1. Introduction

Standard pricing models are incomplete. As discussed in detail in section 2, the popular hedonic pricing model implicitly assumes that consumers are fully informed about all product qualities and prices. In contrast, geographical price dispersion models assume that consumers are fully informed about product qualities but have incomplete information about the prices of a given brand across distribution outlets. Price-signaling models, on the other hand, assume that consumers have incomplete information about product qualities. Thus, manufacturers can use high prices to signal high quality to uninformed consumers. However, price-signaling models unrealistically assume that manufacturers sell directly to consumers (equivalently, the geographical dispersion of prices across stores for a given brand is zero). In practice, many manufacturers use intermediaries (e.g., retailers) to sell their products. And, as casual empiricism shows, prices for any given brand vary across retail outlets.

This paper has three objectives. First, we extend the standard price-signaling model to allow for a two-level distribution channel in which manufacturers sell to retailers who, in turn, sell products to consumers. Consumers have imperfect information about both product quality and retail prices for any given brand. Second, we develop an empirical methodology for testing price-signaling models. To our knowledge, price-signaling models have not been empirically tested before. Third, we test our model using data from consumer reports.

The empirical results strongly support our theory. Specifically, firms use price signals to appeal to uninformed consumers, regardless of whether they market durables or nondurables. Contrary to popular theory, the markets for experience goods (e.g., paper towels) are more efficient than the markets for search goods (e.g., VCRs). Finally, for four of the five product categories analyzed, our model outperforms the hedonic pricing model.

2. Review of previous pricing theories

This section reviews the theoretical foundations of the popular hedonic pricing model, search cost models, and price-signaling models.

The hedonic pricing model

Consider a perfectly competitive market in which all consumers and producers are fully informed about product qualities and prices and firms can enter or exit the market without cost. Then two brands with identical combinations of attributes/
product features must charge identical prices. Rosen (1974) characterizes this type of pricing equilibrium.

Let $p_i$ denote the price of the $i$th brand in a given product category and $(x_{i1}, \ldots, x_{iM})$ denote the levels of the objective attributes provided by the brand, where the attributes can be discrete, continuous, or a combination of both. Then Rosen shows that, in equilibrium:

$$ p_i = f(x_{i1}, \ldots, x_{iM}) $$

where $f$ is nonlinear. Equation (1) provides the theoretical foundation for the standard hedonic regression pricing model.

Rosen’s model (equation (1)) is incomplete because it implicitly assumes that the manufacturer does not use intermediaries (e.g., retailers) to sell its product to consumers. More importantly, the hedonic pricing model unrealistically assumes that all consumers are fully informed about product quality and prices.

**Search cost models**

It is well known that prices for a given brand vary geographically across stores even if the market is competitive (Stigler, 1961). Suppose that consumers are fully informed about product quality but are not fully informed about the prices of different brands across retail outlets. Pratt et al. (1979) develop a search model where consumers know the subjective probability distribution of prices in the market but do not know the price a particular retailer charges for any given brand. The Pratt et al. model postulates that consumers search sequentially across stores to discover the lowest price for a given brand, possibly updating their beliefs about prices, using Bayes’ rule. At any point, consumers can choose to quit the market, make a purchase from the set of sampled brands, or continue searching. Consumers stop searching when the expected benefit from incremental search (i.e., a price reduction) equals the expected cost of additional search. Under these conditions there is a unique price-quality equilibrium in the market. Let $j$ index retail stores. Then:

$$ p_j = f(x_{1j}, \ldots, x_{Mj}) + s_j $$

where the store effect $s_j$ captures the geographical price dispersion in the market.

Note that $f$ represents the “average” retail price of brand $i$ in a market where consumers are fully informed about quality but price information is costly. Furthermore, $s_j$ is not stochastic: it simply describes the equilibrium price dispersion across stores. Interestingly, equation (2) can also be derived from several other search models where consumers are uncertain about sellers’ prices but not about product quality (see Butters, 1977; Salop and Stiglitz, 1977; Braverman, 1980).

In sum, extant search cost models can explain the geographical dispersion of prices in a two-tier distribution structure where firms sell to retailers who, in turn, sell to consumers. However, these search models assume, unrealistically, that all consumers are fully informed about the qualities offered by different brands.

**Price-signaling models**

As discussed above, the hedonic pricing and search cost models assume that all consumers are fully informed about product quality. Suppose more realistically that some consumers are informed about product attributes and others are uninformed. Consider the extreme case where all consumers are uninformed. Then the market will “fail” because only low-quality producers will offer their products on the market (Akerlof, 1970).

In reality, some, but not all, consumers are uninformed about quality. In such cases, in contrast to the competitive model, price can play an informational role. Specifically, firms can attempt to appeal to uninformed consumers by using high prices as a signal of high quality. Thus, firms that signal will gain sales from the segment of uninformed consumers which believes the signal. They will also lose sales from informed consumers who go elsewhere. The interesting question is to determine if an equilibrium exists in such a market and, if so, to characterize it.

The economics literature has examined this problem in detail, assuming that the manufacturer sells directly to consumers. Wolinsky (1983) develops a search model where some, but not all, consumers are uninformed about quality. The Wolinsky model allows for a wide range of consumer information-processing strategies. For example, uninformed consumers can form judgments in “attribute space” and simply perceive the objective attributes with error. Alternatively, they can idiosyncratically map the physical attributes to a set of unobservable perceptual dimensions (e.g., benefits) with error. Thus, the model allows for errors in perception and judgment that can vary across consumers.

In Wolinsky’s model, consumers buy directly from the firm. Their search strategy is as follows. First choose a price level. Then search randomly among brands with this price level and compare marginal benefits and costs. Depending on the signal obtained (this depends on the objective attributes and the biases in consumer information-processing), consumers update their perceptions, decide to buy one of the brands examined, quit the market, or continue sampling brands.
3. A multichannel pricing model

In this section, we develop an equilibrium pricing model in which consumers have incomplete information about both product quality and prices and buy products from intermediaries (e.g., retailers). Suppose the manufacturer uses a two-level distribution channel. For example, manufacturers sell to retailers who, in turn, sell to consumers. The problem is to develop an equilibrium model in which consumers have incomplete product information and incomplete price information across stores.

Consider the following three-stage game. In the first stage, each manufacturer sets a price (e.g., the “list” price) for its brand. In the second stage, consumers decide which brand to purchase. Recall that retail stores are free to set their own prices for each brand. Thus, the retail price for a given brand at any given retail outlet need not coincide with the corresponding list price set by the manufacturer. In the third stage, consumers search among retail outlets and choose which store to purchase from.

Let $f$ denote the number of retailers in the marketplace. For any retailer $j$, let the difference between the list price for a given brand and the store price for that brand be $s_j$, where $s_j$ denotes the store effect. Then, the equilibrium structure of prices across brands and stores is given by:

$$p_j = f(x_{1i}, \ldots, x_{ni}) + u_i + s_j + w_j$$

where $p_j$ denotes the price of brand $i$ in store $j$, $s_j$ denotes the store effect for brand $j$, and $w_j$ is a stochastic disturbance term such that $E(w_j) = 0$. Note that equation (4) reduces to the standard one channel price-signaling model (equation (3)) when the manufacturer sells directly to the consumer (all store effects are equal to zero). Furthermore, equation (4) reduces to the standard geographical price dispersion model (equation (2)) when consumers are fully informed about product quality and manufacturers do not use price as a signal (the $w_j$ are zeros).

To test the theory empirically, we proceed as follows. Suppose we collect price data on different brands using a random sample of stores. Let $\bar{p}_i$ denote the mean price of brand $i$ across stores in the sample and let $E(s_j) = \alpha$. Then equation (4) becomes:

$$\bar{p}_i = f(x_{1i}, \ldots, x_{ni}) + \alpha + u_i + v_j$$

where $v_j$ denotes the new disturbance term capturing $w_j$ and price variance across stores.

The theory can now be tested in a straightforward manner. See the Appendix for details. Estimate equation (5) with the $u_j$s and $v_j$s free (i.e., unconstrained) and with the $u_j$s set
identically to zeroes (i.e. constrained). If the unconstrained model fits the data better than the constrained model, the price signaling hypothesis is supported (i.e. at least one \( u_i \) is positive).

Suppose the price-signaling hypothesis is supported for any given product category. Then the estimates \( \hat{u}_i \geq 0 \) measures the price signals for brand \( i \). To measure the strength of brand \( i \)'s price signal it is necessary to rescale the \( \hat{u}_i \). Thus, a price signal of \( $1 \) for a brand whose average price is \( $20 \) is much weaker than a price signal for a brand whose average price is \( $10 \). We therefore use the quantity \( \hat{u}_i/\hat{\rho}_i \) to measure the strength of brand \( i \)'s price signal[1].

Before testing our model, we review the existing empirical literature on hedonic pricing, search models, and price dispersion.

4. Previous empirical studies

This section reviews previous empirical methodologies that examine market efficiency and price dispersion in the marketplace.

Hedonic pricing models

Numerous empirical studies in economics and marketing have estimated hedonic regression pricing models. However, as discussed, the hedonic pricing model implicitly assumes that all consumers are fully informed about product qualities and prices. Consequently, the hedonic pricing model cannot be used to test if markets are efficient (the focus of our research). The hedonic pricing model is a special case of our model where the \( u_i \)s are zeros in equations (4) or (5).

Price dispersion studies

Pratt et al. (1979) estimate the degree of retail price dispersion for a particular brand. However, their model assumes that consumers are fully informed about product quality but have incomplete information about prices. This model is a special case of our model (equation (4)) in which all signaling effects are identically zeroes.

Empirical studies on market efficiency

Several authors argue that markets are inefficient to the extent that some measure of price dispersion is “large” (Oxenfeldt, 1950; Mayne and Assum, 1982; Geisfeld et al., 1986). This approach can be seriously misleading because it measures price dispersion without controlling for the different levels of attributes provided by competing brands. Other authors measure market efficiency in terms of the Spearman rank-correlation between price and an ordinal measure of quality (Morris and Bronson, 1969; Riesz, 1979; Geisfeld, 1982; Curry and Faulds, 1986; Spores, 1986; Curry and Riesz, 1988). This approach assumes unrealistically that the researcher can correctly rank \( f \) (i.e. the full-information price) across all brands and that \( u_i = 0 \). In addition, it does not account for the equilibrium distribution of prices across stores. Hence weak measured price-quality relationships found in these studies do not necessarily imply that product markets are inefficient.

Hjorth-Andersen (1984) treats quality as a multidimensional construct and uses a simple dominance criterion to determine market efficiency. Specifically, brand \( i \) is inefficient if some other brand in the product category is cheaper but provides a higher or an equal level of all the salient attributes than brand \( i \). This method is in the spirit of equation (3). However, the method does not recognize that the price of a brand varies across retail outlets. In addition, the method does not provide a measure of the degree of market inefficiency for each brand in the product category.

Kamakura et al. (1988) use a linear programming-based method called data envelopment analysis, or DEA, to measure the degree of product market inefficiency (see Charnes et al., 1983). This method is also similar to equation (3). However, in contrast to the Hjorth-Andersen approach, it provides estimates of market inefficiency, \( u_i \). Specifically, DEA estimates separate functions \( f \) for each brand using a piecewise linear approximation. Consequently, DEA allows a high degree of flexibility in fitting the data. However, the DEA method has several limitations. DEA fits different functions for all brands. This is problematic because equilibrium theory requires that the function \( f \) is common across brands. DEA also uses a technical measure of efficiency that implicitly assumes that perceptions are homogeneous (Kamakura et al., 1988, p. 293). Furthermore, DEA does not recognize that there is a geographical dispersion of prices across stores and that the measured price is a random variable. In addition, DEA does not allow for omitted variables.

In summary, previous empirical methods are special cases of our model. Thus, the hedonic regression model cannot be used to test for market efficiency because it assumes that consumers are fully informed about both product quality and prices. Pratt et al. allow for limited market inefficiency because they assume that consumers are fully informed about product quality but have incomplete price information. Other empirical methods (e.g. DEA) cannot be derived from an equilibrium theory of prices. In contrast, our
empirical method (see equation (5)) is based on an equilibrium theory. Furthermore, our model allows consumers to have incomplete information about both product quality and prices.

5. Empirical results

Data
Many previous studies of the price-quality relationship use data published in consumer reports (see Oxenfeldt, 1950; Riesz, 1979; Hjorth-Andersen, 1984; Kamakura et al., 1988; Curry and Riesz, 1988). We follow this tradition and analyze data for several industries: VCRs, toasters, toilet paper, paper towels, and laundry detergents and fabric softeners (Consumer Union, 2003).

As shown in Rosen (1974), the equilibrium price-quality equation is inherently nonlinear even in a simple model in which consumers are fully informed about both product quality and prices. We therefore use logarithms of all continuous variables that were measured on a ratio scale. Categorical variables are treated as dummy variables. Interval-scaled variables (e.g. the "picture quality" of a VCR which is rated on a five-point scale) are incorporated linearly because it is theoretically improper to take logarithms of such variables.

Since we use cross-sectional data, it is important to test for heteroscedasticity (see Appendix for details). In all cases, the hypothesis of homoscedasticity could not be rejected[2]. Thus, we can compare the results for equation (5) for both the constrained and unconstrained models using the data reported by Consumer Union (2003).

VCRs
Data on eight attributes for 36 brands were used. The attributes are:
(1) Picture quality.
(2) Programming ability.
(3) Pause control.
(4) Tape flutter.
(5) Capability of extra-long recording of very high quality up to 24 hours.
(6) Number of heads.
(7) Number of channels.
(8) Format.

Attributes (1) through (4) were measured on rating scales ranging from 1 (poor) to 5 (excellent). Attributes (5) and (8) were measured as dichotomous variables.

Applying the likelihood ratio test, we find that the chi-squared statistic is 0.94, which is not significant for two degrees of freedom (see Table I). Hence the VCR market is efficient: VCR manufacturers do not use prices to signal product quality.

Toasters
Data on four attributes for 25 brands were used. The attributes are:
(1) Overall toasting performance (measured on a five-point scale).
(2) A dummy variable indicating if the toaster is a four-slice toaster or not.
(3) A dummy variable indicating if the unit is a square side-by-side unit or not.
(4) A dummy variable indicating if the unit automatically adjusts the width of the slots or not.

The empirical results are shown in Table II. Applying the likelihood ratio test, we find that the chi-squared statistic is 6.54 which is significant for two degrees of freedom ($p < 0.05$). Hence firms in the toaster market use high prices to signal high quality.

In order to measure the intensity of price signaling, we computed the average strength of the price signals across brands. This value is 20.4 percent. That is, on average, toasters are priced 20.4 percent higher than they would have been in a competitive equilibrium. Hence, the toaster market is inefficient. In contrast to the VCR market, toaster manufacturers use high prices to signal high quality.

Toilet paper
We analyzed 37 brands using four attributes:
(1) Softness.
(2) Absorbency.
(3) Wet strength.
(4) Durability.

All attributes are measured on five-point interval scales, where 5 means "excellent" and 1 means "poor". To correct for package size differences, we measured price (in cents) per roll in the package.

The chi-squared statistic for the likelihood ratio test is 5.2 which is significant for two degrees of freedom ($p < 0.10$). The average strength of the price signal across brands is 19.8 percent. Hence, on average, firms in the toilet paper industry "overprice" by 19.8 percent in order to appeal to the uninformed consumer segment. This result contradicts the popular view that markets for experience goods are efficient (Nelson, 1974) (Table III).

Paper towels
Data on five attributes for 27 brands were used. The attributes are:
(1) Absorption capacity.
Table I Parameters of the hedonic regression and signaling models: VCRs

<table>
<thead>
<tr>
<th></th>
<th>Hedonic regression</th>
<th>t-value</th>
<th>Signaling model</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-28.40</td>
<td>-4.21</td>
<td>-29.39</td>
<td>-4.70</td>
</tr>
<tr>
<td>Picture quality</td>
<td>0.36</td>
<td>1.28</td>
<td>0.36</td>
<td>0.88</td>
</tr>
<tr>
<td>Programming capability</td>
<td>0.25</td>
<td>0.93</td>
<td>0.25</td>
<td>0.69</td>
</tr>
<tr>
<td>Pause control</td>
<td>0.23</td>
<td>1.08</td>
<td>0.22</td>
<td>0.78</td>
</tr>
<tr>
<td>Tape flutter</td>
<td>0.45</td>
<td>1.23</td>
<td>0.45</td>
<td>1.17</td>
</tr>
<tr>
<td>Extra-long recording capability</td>
<td>5.51</td>
<td>4.09</td>
<td>5.52</td>
<td>3.81</td>
</tr>
<tr>
<td>Number of heads</td>
<td>0.75</td>
<td>2.52</td>
<td>0.75</td>
<td>2.79</td>
</tr>
<tr>
<td>Number of channels</td>
<td>5.29</td>
<td>3.85</td>
<td>1.55</td>
<td>4.23</td>
</tr>
<tr>
<td>Format (VHS or beta)</td>
<td>1.55</td>
<td>1.84</td>
<td>0.84</td>
<td>1.83</td>
</tr>
</tbody>
</table>

`adj R^2 = 0.86`  
`s.d. of u = 1.22`
`s.d. of v = 0.075`

Log-likelihood  
-52.86  
-52.39

Note: The average signal strength across brands was not computed because the likelihood ratio test was not statistically significant.

Table II Parameters of the hedonic regression and signaling models: toasters

<table>
<thead>
<tr>
<th></th>
<th>Hedonic regression</th>
<th>t-value</th>
<th>Signaling model</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-4.16</td>
<td>-4.41</td>
<td>-3.21</td>
<td>-6.19</td>
</tr>
<tr>
<td>Overall toasting performance</td>
<td>2.17</td>
<td>7.49</td>
<td>1.59</td>
<td>9.71</td>
</tr>
<tr>
<td>Four-slice toaster</td>
<td>0.65</td>
<td>2.03</td>
<td>1.17</td>
<td>2.43</td>
</tr>
<tr>
<td>Square side-by-side unit</td>
<td>1.66</td>
<td>4.05</td>
<td>1.71</td>
<td>3.76</td>
</tr>
<tr>
<td>Automatically adjusts width of slots</td>
<td>6.48</td>
<td>7.23</td>
<td>5.99</td>
<td>0.83</td>
</tr>
</tbody>
</table>

`adj R^2 = 0.82`  
`s.d. of u = 0.94`
`s.d. of v = 0.03`

Log-likelihood  
-20.30  
-17.03

Average signal strength = 20.4%

Table III Parameters of the hedonic regression and signaling models: toilet paper

<table>
<thead>
<tr>
<th></th>
<th>Hedonic regression</th>
<th>t-value</th>
<th>Signaling model</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.025</td>
<td>2.59</td>
<td>0.012</td>
<td>0.87</td>
</tr>
<tr>
<td>Softness</td>
<td>0.00635</td>
<td>3.00</td>
<td>0.00558</td>
<td>2.312</td>
</tr>
<tr>
<td>Absorbency</td>
<td>0.00254</td>
<td>0.99</td>
<td>0.00212</td>
<td>0.831</td>
</tr>
<tr>
<td>Wet strength</td>
<td>-0.00158</td>
<td>-0.57</td>
<td>0.00798</td>
<td>0.251</td>
</tr>
<tr>
<td>Durability</td>
<td>0.00432</td>
<td>0.00</td>
<td>0.00357</td>
<td>1.103</td>
</tr>
</tbody>
</table>

`adj R^2 = 0.32`  
`s.d. of u = 0.016`
`s.d. of v = 0.005`

Log-likelihood  
-111.73  
-114.33

Average signal strength = 19.0%

(2) Wet strength.
(3) Absorption rate in water.
(4) Absorption rate in oil.
(5) Linting.

All attributes are measured on five-point scales. The results are reported in Table IV. The chi-squared statistic is 8.66 that is significant for two degrees of freedom (p < 0.05) and the average strength of the price signal is 20.1 percent. Hence the results strongly support the price-signaling hypothesis.

Laundry detergents and fabric softeners

Data on six attributes were used:
(1) Anti-deposition on polyester.
(2) Anti-deposition on nylon.
(3) Whitening ability on polyester.
(4) Whitening ability on nylon.
(5) If a fabric softener (dummy variable).
(6) If a liquid (dummy variable).

The log-likelihood statistic is 20.60 which is highly significant for two degrees of freedom (p < 0.001) (see Table V). Furthermore, the average signal strength is 25.0 percent. Hence the laundry...
detergent and fabric softener industry is somewhat more inefficient than the paper towel industry.

To summarize, the results suggest that the markets for big-ticket durables (e.g. VCRs) are efficient. Consequently, manufacturers do not use high prices to signal high quality. In contrast, the markets for cheaper durables (e.g. toasters) are inefficient and manufacturers use high prices to signal quality. These findings are not surprising. The benefit-cost ratio to consumers from search for better quality or lower prices are higher for VCRs (a big-ticket item) than the corresponding ratio for toasters (a cheaper item). Hence the pool of uninformed consumers for VCRs is smaller than the corresponding pool for toasters. The results consistently show that markets for nondurables are inefficient. Thus, firms use price signals in nondurable markets to appeal to uninformed consumers. This occurs for two reasons. First, the benefit-cost ratio to consumers from search is low for nondurables. Hence the pool of uninformed consumers is large. Furthermore, the benefits from product attributes are often subjective (i.e. consumer perceptions are heterogeneous). Hence price signaling is an efficient marketing instrument for the manufacturer selling nondurables.

6. Conclusion

This paper develops and tests an equilibrium pricing model in which consumers have incomplete information about both product quality and prices. Our model includes conventional models (e.g. the hedonic pricing model and standard geographical price dispersion models) as special cases. We propose and test a new empirical methodology for measuring equilibrium prices in markets that are inefficient. The results strongly suggest that firms use high prices to signal high quality, regardless of whether they sell durables or nondurables. Contrary to popular theory, the markets for experience goods appear to be more inefficient than the markets for search goods. In most cases our model explains prices better than the hedonic regression model (a special case of our model).

Several areas remain for future research. Our empirical study was based on cross-sectional data. Future studies should examine how the
distribution of the price signals changes over the product life cycle (see Bagwell and Riordan, 1991). These results should be of interest to the marketing manager and public policy-makers alike. In addition, our model assumes that price is the only signal that the manufacturer can use. Future research should extend the model to the case where the manufacturer can simultaneously use multiple marketing instruments such as price and advertising to signal quality to uninformed consumers in the marketplace (see Milgrom and Roberts, 1986; Engers, 1987).

Notes

1 This measure of signal strength is reasonable if we examine brands of the same type (e.g. national brands). Note that equation (5) uses average store prices because published sources (e.g. Consumer Union, 2003) do not provide raw store-level price data. Thus, equation (5) cannot be used to estimate the geographical dispersion of store prices for different brands.

2 We performed the White (1980) test on the estimated residuals (R²). See appendix for details. The results show that the hypothesis of homoscedasticity cannot be rejected for any of the five product categories examined. The chi-squared values are: 33.2 with 33 df and ρ > 0.45 for VCRs, 9.8 with 7 df and ρ > 0.20 for toasters, 10.1 with 10 df and ρ > 0.76 for toilet paper, 25.5 with 20 df and ρ > 0.18 for paper towels, and 18.8 with 25 df and ρ > 0.81 for fabric softeners and detergents.

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Consumer Union (2003), Consumer Reports, Buying Guide, Yonkers, NY.


Geisfeld, L.V., Mayne, E.S. and Duncan, G.J. (1986), ”Informational imperfections in local consumer markets”, in Olsen, J. (Ed.), Advances in Consumer Research, Association for Consumer Research, Provo, UT.


Appendix. Estimating the multichannel pricing model

Following is an outline of the maximum likelihood procedure for estimating equation (5) using cross-sectional data such as those provided by Consumer Union (2003). Our approach parallels Schmidt (1976).

Let \( \hat{p}_i \) denote the mean price of brand \( i \) in the sample and \( x_i = (x_{i1}, \ldots, x_{in}) \) the \( n \)-dimensional vector of salient attributes where the attributes can be discrete or continuous. Then the price-quality equilibrium relationship is given by:

\[
\hat{p}_i = f(x_i) + \alpha + u_i + \nu_i
\]  
(A1)

where \( u_i \geq 0, \nu_i = (\xi_i - \alpha) + \omega_i \), where \( \xi_i \) denotes the mean store effect in the sample and \( E(\nu_i) = 0 \). Recall that price signaling theory will be supported if and only if the unconstrained model (\( u_i \geq 0 \)) fits better than the constrained model (\( u_i = 0 \)).

For expository convenience, assume that \( f(x_i) \) is linear in the variables. That is, consider:

\[
\hat{p}_i = a_0 + a_1 x_{i1} + \ldots + a_n x_{in} + u_i + \nu_i
\]  
(A2)

where the \( a \) denote parameters. As in standard regression models, the methodology can be used if \( f \) is nonlinear in its arguments provided these effects are linearly separable.

Assume as in standard regression models that Cov(\( u_i, \nu_i \)) = 0 for all \( i \), Cov(\( x_{ij}, u_i \)) = 0 for all \( i \) and \( j \), and Cov(\( x_{ij}, \nu_i \)) = 0 for all \( i \) and \( j \). Let \( u_i \) have a half-normal distribution such that \( u_i \geq 0 \), and \( \nu_i \) be normally distributed. Let \( \epsilon = u + \nu \). In order to form the likelihood function, it is necessary to obtain the density function of the composite error term \( \epsilon \) (Figure A1).

We proceed using the convolution approach. The density function for the sum of a normal and a half-normal distribution is (Aigner et al., 1964):

\[
h(x) = \frac{[2f(x)F(x/\sigma)]}{\sigma}
\]

where

\[
\sigma^2 = \sigma_u^2 + \sigma_\nu^2, \lambda = \frac{\sigma_u}{\sigma_\nu},
\]

\( f \) denotes the density function of a normally distributed random variable, and \( F \) denotes the appropriate distribution function.

Then the log-likelihood function to be maximized is:

\[
\ln(\hat{\beta}/\alpha, \lambda, \sigma^2) = \ln \left[ \left( \frac{2}{\pi} \right) + \sum_{i=1}^{I} \frac{F(\nu)}{\sigma} \right]
\]

\[
+ \ln \left( \frac{1}{\sigma} - \sum_{i=1}^{I} \frac{\nu_i^2}{2\sigma^2} \right)
\]  
(A4)

where \( \hat{\beta} \) denotes the vector of sample mean prices and \( I \) denotes the number of brands.

This maximization problem can be solved using a variety of iterative algorithms (see Aigner et al., 1977). Once the maximum likelihood parameter estimates are obtained, it is straightforward to compute the covariance matrix for the parameters using the appropriate information matrix. All the usual maximum likelihood properties apply (i.e. the estimates are consistent and asymptotically efficient).

Let \( L^* \) denote the solution that maximizes equation (A4). Let \( L^{**} \) denote the solution that maximizes equation (A4) subject to the constraints \( u_i = 0 \).

Suppose the error terms \( (\nu, \xi) \) are homoscedastic (we will discuss tests and empirical procedures for heteroscedasticity later). Then, it is straightforward to use the likelihood ratio test to determine if price is a signal of quality (Maddala, 1977, p. 44). Let \( Q = -2(L^{**} - L^*) \). For large samples, \( Q \) has a chi-squared distribution with two degrees of freedom because the full model requires two additional parameters. Thus, the price-signaling hypothesis is supported if \( Q \) is statistically significant.

Suppose the price-signaling theory is supported. In order to determine the strength of the price signal, we need to estimate \( u_i \) for each brand. To obtain a point estimate of \( u_i \), we use the mean of the conditional distribution of \( u_i \) given \( \epsilon \). This estimate is given by:

![Figure A1 Normal versus half-normal distribution](image)

**Note:** The normal distribution is symmetrical while the half-normal distribution has no representation on the negative side of the number line. On the positive side, the density function of the latter is twice that of the former at each point on the x-axis.
where \( f \) and \( F \) respectively, denote the standard normal and cumulative normal density functions.

Once the \( \mu_i \) have been estimated, it is easy to determine the \( \epsilon_i \) by subtraction from the \( \xi_i \).

Now it is natural to interpret the ratio \( q_i = \frac{u_i}{\tilde{p}_i} \) as the strength of brand \( i \)'s price signal. Let \( q \) denote the average of these values across all brands in the industry. Then \( q \) measures the magnitude of price signaling in the industry. These \( q \) values can be compared across industries to determine which industries are most inefficient in an informational sense.

The tests described above assume homoscedasticity. However, it is possible that the error terms are heteroscedastic. We therefore used the following procedure to test for homoscedasticity. Perform the White test (1980) for heteroscedasticity on the estimated residuals \( (\epsilon_i) \) in equation (5). If the hypothesis of homoscedasticity cannot be rejected, the results from the procedure described above will hold. If the hypothesis of homoscedasticity is rejected, it is necessary to respecify and reestimate the model. This procedure was not necessary for any of the product categories examined in the study.