. \tag{31}
\]
The benefit of being micro-informed relative to being uninformed is given by

\[ J_S/J_U = \exp(\gamma c) \left( 1 + \frac{f_S}{1-f_S} \left( \frac{1}{\rho^2_S} - 1 \right) \right)^{-\frac{1}{2}}. \]  

(32)

Note that because utilities in our model are negative, a decrease in these ratios represents a gain in informed relative to uninformed utility.

Each of the ex ante utility ratios in the proposition is increasing in the corresponding measure of price efficiency — that is, informed investors become progressively worse off relative to uninformed as micro or macro prices become more efficient. But the dependence on \( \rho^2_S \) in (32) differs from the dependence on \( \rho^2_F \) in (31), a point we return to in Section 6.2.

Recalling from Proposition 4.3 that macro and micro price efficiency increase in \( \lambda_M \) and \( \lambda_S \), respectively, we immediately get the following:

**Proposition 5.2** (Benefit of information decreases with number of informed). \( J_S/J_U \) strictly increases (making micro-informed worse off) in \( \lambda_S \). \( J_M/J_U \) strictly increases (making macro-informed worse off) in \( \lambda_M \).

Figure 1 illustrates the results of Propositions 5.1 and 5.2. The figure holds \( \lambda_U \) fixed, and the x-axis is indexed by \( \lambda_M \). As \( \lambda_M \) increases, \( J_M/J_U \) increases, indicating that the macro-informed are becoming worse off. Similarly, at the rightmost point of the graph, \( \lambda_S = 0 \), and as we move to the left, \( J_S/J_U \) increases, indicating that the micro-informed are becoming worse off as more of their type enter the economy.\(^{12}\)

### 5.2 Choice between macro and micro information

At an interior equilibrium, the marginal investor must be indifferent between macro and micro information, in which case equilibrium will be characterized by a \( \lambda_M^* \) such that with that many macro-informed investors and with \( 1 - \lambda_U - \lambda_M^* \) micro-informed investors we will have \( J_M = J_S \), which just sets (31) equal to (32). To cover the possibility of a corner solution, we define an attention equilibrium by a pair of proportions \( \lambda_M \geq 0 \) and \( \lambda_S = 1 - \lambda_U - \lambda_M \geq 0 \) satisfying

\[ J_M < J_S \Rightarrow \lambda_M = 0 \quad \text{and} \quad J_S < J_M \Rightarrow \lambda_S = 0. \]  

(33)

\(^{12}\)Many of our comparisons could be recast as statements about trading intensities, in the sense of Goldstein and Yang (2015). Macro and micro trading intensities are given by \(-b_F/c_F\) and \(-b_S/c_S\) in Proposition 4.2.
The inequalities in this condition are equivalent to \( J_M/J_U > J_S/J_U \) and \( J_S/J_U > J_M/J_U \), respectively, because \( J_U < 0 \).

Recall from Proposition 4.3 that when the fraction of macro- or micro-informed is zero, price efficiency is also 0 (i.e., \( \rho_F^2(\lambda_M = 0) = 0 \) and \( \rho_S^2(\lambda_S = 0) = 0 \)). From (31) and (32), we see that

\[
J_M/J_U(\lambda_M = 0) = e^{\gamma c}\sqrt{1 - f_M} \quad \text{and} \quad J_S/J_U(\lambda_S = 0) = 0. \tag{34}
\]

From Proposition 5.2 we know that \( J_M/J_U \) and \( J_S/J_U \) both increase monotonically (i.e., make the informed worse off) with their respective \( \lambda \)'s. When \( \lambda_M \) is zero, the macro-informed achieve their maximal utility; when \( \lambda_M = 1 - \lambda_U \), the micro-informed achieve their maximal utility. As \( \lambda_M \) increases from zero to \( 1 - \lambda_U \), \( \lambda_S \) decreases, so the macro-informed become progressively worse off and the micro-informed become progressively better off. If at some \( \lambda_M \) the two curves \( J_M/J_U \) and \( J_S/J_U \) intersect, we will have an interior equilibrium, and it must be unique because of the strict monotonicity in Proposition 5.2. This case is illustrated in Figure 1. If there is no interior equilibrium,
then either macro or micro information is always preferred, and no investor will choose the other. Such a scenario is possible in the constrained model of Section 3, though not under the more general information choices in Theorem 2.1.

To make these observations precise, let us define

\[ \tilde{\lambda}_M \equiv (1 - \lambda_U) \frac{1 - \sqrt{\varphi + (1 - \varphi) \frac{\gamma^2 \alpha}{(1 - \lambda_U)^2}}}{1 - \varphi}, \]

where

\[ \varphi = \frac{(1 - f_S) \sigma_X^2}{(1 - f_M) \sigma_M^2 \sigma_X^2} \] and \[ \alpha = \frac{1 - f_M}{f_M} \frac{(1 - f_S) \sigma_M^2}{\sigma_X^2}. \]

Note that \( \varphi \) is the ratio of the total risk arising from the unknowable portion of idiosyncratic supply shocks (the variance of \( \epsilon_i \) times the variance of \( X_i \)) to the total risk arising from macro supply shocks (the variance of \( \epsilon_M \) times the variance of \( X_F \)). The larger \( \varphi \) the more total unknowable risk comes from idiosyncratic rather than systematic sources.

The following proposition characterizes the equilibrium allocation of attention in the economy between macro information and micro information when the total fraction of informed investors \( 1 - \lambda_U \) is fixed.

**Proposition 5.3 (Attention equilibrium).** Suppose \( 0 \leq \lambda_U < 1 \), so some agents are informed.

(i) Interior equilibrium.\(^{13}\) If \( \tilde{\lambda}_M \in [0, 1 - \lambda_U) \), then this point defines the unique equilibrium: at \( \lambda^*_M = \tilde{\lambda}_M \), the marginal informed investor will be indifferent between becoming macro- or micro-informed.

(ii) If \( \tilde{\lambda}_M \notin [0, 1 - \lambda_U) \), the unique equilibrium is at the boundary \( \lambda^*_M = 0 \), where all informed agents are micro-informed.

(iii) In equilibrium, we always have \( \lambda^*_M < 1 - \lambda_U \). In other words, some informed agents will choose to be micro-informed.

\(^{13}\)The first equality in (34) sheds additional light on (e3). Combining (34) with (27) yields \( J_M/J_U < 1 \) at \( \lambda_M = 0 \). But if \( J_M/J_U < 1 \) then some uninformed investors will prefer to become macro-informed, resulting in \( \lambda_M > 0 \). In this sense, (e3) leads to an interior attention equilibrium.

\(^{14}\)This expression has a finite limit as \( \varphi \to 1 \), and we take that limit as the value of \( \tilde{\lambda}_M \) at \( \varphi = 1 \).

\(^{15}\)We refer to (i) as the case of an interior equilibrium, even though it includes the possibility of a solution at the boundary. If \( \tilde{\lambda}_M = \lambda_M = 0 \), then \( J_M = J_S \), and the marginal investor is indifferent between micro and macro information, which is what we mean by an interior equilibrium. If case (ii) holds, then \( \lambda^*_M = 0 \) because micro information is strictly preferred over macro information at all \( \lambda_M \).
It bears emphasizing that an attention equilibrium — regardless of parameter values — precludes all informed agents from being macro-informed. In contrast, it is possible for all informed agents to be micro-informed. We therefore have, as a fundamental feature of the economy, a bias for micro over macro information. This property holds in the constrained model, where informed investors are limited to macro or micro information, and the micro-informed receive all the benefits of providing liquidity to noise traders in individual stocks. We know from Theorem 2.1 that in a setting with greater information choices, an equilibrium will contain both micro- and macro-informed investors.

To get some intuition into the drivers of the attention equilibrium, we consider two special cases in which \( \tilde{\lambda}_M \) simplifies: when \( \gamma = 0 \) and when \( \varphi = 1 \). If \( \gamma = 0 \), then

\[
\tilde{\lambda}_M = \frac{1 - \lambda_U}{1 + \sqrt{\varphi}}.
\]  

(37)

As noted before Proposition 5.3, \( \varphi \) measures the relative magnitude of unknowable idiosyncratic and macro shocks. So, this expression suggests that, at least at low levels of risk aversion, agents favor information about the greater source of uncertainty, with \( \tilde{\lambda}_M \) increasing in macro uncertainty and decreasing in micro uncertainty. At \( \varphi = 1 \),

\[
\tilde{\lambda}_M = \frac{(1 - \lambda_U)^2}{2} \left[ 1 - \frac{\gamma^2 \alpha}{(1 - \lambda_U)^2} \right].
\]

Here it becomes evident that an increase in risk aversion moves investors toward micro information. The benefit to being macro-informed comes from two sources, liquidity provision for the macro supply shocks \( X_F \) and the ability to advantageously trade against the macro-uninformed, whereas the benefit of being micro-informed only comes from liquidity provision for the micro shocks \( X_i \). When \( \gamma \) increases, trade between the informed and uninformed falls, thereby diminishing the advantage of being macro-informed. However, the liquidity discount in micro and macro prices, i.e., \( c_F \) and \( c_S \) from (25) and (26), increases with risk aversion, making being micro-informed relatively more attractive.

Similarly, an increase in micro (macro) volatility, as measured by \( \sigma_S \sigma_X \) (\( \sigma_M \sigma_{X_F} \)), will increase the benefit of information to the micro (macro) informed, and will therefore decrease (increase) \( \lambda_M^* \) when the economy is at an interior equilibrium. We summarize these two results in the following proposition:

**Proposition 5.4** (Effects of risk aversion and risk on the attention equilibrium). We consider the case of an interior equilibrium with \( \lambda_M^* > 0 \).

(i) Risk aversion pushes investors towards micro information: \( d\lambda_M^*/d\gamma < 0 \).
(ii) Increase in micro (macro) risk pushes investors towards micro (macro) information:

\[
\frac{d\lambda^*_M}{d(\sigma_S\sigma_X)} < 0 \quad \text{and} \quad \frac{d\lambda^*_M}{d(\sigma_M\sigma_{X_F})} > 0.
\]

5.3 Impacts of information precision

Recall from (8) and (9) that \(f_M\) and \(f_S\) measure the fraction of variation in \(M\) and \(S_i\) that is known to informed investors. We refer to this as information precision.

Effect of information precision on investor welfare

We first show that, somewhat surprisingly, more precise micro information makes the micro-informed worse off.

**Proposition 5.5** (The micro-informed overtrade on their information). More precise information is worse for the micro-informed in the sense that

\[
\frac{d(J_S/J_U)}{df_S} > 0 \quad \text{(micro informed are worse off)}.
\]

When investors become micro-informed, the more they know about the ultimate idiosyncratic portion of the payout \(S_i\), the less uncertainty they face from owning the stock. From (26) we see that the discount in the stock price due to idiosyncratic supply shocks \(X_i\) will be zero when the micro information is perfect, i.e., when \(f_S = 1\). With no discount in the price, the compensation for liquidity provision goes to zero. Because atomic informed agents cannot act strategically and coordinate to limit their liquidity provision in an optimal (for them) way, uncertainty about the dividend helps them by decreasing the sensitivity of their demand to price shocks, which in turn leads to a higher risk premium in prices. In contrast to the micro-informed, the macro-informed may be better or worse off as their precision, \(f_M\), improves:

**Proposition 5.6** (The macro-informed can be better or worse off with more information). More precise information is better for the macro-informed if and only if

\[
\rho_F^2 < \frac{1}{1 + f_M},
\]

which is equivalent to

\[
\lambda_M < \gamma \sigma_M \sigma_{X_F} \frac{1 - f_M}{f_M}.
\]
In this case, 
\[ \frac{d(J_M/J_U)}{df_M} < 0 \]  
(macro-informed are better off).

To gain intuition into this result recall that at \( f_M = 0 \) we would have \( \rho^2_F = 0 \) (price reveals nothing when nothing about \( M \) is knowable), and at \( f_M = 1 \) we would have \( \rho^2_F = 1 \) (prices are fully revealing when \( M \) is fully known). Furthermore, from Proposition 4.3 we know \( \rho^2_F \) increases monotonically in \( f_M \). So (38) implies that the macro-informed benefit from an increase in the precision \( f_M \) only when \( f_M \) (hence also the price informativeness \( \rho^2_F \)) is low. Using (39) the condition can be reinterpreted as placing a limit on how many macro-informed investors the economy can support before better macro precision begins to make the macro-informed worse off.

The contrast between micro and macro information in Proposition 5.5 and 5.6 can be understood as follows. In the market for the index fund, informed investors trade against uninformed investors as well as taking the other side of price insensitive liquidity shocks, introducing an effect that is absent in the market for individual stocks. With a low signal precision, prices are not very informative, so a small improvement in precision gives the macro-informed an informational edge over the uninformed, allowing the informed to extract rents in trading. However, as the signal precision improves and price efficiency grows, the incremental ability to extract rents from trading against the uninformed diminishes, while the tendency to overtrade on information (as in the case of the market for individual stocks) grows. At some point, determined by \( \rho^2_F = 1/(1 + f_M) \), the overtrading tendency begins to dominate the rent-extraction effect.

Effect of information precision on attention equilibrium

Recall that for a fixed \( \lambda_U \), the equilibrium \( \lambda^*_M \) (proportion of macro-informed) is determined by the condition \( J_M/J_U = J_S/J_U \), in the case of an interior equilibrium. Because \( J_M/J_U \) does not depend on \( f_S \), as \( f_S \) rises and \( J_S/J_U \) increases, \( J_M/J_U \) can increase only if \( \lambda_M \) increases (from Proposition 5.2). An interior equilibrium \( \lambda^*_M \) must therefore increase with \( f_S \). As the benefit of being micro-informed falls due to more precise micro information, the fraction of informed investors that focus on macro information grows. Figure 2 demonstrates this adjustment. For every \( \lambda_M \), a higher \( f_S \) makes the micro-informed worse off, which pushes the equilibrium number of macro-informed higher.

Similarly, since \( J_S/J_U \) does not depend on \( f_M \) but decreases in \( \lambda_M \) (i.e., the micro-informed are better off as there are fewer micro-informed), if \( J_M/J_U \) decreases (increases) in \( f_M \), then \( \lambda^*_M \) must increase (decrease) in \( f_M \). Figure 3 illustrates this phenomenon.
In the figure, the equilibrium $\lambda_M^*$ is sufficiently small so that macro precision makes the macro-informed better off. As macro precision $f_M$ increases, for a range of sufficiently small $\lambda_M$, the macro-informed become better off, which increases $\lambda_M^*$ (i.e., decreases the number of micro-informed, thus making the remaining micro-informed better off). Had the equilibrium $\lambda_M$ been sufficiently high, the effect would have had the opposite sign, as can be seen by the fact the $J_M/J_U$ increases with $f_M$ for high $\lambda_M$.

The preceding arguments establish the following result:

**Proposition 5.7** (Effect of information precision on equilibrium). *In the case of an interior equilibrium with $\lambda_M^* > 0$, the number of macro-informed increases as the micro signal becomes more precise:

$$\frac{d\lambda_M^*}{df_S} > 0.$$*

Condition (38) (or equivalently (39)) is necessary and sufficient for the number of macro-informed to increase as the macro signal becomes more precise:

$$\frac{d\lambda_M^*}{df_M} > 0 \quad (0 < 0) \quad {\text{if and only if}} \quad \rho_F^2 < \frac{1}{1 + f_M} \quad \left(> \frac{1}{1 + f_M}\right).$$
\[ \gamma = 1.5, \ \sigma_M^2 = 0.25, \ \sigma_S^2 = 0.25, \ f_M = 0.7, \ f_S = 0.3, \ \sigma_X^2 = 0.5, \ \sigma_X^2 = 1 \]

\[ \lambda_U = 0.2 \]

\[ J_M / J_U : M \text{ informed to uninformed} \]

\[ J_S / J_U : S \text{ informed to uninformed} \]

\[ J_M / J_U : M \text{ informed to uninformed, higher } f_M \]

**Figure 3:** The effect of increasing macro precision \( f_M \) on the attention equilibrium with fixed \( \lambda_U \).

### 5.4 Equilibrium price informativeness

Define

\[ \tau_M = \frac{f_M/(1 - f_M)}{f_S/(1 - f_S)} \]

To measure the relative informativeness of the macro and micro signals. Under condition (27), \( f_M > f_S \) so \( \tau_M > 1 \); however, in the general setting of the constrained model of Section 3, any \( \tau_M \geq 0 \) is feasible. We therefore explore the full range of possible \( \tau_M \) values but put particular emphasis on the case \( \tau_M > 1 \). We investigate the effect of varying \( f_M \) and \( f_S \) while holding \( \tau_M \) fixed.

In an interior attention equilibrium, the marginal informed investor is indifferent between macro and micro information because \( J_M / J_U = J_S / J_U \). From Proposition 5.1 [27] we see this condition then implies that micro price efficiency is related to macro price efficiency via \( (1 - \rho_S^2) / \rho_S^2 = \tau_M (1 - \rho_F^2) \), which yields

\[ \rho_S^2 = \frac{1}{1 + \tau_M (1 - \rho_F^2)}. \]  

(40)
From this we see that as markets become fully macro efficient, they must also become fully micro efficient, and vice versa. In other words,

\[ \rho_S^2 \to 1 \iff \rho_F^2 \to 1. \]

However, as macro price efficiency tends towards zero, micro price efficiency tends towards \(1/(1 + \tau_M)\). Since both sides of (40) are decreasing as their respective \(\rho^2\) falls, this also represents the lower bound for \(\rho_S^2\) in an interior attention equilibrium. Proposition 5.3 established the possibility of corner attention equilibria with no macro informed, but where \(\lambda_S^* > 0\) – i.e. attention equilibria must have some micro informed traders. In this case, \(\rho_F^2 = 0\) while \(\rho_S^2\) will remain positive, and the relationship in (40) will not hold. The following result makes a more general statement about the relative price efficiency of the two markets at any interior attention equilibrium.

**Proposition 5.8** (When are markets more micro or macro efficient?) If \(\tau_M \leq 1\), then \(\rho_S^2 \geq \rho_F^2\) (markets are more micro efficient). If \(\tau_M > 1\), then \(\rho_S^2 < \rho_F^2\) (markets are more macro efficient) for \(\rho_F^2 \in (1/\tau_M, 1]\) and otherwise \(\rho_S^2 \geq \rho_F^2\).

The difference in price efficiencies \(\rho_S^2 - \rho_F^2\) is illustrated in Figure 4 for two values of \(\tau_M\). We take \(\tau_M > 1\) (\(f_M > f_S\)) for consistency with (27). In each panel, the solid (dashed) portion of the curve represents the region in which micro efficiency exceeds (is less than) macro efficiency. The red dots in the figure represent attention equilibria at a given level of \(\lambda_U\) (i.e. with \(\lambda_M\) given by equation (35)), as \(\lambda_U\) ranges from zero to one. The vertical portion of the curve represents corner equilibria with no macro informed investors.

We make two observations. First, the region of micro efficiency is larger for smaller values of \(\tau_M\). This is intuitive, but it is notable that even with \(f_M > f_S\), i.e. when the macro signal is more informative, there is still a pronounced region of micro efficiency. Second, as the number of uninformed investors grows, markets tend towards micro efficiency with \(\rho_S^2 > \rho_F^2\). With more uninformed, the attention equilibrium will gravitate towards the more valuable information set – which in our model turns out to be micro information.

\(^{16}\)As the number of uninformed goes to 1, both micro and macro efficiency tend towards 0.
Figure 4: The charts show the difference between micro and macro price efficiency, $\rho_S^2 - \rho_F^2$, as a function of macro price efficiency $\rho_F^2$. The solid (dashed) portion of the curve represents the region of micro (macro) efficiency. The two charts are labeled with their respective information precision ratios $\tau_M$. The red points are labeled with the values of $\lambda_U$ corresponding to that particular $\rho_F^2$. The vertical portions of the curves correspond to corner equilibria with $\lambda_M = 0$.

6 Information equilibrium in the constrained model

In Section 5, we examined the choice among informed agents to become macro-informed or micro-informed, holding fixed the total fraction of informed agents. That analysis describes a medium-term equilibrium, over a time scale long enough for informed investors to shift their focus of attention, but not long enough for uninformed investors to acquire the skills needed to become informed.

We now examine a longer-term equilibrium in which the uninformed can become informed by incurring a cost $c$. In other words, while continuing to work within the constrained model of Section 3, we now endogenize not only the choice between micro and macro information, but also the decision to become informed. An equilibrium in this setting — which we refer to as an information equilibrium — is defined by nonnegative proportions $(\lambda_M, \lambda_S, \lambda_U = 1 - \lambda_M - \lambda_S)$ such that no agent of a type in positive proportion prefers switching to a different type. Extending (33), we require that, for any
\( \iota, \iota' \in \{M, S, U\}, \)
\[
J_\iota / J_{\iota'} > 1 \Rightarrow \lambda_\iota = 0. \tag{41}
\]

Recall that our utilities are negative, so the inequality on the left implies that type \( \iota' \) is preferred to type \( \iota \). The ratios \( J_M/J_U, J_S/J_U, \) and \( J_S/J_M \) all have well-defined limits as some or all of \( \lambda_M, \lambda_S, \) and \( \lambda_U \) approach zero. (This follows from the expressions for these ratios in (31) and (32) and the dependence of \( \rho_F^2 \) and \( \rho_S^2 \) on \( \lambda_M \) and \( \lambda_S \) in (29) and (30), respectively.) We may therefore evaluate and compare these ratios even in cases where one or more of the proportions \( \lambda_\iota \) is zero.

6.1 No deviation from information choices

As noted in Section 4.1, if \( \lambda_M, \lambda_S, \) and \( \lambda_U \) are all endogenously positive and \( f_M > f_S \), then the information equilibrium in the constrained model is an equilibrium in the general setting of Theorem 2.1, for \( \kappa \) and \( \delta_F \) satisfying (27). It follows that we can drop the constraint that informed investors must be either micro-informed or macro-informed and allow them to choose more general signals as in Section 2, and yet no investors will choose to deviate from their information choices. The no-deviation argument is the last part of the proof of Theorem 2.1 in the appendix.

6.2 Effect of information cost \( c \) on the equilibrium

Figure 5 helps illustrate the general results that follow. The figure plots the equilibrium proportion of each type of investor in the constrained model as a function of the cost \( c \) of information acquisition. The figure divides into three regions. At sufficiently low costs, all agents prefer to become informed, so \( \lambda_U = 0 \). At sufficiently high costs, no investors choose to be macro-informed, so \( \lambda_M = 0 \). At intermediate costs, we find agents of all three types, and this is the region of overlap with Theorem 2.1. At all cost levels, some fraction of agents choose to be micro-informed.

To justify these assertions and to give an explicit characterization of the information equilibrium at each cost level \( c > 0 \), we first consider the possibility that all three types of agents are present in positive proportions. To be consistent with equilibrium, this outcome requires \( J_M/J_U = J_S/J_U = 1 \). Using the expressions for these ratios in (31) and (32), these equalities imply
\[ \rho_F^2 = 1 - \frac{1 - f_M}{f_M} \left[ e^{2\gamma c} - 1 \right]. \tag{42} \]
Figure 5: Equilibrium proportions of macro-informed, micro-informed, and uninformed agents as functions of the cost of information acquisition $c$.

and

$$\rho_S^2 = \left(1 + \frac{1 - f_S}{f_S} \left[e^{2\gamma c} - 1\right]\right)^{-1}.$$  \hspace{1cm} (43)

Setting these expressions equal to (29) and (30), respectively, we can solve for $\lambda_M$ and $\lambda_S$ to get

$$\lambda_M(c) = \gamma (1 - f_M) \sigma_M \sigma_X \left(\frac{1}{(1 - f_M)(e^{2\gamma c} - 1)} - \frac{1}{f_M}\right)^{1/2}$$ \hspace{1cm} (44)

and

$$\lambda_S(c) = \gamma (1 - f_S) \sigma_S \sigma_X \left(\frac{1}{(1 - f_S)(e^{2\gamma c} - 1)}\right)^{1/2}.$$ \hspace{1cm} (45)

The expression for $\lambda_M(c)$ is valid for $c \leq \bar{c}$, with

$$\bar{c} = -\frac{1}{2\gamma} \log(1 - f_M);$$ \hspace{1cm} (46)

set $\lambda_M(c) = 0$ for $c > \bar{c}$. If $\lambda_M(c) + \lambda_S(c) \leq 1$ with $c \leq \bar{c}$, then $(\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c))$ defines an information equilibrium with $J_M = J_S = J_U$.

Both $\lambda_M(c)$ and $\lambda_S(c)$ increase continuously and without bound as $c$ decreases toward zero, so the equation

$$\lambda_M(c) + \lambda_S(c) = 1,$$
defines the lowest cost at which we can meaningfully set $\lambda_U = 1 - \lambda_M(c) - \lambda_S(c)$. At lower cost levels, we need to consider the possibility of an equilibrium with $\lambda_U = 0$.

Once we fix a value for $\lambda_U$, the split between macro- and micro-informed agents is characterized by Proposition 5.3. Write $\lambda^*_M(0)$ for the value of $\lambda^*_M$ in Proposition 5.3 at $\lambda_U = 0$; this value is given either by the root $\hat{\lambda}_M$ in (35) or zero. Set

$$
\lambda_M, \lambda_S, \lambda_U = \begin{cases}
\lambda^*_M(0), 1 - \lambda^*_M(0), 0, & 0 < c < \zeta; \\
\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c), & \zeta \leq c < \bar{c}; \\
0, \lambda_S(c), 1 - \lambda_S(c), & c \geq \max\{\zeta, \bar{c}\}.
\end{cases}
$$

(47)

Then (47) makes explicit the equilibrium proportions illustrated in Figure 5 for the case $\zeta < \bar{c}$: at large cost levels, $\lambda_M = 0$; at low cost levels, $\lambda_U = 0$ and $\lambda_M$ and $\lambda_S$ are constant; at intermediate cost levels, all three proportions are positive; at all cost levels, $\lambda_S > 0$. We always have $\zeta > 0$ and $\bar{c} < \infty$, so the low cost and high cost ranges are always present; but it is possible to have $\zeta \geq \bar{c}$, in which case the intermediate cost range is absent. This occurs when $\lambda_S(\bar{c}) \geq 1$. By evaluating (45) at (46), we find that $\lambda^*_S(\bar{c}) = \gamma^2 \alpha$, with $\alpha$ as in (36). If $\gamma^2 \alpha \geq 1$, then the root $\hat{\lambda}_M$ in (35) evaluated at $\lambda_U = 0$ is less than or equal to zero if it is real, so $\lambda^*_M(0) = 0$. We summarize these observations in the following result.

**Proposition 6.1** (Information equilibrium in the constrained model). At each $c > 0$, the proportions in (47) define the unique information equilibrium. If $\gamma^2 \alpha < 1$, then $\zeta < \bar{c}$ and all three cases in (47) are present. If $\gamma^2 \alpha \geq 1$, then $\zeta \geq \bar{c}$, the second range in (47) is empty, and no investors choose to be macro-informed at any cost level.

With this result, we can revisit some of the conditions in Theorem 2.1 when applied to the constrained model. Recalling from (27) that $\kappa = 1 - f_M$, condition (e3) implies $c < \bar{c}$, the condition for $\lambda_M(c) > 0$. Combining (e3) with (e4) ensures that we are in the range $\zeta < c < \bar{c}$ and thus that all three investor proportions are positive.

From Proposition 6.1, we can deduce several further properties of the information equilibrium. Let us define $\Pi_M$ as the fraction of informed who are macro-informed, or

$$
\Pi_M \equiv \frac{\lambda_M}{\lambda_M + \lambda_S} = \frac{\lambda_M}{1 - \lambda_U}.
$$

(48)

At an interior attention equilibrium ($\lambda^*_M > 0$), $\Pi_M$ is the coefficient of $(1 - \lambda_U)$ in (35). Differentiating with respect to $\lambda_U$ yields $d\Pi_M/d\lambda_U < 0$, when $\lambda_M > 0$: the more uninformed investors there are in the economy, the greater the fraction of informed investors who choose to be micro-informed. The next result describes the dependence of $\Pi_M$ on $c$. 

32
Corollary 6.1 (Effect of information cost $c$ on information equilibrium). In equilibrium, with a cost of becoming informed given by $c$, the following will hold:

(i) As $c$ increases, the fraction $\Pi_M$ of informed investors who choose macro information falls; moreover, $\Pi_M$ is strictly decreasing in $c$ if $\lambda_M > 0$ and $\lambda_U > 0$.

(ii) As $c$ increases the fraction of investors who are uninformed increases; moreover $\lambda_U$ is strictly increasing in $c$ wherever $\lambda_U > 0$.

(iii) Micro and macro price efficiency are decreasing in $c$.

As $c$ increases and the number of uninformed grows, the benefit to being informed, in the sense that $J_M/J_U$ and $J_S/J_U$ decrease, as shown Proposition 5.2. However, the micro-informed gain disproportionately more than the macro-informed, as will be further discussed Section 6.3. In order to maintain the attention equilibrium at a higher $c$, we need more micro-informed to equilibrate the relative benefits of micro versus macro information. Therefore, $\Pi_M$ must fall when $c$ increases.

6.3 Are markets micro efficient?

A tendency for markets to be more micro than macro efficient has been a recurring theme of many of our results. For example:

- From Propositions 5.3 and 6.1 we get $\lambda^*_S > 0$ — there are always some micro-informed investors. It is, in fact, possible that there are only micro-informed investors, so $\lambda^*_M = 0$. This occurs when information is too costly to satisfy (e3).

- More generally, Corollary 6.1 shows that when information is costlier, a larger fraction of the informed investors choose micro information, and from Proposition 6.1 we know that at a sufficiently high cost of becoming informed, all informed investors choose to be micro-informed.

- From Proposition 5.4 we see that when investors in the economy become more risk averse, there will be more micro-informed.

- From Proposition 5.8 we see that even when the macro signal is more informative (i.e. $f_M > f_S$ and thus $\tau_M > 1$), markets will be micro efficient for a sufficiently
large $\lambda_U$. When non-public information is scarcer, markets tend towards micro efficiency.

A key driver of these results is that the micro-informed are the only investors who can collect surplus from accommodating idiosyncratic supply shocks. This creates a strong incentive for collecting micro information. We see from (34) that the benefit $J_M/J_U$ to being the first macro-informed investor is finite, whereas the benefit to being the first micro-informed is infinite: $J_S/J_U$ takes on its highest possible value — zero — at $\lambda_S = 0$.

7 Applications

7.1 Systematic and idiosyncratic volatility

We define “excess returns” on stock $i$ as $u_i - R P_i$. We decompose this into the systematic return component $(M - R P_F)$ and an idiosyncratic return component given by

$$u_i - R P_i - (M - R P_F) = \frac{\epsilon_i}{\text{Unknowable portion of dividend}} + \frac{\gamma(1 - f_S)\sigma^2_S X_i / \lambda_S}{\text{Adjustment due to idiosyncratic supply shock}}.$$  

(49)

The idiosyncratic return variance can be written as

$$\text{Vol}^2_{idio} = \sigma^2_{\epsilon_S} \left(1 + \frac{\gamma^2 \sigma_{\epsilon_S}^2 \sigma^2_X}{\lambda^2_S} \right) = \sigma^2_{\epsilon_S} \left(1 + \frac{f_S}{1 - f_S} \left[ \frac{1}{\rho^2_S} - 1 \right] \right).$$  

(50)

where the second equation follows from (30). As can be seen from (49), idiosyncratic return volatility consists of two components. The first comes from the unknowable portion of the idiosyncratic dividend, $\epsilon_i$, and the second comes from price adjustment in response to idiosyncratic supply shocks. The knowable portion of the dividend payout $s_i$ does not enter into the idiosyncratic return because of the $s_i/R$ term in $P_i$.

From (50) and the fact that $\sigma^2_{\epsilon_S} = (1 - f_S)\sigma^2_S$ we make two observations about idiosyncratic return volatility. As $f_S$ increases and more of the idiosyncratic portion of the dividend $S_i$ becomes knowable, idiosyncratic return volatility falls (holding all else equal). With an increase in the fraction $\lambda_S$ of micro-informed investors, prices become

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17 In light of Proposition 6.1 we can interpret each feasible point in Figure 4 as representing an information equilibrium for some cost $c$ of becoming informed. Therefore as $c$ increases, $\lambda_U$ increases and we move from right to left along the curve in Figure 4.

18 Since prices can be negative in our model, the return concept is the dollar gain from an investment, rather than a percent return.
less sensitive to idiosyncratic supply shocks, and return volatility again falls.

The systematic variance of stock returns (or equivalently, the variance of index fund returns) follows from (25) and is given by

\[ Vol_{syst}^2 \equiv \text{var}[M - RP_F] = \sigma_m^2 + (1 - Rb_F)^2 \sigma_m^2 + R^2 \epsilon^2 \sigma_X^2. \]  

(51)

We can evaluate (51) numerically. Figure 6 illustrates how systematic volatility changes with \( \Pi_M \), the fraction of informed that are macro-informed, for different levels of precision \( f_M \). Systematic volatility shows the same qualitative dependence on \( f_M \) and \( \lambda_M \) (decreasing in both) as idiosyncratic volatility has on \( f_S \) and \( \lambda_S \). Holding all else equal, it follows that when \( \lambda_M \) and \( \lambda_S \) change, \( Vol_{idio} \) and \( Vol_{syst} \) move in opposite directions.

Figure 6: This figure shows systematic volatility for different sets of macro signal precisions \( f_M \) as a function of the fraction of informed who are macro-informed \( \Pi_M \). The horizontal dashed line shows the volatility of the fundamental portion of the dividend \( \sigma_M \). The shaded area above this line indicates the region of excess covariance.

The relationship in (50) sheds some light on the finding in Bekaert, Hodrick and
Zhang (2012) (at the country level) and in Herskovic et al. (2016) (for US stocks) that idiosyncratic volatility has a strong common component. In a dynamic version of our economy \( f_S \) acts as a state variable, whose value is affected by shocks (technological or regulatory) to the knowable portion of micro uncertainty. In an economy with a high \( f_S \), more information about corporate cash flows will be reflected in prices, investors will demand less compensation for idiosyncratic supply shocks (since prices are more “accurate”), and idiosyncratic return volatility will fall. This will also happen when the cost of becoming informed falls, thereby increasing \( \lambda_S \). Since these dynamics affect all stocks, variation in idiosyncratic volatility induced via this channel should have a strong common component. This channel is quite different from that discussed by Herskovic et al. (2016), who show that idiosyncratic return volatility is related to the volatility of firm-level cash flows, which proxy for non-diversifiable risk faced by households. Our explanation operates not through cash flows but through how much market participants know about cash flows.

### 7.2 Excess volatility and comovement

It has long been recognized that the majority of stock return variability cannot be explained by information about future dividends (LeRoy and Porter 1981 and Shiller 1981 are the classic papers in the area; see also Campbell 1991 and Campbell and Ammer 1993). Later empirical work has documented the fact that stocks also exhibit a higher covariance than is justified by their cash flows (for example, Pindyck and Rotemberg 1993 and Barberis, Shleifer and Wurgler 2005). We show in this section that the excess volatility and excess comovement phenomena are closely related, and can both be explained, at least partially, by the relative micro versus macro efficiency of markets.

In our model, assuming \( N \) is large enough to make \( (2) \) negligible, the covariance between the dividend of stocks \( i \) and \( j \) is given by \( \text{cov}(u_i, u_j) = \sigma_M^2 \). The covariance between excess returns of \( i \) and \( j \) is given by \( \text{cov}(u_i - R_{P_i}, u_j - R_{P_j}) = Vol_{syst}^2 \). If we define excess comovement as a higher covariance of returns than the covariance of dividends, then this reduces to

\[
Vol_{syst}^2 > \sigma_M^2 \iff \text{Excess comovement.}
\]

Return comovement exceeds earnings comovement when index fund returns are more volatile than index fund earnings — excess macro volatility leads to excess covariance.

In the shaded region of Figure 6, \( Vol_{syst} \) is higher than \( \sigma_M \), which is indicated by the
horizontal dashed line.\footnote{We see that when macro signal precision \( f_M \) is high or when the fraction of macro-informed \( \Pi_M \) is large, return volatility is lower than fundamental volatility — we have “insufficient” covariance. This happens because prices reveal so much information about \( M \) that \( u_i - R P_i \) becomes relatively insensitive to \( M \).

Excess volatility and therefore excess covariance arise when the macro signal is very imprecise (\( f_M \) is low) or when there are relatively few macro-informed investors (\( \Pi_M \) is small). The tendency of actual stocks to exhibit excess covariance and excess volatility, as empirical work suggests, lends more evidence to the micro efficiency (and macro inefficiency) of markets.

Peng and Xiong (2006) show that excess comovement can arise in a behavioral representative investor model when investors are sufficiently overconfident in the quality of their information. In Veldkamp (2006) and Mondria (2010), excess comovement results when investors tend to learn the same information as other investors. In Huang and Zeng (2015), firm investment decisions amplify comovements as managers respond to each other’s asset prices. Our model identifies a new and very broad channel of excess comovement — the relative micro versus macro efficiency of the market.

Peng and Xiong (2006) take excess comovement to mean return correlation higher than dividend correlation, which leads to the following characterization in our model:\footnote{Though the shaded excess covariance region is small in the figure to show the full range on systematic volatility behavior, our view is that this is the region most representative of actual markets.}

\[
\frac{\text{Vol}_{\text{syst}}}{\sigma_M} > \frac{\text{Vol}_{\text{idio}}}{\sigma_S} \iff \text{Excess comovement.}
\] (52)

This condition yields the same intuition as the covariance case. Relative micro efficiency (low \( \text{Vol}_{\text{idio}} \) relative to \( \sigma_S \), thus higher \( \rho_S^2 \) in (50)) is associated with excess comovement, as is relative macro inefficiency (high \( \text{Vol}_{\text{syst}} \) relative to \( \sigma_M \)).

Equation (52) can be interpreted in terms of the finding in Vuolteenaho (2002) that firm level market adjusted returns (his analog to \( u_i - R P_i - (M - R P_F) \)) are driven predominantly by cash flow news (Table III in his paper) and that market excess returns (i.e., \( M - R P_F \)) are more driven by discount rate — and not cash flow — news (his Table VII). Since \( \sigma_M \) and \( \sigma_S \) proxy for cash flow news in our model, Vuolteenaho’s results suggest that the ratio on the left hand side of (52) is high, and the ratio on the right hand side is low. This is additional evidence of micro efficiency.

\footnote{Return correlations are given by \( \text{Vol}_{\text{syst}}^2/(\text{Vol}_{\text{syst}}^2 + \text{Vol}_{\text{idio}}^2) \) and cash flow correlations are given by \( \sigma_M^2/(\sigma_M^2 + \sigma_S^2) \). Rearranging and taking square roots and yields (52).}
7.3 Information choices in recessions

Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) study how investors allocate limited attention to macro and micro uncertainty. In their model, as in ours, each stock dividend depends on a macro shock and an idiosyncratic shock. While our work investigates the specialization choices of investors into micro- and macro-informed, they analyze how informed investors, who all receive signals from the same distribution, would choose to allocate the precision of those signals between micro and macro information. The two papers analyze related problems, but from two different, and complementary, vantage points. Therefore contrasting the mechanisms at work in the two theories should be informative for our understanding of the functioning of actual markets.

The main theoretical results in Kacperczyk et al. (2016) follow from their Proposition 1, which states that the informed investors allocate more attention to securities with greater cash flow variance, and their Proposition 2, which states that informed investors allocate more attention to the macro component of cash flows ($M$ in our model) as risk aversion increases. A recession in their model is a time of increased risk aversion and increased macro dividend volatility, $\sigma_M$, which allows them to conclude in Section 2 that

“...in recessions, the average amount of attention devoted to aggregate shocks should increase and the average amount of attention devoted to stock-specific shocks should decrease.”

The corresponding question in our model can be posed as follows: How does $\lambda^*_M$ depend on risk aversion $\gamma$ and dividend volatility (given by $\sigma_M$ and $\sigma_S$)? From Proposition 5.4 we know that (1) increasing risk aversion pushes investors towards micro information, (2) increasing macro dividend risk $\sigma_M$ pushes investors towards macro information, and (3) increasing idiosyncratic dividend risk $\sigma_S$ pushes investors to micro information. Whereas in Kacperczyk et al. (2016) the risk aversion and macro dividend risk effects reinforce each other, in our model they offset. Furthermore if idiosyncratic dividend risk $\sigma_S$ were to increase in a recession, we show that this would push investors further towards micro information.\footnote{Kacperczyk et al. (2016) document that during recessions systematic return volatility increases, while idiosyncratic return volatility increases by much less (and the latter increase is not statistically different from zero). However, because $\sigma_M$ and $\sigma_S$ measure dividend rather than return uncertainty, their behavior in recessions may be different.}

Our models only make the same prediction for recessions characterized by a small increase in $\gamma$, negligible increase in $\sigma_S$, and a large increase in $\sigma_M$.

A key difference in the two models is the fact that increasing risk aversion, or increasing aggregate risk, diminishes one benefit of being macro-informed in our model (trade with
the macro-uninformed falls) but increases the benefit of liquidity provision to idiosyncratic noise trades by the micro-informed. This mechanism is not present in the model of Kacperczyk et al. (2016) and may account for the different predictions.

Kacperczyk et al. (2016) show that funds’ portfolio deviations from the market co-vary more with future changes in industrial production during recessions, and that these portfolio deviations co-vary more with future firm-specific earnings shocks during booms. They interpret this as evidence that funds are more macro-focused during recessions, and more micro-focused during booms. An alternative explanation for this observation is that funds perceive the market to be more micro-efficient during recessions and therefore tailor their portfolios to take advantage of macro opportunities, and similarly during booms funds perceive the market to be more macro-efficient and therefore focus their portfolios on micro opportunities. This latter interpretation would explain the finding in Kacperczyk et al. (2014) that mutual funds seem to be better stock pickers during booms, and better market timers during recessions. Furthermore it is consistent with the implications of our model that investors gravitate towards micro information in recessions characterized by an increase in risk aversion and an increase in both macro and micro dividend risks. Which of these interpretations is a better fit for the combined results of Kacperczyk et al. (2014) and Kacperczyk et al. (2016) is an important area for future work.

8 Conclusion

Professional investors typically specialize along a particular dimension of knowledge. This choice of specialization has important effects on the behavior of market prices. We model an economy where investors can choose from a wide range of signals, yet they choose to specialize in information about either micro or macro fundamentals. An important driver of this choice is the ability for micro-informed investors to accommodate, and thereby extract rents from, the idiosyncratic portion of security supply shocks. This opportunity arises because the micro-uninformed invest in the index fund rather than individual stocks. Our model sheds light on a longstanding question in financial economics — whether markets are better at incorporating information at the micro or macro level. Our framework supports a general tendency of markets towards micro rather than macro efficiency, consistent with Samuelson’s dictum.

If \( \sigma_M \) and \( \sigma_S \) both increase in the same proportion, therefore leaving their ratio and \( \varphi \) from (36) unchanged, then the same logic underlying the result in Proposition 5.4 that \( d\lambda_M^*/d\gamma < 0 \) implies that \( \lambda_M^* \) would fall.
A Appendix

A.1 Proof of Theorem 2.1

The proof of the theorem relies on three lemmas. To compare expected utility under alternative information choices, we will use the following representations, which follow directly from Proposition 3.1 of Admati and Pfleiderer (1987).

**Lemma A.1.** Let $\Psi^{(0,K)}$ denote the covariance matrix of $S_i - R P S_i$, $i = 1, \ldots, K$, and let

$$\Psi^{(1,K)} = \left( \begin{array}{cc} \text{var}[M - R P F] & 0 \\ 0 & \Psi^{(0,K)} \end{array} \right).$$

The squared expected utility of an informed investor who chooses information set $I^i$ is

$$J^2 = e^{2\gamma_c} \times \left\{ \begin{array}{ll} |\hat{\Sigma}^{(0,K)}|/|\Psi^{(0,K)}|, & \iota = 0; \\ \exp(-Q_F)|\hat{\Sigma}^{(i,K)}|/|\Psi^{(1,K)}|, & \iota = 1, 2, \end{array} \right. \quad (A.1)$$

where $Q_F = (E[M - R P F]^2)/\text{var}[M - R P F]$. For an uninformed investor who conditions on $P_F$ and invests only in the index fund, the squared expected utility is given by

$$J^2_U = e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M - R P F]}.$$

(A.2)

Investors can evaluate (A.1)–(A.2) to make their information choices without first observing signals. Combining (A.1) with (5) and (6), we get

$$J^2 \geq e^{2\gamma_c} \times \left\{ \begin{array}{ll} \kappa |\hat{\Sigma}^{(0,K)}|/|\Psi^{(0,K)}|, & \iota = 0; \\ \exp(-Q_F)(\kappa \text{var}[M|P_F]/\delta_F)|\hat{\Sigma}^{(0,K)}|/|\Psi^{(1,K)}|, & \iota = 1; \\ \exp(-Q_F)\kappa |\hat{\Sigma}^{(1,K)}|/|\Psi^{(1,K)}|, & \iota = 2. \end{array} \right. \quad (A.3)$$

For $\iota = 1$, the inequality follows from writing the ratio of determinants in (A.1) as

$$\frac{|\hat{\Sigma}^{(1,K)}|}{|\Psi^{(1,K)}|} = \frac{\text{var}[M|P_F] \cdot |\hat{\Sigma}^{(0,K)}|}{|\Psi^{(1,K)}|},$$

and then applying (6). The expressions in (A.3) hold as equalities when an investor uses the full capacity $\kappa$, which is always possible if $K \geq 1$ or $\iota = 2$, so we will assume this condition holds. Interpret the case $\iota = 0$, $K = 0$ as the option not to invest, in which case the agent effectively consumes the acquired capacity.

The following lemma evaluates the determinants in (A.3).

**Lemma A.2.** For any $K = 1, \ldots, N - 1$,

$$\frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|} = \left( \frac{\text{var}[S_i]}{\text{var}[S_i - R P S_i]} \right)^K.$$

(A.4)
Also,
\[
|\Sigma^{(1,K)}| = \text{var}[M] \cdot |\Sigma^{(0,K)}| \quad \text{and} \quad |\Psi^{(1,K)}| = \text{var}[M - RP_F] \cdot |\Psi^{(0,K)}|.
\]  
(A.5)

**Proof.** Let \(G_K\) be the \(K \times K\) matrix with all diagonal entries equal to 1 and all off-diagonal entries equal to \(-1/(N - 1)\). It follows from (2) that \(\Sigma^{(0,K)} = \sigma^2_S G_K\). In light of (4),
\[
\sum_{i=1}^{N} (S_i - R P_{S_i}) = \sum_{i=1}^{N} S_i - R \sum_{i=1}^{N} (P_i - P_F) = 0,
\]
Under (e1), it follows that, for \(i \neq j\),
\[
\text{cov}[S_i - R P_{S_i}, S_j - R P_{S_j}] = -\text{var}[S_i - R P_{S_i}]/(N - 1),
\]
so the \(S_i - R P_{S_i}\) have the same correlation structure as the \(S_i\) themselves. In other words, \(\Psi^{(0,K)} = \text{var}[S_i - R P_{S_i}] G_K\), and then
\[
\frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|} = \frac{\sigma^2_S G_K}{\text{var}[S_i - R P_{S_i}] G_K} = \frac{\sigma^2_S |G_K|}{(\text{var}[S_i - R P_{S_i}])^K |G_K|},
\]
which yields (A.4). The block structure of \(\Sigma^{(1,K)}\) and \(\Psi^{(1,K)}\) yields (A.5).

**Lemma A.3.** Under the conditions of Theorem 2.1, we have
\[
\frac{\text{var}[M | P_F]}{\text{var}[M]} < \delta_F \quad \text{and} \quad \frac{\text{var}[S_i]}{\text{var}[S_i - R P_{S_i}]} > 1, \quad i = 1, \ldots, N,
\]  
(A.6)
and
\[
e^{-Q_F} \frac{\text{var}[M]}{\text{var}[M - RP_F]} \leq 1.
\]  
(A.7)

The first inequality confirms that making inferences from the price of the index fund consumes less information processing capacity than would be expected from the variance reduction achieved. The reverse inequality would imply a “penalty” in conditioning on the index fund price, a scenario we see as uninteresting and rule out with the conditions in the theorem. The second inequality in (A.6) suggests that individual stock prices are, in a sense, sufficiently informative about fundamentals and not overwhelmed by noise trading. This second condition ensures that being micro informed is not strictly preferable to being uninformed — it is effectively a limit on the benefit of actively trading in individual stocks. As we will see in the proof of Theorem 2.1 this result also makes it suboptimal to have more than one stock in an investor’s consideration set. The inequality in (A.7) confirms that there is a benefit to investing in the index fund.

**Proof of Lemma A.3.** Under condition (e4), an uninformed investor in equilibrium weakly prefers investing in the index fund over becoming informed — in particular, over becoming
We therefore have

\[ J_U^2 \leq e^{2\gamma c} e^{-Q_F} \frac{\text{var}[M|m, P_F]}{\text{var}[M - RP_F]} \equiv J_M^2, \tag{A.8} \]

where \( m = m(f_M) \) is the signal in (8) acquired by an informed investor who allocates all capacity to learning about \( M \). Combining this inequality with (A.2), recalling from (10) that \( \text{var}[M|m, P_F]/\text{var}[M] = \kappa \), we get

\[ \frac{\text{var}[M|P_F]}{\text{var}[M]} \leq e^{2\gamma c} \kappa < \delta_F. \]

We can similarly compare \( J_U^2 \) with the option of becoming micro-informed to get

\[ J_U^2 \leq e^{2\gamma c} e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M - RP_F]} \frac{\text{var}[S_i|s_i, P_{Si}]}{\text{var}[S_i - RP_{Si}]} \equiv J_S^2, \tag{A.9} \]

where \( s_i = s_i(f_s) \) is the signal in (9) acquired by an informed investor who allocates all capacity to \( \{P_F, (s_i, P_{Si})\} \). Recalling from (10) that \( \delta_F \cdot \text{var}[S_i|s_i, P_{Si}]/\text{var}[S_i] = \kappa \), this inequality reduces to

\[ 1 \leq e^{2\gamma c} (\kappa/\delta_F) \frac{\text{var}[S_i]}{\text{var}[S_i - RP_{Si}]} \cdot \]

As the first factor on the right is less than 1, the second inequality in (A.6) must hold.

Suppose no informed investors choose to learn more about \( M \) than its price, meaning that no investor chooses an information set of the type \( \mathcal{I}_K^{(2)} \). Then (e2) implies \( \text{var}[M|P_F] = \text{var}[M] \), which would contradict (A.6) because \( \delta_F < 1 \). It follows that some informed investor weakly prefers an information set \( \mathcal{I}_K^{(2)} \) over an information \( \mathcal{I}_K^{(0)} \) consisting of more precise signals about the same stocks and no information about \( M \). This preference implies

\[ e^{2\gamma c} e^{-Q_F} \frac{\text{var}[M]}{\text{var}[M - RP_F]} \left( \frac{\text{var}[S_i]}{\text{var}[S_i - RP_{Si}]} \right)^K \kappa \leq e^{2\gamma c} \left( \frac{\text{var}[S_i]}{\text{var}[S_i - RP_{Si}]} \right)^K \kappa, \]

from which (A.7) follows.

**Proof of Theorem 2.1.** Now consider the three cases in (A.3). We need to show that for an informed investor, information choices other than \( \{m\} \) or \( \{s_i, P_{Si}\} \) are suboptimal.

**Case of \( \iota = 2 \).** We may take \( K \geq 1 \), since otherwise only the index fund is in the consideration set, in which case the investor cannot do better than \( \{m\} \) because of the
Combining Lemma A.2 and Lemma A.3, we get

\[ J^2 \geq e^{2\gamma c} \exp(-Q_F) \kappa \left| \frac{\sum^{(1,K)}}{\varPsi^{(1,K)}} \right|, \quad \text{using (A.3)}; \]

\[ = e^{2\gamma c} \exp(-Q_F) \kappa \left| \frac{\big| \sum^{(0,K)} \big| \cdot \var[M]}{\varPsi^{(0,K)} \cdot \var[M - RP_F]} \right|, \quad \text{using (A.5)}; \]

\[ = e^{2\gamma c} \exp(-Q_F) \left( \frac{\var[M | m, P_F]}{\var[M - RP_F]} \right) \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right|, \quad \text{using the first equality in (10)}; \]

\[ = J^2_M \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right|, \quad \text{using (A.8)}. \]

Combining Lemma A.2 and Lemma A.3, we get \( |\sum^{(0,K)}|/|\varPsi^{(0,K)}| > 1 \), from which we conclude that \( J^2 > J^2_M \).

**Case of \( \iota = 1 \).** Start with \( K = 0 \). We have \( \var[S_i | s_i, P_{S_i}] = \var[S_i - RP_{S_i} | s_i, P_{S_i}] < \var[S_i - RP_{S_i}] \), and therefore from (A.1)

\[ J^2 = e^{2\gamma c} e^{-Q_F} \frac{\var[M | P_F]}{\var[M - RP_F]} > e^{2\gamma c} e^{-Q_F} \frac{\var[M | P_F]}{\var[M - RP_F]} \frac{\var[S_i | s_i, P_{S_i}]}{\var[S_i - RP_{S_i}]} = J^2_S \]

showing that \( I^{(1)}_1 \) is preferred over \( I^{(1)}_0 \). With \( K \geq 2 \), the squared expected utility is

\[ J^2 \geq e^{2\gamma c} \exp(-Q_F) \kappa \frac{\var[M | P_F] \cdot |\sum^{(0,K)}|}{\delta_F \varPsi^{(1,K)}}, \quad \text{using (A.3)}; \]

\[ = e^{2\gamma c} \exp(-Q_F) \frac{\delta_F \var[S_i | s_i, P_{S_i}]}{\var[S_i]} \left( \frac{\var[M | P_F]}{\var[M - RP_F]} \right) \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right|, \quad \text{using (10) and (A.5)}; \]

\[ = e^{2\gamma c} \exp(-Q_F) \frac{\var[S_i | s_i, P_{S_i}]}{\var[S_i - RP_{S_i}]} \left( \frac{\var[M | P_F]}{\var[M - RP_F]} \right) \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \frac{\var[S_i - RP_{S_i}]}{\var[S_i]} \right| \]

\[ = J^2_S \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right| \frac{\var[S_i - RP_{S_i}]}{\var[S_i]}. \]

With \( K \geq 2 \), multiplying (A.4) by \( \var[S_i - RP_{S_i}]/\var[S_i] \) continues to yield an expression that is greater than 1. It follows that \( J^2 > J^2_S \).

**Case of \( \iota = 0 \).** A nonempty information set requires \( K \geq 1 \), and then

\[ J^2 \geq e^{2\gamma c} \kappa \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right|, \quad \text{using (A.3)}; \]

\[ \geq e^{2\gamma c} e^{-Q_F} \frac{\var[M]}{\var[M - RP_F]} \kappa \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right|, \quad \text{using (A.7)}; \]

\[ = e^{2\gamma c} e^{-Q_F} \frac{\var[M | m, P_F]}{\var[M - RP_F]} \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right|, \quad \text{using (10)}; \]

\[ = J^2_M \left| \frac{\sum^{(0,K)}}{\varPsi^{(0,K)}} \right| > J^2_M, \quad \text{using (A.8) and (A.4)}. \]
No-deviation. We have thus shown that no macro-informed or micro-informed investor would prefer a different information choice. If the proportion of macro-informed or micro-informed investors were zero, \((e2)\) would contradict one of the inequalities in \((A.6)\). The last assertion in the theorem follows.

A.2 Solution of the constrained model

Proof of Proposition 4.1 The analysis is simplified if we allow micro-informed agents to invest in the index fund and in a hedged security paying \(u_i - u_F = S_i\), with price \(P_{S_i} = P_i - P_F\). If we let \(\tilde{q}_F^i\) and \(\tilde{q}_{S_i}^i\) denote the demands in this case, the demands in the original securities are given by \(q_i^F = \tilde{q}_F^i - \beta_i \tilde{q}_{S_i}^i\). Write \(I_i = \{P_F, P_i, s_i\}\). By standard arguments, the modified demands are given by

\[
\begin{bmatrix}
\tilde{q}_F^i \\
\tilde{q}_{S_i}^i
\end{bmatrix} = \frac{1}{\gamma \text{var} \left[ \frac{M}{S_i} | I_i \right]} \left( \text{E} \left[ \frac{M}{S_i} | I_i \right] - R \left( \frac{P_F}{P_i - P_F} \right) \right).
\]

Now

\[
\text{var} \left[ \frac{M}{S_i} | I_i \right] = \left( \text{var}[M|I_i] \right) \left( \text{var}[S_i|I_i] \right) = \left( \text{var}[M|P_F] \right) (1 - f_S) \sigma_S^2 \tag{A.10}
\]

and

\[
\text{E} \left[ \frac{M}{S_i} | I_i \right] = \left[ \text{E}[M|P_F] \right] \tag{s_i}.
\]

Thus, \(\tilde{q}_F^i = q_F^U\), with \(q_F^U\) as given in \((20)\), and

\[
\tilde{q}_{S_i}^i = \frac{s_i - R(P_i - P_F)}{\gamma(1 - f_S) \sigma_S^2}.
\]

As \(q_i^F = \tilde{q}_{S_i}^i\), \((22)\) follows, and then \(q_i^F = \tilde{q}_F^i - \tilde{q}_{S_i}^i = q_F^U - q_i^i\) completes the proof.

Proof of Proposition 4.2 The price \(P_F\) can be derived from first principles, but we can simplify the derivation by reducing it to the setting of Grossman and Stiglitz (1980). The informed \((19)\) and uninformed \((20)\) demands for the index fund and the market clearing condition \((24)\) reduce to the demands in equations (8) and (8') of Grossman and Stiglitz (1980) and their market clearing condition (9), once we take \(\lambda = \lambda_M\) and \(1 - \lambda_M = \lambda_U + \lambda_S\). The coefficients of the price \(P_F\) in \((17)\) can therefore be deduced from the price in their equation (A10). Theorem 1 of Grossman-Stiglitz gives an expression for \(P_F\) in the form \(\alpha_1 + \alpha_2 w_\lambda\), for constants \(\alpha_1\) and \(\alpha_2 > 0\), where, in our notation,

\[
w_\lambda = m - \frac{\gamma(1 - f_M) \sigma_M^2}{\lambda_M} (X_F - X_F).
\]

Comparison with \((17)\) yields the expression for \(c_F/b_F\) in \((25)\). From the coefficient of their
\( \theta \) (our \( m \)) in (A10) of Grossman-Stiglitz, we get
\[
b_F = \frac{1 - \lambda_M}{R} \frac{\lambda_M}{(1 - \lambda_M)\sigma_M^2} \variance{M}{w} + \frac{1 - \lambda_M}{\variance{M}{w} \variance{M}{w}} \frac{f_M \sigma_M^2}{\lambda_M} \frac{1}{R} \frac{\lambda_M}{(1 - \lambda_M)\sigma_M^2} + \frac{1}{\variance{M}{w}} \lambda_M \variance{M}{w} | \mathcal{P} \mathcal{F} \]  \hspace{1cm} (A.11)

Moreover,
\[
\variance{w}{\lambda} = (1 - f_M) \sigma_M^2 + \frac{\gamma^2 (1 - f_M)^2 \sigma_M^2}{\lambda_M} \sigma_X^2,
\]
and \( \variance{M}{w} = \variance{M}{P_F} \). To evaluate \( \variance{M}{P_F} \), note that the only unknown term in (21) is \( K_F b_F \), which we can now evaluate using (25) to get
\[
K_F b_F = \frac{b_F^2 f_M \sigma_M^2}{b_F f_M \sigma_M^2 + c_F \sigma_X^2} = \frac{f_M \sigma_M^2}{f_M \sigma_M^2 + \frac{\gamma^2 (1 - f_M)^2 \sigma_M^2}{\lambda_M} \sigma_X^2}.
\]

This yields an explicit expression for \( \variance{M}{P_F} \) which in turn yields an explicit expression for \( b_F \) through (A.11). An expression for \( c_F \) then follows using (25). Finally, to evaluate the constant term \( a_F \), we can again match coefficients with the expression in (A10) of Grossman-Stiglitz. Alternatively, we can evaluate their (A10) at (using their notation) \( \theta = E \theta^* \) and \( x = E x^* \), which, in our notation yields
\[
a_F = \frac{\bar{m}}{R} - \frac{\bar{X}_F}{R} \left[ \frac{1 - \lambda_M}{\gamma \variance{M}{P_F}} + \frac{\lambda_M}{\gamma (1 - f_M) \sigma_M^2} \right]^{-1}.
\]  \hspace{1cm} (A.12)

Equation (26) follows directly from (22) and (16). In light of (11) and (13), condition (4) is satisfied if and only if \( \xi = 0 \). \( \square \)

### A.3 Attention equilibrium

**Proof of Proposition 5.1.** We use the expressions for \( J_U, J_M, \) and \( J_S \) introduced in (A.2), (A.8), and (A.9), recalling that expected utility is negative. With prices given by Proposition 4.2 \( \variance{M}{P_F, m} = \variance{M}{m} = (1 - f_M) \sigma_M^2 \), so
\[
J_M/J_U = e^{\gamma c} \left( \frac{\variance{M}{P_F}}{(1 - f_M) \sigma_M^2} \right)^{-1/2}.
\]
Combining (21) and (28) yields
\[
\variance{M}{P_F} = f_M \sigma_M^2 (1 - \rho_F^2) + (1 - f_M) \sigma_M^2,
\]
from which (31) follows. Similarly, \( \variance{S_i}{P_{S_i}, s_i} = \variance{S_i}{s_i} = (1 - f_S) \sigma_S^2 \), so
\[
J_S/J_U = e^{\gamma c} \left( \frac{\variance{S_i}{P_{S_i}}}{(1 - f_S) \sigma_S^2} \right)^{-1/2}.
\]
Using first (26) and then (30), we get
\[
\text{var}[S_i - RP_{S_i}] = (1 - f_S)\sigma^2_S + \frac{\gamma^2(1 - f_S)^2\sigma^4_S}{\lambda^2_S}\sigma^2_X
\]
\[
= (1 - f_S)\sigma^2_S + f_s\sigma^2_S \left(\frac{1}{\rho^2_S} - 1\right),
\]
from which (32) follows.

\[\square\]

Proof of Proposition 5.3. As noted in (34), \(J_S/J_U\) approaches zero as \(\lambda_S\) decreases to zero (and \(\lambda_M\) increases to \(1 - \lambda_U\)). We know from (31) that \(J_M/J_U > 0\) for all \(\lambda_M\); in fact, from (34) we know that \(J_M/J_U \geq \sqrt{1 - f_M}\). It follows from the strict monotonicity of \(J_M/J_U\) and \(J_S/J_U\) (Proposition 4.3) that either \(J_M/J_U > J_S/J_U\) for all \(\lambda_M \in [0, 1 - \lambda_U]\) or the two curves cross at exactly one \(\lambda_M\) in \([0, 1 - \lambda_U]\). In the first case, all informed agents prefer to be micro-informed than macro-informed, so the only equilibrium is \(\lambda^*_M = 0\).

In the second case, the unique point of intersection defines the equilibrium proportion \(\lambda^*_M\), as explained in the discussion of Figure 1. We therefore examine at which \(\lambda_M\) (if any) we have \(J_M/J_U = J_S/J_U\). We can equate (31) and (32) by setting
\[
\frac{1 - f_M}{f_M} \left(\frac{1}{1 - \rho^2_F}\right) = \frac{1 - f_S}{f_S} \frac{\rho^2_S}{1 - \rho^2_S}.
\]
Using the expressions for \(\rho^2_F\) and \(\rho^2_S\) in (29) and (30), this equation becomes
\[
\frac{1 - f_M}{f_M} + \frac{\lambda^2_M}{\gamma^2(1 - f_M)\sigma^2_M\sigma^2_X} = \frac{(1 - \lambda_U - \lambda_M)^2}{\gamma^2(1 - f_S)\sigma^2_S\sigma^2_X}.
\]
Thus, \(\lambda_M\) satisfies a quadratic equation, which, with some algebraic simplification, can be put in the form \(A\lambda^2_M + B\lambda_M + C = 0\), where
\[
A = 1 - \varphi, \quad B = -2(1 - \lambda_U), \quad C = (1 - \lambda_U)^2 - \alpha\gamma^2,
\]
with \(\varphi\) and \(\alpha\) as defined in (36). One of the two roots of this equation is given by \(\tilde{\lambda}_M\). Denote the other root by
\[
\eta = \frac{-B + \sqrt{B^2 - 4AC}}{2A}.
\]
We claim that \(\eta \not\in [0, 1 - \lambda_U]\). We may assume \(A \neq 0\), because \(\eta \rightarrow \infty\) as \(A \rightarrow 0\) because \(B < 0\). If \(A < 0\) then either \(\eta\) is complex or \(\eta < 0\), again because \(B < 0\). If \(A > 0\), then \(A < 1\) because \(\varphi > 0\). Then if \(\eta\) is real, it satisfies \(\eta \geq -B/2A > -B/2 = 1 - \lambda_U\).

Combining these observations, we conclude that either \(\tilde{\lambda}_M \in [0, 1 - \lambda_U]\) and the information equilibrium has \(\lambda^*_M = \tilde{\lambda}_M\), or else the equilibrium occurs at \(\lambda^*_M = 0\).

\[\square\]
Proof of Proposition 5.4. Differentiation of $\tilde{\lambda}_M$ with respect to \(\gamma\) yields

\[
\frac{d\tilde{\lambda}_M}{d\gamma} = -\frac{1}{\alpha \sqrt{\varphi (1 - \lambda_U)}} \left[ 1 + \frac{1 - \varphi}{\varphi (1 - \lambda_U)^2} \right]^{-1/2}.
\]

At an interior equilibrium, $\tilde{\lambda}_M$ is real, so the expression on the right is real and negative.

At an interior equilibrium, $\lambda_M$ is the solution to $A\lambda^2 + B\lambda + C = 0$, with the coefficients given by (A.13). Differentiating with respect to some parameter (e.g., $\sigma_S \sigma_X$) yields $\dot{A}\lambda^2 + B\dot{\lambda} + \dot{C} = 0$ (note $\dot{B} = 0$ because $\lambda_U$ is fixed). Solving for $\dot{\lambda}$ yields

\[
\dot{\lambda} = -\frac{\dot{A}\lambda^2 + \dot{C}}{2A\lambda + B}.
\]

We note that $2A\lambda + B < 0$ can be rewritten as $1 - \frac{\phi}{\phi_S \lambda_S} < 0$ which is always true because $\phi > 0$ and $\lambda_M < 1 - \lambda_U$ since $\lambda_S > 0$. Therefore, sgn($\dot{\lambda}$) = sgn($\dot{A}\lambda^2 + \dot{C}$). Differentiating with respect to $\sigma_S \sigma_X$ yields $\dot{A} < 0$ and $\dot{C} < 0$, which implies $\dot{\lambda} < 0$; differentiating with respect to $\sigma_M \sigma_X F$ yields $\dot{A} > 0$ and $\dot{C} = 0$, which implies $\dot{\lambda} > 0$.

Proof of Proposition 5.5. Using the expression for $\rho^2_S$ in (30), we get

\[
\frac{f_S}{1 - f_S} \left( \frac{1}{\rho^2_S} - 1 \right) = \frac{\gamma^2 (1 - f_S) \sigma^2_S \sigma^2_X}{\lambda^2_S}.
\]

This expression is strictly decreasing in $f_S$, so $J_S / J_U$ in (32) is strictly increasing in $f_S$.

Proof of Proposition 5.6. We see from (31) that the derivative of $J_M / J_U$ is negative precisely if the derivative of

\[
\frac{1 - f_M}{f_M} \frac{1}{1 - \rho^2_F} = \frac{1}{\gamma^2 (1 - f_M) \sigma^2_M \sigma^2_{X_F} / \lambda^2_M} + \frac{1}{f_M} - 1
\]

is negative, using the expression for $\rho^2_F$ in (29). Differentiation yields

\[
\frac{1}{\gamma^2 (1 - f_M)^2 \sigma^2_M \sigma^2_{X_F} / \lambda^2_M} - \frac{1}{f^2_M} = \frac{1}{f_M} \left( \frac{1}{1 - \rho^2_F} - 1 \right) - \frac{1}{f^2_M},
\]

which is negative precisely if (38) holds. The equivalence of (39) follows from (29).

Proof of Proposition 5.8. From (40) we see that $\rho^2_S \geq \rho^2_F$ precisely when $\rho^2_F \leq 1 / \tau_M$. If $\tau_M \leq 1$, then we necessarily have $\rho^2_F \leq 1 / \tau_M$. But if $\tau_M > 1$, then markets are more macro efficient whenever $\rho^2_F > 1 / \tau_M$.

A.4 Information equilibrium

Proof of Proposition 6.1. We first show that (47) defines an information equilibrium at each $c > 0$, then verify uniqueness. For all three cases in (47), the specified $\lambda_M$, $\lambda_S$, and $\lambda_F$,
and \( \lambda_U \) are nonnegative and sum to 1, so it suffices to verify (41). For \( \zeta \leq c < \bar{c} \), we have \( J_M = J_S = J_U \) by construction, so the condition holds. For \( c \geq \bar{c} \), we again have \( J_S/J_U = 1 \) by construction. With \( \lambda_M = 0 \), we have \( \rho_M^2 = 0 \), and \( J_M/J_U \) in (31) evaluates to \( \exp(\gamma c)\sqrt{T - f_M} \geq \exp(\gamma \bar{c})\sqrt{T - f_M} = 1 \), so \( J_U/J_M \leq 1 \). Combining the two ratios we get \( J_S/J_M \leq 1 \). Thus, (41) holds.

For \( c < \zeta \), we consider two cases. First suppose case (i) of Proposition 5.3 holds at \( \zeta \). By definition, \( 1 - \lambda_M(\zeta) - \lambda_S(\zeta) = 0 \) and \( J_M/J_U = J_S/J_U = (\lambda_M(\zeta), \lambda_S(\zeta), 0) \), so

\[
\lambda_M(\zeta) = \lambda_M^*(0) \quad \text{and} \quad \lambda_S(\zeta) = 1 - \lambda_M^*(0),
\]

by the definition of \( \lambda_M^* \). Because \( \lambda_M(c) \) and \( \lambda_S(c) \) are strictly decreasing in \( c \), they are strictly greater than \( \lambda_M^*(0) \) and \( 1 - \lambda_M^*(0) \). Decreasing \( \lambda_M \) decreases \( \rho_M^2 \), which decreases \( J_M/J_U \) in (31), and decreasing \( \lambda_S \) similarly decreases \( J_S/J_U \). By construction, \( J_M/J_U = J_S/J_U = 1 \) at \( (\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c)) \), even for \( c < \zeta \), so at \( (\lambda_M^*(0), 1 - \lambda_M^*(0), 0) \) we have \( J_M/J_U < 1, J_S/J_U < 1 \), and \( J_M/J_S = 1 \), confirming (41).

Now suppose case (ii) of Proposition 5.3 holds at \( \zeta \); this includes the possibility that \( \bar{c} \leq \zeta \). Then \( \lambda_M^*(0) = \lambda_M(\zeta) = 0 \), and (17) specifies \( \lambda_M = 0 \) for all \( c < \zeta \). By the monotonicity argument used in case (i), \( J_S/J_U < 1 \) at all \( c < \zeta \). Moreover, Proposition 5.3(ii) entails \( J_S/J_M \leq 1 \), so this also holds for all \( c < \zeta \), and therefore (41) holds.

We now turn to uniqueness. At any \( c \), once we determine which proportions are strictly positive, the equilibrium is determined: if \( \lambda_U = 0 \), the other two proportions are determined by Proposition 5.3 if all three proportions are positive, they must satisfy \( J_M/J_U = J_S/J_U = 1 \) and must therefore be given by (44)–(45); if \( \lambda_M = 0 \) and \( \lambda_U > 0 \), the proportions are determined by the requirement that \( J_S/J_U = 1 \). We know from Proposition 5.3(iii) that \( \lambda_S > 0 \), so these are the only combinations we need to consider.

It therefore suffices to show that at any \( c \), the set of agents with positive proportions is uniquely determined. Suppose we try to introduce uninformed agents into an equilibrium from which they are absent. If we start with \( \lambda_M > 0 \) (and necessarily \( \lambda_S > 0 \)) then \( J_M/J_U \leq 1 \) and \( J_S/J_U \leq 1 \). Increasing \( \lambda_U \) requires decreasing either \( \lambda_M \) or \( \lambda_S \) and therefore decreasing either \( J_M/J_U \) or \( J_S/J_U \), precluding \( \lambda_U > 0 \), in light of (41). If \( \lambda_M = 0 \), the decrease must be in \( \lambda_S \) and the same argument applies. Suppose we try to introduce macro-informed agents into an equilibrium with only micro-informed and uninformed agents. The presence of uninformed agents requires \( J_M/J_U \geq 1 \). Increasing \( \lambda_M \) would increase \( J_M/J_U \), precluding \( \lambda_M > 0 \). Starting from an equilibrium with \( \lambda_S = 1 \) and increasing \( \lambda_M \) while leaving \( \lambda_U = 0 \) fixed is also infeasible because the value of \( \lambda_U \) determines the value of \( \lambda_M \) and \( \lambda_S \) through Proposition 5.3. \( \square \)

Proof of Corollary 6.1. (i) It suffices to consider the range \( \zeta \leq c \leq \bar{c} \) with \( \zeta < \bar{c} \), because \( \Pi_M \) is constant on \( (0, \zeta] \) and identically zero on \( [\bar{c}, \infty) \). It follows from (44) and (45) that

\[
\lambda_M^2(c) = \frac{\gamma^2(1 - f_S)^2}{f_S7M} \left( \frac{\lambda_M^2(c)f_M}{\gamma^2(1 - f_M)^2\sigma_M^2\sigma_X^2} + 1 \right) \equiv a\lambda_M^2(c) + b, \quad a, b > 0.
\]

Because \( \lambda_M(c) \) is strictly decreasing in \( c \), dividing both sides by \( \lambda_M^2(c) \) shows that \( \lambda_M^2(c)/\lambda_M^2(c) \) is strictly increasing in \( c \), hence \( \lambda_M(c)/\lambda_M(c) + \lambda_S(c) \) is strictly decreasing in \( c \). (ii) Follows from (17). (iii) We know from (29) and (30) that \( \rho_F^2 \) and \( \rho_S^2 \) are increasing in \( \lambda_M \) and
\[ \lambda_S, \text{ respectively, so monotonicity of price efficiency follows from monotonicity in } (47). \]

### A.5 Equilibrium without \( \delta_F \)

In this appendix, we briefly consider possible equilibrium behavior in the setting of Section 2 if we remove \( \delta_F \) and assume that the capacity consumed by conditioning on \( P_F \) is \( \text{var}[M]/\text{var}[P_F] \). We drop (e3) and (e4).

Once conditioning on \( P_F \) has no special status, it suffices to consider information sets \( I_K(0) \) and \( I_K(2) \), the latter now including \( I_K(1) \) as a special case. If the information set \( I_K(\iota) \) is feasible, it yields squared expected utility given by

\[
J^2 = e^{2\gamma c} \times \begin{cases} \frac{\kappa}{\text{var}[S]}/(\Psi(0,K)^{1/2}), & \iota = 0; \\ \kappa e^{-QF} \frac{\text{var}[M]}{\text{var}[S-RP_S]} \left( \frac{\text{var}[S] \text{var}[S-RP_S]}{\text{var}[S]} \right)^{K/2}, & \iota = 2, \end{cases}
\]

(A.14)

if informed investors use their full capacity \( \kappa \). Under (e1), these expressions become

\[
J^2 = e^{2\gamma c} \times \begin{cases} \frac{\kappa}{\text{var}[S]}/(\Psi(0,K)^{1/2}), & \iota = 0; \\ \kappa e^{-QF} \frac{\text{var}[M]}{\text{var}[S-RP_S]} \left( \frac{\text{var}[S] \text{var}[S-RP_S]}{\text{var}[S]} \right)^{K/2}, & \iota = 2, \end{cases}
\]

(A.15)

where we have written \( R_S = \text{var}[S]/\text{var}[S-RP_S] \) for the common value of \( \text{var}[S_i]/\text{var}[S_i-RP_{S_i}], \ i = 1, \ldots, N \), and \( R_M = e^{-QF} \text{var}[M]/\text{var}[M-RP_F] \). We discuss necessary conditions for equilibrium for various combinations of \( R_S \) and \( R_M \).

If \( R_S < 1 \), it is evident from (A.15) that investors want to hold as many stocks as possible and therefore learn as little about them as necessary to consume the full capacity \( \kappa \). This will make prices relatively uninformative, which is in turn consistent with \( R_S < 1 \). Such an equilibrium may therefore be feasible, but it would be an equilibrium with noisy prices and little production of information.

If \( R_S = 1 \) and \( R_M < 1 \), then an informed investor allocates all capacity to an information set \( I_K(2) \), and any such set yields \( J^2 = e^{2\gamma c} \kappa R_M \). If \( R_S = 1 \) and \( R_M > 1 \), the investor instead chooses an information set \( I_K(0) \), and any such set yields \( J^2 = e^{2\gamma c} \kappa R_M \). If \( R_M = R_S = 1 \), then all information sets yield the same expected utility, as long as the full capacity \( \kappa \) is consumed. This scenario is similar to the indifference result in Van Nieuwerburgh and Veldkamp (2010); in their partial equilibrium setting, prices are constant, and investors with exponential utility and a variance-ratio capacity constraint are indifferent to the choice of information set. In a general equilibrium setting, this appears to be a knife-edge case in which price variance offsets the covariance between prices and dividends to produce \( R_S = R_M = 1 \).

If prices of individual stocks are sufficiently informative relative to their variance, we expect to have \( R_S > 1 \); this is the outcome in the main body of the paper. If we had \( R_M < R_S \), then the optimal information set would be \( I_K(2) \), and investors would allocate all capacity to learning about \( M \). But then the prices \( P_S \) would be uninformative, contradicting \( R_S > 1 \), so this case appears to be incompatible with equilibrium. If \( R_M > R_S > 1 \), then the optimal information set would be \( I_K(0) \), and investors would allocate...
all capacity to learning about individual stocks. But then $P_F$ would be uninformative, contradicting $R_M > 1$, so this case is also incompatible with equilibrium.

These considerations leave open the case $R_M = R_S > 1$, which we see as the most interesting and relevant outcome. If such an equilibrium exists, it would have some investors choosing $I_0^{(2)}$ and some choosing $I_1^{(0)}$; in other words, some investors would invest only in the index fund, and others would invest only in a single stock. This outcome would be similar to the equilibrium considered in the main body of the paper. In the main body of the paper, a micro-informed investor holds a stock with macro exposure and can hedge the macro risk through a position in the index fund, whereas here the micro-informed invest solely in the idiosyncratic portion of a stock. This distinction is consistent with the interpretation of $\delta_F$ given in Section 4.1 as the information capacity consumed in trading the index fund to hedge part of the macro risk in a position in an individual stock.

References


