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A Reconsideration of Tax Shield Valuation

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Abstract

A quarter-century ago, Miles and Ezzell (1980) solved the valuation problem of a firm that follows a constant leverage ratio $L = D/S$. However, to this day, the proper discounting of free cash flows and the computation of WACC are often misunderstood by scholars and practitioners alike. For example, it is common for textbooks and fairness opinions to discount free cash flows at WACC with beta input $\beta_S = [1 + (1 - \tau)L]\beta_u$, although the latter is not consistent with the assumption of constant leverage. This confusion extends to the valuation of tax shields and the proper implementation of adjusted present value procedures. In this paper, we derive a general result on the value of tax shields, obtain the correct value of tax shields for perpetuities, and state the correct valuation formulas for arbitrary cash flows under a constant leverage financial policy.

Keywords: *tax shield valuation*; *WACC*; *APV*; *cost of capital*; *leveraged beta*.

JEL classifications: *G*13, *G*30, *G*31, *G*32, *G*33

1. A General Result on the Value of Tax Shields under Constant Leverage

In their classic work, Modigliani and Miller (1963) show that when the firm maintains a constant level of debt D and pays a tax rate τ, the value of the tax shield is $VTS = \tau D$. How to value the tax shield in the more interesting case in which the firm maintains a constant leverage ratio is a matter of contention. In recent papers, Fernandez (2004) and Roncaglio and Zanetti (2004) propose a solution for growing perpetuities that contradicts the extant literature. Fiesten *et al.* (2003) provide a critical review of Fernandez's assumptions. In this paper, we attempt to settle the matter by deriving a general result on the value of tax shields under constant leverage as well as a specific formula for perpetuities. In addition, we summarise the correct valuation formulas for arbitrary cash flows when the firm maintains constant leverage and illustrate their application with a numerical example.

To avoid arguments about appropriate discount rates, we shall adopt the modern formulation of asset pricing and assume that the price of an asset that pays a random

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amount x_t at time t is the sum of the expectation of the product of x_t and M_t, the pricing kernel for time t cash flows:

$$P_x = \sum_1^{\infty} E[M_t x_t] \tag{1}$$

We define free cash flow (*FCF*) and equity free cash flow (*EFC*) in the standard way:

$$FCF_t = OPBT_t(1-\tau) - NINV_t \tag{2}$$

$$EFC_t = (OPBT_t - INT_t)(1-\tau) - NINV_t - PP_t, \tag{3}$$

Where *OPBT* is operating profit before tax, *NINV* is net investment (change in net working capital plus capital expenditures minus depreciation), *INT* is interest paid and *PP* is principal payment (which can be positive or negative – a negative value indicates issuance of new debt). We make the standard valuation assumptions (variables without time subscripts are dated time zero).

$$V_u = \sum_1^{\infty} E[M_t FCF_t] \tag{4}$$

$$S = \sum_1^{\infty} E[M_t EFC_t] \tag{5}$$

$$V_L = S + D \tag{6}$$

The value of the tax shield is defined by $VTS = V_L - V_u$. Furthermore, since

$$Taxes_{ut} = (FCF_t + NINV_t)\tau/(1-\tau) \tag{7}$$

$$Taxes_{Lt} = (EFC_t + NINV_t + PP_t)\tau/(1-\tau). \tag{8}$$

Letting G_u and G_L denote the value of these cash flows, respectively, it is immediate that

$$V_u + G_u = V_L + G_L, \text{and hence } VTS = G_u - G_L. \tag{9}$$

Fernandez (2004) points out that the tax cash flows should be discounted at different rates. However, our use of the pricing kernel means that we need not worry about that, and we can discount, directly, $Taxes_{ut} - Taxes_{Lt}$. This leads to the following analysis:

$$VTS = \frac{\tau}{1-\tau}\left[\sum_1^{\infty} E[M_t FCF_t] - \sum_1^{\infty} E[M_t EFC_t] - \sum_1^{\infty} E[M_t PP_t]\right] \tag{10}$$

$$VTS = \frac{\tau}{1-\tau}\left[V_u - S - \sum_1^{\infty} E[M_t PP_t]\right] = \frac{\tau}{1-\tau}\left[D - VTS - \sum_1^{\infty} E[M_t PP_t]\right] \tag{11}$$

Assuming that the only reason the market value of the outstanding debt changes is because of new issues (negative *PP*) or principal payments, $D_t = D_{t-1} - PP_t$. Also, noting that such principal payments or new issues are undertaken to maintain the leverage ratio, we have $D_t = LS_t$. Solving the above leads to:

$$VTS = \tau D - \tau \sum_1^{\infty} E[M_t PP_t] = \tau D + \tau L \sum_1^{\infty} E[M_t(S_t - S_{t-1})] \tag{12}$$

This expression permits us to examine under what conditions $VTS = \tau D$ holds in the absence of growth under constant leverage. The value of the tax shield depends upon the nature of the equity stochastic process, which, in turn, depends upon the free cash flow process. There are two ways that $VTS = \tau D$ can be valid. First, if the free cash flow process is given by $FCF_t = FCF + \varepsilon_t$, where ε_t is random variable such that $E_t[\varepsilon_{t+1}] = 0$, then it is easily verified that $S_t = S$, independent of time, but this corresponds to the original Modigliani and Miller (1963) case with constant debt.

The free cash flow process of the second case is $FCF_{t+1} = FCF_t(1 + \varepsilon_{t+1})$, such that (letting $M_{t,t+1}$ be the one period pricing kernel at time t for cash flows at time $t + 1$) $E_t[M_{t,t+1}\varepsilon_{t+1}] = 0$. In this case, S_t is a constant times FCF_t, and the sum in the second part of the expression above is zero. This corresponds, however, to the case in which the stock under consideration has zero systematic risk in that all cash flows and price changes are uncorrelated with the pricing kernel. In this case, all discount rates for this firm (debt, unlevered equity, levered equity) are equal to the risk free rate.

More interesting is the case in which $FCF_{t+1} = FCF_t(1 + \varepsilon_{t+1})$ with $E_t[M_{t,t+1}\varepsilon_{t+1}] < 0$. This implies that the second term in (12) is negative and $VTS < TD$. For example, in a CAPM world, if $S_t - S_{t-1}$ is positively correlated with the market portfolio, it is negatively correlated with the pricing kernel. In this case, changes in the debt level are also negatively correlated with the kernel M_t, and the value of the tax shield is less than TD.

It should be noted that $\Sigma_1^\infty E_t[M_tPP_t]$ [in expressions (10) and (11)] is simply the value of the cash flow realised by changes in debt principal and does not include subsequent interest payments. It should not be confused with the value of debt, that is, with the value of the cash flows received by debt-holders. For example, consider the cash flow associated with an increase in default-free debt $\Delta D_t = -PP_t$, paying interest at the riskless rate r with principal repayable in a year. The cash flow received by debt-holders is $[-\Delta D_t, \Delta D_t(1 + r)]$, and its value $-E_t[M_t\Delta D_t] + E_t[M_{t+1}\Delta Dr(1 + r)] = E_t[M_t(-\Delta D_t + E_{t+1/t}[M_{t,\,t+1}\Delta D_t(1 + r)]] = E_t[M_t(-\Delta D_t + \Delta D_t)] = 0$. That is, the value of debt at any time is simply D_t and the value of any future debt change and associated interest and amortisation payments is zero. On the other hand, the value of the cash flow generated by principal changes, $\Sigma_1^\infty E_t[M_tPP_t]$, depends on if PP_t is uncorrelated or correlated with the market. That is, $\Sigma_1^\infty E_t[M_tPP_t] < \Sigma_1^\infty E_t[PP_t]/(1 + r)^t$ when $cov[M_t,PP_t] < 0$, and $\Sigma_1^\infty E_t[M_tPP_t] < 0$ when $E_t[PP_t] = 0$.

2. The Value of the Tax Shield under Perpetual Growth: Comparison to Recent Literature

Let us value the tax shield under growth of free cash flows subject to systematic risk with non-negative expectation g. Then, the expected path of debt is $D_t = D(1 + g)^t$ such that $-PP_t = D_t - D_{t-1} = (1 + g)D_{t-1} - D_{t-1}$. Hence, taking into account that D_{t-1} has no systematic risk at time t, (12) becomes

$$VTS = \tau D + \tau D \sum_1^\infty \left[\frac{(1+g)^t}{(1+\rho)^t} - \frac{(1+g)^{t-1}}{(1+r)(1-\rho)^{t-1}} \right] = \frac{\tau r D}{(\rho - g)} \frac{(1+\rho)}{(1+r)} \qquad (13)$$

where ρ is the capitalisation rate for the unlevered free cash flows. It should be noted that (13) is also valid for risky debt, with r replaced by r_D such that $r_D - r > 0$ in the premium for systematic risk.

If one ignores that D_{t-1} has no systematic risk at time t and discounts $-PP_t$ at ρ, (12) yields

$$VTS = \tau D + \tau D\left(\frac{1+g}{\rho - g} - \frac{1}{\rho - g}\right) = \frac{\tau\rho D}{\rho - g} \tag{14}$$

This is the expression obtained by Fernandez (2004) and Roncaglio and Zanetti (2004). Fernandez arrives at (14) by making use of the Modigliani-Miller's expression $VTS = \tau D$, which, as we have shown, does not apply under a constant leverage policy in the presence of systematic risk.[1] As long as ρ exceeds r (or r_D), the value of the tax shield given by (13) is less than (14).

(14) cannot be saved under continuous adjustment of debt because, in that case,

$$VTS = \int_0^\infty \tau\delta D e^{(\gamma-\omega)t} dt = \frac{\tau\delta D}{\omega - \gamma} \tag{15}$$

where $\delta = ln(1 + r)$, $\omega = ln(1 + \rho)$ and $\gamma = ln(1 + g)$. Note that (15) is the limit of (13) when debt changes over the interval $dt \to 0$.

3. The Value of the Tax Shield under Perpetual Growth for an Explicit Free Cash Flow Process

It is useful to recast the previous derivation for an explicit free cash flow process. We calculate the enterprise value explicitly and then subtract the unlevered value from the levered value. Suppose that the free cash flow stochastic process is given by $FCF_{t+1} = FCF_t(1 + g)(1 + \varepsilon_{t+1})$, with $E_t[M_{t,t+1}\varepsilon_{t+1}] = -d/(1 + r)$, where d is a non-negative constant.[2] Notice that $-d$ is merely the risk adjusted expectation of ε. The unlevered value, V_{ut}, must satisfy:

$$\begin{aligned} V_{ut} &= E_t[M_{t,t+1}FCF_{t+1}] + E_t[M_{t,t+1}V_{ut+1}] \\ &= FCF_t(1+g)(1-d)/(1+r) + E_t[M_{t,t+1}V_{ut+1}] \end{aligned} \tag{16}$$

A solution will have $V_{ut} = bFCF_t$, and the above can be solved easily for b:

$$V_{ut} = \frac{(1+g)(1-d)}{(1+r) - (1-d)(1+g)} FCF_t \tag{17}$$

Setting, $(1 - d) = (1 + r)/(1 + \rho)$, leads to the obvious formula, $V_{ut} = FCF_t(1 + g)/(\rho - g)$. The pricing recursion for S_t can be similarly constructed:

$$\begin{aligned} S_t &= E_t[M_{t,t+1}EFC_t] + E_t[M_{t,t+1}S_{t+1}] \\ &= E_t[M_{t,t+1}(FCF_t - rD_t(1-\tau) - D_t + D_{t+1} + S_{t+1})]. \end{aligned} \tag{18}$$

Setting $D_t = LS_t$, $D_{t+1} = LS_{t+1}$, yields

[1] Fernandez's error takes place when he assumes constant leverage but substitutes $VTS = \tau D$ for $g = 0$ into his expression (37). Roncaglio and Zanetti ignore that D_{t-1} has not systematic risk at time t.

[2] Sufficient conditions for the conditional expectation to be constant are that investors have constant relative risk aversion preferences, and that the random growth rate of consumption be i.i.d.

$$S_t[1 + L - L\tau r/(1 + r)] = E_t[M_{t,t+1}FCF_t] + (1 + L)E_t[M_{t,t+1}S_{t+1}]. \quad (19)$$

After noting that $V_{Lt} = (1 + L)S_t$, the above becomes a recursion for V_{Lt}, which can be solved as above:

$$V_{Lt} = \frac{(1+g)(1-d)}{(1+r) - (1+g)(1-d) - \frac{L}{1+L}\tau r} FCF_t \quad (20)$$

Subtracting V_u from V_L and, taking into account that $V_L = (1 + L)D/L$, leads to the formula for the value of the tax shield, VTS:

$$VTS = \tau r D/[(1+r) - (1+g)(1-d)]. \quad (21)$$

Recalling that $(1 - d) = (1 + r)/(1 + \rho)$ yields equation (13):

$$VTS = \frac{\tau r D}{(\rho - g)} \frac{(1+\rho)}{(1+r)}.$$

Furthermore, for $g = 0$, (13) becomes

$$VTS = \frac{\tau r D}{\rho} \frac{(1+\rho)}{(1+r)} < \tau D.$$

Note that, when $g = 0$, the free cash flow process is $FCF_{t+1} = FCF_t(1 + \varepsilon_{t+1})$, where $E_t[FCF_t(1 + \varepsilon_{t+1})] = FCF_t \; for \; E_t[\varepsilon_{t+1}] = 0$. The underlying assumption leading to this result is that expectations about future cash flows are revised on the basis of new information. This is a more realistic representation of uncertain cash flows than assuming that the expectation about future cash flows is a constant independent of cash flow realisations, which is a necessary requirement for $VTS = \tau D$.

4. Enterprise Value under Constant Leverage and Arbitrary Free Cash Flows

We now summarise the correct valuation formulas for arbitrary cash flows when the firm follows a constant leverage policy, and derive the formulas for the cost of equity, *WACC* and CAPM betas. These formulas in conjunction with (13) and (15) are then used to show the consistency of *APV* valuation ($V_L = V_u + VTS$), *WACC* valuation and equity valuation ($V_L = S + D$) in the case of growing perpetuities.

The value of the firm is given by the following recursion:

$$\begin{aligned} V_{Lt} &= E_t[M_{t,t+1}(FCF_{t+1} + \tau r_D D_t + V_{Lt+1})] \\ &= E_t[FCF_{t+1}](1+\rho)^{-1} + \tau r_D L(1+L)^{-1} V_{Lt}(1+r_D)^{-1}] + E_t[M_{t,t+1}V_{Lt+1}] \\ &= \frac{E_t[FCF_{t+1}]}{(1+\rho)[1 - \tau r_D L(1+L)^{-1}/(1+r_D)]} + \frac{E_t[M_{t,t+1}V_{t+1}]}{1 - \tau r_D L(1+L)^{-1}/(1+r_D)} \end{aligned} \quad (22)$$

which has solution

$$V_{Lt} = \sum_{t+1}^{\infty} \frac{E_t[FCF_j]}{(1+\rho)^{j-t}[1 - \tau r_D L(1+L)^{-1}/(1+r_D)]^{j-t}} \quad (23)$$

Is it straightforward to verify that (23) collapses to (20) in the case of perpetual growth. Let us examine the implications of (23) for the correct computation of the weighted average cost of capital. From (23) we can define the capitalisation rate for the levered cash flows as

$$w = (1+\rho)[1 - \tau r_D L(1+L)^{-1}/(1+r_D)] - 1 \quad \rho - \tau r_D L(1+L)^{-1}(1+\rho)/(1+r_D) \tag{24}$$

Alternatively, defining k_S such that

$$S_t = E_t[M_{t,t+1}(FCF_{t+1} - (1-\tau)r_D L S_t + L(S_{t+1} - S_t) + S_{t+1})] \tag{25}$$

$$= \frac{E_t[FCF_{t+1}] - (1-\tau)r_D L S_t - L S_t + (1+L)E[S_{t+1}]}{(1+k_S)} \tag{26}$$

yields

$$S_t(1+k_S) + [1 + (1-\tau)r_D]LS_t = E_t[FCF_{t+1}] + E_t[V_{Lt+1}] \tag{27}$$

or

$$V_{Lt} = \frac{E_t[FCF_{t+1}] + E_t[V_{Lt+1}]}{(1+w)} \tag{28}$$

where w is the weighted average cost of capital

$$w = \frac{S_t k_S + (1-\tau)r_D D_t}{S_t + D_t} = \frac{k_S + (1-\tau)r_D L}{1+L} \tag{29}$$

The recursion (28) has solution

$$V_{Lt} = \sum_{t+1}^{\infty} E_t[FCF_{j-t}](1+w)^{-j+t} \tag{30}$$

Hence, equating the right hand side of this expression to (23) and solving for k_S yields the required return on equity

$$k_S = \rho + (\rho - r_D)L[1 - \tau r_D/(1+r_D)] \tag{31}$$

Furthermore, under CAPM, allowing r_D to contain systematic risk with beta β_D, the beta of levered equity follows from (31):

$$\beta_S = [1 + (1 - \tau r_D/(1+r_D))L]\beta_u - [1 - \tau r_D/(1+r_D)]L\beta_D \tag{32}$$

which, for risk-free debt such that $r_D = r$, becomes $\beta_S = [1 + (1 - \tau r/(1+r))L]\beta_u$.

Under continuous adjustment of capital structure to a constant leverage ratio such that the capitalisation interval $dt \to 0$, the above expressions become

$$\omega = \kappa_u - \tau\kappa_D L/(1+L) = \frac{\kappa_S + (1-\tau)\kappa_D D}{S+D} \tag{33}$$

$$\kappa_S = \kappa_u + (\kappa_u - \kappa_D)L \tag{34}$$

$$\beta_S = (1+L)\beta_u - L\beta_D \tag{35}$$

where ω, κ_v and κ_D are the continuous time $WACC$, cost of equity and cost of debt, respectively.

(33) and (35) are not new, they were derived in several occasions by Miles and Ezzell (1985), Harris and Pringle (1985), Sick (1990), Taggart (1991) and Ruback (2002). Why then the persistent use of $\beta_S = [1 + (1 - \tau)L]\beta_u$ in conjunction with discounting at constant $WACC$? One reason may be a conscious attempt to account for the fact

that debt seems to adjust with a lag to the value of the firm,[3] since that would tend to increase the value of the tax shield and decrease the value of levered beta with respect to the values resulting from the assumption of constant leverage. However, such an ad hoc correction would most likely produce a larger error than the constant leverage model and is inconsistent with the assumption of constant WACC. Another reason may well be that the original results of Hamada (1972) and Rubinstein (1973), which were based upon the Modigliani-Miller's tax shield value τD for constant debt, are wrongly assumed to hold in general.

In Table 1 we compute Fernandez's example for perpetuities under a constant leverage policy (his Tables 3 and 5). We show that equations (17), (20) and (13) give the same result under APV ($V_L = V_u + VTS$), $WACC$ valuation and equity valuation ($V_L = S + D$).

5. Conclusion

A constant target leverage ratio as a characterisation of financial policy is both a realistic approximation to many real life situations and a computationally convenient assumption. In particular, it justifies the estimation of *WACC* using constant weights. However, the very use of constant *WACC* implies that debt adjusts over time as a function of the value of the enterprise and is therefore a function of random cash flow realisations. This inescapable implication determines both the value of the tax shields and the way in which the cost of equity used to estimate *WACC* has to be computed. In addition, it determines the consistent relation between adjusted present value and *WACC* valuation procedures.

One cannot estimate the value of the tax shield generated by a constant leverage policy while ignoring the risk induced on the level of debt and the tax shield by the realisation of uncertain cash flows. When this is done, it leads to errors on the value of tax shields and the appropriate discount rates.

We have shown that in the absence of growth, maintaining constant leverage implies that the value of the tax shield is smaller than τD but for the exceptional cases in which the value of the firm is independent of time and, therefore, debt is constant as in Modigliani and Miller, or in the absence of systematic risk, in which all cash flows are discounted at the riskless rate. We also derived the correct formula for the value of tax shields for growing perpetuities. In addition, we provided a simple derivation of the correct formulas for computing the cost of equity, *WACC* and the CAPM equity betas for arbitrary cash flows when the firm follows a constant leverage policy. Miles and Ezzell anticipated the latter results long ago but we hope that our simple and general derivation will help to clear the confusion about what is right or wrong about valuation under constant leverage.

[3] See, for example, Jalilvand and Harris (1984), Shyam-Sunder and Myers (1999), Ozkan (2000), and Graham and Harvey (2001).

Table 1

Valuation of a hypothetical firm maintaining a constant leverage ratio with and without growth

Capital structure adjustment	Discrete			Continuous		
Growth	g	0.00%	5.00%	$\gamma = ln(1 + g)$	0.00%	4.88%
Risk free rate	r	6.00%	6.00%	$\delta = ln(1 + r)$	5.83%	5.83%
Unlevered beta	β_u	1.00	1.00	β_u	1.00	1.00
Equity premium	p	4.00%	4.00%	π *such that* $\kappa_u = ln(1 + k_u)$	3.70%	3.70%
Unlevered cost of equity	$\rho = r + p\beta_u$	10.00%	10.00%	$\kappa_u = \delta + \pi\beta_u$	9.53%	9.53%
Beta of debt	β_D	0.25	0.25	β_D	0.25	0.25
Cost of debt	$r_D = r + p\beta_D$	7.00%	7.00%	$\kappa_D = \delta + \pi\beta_D$	6.75%	6.75%
Tax rate	τ	40%	40%	τ	40%	40%
Initial debt	D	\$500	\$500	D	\$500	\$500
Intial free cash flow	$FCF_1 = 192 - 2{,}000g$	\$192	\$92	$FCF = FCF_1/(1 + g)$	\$192	\$88
Initial equity cash flow	$ECF_1 = FCF_1 - (1 - \tau)r_D D + gD$	\$171	\$96	$ECF = (FCF - (1 - \tau)\kappa_D D + \gamma D)$	\$172	\$92
Value of unlevered firm	$V_u = FCF_1/(\rho - g)$	\$1,920	\$1,840	$V_u = FCF/(\kappa_u - \gamma)$	\$2,014	\$1,883
Value of tax shield	$VTS = tr_D D(1 + \rho)/[(\rho - g)(1 + r_D)]$	\$144	\$288	$VTS = \rho\kappa_D D/(\kappa_u - \gamma)$	\$142	\$290
Enterprise value (APV)	$V_L = V_u + VTS$	\$2,0630.93	\$2,127.85	$V_L = V_u + VTS$	\$2,156.18	\$2,173.79
Value of equity	$S = V_L - D$	\$1,564	\$1,628	$S = V_L - D$	\$1,656.2	\$1,673.8
Value of equity	$S = ECF_1/(k_S - g)$	\$1,564	\$1,628	$S = ECF/(\kappa_S - \gamma)$	\$1,656.2	\$1,673.8
Levered beta	$\beta_S = [1 + (1 - \tau r_D (1 + r_D)^{-1})D/E]\beta_U - (1 - \tau r_D (1 + r_D)^{-1})D/E]\beta_D$	1.23	1.22	$\beta_S = (1 + D/E)\beta_u - (D/E)\beta_D$	1.23	1.22
Levered cost of equity	$k_S = r + p\beta_S$	10.93%	10.90%	$\kappa_S = \delta + \pi\beta_S$	10.37%	10.36%
Levered cost of equity	$k_S = \rho + (\rho - r_D) L(1 - \tau r_D/(1 + r_D))$	10.93%	10.90%	$\kappa_S = \kappa_u + (\kappa_u - \kappa_D)L$	10.37%	10.36%
Leverage ratio	$L = D/S$	31.97%	30.72%	$L = D/S$	30.19%	29.87%
Levered capitalization rate	$w = \rho - \tau r_D L/ (1 + L)(1 + \rho)/(1 + r_D)$	9.3027%	9.3236%	$\omega = \kappa_u - \tau\kappa_D L/(1 + L)$	8.9046%	8.9097%
WACC	$w = [(1 - \tau)r_D L + k_S]/(1 + L)$	9.3027%	9.3236%	$\omega = [(1 - \tau)\kappa_D L + \kappa_S]/(1 + L)$	8.9046%	8.9097%
Enterprise value @ WACC	$V_L = FCF_1/(w - g)$	\$2,063.93	\$2,127.85	$V_L = FCF/(w - \gamma)$	\$2,156.18	\$2,173.79

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