Investor Information Choice
with Macro and Micro Information

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Abstract

We study information and portfolio choices when securities’ dividends depend on an aggregate (macro) risk factor and idiosyncratic (micro) shocks, and when investors can acquire costly dividend information. We establish a general result that investors endogeneously specialize in either macro or micro information. We construct a specific model with this specialization and study equilibrium information choices and the informativeness of macro and micro prices. Our results favor Samuelson’s dictum: markets are more micro than macro efficient. We calibrate the model and show it reproduces important features of the decomposition of stock return variability into cash flow and discount rate variances.

Keywords: Information choice; asset pricing; price efficiency; attention

JEL Classification: G12, G14

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1 Introduction

Samuelson’s dictum, as discussed in Shiller (2000), is the hypothesis that the stock market is “micro efficient” but “macro inefficient.” More precisely, the dictum holds that the efficient markets hypothesis describes the pricing of individual stocks better than it describes the aggregate stock market. Jung and Shiller (2005) review and add to empirical evidence that supports the dictum, including evidence of macro inefficiency in Campbell and Shiller (1988) and evidence for somewhat greater micro efficiency in Vuolteenaho (2002) and Cohen et al. (2003).

Our goal is to understand conditions under which investor information choices lead markets to show greater micro efficiency than macro efficiency. Our analysis is driven by a simple yet important asymmetry under which it takes more effort to acquire and make inferences from information about individual stocks than to learn from the price level of the overall market. This asymmetry creates large incentives for speculators to gather micro-level information, which leads to micro efficiency. We do not impose this asymmetry as an assumption; we derive it from more primitive assumptions on the costs of acquiring and making inferences from information.

We develop these ideas through a multi-security generalization of the classical model of Grossman and Stiglitz (1980). Our market consists of a large number of individual stocks, each of which is exposed to a macro risk factor and an idiosyncratic risk. The macro risk factor is tradeable through an index fund that holds all the individual stocks and diversifies away their idiosyncratic risks.

We begin with a general formulation in which investors may choose to acquire information processing capacity at a cost. This capacity allows an investor to observe and make inferences from signals about fundamentals. Subject to their capacity constraint, informed investors may choose to learn about the macro risk factor, about the micro (idiosyncratic) risks of individual stocks, or any combination of the two. The capacity constraint limits the fraction of uncertainty about dividends an informed investor can remove from a collection of securities.

In formulating this capacity constraint, we differentiate the index fund from individual stocks, and this is the key asymmetry that drives our results. We posit that the capacity consumed in making inferences from the price of the index fund is fixed, irrespective of the informativeness of the price. This assumption is based on the view that the implications of the overall level of the stock market are widely discussed and accessible in way that does not apply to individual stocks. In particular, few individual stocks get the type of media attention routinely devoted to the overall market. In conditioning demand for the index fund on its
price, an investor allocates a fixed capacity to paying attention to this ambient information. We do not, however, make an assumption on whether this fixed capacity is larger or smaller than the capacity required to make inferences from information about individual stocks.

A second important feature of our model is that we do not assume that investors can effortlessly make inferences from the market prices of securities, even if prices are freely available. Instead, we treat the problem of learning from prices the same way we treat the problem of making inferences from more informative signals: reduction in dividend uncertainty uses information capacity, regardless of the source of this reduction. Moreover, we assume that to invest in a security investors must learn at least the price, though they may choose to acquire additional information. Consequently, the investors in our model cannot effortlessly track hundreds of stocks and rationally condition their demands for these stocks on their prices. Investors instead use their information processing capacity to select a set of securities about which to learn and in which to invest.

Our first main result then shows that investors endogeneously choose to specialize in either macro or micro information. Our investors are ex ante identical, and once they incur the cost of becoming informed they are free to choose general combinations of signals, yet in equilibrium they concentrate in two groups, macro-informed and micro-informed investors. The macro-informed use all their capacity to learn about the macro factor and invest only in the index fund; a micro-informed investor acquires a signal about a single stock and invests in that stock and the index fund; some investors choose to remain uninformed. This outcome — heterogeneous information choices among ex ante identical investors — contrasts with the related literature, as we explain later. In particular, it contrasts with the model of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), in which all informed investors may choose the same type of information, and all investors will focus on the macro factor if it carries significantly more risk than individual stocks.

Having demonstrated that specialization in macro and micro information is a general phenomenon in our framework, we construct a specific model by imposing this specialization as a constraint. In other words, under the conditions of our general result, specialization is a necessary property in equilibrium, and the constrained model demonstrates that such an equilibrium is in fact feasible.

The constrained model has three types of investors: uninformed, macro-informed, and micro-informed, as required by our general result. To solve the model, we first take the fractions of each type as given and solve for an explicit market equilibrium, assuming all agents have CARA preferences. Shares of individual stocks and the index fund are subject to exogenous supply shocks. The exogenous supply shocks themselves exhibit a factor structure.
A common component, reflecting the aggregate level of supply, affects the supply of shares for all firms. In addition, noise trading in individual stocks contributes an idiosyncratic component to the supply of each stock. These micro supply shocks have smaller variance than macro supply shocks; investor specialization provides an incentive to nevertheless acquire information about stock-specific risks. Supply shocks are not observable to investors; as a consequence, equilibrium prices are informative about, but not fully revealing of, the micro or macro information acquired by informed agents.

We then allow informed investors to choose between being micro-informed and macro-informed, and we characterize the equilibrium in which a marginal agent is indifferent between the two types of information. In practice, developing the skills needed to acquire and apply investment information takes time — years of education and experience. In the near term, these requirements leave the total fraction of informed investors relatively fixed. By contrast, we suppose that informed investors can move comparatively quickly and costlessly between being macro-informed or micro-informed by shifting their focus of attention. Kacperczyk, Van Nieuwerburgh, and Veldkamp (2014) show that successful fund managers turn their attention from stock picking (micro information) during booms to market timing (macro information) during recessions. Endogenizing this focus in our model gives rise to an attention equilibrium centered on the choice between macro and micro information.

Over a longer horizon, agents choose whether to gain the skills to become informed, as well as the type of information to acquire. We therefore study an information equilibrium that endogenizes both decisions to determine equilibrium proportions of macro-informed, micro-informed, and uninformed investors. An information equilibrium in the constrained model delivers an explicit case of the necessary specialization established in our general formulation: the investors in this model would not prefer to deviate from their specialization and select other combinations of signals that consume the same capacity.

Working with the constrained model, we find a recurring asymmetry between micro and macro information. For example, we show that the information equilibrium sometimes has no macro-informed agents, but some fraction of agents will always choose to be micro-informed. We show that increasing the precision of micro information makes micro-informed investors worse off — we say that the micro-informed overtrade their information, driving down their compensation for liquidity provision. In contrast, macro-informed investors may be better or worse off as a result of more precise macro information: they are better off when the fraction of macro-informed agents or, equivalently, the informativeness of the price of the index fund is sufficiently low. Similarly, the equilibrium fraction of macro-informed agents always increases with the precision of micro information, but it can move in either direction.
with an increase in the precision of macro information. All these results point towards micro information being extremely valuable to investors. A simple condition on the relative precision of micro and macro information determines whether the market is more micro efficient or more macro efficient.

We use our model to analyze variance ratios that have been widely studied empirically. Campbell (1991) and Vuolteenaho (2002) decompose the variance of index-level and idiosyncratic stock returns into variance from cash flow news and variance from news about discount rates. Their estimates show that the ratio of cash flow variance to discount rate variance is larger for individual stocks than for the aggregate market, a pattern predicted by our model. We also argue that the trends in variance ratios are consistent with a declining cost of becoming informed over the twentieth century combined with increasing indirect index trading, meaning trading in the index that does not involve trading in individual stocks. Jung and Shiller (2005) call the cash flow and discount rate components of returns the efficient market and inefficient market components, respectively. They therefore interpret the larger variance ratio for individual stocks — found empirically and in our model — as evidence for Samuelson’s dictum.

Our work is related to several strands of literature. Our model effectively nests Grossman and Stiglitz (1980) if we take the index fund as the single asset in their model. We draw on the analysis of Hellwig (1980), Admati (1985) and Admati and Pfleiderer (1987) but address different questions; the books by Brunnermeier (2001), Vives (2008), and Veldkamp (2011) provide valuable background. As in Kyle (1985), our noise traders are price insensitive, and gains from trade against them accrue to the informed, which provides an incentive to collect information. We shed light on the discussion in Black (1986) of the crucial role that “noise” plays in price formation by proposing a model in which the factor structure of noise trading plays a key role in determining the relative micro versus macro efficiency of markets.

Van Nieuwerburgh and Veldkamp (2009) analyze how investors’ choices to learn about the domestic or foreign market in the presence of asymmetric prior knowledge may explain the home bias puzzle, and Van Nieuwerburgh and Veldkamp (2010) use related ideas to explain investor under-diversification. Kacperczyk et al. (2016) develop a model of rational attention allocation in which fund managers choose whether to acquire macro or stock specific information before making investment decisions. Their model, like ours, has multiple assets subject to a common cash flow factor; but, in marked contrast to our setting, their model allows a symmetric equilibrium in which all agents make the same information choices. We also compare variance ratios (cash flow variance to discount rate variance) in the two models and find that they show qualitatively different patterns as a result of the differences in
investor information choices.

Peng and Xiong (2006) also use a model of rational attention allocation to study portfolio choice. In their framework, investors allocate more attention to sector or marketwide information and less attention to firm-specific information. Their conclusion contrasts with ours (and with the Jung-Shiller discussion of Samuelson’s dictum and the Maćkowiak and Wiederholt 2009 model of sticky prices under rational intattention) primarily because in their setting a representative investor makes the information allocation decision; since macro uncertainty is common to all securities, while micro uncertainty is diversified away, the representative investor allocates more attention to macro and sector level information. Gârleanu and Pedersen (2018ab) extend the Grossman-Stiglitz model to link market efficiency and asset management through search costs incurred by investors in selecting fund managers and find that micro portfolios are more price efficient than macro portfolios. Schneemeier’s (2015) model predicts greater micro than macro efficiency when managers use market prices in their investment decisions. Bhattacharya and O’Hara (2016) and Glosten, Nallareddy, and Zou (2016) study a Kyle-type model with an ETF, as well as macro- and micro-informed agents. The ETF has higher liquidity than its constituent stocks, which prevents its price from equaling that of the underlying security basket. In this setting, ETF prices can be informative about individual stock prospects, a dynamic which is absent in our model where agents are atomic and trade with no price impact.

As noted above, our model assumes that making inferences from prices consumes some information processing capacity, a point also stressed by Vives and Yang (2018). In their model, boundedly rational investors act on noisy price signals. Our investors observe and learn from prices correctly, but they are constrained in the total information they can acquire. Our investors allocate their information capacity rationally, whereas in Eyster, Rabin, and Vayanos (2018) some investors are simply “cursed” to ignore price information. In an extension of their main model, Kacperczyk et al. (2016) find that when making inferences from prices is costly, investors ignore prices and allocate attention to independent signals. The information choices available to our investors may be more precise than prices, but we do not allow investors to invest in a security without conditioning their demand on (at least) the information in the price.

Section 2 describes our securities and the information choices available to investors, and it then presents our general result showing that investors endogenously specialize in macro or micro information. Section 3 introduces the constrained model and adds additional features (supply shocks and market clearing) that lead to explicit expressions for prices and price efficiency in the market equilibrium of Section 4. Sections 5 and 6 investigate the attention
equilibrium and information equilibrium, respectively, in the constrained model. Section 7 discusses model implications for variance ratios. Proofs are deferred to an appendix.

2 The economy

Securities

We assume the existence \( N \) risky securities — called stocks — indexed by \( i \). There is also an index fund, \( F \), one share of which holds \( 1/N \) shares of each of the \( N \) stocks. There is a riskless security with a gross return of \( R \).

The time 2 dividend payouts of the stocks are given by

\[
u_i = M + S_i, \quad i = 1, \ldots, N.
\] (1)

We interpret \( M \) as a macro factor and the \( S_i \) as idiosyncratic contributions to the dividends. The random variables \( M, S_1, \ldots, S_N \) are jointly normal, with \( \mathbb{E}[M] = \bar{m} \), \( \mathbb{E}[S_i] = 0 \), \( \text{var}[M] = \sigma_M^2 \), \( \text{var}[S_i] = \sigma_S^2 \), and \( \mathbb{E}[MS_i] = 0 \), \( i = 1, \ldots, N \).

To arrive at (1), we start from a representation \( u_i = M' + S_i' \), \( i = 1, \ldots, N \), in which the \( S_i' \) have mean zero and are independent of each other and of \( M' \). As \( N \) increases, the sample mean \( \bar{S}_N' \) approaches zero; but for fixed \( N \) the mean will not be exactly zero, which is to say that \( \bar{S}_N' \) acts like a common component of the nominally idiosyncratic terms \( S_i' \). To remove the common component, we set \( M = M' + \bar{S}_N' \) and \( S_i = S_i' - \bar{S}_N' \) in (1), which still yields \( \mathbb{E}[MS_i] = 0 \) because \( S_i \) is uncorrelated with \( \bar{S}_N' \). The \( S_i \) can be fully diversified away at finite \( N \), in the sense that the sample mean \( \bar{S}_N = 0 \). Removing the common component of the stock-specific terms introduces a small amount of correlation in the residual terms,

\[
\text{corr}(S_i, S_j) = -\frac{1}{N-1}, \quad i \neq j.
\] (2)

These correlations are negligible if \( N \) is 100 or larger, so (2) may be interpreted as approximate independence for large \( N \). The advantage of using this decomposition in (1) is that it avoids the need to keep track of \( \bar{S}_N' \) separately from the macro factor. In particular, since \( \bar{S}_N = 0 \), the index fund \( F \) pays

\[
u_F = \frac{1}{N} \sum_{i=1}^{N} u_i = M + \bar{S}_N = M,
\] (3)

making the index fund a direct investment in the macro factor, consistent with its usual
Prices of individual stocks and of the index fund are realized at time 1; in Section 3, we detail price formation through market clearing, but at this point we keep the setting general. Individual stock prices are given by $P_i$. The index fund price is $P_F$, and precluding arbitrage requires that

$$P_F = \frac{1}{N} \sum_{i=1}^{N} P_i. \quad (4)$$

We also define the price $P_{S_i} = P_i - P_F$, $i = 1, \ldots, N$, of a security paying $u_i - M = S_i$, the idiosyncratic portion of the dividend of stock $i$.

**Agents and information sets**

At time 0, a unit mass of agents maximize expected utility, $-E[\exp(-\gamma \tilde{W}_2)]$, over time time 2 wealth,

$$\tilde{W}_2 = W_1 R + q_F (u_F - R P_F) + \sum_{i=1}^{N} q_i (u_i - R P_i),$$

where $q_F$ and $q_i$ are the shares invested in the index fund and stock $i$, and which are chosen given the information $I$ available to investors at time 1 to maximize $-E[\exp(-\gamma \tilde{W}_2)|I]$. The initial wealth $W_1$ does not affect an investor’s decisions. The risk aversion parameter $\gamma > 0$ is common to all investors.

At time 0, agents can choose to acquire information capacity $\kappa$, $0 < \kappa < 1$, by incurring a cost $c$. This capacity allows an agent to select signals $m'$ about $M$ and signals $s'_i$ about the $S_i$. We measure the informativeness of signals $m'$ and $s'_i$ through the variance reduction ratios $(\text{var}[M] - \text{var}[M|m'])/\text{var}[M]$ and $(\text{var}[S_i] - \text{var}[S_i|s'_i])/\text{var}[S_i]$. Informativeness will be constrained by $\kappa$, and the available signals will allow full use of $\kappa$.

In more detail, for any level of informativeness $f \in [0, 1]$, there is a signal $s_i(f)$, with $s_i(0) = 0$ and $s_i(1) = S_i$. Each $s_i(f)$ has mean zero and variance $f \sigma_{S_i}^2$, with $\text{var}[S_i|s_i(f)] = (1-f) \sigma_{S_i}^2$. Similarly, the signal $m(f)$ has $\mathbb{E}[m(f)] = \bar{m}$, $\text{var}[m(f)] = f \sigma_M^2$, and $\text{var}[M|m(f)] = (1-f) \sigma_M^2$. All macro signals $m(f)$ are independent of signals $s_i(f')$ about idiosyncratic payouts, and all signals and payouts are jointly normal. We henceforth omit the argument $f$ from the signals unless needed for clarity.

An informed investor selects a set of securities about which to acquire signals and in which to invest. The consideration set of securities contains $K$ stocks, $i_1, \ldots, i_K$, for some $0 \leq K \leq N - 1$, and may contain the index fund. We assume that prices are freely available,

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1The same idea is used to formulate the CAPM with a finite number of securities, as in Ross (1978), Chen and Ingersoll (1983), and Kwon (1985), ensuring that idiosyncratic risks are fully diversifiable.
and once an investor chooses to become informed about a security, the investor knows at least the price of the security. But making inferences from prices will consume some capacity in equilibrium (because prices will be informative), and this will act like a fixed cost to following each security.

Together with a set of securities, an investor chooses a corresponding information set

\[ I^{(0)}_K = \{(s'_i, P_{S_i}) : i = 1, \ldots, K\}, \]  
(stocks and micro signals)

\[ I^{(1)}_K = \{P_F, (s'_i, P_{S_i}) : i = 1, \ldots, K\}, \]  
(stocks, micro signals, and index price)

\[ I^{(2)}_K = \{(m', P_F), (s'_i, P_{S_i}) : i = 1, \ldots, K\}, \]  
(stocks, micro signals, index price and macro signal)

depending on whether the index fund is in the consideration set and, if it is, whether the investor learns more than the fund’s price. The information choice thus has both an extensive margin (the set of securities) and an intensive margin (the precision of the information the investor acquires about each security). In any \( I^{(i)}_K \), signals \( s'_i \) and \( s'_k \) may have different precision, but an investor cannot choose to know less than the price of a security in the consideration set.

With additional structure (which we introduce later), prices will reflect investors’ information choices. For now, we keep the discussion general and just assume that prices and signals are jointly normal. We also assume that the fund price \( P_F \) is uncorrelated with the prices \( P_{S_i} \) of the idiosyncratic payouts.

Write \( \Sigma^{(\iota,K)} \) for the unconditional covariance matrix of the payouts \( M, S_{i_1}, \ldots, S_{i_K} \) or \( S_{i_1}, \ldots, S_{i_K} \) in the consideration set, with \( \iota \in \{1, 2\} \) if the index fund is in the set, and \( \iota = 0 \) if it is not. The off-diagonal elements of \( \Sigma^{(0,K)} \) are determined by (2), and we have

\[ \Sigma^{(2,K)} = \Sigma^{(1,K)} = \begin{pmatrix} \text{var}[M] & 0 \\ 0 & \Sigma^{(0,K)} \end{pmatrix}. \]

After observing signals, the investor evaluates the posterior distribution of the security payoffs and evaluates the conditional covariance matrix for the payoffs in the consideration set, which we denote by \( \hat{\Sigma}^{(\iota,K)} \). Because every macro signal \( m' \) is independent of every micro signal \( s'_i \), we assume that \( P_F \) is independent of \( s'_i \) and \( P_{S_i} \) is independent of \( m' \). The conditional
covariance matrices therefore have the form
\[ \hat{\Sigma}^{(1,K)} = \begin{pmatrix} \text{var}[M|P_F] & 0 \\ 0 & \hat{\Sigma}^{(0,K)} \end{pmatrix}, \quad \hat{\Sigma}^{(2,K)} = \begin{pmatrix} \text{var}[M|m', P_F] & 0 \\ 0 & \hat{\Sigma}^{(0,K)} \end{pmatrix}. \]

Investors are constrained in how much information they can acquire, and we model this constraint through a bound on signal precision. Using $|\cdot|$ to indicate the determinant of a matrix, for $\iota = 0$ or $\iota = 2$, we impose the constraint
\[ \frac{|\hat{\Sigma}^{(\iota,K)}|}{|\Sigma^{(\iota,K)}|} \geq \kappa, \quad (5) \]
where $0 < \kappa < 1$ measures the information capacity an investor attains at the cost $c$. Smaller $\kappa$ corresponds to greater variance reduction and thus greater capacity. In the case of diagonal covariance matrices, with $\iota = 0$, (5) would simplify to
\[ \frac{\text{var}[S_i|s_{i1}', P_{S_i}]}{\text{var}[S_i]} \cdots \frac{\text{var}[S_{iK}|s_{iK}', P_{S_{iK}}]}{\text{var}[S_{iK}]} \geq \kappa. \]
This expression makes clear that following a stock consumes capacity of at least $\text{var}[S_i|P_{S_i}]/\text{var}[S_i]$ because we assume that investors know at least the price of any securities in their consideration sets. As discussed in the Introduction, we do not assume that making inferences from prices is costless. Instead, we measure the capacity consumed by making inferences from $P_{S_i}$ the same way we measure the capacity consumed by more precise signals $(s_i', P_{S_i})$.

However, the key asymmetry in our model between macro and micro information lies in investors’ ability to acquire information from the price of the index fund. Here we replace the variance ratio $\text{var}[M|P_F]/\text{var}[M]$ (which depends on the informativeness of the index fund price) with a fixed quantity $\delta_F \in (0,1)$. More precisely, for the case of $\iota = 1$ in (5), meaning a signal set $I_K^{(1)}$ that includes the fund price $P_F$ but no other macro information, we impose the constraint
\[ \delta_F |\hat{\Sigma}^{(0,K)}|/|\Sigma^{(0,K)}| \geq \kappa. \quad (6) \]

The posterior distribution preserves the independence of macro and idiosyncratic sources of risk, but we do not assume that $\hat{\Sigma}^{(0,K)}$ has the same dependence structure as $\Sigma^{(0,K)}$, a point emphasized in Sims (2011), p.167.

The determinant ratio is a multivariate generalization of a variance ratio, and it generalizes one minus a regression $R^2$. The constraint in (5) is very similar to the entropy constraint used by Sims (2003), Mondria (2010), Van Nieuwerburgh and Veldkamp (2010), Hellwig et al. (2012) and others. With no information, $\hat{\Sigma} = \Sigma$ and the determinant ratio is 1, indicating that no capacity is consumed, whereas the entropy measure includes a term depending on the number of assets $K$. When the number of assets is fixed, the measures are equivalent. We take the determinant of empty matrices to be one, so $|\Sigma^{(0,K)}| = |\hat{\Sigma}^{(0,K)}| = 1$, if $K = 0$. 

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This modification treats the index fund price differently from other types of information. We are particularly interested in equilibrium outcomes in which

\[
\frac{\text{var}(M|P_F)}{\text{var}(M)} < \delta_F. \tag{7}
\]

When this inequality holds, making inferences from the price of the index fund consumes less capacity than would be expected from the variance reduction achieved. Our model captures the idea that the implications of the overall state of the market, as measured by the index fund, are widely discussed and publicly disseminated; \( \delta_F \) is the capacity consumed by paying attention to this ambient information. If \( \text{(7)} \) holds, then making inferences from the price of the index fund is at least slightly easier than making inferences from other information, holding fixed the level of variance reduction. We do not assume \( \text{(7)} \); we will show that it follows from more basic assumptions.

If we removed the distinctive treatment of the index fund in \( \text{(6)} \), our results would not go through, and the model would degenerate, with many different information choices potentially leading to the same expected utility; see the discussion in Section 2.1. That outcome would be similar to the partial equilibrium indifference result in Section 2.3 of Van Nieuwerburgh and Veldkamp (2010), where investors are indifferent among all information sets that make full use of their capacity.\(^4\) But that outcome would ignore the contrast between the pervasive availability of information about the overall level of the stock market and the much more specialized nature of information about individual stocks. Condition \( \text{(6)} \) captures that distinction. The main contributions of our analysis lie in showing that this information structure leads investors to specialize in micro or macro information, and that this specialization typically results in greater micro price efficiency — consequences that are not a priori obvious from \( \text{(6)} \) or even \( \text{(7)} \).

Because we condition on prices as well as nonpublic signals in \( \text{(5)} \) and \( \text{(6)} \), our formulation implies that making inferences from prices consumes some of the capacity \( \kappa \). This point merits emphasis. The capacity \( \kappa \) accounts for two types of effort: the effort required to acquire nonpublic signals \( m' \) or \( s'_i \), and also the effort required to make inferences from these signals and from publicly available prices. Price information is freely available, but regularly following the prices of hundreds of stocks and extracting investing implications from these prices consumes attention and effort.

\(^4\)Van Nieuwerburgh and Veldkamp (2010), p.796, note that degeneracy in their partial equilibrium setting can be broken through a capacity constraint in which different variance ratios are raised to different powers. Our constraint \( \text{(6)} \) could be viewed as raising \( \text{var}(M|P_F)/\text{var}(M) \) to a power of zero and scaling it by a constant. We stress however that \( \text{(7)} \) will be an equilibrium outcome for us and not an assumption.
• Agents seek to maximize expected time 1 utility, i.e. \( E\{-E[e^{-\gamma\hat{W}_2}|I]\} \)

• Agents choose whether to acquire capacity \( \kappa \) at cost \( c \)

• Those who become informed choose an information set from \( I_K^{(0)}, I_K^{(1)} \) or \( I_K^{(2)} \) subject to capacity \( \kappa \)

• Signals and prices are realized

• Informed observe their signals from the set \( I_K^{(i)} \)

• Uninformed observe either \( P_F \) or some combination of prices from \( \{P_{S_1}, \ldots, P_{S_N}\} \)

• Agents submit portfolio choices as a function of their information \( I \) to maximize \( -E[e^{-\gamma\hat{W}_2}|I] \)

• Stocks pay dividends \( M + S_i \)

• Index pays dividend \( M \)

• Agents realize utility \( -e^{-\gamma\hat{W}_2} \)

Figure 1: Model timing.

Uninformed investors — those who do not incur the cost \( c \) to acquire the capacity \( \kappa \) — observe market prices. Because making inferences from prices requires some information processing capacity, we endow uninformed investors with capacity \( \delta_F \). This allows the uninformed to invest in the index fund and condition their demand on the price of the index. They may also reallocate this capacity to make inferences from the prices of individual stocks. Figure 1 summarizes the sequence of events in our model.

Equilibrium

Once investors choose their information sets, their optimal portfolios (chosen to maximize expected utility) are determined by the price system. An investor’s strategy thus reduces to a choice of information set. We use the following definition of equilibrium: An equilibrium consists of a collection of investor information choices and a joint distribution (assumed normal) for prices, dividends, and signals, under which no investor can increase expected utilities.

5In Figure 1 investors compare expected utilities at time 0 in deciding whether to become informed, without first incurring the cost \( c \). This raises the question of how the uninformed can evaluate expected utilities. This point merits two comments. First, we will see in the appendix (Lemma A.1) that expected utilities depend on the precision of prices and signals, but not on their realizations. Second, and more fundamentally, a general comment of Lucas (1978), p.1429, about rational expectations applies: “this hypothesis (like utility maximization) is not ‘behavioral’: it does not describe the way agents think about their environment, how they learn, process information, and so forth. It is rather a property likely to be (approximately) possessed by the outcome of this unspecified process of learning and adapting.” Similarly, in our setting, we may interpret the optimal choices made at time 0 as the result of an unspecified process of trial and error that does not require explicit calculations of expected utilities by agents.
utility through a different information choice.\footnote{More precisely, no information set that is selected by a positive fraction of agents is strictly dominated by another information set.}

In Theorem \ref{thm:mmspecialization}, we will show that in any equilibrium satisfying what we consider to be a reasonable set of additional properties, all informed investors will choose to specialize in micro or macro information, and a positive fraction of investors will choose each specialization. Put differently, this says that if an equilibrium exists without this micro-macro specialization, then the equilibrium must fail to have one of the following features:

**(e1)** The joint distribution of prices, dividends, and signals is normal. The joint distribution of the pairs \((S_i, P_{S_i})\), \(i = 1, \ldots, N\), is invariant under permutation of the indices, and every \((m(f), P_F)\) is independent of every \((s_i(f'), P_{S_i})\).

**(e2)** If no investors choose a macro signal \(m(f), f > 0\), then \(\text{var}[M|P_F] = \text{var}[M]\), and if no investors choose any micro signal \(s_i(f), f > 0, i = 1, \ldots, N\), then \(\text{var}[S_j|P_{S_j}] = \text{var}[S_j]\), for all \(j = 1, \ldots, N\).

**(e3)** The information cost and capacity parameters satisfy \(e^{2\gamma_c\kappa} < \delta_F\).

**(e4)** A positive fraction of investors choose to remain uninformed, and a positive fraction of these invest in the index fund.

Condition (e1) restricts attention to equilibria that are symmetric in the individual stocks, which is a reasonable restriction given that their dividends are ex ante identically distributed. This restriction works against finding equilibria in which investors make heterogeneous information choices. The last part of (e1) is consistent with the interpretation of the \(S_i\) as idiosyncratic components. Condition (e2) ensures that prices do not contain exogenous information about dividends — only information acquired by investors.

We have not yet specified how investor information choices affect prices; (e1) and (e2) are minimal consistency properties between prices and information choices. If the micro-macro specialization property in Theorem \ref{thm:mmspecialization} is necessary under minimal assumptions on prices, then it remains necessary when we impose additional structure on price formation, as we do in Section \ref{sec:structure}.

The last two conditions limit us to “interior” equilibria: we will see that (e3) ensures that there is a benefit to becoming informed, whereas (e4) restricts our focus to equilibria in which not all investors become informed. The second half of (e4) requires that some uninformed investors invest in the index fund; it does not preclude the possibility the uninformed also invest in individual stocks, though in equilibrium they will choose not to (see Section \ref{sec:specification}).
The implication of (e4) — the existence of uninformed investors who invest in the index fund — is certainly what one would expect empirically. We do not know if there are equilibria that violate (e4); but we consider that the most interesting equilibria satisfy (e4), and Theorem 2.1 should be understood as describing these equilibria.

To state the main result of this section, we highlight two types of information choices:

i) Call informed investors who choose the information set \( I^{(2)}_0 = \{m, P_F\} \) the macro-informed; and

ii) Call informed investors who choose any information set \( I^{(1)}_1 = \{P_F, (s_i, P_S_i)\}, i = 1, \ldots, N \), the micro-informed.

Here, \( m \) and \( s_i \) are the maximally informative macro and micro signals that can be achieved in these information sets with capacity \( \kappa \).

**Theorem 2.1.** In any equilibrium satisfying (e1)–(e4), all informed investors choose to be either macro-informed or micro-informed, and both types of investors are present in positive proportions.

Under the conditions in the theorem, all informed investors choose one of two types of information. Equivalently, we may say that if an equilibrium exists in which investors do not exhibit the specialization in the theorem, then such an equilibrium must violate at least one of properties (e1)–(e4). In Section 3, we will construct an equilibrium satisfying (e1)–(e4) and the specialization property in the theorem.

Conditions (e3) and (e4) allow us to prove (7) (see Lemma A.3 in the Appendix), so in addition to the inherent plausibility of these restrictions it is worth considering what happens if (7) fails. The reverse inequality in (7) would imply, quite unreasonably, that making inferences from the price of the index fund requires more effort than acquiring and interpreting the information content of the price from other sources. It is unclear if the reverse inequality in (7) is compatible with any equilibrium. With equality in (7), i.e. if \( \delta_F \) is equal to the reduction in dividend uncertainty due to information in the index price, we explain in Section 2.1 that investors would be indifferent across a wide range of information choices, with no special role for the index fund, contrary to what we observe in practice. The conditions in the theorem may therefore be viewed as describing equilibria in which (7) holds, which we consider the most interesting scenario.

It is by no means obvious that (7) leads to the specialization result Theorem 2.1. Indeed, a novel feature of Theorem 2.1 is that it implies heterogeneous information choices by ex ante identical investors. This phenomenon contrasts with most of the related literature.
In a partial equilibrium setting with exogenous prices, Van Nieuwerburgh and Veldkamp (2010) show that investors with exponential utility and a variance-ratio information constraint are indifferent across all feasible information choices: their investors would indeed be indifferent between $I_0^{2}$ and $I_1^{1}$, but they would also be indifferent between these and any other information set that consumed all available capacity. Mondria (2010) finds cases of asymmetric equilibria numerically, but these are outside the scope of his theoretical analysis, which focuses on identical signal choices by investors. In Kacperczyk et al. (2016) all informed investors may choose the same information structure. More precisely, only overall price informativeness is determined in equilibrium; and price informativeness is unchanged if half of investors choose one information set and half choose another or if all investors split their attention evenly between the two sets. In our setting, specialized micro- and macro-informed investors cannot be replaced with identical investors who divide their attention between micro and macro information.

In Goldstein and Yang (2015), the dividend of a single stock depends on two types of fundamentals. Their interpretation is different, but one could think of the two fundamentals as macro and micro sources of uncertainty. In their equilibrium, investors choose to learn about both fundamentals unless the cost of acquiring both types of information is greater than the sum of the costs of acquiring each type of information separately. Their outcome therefore differs from ours, in which investors choose to focus on one source of uncertainty. Investors in Goldstein and Yang (2015) have just one security through which to trade on two types of dividend information, so information about one signal can be inferred from the other, whereas our setting has as many securities as sources of dividend information, which removes the interaction effect. Investors in Van Nieuwerburgh and Veldkamp (2009) specialize, but their specialization depends on differences in prior information.

### 2.1 Intuition for specialization result

The proof of Theorem 2.1 requires several steps, as detailed in the appendix. Here we provide some brief intuition. We show that conditions (e1)–(e4) imply (7), but for (7) to hold, $P_F$ must be informative, which happens only if some investors choose to learn about $M$. Because each security in a portfolio requires a minimum amount of effort, some investors choose to learn only about $M$. Other investors choose to take advantage of the attention “discount” provided by (7). These investors learn from $P_F$ and use their remaining capacity to learn about individual stocks; in equilibrium, they face a fixed cost (in expected utility) to following each additional stock because making inferences from each stock price consumes a discrete amount of capacity. Therefore, investing in a single stock is optimal. These
outcomes use the balance provided by (e3) and (e4), which restrict us to equilibria in which some but not all investors choose to become informed.\footnote{For simplicity, we have formulated Theorem 2.1 under the assumption that all investors who observe a signal about stock $i$ with informativeness $f$ observe the same signal $s_i(f)$. The result extends to private signals of the form $s_i(f) + \varepsilon$, where $\varepsilon$ is an investor-specific, mean-zero noise term.}

In more detail, condition (e3) compares the squared utility cost of becoming informed $e^{2\gamma c}$ with the ratio of the capacity $\kappa$ of the informed and the capacity $\delta_F$ with which the uninformed are endowed. It thus helps ensure that some investors will choose to become informed. When combined with the presence of uninformed required by (e4), it goes further and ensures that the informed have enough capacity to make prices sufficiently informative that $\text{var}[S_i - RP_S] < \text{var}[S_i]$ and condition (7) both hold. When this price-informativeness condition holds, learning from the prices of many securities is costly, but learning from the price of a single security entitles the informed to a $\kappa$ benefit in utility, and this drives investors toward specialization. Reversing the inequality in (e3) would lead to noisy prices, in the sense that $\text{var}[S_i - RP_S] > \text{var}[S_i]$ and the reverse inequality would hold in (7). This scenario drives investors toward holding all securities, eliminating any specialization. But reversing the inequality in (7) would mean that investors are penalized for learning solely from the price of the index fund, leading all investors who invest in the index fund to learn just a little about the macro factor beyond the price to avoid the penalty. This effectively rules out having any genuinely uninformed investors in the index fund, which is an implausible situation. Finally, if these inequalities are replaced with equalities, investors become indifferent across a wide range of information choices.

This last intuition is formalized in Proposition A.1 in the appendix, where we analyze the situation where $\delta_F$ is not fixed, but is given by the equilibrium quantity

$$\delta_F = \frac{\text{var}[M|P_F]}{\text{var}[M]}.$$  

This treats conditioning on the index price identically to conditioning on individual stock prices – investors are charged capacity in proportion to the resultant decrease in dividend uncertainty. The proposition establishes the following results:

i) The information sets $\mathcal{I}^{(1)}_K$ and $\mathcal{I}^{(2)}_K$ lead to the same utility;

ii) If any investor chooses the information set $\mathcal{I}^{(1)}_K$ then $K = 1$ (i.e. the information set of our micro-informed) cannot be strictly optimal;

iii) If any investor weakly prefers information set $\mathcal{I}^{(1)}_K$, then the investor either (a) is indifferent between all choices of $K$ or (b) strictly prefers $K = N - 1$. 

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The proposition thus shows the replacing our fixed $\delta_F$ with the condition in (8) leads to either informational invariance, where all information sets that fully utilize available capacity are equally good, or the result that the optimal information set for any investor is to learn about all securities. Our specialization result disappears.

To summarize, specialization results from the combination of the following main features: the fixed cost $\delta_F$; treating the information cost of inferences from prices the same way we treat inferences from other signals; the presence of some uninformed investors in the market for the index fund; and the constraint $e^{2\gamma c} \kappa < \delta_F$, ensuring that the cost $c$ of becoming informed is not so large as to discourage investors from acquiring additional information to make prices sufficiently informative. Yet none of these modeling choices leads to specialization if $\delta_F$ isn’t fixed but is given by (8).

3 The constrained model

Going forward, we will denote by $m = m(f_M)$ the maximally informative macro signal chosen by a macro-informed investor, $\text{var}[m] = f_M \text{var}[M]$, and we will represent $M$ as

$$M = m + \epsilon_M,$$

where $m$ and $\epsilon_M$ are uncorrelated. Similarly, we will write

$$S_i = s_i + \epsilon_i, \quad i = 1, \ldots, N,$$

where $s_i$ and $\epsilon_i$ are uncorrelated with each other, and where $s_i = s_i(f_S)$, with $\text{var}[s_i] = f_S \text{var}[S_i]$, is the maximally informative micro signal chosen by a micro-informed investor, recalling that the micro-informed also observe the index fund price $P_F$. The information choices of the macro-informed, $\{m, P_F\}$, and of the micro-informed, $\{P_F, (s_i, P_{S_i})\}$, consume the investor’s full capacity, so the constraint in (6) becomes

$$\kappa = \frac{\text{var}[M|m, P_F]}{\text{var}[M]} = \delta_F \frac{\text{var}[S_i\|s_i, P_{S_i}]}{\text{var}[S_i]}.$$

Theorem 2.1 shows that a necessary condition for an equilibrium in our setting is that all informed investors are either macro-informed or micro-informed. We will now show that such an equilibrium does in fact exist. We do so by imposing the necessary condition as a constraint from the outset. In other words, we now consider a market with just three types of investors: uninformed, macro-informed, and micro-informed, with respective fractions $\lambda_U$, $\lambda_M$, and $\lambda_S$. 

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\( \lambda_M \), and \( \lambda_S = 1 - \lambda_U - \lambda_M \). The macro-informed select the signal \( m \) in (9), and a micro-informed investor selects \( P_F \) and a signal \( s_i \) from (10); consistent with Theorem 2.1, no other signals are chosen by any investors. We assume that the mass \( \lambda_S \) of micro-informed investors is evenly divided among the \( N \) stocks, so \( \lambda_S/N \) investors observe each signal \( s_i, i = 1, \ldots, N \), and only these investors invest directly in stock \( i \). Recall from the discussion of consideration sets in Section 2 that investors do not invest in a security without conditioning their demand on (at least) the price of the security, so the macro-informed invest only in the index fund, and the micro-informed invest only in the index fund and a single stock, consistent with Theorem 2.1. We limit the uninformed to investing in the index fund (the alternative of allocating their \( \delta_F \) capacity to condition on individual stock prices is suboptimal under the conditions of Theorem 2.1). Starting with a representation \( u_i = M' + s'_i + \epsilon'_i \), we extend the argument in (1)–(2) to the \( s_i \) and \( \epsilon_i \), so that

\[
\sum_{i=1}^{N} s_i = \sum_{i=1}^{N} \epsilon_i = 0, \tag{12}
\]

and \( \text{corr}(s_i, s_j) = \text{corr}(\epsilon_i, \epsilon_j) = -1/(N - 1) \) for \( i \neq j \). As in our discussion of (1)–(2), (12) should be viewed as the result of removing a common component from a finite number of idiosyncratic terms.

**Supply shocks**

Investor demands for the securities will follow from their utility maximizing decisions. We now detail the supply of the securities. We suppose that the supply has a factor structure similar to that of the dividends in (1), with the supply of the \( i^{th} \) stock given by

\[
\frac{1}{N} (X_F + X_i). \tag{13}
\]

Here, \( X_F \) is the common supply shock, normally distributed with mean \( \bar{X}_F \) and variance \( \sigma^2_{X_F} \). The \( X_i \) are normally distributed idiosyncratic shocks, each with mean 0 and variance \( \sigma^2_X \). Supply shocks are independent of cash flows, and \( X_i \) is independent of \( X_F \) for all \( i \). Following the derivation in (1)–(2), we define the common factor \( X_F \) so that the idiosyncratic

\[8\] Although we do not model fund management, the index fund price may be viewed as a channel through which the uninformed and micro-informed acquire some of the information of the macro-informed. This idea is developed in García and Vanden (2009), where informed investors form mutual funds to sell their private information.
shocks diversify, in the sense that
\[ \sum_{i=1}^{N} X_i = 0, \]  
(14)
and \( \text{corr}(X_i, X_j) = -1/(N - 1) \) for \( i \neq j \). We make the standard assumption that supply shocks are unobservable by the agents.

The idiosyncratic supply shock \( X_i/N \) has standard deviation \( \sigma_X/N \), which is much smaller than the standard deviation \( \sigma_{X_F} \) of the aggregate supply shock if \( \sigma_{X_F} \) and \( \sigma_X \) have similar magnitudes, as they will in our calibration to market data. With \( N \) large and identical investors, the contribution to portfolio risk from idiosyncratic supply shocks would go to zero even in the absence of learning, but investor specialization provides an incentive to acquire information: only \( 1/N \) investors are informed about each stock.\(^9\)

The aggregate portion of supply shocks, \( X_F \), is standard in the literature — as will become clear, it is analogous to the single security supply shock in Grossman and Stiglitz (1980). The idiosyncratic portion of the supply shock, \( X_i \), proxies for price-insensitive noise trading in individual stocks. Some of this noise trading may be liquidity driven (for example, individuals needing to sell their employer’s stock to pay for unforeseen expenditures), and some may originate from incorrect expectations or from other value-irrelevant triggers, such as an affinity for trading or fads. Empirical studies (Brandt, Brav, Graham, and Kumar 2010 and Foucault, Sraer, and Thesmar 2011) document a link from retail trading to idiosyncratic volatility of stock returns. We show in Section 7.2 that the volatility of \( X_i \) enters directly into the idiosyncratic volatility of stock returns.

**Market clearing**

Let us write \( q_U^{j}, q_M^{j}, \) and \( q_i^j \) for the demands of each investor group for security \( j \), which can be one of the \( N \) stocks or the index fund \( F \). For each stock \( i \), \( q_i^j \) denotes the direct demand for stock \( i \) by investors informed about stock \( i \). Each group’s \( F \) demand, \( q_F \), leads to an indirect demand of \( q_F/N \) for every stock \( i \).

Aggregate holdings of the index fund are given by
\[
q_F \equiv \lambda_U q_U^F + \lambda_M q_M^F + \frac{\lambda_S}{N} \sum_{i=1}^{N} q_i^F.
\]  
(15)

\(^9\)In the model of Kacperczyk et al. (2016), individual risk factors are assumed to have the same standard deviation as the aggregate risk factor. We explore the consequences of these contrasting modeling assumptions in Section 7 and Appendix A.3.
The market clearing condition for each stock $i$ is given by

$$\frac{\lambda_i}{N} q_i^i + \frac{q_F}{N} = \frac{1}{N} (X_F + X_i), \quad i = 1, \ldots, N. \quad (16)$$

The first term on the left is the direct demand for stock $i$ from investors informed about that stock; these are the only investors who invest directly in the stock. The second term is the amount of stock $i$ held in the index fund. The right side is the supply shock from (13). The direct and indirect demand for stock $i$ must equal its supply.

As (16) must hold for all $i$, the quantity $\xi \equiv \lambda_i q_i^i - X_i$ cannot depend on $i$, if the market clears. We can therefore write the direct demand for stock $i$ and the total demand for the index fund as

$$\lambda_i q_i^i = X_i + \xi, \quad q_F = X_F - \xi, \quad (17)$$

for some $\xi$ that does not depend on $i$.

We will show in Section 4.1 that in equilibrium $\xi$ must be zero, leading to two important implications. It will follow from (17) that micro-informed investors fully absorb the idiosyncratic supply shock $X_i$, and that the index fund holds the aggregate supply shock. We will interpret the first equation in (17) as liquidity provision by the micro-informed investors in the securities in which they specialize.

4 Market equilibrium in the constrained model

We construct an equilibrium in which the index fund price takes the form

$$P_F = a_F + b_F (m - \bar{m}) + c_F (X_F - \bar{X}_F), \quad (18)$$

and individual stock prices are given by

$$P_i = P_F + b_S s_i + c_S (X_i + \xi), \quad i = 1, \ldots, N. \quad (19)$$

Here, $m$ and $s_i$ are the macro and micro signals in (9) and (10). Equation (18) makes the index fund price linear in the macro shock $m$ and the aggregate supply shock $X_F$. Equation (19) makes the idiosyncratic part of the price of stock $i$, $P_i - P_F$, linear in the micro shock $s_i$ and the idiosyncratic supply shock $X_i + \xi$. These prices satisfy (e1) and, consistent with (e2), the only information they contain about dividends comes from the selected signals $m$ and $s_i$. 
4.1 Model solution

Macro-informed and uninformed investors trade only in the index fund, and a micro-informed investor trades the index fund and one security \( i \). An investor sets a demand at time 1 by maximizing expected utility conditional on an information set \( I \), as illustrated in Figure 1. Here \( I = \{ P_F \} \) for the uninformed, \( I = \{ m, P_F \} \) for the macro-informed, and \( I = \{ P_F, P_S, s_i \} \) for the micro-informed.

By standard arguments, the macro-informed demand for the index fund is given by

\[
q^M_F = \frac{1}{\gamma(1 - f_M)}(m - RP_F),
\]

as in equation (8) of Grossman and Stiglitz (1980), where \( R \) is the risk-free gross return, and uninformed demand for the index fund is given by

\[
q^U_F = \frac{1}{\gamma \text{var}[M|P_F]}(E[M|P_F] - RP_F).
\]

If \( P_F \) takes the form in (18), then \( E[M|P_F] = K_F(P_F - a_F) + \bar{m} \),

\[
\text{var}[M|P_F] = \text{var}[m|P_F] + \text{var}[\epsilon_M] = f_M\sigma^2_M(1 - K_Fb_F) + (1 - f_M)\sigma^2_M,
\]

\[
K_F = \frac{b_F\sigma^2_M}{b_F^2f_M\sigma^2_M + c_F^2\sigma^2_X}.
\]

Demands of the micro-informed agents are given by the following proposition.

**Proposition 4.1.** If the prices \( P_F \) and \( P_i \) take the form in (18) and (19), then the demands of \( i \) informed agents are given by

\[
q^i_i = \frac{R}{\gamma(1 - f_S)}\sigma^2_S(P_F + s_i/R - P_i),
\]

\[
q^i_F = q^U_F - q^i_i.
\]

Equation (24) shows that a micro-informed agent’s demand for the index fund consists of two components. The first component is the demand \( q^U_F \) of the uninformed agents: neither the micro-informed nor the uninformed have any information about \( M \) beyond that contained in \( P_F \). The second term \(-q^i_i\) offsets the exposure to \( M \) that the micro-informed agent takes on by holding stock \( i \). We interpret the second term as the micro-informed’s hedging demand: the micro-informed use the index fund to hedge out excess exposure to \( M \) that they get from speculating on their signal \( s_i \). The net result is that micro-informed and uninformed agents
have the same exposure to \( M \).

Substituting (24) in (15) — which gives the aggregate index fund demand — and combining this with the index fund market clearing condition in (17) yields

\[
(\lambda_U + \lambda_S)q^U_F + \lambda_M q^M_F = X_F. \tag{25}
\]

With the demands (20)–(21) for the index fund and demands (23)–(24) for individual securities, market-clearing prices are given by the following proposition:

**Proposition 4.2.** The market clears at an index fund price of the form (18),

\[
P_F = a_F + b_F(m - \bar{m}) + c_F(X_F - \bar{X}_F), \quad \text{with} \quad \frac{c_F}{b_F} = -\frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M}, \tag{26}
\]

and prices for individual stocks \( i \) of the form (19), given by

\[
P_i = P_F + s_i \frac{R}{\lambda_S R} - \frac{\gamma(1 - f_S)\sigma_S^2}{\lambda_S R} (X_i + \xi). \tag{27}
\]

The no-arbitrage condition (4) is satisfied if and only if \( \xi = 0 \).

The form of the index fund price \( P_F \) follows from Grossman and Stiglitz (1980); explicit expressions for the coefficients \( a_F, b_F, \) and \( c_F \), are derived in the appendix. Comparison of (19) and (27) shows that the ratio \( c_S/b_S \) in the price of stock \( i \) has exactly the same form as \( c_F/b_F \) in the price of the index fund in (26). In fact, if \( \lambda_M = 1 \), then \( b_F = 1/R \) and \( c_F \) has exactly the same form as \( c_S \). The stock \( i \) equilibrium is the direct analog of the index fund equilibrium with only macro-informed agents, making the present model a natural extension of Grossman and Stiglitz (1980) to the multi-security case.

When the proportions \( \lambda_U, \lambda_M, \) and \( \lambda_S \) are all endogenously positive and \( f_M > f_S \), the constrained model solved by Proposition 4.2 realizes the equilibrium conditions of Theorem 2.1 for any choice of cost parameter \( c \) satisfying \( e^{2\gamma c}(1 - f_S) < 1 \). The constrained model is more general in the sense that it does not impose a relationship between the information ratios \( f_M \) and \( f_S \). With prices as in Proposition 4.2 we can drop \( P_F \) and \( P_S, \) from the conditioning in (11) and write (11) as

\[
\kappa = 1 - f_M = \delta_F(1 - f_S). \tag{28}
\]

Here we need \( f_M > f_S \): the informativeness of the macro signal \( m \) is greater than that of the

Using (24) we see that \( N^{-1}\lambda_S \sum_i q^i_F \) in (15) equals \( \lambda_S q^U_F - N^{-1} \sum_i \lambda_S q^i \). Using the first equation in (17) this becomes \( \lambda_S q^U_F - N^{-1} \sum_i (X_i + \xi) = \lambda_S q^U_F - \xi \), and the second equation in (17) then yields (25).
micro signal $s_i$. We will see in Sections 5.2 and 6 that (e3) leads to an interior equilibrium in the constrained model through (28).

The equality $\kappa = \delta_F(1 - f_S)$ suggests an alternative interpretation of $\delta_F$. To trade on their signal $s_i$, micro-informed investors trade stock $i$, which changes their exposure to macro risk, compared with an uninformed investor. We can interpret $\delta_F$ as the capacity consumed by hedging this extra macro risk, leaving informativeness $f_S$ for $s_i$. A fixed $\delta_F$ then means that hedging capacity does not depend on the informativeness of prices.

### 4.2 Price efficiency

We will investigate the extent to which prices reflect available information, and to do so we need a measure of price efficiency. For the case of the index fund, we define price efficiency, $\rho^2_F$, as the proportion of price variability that is due to variability in $m$, the knowable portion of the aggregate dividend. This is the $R^2$ from regressing $m$ on $P_F$.

The squared correlation between $P_F$ in (18) and $m$ is given by

$$\rho^2_F = \frac{b^2_F f_M \sigma^2_M}{b^2_F f_M \sigma^2_M + c^2_F \sigma^2_X}. \tag{29}$$

This equals $b_F K_F$ in (22), so we can use (22) to write $\text{var}[m|P_F] = f_M \sigma^2_M (1 - \rho^2_F)$. As the price efficiency goes to 1, $P_F$ becomes fully revealing about $m$. Dividing both sides of (29) by $b^2_F \sigma^2_M$ and using the expression for $c_F/b_F$ in (26), we get

$$\rho^2_F = \frac{f_M}{f_M + \gamma^2 (1 - f_M)^2 \sigma^2_M \sigma^2_X / \lambda^2_M}. \tag{30}$$

For stock $i$ we define price efficiency as the proportion of the variability of the price that is driven by variability in $s_i$, the knowable part of the idiosyncratic dividend shock, once $P_F$ is known. From the functional form of $P_i$ in (19) and the fact that $\xi = 0$, this is given by

$$\rho^2_S = \frac{b^2_S f_S \sigma^2_S}{b^2_S f_S \sigma^2_S + c^2_S \sigma^2_X} = \frac{f_S}{f_S + \gamma^2 (1 - f_S)^2 \sigma^2_S \sigma^2_X / \lambda^2_S}, \tag{31}$$

using the expression for $c_S/b_S$ in (27). As in the case of the index fund, as $\rho^2_S$ goes to 1, $P_i$ becomes fully revealing about $s_i$.

Differentiating (30) and (31) and straightforward algebra, yields the following results:

**Proposition 4.3** (When are prices more informative?)
(i) Micro (macro) prices are more efficient as either (a) the fraction of micro (macro) informed increases, or (b) as the micro (macro) signal informativeness improves:

\[ \frac{d\rho^2_S}{d\lambda_S} > 0 \quad \text{and} \quad \frac{d\rho^2_F}{d\lambda_M} > 0, \]

and

\[ \frac{d\rho^2_S}{df_S} > 0 \quad \text{and} \quad \frac{d\rho^2_F}{df_M} > 0. \]

(ii) Furthermore, when the fraction of micro (macro) informed is zero, or when the signals are non-informative, price efficiency is zero. In other words, \( \rho^2_F \to 0 \) as either \( \lambda_M \to 0 \) or \( f_M \to 0 \), and \( \rho^2_S \to 0 \) as either \( \lambda_S \to 0 \) or \( f_S \to 0 \).

(iii) When the signals are perfectly informative, prices become fully revealing. In other words, \( \rho^2_F \to 1 \) as \( f_M \to 1 \) if \( \lambda_M > 0 \), and \( \rho^2_S \to 1 \) as \( f_S \to 1 \) if \( \lambda_S > 0 \).

5 Attention equilibrium in the constrained model

In Section 2, we allowed investors to acquire information processing capacity at a cost and then to allocate this capacity. In this section, we focus on the allocation decision, taking the decision to acquire capacity or remain uninformed as given. In other words, we hold \( \lambda_U \) fixed and investigate the equilibrium mix of \( \lambda_M \) and \( \lambda_S \). In the next section, we endogenize \( \lambda_U \) as well.

Part of the cost of becoming informed lies in developing the skills needed to acquire and apply investment information, and this process takes time. In the near term, these requirements leave the total fraction of informed investors \( \lambda_M + \lambda_S \) fixed. Once investors have the skills needed to become informed, we suppose that they can move relatively quickly and costlessly between macro and micro information by shifting the focus of their attention. Kacperczyk et al. (2014) provide empirical evidence of exactly this dynamic. We therefore distinguish a near-term attention equilibrium, in which \( \lambda_U \) is fixed and the split between \( \lambda_M \) and \( \lambda_S \) is endogeneous, from a longer-term information equilibrium, in which the decision to become informed is endogenized along with the choice of information on which to focus. We analyze the attention equilibrium in this section and address the information equilibrium in Section 6.
5.1 Relative utility

Recall that an investor’s ex ante expected utility is given by $J \equiv \mathbb{E}[-\exp(-\gamma \bar{W}_2)]$, where the expectation is taken unconditionally over time 2 wealth. Write $J_M$, $J_S$, and $J_U$ for expected utility of macro-informed, micro-informed, and uninformed investors, respectively.

Fixing the fraction of uninformed, the following proposition establishes the relative benefit of being macro- or micro-informed relative to being uninformed.

**Proposition 5.1.** If the cost of becoming informed is given by $c$, then the benefit of being macro-informed relative to being uninformed is given by

$$J_M/J_U = \exp(\gamma c) \left( 1 + \frac{f_M}{1 - f_M} (1 - \rho_F^2) \right)^{-\frac{1}{2}}.$$  \hspace{1cm} (32)

The benefit of being micro-informed relative to being uninformed is given by

$$J_S/J_U = \exp(\gamma c) \left( 1 + \frac{f_S}{1 - f_S} \left( \frac{1}{\rho_S^2} - 1 \right) \right)^{-\frac{1}{2}}.$$  \hspace{1cm} (33)

Note that because utilities in our model are negative, a decrease in these ratios represents a gain in informed relative to uninformed utility. Each of the ex ante utility ratios in the proposition is increasing in the corresponding measure of price efficiency — that is, informed investors become progressively worse off relative to uninformed as micro or macro prices become more efficient. But the dependence on $\rho^2_S$ in (33) differs from the dependence on $\rho^2_F$ in (32).

Recalling from Proposition 4.3 that macro and micro price efficiency increase in $\lambda_M$ and $\lambda_S$, respectively, we immediately get the following proposition which extends the substitutability result of Grossman and Stiglitz (1980) to our setting:

**Proposition 5.2 (Substitutability).** $J_S/J_U$ strictly increases (making micro-informed worse off) in $\lambda_S$. $J_M/J_U$ strictly increases (making macro-informed worse off) in $\lambda_M$.

Figure 2 illustrates the results of Propositions 5.1 and 5.2. The figure holds $\lambda_U$ fixed, and the x-axis is indexed by $\lambda_M$. As $\lambda_M$ increases, $J_M/J_U$ increases, indicating that the macro-informed are becoming worse off. Similarly, at the rightmost point of the graph, $\lambda_S = 0$, and as we move to the left, $J_S/J_U$ increases, indicating that the micro-informed are becoming worse off as more of their type enter the economy.\footnote{Our numerical examples use the parameters calibrated to market data in Section 7.1.}

\footnote{Many of our comparisons could be recast as statements about trading intensities, in the sense of Goldstein and Yang (2015). Macro and micro trading intensities are given by $-b_F/c_F$ and $-b_S/c_S$ in Proposition 4.2.}
Figure 2: The information equilibrium for a fixed number of uninformed investors. Relative utilities are shown assuming cost of becoming informed is $c = 0$. Parameter values are given in Table 1.

5.2 Choice between macro and micro information

At an interior equilibrium, the marginal investor must be indifferent between macro and micro information, in which case equilibrium will be characterized by a $\lambda_M^*$ such that with that many macro-informed investors and with $1 - \lambda_U - \lambda_M^*$ micro-informed investors we will have $J_M = J_S$, which just sets (32) equal to (33). To cover the possibility of a corner solution, we define an attention equilibrium by a pair of proportions $\lambda_M \geq 0$ and $\lambda_S = 1 - \lambda_U - \lambda_M \geq 0$ satisfying

$$J_M < J_S \Rightarrow \lambda_M = 0 \quad \text{and} \quad J_S < J_M \Rightarrow \lambda_S = 0. \quad (34)$$

The inequalities in this condition are equivalent to $J_M/J_U > J_S/J_U$ and $J_S/J_U > J_M/J_U$, respectively, because $J_U < 0$.

Recall from Proposition 4.3 that when the fraction of macro- or micro-informed is zero, price efficiency is also zero. From (32) and (33), we see that

$$J_M/J_U(\lambda_M = 0) = e^{\gamma c} \sqrt{1 - f_M} \quad \text{and} \quad J_S/J_U(\lambda_S = 0) = 0. \quad (35)$$

From Proposition 5.2, we know that $J_M/J_U$ and $J_S/J_U$ both increase monotonically (i.e., make the informed worse off) with their respective $\lambda$’s. When $\lambda_M$ is zero, the macro-informed achieve their maximal utility; when $\lambda_M = 1 - \lambda_U$, the micro-informed achieve their maximal utility. As $\lambda_M$ increases from zero to $1 - \lambda_U$, $\lambda_S$ decreases, so the macro-informed become progressively worse off and the micro-informed become progressively better off. If at some $\lambda_M$ the two curves $J_M/J_U$ and $J_S/J_U$ intersect, we will have an interior equilibrium, and it must be unique because of the strict monotonicity in Proposition 5.2. This case is illustrated
in Figure 2.

If there is no interior equilibrium, then either macro or micro information is always preferred, and no investor will choose the other. Such a scenario is possible in the constrained model of Section 3, though not under the more general information choices in Theorem 2.1.

To see why the setting of Theorem 2.1 ensures an interior equilibrium, consider how the first equality in (35) is impacted by (e3). Combining (35) with (28) yields that
\[
\frac{J_M}{J_U} = e^{\gamma_c} \sqrt{\kappa} \text{ at } \lambda_M = 0.
\]
From (e3) we have that \(e^{2\gamma_c} \kappa < \delta_F\) and \(\delta_F < 1\) by assumption. Therefore \(J_M/J_U < 1\) at \(\lambda_M = 0\), and some uninformed investors will prefer to become macro-informed, resulting in \(\lambda_M > 0\). In this sense, (e3) leads to an interior attention equilibrium.

To make our concept of attention equilibrium precise, let us define
\[
\tilde{\lambda}_M \equiv (1 - \lambda_U) \frac{1 - \sqrt{\varphi + (1 - \varphi) \frac{\sigma^2_F}{(1 - \lambda_U)^2}}}{1 - \varphi},
\]
where
\[
\varphi = \frac{(1 - f_S) \sigma^2_S \sigma^2_X}{(1 - f_M) \sigma^2_M \sigma^2_{X_F}} \quad \text{and} \quad \alpha = \frac{1 - f_M}{f_M} (1 - f_S) \sigma^2_S \sigma^2_X.
\]
Note that \(\varphi\) is the ratio of the total risk arising from the unknowable portion of idiosyncratic supply shocks (the variance of \(\epsilon_i\) times the variance of \(X_i\)) to the total risk arising from macro supply shocks (the variance of \(\epsilon_M\) times the variance of \(X_F\)). The larger \(\varphi\) the more total unknowable risk comes from idiosyncratic rather than systematic sources.

The following proposition characterizes the equilibrium allocation of attention in the economy between macro information and micro information when the total fraction of informed investors \(1 - \lambda_U\) is fixed.

**Proposition 5.3 (Attention equilibrium).** Suppose \(0 \leq \lambda_U < 1\), so some agents are informed.

(i) Interior equilibrium. If \(\tilde{\lambda}_M \in [0, 1 - \lambda_U)\), then this point defines the unique equilibrium:

- at \(\lambda_M^* = \tilde{\lambda}_M\), the marginal informed investor will be indifferent between becoming macro- or micro-informed.

(ii) If \(\tilde{\lambda}_M \not\in [0, 1 - \lambda_U)\), the unique equilibrium is at the boundary \(\lambda_M^* = 0\), where all informed agents are micro-informed.

---

\(^{13}\)That \(J_S/J_U = 0\) when no one is micro-informed ensures that some agents choose that information set.

\(^{14}\)This expression has a finite limit as \(\varphi \to 1\), and we take that limit as the value of \(\tilde{\lambda}_M\) at \(\varphi = 1\).

\(^{15}\)We refer to (i) as the case of an interior equilibrium, even though it includes the possibility of a solution at the boundary. If \(\tilde{\lambda}_M = \lambda_M = 0\), then \(J_M = J_S\), and the marginal investor is indifferent between micro and macro information, which is what we mean by an interior equilibrium. If case (ii) holds, then \(\lambda_M^* = 0\) because micro information is strictly preferred over macro information at all \(\lambda_M\).
In equilibrium, we always have $\lambda_M^* < 1 - \lambda_U$. In other words, some informed agents will choose to be micro-informed.

It bears emphasizing that an attention equilibrium — regardless of parameter values — precludes all informed agents from being macro-informed. In contrast, it is possible for all informed agents to be micro-informed. We therefore have, as a fundamental feature of the economy, a bias for micro over macro information. This property holds in the constrained model, where informed investors are limited to macro or micro information, and the micro-informed receive all the benefits of providing liquidity to noise traders in individual stocks. We know from Theorem 2.1 that in a setting with greater information choices, an equilibrium will contain both micro- and macro-informed investors.

An increase in micro (macro) volatility, as measured by $\sigma_S \sigma_X$ ($\sigma_M \sigma_{X_F}$), will increase the benefit of information to the micro (macro) informed, and will therefore decrease (increase) $\lambda_M^*$ when the economy is at an interior equilibrium:

**Proposition 5.4** (Effects of risk aversion and risk on the attention equilibrium). We consider the case of an interior equilibrium with $\lambda_M^* > 0$.

(i) Risk aversion pushes investors towards micro information: $d\lambda_M^*/d\gamma < 0$.

(ii) Increase in micro (macro) risk pushes investors towards micro (macro) information:

\[
\frac{d\lambda_M^*}{d(\sigma_S \sigma_X)} < 0 \quad \text{and} \quad \frac{d\lambda_M^*}{d(\sigma_M \sigma_{X_F})} > 0.
\]

### 5.3 Relative price efficiency

Define

\[
\tau_M \equiv \frac{f_M/(1 - f_M)}{f_S/(1 - f_S)}
\]

to measure the relative informativeness of the macro and micro signals. Under condition \[28], $f_M > f_S$ so $\tau_M > 1$; however, in the general setting of the constrained model of Section 3 any $\tau_M \geq 0$ is feasible. We therefore explore the full range of possible $\tau_M$ values but put particular emphasis on the case $\tau_M > 1$.

In an interior attention equilibrium, the marginal informed investor is indifferent between macro and micro information because $J_M/J_U = J_S/J_U$. From Proposition 5.1 we see this condition then implies that micro price efficiency is related to macro price efficiency via $(1 - \rho_S^2)/\rho_S^2 = \tau_M(1 - \rho_F^2)$, which yields

\[
\rho_S^2 = \frac{1}{1 + \tau_M(1 - \rho_F^2)}.
\]
Micro vs macro price efficiency

Figure 3: The figure shows the difference between micro and macro price efficiency, $\rho_S^2 - \rho_F^2$, as a function of macro price efficiency $\rho_F^2$. The solid (dashed) portion of the curve represents the region of micro (macro) efficiency. The red points are labeled with the values of $\lambda_U$ corresponding to that particular $\rho_F^2$. The vertical portion of the curve corresponds to corner equilibria with $\lambda_M = 0$. Parameter values are given in Table 1.

As markets become fully macro efficient ($\rho_F^2 \to 1$), they must also become fully micro efficient ($\rho_S^2 \to 1$), and vice versa. However, as macro price efficiency tends towards zero, micro price efficiency tends towards $1/(1 + \tau_M)$. Since both sides of (38) are decreasing as their respective $\rho^2$ falls, this also represents the lower bound for $\rho_S^2$ in an interior attention equilibrium. When less information is revealed in equilibrium, the economy tends towards micro efficiency – suggesting micro information is in a sense more valuable than macro information.

It follows from (38) that $\rho_S^2 > \rho_F^2$, i.e., markets are more micro efficient, whenever $\rho_F^2 < 1/\tau_M$, which certainly holds if $\tau_M < 1$. For the calibrated parameter values in Section 7.1, $\tau_M = 1.51$, and the difference in price efficiencies $\rho_S^2 - \rho_F^2$ as a function of $\rho_F^2$ is illustrated in Figure 3. The solid (dashed) portion of the curve represents the region in which micro efficiency exceeds (is less than) macro efficiency. The red dots in the figure represent attention equilibria at a given level of $\lambda_U$, with $\lambda_M$ given by equation (36) and $\rho_F^2$ and $\rho_S^2$ determined by (30) and (31) respectively. The vertical portion of the curve represents corner equilibria with no macro informed investors. Unless the number of uninformed investors is implausibly low, the economy is in a region of micro efficiency.
5.4 Impacts of information precision

Recall from (9) and (10) that $f_M$ and $f_S$ measure the fraction of variation in $M$ and $S_i$ that is known to informed investors. We refer to this as information precision. Surprisingly, more precise micro information makes the micro-informed worse off:

**Proposition 5.5** (The micro-informed overtrade on their information). *More precise information is worse for the micro-informed in the sense that*

$$\frac{d(J_S/J_U)}{df_S} > 0 \quad \text{(micro informed are worse off)}.$$  

When investors become micro-informed, the more they know about the ultimate idiosyncratic portion of the payout $S_i$, the less uncertainty they face from owning the stock. From (27) we see that the discount in the stock price due to idiosyncratic supply shocks $X_i$ will be zero when the micro information is perfect, i.e., when $f_S = 1$. With no discount in the price, the compensation for liquidity provision goes to zero. Because atomic informed agents cannot act strategically and coordinate to limit their liquidity provision in an optimal (for them) way, uncertainty about the dividend helps them by decreasing the sensitivity of their demand to price shocks, which in turn leads to a higher risk premium in prices. In contrast to the micro-informed, the macro-informed may be better or worse off as their precision, $f_M$, improves (see Figure 4):

**Proposition 5.6** (The macro-informed can be better or worse off with more information). *More precise information is better for the macro-informed if and only if*

$$\rho_F^2 < \frac{1}{1 + f_M}, \quad \text{or equivalently} \quad \lambda_M < \gamma \sigma_M \sigma_{X_F} \frac{1 - f_M}{f_M}. \quad (39)$$

*In this case,*

$$\frac{d(J_M/J_U)}{df_M} < 0 \quad \text{(macro-informed are better off)}.$$  

To gain intuition into this result recall that at $f_M = 0$ we would have $\rho_F^2 = 0$ (price reveals nothing when nothing about $M$ is knowable), and at $f_M = 1$ we would have $\rho_F^2 = 1$ (prices are fully revealing when $M$ is fully known). Furthermore, from Proposition 4.3 we know $\rho_F^2$ increases monotonically in $f_M$. So (39) implies that the macro-informed benefit from an increase in the precision $f_M$ only when $f_M$ (hence also the price informativeness $\rho_F^2$) is low. Equivalently the condition can be reinterpreted as placing a limit on how many macro-informed investors the economy can support before better macro precision begins to make the macro-informed worse off.
The contrast between micro and macro information in Proposition 5.5 and 5.6 can be understood as follows. In the market for the index fund, informed investors trade against uninformed investors as well as taking the other side of price insensitive liquidity shocks, introducing an effect that is absent in the market for individual stocks. With a low signal precision, prices are not very informative, so a small improvement in precision gives the macro-informed an informational edge over the uninformed, allowing the informed to extract rents in trading. However, as the signal precision improves and price efficiency grows, the incremental ability to extract rents from trading against the uninformed diminishes, while the tendency to overtrade on information (as in the case of the market for individual stocks) grows.

As illustrated in Figure 4, these propositions imply that $\lambda^*_M$ increases in $f_S$, and it also increases in $f_M$ as long as the condition in (39) holds.

6 Information equilibrium in the constrained model

We now examine a longer-term equilibrium in which the uninformed can become informed by incurring a cost $c$. In other words, while continuing to work within the constrained model of Section 3, we now endogenize not only the choice between micro and macro information, but also the decision to become informed. An equilibrium in this setting — which we refer to as an information equilibrium — is defined by nonnegative proportions $(\lambda_M, \lambda_S, \lambda_U = 1 - \lambda_M - \lambda_S)$

---

16 This is discussed in more detail in the Internet Appendix [https://sites.google.com/view/hmamaysky](https://sites.google.com/view/hmamaysky). See Bond and Garcia (2017) for a similar result where increased signal precision can make the informed worse off.
such that no agent of a type in positive proportion prefers switching to a different type. Extending (34), we require that, for any $\iota, \iota' \in \{M, S, U\}$,

$$J_\iota / J_{\iota'} > 1 \Rightarrow \lambda_\iota = 0.$$  \hspace{1cm} (40)

Recall that our utilities are negative, so the inequality on the left implies that type $\iota'$ is preferred to type $\iota$.\textsuperscript{17}

Figure 5 helps illustrate the general results that follow. The figure plots the equilibrium proportion of each type of investor in the constrained model as a function of the cost $c$ of information acquisition. The figure divides into three regions based on the fractions of investor types. At sufficiently low costs, all agents prefer to become informed, so $\lambda_U = 0$. At sufficiently high costs, no investors choose to be macro-informed, so $\lambda_M = 0$. At intermediate costs, we find agents of all three types, and this is the region of overlap with Theorem 2.1. At all cost levels, some fraction of agents choose to be micro-informed.

The figure also shows a vertical line at the largest value of $c$ for which (e3) holds. This condition requires $e^{2\gamma c} K < \delta_F$, which simplifies in the constrained model to $e^{2\gamma c}(1 - f_S) < 1$, in light of (28). The constrained model thus exhibits the conditions of Theorem 2.1 in the region to the left of the vertical line with $\lambda_U > 0$. In other words, for cost parameters within this interval, Theorem 2.1 implies that specialization is inevitable: informed investors would continue to specialize in micro or macro information even if we removed the specialization constraint and allowed them to select other combinations of signals and portfolios.

To give an explicit characterization of the information equilibrium at each cost level $c > 0$, we first consider the possibility that all three types of agents are present in positive proportions. To be consistent with equilibrium, this outcome requires $J_M / J_U = J_S / J_U = 1$. Using the expressions for these ratios in (32) and (33), these equalities imply

$$\rho_F^2 = 1 - \frac{1 - f_M}{f_M} \left[ e^{2\gamma c} - 1 \right].$$ \hspace{1cm} (41)

and

$$\rho_S^2 = \left( 1 + \frac{1 - f_S}{f_S} \left[ e^{2\gamma c} - 1 \right] \right)^{-1}.$$ \hspace{1cm} (42)

Setting these expressions equal to (30) and (31), respectively, we can solve for $\lambda_M$ and $\lambda_S$ to

\textsuperscript{17}The ratios $J_M / J_U$, $J_S / J_U$, and $J_S / J_M$ all have well-defined limits as some or all of $\lambda_M$, $\lambda_S$, and $\lambda_U$ approach zero. (This follows from the expressions for these ratios in (32) and (33) and the dependence of $\rho_F^2$ and $\rho_S^2$ on $\lambda_M$ and $\lambda_S$ in (30) and (31), respectively.) We may therefore evaluate and compare these ratios even in cases where one or more of the proportions $\lambda_\iota$ is zero.
Figure 5: Equilibrium proportions of macro-informed, micro-informed, and uninformed agents as functions of the cost of information acquisition $c$. The vertical, dashed purple line shows the maximum $c$ such that condition (e3), $e^{2\gamma c} K < \delta_F$, holds. Parameter values are given in Table I.

Let

$$\lambda_M(c) = \gamma (1 - f_M) \sigma_M \sigma_X \left( \frac{1}{(1 - f_M)} \left( \frac{1}{(e^{2\gamma c} - 1)} - \frac{1}{f_M} \right) \right)^{1/2}$$

and

$$\lambda_S(c) = \gamma (1 - f_S) \sigma_S \sigma_X \left( \frac{1}{(1 - f_S)} \left( \frac{1}{(e^{2\gamma c} - 1)} \right) \right)^{1/2}.$$  \hspace{1cm} (44)

The expression for $\lambda_M(c)$ is valid for $c \leq \bar{c}$, with

$$\bar{c} = -\frac{1}{2\gamma} \log(1 - f_M);$$  \hspace{1cm} (45)

set $\lambda_M(c) = 0$ for $c > \bar{c}$. If $\lambda_M(c) + \lambda_S(c) \leq 1$ with $c \leq \bar{c}$, then $(\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c))$ defines an information equilibrium with $J_M = J_S = J_U$.

Both $\lambda_M(c)$ and $\lambda_S(c)$ increase continuously and without bound as $c$ decreases toward zero, so the equation

$$\lambda_M(c) + \lambda_S(c) = 1,$$

defines the lowest cost at which we can meaningfully set $\lambda_U = 1 - \lambda_M(c) - \lambda_S(c)$. At lower
cost levels, we need to consider the possibility of an equilibrium with $\lambda_U = 0$.

Once we fix a value for $\lambda_U$, the split between macro- and micro-informed agents is characterized by Proposition 5.3. Write $\lambda^*_M(0)$ for the value of $\lambda^*_M$ in Proposition 5.3 at $\lambda_U = 0$; this value is given either by the root $\tilde{\lambda}_M$ in (36) or zero. Set

$$\lambda^*_M, \lambda^*_S, \lambda^*_U = \begin{cases} \lambda^*_M(0), 1 - \lambda^*_M(0), 0, & 0 < c < \underline{c}; \\ \lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c), & \underline{c} \leq c < \bar{c}; \\ 0, \lambda_S(c), 1 - \lambda_S(c), & c \geq \max\{\underline{c}, \bar{c}\}. \end{cases} \tag{46}$$

Then (46) makes explicit the equilibrium proportions illustrated in Figure 5 for the case $\underline{c} < \bar{c}$: at large cost levels, $\lambda_M = 0$; at low cost levels, $\lambda_U = 0$ and $\lambda_M$ and $\lambda_S$ are constant; at intermediate cost levels, all three proportions are positive; at all cost levels, $\lambda_S > 0$. We always have $\underline{c} > 0$ and $\bar{c} < \infty$, so the low cost and high cost ranges are always present; but it is possible to have $\underline{c} \geq \bar{c}$, in which case the intermediate cost range is absent. This occurs when $\lambda_S(\bar{c}) \geq 1$. By evaluating (44) at (45), we find that $\lambda^2_S(\bar{c}) = \gamma^2 \alpha$, with $\alpha$ as in (37). If $\gamma^2 \alpha \geq 1$, then the root $\tilde{\lambda}_M$ in (36) evaluated at $\lambda_U = 0$ is less than or equal to zero if it is real, so $\lambda^*_M(0) = 0$. We summarize these observations in the following result.

**Proposition 6.1 (Information equilibrium in the constrained model).** **At each** $c > 0$, the proportions in (46) define the unique information equilibrium. **If** $\gamma^2 \alpha < 1$, **then** $\underline{c} < \bar{c}$ **and all three cases in (46) are present. If** $\gamma^2 \alpha \geq 1$, **then** $\underline{c} \geq \bar{c}$, **the second range in (46) is empty, and no investors choose to be macro-informed at any cost level.**

With this result, we can revisit some of the conditions in Theorem 2.1 when applied to the constrained model. Recalling from (28) that $\kappa = 1 - f_M$, condition (e3) implies $c < \bar{c}$, the condition for $\lambda_M(c) > 0$. Combining (e3) with (e4) ensures that we are in the range $\underline{c} < c < \bar{c}$ and thus that all three investor proportions are positive.

From Proposition 6.1, we can deduce several further properties of the information equilibrium. Let us define $\Pi_M$ as the fraction of informed who are macro-informed, or

$$\Pi_M \equiv \frac{\lambda_M}{\lambda_M + \lambda_S} = \frac{\lambda_M}{1 - \lambda_U}. \tag{47}$$

At an interior attention equilibrium ($\lambda^*_M > 0$), $\Pi_M$ is the coefficient of $(1 - \lambda_U)$ in (36). Differentiating with respect to $\lambda_U$ yields $d\Pi_M/d\lambda_U < 0$, when $\lambda_M > 0$: the more uninformed investors there are in the economy, the greater the fraction of informed investors who choose to be micro-informed. The next result describes the dependence of $\Pi_M$ on $c$. 

33
Corollary 6.1 (Effect of information cost $c$ on information equilibrium). In equilibrium, with a cost of becoming informed given by $c$, the following will hold:

(i) As $c$ increases, the fraction $\Pi_M$ of informed investors who choose macro information falls; moreover, $\Pi_M$ is strictly decreasing in $c$ if $\lambda_M > 0$ and $\lambda_U > 0$.

(ii) As $c$ increases the fraction of investors who are uninformed increases; moreover $\lambda_U$ is strictly increasing in $c$ wherever $\lambda_U > 0$.

(iii) Micro and macro price efficiency are decreasing in $c$.

As $c$ increases and the number of uninformed grows, the benefit to being informed increases, in the sense that $J_M/J_U$ and $J_S/J_U$ decrease, as shown Proposition 5.2. However, the micro-informed gain more than the macro-informed. In order to maintain the attention equilibrium at a higher $c$, we need more micro-informed to equilibrate the relative benefits of micro versus macro information. Therefore, $\Pi_M$ must fall when $c$ increases.

7 Model implications: Variance decompositions

To study changes in expected stock returns, Campbell (1991) decomposes the variance of aggregate market returns into variance from cash flow news and variance from news about discount rates. We will use “VR” to abbreviate “variance ratio” in discussing the ratio of cash flow variance to discount rate variance. Vuolteenaho (2002) estimates a similar decomposition for individual firms and finds a much larger VR for individual firms than for the aggregate market. Jung and Shiller (2005) call the cash flow component the efficient market component of returns and they call the discount rate component the inefficient market component. They interpret the larger VR for individual stocks as evidence for Samuelson’s dictum: greater micro efficiency than macro efficiency.

In this section, we show that our model produces results consistent with empirical patterns when calibrated to market data. We also compare our model’s results with historical trends. For this comparison, we argue that the past century has seen a reduction in the cost of becoming informed and an increase in the “indirect” supply of the macro factor, by which we mean trading in the index (through ETFs and derivatives) that does not involve trading in the individual stocks. We then examine how VRs in our model respond to these changes and compare the results with historical trends.

We also show that the micro VR and macro VR can respond differently to an increase in the fraction of informed investors. In particular, our model predicts an increase in the micro
VR and a U-shaped change in the macro VR as functions of the number of informed. These effects follow from the endogenous specialization in investor information choice in our model. They contrast with the model of Kacperczyk et al. (2016), where all informed investors have the same information, and micro and macro VR always decrease or remain unchanged as the fraction of informed investors increases.

7.1 Calibration

For our calibration we normalize the aggregate mean dividend level to equal one by setting \( \bar{m} = \bar{X}_F = 1 \). We think of the one period in our model as representing a year.

(Supply shocks and turnover) We calibrate the share volatilities \( \sigma_{X_F} \) and \( \sigma_X \) to annual turnover. Lo and Wang (2000, Table 3) find that value-weighted stock turnover – shares traded divided by shares outstanding – in the US over the period 1987–1996 averaged 1.25% per week. This implies an annual turnover of \( 52 \times 1.25\% = 65\% \). We update their results by calculating the equal-weighted average turnover for the Dow Jones Industrial average from 1980 to 2018. The Dow Industrials turnover has averaged 76% over this time period. In our model, we measure equal-weighted index turnover as \( \frac{1}{N} \sum_{i=1}^{N} |X_F - \bar{X} + X_i| \). We therefore require

\[
E|X_F - \bar{X} + X_i| = \sqrt{\frac{2}{\pi}} \times \sqrt{\sigma_{X_F}^2 + \sigma_X^2} = 0.76,
\]

using a standard result for the normal distribution. We then regress firm-level on index turnover (annualized, in rolling windows) and find that the average \( R^2 \) in these regression is 47%. If we define stock-level turnover in our model as \( |X_F - \bar{X} + X_i|/N \), then we would like the \( R^2 \) of the regression of stock turnover on index turnover,

\[
|X_F - \bar{X} + X_i| = a + b \sum_{j=1}^{N} |X_F - \bar{X} + X_j| + \text{Noise}, \tag{48}
\]

to be 47%. Using results from Kamat (1958) (see the Internet Appendix), this gives us a second equation in \( \sigma_{X_F} \) and \( \sigma_X \), which we solve to get \( \sigma_{X_F} = 0.805 \) and \( \sigma_X = 0.509 \).

We assume that all idiosyncratic trading demand in an individual stock takes places via trading in the stock itself, and therefore is captured by our firm-level turnover measure. However, we believe that our bottom-up turnover index meaningfully understates actual index

\(^\text{18}\)Lo and Wang (2000, Table 7) show that the first two principal components of turnover-beta sorted portfolios account for close to 90% of portfolio weekly turnover. Our number is lower because we are interested in stock-level turnover.
level liquidity demand. As we argue in the Internet Appendix, turnover of the most liquid stock futures and ETFs is approximately 60% of the market capitalization of the Russell 3000 index. Compared to our 76% bottom-up turnover estimate for the Dow Industrials, the actual liquidity demand for index trading – which is the quantity that $X_F$ proxies for in our model – is conservatively twice as high in the current market.\(^{19}\) To proxy for an increase in this type of indirect index turnover (which does not involve turnover in individual stocks), we use $X_F$ volatility levels of $\ell \times \sigma_{X_F}$, with $\ell = 1, 2$ or 3.

**Dividend volatility** To map our single-period model to a multi-period environment, we interpret the aggregate dividend $M$ paid at the end of the period as the discounted value of $N$ months of future dividends. We show in the Internet Appendix that a dividend series calibrated to S&P500 data has peak per unit volatility of 0.115 at a time horizon of $N = 54$ months. Given our normalization that $E[M] = 1$, we use this for our choice of $\sigma_M$.

Ball, Sadka, and Sadka (2009) and Bonsall, Bozanic, and Fischer (2013) show that in the US, between 60-80% of firm-level earnings variation (at quarterly or annual frequency) can be explained by contemporaneous macro factors. In light of this, we set

$$\frac{\sigma_M^2}{\sigma_M^2 + \sigma_S^2} = 0.75,$$

which implies $\sigma_S^2 = \sigma_M^2 / 3$.

**Dividend forecastability** Fama and French (2000, Table 2) regress year-ahead changes in earnings of US firms on a set of lagged market- and accounting-based explanatory variables (we assume earnings are paid out as dividends). They find that the $R^2$’s of these regressions range from 0.05 to 0.20.\(^{20}\) Under the assumption that informed investors have information superior to a simple regression model, we use a considerably higher degree of predictability by setting $f_S = 0.37$.

To come up with a one-year earnings forecastability benchmark for the S&P500 index we run a forecasting model similar to that in Nissim and Ziv (2001). We use market, analyst and accounting information to forecast one-year ahead S&P500 earnings and dividend growth.

\(^{19}\)Including index options trading or trade in over-the-counter derivatives would further increase turnover. \(^{20}\)Nissim and Ziv (2001, Table III) show that lagged accounting variables and dividend changes are able to explain 14.6% of variation in year-ahead earnings growth. Lev and Nissim (2004) study the effects of changes in accounting standards on earnings predictability. Using lagged accounting and market based variables, they find the $R^2$ of forecasting regressions for year-ahead earnings changes to be between 14-18% (Table 3).
In both sets of regressions (in the Internet Appendix) we forecast

$$\frac{CF[t, t + x] - CF[t]}{Book[t]}$$

where $CF[t, t + x]$ is the average payout (either earnings or dividends) over the next $x$ years, $CF[t]$ is the value of this variable over the prior 12 months, and $Book[t]$ is the current per share book value for the S&P500. For earnings we find that the $R^2$ increases from 0.303 to 0.689 as $x$ increases from 1 to 5 years, whereas for dividends we find that the $R^2$ falls from 0.639 to 0.296. The average $R^2$ between the two regressions is very stable over time, ranging from 0.47 to 0.49. We use $f_M = 0.47$ in our calibrations.

(Summary of calibration) Finally, we use an annual (one-period) interest rate of 2% (which is consistent with our choice discount rates for the dividend calibration in the Internet Appendix), and we set $\gamma = 5.5$. Our choice of risk aversion, interest rate, and aggregate dividend level leads to an annual excess return, $\bar{m}/a_F - R$, in the range of 6% (the risk premium depends on the level of $c$). Our parameters choices are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>5.5</td>
<td>$\bar{m}/a_F - R \approx 6%$</td>
</tr>
<tr>
<td>$R$</td>
<td>1.02</td>
<td>annual risk-free rate of 2%</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>1</td>
<td>normalization of dividend level $\bar{m} \equiv E[M] = 1$</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>1</td>
<td>normalization of share supply</td>
</tr>
<tr>
<td>$f_M$</td>
<td>0.47</td>
<td>$\text{var}[\bar{m}]/\text{var}[M] = 0.47$</td>
</tr>
<tr>
<td>$f_S$</td>
<td>0.37</td>
<td>$\text{var}[S_i]/\text{var}[S_i] = 0.37$</td>
</tr>
<tr>
<td>$\sigma^2_M$</td>
<td>0.115</td>
<td>$\sigma_M/E[M] = 0.115$</td>
</tr>
<tr>
<td>$\sigma^2_S$</td>
<td>0.115$^2/3$</td>
<td>$\sigma_M^2/(\sigma_M^2 + \sigma_S^2) = 3/4$</td>
</tr>
<tr>
<td>$\sigma^2_{X_F}$</td>
<td>0.805$^2$</td>
<td>together with $\sigma^2_X$ ensure $E[X_F - \bar{X} + X_i] = 0.76$</td>
</tr>
<tr>
<td>$\sigma^2_X$</td>
<td>0.509$^2$</td>
<td>together with $\sigma^2_{X_F}$ ensure $R^2$ in (48) equals 47%</td>
</tr>
</tbody>
</table>

Table 1: Parameter values and moment conditions for model calibration. For further details, see the Internet Appendix.

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\textsuperscript{21}The latter finding in consistent with Beeler and Campbell’s (2012) finding that the $R^2$’s of dividend and consumption growth rate forecasting regressions fall with the forecasting horizon. Our $R^2$’s are higher than their Table 6 partly because we are using average dividends paid over the forecast horizon whereas they are using dividend growth – the latter being less predictable.
7.2 Variance decomposition

The variance decompositions in Campbell (1991) and Vuolteenaho (2002) are based on returns. Since our normally distributed prices have a small probability of being negative, returns in our model are not always well defined. As is the convention in this literature (for example, Peng and Xiong 2006, Veldkamp 2006, and Kacperczyk et al. 2016), we instead analyze profits \( M - RP_F \) in the case of the index fund though we refer to them as returns. For the index fund,

\[
M - RP_F = m + \epsilon_M - RP_F = \text{constant} + \epsilon_m + (1 - Rb_F) m - Rc_F X_F. \tag{49}
\]

For time 0 investors, prior to the realization of prices, the variance of \( M - RP_F \) is

\[
Vol^2_{syst} = \sigma^2_{\epsilon_M} + (1 - Rb_F)^2 \sigma^2_m + R^2 c_F^2 \sigma^2_{X_F}. \tag{50}
\]

The ratio of the variance of cash flow news to that of discount rate news is

\[
VR_M \equiv \frac{\sigma^2_{\epsilon_M} + (1 - Rb_F)^2 \sigma^2_m}{R^2 c_F^2 \sigma^2_{X_F}}.
\]

Our market-adjusted individual stock return is

\[
u_i - RP_i - (M - RP_F) = \epsilon_i + \gamma (1 - f_S) \sigma^2_S X_i / \lambda_S, \tag{51}\]

which has the same form as the index return, except \( 1 - Rb_S = 0 \). Note that, as in Vuolteenaho (2002), we are measuring “market-adjusted” (and not raw) stock returns. Since \( \sigma^2_{\epsilon_S} = (1 - f_S) \sigma^2_S \), the variance of idiosyncratic stock returns is given by

\[
Vol^2_{idio} = \sigma^2_{\epsilon_S} \left( 1 + \frac{\gamma^2 \sigma^2_{\epsilon_S} \sigma^2_{X}}{\lambda^2_S} \right). \tag{52}\]

The ratio of cash flow to discount rate news variance for market-adjusted returns is

\[
VR_S \equiv \frac{\lambda^2_S}{\gamma^2 \sigma^2_{\epsilon_S} \sigma^2_{X}}.
\]

The figures in Figure 6 show the index \( (VR_M) \) and firm-level \( (VR_S) \) variance ratios as functions of the equilibrium fraction of informed investors \( \lambda_M + \lambda_S \). Each point is calcu-
lated by first choosing a $\lambda_U$ in the interior equilibrium region$^{22}$ and then calculating the equilibrium $\lambda^*_M$ and $\lambda^*_S$. Since the number of informed investors is falling in the cost $c$ of becoming informed, moving right along the $x$-axis takes us from higher to lower $c$'s. Panel A uses the baseline level of index turnover $\ell = 1$, and Panel B uses the higher level $\ell = 2$ discussed in Section 7.1. The figures thus allow us to compare micro and macro VR (solid versus dashed), the effect of the cost of becoming informed (decreasing from left to right), and the effect of indirect index turnover (increasing from the top panel to the lower panels).

The figures make two main predictions. First, they predict that the single-stock (micro) VR is higher than the index-level (macro) VR, unless the fraction of informed investors and $\ell$ are both low. For the second prediction, we will argue in Section 7.3 that the cost of becoming informed has declined over time (moving us from left to right), and indirect index turnover has increased over time (moving us from the top down). The combined effect can be seen as a move from Point A to Point B in the figure: a large decrease in the macro VR along with a small increase in the micro VR.

We compare these model features to empirical estimates. Campbell (1991) finds that between 1927 and 1951 the VR for the aggregate market was approximately $0.437/0.185 = 2.362$, and that this ratio fell to $0.127/0.772 = 0.165$ in the 1952–1988 time period. Using data from 1954 to 1996 and a similar methodology, Vuolteenaho (2002) finds an aggregate VR of $0.0232/0.0296 = 0.784$, larger than Campbell’s $0.165$ but still quite low relative to the 2.362 ratio from the 20’s to the 50’s$^{23,24}$. These findings support a large decrease over time in the macro VR, as suggested by Points A and B in the figure.

Using 1954–1996 firm-level market-adjusted returns, Vuolteenaho (2002) finds a micro VR of $0.0801/0.0161 = 4.975$, much higher than the index-level measures. We interpret Panel B as reflective of more recent history, and thus consistent with a micro VR larger than the macro VR, though the model value is not as large as the historical estimate.

We do not have evidence on the single-stock VR in the early twentieth century$^{25}$. We know that since the 1950’s this ratio has been close to 5, and a higher value earlier seems implausible. Indeed, our model predicts that the micro VR would have been lower in the earlier period: with fewer informed investors to trade individual stocks (moving to the left

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$^{22}$When $\lambda_u \in [0, 1 - \lambda_S(\max\{\bar{c}, c\})]$, as discussed in Proposition 6.1, we have in equilibrium $\lambda^*_M > 0$ ($\lambda^*_S$ is always positive).

$^{23}$The variance units in Campbell (1991) and Vuolteenaho (2002) are not comparable, but the ratios are.

$^{24}$We only analyze variation in $VR_M$ and $VR_S$ due to investor information choices. Changing variance and correlation of discount rates and cash flows also contribute to changing variance ratios. Therefore our analysis does not generate the empirically observed magnitude of change in $VR_M$.

$^{25}$We are not aware of empirical analysis of individual stock variance ratios in an earlier sample than the 1954–1996 time period in Vuolteenaho (2002). One difficulty with extending the Vuolteenaho (2002) study to an earlier period is that the merged CRSP-Compustat data are not available prior to 1954.
in the figure), liquidity provision would fall, and discount factor variance would rise.

The model of Kacperczyk et al. (2016) makes different predictions. In their setting, the VR decreases (or remains constant) as the fraction of informed investors increases. As all informed investors in their model have the same information and hold the same portfolios, single-stock and index VRs all decrease as the number of informed investors grows. However, unless the number informed (or the total information capacity) is sufficiently large, investors will choose to learn only about the macro factor and not about individual stocks, in which case the micro VR remains constant as the macro VR declines. These are interesting contrasts between our model and that of Kacperczyk et al. (2016). They are discussed in more detail in Appendix A.5.

The increasing micro VR implied by our model is a consequence of the specialization in investor information choice and can be understood as follows. As long as some investors are micro informed ($\lambda_S > 0$), the price of stock $i$ fully incorporates the information $s_i$, and $s_i$ does not enter returns — it is absent from equation (51). However, with few micro informed, there are few investors to absorb the idiosyncratic supply shocks $X_i$, so these shocks lead to large price concessions in order to clear the market. As a result, the innovations in $X_i$ are the key determinant of return variance, making $VR_S$ low when $\lambda_S$ is low. As $\lambda_U$ falls, the number of micro informed grows, meaning $X_i$ shocks can be absorbed by a larger fraction of investors. The requisite price concession therefore decreases, and the proportion of return variance due to cash flow shocks increases.

In contrast, the macro cash flow ratio is a non-monotonic function of $\lambda_M + \lambda_S$. When there are many uninformed investors, little information about $M$ is incorporated into prices, which makes the cash flow contribution to returns in (50) relatively large. At the same time, supply shocks $X_F$ can be absorbed by all investors — since everyone trades in the index fund — therefore preventing the price concession due to $X_F$ from dominating the variance of returns. As $\lambda_U$ decreases and the number of macro informed grows, a larger portion of $m$ is incorporated into the index price, $P_F$, and the ratio of cash flow variation to discount variation falls. At lower $\lambda_U$, with higher $\lambda_M$, the risk effect begins to dominate — as more about $m$ is known (either through prices or through more informed investors), the cash flow risk from owning the index falls, and the supply shock $X_F$ requires less of a price concession to clear the market. As the variation in the price due to $X_F$ falls, the proportion of return variance due to cash flows begins to increase.

The non-monotonicity of the variance ratio for index returns thus reflects an important tradeoff. A decline in uninformed investors makes the index price more informative and

---

26 When $\lambda_M \to 1$, $(1 - Rb_F)^2$ in (50) goes to zero since $b_F \to 1/R$. For low $\lambda_M$, this term is large.
makes index returns less sensitive to cash flow news; but with a more informative prices, the index also becomes less sensitive to supply shocks thus increasing the role of cash flow shocks in index returns. Without uninformed investors – as is the case for individual stocks – the relationship is monotonic.

If the cost of becoming informed continues to drop while index-related trading stays at the $\ell = 2$ level, we would expect the macro VR to eventually increase. However, if $\ell$ continues to increase, then we expect that macro VR continues to fall since supply shock noise increases, while micro VR remains unchanged, as illustrated by Point C in Figure 6.

Our analysis suggests some caution should be used in applying the Jung and Shiller (2005) logic to the VR of the index fund. For single stocks, VRs are monotonic in the number of informed – this conforms with the Jung-Shiller intuition that higher cash flow variance relative to discount rate variance is indicative of higher market efficiency. But the VR-informed relationship is non-monotonic for index funds, suggesting the Jung-Shiller intuition only applies in a range of model parameters – in our example, when the number of informed is sufficiently large.

7.3 Trends in investor composition and the cost of information

Moving from left to right in Figure 6 means increasing the fraction of informed investors and thus decreasing the cost of becoming informed, which we have suggested coincides with experience from the early to the later part of the twentieth century. According to Figure 1 of Philippon (2012), the proportion of the US economy represented by the financial sector increased from 4-5% in the 20’s and 30’s to 8-9% by the 1990’s, with the finance industry share of the economy showing a clear increasing trend throughout the entire twentieth century. Greenwood and Scharfstein (2013) and Philippon and Reshef (2013) provide similar US and international evidence, respectively, for growth in the finance sector share of the economy. We interpret these results as indicating that society has devoted an increasing share of its productive resources, including human capital, to the finance sector over the prior century. We take this as evidence that $\lambda_U$ fell, and $\lambda_M + \lambda_S$ rose, from the 1920’s and 30’s to the 1990’s.

27 The Philippon series splices together the financial sector value added over GDP ratio and the labor compensation share of the finance industry. When both series are available (in the later part of the sample) they track each other very closely.

28 If the number of informed increased over time, this together with our results from the prior section, suggest that individual stocks have become more “efficient” according to our variance ratio measure. Bai, Philippon, and Savov (2016) show that the $R^2$ of regressing firm-level earnings in years $t + 1, \ldots, t + 5$ on year $t$ price to book ratios has been increasing since 1960 for S&P 500 firms. They interpret this finding as indicating that markets have become more informative about fundamentals. On the other hand, Bai et al.
Further evidence of a fall in $\lambda_U$ comes from Lakonishok, Shleifer, and Vishny (1992). They document that from 1955 to 1990 institutional ownership of equities increased from 23% to 53% while equity ownership by individuals fell from 77% to 47%. The trend from 1990 to today appears to go in the same direction. The Investment Company Institute (2018, p.36) documents that the share of household financial assets held in investment companies increased from 3% in 1980 to 24% in 2017. The growth in the finance sector as a share of the economy, together with the growing share of assets under institutional management, both point to a decrease over time in the number of uninformed investors in the market.

Finally, from Figure 5 we see that the cost $c$ consistent with an interior equilibrium (i.e. when all $\lambda$’s are positive) ranges from 0.02 to just under 0.06. Our economy’s “GDP” is $(\bar{X} \times \bar{m})$. The number of informed (who pay cost $c$) ranges from 1 to 0.2, and decreases monotonically with $c$. Hence the information-production share of GDP ranges from 1% to 2%. This seems very plausible in the context of the 4%-9% finance share of GDP documented in the literature.

8 Concluding remarks

A tendency for markets to be more micro than macro efficient has been a recurring theme of many of our results. For example:

- From Propositions 5.3 and 6.1, $\lambda^*_S > 0$ — there are always micro-informed investors in the constrained model. It is, in fact, possible that there are only micro-informed investors, so $\lambda^*_M = 0$. This occurs when information is too costly to satisfy (e3).

- More generally, Corollary 6.1 shows that when information is costlier, a larger fraction of investors who are informed choose micro information.

- Proposition 5.4 shows increasing risk aversion raises the value of micro information.

- Figure 3 shows that for our model calibration, the economy is micro-efficient, unless the number of uninformed investors is implausibly low.

(2016) and Farboodi, Matray, and Veldkamp (2017) show that this same measure of price informativeness has fallen for smaller, non-S&P 500 firms. Our results should be interpreted as applying to large, stable firms.

29 The trend to passive mutual fund strategies which began in the late 1990’s, see Appel, Gormley, and Keim (2016) for example, does not affect our interpretation of Vuolteenaho’s VR results because these results are based on a data sample that ends in 1996.
Figure 6: The panels show the variance decomposition of returns for the index fund, $VR_M$, and for market-adjusted stock returns, $VR_S$. The figure shows only the interior equilibrium region (see Proposition 6.1) where both $\lambda_M$ and $\lambda_S$ are positive. Parameter values are given in Table 1. Volatility of $X_F$ is given by $\ell \times \sigma_{X_F}$.
Finally, Jung and Shiller (2005) argue that micro efficiency is characterized by a larger portion of stock return variation being driven by cash flow news than is the case for index return variation. As Figure 6 shows, this holds in our model. A key driver of these results is that the micro-informed are the only investors who collect surplus from accommodating idiosyncratic supply shocks. This creates a strong incentive to acquire micro information. We see from (35) that the benefit $J_M/J_U$ to being the first macro-informed investor is finite, but the benefit to being the first micro-informed is infinite: $J_S/J_U$ takes on its highest value — zero — at $\lambda_S = 0$. This endogenous monopoly on liquidity provision for idiosyncratic shocks pushes markets towards micro efficiency. These properties are consequences of our general specialization result in Theorem 2.1 and thus result from the asymmetry in the cost of learning from index versus individual stock prices.

A Appendix

A.1 Proof of Theorem 2.1

To help the reader navigate the proof, we first sketch the argument. Through two lemmas, we derive expressions for the squared expected utility resulting from any information choice. These expressions have the general form one would expect in a CARA-normal setting as the products of variance ratios and a risk premium, adjusted for the cost of becoming informed and capacity constraints. Given these expressions, we derive equilibrium properties as consequences of equating utilities across certain information choices. For example, equating the utility $J_U$ of the uninformed and the utility of an informed investor who conditions on the price of the index fund and learns about $K \geq 1$ stocks, $J(I_K^{(1)})$, yields the equation

$$J_U^2 \equiv e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M-RP_F]} = e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M-RP_F]} e^{2\gamma_c \frac{\kappa}{\delta_F}} \left( \frac{\text{var}[S_i]}{\text{var}[S_i-RP_{S_i}]} \right)^K \equiv J^2(I_K^{(1)}).$$

With $e^{2\gamma_c \kappa} < \delta_F$, this equation requires $\text{var}[S_i]/\text{var}[S_i-RP_{S_i}] > 1$, which drives the informed investor to choose the smallest $K$, meaning that the investor specializes in just one stock. We then show through a similar comparison that specialization in macro information achieves the same utility.

To compare expected utility under alternative information choices, we will use the following expressions (A.1)–(A.2) for expected utility, which follow directly from Proposition 3.1 of Admati and Pfleiderer (1987).

**Lemma A.1.** Let $\Psi^{(0,K)}$ denote the covariance matrix of $S_i - RP_{S_i}$, $i = 1, \ldots, K$, and let

$$\Psi^{(1,K)} = \begin{pmatrix} \text{var}[M-RP_F] & 0 \\ 0 & \Psi^{(0,K)} \end{pmatrix}.$$
The squared expected utility of an informed investor who chooses information set $I^\iota_K$ is

$$J^2 = e^{2\gamma c} \times \begin{cases} |\hat{\Sigma}^{(0,K)}|/|\Psi^{(0,K)}|, & \iota = 0; \\ \exp(-Q_F)|\hat{\Sigma}^{(0,K)}|/\Psi^{(1,K)}, & \iota = 1, 2, \end{cases} \quad (A.1)$$

where $Q_F = (E[M - RP_F])^2/\text{var}[M - RP_F]$. For an uninformed investor who invests only in the index fund, the squared expected utility is given by

$$J^2_U = e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M - RP_F]}.$$

Investors can evaluate (A.1)–(A.2) to make their information choices without first observing signals; $\text{var}[M|P_F]$ depends on price informativeness but not on $P_F$ itself. Combining (A.1) with (5) and (6), the expected utility an investor attains by choosing information $I^\iota_K$ is given by

$$J^2 = e^{2\gamma c} \times \begin{cases} \kappa |\Sigma^{(0,K)}|/|\Psi^{(0,K)}|, & \iota = 0; \\ \exp(-Q_F)(\kappa \text{var}[M|P_F]/\delta_F)|\Sigma^{(0,K)}|/|\Psi^{(1,K)}|, & \iota = 1; \\ \exp(-Q_F)\kappa |\Sigma^{(1,K)}|/|\Psi^{(1,K)}|, & \iota = 2. \end{cases} \quad (A.3)$$

For $\iota = 1$, the expression follows from writing the ratio of determinants in (A.1) as

$$\frac{|\hat{\Sigma}^{(1,K)}|}{|\Psi^{(1,K)}|} = \frac{\text{var}[M|P_F] \cdot |\hat{\Sigma}^{(0,K)}|}{|\Psi^{(1,K)}|},$$

and then applying (6). The expressions in (A.3) hold as equalities when an investor uses the full capacity $\kappa$, which is always possible and individually optimal if $K \geq 1$ or $\iota = 2$, so we will assume this condition holds. Interpret the case $\iota = 0, K = 0$ as the option not to invest, in which case the agent effectively consumes the acquired capacity.

The following lemma evaluates the determinants in (A.3).

**Lemma A.2.** For any $K = 1, \ldots, N - 1$,

$$\frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|} = \left(\frac{\text{var}[S_i]}{\text{var}[S_i - RP_{S_i}]} \right)^K. \quad (A.4)$$

Also, $|\Sigma^{(1,K)}| = \text{var}[M] \cdot |\Sigma^{(0,K)}|$ and $|\Psi^{(1,K)}| = \text{var}[M - RP_F] \cdot |\Psi^{(0,K)}|$. \quad (A.5)

**Proof.** Let $G_K$ be the $K \times K$ matrix with all diagonal entries equal to 1 and all off-diagonal entries equal to $-1/(N - 1)$. It follows from (2) that $\Sigma^{(0,K)} = \sigma_S^2 G_K$. In light of (4),

$$\sum_{i=1}^N (S_i - RP_{S_i}) = \sum_{i=1}^N S_i - R \sum_{i=1}^N (P_i - P_F) = 0.$$
Under (e1), it follows that, for \(i \neq j\),

\[
\text{cov}[S_i - RP_{S_i}, S_j - RP_{S_j}] = -\text{var}[S_i - RP_{S_i}] / (N - 1),
\]

so the \(S_i - RP_{S_i}\) have the same correlation structure as the \(S_i\) themselves. In other words, \(\Psi^{(0,K)} = \text{var}[S_i - RP_{S_i}] G_K\), and then, since \(G_K\) is nonsingular for \(K \leq N - 1\),

\[
|\Sigma^{(0,K)}| / |\Psi^{(0,K)}| = |\sigma^2_S G_K| / |\text{var}[S_i - RP_{S_i}] G_K|,\]

which yields [A.4]. The block structure of \(\Sigma^{(1,K)}\) and \(\Psi^{(1,K)}\) yields [A.5].

**Lemma A.3.** Under the conditions of Theorem 2.1, we have

\[
\frac{\text{var}[M|P_F]}{\text{var}[M]} < \delta_F \quad \text{and} \quad \frac{\text{var}[S_i]}{\text{var}[S_i - RP_{S_i}]} > 1, \quad i = 1, \ldots, N, \quad (A.6)
\]

and

\[
e^{-Q_F} \frac{\text{var}[M]}{\text{var}[M - RP_{F}]} \leq 1. \quad (A.7)
\]

The first inequality confirms that making inferences from the price of the index fund consumes less information processing capacity than would be expected from the variance reduction achieved; see the discussion surrounding (7). The reverse inequality would imply a “penalty” in conditioning on the index fund price, a scenario we see as uninteresting and rule out with the conditions in the theorem. The second inequality in (A.6) suggests that individual stock prices are, in a sense, sufficiently informative about fundamentals and not overwhelmed by noise trading. This second condition ensures that being micro informed is not strictly preferable to being uninformed — it is effectively a limit on the benefit of actively trading in individual stocks. As we will see in the proof of Theorem 2.1, this result also makes it suboptimal to have more than one stock in an investor’s consideration set. Had the second inequality in (A.6) gone the opposite way, investors would prefer to learn as little as possible about as many stocks as possible, rendering prices uninformative. The inequality in (A.7) confirms that there is a benefit to investing in the index fund.

**Proof of Lemma A.3.** If investors strictly preferred becoming macro-informed over remaining uninformed and investing in the index fund, then (e4) would be violated. Thus, in any equilibrium satisfying (e4), an uninformed investor weakly prefers investing in the index fund over becoming macro-informed. We therefore have

\[
J_U^2 \leq e^{2\gamma e^{-Q_F}} \frac{\text{var}[M|m, P_F]}{\text{var}[M - RP_{F}]} \equiv J_M^2, \quad (A.8)
\]

where \(m = m(f_M)\) is the signal in (9) acquired by an informed investor who allocates all capacity to learning about \(M\). Combining this inequality with (A.2), recalling from (11)...
that \( \text{var}[M|m, P_F]/\text{var}[M] = \kappa \), we get, by (e3),

\[
\frac{\text{var}[M|P_F]}{\text{var}[M]} \leq e^{2\gamma c \kappa} < \delta_F.
\]

We can similarly compare \( J^2_U \) with the option of becoming micro-informed to get

\[
J^2_U \leq e^{2\gamma c} e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M - R P_F]} \frac{\text{var}[S_i|s_i, P_S_i]}{\text{var}[S_i - R P_{S_i}]} \equiv J^2_S, \quad \text{(A.9)}
\]

where \( s_i = s_i(f_S) \) is the signal in (10) acquired by an informed investor who allocates all capacity to \( \{P_F, (s_i, P_{S_i})\} \). Recalling from (11) that \( \delta_F \text{var}[S_i|s_i, P_{S_i}]/\text{var}[S_i] = \kappa \), this inequality reduces to

\[
1 \leq e^{2\gamma c (\kappa/\delta_F)} \frac{\text{var}[S_i]}{\text{var}[S_i - R P_{S_i}]}.
\]

As \( e^{2\gamma c \kappa} < \delta_F \) under (e3), the second inequality in (A.6) must hold.

Suppose no informed investors choose to learn more about \( M \) than its price, meaning that no investor chooses an information set of the type \( I^{(2)}_K \). Then (e2) implies \( \text{var}[M|P_F] = \text{var}[M] \), which would contradict (A.6) because \( \delta_F < 1 \). It follows that some informed investor weakly prefers an information set \( I^{(2)}_K \) over an information \( I^{(0)}_K \) consisting of more precise signals about the same stocks and no information about \( M \). This preference implies

\[
e^{2\gamma c} e^{-Q_F} \frac{\text{var}[M]}{\text{var}[M - R P_F]} \left( \frac{\text{var}[S_i]}{\text{var}[S_i - R P_{S_i}]} \right) \kappa \leq e^{2\gamma c} \left( \frac{\text{var}[S_i]}{\text{var}[S_i - R P_{S_i}]} \right) \kappa,
\]

from which (A.7) follows.

\[\square\]

\textbf{Proof of Theorem 2.1.} We need to show that for an informed investor, information choices other than \( \{m, P_F\} \) or \( \{s_i, P_{S_i}, P_F\} \) are suboptimal. We show this by considering the three types of information sets \( I^{(i)}_K \), \( i = 0, 1, 2 \). For each case, we evaluate expected utility using (A.3) and show that any information choice other than \( \{m, P_F\} \) or \( \{s_i, P_{S_i}, P_F\} \) is strictly dominated.

\textbf{Case of} \( i = 2 \). We may take \( K \geq 1 \), since otherwise only the index fund is in the consideration set, in which case the investor cannot do better than \( \{m, P_F\} \) because of the capacity constraint. Then

\[
J^2 \geq e^{2\gamma c} \exp(-Q_F) \kappa \frac{\Sigma^{(1,K)}}{\Psi^{(1,K)}}, \quad \text{using (A.3)};
\]

\[
e^{2\gamma c} \exp(-Q_F) \kappa \frac{\Sigma^{(0,K)}}{\Psi^{(0,K)}} \cdot \frac{\text{var}[M]}{\text{var}[M - R P_F]}, \quad \text{using (A.5)};
\]

\[
e^{2\gamma c} \exp(-Q_F) \left( \frac{\text{var}[M|m, P_F]}{\text{var}[M - R P_F]} \right) \frac{\Sigma^{(0,K)}}{\Psi^{(0,K)}}, \quad \text{using the first equality in (11)};
\]

\[
= J^2_M \frac{\Sigma^{(0,K)}}{\Psi^{(0,K)}}, \quad \text{using (A.8)}.
\]
Combining Lemma A.2 and Lemma A.3, we get $|\Sigma^{(0,K)}|/|\Psi^{(0,K)}| > 1$, from which we conclude that $J^2 > J_M^2$. Thus, $I_K^{(2)}$ is strictly dominated by $\{m, P_F\}$.

**Case of $i = 1$.** Start with $K = 0$. We have $\text{var}[S_i|s_i, P_S] = \text{var}[S_i - R P_S|s_i, P_S] < \text{var}[S_i - R P_S]$, and therefore from (A.1)

$$J^2 = e^{2c}e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M - R P_F]},$$

showing that $I_1^{(1)}$ is preferred over $I_0^{(1)}$. With $K \geq 2$, the squared expected utility is

$$J^2 \geq e^{2c} \exp(-Q_F) \frac{\text{var}[M|P_F]}{|\Sigma^{(0,K)}|}, \quad \text{using (A.3)};$$

$$= e^{2c} \exp(-Q_F) \frac{\delta_F \text{var}[S_i|s_i, P_S]}{\text{var}[S_i]} \left( \frac{\text{var}[M|P_F]}{\text{var}[M - R P_F]} \right) \frac{|\Sigma^{(0,K)}|}{\delta_F |\Psi^{(0,K)}|}, \quad \text{using (11) and (A.5)};$$

$$= e^{2c} \exp(-Q_F) \frac{\text{var}[S_i|s_i, P_S]}{\text{var}[S_i - R P_S]} \left( \frac{\text{var}[M|P_F]}{\text{var}[M - R P_F]} \right) \frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|} \frac{\text{var}[S_i - R P_S]}{\text{var}[S_i]};$$

$$= J_M^2 \frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|} \frac{\text{var}[S_i - R P_S]}{\text{var}[S_i]}.$$

With $K \geq 2$, multiplying (A.4) by $\text{var}[S_i - R P_S]/\text{var}[S_i]$ continues to yield an expression that is greater than 1. It follows that $J^2 > J_M^2$, so $I^{(1)}_K$, $K \geq 2$, is strictly dominated.

**Case of $i = 0$.** A nonempty information set requires $K \geq 1$, and then

$$J^2 \geq e^{2c} \frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|}, \quad \text{using (A.3)};$$

$$\geq e^{2c} e^{-Q_F} \frac{\text{var}[M]}{\text{var}[M - R P_F]} \frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|}, \quad \text{using (A.7)};$$

$$= e^{2c} e^{-Q_F} \frac{\text{var}[M|m, P_F]}{\text{var}[M - R P_F]} \frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|}, \quad \text{using (11)};$$

$$= J_M^2 \frac{|\Sigma^{(0,K)}|}{|\Psi^{(0,K)}|} > J_M^2, \quad \text{using (A.8) and (A.4)}.$$

**No-deviation.** We have thus shown that either $\{m, P_F\}$ (macro-informed) or $\{s_i, P_S\}$ (micro-informed) dominates every other information set. If the proportion of macro-informed or micro-informed investors were zero, (e2) would contradict one of the inequalities in (A.6). The last assertion in the theorem follows.

**A.1.1 Investment by the uninformed**

Although not needed for our results, we can confirm that the uninformed choose to invest in the index fund and not in individual stocks. Uninformed investors have capacity $\delta_F$. If they use this capacity to invest in an individual stock then using (A.1) (but with $c = 0$), their
expected utility is
\[ \tilde{J}_U^2 = \frac{\text{var}[S_i|P_{S_i}, s_i]}{\text{var}[S_i - R P_{S_i}]} = \delta_F \frac{\text{var}[S_i]}{\text{var}[S_i - R P_{S_i}]} > \delta_F, \]
where the second step, \( \text{var}[S_i|P_{S_i}, s_i] = \delta_F \text{var}[S_i] \), comes from the capacity constraint of the uninformed, and the third step follows from (A.6). From (A.2), the uninformed who invest in the index fund earn expected utility
\[ J_U^2 = e^{-Q_F} \frac{\text{var}[M|P_F]}{\text{var}[M - R P_F]} < \delta_F \frac{\text{var}[M]}{\text{var}[M - R P_F]} \leq \delta_F, \]
where the second step follows from (A.6) and the third step follows from (A.7). We conclude that \( J_U^2 < \delta_F < \tilde{J}_U^2 \), confirming that the uninformed prefer to invest in the index fund than to stay out of the market or invest in individual stocks.

A.1.2 Information invariance

We now state an informational invariance result in our model which obtains when condition (8) holds, i.e., when \( \delta_F \) is given by \( \text{var}[M|P_F]/\text{var}[M] \) rather than fixed exogenously. In this case, (6) reduces to (5), meaning that we treat inference from the price of the index fund the same way we treat inference from the prices of individual stocks, and we lose the specialization property.

**Proposition A.1.** Suppose (e1), (e2), and (e4) hold. If \( \delta_F \) is given by (8), the following property holds:

(i) For any fixed \( K \), investors have the same utility from information sets \( I_K^{(1)} \) and \( I_K^{(2)} \), and

\[ \delta_F \leq \exp(2\gamma c) \cdot \kappa. \tag{A.10} \]

If there exists an investor who chooses information set \( I_K^{(1)} \) or \( I_K^{(2)} \) with \( K \geq 1 \), the following must hold:

(ii) \( K = 1 \) cannot be strictly optimal; and

(iii) Either all choices of \( K \) yield the same utility and \( \delta_F = \exp(2\gamma c) \cdot \kappa \), or \( K = N - 1 \) is strictly optimal and \( \delta_F < \exp(2\gamma c) \cdot \kappa \).

**Proof of Proposition A.1.**

(i) Since \( |\Sigma^{(1,K)}| = |\Sigma^{(0,K)}| \) from (A.5), when (8) holds, the utility functions in (A.3) imply that for a given \( K \), the \( i = 1 \) and \( i = 2 \) information sets lead to identical utilities.

Using the results on evaluating determinants in (A.4 A.5), we get
\[ J_U^2(K) = e^{2\gamma c} \kappa e^{-Q_F} \frac{\text{var}[M]}{\text{var}[M - R P_F]} \times \left( \frac{\text{var}[S_i]}{\text{var}[S_i - R P_{S_i}]} \right)^K, \tag{A.11} \]
where \( J(K) \) is the utility achieved from using either information set \( I^{(1)}_K \) or \( I^{(2)}_K \). Assumption (e4) implies that being uninformed cannot be dominated by any information set, and in particular not by \( I^{(2)}_0 \), implying that \( J^{(2)}_0 \leq J^{(2)}_0(0) \), and, using the expression for \( J^{(2)}_U \) in (A.2),

\[
\text{var}[M|P_F] \leq e^{2\gamma_c} \kappa \times \text{var}[M],
\]

which using (8) implies that \( \delta_F \leq e^{2\gamma_c} \kappa. \)

(ii) If some investor chooses \( I^{(1)}_K \) or \( I^{(2)}_K \) and (e4) holds, we must have \( J^{(2)}_K = J^{(2)}_U \) for some \( K \), which implies

\[
1 = \frac{e^{2\gamma_c} \kappa}{\delta_F} \times \left( \frac{\text{var}[S_i]}{\text{var}[S_i - RP_{S_i}]} \right)^K
\]

(A.12)

using the utility from (A.11). For \( K = 1 \) to be strictly optimal, we would need to have \( \text{var}[S_i] > \text{var}[S_i - RP_{S_i}] \), because it follows from the form of the utility function in (A.11) that otherwise \( K = 2 \) would either be equally good or better for the marginal investor. In this case we would need \( \delta_F > \exp(2\gamma_c) \cdot \kappa \) which is precluded by (A.10). Hence \( K = 1 \) cannot be strictly optimal.

(iii) Say there exists some \( K < N - 1 \) which is weakly optimal. If \( \text{var}[S_i] \neq \text{var}[S_i - RP_{S_i}] \) then \( K \) cannot be optimal in (A.11), since \( J^{(2)}_K \) can be increased for the marginal investor at some other \( K' \). This implies that \( \text{var}[S_i]/\text{var}[S_i - RP_{S_i}] = 1 \). But then (A.12) implies that \( \delta_F = \exp(2\gamma_c) \cdot \kappa \), and the value of \( J^{(2)}_K \) in (A.11) does not depend on \( K \).

On the other hand, if \( K = N - 1 \) is strictly optimal, then \( \text{var}[S_i] < \text{var}[S_i - RP_{S_i}] \) in (A.11) and we need \( \delta_F < \exp(2\gamma_c) \cdot \kappa \) in order for the equality in (A.12) to hold.

\[ \square \]

A.2 Solution of the constrained model

Proof of Proposition 4.1: The analysis is simplified if we allow micro-informed agents to invest in the index fund and in a hedged security paying \( u_i - u_F = S_i \), with price \( P_{S_i} = P_i - P_F \).

If we let \( q^i_F \) and \( q^i_{S_i} \) denote the demands in this case, the demands in the original securities are given by \( q^i_i = \bar{q}^i_{S_i} \) and \( q^i_F = \bar{q}^i_F - \beta_i \bar{q}^i_{S_i} \). Write \( I_i = \{ P_F, P_i, s_i \} \). By standard arguments, the modified demands are given by

\[
\begin{bmatrix}
q^i_F \\
q^i_{S_i}
\end{bmatrix}
= \frac{1}{\gamma} \text{var} \left[ \frac{M}{S_i} | I_i \right]^{-1} \left( \text{E} \left[ \frac{M}{S_i} | I_i \right] - R \left[ \frac{P_F}{P_i - P_F} \right] \right).
\]

Now

\[
\text{var} \left[ \frac{M}{S_i} | I_i \right] = \left( \frac{\text{var}[M|I_i]}{\text{var}[S_i|I_i]} \right) = \left( \frac{\text{var}[M|P_F]}{(1 - f_S)\sigma^2_S} \right),
\]

(A.13)
and
\[
E \left[ \frac{M}{S_i} \mid \mathcal{I}_i \right] = \left[ E[M|P_F] \right]_i.
\]

Thus, \( \tilde{q}_i^i = q_U^i \), with \( q_U^i \) as given in (21), and
\[
\tilde{q}_S^i = \frac{s_i - R(P_i - P_F)}{\gamma(1 - f_S)\sigma_S^2}.
\]

As \( q_i^i = \tilde{q}_S^i \), (23) follows, and then \( q_F^i = \tilde{q}_F^i - \tilde{q}_S^i = q_U^i - q_i^i \) completes the proof. \( \square \)

Proof of Proposition 4.2. The price \( P_F \) can be derived from first principles, but we can simplify the derivation by reducing it to the setting of Grossman and Stiglitz (1980). The informed (20) and uninformed (21) demands for the index fund and the market clearing condition (25) reduce to the demands in equations (8) and (8') of Grossman and Stiglitz (1980) and their market clearing condition (9), once we take \( \lambda = \lambda_M \) and \( 1 - \lambda_M = \lambda_U + \lambda_S \). The coefficients of the price \( P_F \) in (18) can therefore be deduced from the price in their equation (A10). Theorem 1 of Grossman-Stiglitz gives an expression for \( P_F \) in the form \( \alpha_1 + \alpha_2 w_\lambda \), for constants \( \alpha_1 \) and \( \alpha_2 > 0 \), where, in our notation,
\[
w_\lambda = m - \frac{\gamma(1 - f_M)\sigma_M^2}{\lambda_M} (X_F - \bar{X}_F).
\]

Comparison with (18) yields the expression for \( c_F/b_F \) in (26). From the coefficient of their \( \theta \) (our \( m \)) in (A10) of Grossman-Stiglitz, we get
\[
b_F = \frac{1}{R} \left[ \frac{\lambda_M}{(1 - f_M)\sigma_M^2} + \frac{1 - \lambda_M}{\text{var}[M|w_\lambda]} \right].
\]

Moreover, \( \text{var}[w_\lambda] = (1 - f_M)\sigma_M^2 + \frac{\gamma^2(1 - f_M)^2\sigma_M^4}{\lambda_M^2} \sigma_{X_F}^2 \), and \( \text{var}[M|w_\lambda] = \text{var}[M|P_F] \). To evaluate \( \text{var}[M|P_F] \), note that the only unknown term in (22) is \( K_F b_F \), which we can now evaluate using (26) to get
\[
K_F b_F = \frac{b_F^2 f_M\sigma_M^2}{b_F^2 f_M\sigma_M^2 + c_F^2 \sigma_{X_F}^2} = \frac{f_M\sigma_M^2}{f_M\sigma_M^2 + \frac{\gamma^2(1 - f_M)^2\sigma_M^4}{\lambda_M^2} \sigma_{X_F}^2}.
\]

This yields an explicit expression for \( \text{var}[M|P_F] \) which in turn yields an explicit expression for \( b_F \) through (A.14). An expression for \( c_F \) then follows using (26). Finally, to evaluate the constant term \( a_F \), we can again match coefficients with the expression in (A10) of Grossman-Stiglitz. Alternatively, we can evaluate their (A10) at (using their notation) \( \theta = E\theta^* \) and
$x = E x^*$, which, in our notation yields

$$a_F = \frac{\bar{m}}{R} - \frac{\bar{X}_F}{R} \left[ \frac{1 - \lambda_M}{\gamma \text{var}[M|P_F]} + \frac{\lambda_M}{\gamma (1 - f_M) \sigma_M^2} \right]^{-1}. \quad (A.15)$$

Equation (27) follows directly from (23) and (17). In light of (12) and (14), condition (4) is satisfied if and only if $\xi = 0$.

### A.3 Attention equilibrium

**Proof of Proposition 5.1.** We use the expressions for $J_U$, $J_M$, and $J_S$ introduced in (A.2), (A.8), and (A.9), recalling that expected utility is negative. With prices given by Proposition 4.2, $\text{var}[M|P_F, m] = \text{var}[M|m] = (1 - f_M) \sigma_M^2$, so

$$J_M/J_U = e^{\gamma c} \left( \frac{\text{var}[M|P_F]}{(1 - f_M) \sigma_M^2} \right)^{-1/2}.$$  

Combining (22) and (29) yields

$$\text{var}[M|P_F] = f_M \sigma_M^2 (1 - \rho_F^2) + (1 - f_M) \sigma_M^2,$$

from which (32) follows. Similarly, $\text{var}[S_i|P_S, s] = \text{var}[S_i|s] = (1 - f_s) \sigma_S^2$, so

$$J_S/J_U = e^{\gamma c} \left( \frac{\text{var}[S_i - R P_S]}{(1 - f_S) \sigma_S^2} \right)^{-1/2}.$$  

Using first (27) and then (31), we get

$$\text{var}[S_i - R P_S] = (1 - f_S) \sigma_S^2 + \frac{\gamma^2 (1 - f_s)^2 \sigma_S^4}{\lambda_S^2} \sigma_X^2 = (1 - f_s) \sigma_S^2 + f_s \sigma_S^2 \left( \frac{1}{\rho_S^2} - 1 \right),$$

from which (33) follows.

**Proof of Proposition 5.3.** As noted in (35), $J_S/J_U$ approaches zero as $\lambda_S$ decreases to zero (and $\lambda_M$ increases to $1 - \lambda_U$). We know from (32) that $J_M/J_U > 0$ for all $\lambda_M$; in fact, from (35) we know that $J_M/J_U \geq \sqrt{1 - f_M}$. It follows from the strict monotonicity of $J_M/J_U$ and $J_S/J_U$ (Proposition 4.3) that either $J_M/J_U > J_S/J_U$ for all $\lambda_M \in [0, 1 - \lambda_U]$ or the two curves cross at exactly one $\lambda_M$ in $[0, 1 - \lambda_U]$. In the first case, all informed agents prefer to be micro-informed than macro-informed, so the only equilibrium is $\lambda_M^* = 0$.

In the second case, the unique point of intersection defines the equilibrium proportion $\lambda_M^*$, as explained in the discussion of Figure 2. We therefore examine at which $\lambda_M$ (if any) we have $J_M/J_U = J_S/J_U$. We can equate (32) and (33) by setting

$$\frac{1 - f_M}{f_M} \frac{1}{1 - \rho_F^2} = \frac{1 - f_S}{f_S} \frac{\rho_S^2}{1 - \rho_S^2}.$$
Using the expressions for $\rho^2_F$ and $\rho^2_S$ in (30) and (31), this equation becomes

\[
\frac{1 - f_M}{f_M} + \frac{\lambda^2_M}{\gamma^2(1 - f_M)\sigma^2_M\sigma^2_X} = \frac{(1 - \lambda_U - \lambda_M)^2}{\gamma^2(1 - f_S)\sigma^2_S\sigma^2_X}.
\]

Thus, $\lambda_M$ satisfies a quadratic equation, which, with some algebraic simplification, can be put in the form $A\lambda^2_M + B\lambda_M + C = 0$, where

\[
A = 1 - \varphi, \quad B = -2(1 - \lambda_U), \quad C = (1 - \lambda_U)^2 - \alpha\gamma^2,
\]

(A.16)

with $\varphi$ and $\alpha$ as defined in (37). One of the two roots of this equation is given by $\tilde{\lambda}_M$.

Denote the other root by $\eta = -B + \sqrt{B^2 - 4AC} / 2A$.

We claim that $\eta \notin [0, 1 - \lambda_U]$. We may assume $A \neq 0$, because $\eta \to \infty$ as $A \to 0$ because $B < 0$. If $A < 0$ then either $\eta$ is complex or $\eta < 0$, again because $B < 0$. If $A > 0$, then $A < 1$ because $\varphi > 0$. Then if $\eta$ is real, it satisfies $\eta \geq -B / 2A > -B / 2 = 1 - \lambda_U$.

Combining these observations, we conclude that either $\tilde{\lambda}_M \in [0, 1 - \lambda_U)$ and the information equilibrium has $\lambda^*_M = \tilde{\lambda}_M$, or else the equilibrium occurs at $\lambda^*_M = 0$.

\[\square\]

**Proof of Proposition 5.4.** Differentiation of $\tilde{\lambda}_M$ with respect to $\gamma$ yields

\[
d\tilde{\lambda}_M / d\gamma = -\frac{1}{2A\sqrt{\varphi(1 - \lambda_U)}} \left[ \frac{1 - \varphi}{\sqrt{\varphi}} \frac{\gamma^2\alpha}{(1 - \lambda_U)^2} \right]^{-1/2}.
\]

At an interior equilibrium, $\tilde{\lambda}_M$ is real, so the expression on the right is real and negative.

At an interior equilibrium, $\lambda^*_M$ is the solution to $A\lambda^2 + B\lambda + C = 0$, with the coefficients given by (A.16). Differentiating with respect to some parameter (e.g., $\sigma_S\sigma_X$) yields $\dot{A}\lambda^2 + 2A\lambda\dot{\lambda} + B\dot{\lambda} + \dot{C} = 0$ (note $\dot{B} = 0$ because $\lambda_U$ is fixed). Solving for $\dot{\lambda}$ yields

\[
\dot{\lambda} = -\frac{\dot{A}\lambda^2 + \dot{C}}{2A\lambda + B}.
\]

We note that $2A\lambda + B < 0$ can be rewritten $(1 - \varphi)\lambda < 1 - \lambda_U$ which is always true because $\varphi > 0$ and $\lambda_M < 1 - \lambda_U$ since $\lambda_S > 0$. Therefore, $\text{sgn}(\dot{\lambda}) = \text{sgn}(\dot{A}\lambda^2 + \dot{C})$. Differentiating with respect to $\sigma_S\sigma_X$ yields $\dot{A} < 0$ and $\dot{C} < 0$, which implies $\dot{\lambda} < 0$; differentiating with respect to $\sigma_M\sigma_{X_F}$ yields $\dot{A} > 0$ and $\dot{C} = 0$, which implies $\dot{\lambda} > 0$.

\[\square\]

**Proof of Proposition 5.5.** Using the expression for $\rho^2_S$ in (31), we get

\[
\frac{f_S}{1 - f_S} \left( \frac{1}{\rho^2_S} - 1 \right) = \frac{\gamma^2(1 - f_S)\sigma^2_S\sigma^2_X}{\lambda^2_S}.
\]

This expression is strictly decreasing in $f_S$, so $J_S / J_U$ in (33) is strictly increasing in $f_S$. \[\square\]

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Proof of Proposition 5.6. We see from (32) that the derivative of $J_M/J_U$ is negative precisely if the derivative of
\[
\frac{1 - f_M}{f_M} \frac{1}{1 - \rho_F^2} = \frac{1}{\gamma^2(1 - f_M)\sigma_2^2\sigma_{X_F}^2/\lambda_M^2} + \frac{1}{f_M} - 1
\]
is negative, using the expression for $\rho_F^2$ in (30). Differentiation yields
\[
\frac{1}{\gamma^2(1 - f_M)^2\sigma_2^2\sigma_{X_F}^2/\lambda_M^2} - \frac{1}{f_M^2} = \frac{1}{f_M} \left( \frac{1}{1 - \rho_F^2} - 1 \right) - \frac{1}{f_M^2},
\]
which is negative precisely if the first condition in (39) holds. The equivalence of the second condition in (39) follows from (30). \hfill \Box

A.4 Information equilibrium

Proof of Proposition 6.1. We first show that (46) defines an information equilibrium at each $c > 0$, then verify uniqueness. For all three cases in (46), the specified $\lambda_M$, $\lambda_S$, and $\lambda_U$ are nonnegative and sum to 1, so it suffices to verify (40). For $\bar{c} < c < \bar{c}$, we have $J_M = J_S = J_U$ by construction, so the condition holds. For $c \geq \bar{c}$, we again have $J_S/J_U = 1$ by construction. With $\lambda_M = 0$, we have $\rho_F^2 = 0$, and $J_M/J_U$ in (32) evaluates to $\exp(\gamma c)\sqrt{1 - f_M} \geq \exp(\gamma \bar{c})\sqrt{1 - \bar{f}_M} = 1$, so $J_U/J_M \leq 1$. Combining the two ratios we get $J_S/J_M \leq 1$. Thus, (40) holds.

For $c < \bar{c}$, we consider two cases. First suppose case (i) of Proposition 5.3 holds at $\bar{c}$. By definition, $1 - \lambda_M(\bar{c}) - \lambda_S(\bar{c}) = 0$ and $J_M/J_U = J_S/J_U$ at $(\lambda_M(\bar{c}), \lambda_S(\bar{c}), 0)$, so $\lambda_M(\bar{c}) = \lambda_M^*(0)$ and $\lambda_S(\bar{c}) = 1 - \lambda_M^*(0)$, by the definition of $\lambda_M^*$. Because $\lambda_M(c)$ and $\lambda_S(c)$ are strictly decreasing in $c$, they are strictly greater than $\lambda_M^*(0)$ and $1 - \lambda_M^*(0)$. Decreasing $\lambda_M$ decreases $\rho_F^2$, which decreases $J_M/J_U$ in (32), and decreasing $\lambda_S$ similarly decreases $J_S/J_U$. By construction, $J_M/J_U = J_S/J_U = 1$ at $(\lambda_M(c), \lambda_S(c), 1 - \lambda_M(c) - \lambda_S(c))$, even for $c < \bar{c}$, so at $(\lambda_M^*(0), 1 - \lambda_M^*(0), 0)$ we have $J_M/J_U < 1$, $J_S/J_U < 1$, and $J_M/J_S = 1$, confirming (40).

Now suppose case (ii) of Proposition 5.3 holds at $\bar{c}$; this includes the possibility that $\bar{c} < c \leq \bar{c}$. Then $\lambda_M^*(0) = \lambda_M(\bar{c}) = 0$, and (46) specifies $\lambda_M = 0$ for all $c < \bar{c}$. By the monotonicity argument used in case (i), $J_S/J_U < 1$ at all $c < \bar{c}$. Moreover, Proposition 5.3(ii) entails $J_S/J_M \leq 1$, so this also holds for all $c < \bar{c}$, and therefore (40) holds.

We now turn to uniqueness. At any $c$, once we determine which proportions are strictly positive, the equilibrium is determined: if $\lambda_U = 0$, the other two proportions are determined by Proposition 5.3 if all three proportions are positive, they must satisfy $J_M/J_U = J_S/J_U = 1$ and must therefore be given by (43)–(44); if $\lambda_M = 0$ and $\lambda_U > 0$, the proportions are determined by the requirement that $J_S/J_U = 1$. We know from Proposition 5.3(iii) that $\lambda_S > 0$, so these are the only combinations we need to consider.

It therefore suffices to show that at any $c$, the set of agents with positive proportions is uniquely determined. Suppose we try to introduce uninformed agents into an equilibrium from which they are absent. If we start with $\lambda_M > 0$ (and necessarily $\lambda_S > 0$) then $J_M/J_U \leq
1 and \( J_S/J_U \leq 1 \). Increasing \( \lambda_U \) requires decreasing either \( \lambda_M \) or \( \lambda_S \) and therefore decreasing either \( J_M/J_U \) or \( J_S/J_U \), precluding \( \lambda_U > 0 \), in light of [40]. If \( \lambda_M = 0 \), the decrease must be in \( \lambda_S \) and the same argument applies. Suppose we try to introduce macro-informed agents into an equilibrium with only micro-informed and uninformed agents. The presence of uninformed agents requires \( J_M/J_U \geq 1 \). Increasing \( \lambda_M \) would increase \( J_M/J_U \), precluding \( \lambda_M > 0 \). Starting from an equilibrium with \( \lambda_S = 1 \) and increasing \( \lambda_M \) while leaving \( \lambda_U = 0 \) fixed is also infeasible because the value of \( \lambda_U \) determines the value of \( \lambda_M \) and \( \lambda_S \) through Proposition 5.3.

**Proof of Corollary 6.1** (i) It suffices to consider the range \( \bar{c} \leq c \leq \underline{c} \) with \( \underline{c} < \bar{c} \), because \( \Pi_M \) is constant on \( (0, \underline{c}] \) and identically zero on \( [\bar{c}, \infty) \). It follows from (43) and (44) that

\[
\lambda^2_M(c) = \frac{\gamma^2(1 - fs)^2\sigma^2_X\sigma^2_M}{fs\tau_M} \left( \frac{\lambda^2_M(c)f_M}{\gamma^2(1 - f_M)^2\sigma^2_M}\sigma^2_X + 1 \right) \equiv a\lambda^2_M(c) + b, \quad a, b > 0.
\]

Because \( \lambda^2_M(c) \) is strictly decreasing in \( c \), dividing both sides by \( \lambda^2_M(c) \) shows that \( \lambda^2_M(c)/\lambda^2_M(c) \) is strictly increasing in \( c \), hence \( \lambda^2_M(c)/(\lambda_M(c) + \lambda_S(c)) \) is strictly decreasing in \( c \). (ii) Follows from (46). (iii) We know from (30) and (31) that \( \rho^2_Z \) and \( \rho^2_S \) are increasing in \( \lambda_M \) and \( \lambda_S \), respectively, so monotonicity of price efficiency follows from monotonicity in (46).

### A.5 Variance ratios in Kacperczyk et al. (2016)

In this appendix, we briefly discuss the calculation of variance ratios in Kacperczyk et al. (2016), using notation from their paper, with references to relevant page numbers. The vector of (idiosyncratic and index) returns is given by

\[
\tilde{f} - \tilde{r}p = \Gamma^{-1}\mu + (I - B)z - Cx - A,
\]

where \( z \) is a vector of independent factors, and \( x \) is a vector of independent supply shocks. The matrix \( I - B \) diagonal with entries \( \tilde{\sigma}_i/\sigma_i \leq 1 \) (p.605), with

\[
\tilde{\sigma}_i^{-1} = \sigma_i^{-1} + K_i + \frac{K^2}{\rho^2\sigma_x}.
\]  

(A.17)

Here, \( \sigma_i \) is the prior variance of the payoff of security \( i \), \( \tilde{\sigma}_i \) is the posterior variance, \( \sigma_x \) is a supply variance, \( \rho \) measures risk aversion, and \( K_i \) is the total investor attention allocated to security \( i \). The cashflow variance in the return on the \( i \)th asset is therefore given by \( (\tilde{\sigma}_i/\sigma_i)^2\text{var}[z_i] = \tilde{\sigma}_i^2/\sigma_i \). The total variance of the return is (p.606, equation 29),

\[
V_{ii} = \tilde{\sigma}_i[1 + (\rho^2\sigma_x + K_i)\tilde{\sigma}_i].
\]

The fraction of the total variance given by cashflow news is then

\[
\frac{\tilde{\sigma}_i^2}{V_{ii}\sigma_i} = \frac{\tilde{\sigma}_i}{\sigma_i}[1 + (\rho^2\sigma_x + K_i)\tilde{\sigma}_i]^{-1} = \left( \frac{\sigma_i}{\tilde{\sigma}_i} + (\rho^2\sigma_x + K_i)\sigma_i \right)^{-1}.
\]

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Using (A.17), this becomes

\[
\left( \sigma_i \left[ \sigma_i^{-1} + \bar{K}_i + \frac{\bar{K}_i^2}{\rho^2 \sigma_x} \right] + (\rho^2 \sigma_x + \bar{K}_i) \sigma_i \right)^{-1}. \tag{A.18}
\]

As the proportion of investors who are informed increases, the total attention \( \bar{K}_i \) allocated to an asset does not decrease. (This property is similar to Proposition 1 in Kacperczyk et al. 2016, and we provide a proof in the Internet Appendix.) It follows that (A.18) is decreasing in the fraction of informed. But if the proportion of return variance due to cashflow variance is decreasing, then so is the ratio of cashflow variance to discount rate variance. Thus, VR decreases for all assets, in contrast to our results in Figure 6.

Figure 7 illustrates another contrast between our model and that of Kacperczyk et al. (2016). Recall that in our setting investors endogenously choose to specialize in micro or macro information, and, unless the cost of becoming informed is very high, we have both types of investors in equilibrium. In Kacperczyk et al. (2016), investors allocate all attention to the riskiest asset until they have sufficiently reduced its posterior variance, at which point they also start to allocate attention to the next riskiest asset.

With the calibration parameters of Table 1 the macro factor has greater variance than each micro factor, so investors initially learn only about the macro factor.\footnote{Kacperczyk et al. (2016) assume that supply shocks for all factors have the same variance, but this restriction does not seem to be necessary for their results, so we use our calibrated values \( \sigma_X^2 \) and \( \sigma_X/N \). We take the mean idiosyncratic supply shock to be zero. The capacity level at which investors begin allocating some attention to micro information does not depend on \( N \). See Internet Appendix for details.} In Figure 7, which uses our calibrated parameters and \( N = 100 \) stocks, this initially brings down the macro posterior variance (left) and the macro variance ratio (right) but leaves the corresponding micro quantities unchanged.

The capacity on the horizontal axis is on an arbitrary scale of zero to 100, and varying the capacity is equivalent to varying the fraction of informed (skilled) investors (\( \chi \) in their model) because only their product matters. To interpret the figures, consider that the left panel shows \( 1 - f_M \) and \( 1 - f_S \). Thus, the figure says that \( f_S \) will remain at zero until investors have learned enough about the macro factor to increase \( f_M \) to about 0.76. A market with \( f_M \geq 0.76 \) or \( f_S \approx 0 \) would differ notably from the empirical results discussed in Section 7.1.

The right panel of the figure similarly shows that the micro VR will remain very high until the macro VR has been reduced by a factor of almost 20.\footnote{We have truncated the vertical scale for legibility; at lower capacity levels the micro VR is roughly 30,000. With our notation, the variance ratios at zero capacity are \( 1/\gamma^2 \sigma_X^2 \sigma_{M}^2 \) and \( 1/\gamma^2 (\sigma_X^2/N^2) \sigma_{S}^2 \). The figure shows results for \( \ell = 1 \). The macro-micro contrast is even greater with \( \ell = 2 \).}

These patterns suggest that at many plausible parameters, the model of Kacperczyk et al. (2016) predicts that informed investors learn only about the macro factor, unless \( f_M \) is very high. This contrasts with the equilibrium in our model, in which we always find micro-informed investors and typically find macro-informed investors as well. The model of Kacperczyk et al. (2016) may be more descriptive of the attention allocation problem when several factors are of similar importance and have similar supply variances — factors representing industry sectors, for example, or value and growth factors, or stocks, bonds,
Figure 7: Ratios of posterior to prior variance (left) and ratios of cash flow variance to
discount rate variance (right) as functions of information capacity in the model of Kacperczyk et al. (2016), using Table I parameters with $\ell = 1$ and $N = 100$ stocks.

and commodities. In our setting, a single market factor carries much greater risk than all other individual factors.

References


