A Numerical Simulation Approach to Study Systemic Risk in Banking Systems

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ABSTRACT

We introduce a numerical algorithm to study banking systems subject to credit risk. We start with a description of the banks' balance sheets and incorporate two features to account for connectivity effects: (i) interbank loans, and (ii) correlated-exposures to a common universe of loans. The driving force behind the model is the progressive deterioration of the loan portfolios to which the banks are exposed. This method is useful to identify the weaknesses of a given banking network and assess the merits of different potential interventions from the regulator's viewpoint. An example based on the banking system of a Latin American country demonstrates the advantages of this approach.

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1. INTRODUCTION

The subprime crisis made obvious something that only a handful of authors had acknowledged before the collapse of Lehman in 2008 and the subsequent rescue of AIG: the financial system is really a network whose nodes are linked by complex relationships. Thus, any attempt to regulate or monitor such system by looking at its individual components in isolation is doomed to fail.

Consequently, a successful approach to deal with systemic risk (the possibility that the failure of a handful of components might bring the entire system to a halt, or worse, trigger its collapse) can only be studied using an integrated approach. Recently, this necessity has become more critical as the degree of connectedness has increased substantially (Billio et al.; 2012).

2. PREVIUOUS WORK

Before the subprime crisis only a few authors had focused on the importance of systemic risk. A paper by Sheldon and Maurer (1998) represents one of the first attempts at dealing with this issue. They investigated, albeit with a simplistic model, the Swiss interbank loan system and its response to defaults. Later, Allen and Gale (2000) studied financial contagion for different network configurations and concluded that the likelihood of contagion depends strongly on the structure of the network, namely, the degree of completeness. Elsinger et al. (2006) made an important contribution by bringing into the picture risk management metrics (such as Value-at-Risk or VaR) to estimate the exposure of the central bank in the event of massive bank failures. They demonstrated their approach with reference to the Austrian banking system and brought attention to a key factor: the importance of correlated exposures by banks. In a related paper, Boss et al. (2004) looked more formally at the topology of the interbank network, also using the Austrian market. They concluded that its key features (the clustering coefficient and the average shortest path length) differed dramatically from those exhibited by previous examples treated in the academic literature. Other authors, such as Upper and Worms (2004), Freixas et al. (2000), Eisenberg and Noe (2001), Nier et al. (2008), Iori et al. (2008), van Lelyveld and Liedorp (2006), Furfine (2003), and Wells (2004) investigated different aspects of this problem. They covered a broad range of subjects ranging from the relationship between the topology of a network and the existence of clearing vectors to the specifics of some market segments (e.g. the structure of the Brazilian interbank system).

In retrospect, it might be tempting to dismiss these early efforts as irrelevant as they were often based on assumptions that were clearly naive and unrealistic: banks that exhibit all the same default probability; individual consumers who have all the same preferences; or economies with only one good, and the like. However, their merit is that they identified a critical issue (systemic risk) long before it became trendy.

After the subprime crisis there was a major influx of papers addressing systemic risk. In our view, two papers stand out. The first, by Georg (2013), articulated clearly a general framework to monitor the time-evolution of a banking system subject to external credit shocks. He considered a system with several banks, all with different capital structures. Then, stochastic returns (which are recalculated every time period) trigger updates in the banks' balance sheets which, in turn, might (or might not) cause defaults. Some assumptions, such as the thought that banks optimize their balance sheets according to a common utility maximization function, can probably be challenged. Nevertheless, the author identified important factors relevant to characterizing the resilience of a given network. The most important is perhaps the fact that money-centered networks are more resilient than random networks. A second insightful paper by Elliott et al. (2013) identified, albeit using a simple mechanism, the circumstances under which a network might propagate a shock. The authors investigated how the effects of diversification and integration in the structure of the network affect its capacity to generate cascades. In agreement with Georg (2013), they observed that networks achieve maximum resilience for an intermediate (optimal?) degree of integration. The main merit of their approach is that it provided a more systematic way to identify cascades, proved some theoretical results in terms of the existence and uniqueness of equilibrium solutions, and outlined a simple computational algorithm to identify sequential failures. Additionally, these two papers, together, provide a very comprehensive literature review.

The remaining of the post-crisis papers can be divided into two groups. The first group deals with the topological characteristics of the financial networks under study and how these features affect the dynamic behavior of them. A second group of papers, more empirically oriented, address the dynamic response of some specific financial networks to exogenous credit shocks.

Among the first group, a paper by Haldane and May (2011) is worth mentioning. It brought into the picture an interesting analogy between financial systems and ecological food webs. The paper mentions that prudential regulation has tended to focus on banks individually: an approach which is certainly sure to miss systemic risk. Also, the authors alert us about the danger of having a system in which the banks (again, individually) are highly diversified, but hold very similar portfolios. Other authors who have addressed relevant aspects of the topic at hand are Acemoglu et al. (2014), Cabrales et al. (2013), Acharya (2009), Battiston et al. (2012), Acharya et al. (2010), Blume et al. (2011), Wagner (2010), and Eboli (2013), Uhlig (2010), Gai and Kapadia (2010), Gai et al. (2011), Caccioli et al. (2011), Huang et al. (2009). In general, these papers derive specific results for networks that satisfy special topological conditions. Unfortunately, these findings are unlikely to offer practical guidance to a regulator facing a crisis situation.

In the second group of papers, a few are worth mentioning. Martinez-Jaramillo et al. (2010) used an example based on the Mexican banking system. They introduced a metric to assess the fragility of the system based on the conditional VaR and focused on identifying the bank failures that were caused by an (initial) external shock and those caused by the ensuing contagion process. Dette et al. (2011) explored the "robust-yet-fragile" condition of a network based on an example inspired by interconnections observed among the European countries. They concluded that a default by Greece could be easily absorbed by the network but a joint default by Portugal, Spain and Ireland could trigger a difficult to manage cascade. Cont at al. (2010) argued, based on an analysis of the Brazilian banking system, that capital requirements should be based on banks' exposures rather than balance sheet size. Manna and Schiavone (2012) and Fourel et al. (2013) examined the dynamics of some segments of the Italian and French banking systems respectively.

Finally, a paper by Upper (2011) provides a good assessment of the state-of-the-art at that time in terms of modeling efforts. He remains unconvinced of the usefulness of most models. His skepticism is based on the fact that (in his view): (i) models are not suited to carry out stress testing exercises; (ii) most papers are very theoretical and are not based on empirical evidence; (iii) there is an exaggerated emphasis on idiosyncratic failures of individual banks instead of exploring the effects of common credit shocks; and (iiii) most models lack a behavioral foundation. Unfortunately, notwithstanding recent progress, his observations--almost six years after they were articulated--are still valid.

More specifically, some assumptions made by previous researchers that are at odds with reality--related to (ii) and (iii) above--are the following:

[1] *Banks continuously optimize their balance sheets according to some common utility function.* Reality: most lending decisions are made locally (at the branch level), often due to strategic, relationship-based, or political reasons, and frequently according to general guidelines provided by risk managers and profit-driven executives. There is no evidence that banks run optimization algorithms based on time-independent utility functions to structure their balance sheets;

[2] *Banks, when facing crisis, continuously rearrange their balance sheets to reduce risk.* Reality: the empirical evidence indicates that at times of crises liquidity evaporates, and thus, banks are often forced to sit more or less idly as their balance sheets deteriorate. The subprime crisis provides good evidence of this phenomenon: after the collapse of Lehman in September 2008, the TED spread (the difference between the 3-month T-bill and the LIBOR rate) shot up and stayed at a higher-than-200-basis points-level for almost three months. In short, the TED spread reflected the fact that there was no liquidity in the interbank market as most banks distrusted each other in terms of solvency, and accordingly there was no balance-sheet re-arranging. Notice that in more normal times (e.g., roughly since 2013) the TED spread has fluctuated around the 20-35 basis points range;

[3] *Banks, when facing liquidity or solvency problems, can issue new debt or new/more shares.* Reality: leaving aside that these exercises normally require several months of preparation, banks in trouble can seldom access the market. And when they do, they have to make significant concessions, which, in turn, would probably require a major re-engineering of the simulation model employed up to that point to capture this new reality accurately;

[4] *VaR-related metrics can be useful to reflect systemic risk*. Reality: Financial networks exhibit highly nonlinear behavior: minor changes in key parameters can result in dramatic

changes in their responses. Such phenomena do not lend themselves to be described well by normal or smooth distributions and therefore the VaR (which does not satisfy the sub-additivity condition) is an inappropriate metric.

In conclusion, it is clear that there is room for improvement in terms of understanding better how to identify, and how to analyze, systemic risk in banking networks. More specifically, an important void is the lack of attention that the regulator's viewpoint has received so far. A regulator faces a banking network whose configuration (at least in the short term) he cannot change: it is a given. His interest is, first, to identify potential weaknesses in the system, and second, to have the ability to explore the consequences of several mitigating responses should a bank failure occur. The regulator benefits little, for instance, from knowing that a theoretical network having certain features might have multiple clearing vectors.

With that as background, in this paper we introduce an easy-to-implement numerical simulation algorithm that we hope will serve as a diagnosis tool and also as a device to evaluate potential responses to strengthen a given banking system. Our work can be thought as a continuation of the work by Elliott et al. (2014) and Georg (2013) but it differs in several important ways. We represent the banks using a very detailed and realistic balance sheet structure; additionally, we simulate the external shocks to the system by means of correlated (but different) credit risk exposures to a common universe of loans. This framework offers two advantages. It makes it easy to identify the weakest banks and the potential for cascades. And also, it makes it easy to assess the merits of a number of responses by the regulator (limit leverage, demand an increase in reserves, extend liquidity lines to the weakest banks, etc.).

In what follows we describe the model and the algorithm in detail, we demonstrate the benefits of the approach with an example based on the banking network of a Latin American country, and then we discuss the results. Finally, we present our conclusions.

3. THE PROBLEM AND THE MODEL

We start with a banking system made up of N banks. We also assume that we know, at some given time identified as t=0, the capital structure of each bank and the degree of interconnection among them. These two features are described more fully in what follows. The idea is to explore the evolution of this system as the banks' loan portfolios deteriorate and the banks start to collapse. In summary, the focus is to derive conclusions regarding a specific system under study (what a regulator needs) rather than establishing that systems having some particular properties exhibit certain types of responses.

3.1 Individual banks

The essential elements of a bank's balance sheet can be described as follows. The assets consist of: cash, C; loans to third parties, L; loans to other banks, B; deposits at the central bank, θ D; liquid investments, I; and illiquid investments, A. The liabilities are: deposits, D; debt to central bank, Φ; debt to other banks, H; other debt, G; and equity, E. The factor θ ($0 < \theta < 1$) specifies the fraction of deposits that the banks must keep in liquid assets at the central bank. An index (k) associated to any of these variables refers to the specifics of bank k (k=1, …, N).

3.2 Interbank lending

Direct interconnection among banks is the result of interbank loans, described by a matrix of interconnection coefficients (β's). The set (or vector) $\beta_{k1}, \beta_{k2}, \dots, \beta_{kk}, \dots, \beta_{kN}$ specifies the exposure of bank k to the remaining banks. Clearly, $\beta_{kk} = 0$ (no bank holds its own debt) while β_{ki} refers to the percentage of the debt issued by bank i (H_i) which is held by bank k. Also, $\beta_{1i} + \dots + \beta_{Ni} = 1$ for all i's.

Thus, we have that

$$B_k = \sum_{i=1}^{N} \beta_{ki} \ H_i \tag{1}$$

and also

$$H_k = \sum_{i=1}^{N} \beta_{ik} \ H_k \tag{2}$$

3.3 Equilibrium equation

With the previous elements we can state an equilibrium (assets = liabilities) equation for a given bank, k, as

$$C_k + L_k + B_k + \theta_k D_k + I_k + A_k = D_k + \Phi_k + H_k + G_k + E_k \tag{3}$$

which can be rearranged as

$$E_k = C_k + L_k + B_k + \theta_k D_k + I_k + A_k - D_k - \Phi_k - H_k - G_k \tag{4}$$

and invoking equations (1) and (2), we can write

$$E_k = \alpha_k + \sum_{i=1}^{N} \beta_{ki} \ H_i - \sum_{i=1}^{N} \beta_{ik} \ H_k \tag{5}$$

with

$$\alpha_k = C_k + L_k + (\theta_k - 1)D_k + I_k + A_k - \Phi_k - G_k \tag{6}$$

Obviously, a solvent bank is characterized by a positive value of the equity (E_k).

3.4 Credit risk exposure

Each bank is exposed to a loan portfolio (L_k) subject to credit (default) risk. Thus, the driving force behind the dynamics of the system (network) is the potential deterioration of each bank's loan portfolio.

Let $\varepsilon^T = (\varepsilon_1, \ldots, \varepsilon_N)$ be a random vector whose individual components follow a uniform distribution within the $[0, \delta]$ interval. The parameter δ represents the maximum deterioration in value, due to loan defaults, that the loan portfolio of the k bank can experience in a small time interval. We also assume that the correlations between any two components of the vector ε are identical and equal to ρ. This parameter (ρ) is aimed at capturing the fact that in a realistic situation the banks' credit exposures will exhibit some degree of correlation, either via syndicated loans, or simply because they operate in the same markets, and therefore, are exposed to some common economic drivers.

Hence, if we denote t and t+1 two consecutive points in time, we have that

$$L_k^{t+1} = L_k^t (1 - \varepsilon_k^t) \quad (7)$$

The credit-risk characterization just described is realistic as it is based on empirical evidence. For example, between November 2006 and September 2008, the ABX-HE-BBB index, a market index that reflected the risk associated with subprime mortgages, deteriorated slowly from par (100%) to around 5%. Thus, assuming that banks portfolios deteriorate in market value, gradually, according to the scheme presented is not unreasonable. In any event, as it will be seen later, adapting this framework to apply a violent shock to all (or a group of) banks portfolios is fairly straightforward.

4. SIMULATION ALGORITHM

Suppose that initially (t = 0) all the balance sheet variables are known in addition to δ and ρ (which capture the time deterioration of the aggregate loan portfolios) as well as the interconnection matrix, β.

The algorithm proceeds as follows:

Set p = 1 (a counter).

(i) At time t generate a new vector $(\varepsilon_1^t, \ldots, \varepsilon_N^t)^T$. This random vector can be generated numerically using the Gaussian copula, a technique that allows one to sample from a multivariate distribution that has a given correlation structure. In this case, we have N identical uniform distributions linked by a single correlation factor. This procedure has been described in detail by several authors (see Hawas and Cifuentes, 2014; Cario and Nelson, 1997).

(ii) Update for each bank k, the value of the loan portfolio, using equation (7).

(iii) Recalculate the new equity values (E_k) for each bank using first equation (6) followed by equation (5). If for some bank k, E_k turns out to be zero or negative, it means that bank k has become insolvent. Thus, its debt (H_k) now has a zero value, which in turns requires recalculating the new equity values for all banks by invoking, again, equations (6) and (5).

This process is repeated until no further banks collapse at time t+1. This recalculation process allows one to identify the presence of cascades, that is, distinguishing between primitive bank failures (caused by deterioration of the loan portfolio or L) and cascades (subsequent bank failures triggered by the collapse of other banks or changes in H).

After registering all the banks that collapse at t+1, we update t (t = t+1) and we go to (i) and repeat the process until reaching a time where the entire system (all banks) has collapsed.

(iiii) Once all banks have collapsed (the completion of one feasible realization path) we update p (p = p+1).

We repeat the loop (i)-(ii)-(iii)-(iiii) starting from t=0 again, unless p has reached a maximum pre-established (large) number.

After generating a sufficient large number of feasible paths, by averaging across all paths, we can estimate whatever figure of merit is desired; for instance: time (period) of the first bank failure, time required to collapse all banks, time where the first cascade is observed, probability that the failure of a certain bank can trigger a cascade, etc.

4.1 Implementation considerations

Implicit in the above-mentioned algorithm there are a few simplifications. For example, we are assuming that upon collapsing, the debt of the affected bank (H_k), has no value. Strictly speaking, the debt has probably a non-zero value between 0 and 100%. On the other hand, a case can be made that perhaps due to short-term liquidity constraints treating the debt as having a zero value is realistic. That said, modifying the algorithm to assign such debt a value equal to σH_k instead of zero, where σ is the recovery rate (or market value of the defaulted debt in the secondary market), is trivial. It only reduces to changing H_k by σH_k in equation (5) before re-computing E_k. This issue is explored in the example we present.

Also, when computing α_k in step (iii) by invoking equation (6), we are implicitly assuming that L_k is the only variable that changes from t to t+1. However, it is possible to envision situations in which this might not be true. For instance, suppose that we might also wish to incorporate a random fluctuation of the deposits (D_k), between t and t+1, according to some known stochastic process. This, of course, would imply some adjustments to the balance sheet (presumably a change in the value of I_k) to preserve the assets and liabilities equilibrium.

Alternatively, we could force the banks to increase the deposits they keep at the central bank, which would amount to change the value of θ to a common (and higher) factor for all banks. This, again, would require an adjustment in the value of I_k. Introducing such features is straightforward: it only requires updating the values for the relevant variables in equation (6), at each time step, according to the appropriate formula (or rule), before computing E_k using equation (5). This issue--as well as the potential intervention of the central bank (CB) or regulator--is also explored in the upcoming example.

5. EXAMPLE OF APPLICATION

We consider a banking system with N = 10 banks. This network is based on the actual structure of the banking system of a Latin American country in 2013, after eliminating the six smallest banks. The balance sheet information is given in Appendix A. The figures are expressed in non-dimensional monetary units that reflect the banks' relative sizes. The six configurations described represent the structure of the banking system for six consecutive months. The corresponding interconnection (β) matrices are described in Appendix B. Notice that bank sizes differ significantly (the biggest bank, A, is more than twenty times the size of the smallest bank, J). Notice also that some banks have their debt held by several other banks (several non-zero entries in the corresponding column of the β matrix, for a given configuration). In contrast, bank B--for example-- under several configurations, shows no debt to other banks. Additionally, θ = 15%, a value determined by the regulator, is identical for all banks regardless of their size.

Although the time steps are dimensionless, we interpret them to reflect a month. Thus, δ is set at 1%, to capture a realistic loan portfolio deterioration progression at time of stress. This means that, on average, a bank loan portfolio should lose 5.8% of its value in a year ($[1-0.01/2]^{12}$). Note that bank loan portfolios normally enjoy a 40 to 70% recovery rate. At particularly stressful years (take 2002, for example) bank loans in the U.S. experienced default rates approaching 10% per year. Thus, the value of δ chosen is consistent with stressful, but realistic, market conditions as 0.058/0.1 (expected loss/default probability) equals 58%. This results in a recovery rate of 1-58%=42%, low, but not far-fetched.

The correlation, ρ, is assumed to be 30%. Correlation values are notoriously difficult to estimate (this is the motivation for doing sensitivity analysis regarding this parameter); however, most practitioners employ figures in the 10% to 60% range. The Monte Carlo simulations are carried out using 1,000 samples of feasible failure paths. The results are presented in the next sections. The first group of analysis (5.1 through 5.5) is aimed at identifying the strengths and weaknesses of the different configurations. The second group (5.6 through 5.9) is aimed at assessing the merits of different responses from the regulator.

5.1 Typical failure path

It is illustrative first to consider a typical path of the Monte Carlo simulation for the Base Case (δ = 1%; ρ = 30% and θ = 15%). Table 1 shows such paths for the first three configurations. In reference to configuration 1, we notice that the first bank failure (bank I) occurs at period 26 while it takes 49 periods for the entire system to collapse. Also, banks B and G, which fail at periods 49 and 47 respectively, do so without bringing down other banks. On the other hand, the failure of bank D at period 31, generates a cascade as banks C and E fail during the same period as a consequence of D's failure.

It is important to realize that the goal of this simulation is not to predict what the configuration of the system will look like after, say, three years (36 periods/months) of progressive deterioration of the banks loans portfolios. Or, even more unrealistic, estimate how long would it take for the entire system to collapse. Clearly, no regulator would let such catastrophic scenario unfold without intervening. And since it is impossible to know *a priori* how a regulator would react under such scenario, it is not possible to include this feature in the model. Which, in turn, makes any attempt at predicting the long-term state of the system hopelessly naïve. The real value of the simulation exercise we are doing is to identify the relevant features of the system under study (weak banks, potential for cascades, etc.), to compare several configurations from the resilience point of view, and to test the effectiveness of some mitigating responses. In short, the idea is to employ parameters such as the numbers of months it would take the system to collapse, not as an actual estimate of such event, but as a proxy for the resilience of the system in its current state.

5.2 Time and type of bank failures

Table 2 displays the results of the Monte Carlo simulation (1,000 paths) for the first three configurations only, due to space reasons. The second and third columns of each configuration box show the probability that the bank failure could be the result of: (1) a primitive failure, that is, a failure due to deterioration of the loan portfolio (second column); or (2) a cascade, (third column). There is some consistency across configurations in terms of the type of failure that each bank is likely to suffer. It is interesting to notice that some banks almost never fail a result of a cascade. For example: banks F, G and I in configuration 1; D, F and I in configuration 2; and again D, F and I in configuration 3. In all these banks (see Appendix A) the exposure to other bank loans (parameter B) is quite small compared to L. Intuitively, in these cases one would expect--as the results show--that deterioration of L, rather than a cascade, should be the main cause of failure.

Table 3 shows, for all configurations, the confidence intervals for the periods where the first bank failure occurs and the last bank failure occurs (system collapses). Notice that the time it takes for the entire system to collapse is rather sensitive to the initial conditions (banks' balance sheets and interconnectivity) whereas the first failure period is less sensitive.

5.3 Sensitivity analysis

The algorithm makes it easy to perform sensitivity analyses. Table 4 shows how the system performance (period of first and last bank failure) changes in response to a 10% variation in two key parameters: θ and λ (leverage). The leverage parameter for a bank k is defined as $\lambda_k = (D_k + \Phi_k + H_k + G_k) / E_k$.

As expected, increasing θ has a direct effect on the resilience of the system. It delays both, the period of the first and last failure. Decreasing θ has the opposite effect. Something analogous is observed in relation with the leverage (lower leverage translates into more resilience).

Table 5 explores the effect of changes in δ and ρ. The effect of increasing or decreasing δ is rather obvious. The response of the system to changes in ρ, however, is less intuitive: whether the stability of the response to changes in ρ is a feature of this system or a more general property of these networks should be investigated in more detail.

5.4 Response to credit shocks

Table 6 shows the response of the network to a sudden shock (all banks suffer an identical loan portfolio loss, expressed as a percentage of the current loan exposure, at the same time, t=0) followed by the random deterioration pattern already described. Interpretation of the results is straightforward.

5.5 Risk metrics

It is useful to compute some simple figures of merit to gain insight into the behavior of the system. Consider for instance $\alpha = M/ (N^2-N)$ where M is the number of non-zero entries in the matrix β. This metric attempts to capture the degree of connectivity in the banking network: a value close to 1 corresponds to a highly connected system; 0 indicates no interbank lending.

Table 7 shows the value of α for the six configurations. It also shows the period when the last bank fails (a proxy for the resilience of the system) and P, the probability that a bank failure could be the result of a cascade, which can be estimated directly from the Monte Carlo simulation. The data show that there is a positive (0.54) correlation between P and α. It also shows that the most resilient configurations (1 and 4) correspond to "intermediate" values of α (0.54 and 0.60); whereas the lowest and highest values of α (0.47 and 0.54 respectively) corresponds to some of the weakest configurations (2 and 6). Both tendencies are in agreement with findings of previous authors.

Another useful metric is $\Upsilon_i = (L_i + B_i)/ E_i$. This metric attempts to reflect, for a given bank, the degree of exposure to credit risk in relation to its equity. The higher the value of Υ, the higher the vulnerability of the bank to loan defaults. Table 8 shows the values of Υ for all banks and all six configurations. Clearly, bank I appears to be the weakest bank, whereas banks B and G are the strongest. These observations are in agreement with the results shown in Table 2. For example, in the three cases shown, bank I is always the first to fail. In contrast, banks B and G are the last to fail.

Finally, the parameter $\eta_i = (\beta_{ij} H_j) / E_i$ shows, for a given bank i, whether the collapse of bank j can trigger the failure of bank i. This occurs for values of η_i higher than 1. Table 9 displays the values of η for the first configuration. Since all the entries in this matrix are less than 1, the implication is that no bank, by its own failure (and considering all the other variables unchanged), has the power to trigger the collapse of another bank. Notice that we are not considering here the possibility that the loan portfolio (L) could deteriorate. In essence, this situation reflects something akin to a bank failure due to fraud, where suddenly, due to--for example-- accounting irregularities, it is discovered that a bank is insolvent and $H_i = 0$. This result is interesting for it calls into question the soundness of attempting to detect cascades just by collapsing single banks, one at a time, without incorporating the effects of the interconnections resulting from the common exposure to the economy through L.

Put it in a different way. Had we attempted to detect the potential presence of cascades just by collapsing banks individually, we could have concluded that the answer was no. However, as the previous analyses (Table 2) indicate, there is indeed a significant potential for cascades in these configurations. In essence, just looking at a banking network concentrating on "direct" bank connections through the matrix β could be misleading for it misses all the effects resulting from the "indirect" links arising from the exposure to the loans (the common economic factors affecting all banks).

The previous analyses, taken together, are useful for a regulator: they can help to identify the vulnerabilities of the banking network and get a sense for its overall resilience. In what follows we concentrate on examining several potential responses or interventions by the regulator, leaving aside their feasibility due to legal or political considerations.

5.6 Influence of different values of θ

Consider configuration 1. What would happen if the regulator increases the reserve amount (θ) that the banks must keep at the central bank? Recall that the reference (Base Case) value was 15%. (This, of course, requires adjusting the other parameters of the balance sheet, in this case I, to preserve the equilibrium condition). As expected, Table 10 shows that increasing θ improves the resilience of the system; somehow less expected, the improvement is fairly steady with no abrupt changes and no "saturation" effects (increments in θ always delay bank failures).

5.7 Influence of different values of σ

Consider, again, configuration 1. Recall that we assumed that upon defaulting the value of H (bank debt) was zero. What would happen if the central bank (or a special purpose government entity) sets up a program akin to the TARP initiative enacted during the subprime crisis in the U.S. to buy the impaired loans for a minimum value? That would protect the creditors from being impacted negatively as a result of a fire sale by the issuer (presumably the regulator could dispose these assets later under more benign market conditions).

Curiously, in this case, the analysis shows that such policy would not have any impact on the period of the first bank failure; however, it would significantly delay the collapse of the entire system (see Table 11).

5.8 Influence of different Central Bank (CB) responses to the failure of a specific bank

Consider, again, configuration 1 and the Base Case. Table 2 indicates that bank I is the bank most likely to fail first. And Table 3 shows that the first bank failure is most likely to occur at period 28 while the last failure at period 55.

What would happen if the CB, at the very moment bank I is about to fail, decides to inject some cash to make its equity (which otherwise would be zero or negative) equal to ψ E, where E is the original value of the equity and ψ a parameter between 0 and 1. Such action might be the result of either the political clout of bank I's stockholders or the genuine desire to protect the system by delaying a potentially dangerous bank failure.

Table 12 shows that such action would indeed delay the period where the first bank failure occurs (which, presumably, might provide the regulator with more time to enact other measures). However, values of Ψ higher than 0.3 do not result in any improvements. Also interesting, values of Ψ lower than 0.8 do not improve the overall resilience of the system (as reflected by the number of periods that it takes to collapse all banks).

Table 13 shows how the CB intervention alters the dynamics of the network. For values of Ψ lower than 20% , bank I is always the bank most likely to fail first. Then, for Ψ's larger than 20%, bank A, becomes the bank most likely to fail first, which, in some sense, offers bank I "additional" protection. In fact, values of Ψ approaching 1, not only rescue bank I, but they also make bank I the most resilient in the entire network: the bank most likely to fail last.

This analysis demonstrates that the effects of rescuing a specific bank are far from obvious. First, there seems to be some optimal equity injection amount; and second--and perhaps more important—bailing out a bank not only delays subsequent bank failures, but it can also alter the entire dynamics of the network. This is very relevant for these changes, for example, might render ineffective the entire risk-management decisions of the surviving banks at a moment when the system enjoys little or no liquidity. Such situation would be highly undesirable if not outright dangerous.

5.9 Influence of different CB responses to the potential failure of a bank that can trigger a cascade

Tables 1 and 2 show that the failure of bank D is likely to trigger a cascade as two other banks (C and E) might collapse as a result of D's failure. What would happen if the CB, at the moment bank D is about to fail, decides to inject some cash to make its equity equal to χ E, where E is the original value of the equity and χ is a parameter whose value is between 0 and 1. Such action might represent an attempt by the CB to prevent a cascade from happening simply to avoid a panic-driven crisis.

Table 14 shows, that increasing values of χ, go hand in hand with a decreasing probability of triggering a cascade, as the periods when banks C, D and E fail become more distinct. Note that for χ =0, the table indicates that the period where banks C and E are most likely to fail is 33; whereas bank D is most likely to fail at period 32. This is not a contradiction: the cascade (all three banks failing at period 32) is a likely, but not certain, event as Table 2 shows.

Notice also that the injection of capital to bank D has the effect of improving the survival probability of D, while leaving C and E fairly prone to collapse more or less together at about the same time (periods 33 or 34). It is unclear if such outcome is desirable as not much improvement seems to have occurred overall, even though the resilience of the network has gone from 55 to 66 (period of last bank failure).

In summary, all these analyses, which can be carried out simply by altering two or three lines of code in the general algorithm presented, are useful. Not only they can shed light into what the consequences of a particular regulatory action could be, but they also hint at the existence of a very complex set of behaviors from the network itself (perhaps more than anticipated). In brief, regulatory actions can alter the dynamic behavior of the network substantially. This means that ignoring the regulator's response is likely not to be very enlightening as far as anticipating how the system might evolve long-term. But also--and perhaps more distressing--this reinforces the view suggested earlier that trying to predict how a banking system might evolve throughout a credit crisis (given both the importance and the uncertainty regarding the regulator's behavior) is naïve and hopeless. The best one can hope for is to make reasonable short-to-medium term predictions based on different courses of action by the regulators.

6. CONCLUSIONS

The previous example demonstrates and validates the usefulness of the technique we have presented. For instance, it can help a regulator to identify which banks are more likely to trigger cascades should they fail, which banks are more likely to fail "on their own" (primitive failures) or as a result of cascades, what measures can be more helpful to increase the resilience of the system (increase θ or reduce leverage, for example), etc. In summary, this approach is a useful diagnosis tool. Additionally, the simple metrics presented in this analysis (α, Υ and η) are valuable companions to the simulation exercise as they provide further insights and help to double-check the results, at least intuitively.

Moreover, the model offers three attractive features: (1) it is based on a realistic (factual) representation of a banking system and it does not rely on assumptions whose empirical validity is questionable (banks that adjust their balance sheets according to some dubious utility function, or worse, banking systems whose liquidity is never eroded, even at times of crises); (2) it relies on only two parameters (δ and ρ): this makes it much more manageable and easier to calibrate than more complex (several parameters) models when using empirical data; and (3) it captures the interconnection among the banks in two distinct ways. First, a direct connection via the interbank loan (matrix β) system. And second, an indirect connection through common economic factors captured via the Gaussian copula and the loan (L) portfolios. These features represent an important improvement compared with previous models. In fact, this approach is much more in tune with currently employed methods in the context of analyzing fixed income portfolios subject to credit risk than the idiosyncratic shocks employed by some studies.

Certainly, the model presented can be made more complex by including additional features. For instance, we can assume that I (or D) follows some time-dependent stochastic process whose incorporation into equation (6) should be straightforward. However, caution should be exercised when including too many moving parts as it is not obvious how all these variables are correlated.

Some of our results are in agreement with previous findings. For example, that the highest resilience seems to be achieved for some "intermediate" level of connectivity. And that the presence of cascades seems to be linked to higher levels of connectivity. On the other hand, our simulation shows that attempting to gain insight into the behavior of a banking system by forcing banks to fail individually, but without taking into account their connectivity through the economy can be very misleading. This is probably even more misleading when all η's are lower than 1 as one might be tempted to conclude that there is no danger of having cascades. And obviously, looking at banks in isolation says nothing about the potential danger they pose to the system.

Two interesting extensions of this work, both being investigated by the authors presently are: (i) assume that δ is bank-dependent, in other words, assume that different banks have loan portfolios of different quality; and (ii) assume that the portfolio of banks loans could be divided into a short-term portfolio and a long-term portfolio, both following different deterioration patterns, to further refine the simulation.

And two final thoughts: given the complexity of the problem at hand, we think that a good strategy is to start with a model with few parameters, which describes, albeit in a simple manner, an essential feature of the system (for instance, the nature of the interconnection among banks). This is much better than starting with a multi-parameter model, which, under the umbrella of sophistication, hides too many assumptions that are in direct conflict with the empirical evidence. Two examples that come to mind: (i) algorithms that assume that at times of crises banks optimize their balance sheets according to some untested utility function, or (ii) worse, the very assumption that under extreme market conditions (in essence, the conditions under which we need to understand the behavior of the banking system) financial markets offer enough liquidity to carry out these balance-sheet adjustment exercises.

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Table 1. Typical Simulation Path (Monte Carlo) for Three Configurations.

	Configuration 1		Configuration 2		Configuration 3	
	Type of Failure (1 or 2)	Period of Failure	Type of Failure (1 or 2)	Period of Failure	Type of Failure (1 or 2)	Period of Failure
Bank A	1	29	1	29	1	29
Bank B	1	49	1	42	2	38
Bank C	2	31	2	31	1	33
Bank D	1	31	1	31	1	31
Bank E	2	31	1	32	2	31
Bank F	1	38	1	39	1	39
Bank G	1	47	2	39	2	38
Bank H	2	38	2	39	1	38
Bank I	1	26	1	26	1	26
Bank J	1	34	1	37	2	33
	1= primitive (L)					
	2= cascade (H)					

Table 2. Results of Monte Carlo Simulation for Three Configurations: Expected Period of Failure (for Each Bank) and Types of Failures (Probabilities).

Period of Bank Failure (Expected Value) Based on Monte Carlo Simulation--1000 paths

		Config. 1			Config. 2			Config. 3	
	Period of Failure	Type of Failure (1) (%)	Type of Failure (2) (%)	Period of Failure	Type of Failure (1) (%)	Type of Failure (2) (%)	Period of Failure	Type of Failure (1) (%)	Type of Failure (2) (%)
Bank A	33	0.802	0.198	33	0.799	0.201	32	0.56	0.44
Bank B	55	0.632	0.368	47	0.852	0.148	39	0.367	0.633
Bank C	33	0.519	0.481	33	0.365	0.635	35	0.858	0.142
Bank D	32	0.877	0.123	33	0.967	0.033	33	0.944	0.056
Bank E	32	0.725	0.275	33	0.823	0.177	32	0.607	0.393
Bank F	41	0.955	0.045	42	1.000	0.000	41	1.000	0.000
Bank G	53	0.989	0.011	41	0.044	0.956	41	0.482	0.518
Bank H	38	0.625	0.375	40	0.595	0.405	39	0.721	0.279
Bank I	28	1.000	0.000	27	0.945	0.055	27	0.942	0.058
Bank J	35	0.702	0.298	37	0.7	0.3	35	0.440	0.560
	1= primitive (L)								
	2= cascade (H)								

Table 3. Confidence Intervals from the Monte Carlo Simulation: Period of First and Last Bank Failure.

Period of Failure--Expected Value-Lower Bound (95%)--Upper Bound (95%)

	Configuration 1			Configuration 2			Configuration 3		
	Period of Failure (average)	Interval Lower End	Interval Upper End	Period of Failure (average)	Interval Lower End	Interval Upper End	Period of Failure (average)	Interval Lower End	Interval Upper End
First Failure	28	23	32	27	22	33	27	22	33
Last Failure	55	49	62	47	40	55	42	36	50

	Configuration 4			Configuration 5			Configuration 6		
	Period of Failure (average)	Interval Lower End	Interval Upper End	Period of Failure (average)	Interval Lower End	Interval Upper End	Period of Failure (average)	Interval Lower End	Interval Upper End
First Failure	27	22	33	28	22	33	27	22	33
Last Failure	55	47	64	46	39	54	40	35	47

Table 4. Sensitivity Analysis: System Response (Period of First and Last Failure) to a 10% Variation in θ and λ.

	Config. 1	Config. 2	Config. 3	Config. 4	Config. 5	Config. 6
Base Case						
First Failure	28	27	27	27	28	27
Last Failure	55	47	42	55	46	40
θ (+10%)						
First Failure	30	31	31	31	31	30
Last Failure	60	51	46	60	50	44
θ (-10%)						
First Failure	24	24	24	24	24	24
Last Failure	50	43	39	49	42	37
λ (+10%)						
First Failure	25	25	25	25	25	25
Last Failure	48	41	38	49	40	36
λ (-10%)						
First Failure	27	31	31	31	31	30
Last Failure	55	55	48	63	54	46

Table 5. Sensitivity Analysis: System Response (Period of First and Last Failure) to a 10% Variation in δ and ρ.

	Config. 1	Config. 2	Config. 3	Config. 4	Config. 5	Config. 6
Base Case						
First Failure	28	27	27	27	28	27
Last Failure	55	47	42	55	46	40
δ (+10%)						
First Failure	25	25	25	25	25	25
Last Failure	50	43	39	39	42	37
δ (-10%)						
First Failure	30	30	30	30	30	30
Last Failure	61	52	47	61	51	45
ρ (+10%)						
First Failure	27	28	27	27	27	27
Last Failure	54	47	42	55	46	40
ρ (-10%)						
First Failure	27	27	27	27	27	28
Last Failure	55	47	42	55	46	41

Table 6. Response to a Shock Applied Simultaneously to the Loan Portfolio of All Banks at t=0, Followed by Gradual Deterioration of Their Portfolios, (Period of First and Last Failure).

		Config. 1	Config. 2	Config. 3	Config. 4	Config. 5	Config. 6
Base Case	**First Failure**	28	27	27	27	28	27
	Last Failure	55	47	42	55	46	40
shock=5%	**First Failure**	16	16	14	16	16	16
	Last Failure	40	34	28	38	34	29
shock=10%	**First Failure**	7	7	4	7	7	7
	Last Failure	30	26	16	29	24	18
shock=15%	**First Failure**	2	2	2	2	2	2
	Last Failure	21	15	5	19	14	7

Table 7. Value of α for All Configurations, Period of Last Failure, and Probability (P) that a Bank Failure Could Be the Result of a Cascade.

	Config. 1	Config. 2	Config. 3	Config. 4	Config. 5	Config. 6
α	0.54	0.47	0.58	0.60	0.56	0.64
Period of Last Failure	55	47	42	55	46	40
P (cascade)	0.22	0.29	0.31	0.29	0.35	0.39

Table 8. Value of ϒ for All Banks and for All Configurations.

Value of ϒ	Config. 1	Config. 2	Config. 3	Config. 4	Config. 5	Config. 6	Avrg Bank
Bank A	6.8	7.0	7.6	7.1	6.8	6.9	7.0
Bank B	3.7	3.8	5.4	4.4	3.8	3.9	4.2
Bank C	6.6	7.1	6.3	6.4	7.6	6.6	6.8
Bank D	7.0	6.9	6.9	6.9	6.9	7.3	7.0
Bank E	7.0	7.0	7.5	8.0	8.0	7.8	7.6
Bank F	5.6	5.5	5.5	5.6	5.5	5.6	5.6
Bank G	3.9	4.2	4.1	3.8	4.9	4.7	4.3
Bank H	5.9	5.5	5.4	5.5	5.5	5.6	5.6
Bank I	8.3	9.0	9.0	9.5	9.7	9.7	9.2
Bank J	6.1	6.1	6.2	6.2	6.1	6.0	6.1

Table 9. Value of η for All Banks (Configuration 1).

Configuration 1

η	Bank A	Bank B	Bank C	Bank D	Bank E	Bank F	Bank G	Bank H	Bank I	Bank J
Bank A	0.00	0.00	0.00	0.05	0.01	0.05	0.04	0.00	0.00	0.00
Bank B	0.01	0.00	0.00	0.03	0.00	0.08	0.08	0.00	0.00	0.00
Bank C	0.08	0.00	0.00	0.05	0.00	0.19	0.02	0.01	0.00	0.00
Bank D	0.03	0.00	0.00	0.00	0.00	0.05	0.00	0.02	0.00	0.01
Bank E	0.05	0.00	0.00	0.02	0.00	0.09	0.02	0.02	0.00	0.02
Bank F	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.00	0.01
Bank G	0.02	0.00	0.00	0.01	0.00	0.12	0.00	0.00	0.00	0.01
Bank H	0.02	0.00	0.00	0.05	0.03	0.17	0.41	0.00	0.00	0.00
Bank I	0.00	0.00	0.00	0.00	0.00	0.06	0.01	0.00	0.00	0.00
Bank J	0.02	0.00	0.00	0.08	0.00	0.04	0.08	0.02	0.00	0.00

Table 10. Configuration 1: Period of First and Last Bank Failures, and Their Corresponding Confidence Intervals, for Different Values of θ.

	Period of First Bank Failure			Period of Last Bank Failure		
θ	**Mean**	**5%**	**95%**	**Mean**	**5%**	**95%**
8%	13	10	16	36	31	42
9%	15	12	19	38	33	44
10%	17	14	21	41	36	47
11%	19	16	23	43	38	50
12%	21	17	25	46	40	53
13%	23	19	27	49	43	55
14%	25	21	30	52	46	59
15%	28	23	32	55	49	62
16%	30	25	34	58	52	65
17%	32	27	36	61	55	68
18%	34	29	39	64	58	72
19%	36	31	41	68	61	75
20%	38	33	43	71	64	79

Table 11. Configuration 1: Period of First and Last Bank Failures, and Their Corresponding Confidence Intervals, for Different Values of σ.

	Period of First Bank Failure			Period of Last Bank Failure		
σ	**Mean**	**5%**	**95%**	**Mean**	**5%**	**95%**
0%	28	23	32	55	49	62
10%	28	23	32	56	50	63
20%	28	23	32	58	52	65
30%	28	23	32	59	53	66
40%	28	23	32	60	54	68
50%	28	23	32	62	55	69
60%	28	23	32	63	57	70
70%	28	23	32	65	58	72
80%	28	23	32	66	59	74
90%	28	23	32	68	61	76

Table 12. Period of First and Last Bank Failures, and Their Corresponding Confidence Intervals, for Different Values of Ψ (Relative Size of Equity Injection to Bank I).

	Period of First Bank Failure			Period of Last Bank Failure		
Ψ	**Mean**	**5%**	**95%**	**Mean**	**5%**	**95%**
0%	28	23	32	55	49	62
10%	30	26	34	55	49	62
20%	31	27	35	55	49	62
30%	31	27	35	55	49	62
40%	31	27	35	55	49	62
50%	31	27	35	55	49	62
60%	31	27	35	55	49	62
70%	31	27	35	55	49	62
80%	31	27	35	55	49	62
90%	31	27	35	56	50	62
100%	31	27	35	57	51	64

Table 13. For Different Values of Ψ (Relative Size of Equity Injection to Bank I), the Table Shows the Frequency with Which Each Bank Fails First and Last.

Frequency with which a specific bank fails first

					Banks					
Ψ	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
0	0.077	0.000	0.013	0.067	0.046	0.000	0.000	0.000	0.797	0.000
0.1	0.210	0.000	0.045	0.181	0.135	0.000	0.000	0.000	0.428	0.001
0.2	0.322	0.000	0.075	0.250	0.196	0.000	0.000	0.000	0.154	0.003
0.3	0.389	0.000	0.090	0.269	0.215	0.000	0.000	0.000	0.034	0.003
0.4	0.410	0.000	0.093	0.272	0.220	0.000	0.000	0.000	0.002	0.003
0.5	0.410	0.000	0.093	0.272	0.221	0.000	0.000	0.000	0.001	0.003
0.6	0.411	0.000	0.093	0.272	0.221	0.000	0.000	0.000	0.000	0.003
0.7	0.411	0.000	0.093	0.272	0.221	0.000	0.000	0.000	0.000	0.003
0.8	0.411	0.000	0.093	0.272	0.221	0.000	0.000	0.000	0.000	0.003
0.9	0.411	0.000	0.093	0.272	0.221	0.000	0.000	0.000	0.000	0.003
1	0.411	0.000	0.093	0.272	0.221	0.000	0.000	0.000	0.000	0.003

Frequency with which a specific bank fails last

					Banks					
Ψ	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
0	0.000	0.901	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000
0.1	0.000	0.901	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000
0.2	0.000	0.901	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000
0.3	0.000	0.901	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000
0.4	0.000	0.901	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000
0.5	0.000	0.901	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000
0.6	0.000	0.897	0.000	0.000	0.000	0.000	0.099	0.000	0.004	0.000
0.7	0.000	0.873	0.000	0.000	0.000	0.000	0.097	0.000	0.030	0.000
0.8	0.000	0.779	0.000	0.000	0.000	0.000	0.094	0.000	0.127	0.000
0.9	0.000	0.592	0.000	0.000	0.000	0.000	0.088	0.000	0.320	0.000
1	0.000	0.367	0.000	0.000	0.000	0.000	0.068	0.000	0.565	0.000

Table 14. System Response to Different Values of χ (Relative Size of Equity Injection to Bank D). The Table Shows the Bank Most Likely to Fail First and the Period When Such Failure Happens, the Bank Most Likely to Fail Last and the Period When Such Failure Happens, and the Periods Where D, C and E Fail (Which Indicate Whether a Cascade Has Occurred).

	Period Where First Failure Occurs	Bank Most Likely to Fail First	Period Where Last Failure Occurs	Bank Most Likely to Fail Last	Period Where Bank D Fails	Period Where Bank C Fails	Period Where Bank E Fails
χ	Mean		Mean		Mean	Mean	Mean
0.0	28	I	55	B	32	33	33
0.1	28	I	55	B	36	34	33
0.2	28	I	55	B	38	34	33
0.3	28	I	55	B	41	34	33
0.4	28	I	55	B	44	34	33
0.5	28	I	55	B	47	34	33
0.6	28	I	55	B	51	34	33
0.7	28	I	57	B	55	34	33
0.8	28	I	59	D	58	34	33
0.9	28	I	62	D	62	34	33
1.0	28	I	66	D	66	34	33

Appendix A. Balance Sheet Information for All Ten Banks (Each Configuration Reflects the Balance Sheet on a Given Consecutive Month).

Balance Sheet, Bank - A	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	32.9	32.9	32.9	32.9	32.9	32.9
Third-party loans, L	1568.7	1568.7	1568.7	1568.7	1568.7	1568.7
Loans to other banks, B	35.8	78.3	217.8	104.1	28.0	49.2
Deposits at Central Bank, θ D	267.4	267.4	267.4	267.4	267.4	267.4
Liquid investments, I	489.3	489.3	489.3	489.3	489.3	489.3
Illiquid investrments, A	16.2	16.2	16.2	16.2	487.5	226.1
Total Assets	**2410.3**	**2452.8**	**2592.3**	**2478.7**	**2873.8**	**2633.6**
Liabilities						
Deposits, D	1782.8	1782.8	1782.8	1782.8	1782.8	1782.8
Debt to Central Bank, ϕ	0	8.9	20.7	1.8	14.2	54.6
Debt to other banks, H	26.2	2.4	13.72	5.5	301.7	117.1
Other debt, G	366.5	424	540.3	453.8	540.3	444.3
Equity, E	234.8	234.8	234.8	234.8	234.8	234.8
Total Liabilities	**2410.3**	**2452.8**	**2592.3**	**2478.7**	**2873.8**	**2633.6**

Balance Sheet, Bank - B	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	28.3	28.3	28.3	28.3	28.3	28.3
Third-party loans, L	756.8	706.8	756.8	756.8	701.5	756.8
Loans to other banks, B	44.8	63.0	117.6	194.1	66.8	94.9
Deposits at Central Bank, θ D	112.8	112.8	112.8	112.8	112.8	112.8
Liquid investments, I	264.4	264.4	264.4	264.4	319.7	17.4
Illiquid investrments, A	17.4	71.8	17.4	17.4	68.3	264.4
Total Assets	**1224.4**	**1247.0**	**1297.3**	**1373.8**	**1297.4**	**1274.5**
Liabilities						
Deposits, D	752.0	752.0	752.0	752.0	752.0	752.0
Debt to Central Bank, ϕ	0.0	0.0	0.0	0.0	0.1	0.0
Debt to other banks, H	0.0	2.1	0.0	0.0	0.0	0.0
Other debt, G	256.2	276.7	329.0	405.5	329.0	306.3
Equity, E	216.3	216.3	216.3	216.3	216.3	216.3
Total Liabilities	**1224.4**	**1247.0**	**1297.3**	**1373.8**	**1297.4**	**1274.5**

Balance Sheet, Bank - C	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	36.0	36.0	36.0	36.0	36.0	36.0
Third-party loans, L	1142.8	1142.8	1142.8	1142.8	1142.8	1142.8
Loans to other banks, B	65.5	152.3	17.6	33.7	248.7	59.7
Deposits at Central Bank, θ D	191.9	191.9	191.9	191.9	191.9	191.9
Liquid investments, I	321.2	321.2	321.2	321.2	321.2	86.1
Illiquid investrments, A	17.4	17.4	404.5	45.2	17.4	321.2
Total Assets	**1774.7**	**1861.6**	**2114.0**	**1770.7**	**1957.9**	**1837.7**
Liabilities						
Deposits, D	1279.2	1279.2	1279.2	1279.2	1279.2	1279.2
Debt to Central Bank, ϕ	0.0	3.7	95.8	34.8	0.0	27.0
Debt to other banks, H	0.0	0.0	160.2	81.2	24.8	77.3
Other debt, G	312.2	395.3	395.3	192.2	470.6	270.9
Equity, E	183.3	183.3	183.3	183.3	183.3	183.3
Total Liabilities	**1774.7**	**1861.6**	**2114.0**	**1770.7**	**1957.9**	**1837.7**

Balance Sheet, Bank - D	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	30.7	30.7	30.7	30.7	30.7	30.7
Third-party loans, L	714.0	714.0	714.0	714.0	714.0	714.0
Loans to other banks, B	11.5	1.5	3.7	0.0	6.1	46.2
Deposits at Central Bank, θ D	99.9	99.9	99.9	99.9	99.9	99.9
Liquid investments, I	59.9	59.9	59.9	59.9	59.9	12.7
Illiquid investrments, A	22.1	99.8	12.7	77.3	12.7	59.9
Total Assets	**938.1**	**1005.8**	**920.9**	**981.8**	**923.3**	**963.4**
Liabilities						
Deposits, D	665.9	665.9	665.9	665.9	665.9	665.9
Debt to Central Bank, ϕ	6.3	16.5	8.8	9.1	8.0	0.0
Debt to other banks, H	32.3	70.2	51.5	61.1	4.4	5.6
Other debt, G	129.9	149.5	91.1	142.0	141.2	188.2
Equity, E	103.7	103.7	103.7	103.7	103.7	103.7
Total Liabilities	**938.1**	**1005.8**	**920.9**	**981.8**	**923.3**	**963.4**

Balance Sheet, Bank - E	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	9.4	9.4	9.4	9.4	9.4	9.4
Third-party loans, L	467.9	467.9	467.9	467.9	467.9	467.9
Loans to other banks, B	14.8	15.8	53.9	86.6	87.8	74.6
Deposits at Central Bank, θ D	59.8	59.8	59.8	59.8	59.8	59.8
Liquid investments, I	48.7	48.7	48.7	48.7	48.7	8.7
Illiquid investrments, A	37.8	4.7	4.7	4.7	4.7	48.7
Total Assets	**638.3**	**606.3**	**644.4**	**677.1**	**678.3**	**669.1**
Liabilities						
Deposits, D	398.8	398.8	398.8	398.8	398.8	398.8
Debt to Central Bank, ϕ	1.8	0.6	0.6	0.2	2.1	0.4
Debt to other banks, H	4.1	2.2	0.8	0.8	12.0	3.6
Other debt, G	164.2	135.3	174.8	208.0	196.1	196.8
Equity, E	69.4	69.4	69.4	69.4	69.4	69.4
Total Liabilities	**638.3**	**606.3**	**644.4**	**677.1**	**678.3**	**669.1**

Balance Sheet, Bank - F						
	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	9.8	9.8	9.8	9.8	9.8	9.8
Third-party loans, L	246.7	246.7	246.7	246.7	246.7	246.7
Loans to other banks, B	1.6	0.0	0.0	4.7	0.9	1.8
Deposits at Central Bank, θ D	35.7	35.7	35.7	35.7	35.7	35.7
Liquid investments, I	73.1	73.1	73.1	73.1	73.1	78.4
Illiquid investrments, A	46.4	132.1	214.7	109.9	57.0	73.1
Total Assets	**413.3**	**497.4**	**580.0**	**480.0**	**423.2**	**445.5**
Liabilities						
Deposits, D	238.2	238.2	238.2	238.2	238.2	238.2
Debt to Central Bank, ϕ	4.7	25.3	17.6	34.3	2.8	39.2
Debt to other banks, H	87.4	161.3	121.8	126.5	108.0	115.4
Other debt, G	38.4	28.0	157.8	36.4	29.5	8.1
Equity, E	44.6	44.6	44.6	44.6	44.6	44.6
Total Liabilities	**413.3**	**497.4**	**580.0**	**480.0**	**423.2**	**445.5**

Balance Sheet, Bank - G						
	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	5.0	5.0	5.0	5.0	5.0	5.0
Third-party loans, L	131.3	131.3	131.3	131.3	131.3	131.3
Loans to other banks, B	5.7	18.1	13.0	4.6	41.2	33.3
Deposits at Central Bank, θ D	26.4	26.4	26.4	26.4	26.4	26.4
Liquid investments, I	42.5	42.5	73.4	42.5	42.5	111.9
Illiquid investrments, A	66.0	24.7	25.6	163.8	67.6	42.5
Total Assets	**277.0**	**248.0**	**274.7**	**373.6**	**314.0**	**350.3**
Liabilities						
Deposits, D	175.9	175.9	175.9	175.9	175.9	175.9
Debt to Central Bank, ϕ	1.5	2.2	12.4	29.2	3.9	31.4
Debt to other banks, H	47.2	32.1	48.5	130.7	16.6	27.3
Other debt, G	17.0	2.5	2.5	2.5	82.4	80.4
Equity, E	35.3	35.3	35.3	35.3	35.3	35.3
Total Liabilities	**277.0**	**248.0**	**274.7**	**373.6**	**314.0**	**350.3**

Balance Sheet, Bank - H						
	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	3.8	3.8	3.8	3.8	3.8	3.8
Third-party loans, L	163.2	163.2	163.2	163.2	163.2	163.2
Loans to other banks, B	20.8	8.4	5.6	7.5	6.1	10.3
Deposits at Central Bank, θ D	33.1	33.1	33.1	33.1	33.1	33.1
Liquid investments, I	64.8	64.8	64.8	64.8	64.8	29.5
Illiquid investrments, A	28.8	65.5	65.5	40.0	32.1	64.8
Total Assets	**314.4**	**338.8**	**336.0**	**312.4**	**303.0**	**304.7**
Liabilities						
Deposits, D	220.4	220.4	220.4	220.4	220.4	220.4
Debt to Central Bank, ϕ	0.0	6.3	2.4	5.5	0.9	11.4
Debt to other banks, H	4.6	80.9	48.6	52.0	45.6	41.7
Other debt, G	58.4	0.1	33.6	3.4	5.1	0.1
Equity, E	31.0	31.0	31.0	31.0	31.0	31.0
Total Liabilities	**314.4**	**338.8**	**336.0**	**312.4**	**303.0**	**304.7**

Balance Sheet, Bank - I	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	2.6	2.6	2.6	2.6	2.6	2.6
Third-party loans, L	152.5	152.5	152.5	152.5	152.5	152.5
Loans to other banks, B	1.3	14.6	14.3	23.7	27.9	27.5
Deposits at Central Bank, θ D	21.2	21.2	21.2	21.2	21.2	21.2
Liquid investments, I	15.1	15.1	15.1	15.1	15.1	3.9
Illiquid investrments, A	15.4	3.9	4.2	3.9	3.9	15.1
Total Assets	**208.2**	**209.9**	**210.0**	**219.1**	**223.2**	**222.9**
Liabilities						
Deposits, D	141.4	141.4	141.4	141.4	141.4	141.4
Debt to Central Bank, ф	0.0	0.0	0.3	0.0	0.0	0.0
Debt to other banks, H	0.0	0.1	0.4	0.1	0.2	0.0
Other debt, G	48.1	49.8	49.3	58.9	63.0	62.8
Equity, E	18.6	18.6	18.6	18.6	18.6	18.6
Total Liabilities	**208.2**	**209.9**	**210.0**	**219.1**	**223.2**	**222.9**

Balance Sheet, Bank - J	Configuration 1	Configuration 2	Configuration 3	Configuration 4	Configuration 5	Configuration 6
Assets						
Cash, C	0.4	0.4	0.4	0.4	0.4	0.4
Third-party loans, L	76.0	76.0	76.0	76.0	76.0	76.0
Loans to other banks, B	3.1	4.0	5.4	4.5	2.9	1.6
Deposits at Central Bank, θ D	3.9	3.9	3.9	3.9	3.9	3.9
Liquid investments, I	4.0	4.0	4.0	4.0	4.0	13.6
Illiquid investrments, A	5.0	5.9	1.5	5.1	4.0	4.0
Total Assets	**92.4**	**94.2**	**91.1**	**93.8**	**91.1**	**99.5**
Liabilities						
Deposits, D	26.1	26.1	26.1	26.1	26.1	26.1
Debt to Central Bank, ф	0.8	4.4	2.6	3.1	0.5	1.1
Debt to other banks, H	3.1	4.8	3.4	5.8	3.1	11.1
Other debt, G	49.4	45.9	45.9	45.9	48.4	48.2
Equity, E	13.0	13.0	13.0	13.0	13.0	13.0
Total Liabilities	**92.4**	**94.2**	**91.1**	**93.8**	**91.1**	**99.5**

Appendix B. Interconnection (β) Matrices for All Six Configurations.

	Configuration 1--Matrix (β) of Interconnections									
	Bank A	**Bank B**	**Bank C**	**Bank D**	**Bank E**	**Bank F**	**Bank G**	**Bank H**	**Bank I**	**Bank J**
Bank A	0.00	0.00	0.00	0.35	0.38	0.15	0.21	0.06	0.00	0.00
Bank B	0.10	0.00	0.00	0.20	0.13	0.20	0.36	0.00	0.00	0.04
Bank C	0.59	0.00	0.00	0.30	0.13	0.39	0.09	0.21	0.00	0.11
Bank D	0.14	0.00	0.00	0.00	0.12	0.06	0.00	0.34	0.00	0.21
Bank E	0.12	0.00	0.00	0.05	0.00	0.07	0.02	0.26	0.00	0.47
Bank F	0.00	0.00	0.00	0.00	0.05	0.00	0.02	0.08	0.00	0.08
Bank G	0.02	0.00	0.00	0.02	0.00	0.05	0.00	0.00	0.00	0.09
Bank H	0.02	0.00	0.00	0.05	0.19	0.06	0.27	0.00	0.00	0.00
Bank I	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00
Bank J	0.01	0.00	0.00	0.03	0.00	0.01	0.02	0.05	0.00	0.00
Total	1.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00

	Configuration 2--Matrix (β) of Interconnections									
	Bank A	**Bank B**	**Bank C**	**Bank D**	**Bank E**	**Bank F**	**Bank G**	**Bank H**	**Bank I**	**Bank J**
Bank A	0.00	0.00	0.00	0.18	0.00	0.23	0.25	0.25	0.00	0.00
Bank B	0.00	0.00	0.00	0.18	0.79	0.13	0.25	0.22	0.00	0.45
Bank C	0.33	0.00	0.00	0.47	0.00	0.45	0.32	0.44	1.00	0.00
Bank D	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank E	0.00	0.00	0.00	0.05	0.00	0.05	0.13	0.00	0.00	0.13
Bank F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank G	0.23	0.00	0.00	0.02	0.09	0.07	0.00	0.04	0.00	0.38
Bank H	0.00	1.00	0.00	0.02	0.00	0.03	0.00	0.00	0.00	0.00
Bank I	0.00	0.00	0.00	0.07	0.12	0.04	0.05	0.01	0.00	0.04
Bank J	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.04	0.00	0.00
Total	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

	Configuration 3--Matrix (β) of Interconnections									
	Bank A	**Bank B**	**Bank C**	**Bank D**	**Bank E**	**Bank F**	**Bank G**	**Bank H**	**Bank I**	**Bank J**
Bank A	0.00	0.00	0.43	0.47	0.00	0.62	0.40	0.57	0.67	0.35
Bank B	0.63	0.00	0.32	0.29	0.67	0.12	0.40	0.18	0.00	0.15
Bank C	0.14	0.00	0.00	0.08	0.00	0.05	0.01	0.11	0.33	0.00
Bank D	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank E	0.11	0.00	0.14	0.08	0.00	0.15	0.07	0.06	0.00	0.05
Bank F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank G	0.04	0.00	0.04	0.00	0.00	0.02	0.00	0.03	0.00	0.31
Bank H	0.05	0.00	0.01	0.03	0.00	0.01	0.02	0.00	0.00	0.00
Bank I	0.00	0.00	0.02	0.04	0.00	0.02	0.09	0.03	0.00	0.14
Bank J	0.03	0.00	0.02	0.01	0.33	0.01	0.01	0.02	0.00	0.00
Total	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

	Configuration 4--Matrix (β) of Interconnections									
	Bank A	**Bank B**	**Bank C**	**Bank D**	**Bank E**	**Bank F**	**Bank G**	**Bank H**	**Bank I**	**Bank J**
Bank A	0.00	0.00	0.17	0.14	0.00	0.27	0.13	0.55	0.00	0.08
Bank B	0.29	0.00	0.63	0.51	0.42	0.38	0.38	0.21	0.00	0.29
Bank C	0.38	0.00	0.00	0.18	0.28	0.05	0.08	0.06	0.00	0.05
Bank D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank E	0.26	0.00	0.11	0.09	0.00	0.19	0.33	0.08	1.00	0.09
Bank F	0.07	0.00	0.02	0.02	0.00	0.00	0.01	0.01	0.00	0.03
Bank G	0.00	0.00	0.01	0.02	0.30	0.00	0.00	0.00	0.00	0.32
Bank H	0.00	0.00	0.03	0.02	0.00	0.03	0.00	0.00	0.00	0.00
Bank I	0.00	0.00	0.02	0.01	0.00	0.07	0.06	0.08	0.00	0.14
Bank J	0.00	0.00	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00
Total	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

	Configuration 5--Matrix (β) of Interconnections									
	Bank A	**Bank B**	**Bank C**	**Bank D**	**Bank E**	**Bank F**	**Bank G**	**Bank H**	**Bank I**	**Bank J**
Bank A	0.00	0.00	0.42	0.00	0.11	0.13	0.13	0.00	0.00	0.00
Bank B	0.10	0.00	0.38	0.00	0.04	0.19	0.02	0.15	0.00	0.00
Bank C	0.57	0.00	0.00	0.83	0.43	0.31	0.40	0.55	0.00	0.00
Bank D	0.00	0.00	0.00	0.00	0.04	0.04	0.04	0.00	1.00	0.29
Bank E	0.19	0.00	0.07	0.00	0.00	0.15	0.14	0.18	0.00	0.55
Bank F	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
Bank G	0.09	0.00	0.00	0.00	0.16	0.08	0.00	0.10	0.00	0.16
Bank H	0.01	0.00	0.04	0.00	0.00	0.01	0.11	0.00	0.00	0.00
Bank I	0.04	0.00	0.09	0.14	0.17	0.08	0.16	0.01	0.00	0.00
Bank J	0.00	0.00	0.00	0.03	0.02	0.01	0.00	0.01	0.00	0.00
Total	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

	Configuration 6--Matrix (β) of Interconnections									
	Bank A	**Bank B**	**Bank C**	**Bank D**	**Bank E**	**Bank F**	**Bank G**	**Bank H**	**Bank I**	**Bank J**
Bank A	0.00	0.00	0.25	0.00	0.45	0.17	0.19	0.06	0.00	0.08
Bank B	0.07	0.00	0.56	0.40	0.16	0.15	0.33	0.33	0.00	0.13
Bank C	0.22	0.00	0.00	0.45	0.12	0.16	0.10	0.19	0.00	0.03
Bank D	0.22	0.00	0.06	0.00	0.08	0.09	0.02	0.08	0.00	0.16
Bank E	0.22	0.00	0.07	0.00	0.00	0.24	0.27	0.13	0.00	0.26
Bank F	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Bank G	0.12	0.00	0.04	0.12	0.04	0.07	0.00	0.09	0.00	0.31
Bank H	0.04	0.00	0.01	0.00	0.00	0.05	0.00	0.00	0.00	0.00
Bank I	0.10	0.00	0.01	0.00	0.15	0.06	0.08	0.11	0.00	0.02
Bank J	0.00	0.00	0.00	0.03	0.00	0.01	0.01	0.01	0.00	0.00
Total	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00