# Bank Liability Structure

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**Abstract**

This paper develops a dynamic continuous-time model of optimal bank liability structure that incorporates the liquidity services on deposits, deposit insurance, regulatory closure, and endogenous default in banks' financing decisions. Nesting the classic model for non-financial firms as a special case, the model clearly explains why banks use higher leverage than non-financial firms. The model shows that a value-maximizing bank balances between deposits and debt so that its endogenous default coincides with the regulatory closure in order to maximize the tax benefits of debt and minimize the protection for deposits. Banks' optimal responses to regulatory changes often counteract regulators' objectives.

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# 1 Introduction

Bank leverage has drawn much attention from regulators and the public after many crises experienced by the banking industry. Regulators around the world have gradually rolled out regulations on bank capital structure, and the shape of bank regulation is still evolving.[1] Banks have been readjusting their capital structure, and academics have been grappling with the questions about the level and composition of capital that banks should hold.[2] Proposals of further regulation are abundant in the literature. There have been arguments for restricting bank leverage to a level similar to non-financial firms.[3] There are also antithetical views on whether banks should hold long-term debt.[4] The FDIC has recently been required to reform deposit insurance policy in order to reduce or eliminate insurance subsidy, which is believed to give banks incentive for high leverage.[5] There are also proposals on tax reforms for banks because the tax benefits of interest expenses are believed to be a major reason for leverage.[6]

The foregoing debate on bank capital regulation calls for a better understanding of bank leverage and its factors. Each regulatory mandate typically attempts to fix a particular broken factor observed in bank liability structure.[7] Arguments for a regulatory mandate on the broken factor often implicitly assume that other factors will remain unchanged, ignoring the overall response of banks that optimally adjust various parts of their liability structure. It is unclear whether banks' optimal response will undo or significantly diminish the intended effects of a regulatory mandate. It is even possible that a regulation may result in unintended consequences.

Before working out banks' optimal response to a change of regulation, we first need to understand how a bank chooses leverage and liability structure when it maximizes its value. Value maximization is a fiduciary responsibility of bank management: acting in the interest of its claim-holders.[8] Banks do not maximize social welfare, such as reducing systemic risk or increasing banking services. An analysis of social welfare implications of bank leverage is

[1] After the frequent bank runs during the Great Depression, the Banking Act of 1933 created the Federal Deposit Insurance Corporation (FDIC). After the financial crisis during the Great Recession, the Dodd-Frank Act of 2010 brought sweeping regulatory reforms ranging from FDIC deposit insurance to stress tests of banks' capital adequacy. Worldwide regulators agreed on Basel III in 2011 to strengthen restrictions on bank leverage.

[2] See Thakor (2014) for a review of the debate on bank capital.

[3] See the book by Admati and Hellwig (2013).

[4] Bulow and Klemperer (2013) argue that banks should hold no debt, besides equity and securities convertible to equity. The Fed governor, Daniel Tarullo (2013), goes in the opposite direction by arguing for requirement of holding more long-term debt, which he thinks will improve the capital structure and the resolution of banks.

[5] See section 331 of the Dodd-Frank Act.

[6] For example, Fleischer (2013) proposes cutting corporate tax rate for banks to make them safer.

[7] For this reason, Santos (2000) motivates regulation as a policy arising out of market failure.

[8] Our focus on bank value maximization sets aside the principal-agent problem such as management's conflict of interests with stake holders. This problem may play a role in bank choices of liability structure. For example, Admati, DeMarzo, Hellwig, and Pfleiderer (2013) discuss how conflict of interests leads bank management to use excessive leverage even if it destroys bank value.

unquestionably important, but understanding of the optimal choices of liability structure by value-maximizing banks is necessary for a proper social welfare analysis of bank regulation.

Banks distinguish themselves from other firms by taking deposits. Deposits are different from other forms of debt partly because banks earn income from the provision of account services and liquidity services to depositors.[9] Other important features of deposits are that deposits are typically insured and that deposit-taking banks are subject to regulatory closure by charter authorities. The risk exposure of deposits covered by deposit insurance should be reflected in insurance premium, unless the insurance is subsidized by the insurance provider.

We develop a dynamic structural model that incorporates the institutional features of banks explicitly and nests the structural model of non-financial firms pioneered by Merton (1974, 1977) and Leland (1994) as a special case. The consistency of our model of banks with the classic model of firms is important because it shows how the special institutional features distinguish banks from non-financial firms in capital structure decisions. The connection and distinction between banks and non-financial firms in our model clearly explains why banks use higher leverage than non-financial firms.

We analytically solve for the optimal liability structure of banks that issue long-term debt and common equity while taking deposits and providing account services. The solution to our model offers new perspectives on bank liability structure. We find it optimal for a value-maximizing bank to choose deposits and debt so that its endogenous default coincides exactly with the regulatory closure. With this optimal choice of liability structure, the distance to default is the same as the distance to regulatory closure. This optimal structure of liabilities minimizes the debt's protection against deposits and results in leverage higher than that optimal for a firm that does not serve deposits.

The above property of optimal leverage has an intuitive economic reason. Because of the income from account services, deposits are cheaper than debt as financing sources. A bank should generally prefer deposits to debt when balancing the benefits of debt against the potential loss to bankruptcy. Given the amount of deposits, however, debt does not affect bankruptcy risk as long as the endogenous default does not happen before the regulatory closure. A bank should therefore take as much debt as possible for availing of the tax benefits but avoid making a default happen before the regulatory closure. Consequently, the optimal debt sets the endogenous default and the regulatory closure concurrent.

Another new perspective offered by our model is the optimal response of banks to FDIC insurance. Deposit insurance allows a bank to take more deposits and issue less debt than it would have done without the insurance. The optimal mix of deposits and debt ensures

[9] In the literature, account and liquidity services to depositors are also referred to as production of liquidity. The income from these services is sometimes referred to as the liquidity premium of deposits. We refer to it as account service income.

that the endogenous default remain coincident with the regulatory closure and minimizes the debt's protection of the insurer against losses in its insurance obligation. Although deposit insurance causes the bank holds less debt, the increase in deposits leads to higher overall leverage than what would have been optimal for a comparable uninsured bank. This optimal response by banks to FDIC insurance prevents the insurance program from reducing the expected bankruptcy loss. As a result, deposit insurance raises the optimal leverage.

The link between insurance premium and liability structure is another new perspective offered by our model. On one hand, a bank's choice of leverage and liability structure affects the insurance premium the bank needs to pay. On the other hand, the insurance premium affects the bank's decision on leverage and liability structure. We explicitly model this feedback channel, which is crucial in assessing regulatory policies pertaining to bank capital structure. Our model shows that an insurance subsidy increases bank leverage, as expected. More importantly, our model shows that an FDIC-insured bank uses higher leverage than a comparable uninsured bank does even when the insurance premium is not subsidized.

Our analysis shed light on the regulatory treatment of long-term debt. If such debt is a claim ranked lower than deposits, it is naturally viewed as a source of capital that protects deposits. Reflecting this view, regulators treat certain long-term unsecured debt as Tier 2 regulatory capital. However, if a bank adjusts its liability structure so that the endogenous default coincides with the regulatory closure, the debt held by the bank does not offer more benefits of deposit protection than that required by the regulatory closure rule. In this sense, banks minimize debt's protection for deposits. A large body of academic literature debates whether long-term debt provides a market discipline on bankruptcy risk.[10] The optimal choice of debt in bank liability structure is especially relevant to this debate and should not be ignored.

Since banks use much higher leverage than non-financial firms do, the corporate tax benefits of debt are particularly important for bank liability structure. Apart from showing that bank leverage is lower in an economy with lower tax benefits, our model shows that it is optimal for banks to shrink their non-deposit debt more than deposits if the tax benefits are lowered. More importantly, the model shows that banks remain substantially leveraged even when the tax benefits are nearly zero. While a full general equilibrium analysis is needed for a thorough welfare analysis, our model of the optimal response of bank liability structure should lay a stepping stone for evaluating the benefits and costs of tax policy reforms in the context of bank leverage.

Our theory directly contributes to the literature of bank leverage. A number of papers have studied the reason for high leverage of banks, going back to Buser, Chen and Kane (1981), who conceptually, not analytically, discuss banks that optimize deposits in the presence of

[10]Flannery and Serescu (1996) contend that debt price rationally reflects the risk in a bank. Gorton and Santomero (1990), however, opine the opposite.

FDIC insurance. Song and Thakor (2007) examine banks' choice between deposits and debt as financing sources in a discrete two-period model, which does not reveal the concurrence of the endogenous default and regulatory closure in an optimal liability structure. DeAngelo and Stulz (2014) provide a rationale for bank leverage by assuming that households and firms value a hedge against liquidity shocks and are willing to pay a premium to banks that do not borrow debt but hedge asset risk to zero. Garnall and Strebulaev (2013) posit that high leverage arises from low volatility of bank assets due to diversification, while assuming that the relative mix of deposits and debt are exogenously given and that banks pay no premium on deposit insurance. Allen and Carletti (2013) rationalize high leverage of banks that hold only deposits and equity by assuming that deposits are cheaper than equity as a financing source because they are traded in segmented markets. Our work complements these insights to formalize the endogenous decision of leverage, which combines deposits and debt optimally, for banks that face the risk of a regulatory closure and must pay premium for FDIC insurance.

Our model is along the line of the literature that attempts to apply the structural framework of Merton (1974, 1977) and Leland (1994) to bank capital structure.[11] Rochet (2008) applies Merton's model and Leland's concept of endogenous default to the monitoring problem of banks. Unlike our model, Rochet's analysis takes bank deposits or debt as given, setting aside the optimal choice of bank liability structure. Harding, Liang and Ross (2009) set out a structural model for banks in which liabilities consist of only deposits, treating banks as the non-financial firms in Leland (1994) and ignoring the special features of banks. Hugonnier and Morellet (2015) use a continuous-time model to analyze role of liquidity reserves in banks in bank liability structure absent deposit insurance. We broaden these models by incorporating the endogenous bank decision on the choice of deposits and debt.

Another contribution of our paper is the explicit modeling of deposit insurance and closure policy of charter authorities. In a setting with these detailed institutional features, we derive the endogenous FDIC insurance premium, taking into account the optimal liability structure. While a number of papers, notably Merton (1977) and Ronn and Verma (1986), have derived risk-adjusted FDIC insurance policies, our work extends their insights to the endogenous decisions on both the default and closure boundaries when a bank liability structure optimally responds to FDIC insurance.

The road map for the rest of the paper is as follows. Section 2 develops the model of bank liability structure. Section 3 characterizes the bank optimal liability structure. Section 4 illustrates quantitatively the liability structures of banks and compare them with banks that are not insured and firms that do not serve deposits. Section 5 analyzes the comparative

[11] Leland considers both unprotected debt and protected debt and interprets protected debt as rolling short-term debt. He analyzes each type of the debt separately, whereas we model the endogenous choice of deposits and unprotected debt simultaneously.

statics, which shed light on the optimal responses of bank liability structure to the changes in exogenous factors, which include the changes of regulatory rules. Section 6 concludes and discusses potential applications and extensions.

# 2 Bank Liability Structure

Banks share some common characteristics with non-financial firms: both have access to cash flows generated by their assets and both finance their assets by issuing debt and equity. Banks, however, differ from non-financial firms in that they take deposits and provide liquidity services to their depositors through check writing, ATMs, and other transaction services such as wire transfers, bill payments, etc. The banking business of taking deposits and serving accounts is heavily regulated in most countries. In the U.S., a large part of deposits is insured by the FDIC, which charges insurance premium and imposes regulations on banks. The model of FDIC insurance has gained popularity outside the U.S., and an increasing number of countries have started to offer deposit insurance.[12] Deposits and the associated services, deposit insurance, and the regulations on opening and closing banks distinguish banking business from other non-financial corporate business and set the financial decisions of banks apart from those of the other firms.

Firms operate in a market with two frictions: corporate taxes and bankruptcy costs. These frictions are crucial for firms in their choice of liability structure, as recognized in the literature originating from Modigliani and Miller (1963) and Baxter (1967) and analyzed in structural models by Leland (1994). Banks face these frictions too, but they have to incorporate simultaneously other considerations, such as FDIC insurance and regulatory closure, in determining their optimal leverage and liability structure. Figure 1 illustrates the liability structure of a typical bank. In Section 2.1, we discuss each part of the structure in detail.

## 2.1 Assets and Liabilities

A typical bank owns a portfolio of risky assets that generate cash flows. The portfolio of assets is valued at $V$, which is the major part of Figure 1. The asset is risky, and its value follows a stochastic process.[13] The instantaneous cash flow of the assets is $\delta V$, where $\delta$ is the rate of cash flow, which are paid as either dividend to equity holders or liabilities to other stakeholders. In a non-financial firm, $\delta V$ is the total earnings before interests, but in a bank, $\delta V$ represents only

[12] The International Association of Deposit Insurers (IADI) was formed on May 6, 2002 to enhance the effectiveness of deposit insurance systems by promoting guidance and international cooperation. As of the end of 2014, IADI represents 79 deposit insurers from 76 countries and areas.

[13] Following Merton (1974) and Leland (1994), we assume that the stochastic process is a geometric Brownian motion, which is described by equation (16) in Appendix.

<table>
<tr><th>Asset Side</th><th>Liability Side</th></tr>
<tr><td rowspan="3">Assets: V<br>Volatility: $\sigma$<br>Cash flow: $\delta$</td><td>Deposits: D<br>Benefit: deduct tax $\tau$<br>Benefit: service income $\eta$<br>Cost: bankruptcy $\alpha$<br>Cost: insurance premium $I$</td></tr>
<tr><td>Debt: $\mathbf{D_1}$<br>Benefit: deduct tax $\tau$<br>Cost: bankruptcy $\alpha$</td></tr>
<tr><td>Tangible equity: $V-(D+D_1)$</td></tr>
<tr><td>Charter value: $\mathbf{F-V}$</td><td>Equity: E</td></tr>
</table>

**Bank value: $\mathbf{F=D+D_1+E}$**

Figure 1: An Illustration of Bank Liability Structure

the earnings from bank assets such as loans, not including the income from serving deposit accounts. The asset portfolio is risky, and the risk is characterized by the volatility of asset value and denoted by $\sigma$. Notice that $\sigma$ is also the volatility of asset cash flow. We focus on the liability structure for a given portfolio of assets.[14] Our study may be viewed as an analysis of the optimal liability structure of the bank, which has already optimally chosen its asset portfolio. Following Merton (1974) and Leland (1994), we assume that investors have full information about asset value.[15]

Banks take deposits from households and businesses and provide account and liquidity services to depositors. Deposits, the first part on the liability side in Figure 1, are the most important source of funds for banks to finance their assets. Let $D$ denote the amount of deposits that a bank takes. Deposits are rendered safe if banks purchase insurance that guarantees depositors in full. This requires banks to pay an insurance premium, which will be discussed in the next subsection. If deposits are risk-free, the fair interest rate on deposits is the risk-free rate, denoted by $r$. However, banks typically pay lower interest rates on deposits.[16] Depositors

[14]This focus rules out interesting issues of endogenous asset substitution. The literature has pointed out that debt may create incentives to substitute assets with higher risk (e.g., Green, 1984, and Harris and Raviv, 1991) and FDIC insurance may also make for such incentive (e.g., Pennacchi, 2006, and Schneidar and Tornell, 2004). Clearly, modeling the endogenous asset choice along with the liability choice is an important direction for future research.

[15]In reality, active investors use all available information to assess bank asset value and cash-flows although only accounting values of assets are directly observable in quarterly frequency. The full-information assumption sets aside the disparity between accounting value and intrinsic value. We may therefore interpret $V$ as the fair accounting value. If the assets are of same risk category, we may interpret $V$ as the value of risk-weighted assets.

[16]The Banking Act of 1933, known as the Glass-Steagall Act, prohibited banks from paying interest on demand deposits and gave the Fed the authority to impose ceilings on interest rates paid on time deposits. The prohibition

accept a lower interest rate, say $r - \eta_1$ where $\eta_1 > 0$, because customers receive liquidity services associated with maintaining accounts and transacting normal payments. Banks also charge fees for services such as money transfers, overdrafts, etc. Let $\eta_2$ be the banks' fee income on each dollar of deposits. A bank's net liability on deposits is $C = (r - \eta_1)D - \eta_2 D$, excluding deposit insurance premium. Let $\eta = \eta_1 + \eta_2$, which is the net income on each dollar of deposits. The net deposit liability is $C = (r - \eta)D$, excluding deposit insurance premium. The bank's total liability on deposits is $I + C$, where $I$ is the insurance premium. The parameter $\eta$ plays a crucial role in our model of banks. It represents a sacrifice in the required rate of return that the households are willing to accept for the services provided by the bank. This sacrifice is a distinctive character of deposits.

Another important source of banksÕ funding is corporate debt, on which banks do not provide account and payment services. Debt is the second part on the liability side in Figure 1. In our model, debt is a metaphor for all non-deposit liabilities. Debt pays interests until bankruptcy, at which it has a lower priority than deposits in claiming the liquidation value of bank assets. The lower priority potentially protects deposits at bankruptcy. For that reason, certain unsecured debt is treated as Tier 2 capital in bank capital regulation. Debt comes with a cost: its yield contains a credit spread, denoted by $s$, over the risk-free rate to compensate debt holders for bearing the risk of bankruptcy. The credit spread arises endogenously in our model; it depends on the risk of assets and the liability structure of the bank. Thus, a bank's choice of liability structure affects the credit spread, which we will solve endogenously along with the value of debt. The liability on debt is $C_1 = (r + s)D_1$, where $D_1$ is the value of the debt at issuance (face value).

The common equity holders garner all the residual value and earnings of the bank after paying the contractual obligations on deposits and debt. The first slice of value that equity owners lay claim to is the asset value exceeding deposits and debt: $V - (D + D_1)$. This slice, also on the liability side in Figure 1, is referred to as tangible equity or book-value of equity. This is the value equity holders would receive if bank assets are liquidated at fair value and all deposits and debt are paid off at par. A larger book value of equity means a smaller loss for depositors and debt holders after liquidation. Hence, regulators regard it as bank capital of the highest quality, the core Tier 1 capital.

Since equity holders receive all future earnings of the bank, the present value of the future earnings is the bank's charter value, which is the bottom part of the equity in Figure 1. The earnings contain the savings from corporate tax. Since interest expenses are deductible from earnings for tax purposes, the flow of tax savings is $\tau(I + C + C_1)$. The dividend paid to equity

and ceiling of interest rate on deposits were removed after the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Depository Institutions Act of 1983, the latter of which is known as the Garn-St. Germain Act.

holders is the difference between the asset cash flow and the after-tax liability associated with deposits and debt: $\delta V - (1-\tau)(I + C + C_1)$. Since equity value depends on its dividend, it is affected by the liability structure. The triplet $(I, C, C_1)$ therefore characterizes the liability structure of a bank.

## 2.2 The FDIC and Charter Authority

A consequence of borrowing through deposits is the risk that depositors may run, a major challenge commonly faced by banks but not by non-financial firms. As experienced in the crises of the U.S. banking history and theorized in the academic literature, depositors may run from a bank if they believe it has difficulty in repaying their deposits promptly upon their demand. When depositors run, the bank will be closed, unless it is recapitalized to stop the run, and its assets will be liquidated.[17]

The establishment of the FDIC is to deter bank runs by insuring that deposits be paid when a bank closes.[18] With FDIC insurance, a bank is closed by its charter authority, which is typically either the bank's state banking commission or the Office of the Comptroller of the Currency (OCC). The charter authority closes a bank if the bank is insolvent or if the bank's capital is deemed to be too low to be sustainable. For example, a bank is categorized by regulators as critically under-capitalized when the total capital that protects deposits drops to a threshold, which is a fraction $\beta$ of its asset value (say, $\beta = 2\%$).[19] The total capital is the sum of Tier 1 and Tier 2 capital. In our model, it is the sum of tangible equity and debt and amounts to $[V - (D + D_1)] + D_1 = V - D$. Let $V_a$ be the threshold when the charter authority closes the bank. Then, $V_a - D = \beta V_a$ implies $V_a = D/(1-\beta)$. This means that the bank is closed when its asset value reaches $V_a = \kappa D$, where $\kappa = 1/(1-\beta) \geq 1$.

The closure rule in our model may also be intuitively interpreted as capital requirement, the minimum capital for a bank to operate, as modeled in Rochet (2008). Under capital requirement, the charter authority shuts down the bank, when the capital falls below the capital requirement. Capital requirement is typically specified as a ratio to the asset value of the bank. If the requirement of total capital is $\beta = 10\%$, then $V_a = \kappa D$, where $\kappa = 1/(1-\beta) = 1/0.90$.

The FDIC functions as a receiver of the closed banks and also as an insurer of the deposits. As a receiver, the FDIC liquidates the assets of a closed bank in its best effort to pay back

[17]The bank run literature was pioneered by Diamond and Dybvig (1983), who construct a model in which bank run emerges as an equilibrium. The literature has been extended significantly by Allen and Gale (1998) and others.

[18]To deter bank runs during the credit crisis of 2007–2009, the FDIC deposit insurance limit was raised from \$100,000 to \$250,000 on October 3, 2008.

[19]In principle, the FDIC categorizes a bank as critically under capitalized when the total capital drops to 2% of its asset value. The FDIC closes the bank if the bank cannot be recapitalized. For a review of the rules for the list of critically under-capitalized banks, we refer readers to Shibut, Critchfield and Bohn (2003).

the bank's creditors. Suppose the liquidation cost is $\alpha V_a$, proportional to the asset value $V_a$ when the bank is closed. The cost of liquidation by the FDIC may be different from the costs of liquidation through bankruptcy courts. Since the FDIC does not go through the lengthy procedure of bankruptcy, it is likely that the FDIC liquidation cost is smaller than the typical bankruptcy cost in the private sector.[20]

As an insurer, the FDIC pays $D$ to depositors when the bank is closed. The insurance corporation loses $D-(1-\alpha)V_a$ if $(1-\alpha)V_a < D$ or nothing otherwise. The loss function is $[D-(1-\alpha)V_a]^+$, where $[x]^+ = x$ if $x \geq 0$ and $[x]^+ = 0$ if $x < 0$. Since $V_a = \kappa D$, the loss function is positive if $\kappa < 1/(1-\alpha)$, in which case the FDIC expects to suffer a loss after bank closure.[21] To cover the loss, the FDIC charges insurance premium on banks. In 2006, Congress passed reforms that permit the FDIC to charge risk-based premium. For deposit insurance assessment purposes, an insured depository institution is placed into one of four risk categories each quarter, depending primarily on the institution's capital level and supervisory evaluation. Hence, a riskier bank pays higher insurance premium than a safer bank does. Recall that $I$ denotes the deposit insurance premium a bank pays.[22]

If deposits in banks are not insured, the liquidation occurs through bankruptcy courts. The costs of liquidation will include dead-weight losses due to liquidation discount and legal expenses. This can also be modeled as a fraction $\alpha$ of the asset value $V_a$ at bankruptcy. The asset value after bankruptcy is $(1-\alpha)V_a$. Deposits carry with them the risk of bank closure. With full information, it is rational for depositors to run before the bank value drops below $D/(1-\alpha)$. It is also reasonable that depositors may actually wish to run earlier than $D/(1-\alpha)$, worrying about delay of payments when the bank files for bankruptcy. Therefore, we assume that an uninsured bank is closed when its asset value drops to a threshold: $V_a \geq \kappa D$, where $\kappa \geq 1/(1-\alpha)$.

The FDIC and the charter authority together play two important economic roles in our model. First, the FDIC insurance and the charter authority regulation are designed to reduce the probability of bank closure by insuring depositors and preventing a bank run. It prolongs

[20]Title II of the Dodd-Frank Act is perhaps a reflection of the belief that the cost of FDIC liquidation is lower than the cost of bankruptcy procedures. Title II authorizes the FDIC to receive and liquidate failed large financial institutions in order to avoid lengthy and costly bankruptcy procedures, which are supposed to be harmful for the stability of financial system.

[21]In practice, the FDIC always expects a chance of loss because liquidation cost is uncertain. To keep analysis tractable, we assume a fixed $\alpha$ and $\kappa < 1/(1-\alpha)$ so that the FDIC expects a loss.

[22]Until 2010, the FDIC assesses insurance premium based on total deposits. The assessment rate is $a$ such that $I = aD$. There have long been concerns that banks shift deposits out of balance sheet temporally at quarter-ends to lower the assessment base. Since April 2011, the FDIC has changed the assessment base to the difference between the risk-weighted assets and tangible equity, as required by the Dodd-Frank Act (Section 331). If $V$ equals the asset value, the new assessment base equals $D+D_1$, which implies that assessment rate is $b$ such that $I = b(D+D_1)$. The actual premium calculations may also depend on credit rating and the proportion of long-term debt to deposits. See Federal Deposit Insurance Corporation (2011) for more details.

the life of the bank, whereby the bank pays the insurance premium in "good state" when it is solvent in exchange for an indemnity against the loss of deposits in "bad states." The exchange of payments across states improves the value of the bank, which earns more service income and receives additional tax shields. Second, these institutional arrangements are also designed to reduce expected dead-weight losses associated with a bank failure by replacing an uncoordinated bank run with an orderly regulatory closure. The reduction in dead-weight losses also increases the value of the bank.

In section 4.2, we will show that in response to these institutional arrangements, the bank optimally increases leverage, accepting a slightly higher closure probability and a slightly larger dead-weight loss, so that it can take more advantage of service income and tax shields. The higher closure probability and larger dead-weight losses due to the bank's optimal adjustment of its liability structure turn out to be opposite to the original objectives of the FDIC and charter authority. Our analysis suggests that additional restrictions on leverage might be necessary in order to lower expected dead-weight losses in the presence of FDIC and the charter authority.

Equity holders can choose to default before bank closure. Absent bank closure, there is an optimal point for equity holders to default. The default decision maximizes equity value. The optimal default of debt is referred to as *endogenous default* and derived by Leland (1994) for firms without deposits. In section 3.1, we provide the formula of endogenous default in the presence of both deposits and debt. Let $V_d$ be the point of endogenous default, i.e., equity holders choose to default if and only if asset value $V$ reaches or drops below $V_d$, in the absence of bank closure. Then bankruptcy happens if the liabilities of the bank are defaulted by equity holders endogenously or if the bank is to be closed. Mathematically, the point of bankruptcy is $V_b = \max\{V_d\,,\,V_a\}$. When bank assets are liquidated after bankruptcy,[23] depositors are paid first, and debt holders are paid the next if there is value left. Since $\alpha V_b$ is the bankruptcy cost, the payoff to debt holders is $\left[(1-\alpha)V_b - D\right]^+$.

# 3 Valuation and Optimization

Table 1 summarizes the exogenous parameters in the model and the assumptions on them. In the table, service income is positive but with a rate smaller than the risk-free rate: $0 < \eta < r$. Corporate tax is present: $0 < \tau < 1$. The liquidation by the FDIC or at default is costly: $0 < \alpha < 1$. These assumptions are not only realistic but also the requisite mathematical conditions to carry out valuation and optimization.

[23]We can even allow the FDIC liquidation and endogenous default to have different dead-weight losses by letting $\alpha$ be $\alpha_a$ or $\alpha_d$ depending on the type of bankruptcy and still derive the same result. However, we present the model with a single bankruptcy cost $\alpha$ to avoid making notations too complicated.

| Parameter | Notation | Allowed range |
|---|---|---|
| Asset volatility | $\sigma$ | $(0,\infty)$ |
| Asset cash flow | $\delta$ | $[0,\infty)$ |
| Asset value | $V$ | $(0,\infty)$ |
| Risk-free interest rate | $r$ | $(0,\infty)$ |
| Bank service income | $\eta$ | $(0,r)$ |
| Corporate tax benefit | $\tau$ | $(0,1)$ |
| Liquidation cost | $\alpha$ | $(0,1)$ |
| Regulatory closure | $\kappa$ | $[1,\infty)$ |
| Insurance subsidy | $\omega$ | $[0,1)$ |

Table 1: Exogenous parameters are pre-specified, not determined by either valuation or optimization in the model. The allowed ranges are assumptions of the model.

## 3.1 Bank Valuation and Insurance Premium

Since deposits are insured, the value of deposits is the amount of deposits $D$. Because the net liability on deposits is $C = (r-\eta)D$, the value of deposits is related to its liability by $D = C/(r-\eta)$.

The values of debt and equity are affected by the risk of bankruptcy. The Arrow-Debreu state price of bankruptcy plays a key role in our bank valuation. Consider a security that pays \$1 when bankruptcy occurs, and pays nothing otherwise. The price of this security is the state price of bankruptcy. The state price is $P_b = [V_b/V]^{\lambda}$, where $\lambda$ is the positive root of a quadratic equation, which is given by (18) in Appendix. The quadratic equation implies that $\lambda$ is an increasing function of $r$ and a decreasing function of $\delta$ and $\sigma$. If the cash flow of assets is zero, i.e., $\delta = 0$, we have $\lambda = 2r/\sigma^2$, which is proportional to $r$ and inversely proportional to $\sigma^2$. The state price $P_b$ is a solution to Merton's (1974) pricing restriction, which is equation (17) in Appendix.

Bank value depends on its liability structure $(I, C, C_1)$ because the liabilities affect bankruptcy boundary and its state price. The following theorem, derived in Appendix A.1, summarizes the relation between bank value and liability structure.

**Theorem 1** *Given a liability structure $(I, C, C_1)$, the boundary of regulatory closure and the boundary of endogenous default are, respectively,*

$$V_a = \kappa C/(r-\eta) \tag{1}$$

$$V_d = (1-\tau)[\lambda/(1+\lambda)](I + C + C_1)/r\,. \tag{2}$$

*The bankruptcy boundary is $V_b = \max\{V_a, V_d\}$. The equity, debt, and bank values are, respectively,*

$$D_1 = (1 - P_b)C_1/r + P_b\big[(1-\alpha)V_b - D\big]^+ \tag{3}$$

$$E = V - (1-\tau)(1-P_b)(I + C + C_1)/r - P_b V_b \tag{4}$$

$$\begin{aligned} F = {} & V - P_b \min\{\alpha V_b\,,\, V_b - D\} \\ & +\ (1-P_b)\big[C\eta/(r-\eta) + \tau(I + C + C_1) - I\big]/r\,. \end{aligned} \tag{5}$$

In equation (3), debt value is the sum of the expected value of interests before bankruptcy and the expected value of recovery at bankruptcy. In equation (4), equity value is the residual asset value after subtracting the expected after-tax liabilities on insurance, deposits, and debt and the expected value of bankruptcy loss. In equation (5), which is for bank value, the first term is asset value, the second term reflects the value of expected loss in bankruptcy, and the last term shows the value of expected service income and tax benefit after paying insurance premium. All these formulas apply to uninsured banks if we set $I = 0$ and $\kappa \geq 1/(1-\alpha)$.

Theorem 1 shows the role of service income and deposit insurance in bank valuation. Along with the tax benefits of interest expenses, account service income ($\eta$) increases bank value as shown by the last term on the right-hand side of equation (5). The ability of a bank to attract deposits at a rate lower than the risk-free rate comes at a price: the bank has to incur bankruptcy cost if the charter authority closes the bank. Moreover, insurance premium reduces bank value, which is evident in the last term of the equation.

We obtain the endogenous credit spread of debt from Theorem 1. The endogenous credit spread is $s = C_1/D_1 - r$, where $D_1$ is a function of $C_1$ as given by equation (3). The credit spread takes the probability of bankruptcy into account through state price $P_b$; at the same time, the state price is affected by the liability structure. Insurance premium affects the credit spread even though $I$ does not appear in equation (3) explicitly. The premium affects the endogenous default boundary in equation (2), which in turn affects bankruptcy boundary $V_b$ and its state price. The bankruptcy boundary and its state price affect the credit spread directly.

A comparison of our model of banks with the model of non-financial firms in Leland (1994) shows the connection and distinction between banks and non-financial firms. If we set $C = I = 0$ but $C_1 > 0$, the formulas in Theorem 1 reduce to those in Leland (1994) for a firm with only equity and unprotected debt. Leland's capital structure theory is about non-financial firms, which is not applicable to banks that take deposits, earn service income, pay deposit insurance premium, and face the risk of bank closure. Our model extends Leland's model to banks and offers a consistent framework for understanding the similarities and differences between banks and non-financial firms.

While deposit insurance premium is exogenously given in Theorem 1, it should endogenously depend on the amount of deposits under insurance and the risk involved. In principle, an insurance corporation should charge each bank a fair insurance premium. A fair premium makes the insurance contract worth zero to each party of the contract. The next theorem, derived in Appendix A.2, characterizes the fair insurance premium.

**Theorem 2** *Given $D$ dollars of deposits, the fair insurance premium is*

$$I^\circ = r[1-(1-\alpha)\kappa]^+ DP_a/(1-P_a), \tag{6}$$

*where $P_a = [\kappa D/V]^\lambda$ is the state price of bank closure.*

An alternative way to write the insurance pricing equation is

$$(1-P_a)(I^\circ/r) = P_a[1-(1-\alpha)\kappa]^+ D, \tag{7}$$

which says that the expected present value of insurance premium paid to the insurance corporation equals the expected present value of the insurance obligations at bank closure. If $\kappa < 1/(1-\alpha)$, the fair premium $I^\circ$ is positive. It converges to zero as $\kappa$ rises to $1/(1-\alpha)$. If $\kappa \geq 1/(1-\alpha)$, the fair premium is zero because the bank will be closed with enough asset value to cover deposits in full.

The fair insurance premium $I^\circ$ increases with $D$. If deposits increase, not only the insurance premium increases, the assessment rate of insurance premium, which is the premium for insuring one dollar deposits, also increases. By Theorem 2, the assessment rate is

$$h \equiv I^\circ/D = r[1-(1-\alpha)\kappa]^+ P_a/(1-P_a). \tag{8}$$

The rate is increasing with $D$ because $P_a$ is bigger for a larger $D$. The positive relation between $h$ and $D$ makes sense because an expansion of deposits exposes the insurance corporation to a bigger risk.

Some academics have argued that the FDIC does not charge enough insurance premium to cover its risk exposure.[24] A premium lower than the fair rate provides subsidized insurance to banks. To allow for subsidized insurance premium, we assume that the FDIC insurance premium is $I = (1-\omega)I^\circ$, where $\omega = 0$ represents a fair premium and $\omega > 0$ represents a subsidized premium. Relating to the net cash outflow on deposits by $D = C/(r-\eta)$, we have $I = iC$, where

$$i = (1-\omega)[1-(1-\alpha)\kappa]^+[r/(r-\eta)]P_a/(1-P_a). \tag{9}$$

If the FDIC subsidizes deposit insurance, it increases the bank value because the bank pays lower insurance premium for enjoying the risk-free value of deposits. Even with the subsidy, the assessment rate and the total premium a bank pays still endogenously depends on the amount of deposits and the bank's risk profile.

With endogenous insurance premium, a liability structure is characterized by the pair $(C, C_1)$

[24]See Duffie, Jarrow, Purnanandam, and Yang (2003) for evidence of FDIC insurance subsidy. On the other hand, one may argue that a lower premium is necessary to compensate the insured banks for the costs of reporting requirements and tight regulation.

because $C$ determines $D$, which in turn determines $I$. Imposing $I = iC$ in the bank value formula (5), we obtain

$$F = V \ + \ (1-P_b)[\eta/(r-\eta)+\tau-(1-\tau)i]C/r \ + \ (1-P_b)\tau C_1/r \\ - P_b \min\{\alpha V_b\,,\, V_b - D\}. \tag{10}$$

On the right-hand side of equation (10), the second term is the value of tax benefits and account service income from deposits, netted off against the insurance premium. The third term is the value of tax benefits to the bank for its interest expense on debt. The last term is the expected value of bankruptcy loss.

## 3.2 Optimal Liability Structure

Now we proceed to examine how a value-maximizing bank chooses its liability structure. As pointed out earlier, a liability structure of a bank with endogenous insurance premium is described by the pair $(C, C_1)$. An optimal liability structure is the deposit liability $C^*$ and debt liability $C_1^*$ that maximize bank value in equation (10) subject to equation (9). The value-maximizing bank in our framework is fully aware that any decision pertaining to liability structure has a consequence on the FDIC insurance premium. The bank should therefore be mindful of this channel in its choice of liability structure. The endogenous determination of FDIC premium and liability structure captures the feedback channel from the FDIC to the banks and vice versa.

The next theorem, derived in Appendix A.3, provides a characterization of the optimal liability structure for a bank that pays endogenous insurance premium and is subject to regulatory closure.

**Theorem 3** *Suppose* $0 < \eta < r$, $0 < \tau < 1$, $1 < \kappa < 1/(1-\alpha)$, *and the FDIC insurance premium is* $I = (1-\omega)I^\circ$, *where* $I^\circ$ *is defined in Theorem 2 and* $0 \leq \omega < 1$. *A liability structure with* $V_d < V_a$ *is never optimal for an FDIC-insured bank. There exists* $\kappa^* \in [1, 1/(1-\alpha))$ *such that for all* $\kappa \in (\kappa^*, 1/(1-\alpha))$, *the optimal structure is unique and satisfies* $V_d^* = V_a^*$. *In such optimal structure, the state price of bankruptcy is*

$$P_b^* = \frac{1}{1+\lambda} \cdot \frac{\eta(1-\tau)\lambda + r\tau\kappa(1+\lambda)}{\eta(1-\tau)\lambda + r\tau\kappa(1+\lambda) + r(1-\tau)\lambda\{\kappa - 1 + (1-\omega)[1-(1-\alpha)\kappa]^+\}}. \tag{11}$$

*The optimal deposit liability and debt liability are, respectively,*

$$C^* = (r-\eta)VP_b^{*1/\lambda}\big/\kappa \tag{12}$$

$$C_1^* = rVP_b^{*1/\lambda}\left[\frac{1+\lambda}{(1-\tau)\lambda} - \frac{r-\eta}{r\kappa} - (1-\omega)[1/\kappa - (1-\alpha)]^+ \frac{P_b^*}{1-P_b^*}\right]. \tag{13}$$

The theorem characterizes the optimal liability structure of a bank that faces corporate tax, bears the risk of costly bankruptcy, takes deposits to earn service income, and pay endogenously determined insurance premium. Formula (11) shows that the optimal state price $P_b^*$ is a function of the following exogenous parameters: $r$, $\sigma$, $\delta$, $\tau$, $\eta$, $\alpha$, $\kappa$, and $\omega$. Combining this theorem with Theorems 1 and 2, we obtain analytical solutions, which we omit to save space, for the deposits $D^*$, debt value $D_1^*$, equity value $E^*$, bank value $F^*$, bankruptcy boundary $V_b^*$, credit spread $s^*$, and insurance premium $I^*$ in the optimal capital structure of the FDIC-insured bank.

The theorem states that the optimal amount of debt makes the endogenous default boundary coincide with the bank closure boundary. Deposits attract a discount in the deposit rate as well as service fees, in addition to tax savings. The cost of taking deposits is insurance premium and the expected loss due to a bank closure. By contrast, debt brings tax savings but produces no account services or fee income; its cost is the expected loss due to bankruptcy. Therefore, at the margin, the bank should use deposits, not debt, to balance the benefits of leverage with the loss to bankruptcy. With this balance, the bank should take as much debt as possible for availing the tax benefits but should avoid the expected bankruptcy cost resulting from endogenous default. To avoid the expected bankruptcy cost associated with endogenous default, the bank should not set the endogenous default boundary above the bank closure boundary. As a result, the optimal debt should make default occur at exactly the same point at which bank closure takes place.

The services associated with deposits are important for Theorem 3. If $\eta = 0$ while $\tau \in (0, 1)$, the optimal liability structure is not unique. If we set $\eta = 0$ in formulas (11)–(13), we obtain the optimal structure with the maximum deposits, but every $C$ with $0 \leq C \leq C^*$ leads to the same maximum bank value. The optimal capital structure is not unique for a bank with $\eta = 0$ because deposits and debt have the same tax benefits and bankruptcy costs in the absence of account service.[25] However, setting $\eta = 0$ and $C = 0$ in Theorem 3 gives the unique optimal liability structure for a firm that takes no deposits. This corresponds to the unique optimal capital structure of a firm as modeled by Leland (1994). Obviously, a firm holding no deposits and providing no account service is not a bank. Hence, a structural model without considering deposits and bank services is not appropriate for understanding banks' optimal liability structure and leverage.

In theory, if asset volatility $\sigma$ and liquidation cost $\alpha$ are high enough, it is possible for a liability structure with $V_d > V_a$ to be optimal for some low $\kappa$ close to 1. We have confirmed this possibility by both mathematical derivations and numerical optimization. If $\sigma$ and $\alpha$ are very high and $\kappa$ is very low, the fair insurance premium rate $i$ may be very high, making deposits

[25]Without service income, deposits in our model bears resemblance to the secured debt in Leland's (1994) because deposits are protected by bank closure. Leland considers the optimal capital structure of firms that take either protected or unprotected debt but not the optimal mix of the two. He analytically solves for the optimal capital structure of firms that take unsecured debt, but for a firm that takes secured debt, he solves for the optimal structure numerically.

too expensive as source of funds compared to debt. When that happens, reducing deposits to have $V_a < V_d$ may be optimal. Preventing $i$ from being too high is the reason for $\kappa$ to be higher than a threshold $\kappa^*$ in the theorem. Nevertheless, for all the asset volatility and liquidation cost we consider in later sections, we find $\kappa^* = 1$. That is, $V_a^* = V_d^*$ and the formulas in Theorem 3 hold for all $\kappa \in (1, 1/(1-\alpha))$.

Theorem 3 incorporates the endogenous insurance premium in optimal liability structure. Besides considering the tradeoff between tax benefits, account service income, regulatory closure, and bankruptcy costs, banks take the cost of deposit insurance into account. If we set $\kappa \geq 1/(1-\alpha)$, the insurance premium becomes zero. In this case, the formulas in this theorem are valid for uninsured banks. With a general $\kappa$ in this theorem, the assessment rate $(1-\omega)h$ of insurance premium is an increasing function of $D$, and thus $i$ increases with $C$. Therefore, under the assumption of Theorem 3, banks consider both the increase in insurance premium caused directly by the expansion of deposits as well as the increase caused indirectly through the rise of assessment rate. In section 5.2, we will show the impact of endogenous insurance premium on banks' optimal choice of liability structure.

Corporate tax plays an important role in Theorem 3. The empirical relevance of taxes in bank capital structure has been examined by Schepens (2014) who exploits a recent change of the tax code in Belgium, which permitted tax advantages to equity for the first time (thereby reducing the tax discrimination between bank debt and bank equity). He found that the changes in tax led the banks to issue more equity and improved their capital ratios. This evidence shows that banks do respond to tax changes by altering their liability structure.

If there is no corporate tax, the bank can maximize its value without issuing debt. In the absence of tax benefits, increasing debt from zero does not alter bank value, and thus the optimal level of debt is indeterminate. However, if a bank finances its operations only by taking deposits, then there is a unique optimal level of deposits when tax rate is zero, as stated in the theorem below.

**Theorem 4** *Suppose $\tau = 0$ and $0 < \eta < r$. The optimal liability structure is unique for an uninsured bank that finances its operations by deposits and equity. In the optimal liability structure, the bank closure boundary is higher than the endogenous default boundary: $V_a^* > V_d^*$. The state price of bankruptcy is*

$$P_a^* = \frac{1}{1+\lambda} \cdot \frac{\eta}{\eta + r(\kappa - 1)}, \tag{14}$$

*and the optimal deposit liability is*

$$C^* = (r-\eta)V P_a^{*1/\lambda}/\kappa. \tag{15}$$

The proof of this theorem is shown in Appendix A.4. This optimal liability structure in the absence of corporate tax is especially relevant to banks as the motive to serve deposits arises from liquidity provision, a feature that is unique to banks.

Our result in Theorem 4 may help explain why banks in the 19th century used substantial leverage in the form of deposits when there were no tax advantages or deposit insurance, as documented by Calomiris and Carlson (2015). Theorem 4 shows that leverage is desired by banks despite the absence of corporate tax benefits.

Moreover, the inequality $V_a^* > V_d^*$ in Theorem 4 explains why equity holders never chose to default before bank closure as observed in history. A bank described in this theorem provides account and liquidity services on deposits but receives no tax benefit. For this reason, tax benefit is typically ignored in the literature that focuses on bank account services. Models of banks without tax benefit and debt are nevertheless largely inconsistent with the typical bank capital structure in today's world.

# 4 A Quantitative Illustration

The model just developed paves a way to characterize quantitatively the optimal bank liability structure. To provide numerical examples, we first need to specify exogenous parameters in our model. While the theoretical range of the exogenous parameters are as wide as those listed in Table 1, the practical range of the parameters should be much narrower. For the numerical illustration in this section and the analysis of comparative statics in the next section, we choose ranges that are practically conceivable and interesting.

## 4.1 A Choice of Exogenous Parameters

As our model inherits the major advantage of structural models that coherently connects the risk of debt and equity to the risk of assets, the risk comes from asset volatility ($\sigma$), which is one of the most important parameters to affect bank leverage and liability structure. Since asset volatility is not directly observable, asset volatility is typically inferred from accounting data and market prices. Moody's KMV provides estimates of asset volatility for a large number of companies across a wide range of industries.

In panel A of Figure 2, we present the average, median, and the 10- and 90-percentiles of Moody's estimates of bank asset volatilities. As a comparison, in panel B we present Moody's estimates of manufacturing-firm asset volatilities. The figure shows a difference between the assets held by banks and those owned by manufacturing firms: bank assets have much lower volatility. The average asset volatility is around 10% for banks, whereas it is 40 $\sim$ 50% for manufacturing firms. Although bank asset volatility fluctuates over time, the median is around 5% for 2001–2012. The 90 percentile of bank asset volatilities is well below 15% for 2001–2007, and it stays below 25% even for the period of 2007–2012. In view of these facts, we let

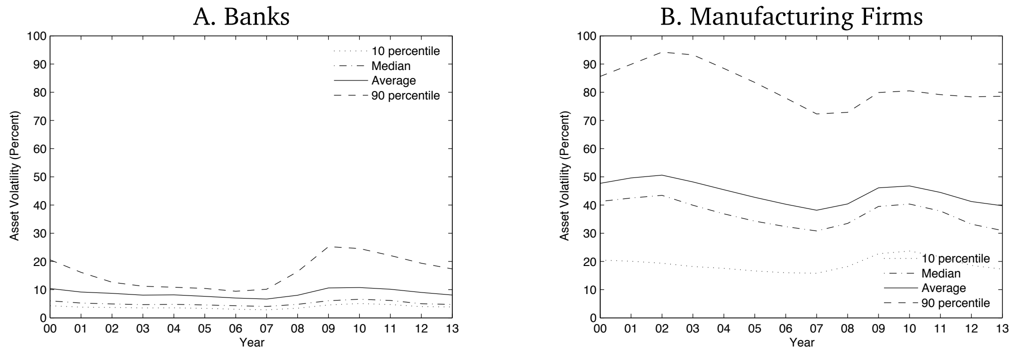


Figure 2: Plots of the average, median, and 10- and 90-percentiles of asset volatilities of banks (panel A) and manufacturing firms (panel B) from 2000 to 2013. Moody's KMV Investor Service provided the estimates of asset volatilities.

$\sigma \in [0.03, 0.20]$ in our study of comparative statics.

Another parameter of bank assets is its rate of cash flows ($\delta$). If the assets contain only commercial and consumer loans, the cash flows are interest and principal payments of the loans. In the numerical illustration and comparative statics, we set the cash-flow rate to 8%, which is the average mortgage rate in the U.S. during 1984–2013. Correspondingly, we also set the risk-free rate to the average federal funds rate during the same period; this gives $r = 5\%$. We choose this period because we would like to make our numbers broadly comparable to the aggregate balance sheet data of FDIC-insured commercial banks and savings institutions. The FDIC made the balance sheet data for this period available, and both the mortgage rate and federal funds rate data are obtained from Table H.15 published by the Federal Reserve.

The income from deposit services is important in bank liability structure. The net income from deposit services should be determined in a competitive market for deposits. In a perfect competitive market with free entry, the net income would be driven to zero, or it should just cover the insurance premium if a bank has deposit insurance. At least in the U.S., new entry of banks into the market is regulated by charter authorities.[26] Without free entry, deposit rents arise from the market power enjoyed by the bank (De Nicolo and Rurk Ariss, 2010). The profitability should depend on the amount of deposits and the bank. Thus, parameter $\eta$ may differ across banks and should be a function of $D$. We do not explicitly model the market equilibrium of deposits or the demand function of deposits, $D(\eta)$, in order to keep the model tractable and to focus on the choice of liability structure. We instead assume $\eta$ to be a constant but allow it to be different across banks. A range of values, $\eta \in [0.02, 0.04]$, are examined in our analysis of comparative statics.

[26] Founders of a new bank have to show their integrity and ability to manage the bank. In addition, the regulators demand evidence of need for a new bank before granting a charter. Peltzman (1965) documents the restriction on entry of commercial banking. Jayaratne and Strahan (1998) examines the effects of entry restrictions on bank efficiency.

Since a benefit of leverage is the tax deductibility of interest expenses, corporate tax rate is an important parameter in determining liability structure. The statutory corporate tax rate in the U.S. ranges up to 35%. The U.S. Department of Treasury (2007) reports that the effective marginal tax rate on investment in business varies substantially by business sectors. The academic literature suggests that the effective corporate tax rate is around 10% for non-financial firms (Graham, 2000) but can be more important for banks (Heckemeyer and Mooij, 2013). In our analysis of comparative statics, we consider a wide range, $\tau \in [0.01, 0.20]$.

Bankruptcy cost is an important countering force to the benefit of leverage, but the task of measuring the cost has always been a challenge. A well-known reference is the study of Altman (1984), which examines a sample of 19 industrial firms that went bankrupt over the period of 1970–1978. The estimated bankruptcy cost is 19.7% of the firm's asset value just prior to its bankruptcy. Bris, Welch and Zhu (2006), however, show that bankruptcy cost varies across firms and ranges between 0% and 20% of firm assets. Banks experienced higher bankruptcy costs. Based on 791 FDIC-regulated commercial banks failed during 1982–1988 (the Savings and Loan Crisis), James (1991) estimates that bankruptcy cost is 30% of a failed bank's assets. Based on 325 insured depository institutions failed during 2008–2010 (the Great Recession), Flannary (2011) estimates that bankruptcy cost is about 27% of a failed bank's assets. In light of these estimates, we choose 27% as the mid-point of the range. The range we consider is $[0.17, 0.37]$.

The other exogenous parameters are as follows. A state banking regulatory agency closes a bank when it is unable to meet its obligations to depositors. The parameter $\kappa$ should not be lower than 1. When a bank's total capital is less than 2% of its assets, the FDIC classifies it as "critically undercapitalized," and the charter authority typically closes the bank. In view of these institutional arrangements, we set the regulatory closure rule as $\kappa = 1/(1-0.02) \approx$ 102%. For insurance subsidy ($\omega$), we examine a wide range from zero to 40%.

## 4.2 An Example of Endogenous Liability Structure

The optimal liability structure is characterized by a set of ratios endogenously determined by bank management.[27] The first endogenous variable of our interest is the ratio of deposits to assets, $D/V$. The next is the ratio of debt to assets, $D_1/V$. The leverage of the bank is measured by the ratio of tangible equity to assets, which is referred to as Tier 1 ratio in bank regulation. Since the tangible equity is $V-D-D_1$, the ratio of tangible equity to assets is $(V-D-D_1)/V$. Higher leverage corresponds to lower tangible equity.[28]

[27] Even though the amount of deposits is partially determined by the supply in the markets, banks have control of the ratios because they can adjust the level of assets to achieve their desired capital structure.

[28] It is useful to point out that tangible equity of a bank can be negative in practice. In its December 2011 filing, last time as a bank holding company, the U.S. operations of Deutsche Bank had total assets of $355 billion and

The amount of deposits and the level of debt determine several other endogenous variables. They determine the boundary of bank closure, $V_a/V$, and the boundary of endogenous default, $V_d/V$, relative to asset value. The two boundaries determine the distance to bankruptcy. The higher of the two is the bankruptcy boundary $V_b/V$, which influences the expected value of bankruptcy loss, $P_b\alpha V_b/V$, another endogenous variable of our interest. The possibility of bankruptcy causes the bank to pay a credit spread, $s$, on debt. The credit spread is observable in the market. The insurance premium $I$ is also endogenously determined by the liability structure. The income from serving deposits and the tax benefits of interest expenses create charter value of the bank, which is the difference between bank value and asset value. We measure charter value as a percent of asset value: $(F-V)/V$.

Table 2 presents the optimal liability structure and related endogenous variables for an FDIC-insured bank and an uninsured bank, as well as a non-financial firm that does not take deposits. The parameters used for generating the optimal liability structure are chosen from the ranges discussed in section 4.1. We later vary these parameters to examine the effect of each.

| Endogenous optimal ratio | Mathematical definition | FDIC-insured bank | Uninsured bank | Non-financial firm |
|---|---|---|---|---|
| Deposit | $D/V$ | 45.30 | 32.22 | 0.00 |
| Debt | $D_1/V$ | 46.48 | 52.07 | 60.03 |
| Tangible equity | $(V-D-D_1)/V$ | 8.23 | 15.71 | 39.97 |
| Closure boundary | $V_a/V$ | 46.20 | 44.14 | 0.00 |
| Default boundary | $V_d/V$ | 46.20 | 44.14 | 35.28 |
| Bankruptcy loss | $P_b\alpha V_b/V$ | 3.89 | 3.47 | 1.98 |
| Credit spread | $s$ | 2.27 | 2.05 | 0.75 |
| Insurance premium | $I/V$ | 0.24 | 0.00 | 0.00 |
| Charter value | $(F-V)/V$ | 24.50 | 19.42 | 6.23 |

Table 2: The optimal liability structures in an FDIC-insured bank, and an uninsured bank, and a non-financial firm that does not take deposits. All values are reported in percentage points. In the calculation of endogenous variables, the exogenous parameters are $\sigma=0.05$, $\eta=0.03$, $\tau=0.15$, $\omega=0.1$, and $\alpha=0.27$. In addition, $\kappa=1/0.98$ for the FDIC-insured bank, but $\kappa=1/(1-\alpha)$ for the uninsured bank.

Table 2 reveals some distinctive characteristics of bank optimal liability structure. The most striking is the high level of leverage in the FDIC-insured bank, which is typical in banks but not common in non-financial firms. In the FDIC-insured bank, the optimal deposit-to-asset ratio is 45.30%, and the optimal debt-to-asset ratio is 46.48%. This liability structure leaves tangible equity to be only 8.23% of asset value. Although this liability structure is for illustrating our theory, it is broadly comparable to the average liability structure of FDIC-insured banks. In Table 3, we present the statistics of liability structure of all FDIC-insured commercial banks and savings institutions from 1984 to 2013.[29]

Tier 1 capital of negative $5.68 billion.

[29]Since our model considers only two kinds of leverage: deposits that can run and earn service/liquidity pre-

In Table 2, we also consider a hypothetical bank that operates without deposit insurance and regulatory closure. In practice, many non-U.S. banks face the risk of a bank run because their governments do not provide deposit insurance.[30] Some U.S. banks are also not covered by FDIC deposit insurance. In theory, an uninsured bank serves as a counterfactual for the majority of U.S. banks, which are covered by FDIC insurance and subject to regulatory closure.

| | Average | Median | StDev | Maximum | Minimum |
|---|---|---|---|---|---|
| Demand/Savings Deposits | 45% | 44% | 5% | 59% | 39% |
| Other Liabilities | 47% | 49% | 6% | 53% | 30% |
| Total Liabilities | 92% | 92% | 2% | 95% | 89% |
| Total Equity | 8% | 8% | 2% | 11% | 5% |

Table 3: Statistics of the aggregate liability structure of all FDIC-insured commercial banks and savings institutions from 1984 to 2013. Source: The Federal Deposit Insurance Corporation.

Consideration of uninsured banks helps answer an important question: are banks fundamentally different from non-financial firms regardless of FDIC insurance and regulatory closure? This question is also important for understanding the effects of deposit insurance and regulatory closure, which will be discussed in the next subsection. Table 2 shows that high level of leverage is optimal for a bank even without deposit insurance. The sum of deposits and debt amounts to about 84.29% of the asset value, leaving tangible equity to be only 15.71% of the asset value. This means that the FDIC insurance is not the sole reason for high leverage of banks.

High leverage in uninsured banks is likely related to a fundamental factor that distinguishes banks from other firms: taking deposits to provide account and liquidity services, besides earning cash flows from low-volatility assets such as loans. Without the insurance and regulation of FDIC, the optimal liability structure of the uninsured bank in Table 2 consists of substantial amount of deposits, which is 32.22 percent of asset value, although is less than the comparable insured bank. By contrast, the uninsured bank takes up more debt, which is 52.07 percent of the asset value.

A noticeable characteristic of the optimal liability structures in Table 2 is the significance of debt.[31] The optimal debt is 46.48% of the asset value in the insured bank (and 52.07% in the

mium and debt that cannot run or earn service/liquidity premium, Table 3 distinguishes only demand and savings deposits from the rest of bank liabilities. The actual liability structures of banks are far more complex than the structure presented in the table. The complexity of full array of bank leverage is beyond the scope of this paper.

[30] In September 2007, Northern Rock, a U.K. Bank, experienced a run on its deposits, and had to be nationalized in 2008. See Shin (2008) for a cogent analysis of the Northern Rock bank run. Bank runs have happened in Europe and Asia even after the recent financial crisis. In 2010, depositors "ran" from two Swedish banks, Swedbank and SEB, and a Chinese bank, Jiangsu Sheyang Rural Commercial Bank. In 2013, depositors ran from Cypriots banks and forced the country to close its banks for many days. In 2014, depositors ran from two Bulgarian banks, Corporate Commercial Bank and First Investment Bank.

[31] In practice, debt has always been an important source of funding for banks. Avdjiev, Katasheva and Bogdanova

uninsured bank) in Table 2. As noted in Theorem 3, banks optimally set debt to a level such that the default boundary is exactly the same as the closure boundary. With this strategy, banks maximize tax deduction without further increasing the probability of bankruptcy. In Table 2, the endogenous default boundary is exactly same as the closure boundary of the bank.

In order to better understand the nature of bank liability structure, Table 2 presents in the last column the optimal leverage of a non-financial firm that does not take deposits. The calculation of optimal leverage in the firm uses the same parameters as those used for banks. Without taking deposits, the firm drastically reduces leverage. The tangible equity in the firm is nearly 40 percent of asset value, much higher than the tangible equity in either the insured bank or the uninsured bank in the table. Debt in the firm is slightly higher than debt in the comparable bank. This level of firm leverage is still rather high, but this is due to the low volatility for bank assets in Table 2. Recall that Figure 2 shows that the median asset volatility of manufacturing firms ranges from 30% to 50%. If we increase asset volatility to 30%, the optimal debt-to-asset ratio in the firm reduces to 45%, and the tangible equity ratio rises to 55%, which is typical for non-financial firms.

A major objective of FDIC is to reduce the probability of bank failure. The optimal closure boundary of the FDIC-insured bank is 46.20% of asset value, as shown in Table 2. This boundary is slightly higher than the 44.14% bank closure boundary of the comparable uninsured bank in the table. The higher closure boundary leads to a larger expected bankruptcy loss. In the table, the expected bankruptcy loss of the FDIC-insured bank is 3.89% of asset value, slightly larger than the 3.47% in the comparable uninsured bank. The larger expected bankruptcy loss results in 22-bps increase in credit spread. By contrast, the credit spread for the debt in the firm that does not take deposits is only 0.75%. The higher closure boundary, higher expected bankruptcy loss, and higher credit spread of insured banks are not necessarily consonant with the FDIC mandates. This outcome is a result of the bank's optimal response when it counteracts the intended impact of the FDIC by taking more deposits.

FDIC insurance plays a part in driving banks to high leverage. In Table 2, the tangible equity ratio in the insured bank is 8.23%, substantially lower than the 15.71% ratio in the comparable uninsured bank. With deposit insurance, the bank holds more deposits and less debt, but its total leverage is higher. Intuitively, deposit insurance, accompanied by regulatory closure, allows the insured bank to take more deposits and keep the closure boundary from becoming too high. Thus, a major benefit banks receive from the FDIC is that it prevents a sharp increase in the probability of bank closure when they increase deposits. With this benefit, the banks use more leverage.

Although driving up bank leverage, the FDIC brings two benefits to the banking industry,

(2013) report that during the 2009–2013 period alone, banks around the world have issued \$4.1 trillion of unsecured long-term debt.

according to our model. The first benefit is to allow banks to provide more deposit services. In Table 1, the amount of deposits in the FDIC-insured bank is 45.30% of asset value, 13.08 percentage points higher than the deposits in the comparable uninsured bank. The second benefit is to increase the value of banks. The charter value of the FDIC-insured bank in Table 2 is 24.50% of asset value, about 5 percentage points higher than the value of the comparable uninsured bank. Only a small part of the increase in charter value is due to the subsidy of FDIC insurance. If we assume that the FDIC charges a fair insurance premium ($\omega = 1$), the charter value would still be about 24% of asset value. The FDIC increases bank value because it allows the bank better take advantage of the income from serving deposits.

# 5 Comparative Statics

Bank optimal liability structure depends on the characteristics of bank business. The important characteristics are the presence of deposit service income, the low asset volatility, and the bankruptcy cost associated with liquidation by the FDIC. In Section 5.1, we examine the effects of these three factors by considering various values of $\eta$, $\sigma$, and $\alpha$.

Keeping the bank business characteristics fixed, the FDIC policies on insurance and regulatory closure also affect the optimal bank liability structure. The FDIC has modified its policies multiple times and may make more changes in the future. Optimal responses of bank liability structure to policy changes are especially important in understanding the effects of the FDIC and other regulations because the responses may counteract the intended objectives of the regulators. In Section 5.2, we re-calculate the optimal bank liability structure under various assumptions about bank closure policy ($\kappa$) and insurance subsidy ($\omega$).

Another well-known reason for leverage in all firms, not just in banks, is tax deductibility of interest expenses. Although the tax benefit of leverage is not a unique reason for banks to be different from other companies, corporate tax is more important for bank liability structure because banks use more leverage than non-financial firms do. Observing the importance of tax benefit to banks, several recent papers have attempted to measure empirically the link between leverage and taxes. These papers include Heckemeyer and de Mooij (2013), Keen (2011), Schepens (2013), and Schandlbauer (2013). In Section 5.3, we look at the optimal bank liability structure under alternative rates of tax benefits.

## 5.1 Effects of Bank Business Characteristics

Service income appears to be a driver of bank leverage, as shown in Table 4. The tangible equity is lower in a bank with higher service income. When the service income rate changes from 2 percent to 4 percent, the tangible equity-to-asset ratio decreases from 16.52% to 0.55%.

The optimal leverage increases with the service income because not only the optimal amount of deposits goes up and the optimal debt also goes up. It is interesting to notice that an increase in service income rate does not have a substitution effects between deposits and debt. They both went up because the additional deposits raise the closure boundary, giving more room for debt. As a result, the optimal liability structure consists of more deposits and debt if the service income is higher.

| Endogenous | Account service income rate ($\eta$) | | | | |
|---|---|---|---|---|---|
| optimal ratio | 2.00 | 2.50 | 3.00 | 3.50 | 4.00 |
| Deposit | 43.52 | 44.50 | 45.30 | 45.96 | 46.53 |
| Debt | 39.96 | 43.23 | 46.48 | 49.71 | 52.93 |
| Tangible equity | 16.52 | 12.27 | 8.23 | 4.33 | 0.55 |
| Closure boundary | 44.39 | 45.39 | 46.20 | 46.88 | 47.46 |
| Default boundary | 44.39 | 45.39 | 46.20 | 46.88 | 47.46 |
| Bankruptcy loss | 3.52 | 3.72 | 3.89 | 4.04 | 4.16 |
| Credit spread | 2.08 | 2.18 | 2.27 | 2.34 | 2.40 |
| Insurance premium | 0.21 | 0.22 | 0.24 | 0.25 | 0.26 |

Table 4: Effects of account service income on the optimal structure of an FDIC-insured bank. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $\eta$ varies, the other parameters are fixed at $\sigma = 0.05$, $\tau = 0.15$, $\omega = 0.1$, $\alpha = 0.27$, and $\kappa = 1/0.98$

The effects of account service income on bank leverage is consistent with DeAngelo and Stulz (2014), who suggest that premium on liquidity production is a reason for high leverage in banks that take deposits, although the banks in their paper do not have FDIC insurance and regulation. A major distinction of our analysis from DeAngelo and Stulz's is that their banks do not use debt to finance assets, whereas our model predicts that the optimal debt is also positively related to the service income.

It is important to observe that the credit spread is positively related with the account service income rate. The bank regulations during 1930s through 1970s prohibited banks from paying interests on demand and savings deposits and limited bank competition. It was thought that making deposit service more profitable would reduce the probability of bank failure. This way of thinking ignores bank's optimal response to profitability of deposit service. Table 4 shows that when deposit service income is higher, bank's optimal response is to take more deposits, as well as debt. This raises the closure boundary, which increases the probability of bank failure. As a result, expected bankruptcy loss is higher, and credit spread is higher, opposite to what the early regulators thought. The comparative statics of the account service income further underscores the importance of incorporating banks' optimal response.

Bank asset volatility is typically low, as noted previously, and low asset volatility appears to be an important driver of bank leverage, as shown in Table 5. If volatility is as low as 3%, the optimal tangible equity is only 6.75%. By contrast, for an asset volatility of 20%, the tangible

| Endogenous | Asset volatility ($\sigma$) | | | | |
|---|---|---|---|---|---|
| optimal ratio | 3.00 | 4.00 | 5.00 | 10.00 | 20.00 |
| Deposit | 46.28 | 45.84 | 45.30 | 41.90 | 35.00 |
| Debt | 46.96 | 46.74 | 46.48 | 44.82 | 41.54 |
| Tangible equity | 6.75 | 7.42 | 8.23 | 13.29 | 23.45 |
| Closure boundary | 47.21 | 46.75 | 46.20 | 42.74 | 35.70 |
| Default boundary | 47.21 | 46.75 | 46.20 | 42.74 | 35.70 |
| Bankruptcy loss | 3.82 | 3.85 | 3.89 | 4.10 | 4.38 |
| Credit spread | 2.14 | 2.20 | 2.27 | 2.76 | 4.17 |
| Insurance premium | 0.23 | 0.23 | 0.24 | 0.27 | 0.34 |

Table 5: Effects of asset volatility on the optimal liability structure of an FDIC-insured bank. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $\sigma$ varies, the other parameters are fixed at $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.1$, $\alpha = 0.27$, and $\kappa = 1/0.98$.

equity ratio is 23.45%. A bank with 20% asset volatility takes much less deposits, which is 35% of asset value. In view of the inverse relationship between volatility and leverage, a manufacturing company with asset volatility higher than 30% or 40% is unlikely to use the level of leverage that banks use. Thus, low asset volatility is crucial for banks to use high leverage. When a high leverage consists of a large amount of deposits, it allows a bank to engage in service on deposits as its major business. Therefore, low asset volatility is important for banking business.

| Endogenous | Cost of FDIC Liquidation ($\alpha$) | | | | |
|---|---|---|---|---|---|
| optimal ratio | 17.00 | 22.00 | 27.00 | 32.00 | 37.00 |
| Deposit | 47.87 | 46.54 | 45.30 | 44.14 | 43.05 |
| Debt | 48.24 | 47.34 | 46.48 | 45.64 | 44.84 |
| Tangible equity | 3.90 | 6.12 | 8.23 | 10.22 | 12.11 |
| Closure boundary | 48.83 | 47.47 | 46.20 | 45.02 | 43.91 |
| Default boundary | 48.83 | 47.47 | 46.20 | 45.02 | 43.91 |
| Bankruptcy loss | 2.81 | 3.39 | 3.89 | 4.32 | 4.69 |
| Credit spread | 2.56 | 2.41 | 2.27 | 2.14 | 2.03 |
| Insurance premium | 0.17 | 0.21 | 0.24 | 0.26 | 0.28 |

Table 6: Effects of liquidation costs on the optimal liability structure of an FDIC-insured bank. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $\alpha$ varies, the other exogenous parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.1$, and $\kappa = 1/0.98$.

Table 6 demonstrates the effects of liquidation cost on bank optimal liability structure. We vary liquidation cost $\alpha$ in a range from 17% to 37%. Both deposits and debt vary inversely with liquidation cost. The tangible equity ratio is lower if liquidation costs less. The higher leverage has a consequence in credit spread. Without considering the optimal response of banks, one would expect credit spread to be lower if liquidation cost is lower. To the contrary, credit spread goes up slightly. The apparently counterintuitive change in the credit spread is due to the optimal increase of leverage.

## 5.2 Effects of Closure Rule and Insurance Subsidy

We first consider the potential effects of changing the regulatory closure rule. In fact, the Dodd-Frank Act requires bank regulators to consider increasing the threshold for banks to be closed. In the meantime, Basel III and new rules in the U.S. increased the capital requirement for banks to operate. Recall that the closure rule in our model may be interpreted as a capital requirement, which is the minimum capital for a bank to operate, as modeled in Rochet (2008). Then, an increase in $\kappa$ can be interpreted as a raise in capital requirement.

| Endogenous | Regulatory Closure Rule ($\kappa$) | | | | |
|---|---|---|---|---|---|
| optimal ratio | 100.00 | 102.00 | 110.00 | 120.00 | 130.00 |
| Deposit | 46.35 | 45.30 | 41.52 | 37.55 | 34.23 |
| Debt | 45.99 | 46.48 | 48.18 | 49.90 | 51.27 |
| Tangible equity | 7.66 | 8.23 | 10.30 | 12.55 | 14.50 |
| Closure boundary | 46.35 | 46.20 | 45.67 | 45.06 | 44.50 |
| Default boundary | 46.35 | 46.20 | 45.67 | 45.06 | 44.50 |
| Bankruptcy loss | 3.92 | 3.89 | 3.78 | 3.65 | 3.54 |
| Credit spread | 2.28 | 2.27 | 2.21 | 2.15 | 2.09 |
| Insurance premium | 0.26 | 0.24 | 0.16 | 0.09 | 0.03 |

Table 7: Effects of regulatory closure rule on the optimal liability structure of an FDIC-insured bank. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $\kappa$ varies, the other exogenous parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.1$, and $\alpha = 0.27$.

In Table 7, we vary $\kappa$ from 100% to 130%. Tightening the closure rule reduces bank leverage. As $\kappa$ changes from 100% to 130%, optimal tangible equity increases from 7.66% of asset value to 14.50%. The reduction of leverage is due to the drop in deposits: the deposit-to-asset ratio drops from 46.35% to 34.23%. Therefore, tightening of closure rule causes banks to reduce deposit services. In the meantime, it causes banks to increase debt from 45.99% to 51.27% of the asset value.

Perhaps a striking observation in Table 7 is that the closure boundary in the optimal liability structure actually decreases if regulators tighten the closure rule. As $\kappa$ increases from 102% to 110%, the closure boundary gets lower, dropping from 46.20% to 45.67% of asset value. This reverse relation between the closure boundary and closure rule is due to the optimal response of bank liability structure. For a higher $\kappa$, banks take fewer deposits. The drop of deposits is so large that it entirely counteracts the increase of $\kappa$.

Figure 3 shows the influence of optimal response on the relation between the closure boundary and the closure rule. In the figure, a bank first optimizes its liability structure for $\kappa = 102\%$, and then $\kappa$ changes. If the bank does not adjust its liability structure for the change in $\kappa$, the

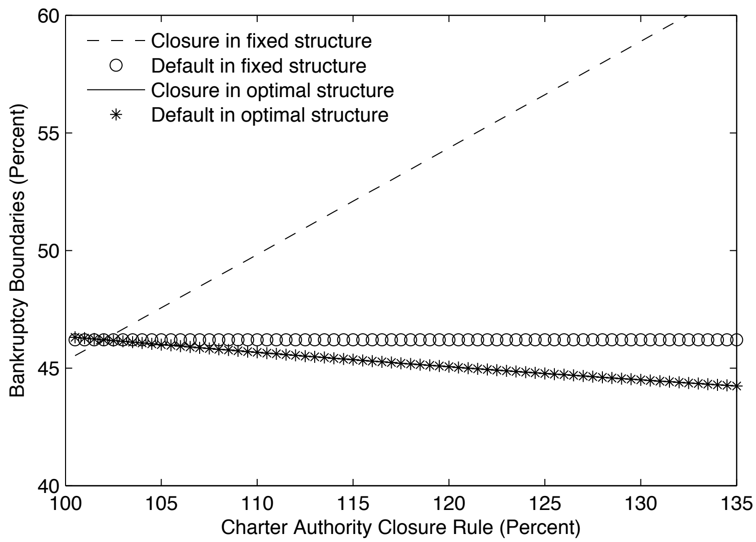


Figure 3: The bankruptcy boundary of an FDIC-insured bank. The bank first optimizes its liability structure for $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.1$, $\alpha = 0.27$, and $\kappa = 1.02$. When the closure rule $\kappa$ varies in the range from 100% to 135%, the capital structure is either kept fixed or re-optimized for new $\kappa$. The panel plots the closure boundary $V_a/V$ and default boundary $V_d/V$ for each value of $\kappa$ in the fixed or re-optimized liability structure.

| Endogenous | Insurance Subsidy ($\omega$) | | | | |
|---|---|---|---|---|---|
| optimal ratio | 0.00 | 10.00 | 20.00 | 30.00 | 40.00 |
| Deposit | 44.64 | 45.30 | 45.98 | 46.68 | 47.42 |
| Debt | 46.01 | 46.48 | 46.95 | 47.44 | 47.94 |
| Tangible equity | 9.35 | 8.23 | 7.07 | 5.88 | 4.65 |
| Closure boundary | 45.54 | 46.20 | 46.90 | 47.62 | 48.37 |
| Default boundary | 45.54 | 46.20 | 46.90 | 47.62 | 48.37 |
| Bankruptcy loss | 3.75 | 3.89 | 4.04 | 4.20 | 4.36 |
| Credit spread | 2.20 | 2.27 | 2.34 | 2.42 | 2.51 |
| Insurance premium | 0.25 | 0.24 | 0.22 | 0.20 | 0.18 |

Table 8: Effects of insurance on the optimal liability structure of an FDIC-insured bank. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $\omega$ varies, other exogenous parameters are fixed: $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\kappa = 1.02$, and $\alpha = 0.27$.

closure boundary, $V_a = \kappa D$, should be a linear function of $\kappa$ plotted as a dashed line in the figure, and the default boundary $V_d$ should be independent of $\kappa$, as marked by circles in the figure. If the bank optimally adjusts its liability structure to the change in $\kappa$, however, the relation between the closure boundary and $\kappa$ is completely different; the closure boundary decreases as $\kappa$ increases. The closure boundary $V_a^* = \kappa D^*$ in the optimal liability structure is not a linear function of $\kappa$ anymore because the bank optimally reduces deposits in response to the increase in $\kappa$. The endogenous default boundary $V_d^*$ is related to $\kappa$ the same way as $V_a^*$ because $V_d^* = V_a^*$ for all $\kappa$, as shown in the figure.

In the preceding analysis, we assume that the subsidy in insurance premium is $\omega = 10\%$. Under this assumption, the insurance premium of the insured bank in Table 2 is 24 bps on assets. In Table 8, we consider various magnitudes of the subsidy, ranging from zero to 40%, and look at its impact on the optimal liability of insured banks. Clearly, the FDIC subsidy encourages bank leverage. As the subsidy increases from zero to 40%, tangible equity ratio drops from 9.35% to 4.65%. More interestingly, the subsidy not only increases the optimal deposit-to-asset ratio but also increases the optimal debt ratio. While the deposit ratio goes up from 44.64% to 47.42%, the debt ratio rises from 46.01% to 47.94%. In the meantime, both the closure boundary and the credit spread are higher. As expected, though, the insurance premium drops as the insurance is more subsidized. These effects of insurance subsidy appear to support the FDIC reforms in its effort to charge banks fair insurance premium.

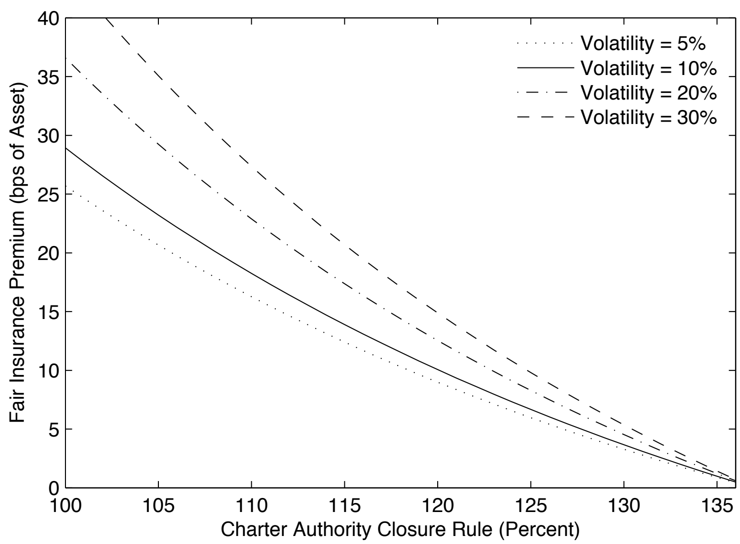


Figure 4: Insurance premium, closure rule, and volatility. An insured bank optimizes its liability structure for each $\kappa$, ranging from 100% to 135%, and each $\sigma$, taking values of 5%, 10%, 15%, and 25%. The figure plots the fair insurance premium in the optimal liability structure for various values of $\kappa$ and $\sigma$. The other exogenous parameters are fixed at $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.1$, and $\alpha = 0.27$.

The fair insurance premium depends on how risky the bank assets are and how early the charter authority closes the bank, besides the liability structure of the bank. Figure 4 shows how the insurance premium for the optimal structure depends on its two important factors: the closure rule and asset volatility. The range of insurance premium in the figure is broadly in line with the assessment rates published by the FDIC. The FDIC (2011) reports that the initial base assessment rates are 12–16, 22, 32, 45 bps for banks in four risk categories. The insurance premium is a cost to the bank, but the insured bank is able to benefit from FDIC insurance because of its optimal response, which in turn reduces the effect of FDIC on the bank's probability of bankruptcy. The endogenous relation between insurance premium and liability structure has been overlooked in the literature.

## 5.3 Effects of Corporate Tax Benefits

Our model offers a coherent framework to examine the link between tax rate and leverage. The asset value $V$ in Leland's (1994) model, and also in our model, is the value of an all-equity firm that owns the assets and faces corporate tax. In other words, $V$ is the after-tax value of assets. If the before-tax value of assets is $V^*$, then the after-tax value is $V = (1 - \tau)V^*$. If the government lowers corporate tax rate from $\tau$ to $\tau'$, the after-tax value of assets should be higher. Let $V'$ be the after-tax value under corporate tax $\tau'$. Then, the after-tax values of assets in the two tax regimes are related by $V' = V(1 - \tau')/(1 - \tau)$. Incorporating the effect of a tax rate change on the asset value is necessary when analyzing the relation between tax rate and capital structure, as Goldstein et al. (2001) point out. Otherwise, one would erroneously conclude that government can increase firm value by raising corporate tax rate.

| Endogenous | Alternative corporate tax benefits | | | | |
|---|---|---|---|---|---|
| optimal ratio | 1.00 | 5.00 | 10.00 | 15.00 | 20.00 |
| Deposit | 42.54 | 43.41 | 44.40 | 45.30 | 46.11 |
| Debt | 37.24 | 39.63 | 42.87 | 46.48 | 50.50 |
| Tangible equity | 20.22 | 16.96 | 12.72 | 8.23 | 3.38 |
| Closure boundary | 43.39 | 44.28 | 45.29 | 46.20 | 47.03 |
| Default boundary | 43.39 | 44.28 | 45.29 | 46.20 | 47.03 |
| Bankruptcy loss | 3.32 | 3.50 | 3.70 | 3.89 | 4.07 |
| Credit spread | 1.98 | 2.07 | 2.17 | 2.27 | 2.36 |
| Insurance premium | 0.19 | 0.21 | 0.22 | 0.24 | 0.25 |

Table 9: Effects of corporate tax benefits on the optimal liability structure of an FDIC-insured bank. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $\tau$ varies, other exogenous parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\omega = 0.1$, $\alpha = 0.27$, and $\kappa = 1/0.98$.

Table 9 shows the optimal response of bank liability structure to changes of corporate tax policy. If corporate tax rate is lowered from 15% to 10%, the bank reduces both ratios of deposits and debt to assets. The ratio of deposits to assets drops by less than 1 percentage point, from 45.30% to 44.40%. By contrast, the ratio of debt to assets reduces by nearly 3.4 percentage points. Since debt does not bring service income, the reduction of debt is larger than the reduction of deposits. Overall, banks use less leverage in a regime of lower tax rate. In Table 9, tangible equity is up by nearly 4.5 percentage points when the rate of tax benefits is lowered from 15% to 10%. Because the leverage drops, the expected bankruptcy loss and the credit spread both drop when the tax rate is lowered.

Although the association of lower leverage with lower tax rate appears to support those who argue for lowering corporate tax in order to stabilize banks, we need to be cautious about this policy proposal. If lowering corporate tax leads to a loss in tax revenue, then it may be an expensive policy change for the public to achieve a greater stability of banking industry. An alternative is to lower corporate tax just for banks as suggested by Fleischer (2013). Lowering tax for banks, not for other firms, will make banking a subsidized business, begging the question of fairness of corporate tax policy and the question of distortions in the economy. These important issues are beyond the scope of this paper, but the link between the tax rate and bank leverage in our model lays a stepping-stone for a welfare analysis of the benefit and cost of tax policy reforms. As noted earlier, Schepens (2014), who exploits a recent tax code change permitting some tax advantages to equity, shows that banks do respond to changes in the tax code by altering their liability structure. Collectively, the current research suggests that a welfare analysis of tax policy must not ignore banks' optimal responses.

Although tax policy affects bank leverage, the effect appears to be limited. The first column of Table 9 shows the optimal liability structure of the bank that enjoys only 1% tax benefit on debt liability. Even with such low tax benefit, it is still optimal for the bank to use substantial leverage. The ratio of deposits to assets is still above 42%, and the ratio of debt to assets is about 37%. With these levels of deposits and debt, the bank's optimal tangible equity is about 20%. In fact, as the tax rate approaches to zero, the optimal deposit level is still about 42% of asset value, and the optimal debt-to-asset ratio approaches a limit above 36%. These numerical results indicate the limitation of tax policy as incentives for banks to significantly lower leverage.

It is reasonable to question whether the limitation of tax effects on leverage is due to FDIC insurance. To address this issue, Table 10 presents the optimal liability structure of an uninsured bank for various assumptions of tax benefits. When the tax benefit is only 1%, the optimal deposit ratio of the uninsured bank is under 30%, which is about 13 percentage points lower than the FDIC-insured bank in Table 9. However, the debt ratio is slightly higher. Overall, the uninsured bank is still substantially leveraged when the tax benefit is only 1%, with tangible equity being under 30%. Therefore, the FDIC is part of the reason for leverage, but it is not necessarily a major reason.

Calomiris and Carlson (2015) report that tangible equity ratio in the national banks in the 1890s, were no corporate tax or FDIC insurance, ranges from 8 to 76 percent, and most of the leverage in those banks are in the form of deposits. Since the parameters used in Table 10 are based on recent historical observations, it is difficult to compare the numbers in the table to the empirical results reported by Calomiris and Carlson. Nonetheless, our model's implication that substantial leverage is optimal for value-maximizing banks in the absence of substantial tax benefit is qualitatively consistent with their empirical findings.

| Endogenous | Alternative corporate tax benefits | | | | |
|---|---|---|---|---|---|
| optimal ratio | 1.00 | 5.00 | 10.00 | 15.00 | 20.00 |
| Deposit | 29.14 | 30.15 | 31.25 | 32.22 | 33.07 |
| Debt | 41.53 | 44.36 | 48.08 | 52.07 | 56.43 |
| Tangible equity | 29.33 | 25.49 | 20.67 | 15.71 | 10.50 |
| Closure boundary | 39.91 | 41.30 | 42.81 | 44.14 | 45.31 |
| Default boundary | 39.91 | 41.30 | 42.81 | 44.14 | 45.31 |
| Bankruptcy loss | 2.70 | 2.94 | 3.21 | 3.47 | 3.70 |
| Credit spread | 1.67 | 1.79 | 1.93 | 2.05 | 2.17 |
| Insurance premium | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 10: Effects of corporate tax benefits on an uninsured bank. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $\tau$ varies, other exogenous parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\alpha = 0.27$, and $\kappa = 1/(1-\alpha)$.

# 6 Conclusion

Our theory of bank liability structure explicitly models an array of factors that affect the optimal leverage and liability composition of banks. The factors include service income from deposits, FDIC insurance, and the risk of bank closure, besides asset volatility and corporate tax benefits. Since our model is structural and solved analytically, it provides a convenient setting for quantifying bank capital structure and designing practical capital strategies.[32] A salient feature of our model is the interaction between deposits and debt: a bank uses as much debt as possible to take advantage of tax benefits but not as much to offer unnecessary protection for deposits. Optimal liability structure of any bank is likely to have the features derived in this paper because leverage and liability composition should be optimal relative to bank assets even if bank assets are optimized simultaneously with the liability structure.[33]

Our theory of bank liability structure has important implications to bank regulation. Since the financial crisis of 2007–2009, regulators have decided to raise capital requirement for banks. Basel III lifts the required equity ratio from 4%, which was set in Basel II, to 7% for all banks and to nearly 10% for large banks that are designated as systemically important.[34] As we have noted in footnote 1, European and U.S. regulators have laid out higher capital requirements for banks, and academics have proposed raising equity ratio to as high as 20%. Two important questions for regulators are: how will banks adjust their liability structure in response to the increase in equity ratio requirement, and what are the potential unintended consequences? It is important to know whether a value-maximizing bank cuts down deposits or

[32]Models with a few time steps and discrete asset value are common in banking studies. Those models are useful for certain conceptual issues, but not for the issues investigated in this paper or for practical applications.

[33]As in Leland (1994), this paper does not have a complete model of capital structure that includes the optimal choice of assets.

[34]Hirtle (2011a, 2011b) explains the rationale for these capital requirements.

debt, or both, when reducing leverage to meet a capital requirement.[35] Cutting back deposits reduces banking service, and lowering debt shrinks an important funding source for banks. If we interpret capital requirement as the minimum capital for a bank to operate without being closed, our results show that banks respond to an increase in capital requirement by lowering deposits and increasing debt.

The model of bank liability structure developed in this paper provides tools for studying alternative bank liability structures that include securities beyond deposits and debt. Perhaps the most controversial security since the crisis of 2007–2009 is contingent capital, which is debt that converts to equity when the bank becomes under-capitalized, as advocated by Flannery (2009). Sundaresan and Wang (2015) analyze the intricate issues in designing contingent capital. Extension of our model to include convertible debt can shed light on the interactions of convertible debt with deposits, the traditional debt, and FDIC insurance. These interactions are critical for figuring out whether convertible debt helps stabilize banks. Another important component in bank capital structure is liquidity reserves. Hugonnier and Morellec (2015) examine liquidity reserves in banks that do not have deposit insurance but depositors can run timely so that their deposits are safe.

Our model can be extended to a setting in which banks dynamically change the liability structure as well as its composition of assets. This is especially important as banks, which are able to borrow below the risk-free rate in the form of deposits, can dynamically increase leverage in order to increase bank value. In reality, this prospect is not always available as the supply of deposits is finite. Furthermore, in a depressed economy, the risk-free rate itself can be very low, limiting the ability of banks to gain from this channel. Goldstein et al. (2001) have developed a dynamic framework for a corporate borrower. In their model, firms consider the opportunity of issuing additional debt when they optimize capital structure. By contrast, an important issue for banks is the ability to reduce leverage when it becomes poorly capitalized after losses. Another important issue for banks is their dynamic adjustment of their asset structure in response to changes in risk.[36] Extension of our model to a dynamic setting will be a useful, although challenging, project that warrants further research.

# A Appendix

Following Merton (1974) and Leland (1994), we assume bank asset value follows a geometric Brownian motion. In the risk-neutral probability measure, the stochastic process of asset value

[35] Subramanian and Yang (2013) consider the question of prudential regulation in a structural model, in which firms issue only perpetual debt and do not take deposits, as in Leland's (1994a) model.

[36] Adrian and Shin (2010) document that financial intermediaries adjust balance sheets to their forecast of risk.

is

$$dV = (r - \delta)V dt + \sigma V dW. \tag{16}$$

where $r$ is the risk-free interest rate, $\delta$ is the rate of cash flow, $\sigma$ is the volatility of asset value, and $W$ is a Wiener process.[37] Following Leland (1994), $V$ denotes the after-tax value of assets, and thus $\delta V$ is the after-tax cash flow.[38]

For a given bankruptcy boundary $V_b$, consider a security that pays one dollar if and only if $V$ hits $V_b$ for the first time. The price of this security, denoted by $P_b$, is referred to as the state price of $V_b$. According to Merton (1974), $P_b$ satisfies a differential equation:

$$\frac{1}{2}\sigma^2 V^2 P_b'' + (r - \delta)V P_b' - r P_b = 0, \tag{17}$$

where $P_b'$ and $P_b''$ are the first and second partial derivatives of $P_b$ with respect to asset value $V$. The general solution to the equation is $P_b = a_1 V^{-\lambda} + a_2 V^{-\lambda'}$, where $\lambda > 0$ and $\lambda' < 0$ are the two roots of quadratic equation

$$\frac{1}{2}\sigma^2 \lambda(1 + \lambda) - (r - \delta)\lambda - r = 0. \tag{18}$$

The boundary conditions are $P_b(V_b) = 1$ and $\lim_{V \to \infty} P_b(V) = 0$. The conditions imply $a_2 = 0$ and $a_1 = V_b^{\lambda}$, which give $P_b = (V_b/V)^{\lambda}$.

Equity holders earn dividend, $\delta V - (1 - \tau)(I + C + C_1)$, until bankruptcy, for given insurance premium $I$, deposit liability $C$, and debt liability $C_1$. The pricing equation of equity value before bankruptcy is

$$\frac{1}{2}\sigma^2 V^2 E'' + (r - \delta)V E' - rE + \delta V - (1 - \tau)(I + C + C_1) = 0, \tag{19}$$

where $E'$ and $E''$ are partial derivatives respect to asset value $V$. We assume $I \geq 0$ in general, but setting $I = 0$ gives the valuation without FDIC. There are two boundary conditions. Since bankruptcy wipes out equity, we have $E(V_b) = 0$. If $V \to \infty$, bankruptcy is remote, and $E(V)$ approximately equals to $V - (1 - \tau)(I + C + C_1)/r$.

The pricing equation of debt $D_1$ is

$$\frac{1}{2}\sigma^2 V^2 D_1'' + (r - \delta)V D_1' - r D_1 + C_1 = 0, \tag{20}$$

where $D_1'$ and $D_1''$ are partial derivatives with respect to $V$. There are also two boundary conditions. Debt holder receives $\left[(1 - \alpha)V_b - D\right]^+$ at bankruptcy. If $V \to \infty$, the debt becomes risk-free, and $D_1$ approaches $C_1/r$.

[37]Notice that cash flow $\delta V$ also follows a geometric Brownian motion with volatility $\sigma$. One may start with the assumption that asset cash flow follows a geometric Brownian motion with volatility $\sigma$ and then show that asset value follows process (16).

[38]Alternatively, one may specify the before-tax value of the assets, $V^*$, as in Goldstein et al. (2001). Then, $\delta V^*$ is earnings before interests and tax (EBIT), and the after-tax asset value is $V = (1 - \tau)V^*$. Recovery value of bankruptcy is then $(1 - \phi)(1 - \tau)V_b^*$, which is equivalent to $(1 - \phi)V$. All the results in this paper can be derived and presented accordingly.

Theorem 1 in Section 3.1 presents the solutions to equations (19) and (20) and their boundary conditions. The solutions can be derived similarly to those in Leland (1994) and Goldstein et al. (2001). Section A.1 provides the details.

To simplify the derivation of optimal liability structure, we introduce the following notations:

$$x = C_1/C, \quad c = C/(rV), \quad v_a = rV_a/C, \quad v_d = rV_d/C, \quad v_b = rV_b/C \tag{21}$$

$$\iota = \eta/(r-\eta), \quad \theta = (1-\tau)\lambda/(1+\lambda). \tag{22}$$

We refer to $x$ as the liability ratio and $c$ as the deposit liability scaled by asset value. The state price of bankruptcy is then simplified to $P_b = (v_b c)^{\lambda}$. By Theorem 1, the scaled boundaries are

$$v_a = \kappa(1+\iota), \quad v_d = \theta(1+i+x), \quad v_b = \max\{\kappa(1+\iota), \theta(1+i+x)\}. \tag{23}$$

Notice that $V_a < V_d$ if and only if $v_a < v_d$. Furthermore, equation (9) can be written as a function of $c$

$$i = (1-\omega)[1-(1-\alpha)\kappa]^{+}(1+\iota)(v_a c)^{\lambda}/[1-(v_a c)^{\lambda}]. \tag{24}$$

We can also express the bank value in Theorem 1 as a ratio to asset value:

$$\begin{aligned} f(x,c) = F/V = 1 + [1-(v_b c)^{\lambda}][\iota + \tau(1+i+x) - i]c \\ -\ (v_b c)^{\lambda} \min\{\alpha v_b, v_b - (1+\iota)\}c. \end{aligned} \tag{25}$$

Choosing $(C, C_1)$ to maximize bank value $F$ is equivalent to choosing the duplet $(x, c)$ to maximize $f$. Once we obtain the optimal $(x^*, c^*)$, the optimal liabilities $(C^*, C_1^*)$ can be obtained easily as $C^* = c^* rV$ and $C_1^* = x^* C^*$.

## A.1 Proof of Theorem 1

The general solution to pricing equation (19) is

$$E = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + V - (1-\tau)(I + C + C_1)/r \tag{26}$$

where $\lambda > 0$ and $\lambda' < 0$ are the two solutions to equation (18), and $a_1$ and $a_2$ are arbitrary constants. The boundary conditions of $E$ imply $a_2 = 0$ and $a_1 = -[V_b - (1-\tau)(I+C+C_1)/r]V_b^{\lambda}$, which give equation (4).

If $V_b = V_a = \kappa D$, then $C = (r-\eta)D$ gives equation (1). To prove $V_d$ in equation (2) is the endogenous default boundary, we need to show that $V_b$ maximizes the equity value when $V_b = V_d$. Differentiation of equation (4) with respect to $V_b$ leads to

$$\partial E/\partial V_b = [(1+\lambda)/V_b](V_b/V)^{\lambda}\big(V_d - V_b\big). \tag{27}$$

Since the above is positive if $V_b < V_d$ and negative if $V_b > V_d$, we know $V_b = V_d$ maximizes the equity value. Notice that $V_d$ is independent of $V$. Equity holders choose to default before bank closure if $V$ drops to $V_d$ before $V_a$. The bank is closed before endogenous default if $V$ drops to $V_a$ first. Therefore, the bankruptcy boundary is $V_b = \max\{V_a, V_d\}$.

The general form of solution to pricing equation (20) is

$$D_1 = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + C_1/r\,, \tag{28}$$

where $a_1$ and $a_2$ can be any constants. The boundary conditions of $D_1$ imply $a_2 = 0$ and $a_1 = \big\{[(1-\alpha)V_b - D]^+ - C_1/r\big\}V_b^{\lambda}$, which give equation (3).

Bank value is $F = D + D_1 + E$. We obtain equation (5) by substituting equations (3) and (4), and using $D = C/(r-\eta)$.

## A.2 Proof of Theorem 2

Let $Q$ be the value of deposit insurance to banks. Its pricing equation is

$$\frac{1}{2}\sigma^2 V^2 Q'' + (r-\delta)VQ' - rQ - I = 0\,, \tag{29}$$

where $Q'$ and $Q''$ denote the first and second partial derivatives of $Q$ with respect to $V$. The general solution to the equation is $Q(V) = -I/r + a_1 V + a_2 V^{-\lambda}$, where $a_1$ and $a_2$ can be any constants. The boundary conditions of the value of the insurance product are $\lim_{V\to\infty} Q = -I/r$ and $Q(V_a) = [D-(1-\alpha)V_a]^+$, where $V_a = \kappa D$. They imply $a_1 = 0$ and $-I/r + a_2 V_a^{-\lambda} = [D-(1-\alpha)V_a]^+$, which give $Q(V) = -(1-P_a)I/r + [D-(1-\alpha)V_a]^+ P_a$, where $P_a = [V_a/V]^{\lambda}$. The insurance premium $I^\circ$ is fair iff $Q(V) = 0$. It follows that $(1-P_a)I^\circ = r[D-(1-\alpha)V_a]^+ P_a$. We obtain equation (6) by substituting $V_a = \kappa D$ and factoring $D$ out.

## A.3 Proof of Theorem 3

We first show that $v_d < v_a$ is not optimal. If $v_d < v_a$, then $\theta(1+i+x) < (1+\iota)\kappa$, $v_b = v_a = (1+\iota)\kappa$, and

$$f(x,c) = 1 + c\Big\{[\iota - i + \tau(1+i+x)][1-(v_a c)^{\lambda}] - (\kappa-1)(1+\iota)(v_a c)^{\lambda}\Big\}\,. \tag{30}$$

We then obtain $f_x'(x,c) = \tau[1-(v_a c)^{\lambda}]c > 0$, which implies that the current $x$ is not optimal.

It follows from equation (24) that $i_c' = \partial i/\partial c = \lambda i c^{-1}/[1-(v_a c)^{\lambda}]$. Since $\kappa < 1/(1-\alpha)$, both $i$ and $i_c'$ are positive. We also have $c i_c'[1-(v_b c)^{\lambda}] \le \lambda i$ because $v_b \ge v_a$. The equality, $c i_c'[1-(v_b c)^{\lambda}] = \lambda i$ holds when $v_b = v_a$. Both $i$ and $i_c'$ converge to zero when $\kappa$ rises to $1/(1-\alpha)$ while other parameters and variables are fixed. Thus, given any $v > v_a$, there exists $\kappa^* \in [1, 1/(1-\alpha))$ such that $\kappa \in (\kappa^*, 1/(1-\alpha))$ implies $i + c i_c' < \iota$ for all $c$ in $[0, 1/v]$.

If $v_d > v_a$, we have $\theta(1+i+x) > (1+\iota)\kappa$ and $v_b = \theta(1+i+x)$. Notice that

$$\min\{\alpha v_b\,,\, v_b - (1+\iota)\} = \begin{cases} v_b - (1+\iota) & \text{if} \quad v_b \le (1+\iota)/(1-\alpha) \\ \alpha v_b & \text{if} \quad v_b > (1+\iota)/(1-\alpha). \end{cases} \tag{31}$$

If $v_a < v_d \leq (1+\iota)/(1-\alpha)$, equations (31) and (25) give

$$f_x'(x,c) = c\Big[\tau - (\tau+\lambda)(v_d c)^\lambda + \lambda(v_d c)^\lambda \frac{1+i}{1+i+x}\Big] \tag{32}$$

$$\begin{aligned} f_c'(x,c) &= 1+\iota-(1+i+ci_c')\big[1-(v_d c)^\lambda\big] \\ &\quad + \ (1+i+x+ci_c')\Big\{\tau-(\tau+\lambda)(v_d c)^\lambda+\lambda(v_d c)^\lambda\frac{1+i}{1+i+x}\Big\}. \end{aligned} \tag{33}$$

Let $c_x$ be the optimal $c$ for given $x$, then equation (33) implies

$$\tau-(\tau+\lambda)(v_d c_x)^\lambda+\lambda(v_d c_x)^\lambda\frac{1+i}{1+i+x} = -\frac{1+\iota-(1+i+c_x i_c')[1-(v_d c_x)^\lambda]}{1+i+x+c_x i_c'}. \tag{34}$$

For $\kappa \in (\kappa^*, 1/(1-\alpha))$, we have $i + c_x i_c' < \iota$, which implies that the numerator is positive. Substitution of the above into equation (32) shows $f_x'(x,c_x) < 0$. Thus, $v_a < v_d < (1+\iota)/(1-\alpha)$ is not optimal because lowering $x$ and $v_d$ increases $f(x,c_x)$.

If $v_b \geq (1+\iota)/(1-\alpha)$, equation (31) and (25) give

$$f_x'(x,c) = c\Big\{\tau-\big[(1+\lambda)(\tau+\alpha\theta)+\lambda(\iota-i)/(1+i+x)\big](v_d c)^\lambda\Big\}. \tag{35}$$

$$\begin{aligned} f_c'(x,c) &= (\iota-i-ci_c')[1-(v_d c)^\lambda] \\ &\quad +(1+i+ci_c'+x)\cdot\Big\{\tau-\Big[(1+\lambda)(\tau+\alpha\theta)+\frac{\lambda(\iota-i)}{1+i+x}\Big](v_d c)^\lambda\Big\}. \end{aligned} \tag{36}$$

Let $c_x$ be the optimal $c$ relative to $x$. Then, equation (36) implies

$$\tau-(1+\lambda)(\tau+\alpha\theta)+\frac{\lambda(\iota-i)}{1+i+x} = -\frac{[\iota-i-c_x i_c'][1-(v_d c_x)^\lambda]}{1+i+x+c_x i_c'}. \tag{37}$$

For $\kappa > \kappa^*$, we have $i + ci_c' \leq \iota$. Then, the numerator is positive. Substitution of the above into equation (35) shows $f_x'(x,c_x) < 0$, which means the current $x$ and $v_d$ is not optimal because lowering $x$ and $v_d$ increases $f(x,c_x)$.

The above two cases show that there exists $\kappa^*$ such that for $\kappa^* < \kappa < 1/(1-\alpha)$, we have $f_x' < 0$ for all $x$ satisfying $\theta(1+i+x) > (1+\iota)\kappa$, if $c$ is kept to be optimal relative to $x$. Therefore, $\theta(1+i+x) > (1+\iota)\kappa$ cannot be optimal because reducing $x$ adds value to the bank. Consequently, the optimal $x^*$ and $c^*$ must satisfy $\theta(1+i^*+x^*) = (1+\iota)\kappa$, which implies $v_d^* = (1+\iota)\kappa$ and thus $v_b^* = v_d^* = v_a$.

With $v_a^* = v_d^* = v_b^*$, the state price of bankruptcy is: $\pi = (v_a^* c^*)^\lambda = (v_b^* c^*)^\lambda$. This equation implies $v_a^* = \pi^{1/\lambda}/c^*$. In view of equation (23), we have $(1+\iota)\kappa = \pi^{1/\lambda}/c^*$. It follows that $c^* = \pi^{1/\lambda}/[(1+\iota)\kappa]$, $i^* = (1+\iota)[1-(1-\alpha)\kappa]^+\pi/(1-\pi)$, and $x^* = (1+\iota)\{\kappa/\theta-(1-\omega)[1-(1-\alpha)\kappa]^+\pi/(1-\pi)\}-1$.

Let $x_c = v_a/\theta-(1+i)$ for any $c \in [1, 1/v_a]$ and $g(c) = f(x_c, c)$. It follows from equation (30) that

$$g(c) = 1+c\Big\{[\iota-i+\tau v_a/\theta][1-(v_a c)^\lambda]-(\kappa-1)(1+\iota)(v_a c)^\lambda\Big\}. \tag{38}$$

This function is differentiable in $c$, and

$$g'(c) = [\iota - i + \tau v_a/\theta]\big[1-(1+\lambda)(v_a c)^\lambda\big] \\ - (\kappa-1)(1+\iota)(1+\lambda)(v_a c)^\lambda - c i'_c\big[1-(v_a c)^\lambda\big]. \tag{39}$$

With equation (24) and the formula of $i'_c$, the above simplifies to

$$g'(c) = \iota + \tau v_a/\theta - \big\{\iota + \tau v_a/\theta + (\kappa-1)(1+\iota) \\ + (1-\omega)[1-(1-\alpha)\kappa]^+(1+\iota)\big\}(1+\lambda)(v_a c)^\lambda. \tag{40}$$

If $(x^*, c^*)$ is a maximum, $c^*$ must maximizes $g(c)$, and thus $g'(c^*) = 0$. Letting $\pi = (v_a c^*)^\lambda$ and setting equation (40) to zero, we obtain

$$\pi = \frac{1}{1+\lambda} \cdot \frac{\iota\theta + \tau(1+\iota)\kappa}{\iota\theta + \tau(1+\iota)\kappa + (1+\iota)\{\kappa - 1 + (1-\omega)[1-(1-\alpha)\kappa]^+\}\theta}. \tag{41}$$

Finally, we complete the proof by substituting the original parameters into (41) and the original variables into the formulas for $c^*$, $i^*$, and $x^*$.

## A.4 Proof of Theorem 4

Setting $\tau = 0$, $I = 0$, and $C_1 = 0$ in Theorem 1, we obtain $V_a = \kappa C/(r-\eta)$, $V_d = [\lambda/(1+\lambda)]C/r$. Observing that $\lambda/(1+\lambda) < 1$ and $\kappa[r/(r-\eta)] > 1$, we know $V_b = \max\{V_a, V_d\} = V_a$. Also from Theorem 1 we obtain the equity value as $E = V - (1-P_a)C/r - P_a V_a$, where $P_a = [V_a/V]^\lambda$. Thus, the bank value is

$$F = D + E = V + [\eta/(r-\eta)]C/r + (1-v_a)P_a C/r, \tag{42}$$

where $v_a = \kappa r/(r-\eta)$. Dividing the above by $V$ and letting $c = C/(rV)$ and $f = F/V$, we have

$$f = 1 + [\eta/(r-\eta)]c + (1-v_a)(v_a c)^\lambda c. \tag{43}$$

The first order condition for choosing $c$ to maximize $f$ is

$$f'_c = \eta/(r-\eta) + (1+\lambda)(1-v_a)(v_a c)^\lambda = 0, \tag{44}$$

which gives

$$P_a^* = (v_b c)^\lambda = \frac{1}{1+\lambda}\frac{\eta/(r-\eta)}{v_a - 1}. \tag{45}$$

Then, we obtain equation (14) by substituting $v_a = \kappa r/(r-\eta)$. Furthermore, we obtain equation (15) by solving $C^*$ from $P_a^* = [(v_a C^*/r)/V]^\lambda$.

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