An Explanation of Negative Swap Spreads:

*Demand for Duration from Underfunded Pension Plans*

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Abstract

The 30-year US swap spreads have been negative since September 2008. We offer an explanation for this persistent anomaly. Through a model, we show that the demand for swaps arising from duration hedging needs of underfunded pension plans, coupled with balance sheet constraints of swap dealers, can drive swap spreads to become negative. We construct an empirical measure of the aggregate funding status of Defined Benefits (DB) pension plans from the Federal Reserve’s financial accounts of the United States and show that this measure is a significant explanatory variable of 30-year swap spreads, but not for swaps with shorter maturities.

**Keywords:** Pension funds, liability-driven investment, swap spreads, pension protection act, swap rates, limits of arbitrage; **JEL:** G11, G12, G13, G22, G23

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1 Introduction

In September 2008, shortly after the default of Lehman Brothers, the difference between the swap rate (which is the fixed-rate in the swap) of a 30-year interest rate swap (IRS) and the yield of a Treasury bond with the same maturity, commonly referred to as swap spread, dropped sharply and became negative. As we explain in more detail later, this is a theoretical arbitrage opportunity and a pricing anomaly. In contrast to other crises phenomena, the 30-year negative swap spread is very persistent and still at around -40 basis points as of December 2015. In this paper, we examine the persistent negative 30-year swap spread and offer a new perspective on the possible reasons behind this anomaly. Our hypothesis is that demand for duration hedging by underfunded pension plans coupled with balance sheet constraints faced by swap dealers puts pressure on long-term swap fixed rates and ultimately turned the 30-year swap spread negative.

Negative swap spreads are a pricing anomaly and present a challenge to views that have been held prior to the financial crisis that suggested that swap spreads are indicators of market uncertainty, which increase in times of financial distress. This is because the fixed payment in an IRS is exchanged against a floating payment, which is typically based on Libor, and entails credit risk. Hence, even though IRS are collateralized and viewed as free of counterparty credit risk, the swap rate should be above the (theoretical) risk-free rate because of the credit risk that is implicit in Libor. Therefore, swap spreads should increase in times of elevated bank credit risk (see Collin-Dufresne and Solnik [2001] for a treatment of this and related issues). Additionally to that, treasuries (which are the benchmarks against which swap spreads are computed) have a status as "safe haven", i.e., assets that investors value for their safety and liquidity. In times of financial distress, investors value the convenience of holding safe and liquid assets even more, which decreases the treasury yield and makes
them trade at a liquidity premium or convenience yield (see, for instance, Longstaff 2004, Krishnamurthy and Vissing-Jorgensen 2012, or Feldhütter and Lando 2008). In summary, these arguments show that the 30-year swap spread should have increased around the default of Lehman Brothers.

**Contributions of the paper**

We offer a demand-driven explanation for negative swap spreads. In particular, we develop a model where underfunded pension plans’ demand for duration hedging leads them to optimally receive the fixed rate in IRS with long maturities. Pension funds have long-term liabilities in the form of unfunded pension claims and invest in a portfolio of assets, such as stocks, as well as in other long-term assets, like government bonds. They can balance their asset-liability duration by investing in long-term bonds or by receiving fixed in an IRS with long maturity. Our theory predicts that, if pension funds are underfunded, they prefer to hedge their duration risk with IRS rather than buying Treasuries, which may be not feasible given their funding status. The preference for IRS to hedge duration risk arises because the swap requires only modest investment to cover margins, whereas buying a government bond to match duration requires outright investment. Thus, the use of IRS allows the underfunded pension funds to invest their scarce funds in assets (such as stocks) with higher expected return.

Greenwood and Vayanos (2010) show that pension funds’ demand for duration hedging in the UK can affect the term structure of British gilts by lowering long-term rates. In this sense, our paper bears a close relationship to their work. However, our approach differs from theirs since we focus on underfunded pension funds’ optimal preference for the use of IRS for duration hedging. The model that we develop shows that the demand for IRS increases
as the fund becomes more underfunded, and the sponsor combines the IRS positions with positions in the (risky) stock portfolio in the hope of potentially overcoming the underfunded status.

We provide non-parametric evidence suggesting that the swap spreads tend to be negative in periods when DB plans are underfunded. We thus illustrate a new channel that may be at work in driving long-term swap spreads down. Using data from the financial accounts of the United States (former flow of funds table) from the Federal Reserve, we construct a measure of the aggregate under-funded status of DB plans (both private and public) in the United States. We then use this measure to test the relationship between the underfunded ratio (UFR) of DB plans and long-term swap spreads in a regression setting. Even after controlling for other common drivers of swap spreads, recognized in the literature, such as the spread between LIBOR and repo rates, Debt-to-GDP ratio, dealer-broker leverage, market volatility, and level as well as the slope of the yield curve, we show that the UFR is a significant variable in explaining 30-year swap spreads. In line with our narrative, we also show that swap spreads of shorter maturities are not affected by changes in UFR. We use stock prices as an instrumental variable in a two-stage least square setting to address possible engodeneity concerns and to further show the robustness of our conclusions.

Related Literature

As mentioned above, Greenwood and Vayanos (2010) show that the demand pressure by pension funds lowers long-term yields of British gilts. Additionally to that, Greenwood and Vayanos (2010) mention that pension funds also fulfill their demand for long-dated assets by using derivatives to swap fixed for floating payments. They note that pension funds have “swapped as much as £50 billion of interest rate exposure in 2005 and 2006 to increase the
duration of their assets” but do not investigate the impact of such demand on swap spreads any further. Their focus was on U.K. Gilt markets. Hence, our paper complements their analysis by showing that underfunded pension funds’ demand for long-dated assets can have a strong impact on swap rates.

More generally, swap rates and treasury yields have been studied extensively in the previous literature. A stream of literature calibrates dynamic term-structure models to understand the dynamics of swap spreads (see Duffie and Singleton, 1997, Lang, Litzenberger, and Liu, 1998; Collin-Dufresne and Solnik, 2001; Liu, Longstaff, and Mandell, 2006; Johannes and Sundaresan, 2007; and Feldhütter and Lando, 2008 among others). Amongst these papers, the paper close in spirit to our paper is Feldhütter and Lando (2008). They decompose swap spreads into three components, credit risk in Libor, the convenience yield of government bonds, and a demand-based component. In contrast to our paper, their study focuses on maturities between one and ten years and they link the demand-based component to duration hedging in the mortgage market.

The usage of swaps by non-financial companies has been studied by, among others, Faulkender (2005), Chernenko and Faulkender (2012), Jermann and Yue (2013). We focus on pension funds’ underfunding issues, which have been studied by, among others, Sundaresan and Zapatero (1997) and Ang, Chen, and Sundaresan (2013). We add to this literature by linking changes in swap spreads to changes in pension fund underfunding.

We note that any demand-based explanation would be incomplete if there were no financial frictions for the supply of IRS. Hence, we also build on the literature of limits of arbitrage (Shleifer and Vishny, 1997, Gromb and Vayanos, 2002, Liu and Longstaff, 2004, Gromb and Vayanos, 2010, Gärleanu and Pedersen, 2011, among many others) and especially the literature on dealer constraints and demand pressure in the derivatives market
To the best of our knowledge, we are the first to offer a demand-based explanation for negative swap spreads. In contrast to our demand-based explanation for negative swap spreads, Jermann (2016) studies the negative swap spreads, offering frictions for holding long-term bonds as an explanation. In contrast to our paper, Jermann (2016) takes the demand for long-dated swaps as exogenously given and focuses explicitly on the risks of holding long-dated bonds to hedge the cash flows of long-dated swaps. In his model, a risk-averse derivatives dealer chooses his optimal investment in short-term government bonds, long-term government bonds, and long-dated swaps. Jermann (2016) assumes that holding bonds is costly and shows that as costs increase, the swap rate converges to the Libor rate. Since long-term Treasury yields are typically above Libor, his model predicts that there is a negative relationship between swap spreads and term spreads, where term spreads are proxied as the difference between long-dated treasuries and short-dated treasuries. He provides some empirical evidence showing the link between term spreads and swap spreads. Our explanation is distinct from his work, as the $UFR$ measure of underfunded status of DB pension plans is a significant variable in explaining 30-year swap spreads but not for swap spreads with other maturities. Furthermore, controlling for term spreads leaves our main results unchanged. Holding outright long positions in bonds for under-funded pension plans to match duration has an opportunity cost in practice and this is what we stress in our work. Lou (2009) also offers derivatives dealers’ funding costs as an explanation of negative swap spreads.

Finally, there is a wide range of industry research offering a variety of different reasons for the persistent negative 30-year swap spread. One frequently used explanation is the poten-
tial credit risk of US Treasuries. The problem with this argument is that while Treasuries are linked to the credit risk of the US, swap rates are linked to the average credit risk of the banking system and a default of the US government would most likely cause defaults in the banking system as well. A second, commonly-offered explanation, is the different funding requirements of swaps and treasuries. Long-term Treasury holdings are outright cash position while engaging in IRS requires only modest capital for initial collateral, typically a small fraction of the Treasury bond principal. Sophisticated investors can use repo agreements to purchase/finance Treasuries, although financing Treasury securities for 30 years would require open repo positions, which need to be rolled over for a long duration. The risk with such a strategy is that the cash lenders may refuse to renew the repo agreement. These considerations may cause pension funds to prefer swaps as opposed to a repo-financed positions in government bonds.

The roadmap of the paper is as follows. Section 2 of the paper provides some motivating evidence. In section 3 we present the swap spreads and the underlying drivers for the demand for receiving fixed rates in long-term swaps from pension funds. In section 4 we develop a dynamic model with stochastic interest rates, which shows that the need to match the duration of assets and liabilities can lead to a demand for receiving fixed in long-term swaps, when the pension plan is underfunded. Section 5 contains our empirical results. Section 6 concludes.

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2 See, for instance, Van Deventer (2012).
2 Motivating Evidence

We motivate our model, by documenting a few stylized facts: we first show in Figure 1 that the 30-year swap spreads became negative following the bankruptcy of Lehman Brothers, and has been in the negative territory since then.

Figure 1: Term structure of interest rate swap spreads: The graph shows the history of swap spreads from May 1994 until December 2015. The grey shaded areas represent US recession periods. The source for our data is Bloomberg. The differences in market conventions have been taken into account in computing the spreads.

We can see from Figure 1 that the term structure of swap spreads track each other closely until the end of 2007 when long-term swap spreads start decreasing relative to short-term spreads. Since then, the dynamics of the 30-year swap spreads have decoupled from the dynamics of the other tenures. In the month after the default of Lehman Brothers, highlighted by the first vertical line, the 30-year swap spread drops sharply and turns negative. During that period, there is also a decline in the 10-year swap spread, while swap spreads of shorter maturities increase. Between 2008 and 2014 the 30-year swap spread slowly converges close
to 0 and starts decreasing again in 2015. In August 2015, highlighted by the second vertical line, the Libor-Repo spread turns negative, which causes a decrease in swap spreads of all maturities³.

We perform a principal components analysis (PCA) of swap spreads before and after September 2008 to see if there is a significant change in the PCs driving the swap spreads after the crisis, relative to the drivers prior to the crisis. The results of our PCA are shown in Table 1 next. We present the loadings of each PC before and after September 2008 as well as the proportion of the spreads explained by each PC. Note that prior to the crisis, the first PC explained more than 80% of the variations in swap spreads for all maturities except the 30-year swap spreads. The second PC had a much more important role for the 30-year swap spreads when compared to swap spreads of other tenures. After the crisis, the first PC

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³The prolonged drop in interest rates, following the crisis of 2008, increased the duration of pension liabilities and the monetary policy of the Fed also might have contributed to the overall drop in other interest rates and spreads.
became even more important in explaining the swap spreads of maturities up to five years, and less so for maturities from seven to thirty years. But the drop in its explanatory power for the 30-year swap spreads is dramatic: it fell from 70.8% to just 5.9%. In fact, the second PC became the dominant component in explaining the swap spreads for 30-year maturity, in sharp contrast with swap spreads associated with other maturities. Our results in Table I demonstrate that the determinants of 30-year swap spreads underwent a big change after September 2008. This change appears to be unique for 30-year swap spreads. To a lesser extent, we see a similar effect for the ten year swap spreads as well.

Taken together, Figure 1 and Table I suggest that the 30-year swap spreads behaved qualitatively different from the rest of the swap spreads after September 2008. This provides the motivation for both our theory and empirical work. We provide next a possible link between the above evidence and the funding status of defined benefit (DB) pension plans. DB Pension funds have long-dated liabilities and they use long-term interest rate swaps to hedge their duration risk in swap overlay strategies. Adams and Smith (2009) show how interest rate swaps are used by pension funds to manage their duration risk. Furthermore, CGFS (2011) documents that insurance companies and pension funds need to balance asset-liability durations and can do so using swaps.

In theory, a sophisticated investor with full access to repo financing, can buy Treasury bonds and use the repo market to obtain an almost unfunded position. This repo transaction requires an initial funding of approximately 6%. At the same time, engaging in an IRS could also require an initial margin and regular collateral posting. With the implementation of mandatory central clearing this is becoming more of an issue recently. Nevertheless, as noted

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4This number is a first approximation that we obtained from http://www.cmegroup.com/clearing/financial-and-collateral-management/. They analyze haircuts for securities posted as collateral in cleared derivatives transactions. However, market participants confirm that 6% is a reasonable proxy for haircuts of Treasuries with 30 years to maturity.
earlier, financing a long-term bond for thirty years remains a less practical proposition than merely entering into an interest rate swap. As noted in a recent Bloomberg article (see Leising, 2013), US pension funds use IRS markets. Overall, pension funds may find long-term IRS as a simpler vehicle to take leverage than utilizing the repo market for duration hedging purposes.\(^5\)

Further anecdotal evidence of pension funds’ demand for IRS and resulting demand pressure is best summarized by the following quote from a recent Bloomberg article: “Pension funds need to hedge long-term liabilities by receiving fixed on long-maturity swap rates. When Lehman dissolved, pension funds found themselves with unmatched hedging needs and then needed to cover these positions in the market with other counterparties. This demand for receiving fixed in the long end drove swap spreads tighter.”\(^6\)

We provide next some motivating evidence that suggests a strong association between the funded status of pension plans and thirty year swap spreads. The size of pension funds in the United States is significant relative to the GDP of the US economy.\(^7\) To make the case that the demand by pension funds to receive fixed in the long-term swap contracts can potentially influence the 30-year swap spread, Figure\(^2\) offers a comparison between the size of the interest rate swap market to the value of pension funds’ liabilities. The solid line indicates the mark-to-market value of USD interest rate swap contracts with a maturity of more than 5 years. The dashed line illustrates the time series of the total pension liabilities in the US defined benefits plans, which are the focus of our paper.

\(^5\)There may be other frictions such as taxes that may also favor IRS relative to repo. In the US, Internal Revenue Service views repo as financing that would subject the pension plan to tax filings as Unrelated Business Income (UBI). Most US pension plans will therefore avoid UBI taxes by avoiding repo and relying on IRS, which does not invoke UBIT. We thank Scott McDermott for alerting us to this point.


\(^7\)The size of pension plan assets in the US is about $13.60 trillion dollars as of the first quarter of 2015.
Figure 2: **Size of pension liabilities and long-term Interest Rate Swaps:** This plot illustrates that the total unfunded liabilities of private as well as state and local government employee defined-benefit (DB) pension plans are qualitatively similar to the gross market value of interest rate swaps denominated in US dollars with maturity greater than five years. The amounts are in billions of dollars, not seasonally adjusted. (Source: BIS and financial accounts of the U.S.)

3 **Demand for and Supply of Duration**

In this section we discuss pension funds, their duration matching needs and how underfunding affects their demand for long-dated IRS. We briefly review the implications of regulations such as the pension protection act of 2006 and the diminished incentives to overfund pension plans, due to some tax policy developments. We conclude with an overview of the demand for receiving fixed in long-dated IRS as well as the supply of long-dated IRS.
3.1 Pension Funds’ Duration Matching Needs

The most important customers in the long end of the swap curve are pension funds and insurance companies, who have a natural demand for receiving fixed for longer tenors. Pension funds have long-term liabilities towards their clients and the Pension Protection Act of 2006 requires them to minimize underfunding by stipulating funding standards and remedial measures to reduce under-funded status. This promotes the incentive to match the duration of their asset portfolios with the duration of these liabilities: any duration mismatch can produce future shortfalls. Increasing the duration of their asset portfolios could be achieved by receiving fixed in an IRS or by buying bonds with long maturities. Greenwood and Vayanos (2010) provide evidence from the 2004 pension reform in the United Kingdom where pension funds started buying long-dated gilts and more recently Domanski, Shin, and Sushko (2015) show that German insurance companies increased their holdings of German long-term bonds significantly over the past years. In line with previous research (see, for instance, Ang et al., 2013 or Ring, 2014, among many others) we document that many US pension funds are underfunded and therefore tend towards more risky investments. Using IRS instead of long-dated Treasuries for duration hedging allows pension funds to use their limited funding to invest in more risky assets such as stocks.

To illustrate that pension funds are indeed using swaps, we collect survey data from the Chief Investment Officer magazine, who conducts regular surveys on US pension funds and their investment strategies. In 2013, 2014, and 2015 they surveyed more than 100 US pension fund managers on their investment strategies. The question most relevant to this paper was whether the plans are using derivatives. A majority of 64.6%, 63%, and 70% of the respondents in 2013, 2014, and 2015, respectively, stated that they were currently

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8 These surveys are available under [http://www.ai-cio.com/surveys/](http://www.ai-cio.com/surveys/)
using derivatives. In 2013 and 2014 the respondents provided additional details on their derivatives usage. In 2013 and 2014 80.9% and 79% stated that they were using interest rate swaps, among other derivatives. Furthermore, 25.4% (29%) of the respondents in 2013 (2014) stated that they were using derivatives to obtain leverage and 49.2% (39%) stated that they are using derivatives for capital/cash efficiency.

Pension Funds’ Aversion to Over-funding after 1990

During the period 1986-1990, laws were enacted in the US to discourage “pension reversions” whereby, a pension plan with excess assets can be tapped into by the sponsoring corporation to draw the assets back into the corporation. In 1986, the reversion tax rate was 10% but by 1990, this tax rate had increased to 50%. In addition, the sponsoring firm was also required to pay corporate income tax on reversions. These changes in tax policies meant that the US corporations have dramatically lower incentives to overfund their pension plans since 1990 than was the case before. This is important to note because pension plans were generally not significantly overfunded before the onset of the credit crisis of 2008 which made them vulnerable to becoming underfunded should there be a big correction in equity markets or a protracted fall in discount rates, which can cause the pension liabilities to increase (both these developments occurred after the credit crisis of 2008).

Once the plans become underfunded and the rates fall (as was the case after 2008), the plan sponsors are faced with two objectives: first to match the duration of assets with liabilities to avoid future underfunding due to market movements, and second to find assets which can provide sufficiently high returns to get out of their underfunded status. This is the context in which the long-term swaps play a role: they enable the sponsors to match duration without setting aside any explicit funding and the sponsor can then use the limited
funding to invest in riskier assets in the hope of earning higher returns.

3.2 The Supply of Long-Dated Swaps

Investors

In general, investors could either have a demand for receiving fixed in an IRS or for paying fixed in an IRS and may use IRS for speculative and hedging purposes. In any case, the demand for IRS can depend on the level and the slope of the yield curve. The level of the yield curve matters, for instance, for agencies issuing Mortgage-Backed Securities (MBS). Agencies aim to balance the duration of their assets and liabilities. When interest rates fall, mortgage borrowers tend to execute their prepayment right, thereby lowering the duration of the agencies’ mortgage portfolio. Hence, agencies want to receive fixed in an IRS to hedge this mortgage prepayment risk (see [Hanson, 2014]). The slope of the yield curve may also matter for non-financial firms. According to [Faulkender, 2005], these firms tend to use IRS mostly for speculation, preferring to pay floating when the yield curve is steep. [Faulkender, 2005] also finds that firms tend to prefer paying fixed when macro-economic conditions worsen. Overall, these papers show that there could be demand and supply effects from other investors. However, as these examples suggest, it is hard to conclude that non-financial firms have a large demand for long-dated IRS with a maturity of 30 years. We therefore conclude that the demand for receiving fixed in long-dated IRS by pension funds has to be met largely by derivatives dealer-brokers.

[Feldhüter and Lando, 2008] argue that using IRS is the predominant way for doing this as opposed to using Treasuries.

[10] Insurance companies could be another big demanders for receiving fixed rate on long-term swaps, but we have no data available to characterize their demand. Additional to insurance companies, recent long-term corporate bond issuance might also create a demand for receiving fixed in long-dated interest rate swaps. This is because companies may hedge the duration risk of their bond issuance.
Dealer-Brokers

A dealer-broker paying fixed in a long-dated IRS, thereby taking the opposite position than a pension fund would generally aim to hedge the interest-rate risk of his position. He can either do so by finding another counterparty willing to pay fixed or by following the hedging strategy described in Table 6 in the appendix where he purchases a 30-year treasury bond financed with a short-term repo transaction in order to hedge the duration risk. We discussed above that finding a counterparty willing to pay fixed in long-dated swaps is difficult and now highlight several constraints with this hedging strategy that limit the supply of long-dated IRS.

The first issue has to do with margin requirements. Financing the purchase of the long-dated government bond with short-term borrowing is subject to the risk of increasing margin requirements. For instance, Krishnamurthy (2010) documents that haircuts for longer-dated government bonds increased from 5% to 6% during the crisis. The haircut for 30-year bonds conceivably increased even more. Furthermore, Musto, Nini, and Schwarz (2014) document that the amount of repo transactions decreased sharply during the financial crisis. One possible reason for this observation is that the supply of repo financing deteriorated and hence borrowing at repo was not always possible, especially for long-term swaps. Hence, the arbitrage strategy is subject to a severe funding risk. Furthermore, engaging in an IRS requires an initial margin as well. This margin requirement increased after the financial crisis. Hence the dealer may be forced to offer a lower fixed rate on long-term swaps.

The second issue is a standard limits of arbitrage argument. As pointed out by Shleifer and Vishny (1997), Liu and Longstaff (2004), and many others, arbitrage opportunities are subject to a risk: it is the possibility that the mispricing increases before it vanishes, thereby forcing the arbitrageur out of his position at a loss. With negative 30-year swap
spreads arbitrage, we know that the mispricing vanishes after 30 years, but we do not know whether it will vanish within a much shorter and practical horizon. To benefit from negative swap spreads arbitrage a high amount of leverage is required and arbitraging negative swap spreads can therefore be seen as a case of “picking up Nickels in front of a steamroller” (Duarte, Longstaff, and Yu 2007).

4 Model

In this section, we develop a model that links pension funds’ underfunding to swap spreads, proceeding in three steps. First, we show that underfunded pension plans optimally take a long position in interest rate swaps, by receiving fixed and paying floating. Second, we model the supply of long-dated swaps in reduced form, assuming that derivatives dealers provide fixed rates in swaps elastically up to a certain threshold at the arbitrage-free rate, and then require additional compensation in the form of negative swap spreads above this threshold, due to their balance sheet constraints. Third, bringing the demand and supply side of our model together, we show how pension funds’ underfunding leads to decreasing swap spreads.

\[11\] Another friction we abstract away from in this project is the possible presence of credit risk in US Treasuries. The reason for doing so is that it is not obvious how an increase in credit risk in US Treasuries might affect the swap spread. Clearly, an increase in treasury credit risk would increase the treasury yield and assuming all else equal, a decrease in the swap spread would result. However, it is not obvious that swap rates would be unaffected by the increase in treasury credit risk since interbank lending rates would presumably increase sharply when US credit risk increases. Therefore, it is just as likely that the swap rate would be elevated.
4.1 Demand For Long-Dated Swaps by Pension Funds

Model with Swaps and Safe Assets

We now present a simple model of an under-funded pension plan, which has assets $A_t$ and a flow rate of liabilities $L$ per unit time, that live forever. The pension plan is underfunded, i.e., $A_0 < PV(L)$. We make this assumption to explicitly model under-funded pension plans. In addition, this implies that the fund cannot buy a perpetual bond to match the cash flows. Hence, there is a natural role for interest rate swaps, which are funded as a floating rate. The fund can also contribute at a rate $y_t$ per unit time. Formally, the sponsor’s value function is given as:

$$G(t, A) := \min_{\{y_t, m_t\}} \mathbb{E}_t \left[ \int_t^T e^{-\rho s} u(y_s) ds \right]$$

(1)

where we assume CRRA utility with risk-aversion coefficient $\beta > 1 : u(y) = y^\beta$ and where the fund optimally chooses its funding rate and asset allocation between interest rate swaps and safe assets to reach a fully funded status at a future point.\footnote{We have also solved the model under the alternative assumption that the pension fund can invest in a perpetual bond by borrowing short-term. The results for this assumption are available upon request.} Once the plan is fully funded, it simply dedicates the cash flows from its portfolio to meet its liabilities.

We define a swap as one in which the pension plan receives a fixed dollar amount of 1 per unit time, and pays a floating rate of $r_t$ per unit time. The derivatives dealer, who is the counterparty to this IRS, will be introduced in the following section. We abstract from credit risk, which implies that the fixed rate of the swap can be funded at the risk-free floating rate. The value of this swap is $P - 1$, where $P$ is the value of a perpetuity which pays $1$ forever. The value of floating payments $\{r_s, s \geq t\}$ is simply $1$\footnote{See, Cox, Ingersoll, and Ross [1985] for a proof of this assertion.}. This is a stylized representation
of an interest rate swap, which differs from a newly minted interest rate swap, which will always be valued at zero. Our stylized representation of the swap provides tractable and simple closed form solutions\(^{14}\) The swap can be regarded as a seasoned swap, which the fund enters into for duration matching purposes. The pension funds can buy \(m\) swaps which cost \(m[P - 1] < A_t < PV[L]\), where \(A_t\) is the value of the assets held by the pension funds at time \(t\). The remaining funds \(A_t - m[P - 1]\) are invested in \(r_t\) at each time. It should be noted the fund will pay the intermediation costs, \(\delta\), per unit time, which affects the cash flows. The cash flows from the swap position is: \(m(1 - r - \delta)\) per unit time.

The term structure model is a simple one-factor model where the instantaneous interest rate \(r\) follows a diffusion process, as in Constantinides and Ingersoll (1984):\(^{15}\)

\[
dr = \alpha r^2 dt + sr^{3/2}dw_1
\]

(2)

For this process, the consol bond price \(P\) is:

\[
P = \frac{1}{(1 + \alpha - s^2)r} = H/r,
\]

(3)

where \(H \equiv \frac{1}{1 + \alpha - s^2}\). We can derive the dynamics of the pension fund’s asset value as:

\[
dA = [Ar + m(1 - \delta) + y - L]dt - mPs\sqrt{r}dw_1.
\]

(4)

The dynamic problem facing the pension sponsor is specified below: we choose \(A\) to be our state variable, and formulate the HJB equation associated with the funds’ optimization

\(^{14}\)Modeling a swap that is zero valued is feasible, but may likely require a numerical approach in the context of our model, where the fund has to also choose asset allocation.

\(^{15}\)In Appendix A we briefly characterize the term structure of zero coupon yields implied by this model.
problem next:

$$0 = \inf_{y_t,m} \left[ y^\beta - \rho G + G_A [m(1 - \delta) + rA + y - L] + \frac{1}{2} G_A m^2 P^2 s^2 r \right]. \quad (5)$$

The fund will close out the swaps, payoff any loans, and stop contributing when the assets are sufficient to meet the present value of the liabilities. Note that when \( m = y = 0 \), the dynamics of assets are:

$$dA = [Ar - L]dt = r[A - \frac{L}{r}]dt = r[A - LP(1 + \alpha - s^2)]dt \quad (6)$$

When \( A \uparrow A^* \) where \( A^* = LP(1 + \alpha - s^2) \), note that \( A^* r = L \), and the fund can meet its liabilities from its assets, and the value function goes to zero. This leads to the following boundary condition for the HJB equation above:

$$G(A \uparrow A^*) = 0. \quad (7)$$

The above condition follows from the fact that the cost of funding goes to zero when the assets are sufficient to meet the liabilities. Let us define \( \Psi \equiv A^* - A \). We can now characterize the demand functions of the pension fund and the optimal funding policy.
Demand Functions

**Proposition 1.** The sponsor’s optimal contribution and optimal asset allocation are given as

\[
y = r\beta - \rho - \frac{1}{2}\beta \left[ \frac{(1-\delta)^2}{(\beta-1)P^2s^2r} \right] \Psi \quad \text{(8)}
\]

and

\[
m^* = \frac{(1-\delta)(1+\alpha - s^2)\Psi}{(\beta-1)Ps^2}. \quad \text{(9)}
\]

The proof of Proposition 1 can be found in the appendix. Note that \(\frac{\partial m^*}{\partial \delta} < 0\) and \(\frac{\partial m^*}{\partial P} > 0\), which makes intuitive sense. The fund’s demand for IRS falls when the intermediation costs (negative swap spreads) are higher. As interest rates go down, \(P\) increases, and this leads to a higher demand for IRS.

When the underfunding is high, the fund uses more IRS and funds more aggressively: this is in fact the basic implication of our model. As \(\beta\) increases, the pension fund increases \(y\) much more and reduces its positions in IRS: this suggests that funding requirements imposed by regulators may have beneficial implications for the way in which pension assets are managed by the sponsors.

**Model with Stocks, Swaps and Safe Assets**

We now extend our model to allow the pension fund to additionally invest in a risky asset. To keep the model simple, we introduce a generic risky asset which can be interpreted as

\[16\text{We require that } r\beta - \rho - \frac{1}{2}\beta \left[ \frac{(1-\delta)^2}{(\beta-1)P^2s^2r} \right] > 0. \text{ In addition, the intermediation costs represented by } \delta \text{ cannot be too high, i.e., } \delta < 1.\]
stock portfolio. The price of the stock portfolio follows a geometric Brownian motion:

\[ dS = S\mu dt + S\sigma dw_2. \]  \hspace{1cm} (10)

We allow the processes \( \{w_1, w_2\} \) to be correlated with each other with correlation coefficient \( R \) and introduce the notation \( \sigma_{12} := s\sigma R \). The fund invests in \( n \) shares of the stock portfolio, \( m \) swaps and places the remainder in risk-free asset.

**Proposition 2.** The sponsor’s optimal contribution and optimal asset allocation are given as:

\[ y = g^\frac{1}{\beta-1} \Psi, \]  \hspace{1cm} (11)

\[ nS = \frac{\left[ \left( \frac{\mu-r}{\sigma^2} \right) + P \frac{\sigma_{12}}{\sigma_1^2} \sqrt{r} \left( \frac{1-\delta}{P\sigma_2^2} \right) \right]}{(\beta - 1) \left( 1 - \frac{\sigma_{12}^2}{\sigma_1^2\sigma_2^2} \right) \Psi} \equiv \lambda_1 \Psi, \]  \hspace{1cm} (12)

and

\[ m = \frac{\left[ \left( \frac{1-\delta}{P\sigma_2^2} \right) + \frac{\sigma_{12}}{P\sigma_1^2} \sqrt{r} \left( \frac{\mu-r}{\sigma^2} \right) \right]}{(\beta - 1) \left( 1 - \frac{\sigma_{12}^2}{\sigma_1^2\sigma_2^2} \right) \Psi} \equiv \lambda_2 \Psi, \]  \hspace{1cm} (13)

where \( g \) is given as:

\[ g = \left[ \frac{1}{1-\beta} \left( \rho + \beta (\lambda_1 (\mu - r) + \lambda_2 (1 - \delta) - r) \right) - \frac{1}{2} \beta (\beta - 1) \left( \lambda_1^2 \sigma^2 + \lambda_2^2 P^2 s^2 r - 2 \lambda_1 \lambda_2 P \sqrt{r} \sigma_{12} \right) \right]^{\beta-1}. \]  \hspace{1cm} (14)

The proof of Proposition 2 can be found in the appendix. We now illustrate the pension
Figure 3: Pension funds’ optimal holdings of stocks and swaps. UFR is computed as $(L/r - A)/A$. Parameter choices are: $\beta = 10$, $\rho = 0.05$, $\mu = 0.06$, $\sigma = 0.2$, $\alpha = 0.1$, $s = 0.3$, $R = 0$, $P = 50$, and $\delta = 50 \cdot 10^{-4}$.

As we can see from Figure 3, both, risky asset holdings and the amount of swaps held increase with UFR. For $UFR \downarrow 0$ the fund closes out his risky asset and swap holdings and pays off any loans. It is important to note that the pension fund increase both the exposure to swaps and risky assets as the fund gets more underfunded: the increase in risky asset is due to a desire to get out of the underfunded status in the future. On the other hand, the increase in swap position is to manage the duration risk to prevent future losses arising from interest rate changes.
4.2 Supply of Long-Dated Swaps by Dealer-Brokers

On the supply side of long-dated IRS, we assume that derivatives dealers decide on the amount $S$ that they supply, by maximizing their discounted stream of profits. The supply is per unit time and we assume that there is a base amount $S_0$ up to which the dealers supply fixed payments at the arbitrage-free rate. In this situation, the dealer pays $1$ per unit time and hedges himself by financing a perpetuity with time-$t$ price $P_t$. The cost of financing this position is $Pr_s$ for $s \geq t$, which corresponds to the cashflows received from the pension fund. Hence, this position has a value equal to zero. In this case, the swap spread is simply equal to zero and the equilibrium swap rate is given as the yield of the perpetuity.

If the demand for receiving fixed exceeds $S_0$, the dealers are not able to provide swaps at the frictionless rate anymore. The intuition behind this assumption is that dealers are facing balance sheet constraints. In our model, they dedicate a certain part of their balance sheet to swap trading. If they decide on supplying more swaps than the dedicated amount they face the risk of future costs due to binding balance sheet constraints. For instance, a dealer supplying a large amount of IRS might either not be able to take advantage of an attractive arising investment opportunity due to binding balance sheet constraints or would need to face costly unwinding of his swap positions. We model this risk using a random variable $\xi \sim \mathcal{N}(0, \sigma^2)$. Assuming that dealers have CARA utility, with risk-aversion $\lambda$, the cost of supplying an amount $S > S_0$ of IRS is given as $\frac{\lambda}{2} ([S - S_0]P)^2 \sigma^2$. To compensate for this risk, dealers charge a fee of $\delta$ on the present value of the stream of fixed payments per unit time, thereby earning a profit of $\delta(S - S_0)P$. Hence, the dealer is maximizing

$$\delta(S - S_0)P - \frac{\lambda}{2} ([S - S_0]P)^2 \sigma^2.$$
Taking first-order conditions leads to the following supply of IRS:

\[ S^* = S_0 + \frac{\delta}{\lambda \sigma_\xi^2 P}. \]  \hspace{1cm} (15)

We provide the detailed derivation of the supply function in the appendix.

4.3 Equilibrium Swap Spreads: Numerical Example

Equating Equations (13) and (15) gives the equilibrium swap spread. To illustrate the
effect of pension fund’s underfunding on the equilibrium swap spread, we continue the nu-
merical example from Section 4.1. For the supply, we choose the following parameters:
\( S_0 = 0.05, \sigma_\xi = 0.2, \) and \( \lambda = 0.2. \) Furthermore, we choose three different values for UFR,
15\%, 25\%, and 35\%. The equilibrium swap rate in the three different UFR regimes is illus-
trated in Figure 4.
Figure 4: **Supply and demand of IRS.** This graph illustrates the demand for IRS by pension funds for different UFR and the supply of IRS by derivatives dealers. UFR is computed as \((L/r - A)/A\). Parameter choices are: \(\beta = 10\), \(\rho = 0.05\), \(\mu = 0.06\), \(\sigma = 0.2\), \(\alpha = 0.1\), \(s = 0.3\), \(R = 0\), \(P = 50\), \(\sigma_\xi = 0.2\), \(\lambda = 0.2\), and \(S_0 = 0.005\), which corresponds to a frictionless swap spread of zero as long as \(UFR < 10\%\). The amount of IRS traded is given under the assumption that pension fund’s total assets are normalized to 1.

As we can see from the figure, our model supports the intuition that higher underfunding of pension funds leads to more demand for IRS which decreases swap spreads and eventually pushes them into negative territory. Note that the pension funds’ demand for IRS is almost unaffected by changes in the swap spread.

## 5 Empirical Analysis

In this section, we first describe our approach to measuring pension fund underfunding and constructing an aggregate measure for the underfunded ratio \((UFR)\) of US pension funds. We then show that 30-year swap spreads differ in different funding regimes, using
a Kolmogorov-Smirnov test. Subsequently, we run OLS regressions to test the relationship between the 30-year swap spread and $UFR$. We conclude this section with addressing the possible endogeneity concern that the level of the yield curve can drive both the swap spread and $UFR$. To account for this, we run a 2-stage least squares regression, where we use stock returns as an instrument.

5.1 Measuring Pension Fund Underfunding

To test our hypotheses, we first construct a measure of pension fund underfunding. We obtain quarterly data on two types of defined benefit (DB) pension plans, private as well as public local government pension plans, from the financial accounts of the US (former flow of funds) tables L.118b and L.120b. We exclude defined contribution pension plans since they cannot become underfunded and also exclude public federal DB pension plans since they are only allowed to invest in government bonds. We first note that the overall size of the pension funds’ balance sheet is 8,235 billion US dollar (as of Q3 2015), thereby capturing approximately 45% of the total assets held by all US pension funds. Furthermore, comparing the size of the pension funds’ balance sheet to the size of the US dealer-brokers’ balance sheet shows that it is approximately 2.5 times as large.

Table 2 shows the aggregate pension fund balance sheet for the third quarter of 2015. As we can see from the table, the liabilities of these pension funds consist only of pension entitlements. On the asset side, there are three major positions. First, corporate equities, which make up more than one third of the balance sheet. Second, claims of pension fund on sponsor, which account for almost one quarter of the pension funds assets. As we describe below, these claims on sponsor are our main proxy for underfunding.\(^{[17]}\) Third, debt securities,\(^{[17]}\)

\(^{[17]}\)The Financial Accounts report assets and liabilities (and corresponding financial flows) for both private
Table 2: **Aggregate pension fund balance sheet as of Q3 2015.** This table presents the assets and liabilities of private as well as state and local government employee defined-benefit (DB) pension plans. The amounts are in billions of dollars, not seasonally adjusted. (Source: Financial accounts of the United States)

<table>
<thead>
<tr>
<th>DB Pension Fund Assets (billions)</th>
<th>DB Pension Fund Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>checkable deposits and currency</td>
<td>$14.42</td>
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<tr>
<td>total time and savings deposits</td>
<td>$75.67</td>
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<tr>
<td>money market mutual fund shares</td>
<td>$85.45</td>
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<tr>
<td>security repurchase agreements</td>
<td>$7.11</td>
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<td>debt securities</td>
<td>$1,744.41</td>
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<tr>
<td>commercial paper</td>
<td>$82.26</td>
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<tr>
<td>Treasury securities</td>
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<tr>
<td>agency- and GSE-backed securities</td>
<td>$191.15</td>
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<tr>
<td>corporate and foreign bonds</td>
<td>$1,101.75</td>
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<tr>
<td>municipal securities</td>
<td>$4.05</td>
</tr>
<tr>
<td>total mortgages</td>
<td>$20.00</td>
</tr>
<tr>
<td>corporate equities</td>
<td>$3,141.21</td>
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<tr>
<td>mutual fund shares</td>
<td>$613.29</td>
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<tr>
<td>miscellaneous assets</td>
<td>$2,533.58</td>
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<tr>
<td>unallocated insurance contracts</td>
<td>$58.02</td>
</tr>
<tr>
<td>pension fund contributions receivable</td>
<td>$47.28</td>
</tr>
<tr>
<td>claims of pension fund on sponsor</td>
<td>$2,044.53</td>
</tr>
<tr>
<td>unidentified miscellaneous assets</td>
<td>$383.76</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$8,235.12</td>
</tr>
</tbody>
</table>
which consist mainly of fixed-rate securities, like corporate bonds.

We use claims of pension funds on sponsors as our measure of pension funds’ underfunding ratio ($UFR$). $UFR$ in quarter $t$ is computed as:

$$UFR_t = \frac{\text{Private DB claims on Sponsor}_t + \text{Public DB claims on Sponsor}_t}{\text{Private DB total financial assets}_t + \text{Public DB total financial assets}_t}. \quad (16)$$

The claims of pension fund on sponsors represents the difference between actuarial liabilities and pension fund assets. It reflects the amount of underfunding or overfunding of the plans. These claims (which can be positive or negative) are treated as an asset of the pension funds sector and a liability of the sponsors of the plans. If claims of pension fund on sponsor is positive, pension funds are underfunded. Since our hypothesis is that $UFR$ has a more significant impact on swap spreads if it is negative, we introduce the notation $UFR_t^+ := \max(UFR_t, 0)$ and $UFR_t^- := \min(UFR_t, 0)$ for the positive and negative part of the underfunding ratio respectively. Since we are using changes in $UFR$ in our regression analysis, we also introduce the notation $\Delta UFR_t^+ := \Delta UFR_t \mathbb{1}_{UFR_t > 0}$ and $\Delta UFR_t^- := \Delta UFR_t \mathbb{1}_{UFR_t \leq 0}$. Note that the way we define $\Delta UFR^+$ means that the measure includes a change from fully funded to underfunded periods but not from underfunded to fully-funded (this change is included in $UFR^-$).

and public DB pension funds. Prior to September 2013, the assets and liabilities of DB pension plans were reported using cash accounting principles, which record the revenues of pension funds when cash is received and expenses when cash is paid out. Under this treatment, there was no measure of a plan’s accrued actuarial liabilities. Rather, the liabilities in the Financial Accounts were set equal to the plans’ assets. As a result, the Financial Accounts did not report any measure of underfunding or overfunding of the pension sector’s actuarial liabilities, as would occur if the assets held by the pension sector fell short of or exceeded the liabilities. Starting with the September 2013 release, the Financial Accounts treat DB pensions using accrual accounting principles, whereby the liabilities of DB pension plans are set equal to the present value of future DB benefits that participants have accumulated to date, which are calculated using standard actuarial methods. This new measure is retroactively made available. Throughout, we use the accrual measures of the claims of pension funds on sponsors.

\[^{18}\text{See Stefanescu and Vidangos (2014) for further details.}\]
5.2 Swap Spreads in Different Underfunding Regimes

It should be noted for the end of quarter \( t \), the Fed’s flow of funds report the pension sponsors’ funding status resulting from events during the end-quarter \( t - 1 \) to end-quarter \( t \). This is reported roughly 2 weeks after end of quarter \( t \). The swap spreads that we use in the paper are calculated precisely at the end of quarter \( t \). In this sense our measure of funding status for quarter end \( t \), \( UFR_t \), which is based on the information from end-quarter \( t - 1 \) to end-quarter \( t \) is effectively a lagged measure relative to the time at which the swap spreads are collected.

Using the measure constructed above, we provide some preliminary evidence on the proposition that the demand by a significant subset of pension sponsors to receive fixed in long-term swaps has an effect on long-term swap spreads.

The top panel of figure 5 shows a scatter plot of the 30-year swap spreads in basis points against our measure of aggregate funding status, \( UFR_t \), and gives a first overview of the results. The time period is between Q2 1994 and Q3 2015. The swap spreads are quarter-end observations and we are distinguishing between the negative part (solid dots) and positive part (circles) of the \( UFR \). [It is worth recalling that when \( UFR > 0 \), there is underfunding]. The dashed lines indicate 95% confidence intervals. As we can see from the figure, the level of the swap spread is negatively related to the \( UFR \) for both funded and underfunded regions. However, in line with our theory, the dots are less scattered around the solid line if the \( UFR \) is positive, indicating a stronger correlation when pension funds are underfunded. As we can also see from the top panel, the intercepts of the two lines differ. Unfortunately, there are not enough data points around this cutoff to provide a powerful empirical analysis of the relationship at the cusp where the funds are just fully funded. Instead, we test below whether the distribution of swap spreads is different when pension
Figure 5: Relationship between 30-year swap spreads and the aggregate funding status of DB pension plans. The lower panel shows the time series of the two variables, wherein the black solid line is the 30-year swap spread (left-hand axis) and the blue line with dots is pension funds’ underfunded ratio (right-hand side). The grey shaded areas indicate periods where pension funds are fully funded or over-funded. Data on pension fund underfunding ratios are obtained from the financial accounts of the U.S. and the underfunding ratio is computed as indicated in Equation 16.

funds are underfunded when compared to regimes in which they are fully funded. The lower panel of Figure 5 shows the time series plot of the same variables, illustrating that both variables are relatively volatile without an obvious trend component. The grey shaded areas indicate periods where pension funds are fully funded or over-funded. The U.S. Economy
was generating a surplus during the end this (shaded) period, with a drop in the supply of long-term government bonds, which might have partially accounted for the increase in swap spreads. The stock market boom during this period could have partially accounted for the over-funded status of the pension plans.

We next test whether the 30-year swap spread behaves significantly different during time periods when pension funds are underfunded as opposed to time periods when pension funds are fully funded. To that end we use monthly month-end observations of the 30-year swap spread and divide the sample into periods where $UFR \leq 0$ and periods where $UFR > 0$. Figure 6 shows the kernel density of monthly 30-year swap spreads in the two different regimes, illustrating that there is a difference in the swap spread distribution in the two regimes. To formally test this hypothesis, we run a Kolmogorov-Smirnov test, which results in a test statistic of $D = 0.44219$, corresponding to a p-value below 0.1%.
Figure 6: Association between the funding status of DB pension plans and long-term swap spreads. The plot shows the kernel density of swap spreads when $UFR > 0$ and when $UFR < 0$. The shaded region illustrates the bootstrapped standard errors. We can formally reject the hypothesis that these two kernel densities are identical with a $p$-value below 0.1%.

The evidence presented in this section helps to motivate why the funding status of pension plans, as suggested by our theory, may be a channel that could be at work in driving the swap spreads down to negative levels. We next use regression analysis to further explore this channel.

5.3 Regression Analysis

To shed additional light on the relationship between $UFR$ and swap spreads we next run a regression analysis of changes in 30-year swap spreads on changes in $UFR$. Motivated by the no-arbitrage argument in Table 6, we control for the change in the difference between the

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\footnote{We use changes in these variables since both are highly serially correlated. A regression of the level of the 30-year swap spread (level of $UFR$) on the lagged level of the 30-year swap spread ($UFR$) gives a highly significant coefficient of 0.95 (0.97).}
3-months Libor rate and 3-month general collateral repo rate ($\Delta LR\; spread_t$) in all regression specifications. Panels (1) and (2) of Table 3 show that, without additional control variables, pension fund underfunding is a significant explanatory variable for 30-year swap spreads.
Table 3: 30-year swap spreads and pension fund underfunding. This table reports results from regressions of quarterly changes in the 30-year swap spread on the indicated variables. $\Delta UFR_t$ is the change in the underfunding ratio of private and local government defined benefit pension funds, as defined in Equation (16), $\Delta UFR_t^+$ ($\Delta UFR_t^-$) is the change in $UFR_t$ if $UFR_t > 0$ ($UFR_t \leq 0$) and zero otherwise. $\Delta LR Sprd_t$ is the change in the quarter-end difference between the 3-month Libor and 3-month General Collateral repo rate. $\Delta Debt/GDP_t$ is the change in the ratio of US public debt to GDP, $\Delta EDF_t$ is the change in the Moody’s expected default frequency of the 14 largest derivatives-dealing banks (G14 banks), $\Delta Move_t$ is the change in the 1-month implied volatility of US Treasuries with 2, 5, 10, and 30 years to maturity , $\Delta TERM_t$ measures changes in the slope of the yield curve, approximated as the difference between 30-year and 3-month treasury yields. In panels (5) and (6) five additional controls are added: changes in the level of the 30-year treasury yield, changes in the mortgage refinancing rate, obtained form the mortgage bankers association, the broker-dealer leverage factor provided by Adrian, Etula, and Muir (2014), changes in VIX, and changes in the 10-year on-the-run-off-the-run spread. All variables are quarter-end. The numbers in parenthesis are heteroskedasticity-robust $t$-statistics. $^{***}$, $^{**}$, and $^*$ indicate significance at a 1%, 5%, and 10% level respectively. The observation period is Q3 1994 – Q4 2015 with 5 missing observations between Q4 1997 and Q4 1998 due to missing repo rates.

<table>
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<th>(5)</th>
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<tr>
<td>$\Delta UFR_t$</td>
<td>-1.09***</td>
<td>-0.96**</td>
<td>-0.76</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.99)</td>
<td>(-2.20)</td>
<td>(-1.56)</td>
<td></td>
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</tr>
<tr>
<td>$\Delta UFR_t^+$</td>
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<td>-1.32***</td>
<td>-1.27***</td>
<td>-1.25***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-3.49)</td>
<td>(-3.17)</td>
<td>(-2.68)</td>
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<tr>
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<td>-0.35</td>
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<tr>
<td></td>
<td>(-0.83)</td>
<td>(-0.60)</td>
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<tr>
<td>$\Delta LR Sprd_t$</td>
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<td>0.06</td>
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<td>(0.26)</td>
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<tr>
<td>$\Delta Move_t$</td>
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<td>0.17**</td>
<td>0.19**</td>
<td>0.19**</td>
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<td>(2.45)</td>
<td>(2.49)</td>
<td>(2.23)</td>
<td>(2.09)</td>
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<tr>
<td>$\Delta TERM_t$</td>
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<td>-0.06**</td>
<td>-0.08*</td>
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<td>0.15</td>
<td>0.27</td>
<td>0.28</td>
<td>0.26</td>
<td>0.27</td>
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</table>


Panel (1) shows that $UFR$ for the entire sample period is statistically significant at a 1% level with a coefficient of $-1.09$ ($t$-statistic of $-2.99$). More importantly and in line with our theory, panel (2) shows that $UFR$ is even more significant when only considering underfunded regimes and insignificant when pension funds are fully funded. For underfunded periods, $UFR$ is statistically significant at a 1% level with a coefficient of $-1.32$ ($t$-statistic of $-3.49$) and for funded periods $UFR$ is insignificant with a coefficient of $-0.57$ ($t$-statistic of $-0.83$). Note that a coefficient of -1 indicates that swap spreads fall by one basis point when pension fund underfunding increases by 1%.

We next check whether our results are robust to controlling for other factors that are likely to affect swap spreads. We start by adding four control variables, the US debt-to-GDP ratio as a proxy for the “convenience yield” of US Treasuries (Krishnamurthy and Vissing-Jorgensen, 2012), the average Moody’s expected default frequency (EDF) of the 14 largest derivatives-dealing banks, the implied volatility in US Treasuries as proxied by the Move index, and a term factor, measuring the slope of the yield curve. These variables (as well as all other data used in our analysis) are described in more detail in Appendix D. The results of these regressions are reported in Panels (3) and (4) of Table 3.

As we can see from the table, Debt-to-GDP is insignificant but with the expected sign: An increase in Debt-to-GDP lowers the convenience yield of treasuries, thereby lowering the swap spread. $\Delta EDF_t$ is statistically significant and an increase in derivatives dealers’ expected default frequency lowers the swap spread, indicating that, as dealers become more constrained, swap spreads decrease. $\Delta Move_t$ and $\Delta TERM_t$ are both significant and an increase in uncertainty, as captured by Move, increases the swap spread. Most importantly, as we can see from Panels (3) and (4) of Table 3, controlling for these variables leads to a small drop in the statistical and economic significance of $UFR$, but leaves our main result
unchanged. Panel (3) shows that \( UFR \) for the full sample period is still significant at a 5% level with a coefficient of \(-0.96\) (\(t\)-statistic of \(-2.20\)). More importantly, Panel (4) shows that \( UFR \) during times of underfunding is still statistically significant at a 1% level with a coefficient of \(-1.27\) (\(t\)-statistic of \(-3.17\)).

In panels (5) and (6) we add five additional controls to check whether our results remain robust to including more potential drivers of swap spreads. These five controls are the level of the 30-year treasury yield, the mortgage refinancing rate, the broker-dealer leverage factor by [Adrian et al. (2014)](http://example.com), the VIX index, and the 10-year on-the-run off-the-run spread. We do not report the coefficient estimates for these variables for brevity and note that none of these additional variables is statistically significant. Moreover, as we can see by comparing the \( R^2 \) values from panels (3) and (4) with the \( R^2 \) values from panels (5) and (6), adding these additional controls does not improve the explanatory power of our analysis. Furthermore, adding these controls leads to a minor drop in statistical and economical significance of \( UFR + \).

As a next step, we check whether \( UFR \) is a significant explanatory variable for swap spreads with shorter maturities. To that end, we regress changes in the 2-year, 5-year, 10-year, and 30-year swap spread on the positive and negative part of changes in \( UFR \), controlling for changes in the Libor-repo spread.\(^{20}\) The results of this regression are exhibited in Table 4.

In line with our theory, \( UFR + \) is only significant for the 30-year swap spread and insignificant for swap spreads with shorter maturities. We note that, in line with the no-arbitrage argument from Table 6, the Libor-repo spread is a significant explanatory variable for swap spreads with shorter maturities (2-year and 5-year).

\(^{20}\)We focus on 2, 5, 10, and year swap spreads since there are no missing observations for these data and we do not need to supplement them with data from the FED H.15 reports.
Table 4: **Pension fund underfunding and swap spreads with different maturities.** This table reports results from regressions of quarterly changes in swap spread with 2, 5, 10, and 30 years to maturity on the indicated variables. $\Delta UFR_t^+ (\Delta UFR_t^-)$ is the change in the underfunding ratio of private and local government defined benefit pension funds as defined in Equation (16), conditional on pension funds being underfunded (funded) at time $t$. $\Delta LR Sprd_t$ is the change in the quarter-end difference between the 3-month Libor rate and 3-month General Collateral repo rate. The numbers in parenthesis are heteroskedasticity-robust $t$-statistics. \*, \**, and \*** indicate significance at a 1\%, 5\%, and 10\% level respectively. The observation period is Q3 1994 – Q4 2015 with 5 missing observations between Q4 1997 and Q4 1998 due to missing repo rates.

<table>
<thead>
<tr>
<th></th>
<th>2 Year</th>
<th>5 Year</th>
<th>10 Year</th>
<th>30 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.01</td>
<td>−0.46</td>
<td>−0.73</td>
<td>−1.05</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(−0.47)</td>
<td>(−0.66)</td>
<td>(−0.85)</td>
</tr>
<tr>
<td>$\Delta UFR_t^+$</td>
<td>−0.05</td>
<td>0.04</td>
<td>−0.31</td>
<td>−1.32***</td>
</tr>
<tr>
<td></td>
<td>(−0.12)</td>
<td>(0.10)</td>
<td>(−0.81)</td>
<td>(−3.49)</td>
</tr>
<tr>
<td>$\Delta UFR_t^-$</td>
<td>0.07</td>
<td>0.04</td>
<td>−0.15</td>
<td>−0.57</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.11)</td>
<td>(−0.23)</td>
<td>(−0.83)</td>
</tr>
<tr>
<td>$\Delta LR Sprd_t$</td>
<td>0.35***</td>
<td>0.19***</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(5.42)</td>
<td>(3.15)</td>
<td>(1.26)</td>
<td>(1.30)</td>
</tr>
<tr>
<td>Observations</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.56</td>
<td>0.25</td>
<td>0.01</td>
<td>0.15</td>
</tr>
</tbody>
</table>
5.4 Two-Stage Least Squares Regression Results

To mitigate endogeneity concerns, we next run a 2-stage least squares regression. In a first stage, we regress $\Delta UFR_t$ on US stock returns proxied by the first Fama-French factor. In panels (2), (4), and (6) we drop fully funded periods and only regress $\Delta UFR_t^+$ on stock returns. Stock returns affect $UFR$ since pension funds are heavily invested in corporate equity (almost half their assets are invested in corporate equity according to Table 2) and therefore decreasing stock returns increase $UFR$.

At the same time, there is no obvious connection between the 30-year swap spread and stock returns. We therefore argue that the exclusion restriction is fulfilled. Furthermore, the results from a weak instrument test give a p-value far below 0.1% for all six regression specifications. Additionally to that, the results from a Hausman test give a p-value above 0.6 (ranging from 0.619 for specification (1) to 0.934 for specification (6)) for all six specifications. Hence, we can reject the hypothesis that stock returns are a weak instrument and we cannot reject that the 2 SLS is as consistent as the OLS regression.

Table 5 shows the results of the second stage, where we use the projected $UFR$ as explanatory variable. Overall the results from the second stage are similar to those from the OLS regression discussed before. The projected $UFR$ is significant at a 1% level and decreases in significance as we add controls. More importantly, the projected underfunded ratio in regimes when pension funds are underfunded is even more significant ($t$-statistic of -3.31 without controls) and remains significant even after adding several controls ($t$-statistic of -2.71).
Table 5: 30-year swap spreads and pension fund underfunding (2-stage least squares). This table reports results from a second stage regressions of quarterly changes in the 30-year swap spread on the indicated variables. In the first stage, the change in the underfunding ratio of private and local government defined benefit pension funds, $\Delta UFR_t$ ($\Delta UFR^+_t$) is regressed on excess stock returns, measured by the first Fama-French factor, controlling for the other indicated variables. $\Delta LR Sprd_t$ is the change in the quarter-end difference between the 3-month Libor rate and 3-month General Collateral repo rate, $\Delta Debt/GDP_t$ is the change in the ratio of US public debt to GDP, $\Delta EDF_t$ is the change in the Moody's expected default frequency of the 14 largest derivatives-dealing banks (G14 banks), $\Delta Move_t$ is the change in the 1-month implied volatility of US Treasuries with 2, 5, 10, and 30 years to maturity, $\Delta TERM_t$ measures changes in the slope of the yield curve, approximated as the difference between 30-year and 3-months treasury yields. In panels (5) and (6) five additional controls are added: changes in the level of the 30-year treasury yield, changes in the mortgage refinancing rate, obtained from the mortgage bankers association, the broker-dealer leverage factor provided by Adrian et al. (2014), changes in VIX, and changes in the 10-year on-the-run-off-the-run spread. All variables are quarter-end. The numbers in parenthesis are small-sample and heteroskedasticity-robust t-statistics. *** , **, and * indicate significance at a 1%, 5%, and 10% level respectively. The observation period is Q3 1994 – Q4 2015 with 5 missing observations between Q4 1997 and Q4 1998 due to missing repo rates.

<table>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
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<td>-0.60</td>
<td>-1.18</td>
<td>-0.48</td>
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<tr>
<td></td>
<td>(-0.91)</td>
<td>(-1.45)</td>
<td>(-0.42)</td>
<td>(-1.00)</td>
<td>(-0.36)</td>
<td>(-0.85)</td>
</tr>
<tr>
<td>$\Delta UFR^+_t$</td>
<td>-1.05***</td>
<td>-0.93**</td>
<td>-0.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.02)</td>
<td>(-2.29)</td>
<td>(-1.62)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta UFR_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta LR Sprd_t$</td>
<td>0.06</td>
<td>0.08*</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(1.16)</td>
<td>(1.87)</td>
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<td>(0.79)</td>
<td>(0.20)</td>
<td>(0.40)</td>
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<tr>
<td>$\Delta Debt/GDP_t$</td>
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<td>-0.78</td>
<td>-0.78</td>
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<tr>
<td></td>
<td>(-1.30)</td>
<td>(-1.11)</td>
<td>(-1.07)</td>
<td>(-1.07)</td>
<td>(0.22)</td>
<td></td>
</tr>
<tr>
<td>$\Delta EDF_t$</td>
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<td>-0.05**</td>
<td>-0.06***</td>
<td>-0.06**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td>(-3.08)</td>
<td>(-3.08)</td>
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<tr>
<td>$\Delta Move_t$</td>
<td>0.16**</td>
<td>0.14**</td>
<td>0.19**</td>
<td>0.16***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.49)</td>
<td>(2.39)</td>
<td>(2.21)</td>
<td>(2.21)</td>
<td>(2.80)</td>
<td></td>
</tr>
<tr>
<td>$\Delta TERM_t$</td>
<td>-0.05*</td>
<td>-0.05**</td>
<td>-0.08*</td>
<td>-0.09**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.76)</td>
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<td>(-1.88)</td>
<td>(-2.55)</td>
<td></td>
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</tr>
<tr>
<td>Add. Controls?</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>81</td>
<td>62</td>
<td>81</td>
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</tbody>
</table>

39
6 Conclusion

We provide a novel explanation of persistent negative 30-year swap spreads. Our explanation is based on the funding status of DB pension plans. Specifically, we argue that under-funded pension plans prefer to meet the duration needs arising from their unfunded pension liabilities through receiving fixed payments in 30-year interest rate swaps. This allows them to use their scarce funding to invest in risky assets with the hope of improving their future funding status. We present empirical evidence, which supports the view that the under-funded status of DB pension plans has a significant explanatory power for 30-year swap spreads, even after controlling for several other drivers of swap spreads, commonly used in the swap literature. Moreover, we show that the funding status does not have any explanatory power for swap spreads associated with shorter maturities.

References


Van Deventer, D. R. (2012). Why is the 30 year swap spread to treasuries negative?
A Characterizing the Term Structure

In this section, we illustrate several properties of the short rate, which we assume to follow a process suggested by Constantinides and Ingersoll (1984) with dynamics given by Equation (2). To illustrate the main properties of the process and get an understanding of the term structure of interest rates, we simulate the process using the following set of parameters:

\[
\begin{align*}
\alpha & \in \{0.1, 0.5, -0.5\} \\
 s & \in \{0.2, 0.6\} \\
r_0 &= 0.02.
\end{align*}
\]

and characterize the term structure of interest rates under the process specified in Equation (2). Recall that the price of a zero coupon bond is given as:

\[
p(0, T) = \mathbb{E} \left[ e^{-\int_0^T r_s ds} \right]
\]

and the zero coupon yield can be computed as:

\[
y(0, T) = -\frac{\ln (p(0, T))}{T}.
\]

We compute the zero-coupon yield using the simulated processes considering the following five parameter choices using \( r_0 = 0.02 \) in all specifications:

(i) the base case with \( \alpha = 0.1 \) and \( s = 0.2 \), (ii) the case where \( s = 0.6 \) is increased while \( \alpha = 0.1 \) is kept fixed, (iii) the case

\[21\] When simulating the process, we use 10 years of observations, 10,000 simulations and 1,000 time steps per year to discretize the sample period.

\[22\] For each simulated sample path we approximate \( \int_0^T r_s ds \) using the simulations and then compute the expected value of the exponential as average of all sample paths.
where $\alpha = 0.5$ is increased while the $s = 0.2$ is kept fixed, (iv) the case where both $\alpha = 0.5$ and $s = 0.6$ are increased, (v) and the case where $\alpha = -0.5$ is set to a negative number with $s = 0.6$.

Figure 7 shows the term structure of interest rates under the five parameter specifications. As we can see from the figure, the parameter $\alpha$ is key in characterizing the slope of the yield curve. For cases (i) and (ii) where we keep $\alpha = 0.1$ we see that the term structure of interest rates is upward sloping but almost flat, while for cases (iii) and (iv), where we increase $\alpha = 0.5$, the term structure becomes steeper. In case (v), where $\alpha = -0.5$ is set to a negative number, the term structure is downward-sloping. In contrast to $\alpha$, the volatility parameter has a minor effect on the term structure. Comparing cases (i) and (ii) as well as (iii) and (iv), we can see that an increase in $s$ results in a flatter the term structure of interest rates.
Figure 7: **Term structure of interest rates for the short rate.** The short rate follows a process suggested by Constantinides and Ingersoll as characterised by Equation (2) with the indicated parameters. The initial short rate is set to $r_0 = 2\%$. We use 10,000 sample paths and 1,000 time steps per year to discretize the time interval.
B Proofs and Additional Theoretical Results

Proof of Propositions 1 and 2

We first note that Proposition 1 is a special case of Proposition 2 where $\sigma_{12} = 0$ and $n = 0$. Hence, it remains to show that Proposition 2 holds. To show this result, we first state the pension fund’s HJB in the general setting with stocks and swaps:

$$0 = \inf_{y,m,n} \left[ y^2 - \rho G + G_A (nS(\mu - r) + m(1 - \delta) + Ar + y - L) 
+ \frac{1}{2} G_{AA} \left( n^2 \sigma^2 + m^2 PS^2 - 2mnSP\sqrt{\tau}\sigma_{12} \right) \right].$$

Taking derivatives with respect to $y$ and $m$ leads to the following first order conditions (FOC):

$$y = \left(-\frac{1}{\beta} G_A \right)^{(\beta-1)}$$
$$nS = -\frac{G_A}{G_{AA}} \frac{\mu - r}{\sigma^2} + m \frac{P\sqrt{\tau}\sigma_{12}}{\sigma^2}$$
$$m = -\frac{G_A}{G_{AA}} \frac{1 - \delta}{P^2 S^2 r} + nS$$

Plugging $m$ into the expression for $n$ and vice versa gives the optimal controls:

$$nS = -\frac{G_A}{G_{AA}} \frac{\mu - r}{\sigma^2} + \frac{(1 - \delta)\sigma_{12}}{s^2 P\sqrt{\tau}\sigma^2} \left(1 - \frac{s_{12}^2}{s^2 \sigma^2}\right)$$
$$m = -\frac{G_A}{G_{AA}} \frac{1 - \delta}{P^2 S^2 r} + \frac{\mu - r}{\sigma^2} \frac{\sigma_{12}}{Ps^2 \sqrt{\tau}} \left(1 - \frac{s_{12}^2}{s^2 \sigma^2}\right).$$
We now guess and verify that the value function is of the following form:

\[ G = g(P)(LP(1 + \alpha - s^2) - A)^\beta = g\Psi^\beta, \]

where we introduce the notation \( \Psi := (LP(1 + \alpha - s^2) - A) \) and short hand \( g := g(P) \) to simplify notations. Taking partial derivatives of the value function gives:

\[
G_A = (-\beta g\Psi^{\beta - 1}) \\
G_{AA} = (\beta(\beta - 1)g\Psi^{\beta - 2}) \\
G_A G_{AA} = (-\beta g\Psi^{\beta - 1}) (\beta(\beta - 1)g\Psi^{\beta - 2}) = -\Psi \beta - 1.
\]

Hence, under the guess, the optimal controls are given as:

\[
y = \left( -\frac{1}{\beta} (-\beta g\Psi^{\beta - 1}) \right)^{1/(\beta - 1)} = g^{1/(\beta - 1)}\Psi \\
nS = \frac{\left( \frac{\mu - r}{\sigma^2} + \frac{(1 - \delta)\sigma_{12}}{\sigma^2 P \sqrt{\sigma^2}} \right)}{(\beta - 1) \left( 1 - \frac{\sigma_{12}^2}{\sigma^2 \sigma^2} \right)} \Psi =: \lambda_1 \Psi \\
m = \frac{\left( \frac{1 - \delta}{P^2 s^2 \sigma^2} + \frac{\mu - r}{\sigma^2 P s^2 / \sqrt{\sigma^2}} \right)}{(\beta - 1) \left( 1 - \frac{\sigma_{12}^2}{\sigma^2 \sigma^2} \right)} \Psi =: \lambda_2 \Psi,
\]

where the last two Equations correspond to Equations (12) and (13). It remains to show that the function \( g(P) \) stated in Equation (14) solves the HJB. To see this, we plug both,
the guess and the optimal controls into the HJB:

\[
0 = g^{\beta/(\beta-1)}\Psi - \rho g^\beta - \beta g^{\beta-1} (\lambda_1 \Psi (\mu - r) + \lambda_2 \Psi (1 - \delta) - r (L/r - A) + g^{1/(\beta-1)} \Psi) \\
= \left[ (1 - \beta) g^{1/(\beta-1)} - \rho - \beta \lambda_1 (\mu - r) - \beta \lambda_2 (1 - \delta) + \beta r \\
+ \frac{1}{2} \beta (\beta - 1) (\lambda_1^2 \sigma^2 + \lambda_2^2 P^2 s^2 r - 2 \lambda_1 \lambda_2 P \sqrt{r} \sigma_{12}) \right] g \Psi
\]

Dividing by \( g \Psi \) and solving for \( g \) leads to Equation \((14)\), which completes the proof. ■

The Supply Function

**Proposition 3.** Assume that derivatives dealers can provide swaps at the frictionless rate up to a level \( S_0 \). Afterwards, they face a random cost of \( \xi \sim \mathcal{N}(0, \sigma_\xi^2) \) and are risk-averse with aversion coefficient \( \lambda \) towards that cost. Then, the Dealer’s optimal supply of IRS is given by Equation \((15)\).

**Proof of Proposition 3**

Under risk-aversion \( \lambda \), the dealer’s expected profits of providing swaps \( S > S_0 \) are given as:

\[
\delta (S - S_0) P - \frac{\lambda}{2} \left( [S - S_0] P \right)^2 \sigma_\xi^2.
\]

More formally, the dealer is facing the following HJB:

\[
0 = \max_S \left( \delta (S - S_0) P - \frac{\lambda}{2} \left( [S - S_0] P \right)^2 - \rho J + J_r \sigma^2 + \frac{1}{2} J_{rr} s^2 r^3 \right).
\]
Taking FOC then leads to the optimal strategy in Equation (15) which completes the proof.

**C What keeps Arbitrageurs Away?**

Table 6: The arbitrage relationship between interest rate swaps and Treasuries. This table provides an arbitrage argument for positive swap spreads. $s_0$ denotes the fixed rate in an interest rate swap with maturity $T$, $l_t$ denotes the variable Libor rate in month $t$, $c_0$ denotes the coupon of a treasury bond with maturity $T$, and $r_t$ denotes repo rate in month $t$. Since the difference between Libor and Repo rate is usually positive, the difference between swap rate and treasury yield should be positive too.

<table>
<thead>
<tr>
<th>t</th>
<th>Pay fixed rate $s_0$ in IRS</th>
<th>Receive Libor $l_t$ from IRS</th>
<th>Buy bond with coupon $c_0$</th>
<th>Borrow at repo rate $r_t$</th>
<th>Payoff</th>
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<tbody>
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<td>0</td>
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<td>$0$</td>
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<tr>
<td>$t = 1$</td>
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<td>$+l_t$</td>
<td>$+c_0$</td>
<td>$-r_t$</td>
<td>$-(s_0 - c_0)$</td>
</tr>
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<td>...</td>
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</tr>
<tr>
<td>$t = T$</td>
<td>$-s_0$</td>
<td>$+l_T$</td>
<td>$+1 + c_0$</td>
<td>$-1 - r_T$</td>
<td>$-(s_0 - c_0)$</td>
</tr>
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</table>

In this section we show that even if negative swap spreads are a textbook arbitrage opportunity, assuming no transaction costs and institutional frictions, the arbitrage strategy explained in Table 6 is still a risky strategy.\(^{23}\) As pointed out by Shleifer and Vishny (1997), Liu and Longstaff (2004) and many others, even textbook arbitrage opportunities are subject to a risk, namely the possibility that the mispricing increases before it vanishes, thereby forcing the arbitrageur out of his position at a loss. With negative swap spreads arbitrage, we know that the mispricing vanishes after 30 years, but we do not know whether it vanishes within a much shorter and practical horizon.

As pointed out by Shleifer and Vishny (1997), Liu and Longstaff (2004) and many others, even textbook arbitrage opportunities are subject to a risk, namely the possibility that the mispricing increases before it vanishes, thereby forcing the arbitrageur out of his position at a loss. With negative swap spreads arbitrage, we know that the mispricing vanishes after 30 years, but we do not know whether it vanishes within a much shorter and practical horizon.

To illustrate this point we provide some stylized sample calculations to approximate the

\(^{23}\) We ignore potential issues with leverage constraints or frictions in the repo market and illustrate the returns to swap spreads arbitrage in a “best case”.  

51
excess returns of an arbitrageur engaging in the strategy, described in Table 6. We assume that the arbitrageur unwinds his position before maturity and consider two cases. In the first case, we assume that the arbitrageur unwinds the position after 3 months, in the second case we assume that he unwinds after 12 months. In both cases he receives a positive carry from the strategy but is exposed to the risk that the swap spread becomes even more negative. For simplicity, we ignore the ageing of the treasury and swap and simply assume that the arbitrageur unwinds the position by engaging in an opposite transaction where he sells a treasury bond with 30-years to maturity and receives fixed in an IRS with 30-years to maturity.\footnote{This simplification leads to a duration mistake of 3 months in case one and 1 year in case two. Since swap and treasury originally have 30 years to maturity this ageing effect is neglect-able for our approximation.} Every month, the arbitrageur observes the 30-year swap spread and engages in the transaction if the swap spread is negative. We illustrate the resulting excess returns of the two strategies in Figure 8. The Sharpe ratio for the 3-month and 12-month strategies are 0.86\% and 5.03\% respectively. Note that the Sharpe ratio for investing in the US stock market for the same time period is 29.39\%.

D Data Description

This appendix provides additional details about the data used for our analysis.

1. Swap Spreads: Swap rates and treasury yields for 2, 3, 5, 10, and 30 years to maturity are obtained from the Bloomberg system. The swap rates are the fixed rates an investor would receive on a semi-annual basis at the current date in exchange for quarterly Libor payments. The treasury yields are the yields of the most recently auctioned issue and adjusted to reflect constant time to maturity. For 3-year and 7-year treasury yields, we supplement the Bloomberg data with treasury yields from the
Figure 8: **Returns from 30-year swap spread arbitrage.** The Figure shows the returns from engaging in swap spreads arbitrage. The Sharpe ratio of the two strategies are 0.86% and 5.03% respectively.
FED H.15 reports due to several missing observations in the Bloomberg data. Swap spreads are computed as the difference between swap rate and treasury yield, where the swap rate is adjusted to reflect the different daycount conventions which are actual/360 for swaps and actual/actual for treasuries.

2. **Underfunded Ratio** (*UFR*): Quarterly data on two types of defined benefit (DB) pension plans, private as well as public local government pension plans, are obtained from the financial accounts of the US (former flow of funds) tables L.118b and L.120b. *UFR* in quarter *t* is then computed using Equation (16). Next, positive and negative part are defined as $UFR^+_t := \max(UFR_t, 0)$ and $UFR^-_t := \min(UFR_t, 0)$. Changes in *UFR* in the different regimes are computed as $\Delta UFR^+_t := \Delta UFR_t 1_{\{UFR_t > 0\}}$ ($\Delta UFR^-_t := \Delta UFR_t 1_{\{UFR_t \leq 0\}}$).

3. **Libor-repo spread**: The 3-month Libor rate as well as the 3-month general collateral repo rate are obtained from the Bloomberg system. The Libor-repo spread is then computed as the difference between these two variables.

4. **Debt-to-GDP ratio**: Quarterly data on the US debt-to-GDP are obtained from the federal reserve bank of St. Louis which provides a seasonally-adjusted time series.

5. **Dealer-Broker EDF**: Expected default frequencies are provided by Moody’s analytics and we use the equally-weighted average of the 14 largest derivatives dealing banks (G14 banks). These 14 banks are: Morgan Stanley, JP Morgan, Bank of America, Wells Fargo, Citigroup, Goldman Sachs, Deutsche Bank, Societe Generale, Barclays, HSBC, BNP Paribas, Credit Suisse, Royal Bank of Scotland, and UBS.

6. **Move Index**: The Move index is computed as the 1-month implied volatility of US treasury bonds with 2, 5, 10, and 30 years to maturity. Index levels are obtained from the Bloomberg system.
7. **Term Factor:** This factor captures the slope of the yield curve, measured as the difference between the 30-year treasury yield and the 3-month treasury yield. A description of these yields can be found under point 1 (swap spreads).

8. **Level:** The level of the yield curve is captured by the 30-year treasury yield. For a description of this yield see point 1 (swap spreads).

9. **VIX:** Is the implied volatility of the S&P 500 index and data on VIX are obtained from the Bloomberg System.

10. **On-the-run spread:** The spread is computed for bonds with 10-years to maturity because estimates of the 30-year spread are noisy and suffer from the 2002-2005 period where the US treasury reduced its debt issuance. The 10-year on-the-run yield is obtained from the FED H.15 website and the 10-year off-the-run yield is constructed as explained in Gürkaynak, Sack, and Wright (2007) and data are obtained from [http://www.federalreserve.gov/pubs/feds/2006](http://www.federalreserve.gov/pubs/feds/2006).

11. **Dealer Broker Leverage:** This variable captures the leverage of US broker-dealers and is described in more detail in Adrian et al. (2014). Until Q4 2009, data on this variable are obtained from Tyler Muir’s website. Since the data ends in Q4 2009, we use the financial accounts of the US data, following the procedure described in Adrian et al. (2014) to supplement the time series with more recent observations for the Q1 2010 – Q4 2015 period.

12. **Mortgage Refinancing:** Quarterly mortgage origination estimates are directly obtained from the Mortgage Bankers Association website. We use mortgage originations due to refinancing as a proxy for the mortgage refinancing rate.