Facilitating the search for partners on matching platforms: Restricting agent actions*

Yash Kanoria[†] Daniela Saban[‡]

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Abstract

Two-sided matching platforms, such as those for labor, accommodation, dating, and taxi hailing, can control and optimize over many aspects of the search for partners. To understand how the search for partners should be designed, we consider a dynamic model of search by strategic agents with costly discovery of pair-specific match value. We find that in many settings, the platform can mitigate wasteful competition in partner search via restricting what agents can see/do. Surprisingly, simple restrictions can improve social welfare even when screening costs are small, and agents on each side are ex-ante homogeneous. In asymmetric markets where agents on one side have a tendency to be more selective (due to smaller screening costs or greater market power), the platform should force the more selective side of the market to reach out first, by explicitly disallowing the less selective side from doing so. This allows the agents on the less selective side to exercise more choice in equilibrium. When agents are vertically differentiated, forcing one side of the market to propose results in a significant increase in welfare even in the limit of vanishing screening costs. Furthermore, a Pareto improvement in welfare is possible in this limit; the weakest agents can be helped without hurting other agents. In addition, in this setting the platform can further boost welfare by hiding quality information.

Keywords: matching markets, market design, search frictions, stationary equilibrium, sharing economy, platforms.

1 Introduction

During the last decade there has been rapid growth in the number of online platforms for two-sided matching in the contexts of dating, labor markets, accommodation, and taxi services, among others. Although these markets appear to fit a traditional supply-demand setting at first glance, a key differentiating feature is that agents on *both* sides of the market (e.g. hosts and guests in accommodation, workers and employers in online labor markets) have heterogeneous

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[†]Decision, Risk and Operations Division, Columbia Business School, Email: `ykanoria@gsb.columbia.edu`

[‡]Operations, Information & Technology, Stanford Graduate School of Business, Email:`dsaban@stanford.edu`

preferences over people/places/jobs they could match to in many of these marketplaces. Thus, in sharp contrast to a traditional buyers-sellers setting in which sellers are typically indifferent between buyers, accounting for the role of heterogeneous preferences on both sides of the market becomes crucial when designing matching platforms.

Whenever it is possible to obtain or elicit adequate information regarding the agents' preferences, the platform can use this information to determine matches centrally and save the agents the effort needed to find a match. However, obtaining such information is not always possible as there might be significant heterogeneity in agents' preferences (a "beauty lies in the eye of the beholder" component to preferences) which cannot be easily uncovered by the platform. In such situations, the platform can instead provide agents with information about potential partners based on which they can decide whether to interact and eventually match with them. However, this flexibility comes at the cost of search and screening effort required on the part of market participants. As an example, consider a guest looking to rent a place on Airbnb for a short stay. The platform provides a few filters to help potential guests to narrow their search (such as accommodation type, price range, etc.), but even after filtering, tens or hundreds of options may remain. Guest preferences over these remaining options may involve guest and trip-specific trade-offs between location, reviews, pictures, house rules, etc., that the platform may have difficulty uncovering. Presumably this is why the platform allows guests to take a closer look at the filtered listings before choosing, though this may be a costly exercise for guests.

In addition to allowing guests to screen options before requesting to book, a specific choice Airbnb has made here is to require guests to browse listings by hosts, and not the other way around. It is natural to ask whether this choice is consequential (evidence from TaskRabbit suggests that it is [10]) and whether it is the right one. The current paper studies the question of how a platform should choose its "search environment" design, i.e., the framework within which agents can acquire more information about potential matches, contact them, and match with them.

The "search environment" designs typically used in practice roughly fall into three categories: centralized, one-sided search, and two-sided search. In a *centralized matching* design (as used by Uber and Lyft), the marketplace chooses the matches and thus agents do not engage in active search for partners. In a *one-sided search* design, the platform allows only one side to search through available options and pick a suitable match. This match can then quickly be secured, as agents on the other side play a passive role and do not exercise choice. Examples in this category are "Instant Book" on AirBnb, and "Quick Assign" on TaskRabbit. Finally, in a *two-sided search* design, agents on both sides of the market are able to screen potential partners, and a match results only upon approval by both sides of the market. In this setting, the platform may provide both sides of the market with the ability to reach out to potential partners (e.g. OkCupid, Upwork), or implement *directional search*, in which one side of the market is required to reach out/apply/request, leaving the other side of the market to accept or reject the application (e.g., guests must "Request to Book" on Airbnb, and women must send the first message on the dating platform Bumble).

The evidence strongly suggests that the choice of search design can be critical for the success of the platform in the presence of search frictions. In fact, the impact of search frictions has been documented in the context of AirBnB [12], Upwork [18] and TaskRabbit [10]. [10] further demonstrates the importance of search environment design; it finds a significant reduction of search costs and increase in the number of matches formed ("match efficiency") as a consequence of re-design of TaskRabbit in the spring of 2014, a key component of which was a change from taskers bidding on jobs, to clients inviting taskers. In this paper, we aim to provide a theoretical framework to understand the impact that different design choices have on the overall welfare of market participants, and hence guide platform design. In particular, we take the point of view of a monopolistic platform that, given a set of market characteristics (i.e., volume of agents on each side, screening costs, agent types, variation in preferences, etc.), chooses a design to maximize welfare.[1]

Our model. Our first contribution is our model of two-sided matching without transfers, mediated by a platform (Section 2). Our model, while stylized, captures two distinctive features of these markets: dynamic arrivals and departures, and strategic agents on each side of the market who have idiosyncratic values for matching which that can only be discovered after incurring a cost; this allows us to capture agent-pair level heterogeneity in preferences ("beauty lies in the eye of the beholder") that the platform cannot uncover a priori, which is a key feature of our model. Agents on both sides of the market can choose whether to issue proposals, and can also choose whether to screen before issuing/accepting proposals. If a match occurs, both agents leave the market. Agents unable to match over an extended period (exogenously) leave the market. We consider stationary equilibria [17], where agent best responses are utility maximizing solutions to their optimal stopping problem in the steady state of the market.

We study the welfare achieved under the equilibria that arise in this *no platform intervention* setting, in which both sides are allowed to propose and uncover the idiosyncratic match values via screening. Next, we consider the equilibria that arise under simple platform interventions corresponding to the search environment designs discussed earlier. Specifically, we allow the platform to block one or both sides from screening (leading to one-sided search or centralized matching respectively), or to block a side from proposing (leading to a directional search design), and ask if the platform can boost welfare via such interventions.

Main findings. We start by characterizing equilibria in the no intervention setting, and find that three equilibrium regimes arise under no intervention: when screening costs are small relative to idiosyncratic variation in match values, one side screens and proposes, and the other side screens and accepts/reject incoming proposals; with medium-sized screening costs, one side proposes without screening and the other side screens and accepts/rejects (similar to what

[1]We suppress issues of pricing and revenues here. If each agent (on either side) operates in a particular price range (e.g., $25-30/hour on Upwork), it may be reasonable to think of the overall market as composed of a number of submarkets, each corresponding to a relatively narrow price range, within which prices charged by agents do not play a significant role. There may still be some tension between maximizing revenues and maximizing user welfare, which we suppress, noting only that agent welfare is crucial even for a revenue maximizing platform, since user retention depends on it.

occurs under a one-sided search design); finally, with large screening costs, both sides of the market propose and accept proposals without screening (equivalent to the outcome of centralized matching).

Our main contribution is to show that, even with the simple interventions considered, *suitable interventions can significantly boost welfare in markets with asymmetries across the two sides of the market*, even if the agents on each side are ex-ante homogeneous. Externalities resulting from selectivity by agents, both same-side and cross externalities, play an important role in determining the welfare under different equilibria and designs. Selectivity by proposal recipients can have a *negative cross externality* due to wasted screening effort by proposers. On the other hand, selectivity in matching with scarce agents has a *positive same-side externality* because it increases the effective availability of options on the same side. We next describe how the interventions help in different settings.

In markets where agents on opposite sides of the market have different screening costs (Section 3.1), but the two sides are otherwise identical, the platform is able to boost welfare by *selecting* a high welfare equilibrium : (i) In a small screening cost regime, the platform should block the side with higher screening costs from proposing, forcing the side with lower screening costs to do so. This allows the side with higher screening cost to be somewhat selective, improving their welfare, at a small cost to the lower-screening-cost side that now faces occasional rejection, (ii) In a medium-sized screening costs regime, the platform should implement a one-sided search design in which the side with lower screening costs chooses. The reason is that the benefit to the higher-screening cost side of allowing it to screen is less than the consequent negative externality on the the lower-screening cost side.

In unbalanced markets (Section 3.2), where the arrival rate of agents is faster on one side (the long side) than the other side (the short side), but the two sides are otherwise identical, we find (under mild assumptions) that the long side proposes in *all* equilibria. (This finding is facilitated by our consideration of stationary equilibria in a setting with dynamic arrivals and departures.) Proposals by agents on the long side are accepted only rarely, hence agents on the long side cannot afford to be too selective, which hurts them in addition to the risk they face of dying without matching. We find that the platform can significantly boost welfare by preventing the long side from proposing. This *creates* new equilibria in which the long side is able be more selective when considering incoming proposals, as the risk of rejection is eliminated (though the risk of dying without matching remains). This intervention provides a significant welfare boost to the long side at a small cost to the short side (which now faces infrequent rejection).

Finally, we study markets with vertical differentiation (Section 4) by allowing the long side of the market (call them workers) to have two quality levels, top and bottom, where top workers are fewer than employers (employers are ex ante identical). The platform knows the quality of each worker and, under no intervention, makes this information visible to the employers. In the resulting equilibrium, the bottom workers suffer low welfare due to two equilibrium features, which hold even when screening costs vanish: bottom workers propose *without screening* since most of their proposals are *ignored* by employers waiting for a dream match to a top worker, and further, some of these employers who are waiting for a dream match end up leaving unmatched, reducing the number of employers available to match with bottom workers. Our dynamic model, with populations of different agents who are present in the market being *endogenously determined*, enables us to identify these phenomena. We next identify a number of different interventions that the platform can employ to improve welfare. As in the case of unbalanced markets with no vertical differentiation, the platform is able to boost to welfare by preventing the workers from proposing, thus allowing bottom workers to be somewhat selective. The platform can further increase welfare by *hiding the types of workers* from employers, because employers no longer wastefully die while waiting to match with a top worker. Importantly, the boost in welfare of bottom workers is possible without significantly hurting any other type of agent; a Pareto improvement of welfare occurs in the limit of vanishing screening costs.

1.1 Related Literature

The present work draws inspiration from the literatures on search frictions in labor markets (which motivates our model of search), work on two-sided platforms (which highlights the role of externalities), and the stable marriage literature (which studies heterogeneous agent preferences and market design concerns). We discuss each of these in turn.

The search frictions literature has traditionally focused on macro level job growth and unemployment under search frictions, using relatively crude models of agent level behavior (e.g., see [24]). In typical models, agents meet one another randomly (in direct proportion to their mass in the unmatched pool). Each agent is assumed to have an inherent quality which represents her desirability as a match partner, and this leads to assortative matching in equilibrium. Search is costly due to the *time it takes to meet* a potential partner such that a match can occur. In the current work, we incorporate agent qualities (Section 4) but focus on heterogeneity in agent preferences, the key search cost being the *cost of screening* to learn the idiosyncratic utility for a potential partner. We study not only the structure of the equilibria that arise, but also what design a matching platform can deploy to improve the equilibrium welfare of agents, in other words, we are focused on the *impact of the matching technology ("search environment") on market performance.*

The literature on platforms (e.g. [25, 15]) has focused on platform-level effects of features like participation, and issues of attracting users and competition between platforms, and pricing. This line of work zooms in on the role of externalities, particularly cross-side externalities, while using crude agent-level models. In particular, it suppresses the search for partners, and does not lend itself to understanding the role of search environment design.

The stable marriage literature [13, 28, 29, 2] has focused on two-sided heterogeneity in agent preferences ("beauty lies in the eye of the beholder") and market design issues, but suppressed search related concerns. There has been some work on signaling in matching markets when there is a constraint on the number of signals (e.g., [9, 8]). In contrast, we explicitly model search costs.

The operations literature (including work on inventory management [30], revenue management [31], dynamic programming [6] and queueing [3]) has built a deep understanding of dynamics and decision making, but has traditionally focused on cases in which units on one side of the market (usually representing inventory/products/servers) do not have strategic considerations (e.g., inventory units can be purchased and sold, whereas agents may decide whether to participate and whom to match with). Recent papers explore how operational decisions can be used by a platform to improve its performance, e.g., [5, 14]. Perhaps the one most related to our work is the paper by Allon et al [1]. They find that improving operational efficiency of a platform may reduce market efficiency due to negative externalities, similar in spirit to some of our findings albeit in a very different setting; while they consider a buyer-seller setting, where the seller sets a price and is indifferent as to whom he is serving, we consider a two-sided matching market, where agents on both sides have heterogeneous preferences over agents on the other side. Our equilibrium concept draws upon the notion of mean field equilibrium which has been effectively employed in the operations literature to study complex dynamic games involving many players [19, 4].

One of the interventions we suggest in the current paper involves hiding information. Several papers [23, 21] find that hiding information about market participants can serve to prevent the market from unraveling or to reduce cherry-picking. A recent concurrent paper [27] also explores the benefit of hiding information, albeit in a buyers-sellers setting. Our findings on the benefits of hiding information are somewhat similar in spirit; we find that the platform can induce agents to consider a larger set of potential partners by hiding tier information.

2 Model

We model a dynamic two-sided matching market without transfers, mediated by a platform, with agents on each side being ex-ante homogeneous. (Later, in Section 4, we describe and study a model with vertical differentiation, where the platform knows agent quality.)

We first informally describe a setting with discrete agents, describing events and actions at the level of individual agents in Section 2.1. Section 2.2 specifies our formal model, which defines the system level evolution of a continuum of agents based on the motivation in Section 2.1.

2.1 Informal motivation

We refer to the two sides of the market as workers and employers. Workers arrive at rate λ_w and employers arrive at a rate λ_e. When a match forms, the concerned agents leave the market immediately; we describe the dynamics of search and match formation below. Unmatched agents leave due to "death" exogeneously at a (small) rate $\mu > 0$, common across all agents.

Each agent i has an idiosyncratic match value u_{ij} for every agent j on the other side. (This is different from u_{ji}, the value that j would derive if matched with i. In our benchmark model, u_{ij} is independent of u_{ji}.) We assume that the u_{ij}'s are independent identically distributed (i.i.d.) with distribution F for workers over employers, and i.i.d. with distribution G for employers over workers. For simplicity, we focus on the case where both F and G are Uniform$(0, 1)$. We assume that F and G are common knowledge among agents and the platform, whereas u_{ij}'s are unknown a priori. Given the opportunity, an agent i can *privately* learn u_{ij} for any j on the other side by spending a *screening cost*, which is c_w if i is a worker and c_e if i is an employer.

Dynamics of search and matching, and agents' strategies. Each agent has a Poisson "opportunity" clock of rate 1. Each time an agent i's clock ticks, she can (costlessly) request to view a "candidate" –an available agent on the other side, unexplored by i thus far–, in which case the platform shows i a uniformly random candidate (recall agents are ex ante homogeneous), if any is available. Agent i can spend the screening cost to learn her idiosyncratic valuation u_{ij} for j, or not screen. Either way, agent i then decides whether to propose to j. If i proposes, her proposal is conveyed to j. In turn, agent j decides whether to spend the screening cost to learn u_{ji}, and then whether to accept i's proposal. If j accepts, a match is formed and the pair leaves the market, else both agents stay. All these events occur instantaneously after a clock ring.

An agent's strategy is specified by: (i) Does she request a candidate when an opportunity arises? Does she screen candidates? (ii) If she receives a proposal, does she consider it? Does she screen the proposer? (iii) When she screens, what is her minimum acceptable match utility (her "threshold")?

Equilibrium concept. We study stationary equilibria (see Section 2.3), in which all agents are playing best responses to the steady state volume of agents following each strategy.[2] (In a large market, all stationary equilibria can be captured by considering the case where individual agents play deterministic time-invariant strategies, so we restrict attention to this subclass of agent strategies. Also, any agent best response will involve an acceptability threshold equal to the continuation value, and hence we restrict attention to agent strategies where a single threshold is used post-screening: whether for proposing or for accepting/rejecting an incoming proposal.) We further consider an equilibrium refinement ("evolutionary stability") that rules out implausible equilibria.

Platform interventions considered. We allow the platform to intervene by constraining agents' actions in specific ways:

- *Shutting down screening*: The platform can prevent agents on one or both sides of the market from screening, which we term a *one-sided search* or a *centralized matching* design respectively (see Section 1).
- *Directional search*: When agents on both sides of the market are allowed to screen, this is a *two-sided search* design. Even here, the platform may constrain agents by preventing agents on one side of the market from proposing.

In each setting considered, we characterize the equilibria that arise under no intervention, and compare them with equilibria under each of the considered platform designs. We formally

[2]The positive death rate ensures that the market reaches a steady state. The clock speed of 1 and death rate of μ together lead to each agent having a large number Geometric$(\mu/(\mu+1)) - 1 = \Theta(1/\mu)$ opportunities during their lifetime. In the case of unbalanced markets, we will find that agents on the long side can see only Geometric$(\beta_\mu) - 1$ where $\lim_{\mu \to 0} \beta_\mu = \beta > 0$ candidates during their lifetime, due to unavailability on the other side, leading to a probability of matching before dying that is bounded away from 0 and 1, as expected.

define the model and the equilibrium concept next. (The initial part until Eq. (1) is most significant.)

2.2 Formal description

There is a continuum of workers and employers, with arrival flow λ_w and λ_e respectively. Agents select a deterministic strategy upon arrival, which is assumed to be fixed throughout their lifetime (see Section 2.1).

Definition 1 (Agents' strategies)**.** *We consider agent strategies $s = (a, \theta)$ defined by:*

1. *A deterministic selection $a = (a^{\mathrm{i}}, a^{\mathrm{o}})$, chosen from among $3 \times 3 = 9$ possibilities:*
   - *(i) The selection $a^{\mathrm{i}} \in \{\mathrm{I}, \mathrm{A}, \mathrm{S{+}A/R}\}$ specifies how incoming proposals are handled: whether the agent ignores them (I), accepted without screening (A), or screened and then accepted/rejected (S+A/R).*
   - *(ii) The selection $a^{\mathrm{o}} \in \{\mathrm{N}, \mathrm{P\ w/o\ S}, \mathrm{S{+}P}\}$ specifies how opportunities are handled: whether the agent passes on the opportunity (N), or proposes without screening (P w/o S), or screens and then proposes or not (S+P).*
2. *A deterministic threshold θ used to screen participants: the agent proposes/accepts a proposal if and only if the match value she learns from screening exceeds θ.*

That is, besides the choice of threshold, a strategy involves a choice a from a finite set of options. Note that an intervention in our setting will result in a restriction on the allowed a's on one or both sides. We denote the finite sets of allowed a's for workers and employers (under the chosen platform intervention, if any) by $\mathcal{S}_w$ and $\mathcal{S}_e$ respectively.

We next describe the dynamic evolution of the system. We assume up front that all agents on the same of the market whose strategy involves screening (either to issue a proposal or to accept one) will use the same threshold. (Again, we can do this because an agent's threshold should match her continuation value in any best response, cf. Section 2.1, and since agents on a particular side are ex-ante symmetric, their continuation values must agree in any equilibrium.) We further find it convenient to fix θ_w and θ_e (the thresholds for workers and employers respectively), and suppress the fact that θ is part of the strategy by identifying s with a. For a pair of thresholds (θ_w, θ_e), for each $s_w \in \mathcal{S}_w$ let $f_w(s_w)$ denote the fraction of workers who adopt strategy s_w upon entering (this fraction does not change with time), and define $f_e(s_e)$ analogously.

Dynamic evolution of the system. Let $N_w(s_w)$ be the mass of workers in the system following strategy s_w, and let $N_w = \sum_{s_w \in \mathcal{S}_w} N_w(s_w)$ denote the total mass of workers in the system. Define $N_e(s_e)$ and N_e analogously. Further, let $\bar{N}_w = (N_w(s_w))_{s \in \mathcal{S}_w}$, $\bar{N}_e = (N_e(s_e))_{s \in \mathcal{S}_e}$, and let $\bar{N} = (\bar{N}_w, \bar{N}_e)$.

The rate of change of $N_w(s_w)$ is given by

$$\frac{\mathrm{d}N_w(s_w)}{\mathrm{d}t} = f_w(s_w)\lambda_w - N_w(s_w)\mu - \rho_w(s_w;\bar{N}) \qquad \forall s_w \in \mathcal{S}_w\,, \tag{1}$$

where the first term on the right captures the arrival flow of agents following strategy s_w, the second term captures the departure-upon-death flow of such agents, and the last term, $\rho_w(s_w;\bar{N})$, denotes the flow of matches formed involving such agents, which we describe next. (The rate of change of $N_e(s_e)$ in terms of $\rho_e(s_e;\bar{N})$ is analogous.)

Matching formation rates. We define $\eta_w(s_w,s_e)$ to capture the probability that, when a worker following s_w is presented with a candidate employer following s_e, this results in a match.

$$\eta_w(s_w,s_e) = \big(\mathbb{I}(s_w \text{ involves P}) - F(\theta_w)\mathbb{I}(s_w \text{ involves S+P})\big)\big(\mathbb{I}(s_e \text{ involves A}) - G(\theta_e)\mathbb{I}(s_e \text{ involves S+A/R})\big). \tag{2}$$

The first term on the right captures the probability that, when presented with a candidate, a worker following strategy s_w proposes; the second term captures the probability that an employer following s_e will accept such a proposal (recall that we have assumed u_{ij} independent of u_{ji}).

Let $\rho_w(s_w,s_e;\bar{N})$ denote the flow of matches formed between workers following s_w and employers following s_e *as a result of proposals issued by the workers*; $\rho_e(s_e,s_w;\bar{N})$ is analogous. Clearly $\rho_w(s_w;\bar{N}) = \sum_{s_e\in\mathcal{S}_e}\big(\rho_w(s_w,s_e;\bar{N}) + \rho_e(s_e,s_w;\bar{N})\big)$, and similarly for $\rho_e(s_e;\bar{N})$. We now define $\rho_w(s_w,s_e;\bar{N})$ and $\rho_e(s_e,s_w;\bar{N})$. Consider the following cases:

1. $\mathbf{N_w > 0}$ and $\mathbf{N_e > 0}$: In this case, workers following s_w collectively are offered opportunities (employers) following strategy s_e at flow rate $N_w(s)\frac{N_e(s_e)}{N_e}$ (consistent with each agent being presented with a candidate on the other side uniformly at random at a rate of 1), and a fraction $\eta_w(s_w,s_e)$ of this flow results in matches. Therefore,

$$\rho_w(s_w,s_e;\bar{N}) = N_w(s_w)\frac{N_e(s_e)}{N_e}\eta_w(s_w,s_e), \quad \text{and similarly for } \rho_e(s_e,s_w;\bar{N})\,. \tag{3}$$

2. $\mathbf{N_e = 0}$: Suppose $N_e = 0$ (the case where $N_w = 0$ is analogous). Since $N_e = 0$, the flow of opportunities to employers is 0 so the flow of matches formed due to proposals by employers is 0, implying $\rho_e(s_e,s_w;\bar{N}) = 0$ for all s_e, s_w.

   Suppose that N_e remains zero for a non-zero interval of time. Then it must be that the flow of employer candidates following s_e shown to workers is

$$\begin{aligned}&\lambda_e f_e(s_e)/(\text{Fraction of these occurrences leads to a match})\\ &= \lambda_e f_e(s_e)\frac{N_w}{\sum_{s_w\in\mathcal{S}_w} N_w(s_w)\eta_w(s_w,s_e)}\,.\end{aligned}$$

Since a fraction $N_w(s_w)/N_w$ of this flow goes to workers following s_w, we define

$$\rho_w(s_w, s_e; \bar{N}) = \lambda_e f(s_e) \frac{N_w(s_w)\eta_w(s_w, s_e)}{\sum_{s_w \in \mathcal{S}_w} N_w(s_w)\eta_w(s_w, s_e)} \Rightarrow \rho_e(s_e; \bar{N}) = \lambda_e f(s_e). \tag{4}$$

Now, the overall flow of employer candidates to workers should not exceed N_w. Thus, the necessary condition for N_e remaining zero is

$$\sum_{s_e} \frac{\lambda_e f_e(s_e) N_w}{\sum_{s_w} N_w(s_w)\eta_w(s_w, s_e)} \leq N_w \quad \Leftrightarrow \quad \sum_{s_e} \frac{\lambda_e f_e(s_e)}{\sum_{s_w} N_w(s_w)\eta_w(s_w, s_e)} \leq 1\,. \tag{5}$$

It is easy to check that this condition is also sufficient to ensure that N_e remains zero. The complementary case $\frac{\mathrm{d}N_e}{\mathrm{d}t} > 0$ arises if and only if condition (5) is violated. The definition of $\rho_w(s_w, s_e; \bar{N})$ in this case is deferred to Appendix A.1.

Agents' utilities. For every $s_w \in \mathcal{S}_w$, let $L_w(s_w) \geq 0$ denote the *steady state mass* of workers in the system using strategy s_w induced by these thresholds and fractions, and define $L_e(s_e)$ similarly for employers. Further, let $\bar{L}_w = (L_w(s_w))_{s \in \mathcal{S}_w}$, $\bar{L}_e = (L_e(s_e))_{s \in \mathcal{S}_e}$, and let $\bar{L} = (\bar{L}_w, \bar{L}_e)$. In other words, $\bar{L}$ is a fixed point of Eq. (1).

We define *expected utility* $U_w(s_w; \bar{L})$ of workers following strategy $s_w \in \mathcal{S}_w$ in steady-state based on the informal model discussed in Section 2.1, in terms of the L's and ρ's. In particular, the expected utility is defined based on:

- *Expected benefit from matching*: the expected match utility conditional on matching, times the fraction of workers who match (as opposed to dying). We separately consider matches due to incoming and outgoing proposals. The fraction of workers following strategy s_w who match as a result of *issuing* a proposal is $\frac{\sum_{s_e \in \mathcal{S}_e} \rho_w(s_w, s_e; \bar{L})}{\lambda_w f_w(s_w)}$, and their expected match utility (conditional on matching) is

  $$\gamma_w(s_w) = \mathbb{I}(s_w \text{ involves S + P}) E[u_{we} | u_{we} \geq \theta_w] + (1 - \mathbb{I}(s_w \text{ involves S + P})) E[u_{we}],$$

  That is, if they are screening and match, their expected utility is the expected value u_{we} given that u_{we} exceeds the threshold; otherwise, it is just the expectation of u_{we}. On the other hand, the fraction of workers following strategy s_w who match as a result of an *incoming* proposal is $\frac{\sum_{s_e \in \mathcal{S}} \rho_e(s_e, s_w; \bar{L})}{\lambda_w f_w(s_w)}$, and their expected match utility conditional on matching is

  $$\hat{\gamma}_w(s_w) = \mathbb{I}(s_w \text{ involves S + A/R}) E[u_{we} | u_{we} \geq \theta_w] + (1 - \mathbb{I}(s_w \text{ involves S + A/R})) E[u_{we}].$$

  Then, the expected utility from matching is

  $$\frac{\sum_{s_e \in \mathcal{S}_e} \rho_w(s_w, s_e; \bar{L})}{\lambda_w f_w(s_w)} \gamma_w + \frac{\sum_{s_e \in \mathcal{S}} \rho_e(s_e, s_w; \bar{L})}{\lambda_w f_w(s_w)} \hat{\gamma}_w,$$

i.e., the match utility contribution from outgoing proposals plus that of incoming proposals.

- *Expected cost of screening*: the mass of employers screened per unit mass of workers during their lifetime in the system (notionally, the expected number of employers screened by a worker during his lifetime), times the screening cost c_w. We separately consider costs incurred by screening opportunities from those due to incoming proposals. For each match that resulting from an incoming proposal, workers screen $\frac{1}{1-F(\theta_w)\mathbb{I}(s_w \text{ involves S + A/R})}$ proposals in expectation. Further, workers following strategy s_w match due to incoming proposals at rate $\sum_{s_e \in \mathcal{S}} \rho_e(s_e, s_w, \bar{L})$. Hence, a mass $\lambda_w f_w(s_w)$ of workers following s_w will collectively screen a mass

$$\hat{\upsilon}_w(s_w) = \frac{\sum_{s_e \in \mathcal{S}} \rho_e(s_e, s_w, \bar{L})}{1 - F(\theta_w)\mathbb{I}(s_w \text{ involves S + A/R})}$$

of incoming proposals at a cost c_w per unit. Thus, the expected cost of screening incoming proposals for workers following strategy s_w is $\frac{c_w}{\lambda_w f(s_w)} \hat{\upsilon}_w(s_w)$.

We now consider the cost of screening candidates when opportunities arise. Let $\mathcal{S}'_e$ be the set of employer strategies that consider incoming proposals. Note that since workers are now proposing, each such match with an employer following strategy $s_e \in \mathcal{S}'_e$ requires a worker to screen $\frac{1}{\eta_w(s_w, s_e)}$ opportunities in expectation. Therefore, the flow rate at which workers following s_w screen candidates who would consider their proposals is given by $\sum_{s_e \in \mathcal{S}'_e} \frac{\rho_w(s_w, s_e; \bar{L})}{\eta_w(s_w, s_e)}$. However, workers might also waste search effort in screening employers who ignore all incoming proposals. In fact, a fraction $\frac{\sum_{s_e \in \mathcal{S}_e \setminus \mathcal{S}'_e} L_e(s_e)}{L_e}$ of the proposals get ignored, and the rate at which proposals are actually screened is then[3]

$$\upsilon_w(s_w) = \left(\sum_{s_e \in \mathcal{S}'_e} \frac{\rho_w(s_w, s_e; \bar{L})}{\eta_w(s_w, s_e)} \right) \left(\frac{L_e}{L_e - \sum_{s_e \in \mathcal{S}_e \setminus \mathcal{S}'_e} L_e(s_e)} \right).$$

Overall, the expected cost for screening opportunities is $\frac{c_w}{\lambda_w f(s_w)} \upsilon_w(s_w)$.

By the preceding discussion, the expected utility of workers following strategy $s_w \in \mathcal{S}_w$ can be formally defined as:

$$U_w(s_w; \bar{L}) = \frac{\sum_{s_e \in \mathcal{S}_e} \rho_w(s_w, s_e; \bar{L})}{\lambda_w f_w(s_w)} \gamma_w + \frac{\sum_{s_e \in \mathcal{S}_e} \rho_e(s_e, s_w; \bar{L})}{\lambda_w f_w(s_w)} \hat{\gamma}_w - \frac{c_w}{\lambda_w f(s_w)} \left(\upsilon_w(s_w) + \hat{\upsilon}_w(s_w) \right) \tag{6}$$

Finally, *average welfare* in steady state, the performance metric used in the paper, is defined as the average expected utility of arriving agents across the two sides.

$$\text{Average welfare}(\bar{L}) = \frac{\lambda_e \sum_{s_e \in \mathcal{S}_e} f_e(s_e) U_e(s_e; \bar{L}) + \lambda_w \sum_{s_w \in \mathcal{S}_w} f_w(s_w) U_w(s_w; \bar{L})}{\lambda_e + \lambda_w}. \tag{7}$$

[3] If $L_e = \sum_{s_e \in \mathcal{S}_e} L_e(s_e)$, all proposals are ignored and thus the workers' match rate as a result of outgoing proposals is zero. We then assume that, in that case, the whole expression is zero.

2.3 Equilibrium concept: Evolutionarily Stable Stationary Equilibria

We draw upon the notions of mean field equilibrium (effectively employed in the operations literature to study complex dynamical games involving many players [34, 19, 4, 7]), and stationary equilibrium, introduced by Hopenhayn [17], which considers game-theoretic equilibria corresponding to dynamical steady state (again in a large market limit). These equilibrium notions relax the informational requirements of agents, requiring them only to know the aggregate description of the system, which makes them behaviorally appealing and tractable.

Definition 2 (Stationary equilibrium)**.** *Fix the distributions of agents' strategies $\{f_w(s)\}_{s\in\mathcal{S}_w}$ and $\{f_e(s)\}_{s\in\mathcal{S}_e}$ and the thresholds θ_w and θ_e, and suppose $\bar{L}_w = (L_w(s_w))_{s\in\mathcal{S}_w}$, $\bar{L}_e = (L_e(s_e))_{s\in\mathcal{S}_e}$, $\bar{L} = (\bar{L}_w, \bar{L}_e)$ is a steady state of Eq.* (1)*. Then, $\{\{f_w(s)\}_{s\in\mathcal{S}_w}, \{f_e(s)\}_{s\in\mathcal{S}_e}, \theta_w, \theta_e, \bar{L}\}$ constitute a* stationary equilibrium (SE) *if for each $s_w \in \mathcal{S}$ such that $f_w(s_w) > 0$, it holds that s_w is a best response for a worker assuming that the system is in steady state $\bar{L}$, i.e. $s_w \in \arg\max_{s\in\mathcal{S}} U_w(s_w; \bar{L})$, and similarly for employers.*

Note that the definition requires a steady-state. Thus, each of our results claiming a stationary equilibrium characterizes the corresponding steady state as part of the proof.

We further refine our equilibrium concept to focus on the subset of stationary equilibria that are *evolutionarily stable* (a classic reference is [26]). Intuitively, an evolutionary stable equilibrium is robust in the sense that, if the mix of agents $\bar{N}$ deviates slightly from the steady state value of $\bar{L}$ (slightly changing the utility derived from different strategies) and incoming agents choose their strategies as a best response to the current environment, this reaction should push the system back towards $\bar{L}$. This refined rules out implausible equilibria under which both sides of the market mix between proposing and not proposing (starting at such an equilibrium, a tiny perturbation leading to an increase in the volume of workers who are proposing will make it a best response for employers to not propose, which, in turn, will make it a best response for workers to propose). We formalize the notion of evolutionary stability in Appendix A.2.

3 Ex ante homogeneous agents on each side

As per the model in Section 2, we consider ex-ante homogeneous agents on each side. Throughout the rest of the section, we think about the agents' strategies in equilibrium as a function of the screening cost. For every fixed pair of screening costs, we consider the limit of small death rate $\mu \to 0$.

We first consider the equilibria in the fully symmetric market (same arrival rate λ, screening cost c, and valuation distribution Uniform$(0, 1)$ on both sides of the market) which allows us to illustrate key features of our model. (In the interest of space, we defer the full discussion to Appendix B and only summarize the main findings.)

- In each equilibrium (except when c is large), one side proposes and the other side waits for incoming proposals. (The requirement of evolutionary stability rules out equilibria where one or both sides mix between proposing and waiting for proposals.)

- Both sides screen when the search cost is small, whereas only the side receiving proposals screens for intermediate search costs. When screening costs are large, both sides propose without screening and also accept incoming proposals without screening.
- The utility of the side proposing (say workers) is smaller than the utility of the side waiting for proposals (employers). The reason is that, when proposing, there is a risk that the proposal gets rejected if the other side is screening, i.e., there is a *negative externality of screening by recipients on proposers*. The workers can internalize the impact of rejections by considering an inflated *effective screening cost*, which is c times the expected number of proposals that must be issued until one is accepted (see Lemma 1).
- In this setting, the platform *cannot increase average welfare* by implementing one of the proposed interventions. If the platform blocks a side from issuing proposals, the equilibrium that arises is its symmetric counterpart, and due to market symmetry, the average welfare is unchanged. If the platform blocks one side from screening, the welfare decreases (if both sides were screening) or remains unchanged (if one side was not screening).

In the next subsections we see that, when markets are asymmetric, interventions can be useful to either *select* the highest welfare equilibrium, or *create* equilibria with higher welfare.

3.1 Different screening costs on the two sides

We now consider the case where the two sides face different screening costs. Without loss of generality, we assume that the screening cost for workers is greater than that for employers; let $c_w = \alpha c_e$ for some $\alpha \geq 1$. We assume a balanced market as before: workers and employers arrive at the same rate λ, and match values are i.i.d. Uniform$(0, 1)$ on both sides. As usual, we are interested in the limiting description of the equilibria as $\mu \to 0$, for each fixed α and c_e. Note that as $\mu \to 0$, all workers and employers will leave the market matched. Hence, the difference between equilibria is in which side proposes and whether each side screens or not.

As workers face a larger screening cost, all else being equal, they cannot afford to be as selective as employers. This is consistent with our finding (Corollary 1 below) that when screening costs are small, the average welfare is higher if employers (and not workers) propose, as then workers do not face the risk of rejection, allowing them to be at least somewhat selective, while inflating the effective screening cost faced by employers by only a small factor. However, for medium sized screening costs, the highest welfare equilibrium is one in which only the employers screen (whereas the workers propose without screening).

The following theorem characterizes the equilibria for different values of screening cost c_e (holding α fixed). (In the interest of readability, we omit the characterization of the steady state $\bar{L}$ from the statements of our theorems and defer it to the proofs. Thresholds θ_w and θ_e are identical to expected utility for workers and employers respectively.)

Theorem 1 (No intervention equilibria)**.** *Consider a market with $\lambda_w = \lambda_e = \lambda$ and $c_w = \alpha c_e$ for some $\alpha \geq 1$. For agents on both sides, their valuations for potential partners are drawn i.i.d.*

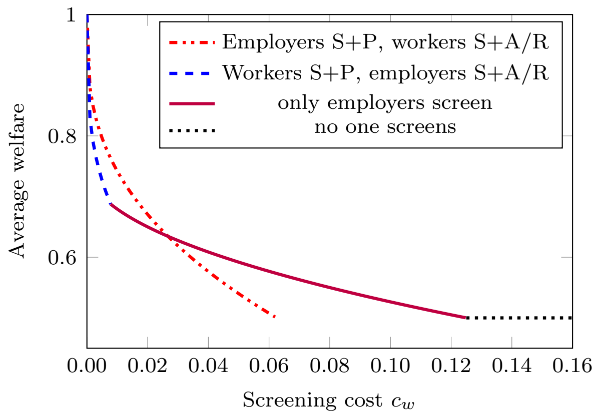

Figure 1: Welfare of equilibria with unequal screening costs $c_w = 2c_e$, same arrival rate on both sides, and i.i.d. $U(0,1)$ valuations on both sides of the market. In the legend, S+P=screen and propose, S+A/R=screen and then accept or reject.

from a U(0,1) distribution. Fix α*, and consider the limit* $\mu \to 0$ *for each fixed* c_e*. Then, the following are the stable stationary equilibria as a function of* c_e*:*

1. *(employers screen + propose, workers screen + accept/reject) with thresholds* $\theta_e = 1 - (2c_e/\alpha)^{1/4}$ *and* $\theta_w = 1 - \sqrt{2\alpha c_e}$*. This is an equilibrium for* $c_e \in \left(0, \min\left(\frac{1}{8\alpha}, \frac{\alpha}{32}\right)\right)$*.*

2. *(employers propose w.o. screening, workers screen + accept/reject) with threshold* $\theta_w = 1-\sqrt{2\alpha c_e}$*. Employers get an expected utility of* $1/2$*. This is an equilibrium for* $c_e \in \left[\frac{\alpha}{32}, \frac{1}{8\alpha}\right)$ *if the interval is non-empty (*$\alpha < 2$*).*

3. *(employers screen + propose, workers accept w.o. screening) with threshold:* $\theta_e = 1-\sqrt{2c_e}$*. Workers get expected utility* $1/2$*. This is an equilibrium for* $c_e \in \left[\frac{1}{8\alpha}, \frac{1}{8}\right)$*.*

4. *(workers screen + propose, employers screen + accept/reject) with thresholds:* $\theta_w = 1 - (2\alpha^2 c_e)^{1/4}$ *and* $\theta_e = 1 - \sqrt{2c_e}$*. This is an equilibrium for* $c_e \in \left(0, \frac{1}{32\alpha^2}\right)$*.*

5. *(workers propose w.o. screening, employers screen + accept/reject) with threshold:* $\theta_e = 1 - \sqrt{2c_e}$*. Workers get expected utility* $1/2$*. This is an equilibrium for* $c_e \in \left[\frac{1}{32\alpha^2}, \frac{1}{8}\right)$*.*

6. *Agents on both sides propose without screening, and accept all incoming proposals without screening. This happens when* $c_w \geq \frac{1}{8}$*. Workers and employers get expected utility* $1/2$*.*

The proof of Theorem 1 can be found in Appendix C. As a reference, Figure 1 illustrates the average agent welfare under the different equilibria when $\alpha = 2$.

When c is close to zero, the only equilibria are those in which one side proposes and both sides screen. When both these equilibria exist, the average welfare is higher when employers play the role of proposers. To see why, compare the thresholds of workers and employers when they are the ones waiting for incoming proposals: employers use a higher threshold than workers, as employers' screening cost is smaller. Hence, when workers are proposing, not only do they face a higher cost per opportunity screened, but also a smaller likelihood of their proposals being accepted. These two effects cause workers' selectivity and expected utility to decline rapidly with c_e (recall that c_w/c_e is fixed) when they are proposing, and for $c_e \geq 1/(32\alpha^2)$ workers propose without screening.

For intermediate values of c_e, in particular for $c_e \in \left[\frac{1}{32\alpha^2}, \min\left(\frac{1}{8\alpha}, \frac{\alpha}{32}\right)\right)$, equilibria 1 and 5 co-exist: (employers screen + propose, workers screen + accept/reject), and (workers propose w.o. screening, employers screen + accept/reject). For values of c_e just above $1/(32\alpha^2)$, the welfare is larger under the former equilibrium; workers screen incoming proposals and this improves worker's welfare (relative to not screening) more than it hurts employers (recall the negative externality on proposers of selectivity by recipients). However, as c_e increases, the negative externality on employers dominates and the equilibrium (workers propose w.o. screening, employers screen + accept/reject) has a larger average welfare.[4]

From the previous discussion, it should be clear that for small and medium values of c and $\alpha > 1$, the platform can use an appropriate intervention to *select* a high welfare equilibrium.

Corollary 1 (Selecting good equilibria via design)**.** *Consider again the setting of Theorem 1 and the equilibria described there.*

- *For $c_e \leq c_*$, the welfare maximizing equilibrium is Equilibrium 1. The platform can eliminate other equilibria by preventing workers from proposing.*
- *For $c_e \in (c_*, 1/8)$, Equilibrium 5 maximizes welfare. The platform can implement one sided search where only employers choose to obtain this welfare (and outcome) in the unique resulting equilibrium.*
- *For $c_e \geq 1/8$, equilibrium 6 maximizes welfare. The platform can implement centralized matching (agents do not choose) to obtain this welfare (and outcome).*

Here c_* is defined as

$$c_* = c_*(\alpha) = \frac{1}{32}\left(\sqrt{2 + \left(1 - \alpha^{-1/4}\right)^2} + 1 - \alpha^{-1/4}\right)^4 \tag{8}$$

Here, c_* is chosen such that Equilibrium 1 and Equilibrium 5 have identical welfare when $c_e = c_*$. Note that implementing the suggested interventions never decreases the average welfare. Furthermore, the improvement in average welfare can be substantial, for instance we get a 14.6% average welfare improvement when $\alpha = 2$ and $c_e = 1/16$.

3.2 Unbalanced markets

We now study the effect of unequal arrival rates on the two sides of the market. Assume, without loss of generality, that workers arrive faster than employers. We find that if the market imbalance is not too small, *workers (the long side) propose in all equilibria* under no intervention.

[4]If $\alpha < 2$, Equilibrium 2 coexists with Equilibrium 5 for $c_e \in \left[\frac{\alpha}{32}, \frac{1}{8\alpha}\right)$, but has smaller welfare. Further, Equilibrium 3 coexists with Equilibrium 5 for $c_e \in \left[\frac{1}{8\alpha}, \frac{1}{8}\right)$ but has identical welfare. So Equilibrium 5 is the highest welfare equilibrium for c_e values all the way until $1/8$.

(We highlight that we are only able to capture this phenomenon due to the dynamic nature of our model.) Therefore, workers face a higher effective screening cost, which results in them not being able to be selective, in addition to the risk of not matching. However, the platform can significantly alleviate this issue by preventing workers from proposing. This *creates* an equilibrium where employers propose, allowing workers to be more selective and boosting worker welfare significantly, at a small cost to employers.

We consider an unbalanced market that has identical screening costs and valuation distributions on the two sides.

Theorem 2 (No intervention equilibria)**.** *Consider a market with $\lambda_e = 1$ and $\lambda_w = \lambda$ for $\lambda > 1$. For agents on both sides, their valuations for potential partners are drawn i.i.d. from a Uniform(0,1) distribution. Both sides face the same screening cost of $c > 0$. Then, the following is the limiting description of a subset of stable stationary equilibria as a function of c (considering $\mu \to 0$ for each fixed c):*

1. *(workers screen + propose, employers screen + accept/reject) with thresholds $\theta_w = \xi(\lambda, \sqrt{\frac{c}{2}})$ and $\theta_e = 1 - \sqrt{2c}$. This is an equilibrium for $c \in (0, 2\bar{c}^2)$.*
2. *(workers propose w.o. screening, employers screen + accept/reject) with threshold $\theta_e = 1 - \sqrt{2c}$. Worker expected utility is $1/(2\lambda)$. This is an equilibrium for[5] $c \in \left[2\underline{c}^2, \frac{1}{8}\right)$.*
3. *Agents on both sides propose without screening, and accept all incoming proposals without screening. This happens when $c \geq \frac{1}{8}$. Employer expected utility is $1/2$ and worker expected utility is $1/(2\lambda)$, where*

$$\xi(\lambda, c) = \frac{\lambda - \sqrt{\lambda^2 - 2\lambda(1-2c) + (1-2c)}}{2\lambda - 1} = \frac{\lambda - \sqrt{(\lambda-1)^2 + 2c(2\lambda-1)}}{2\lambda - 1} \tag{9}$$

(the function $\xi(\lambda, c)$ captures the equilibrium threshold for accepting a proposal used by the workers when employers are proposing),

$$\bar{c} = \frac{1}{4}\left(1 - \frac{\lambda}{1 + \sqrt{1 + (\lambda-1)^2}}\right), \text{ and } \underline{c} = 1/(8\lambda^2). \tag{10}$$

Furthermore, if $\lambda \geq 1.25$, there are no other stable equilibria.

The proof of Theorem 2 can be found in Appendix D. (We also state and prove Theorem 4, that captures additional equilibria in this setting for $\lambda < 1.25$.) We describe the main findings next. As a reference, Figure 2 illustrates the welfare under the different equilibria when $\lambda = 2$.

Under the equilibria described, the expected utility of employers is greater than the expected utility of the workers. An obvious reason for this is that all employers match (as $\mu \to 0$), while only a fraction $1/\lambda$ of workers match. The risk a worker faces of dying without matching has

[5] Depending on λ and c, employers may or may not want to propose if they are given the chance. However, this does not play a significant role because all but a vanishing fraction of employers will match due to incoming proposals. In particular, as $\mu \to 0$, employers would only propose if workers do not screen the incoming proposals.

the additional consequence that even though both sides are allowed to propose, *there is no equilibrium where only employers propose* for $\lambda > 1.25$. The intuition is that it is always a best response for a worker to reach out to an employer if he gets the opportunity to propose, as this would increase his chances of matching before dying. Given that workers are thus active, employers would rather wait for incoming proposals than screen and propose. (We remark that our approach that considers the endogenously determined steady state, as opposed to an exogenously fixed ratio of workers to employers present as in flow economy models [22], is instrumental in obtaining this result.) Second, competition for scarce employers prevents workers from being too selective in equilibrium, which is further exacerbated by the fact that workers are proposing and thus facing higher effective screening costs (see Lemma 1). As a result, for all but very small values of c (in particular, for $c \geq 2\bar{c}^2$) workers propose *without screening* and get no better than a random employer when they are lucky enough to match.

We find that the platform is, in most settings, able to significantly boost the welfare via intervention by blocking workers from proposing and thus forcing the employers to propose. In contrast with the case with different screening costs, this intervention *creates* new equilibria if the imbalance is not too small ($\lambda \geq 1.25$). These new equilibria are characterized next.

Theorem 3 (Intervention equilibria)**.** *Consider the market described in the statement of Theorem 2. If workers are not allowed to propose, the following are all the stable stationary equilibria as a function of c (as $\mu \to 0$ for each fixed c.):*

1. *(employers screen + propose, workers screen + accept/reject) with thresholds $\theta_w = \xi(\lambda, c)$ and $\theta_e = 1 - \sqrt{2c/(1-\theta_w)}$. This is an equilibrium for $c \in (0, \min(\bar{c}, \hat{c}))$.*

2. *(employers screen + propose, workers accept w.o. screening) with threshold $\theta_e = 1 - \sqrt{2c}$. Each worker earns expected utility $1/(2\lambda)$. This is an equilibrium for $c \in [\underline{c}, \frac{1}{8})$.*

3. *(employers propose w.o. screening, workers screen + accept/reject) with threshold $\theta_w = \xi(\lambda, c)$. Each employer earns expected utility $1/2$. This is an equilibrium for $c \in [\hat{c}, \bar{c})$, and only exists if $\hat{c} < \bar{c}$ which occurs for $\lambda < 1.46$.*

4. *(employers propose w.o. screening, workers accept w.o. screening). This is an equilibrium for $c \geq 1/8$. Agents on both sides propose without screening, and accept all incoming proposals without screening. Employer expected utility is $1/2$ and worker expected utility is $1/(2\lambda)$. This happens when $c \geq \frac{1}{8}$.*

where , $\bar{c}$ and $\underline{c}$ are as per Eq. (10), $\xi(\lambda, c)$ is defined in Eq. (9), and

$$\hat{c} = \frac{8\lambda - 7}{32(2\lambda - 1)} \,. \tag{11}$$

However, it might not always be beneficial for the platform to implement the proposed intervention of blocking workers from proposing. Based on Theorems 2 and 3, our design recommendation is as follows.

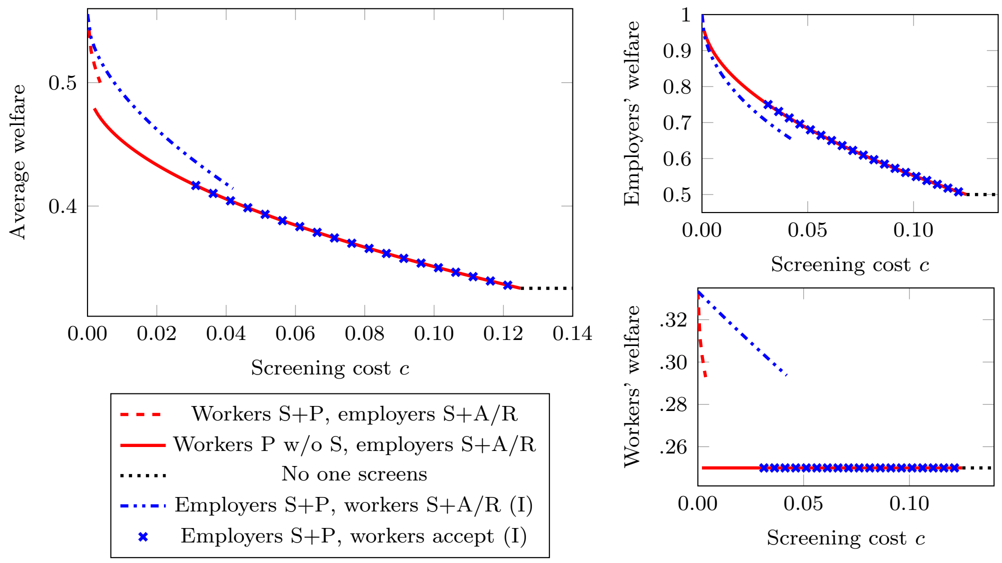

Figure 2: Equilibria with symmetric screening cost c, workers arriving $\lambda = 2$ times as fast as employers, and i.i.d. Uniform(0, 1) valuations on both sides of the market. (*Left*) Average agent welfare, see (7) for the definition. (*Top Right*) Employers' welfare. (*Bottom Right*) Workers' welfare. In the legend, S+P = screen and propose, S+A/R = screen and then accept/reject, and (I) denotes that the equilibrium *only exists under the proposed intervention.*

Corollary 2. *The platform can boost average welfare by preventing workers from proposing if Theorem 3 Equilibrium 1 exists when workers are blocked from proposing, and the average welfare*[6] *under this equilibrium is larger than that under the equilibria that can exist under no intervention (Theorem 2 Equilibria 1 and 2 are the candidates).*

We characterize the pairs (c, λ) for which the intervention helps in Appendix D.1. However, it is worth noting that the welfare of employers slightly decreases under our intervention, as they must now propose and risk rejection. On the other hand, the welfare of the workers increases, since their effective screening cost decreases and they can thus be more selective, which further turns out to have a positive externality on other workers.

Remark 1. *We find that* selectivity by agents on the long side has a positive same-side externality*; when a worker rejects an employer, this makes the employer available to match with other workers. This externality leads to virtuous cycle in which selectivity by other workers increases the availability of options for a particular worker, and allows her to be more selective. This boosts the benefit from our proposed intervention. Another consequence of this externality is that Equilibria 1 and 2 in Theorem 2 (and similarly in Theorem 3) co-exist for some c's, with Equilibrium 1 where workers screen resulting in higher welfare for workers.*

Again consider $\lambda = 2$. The utility loss incurred by employers is less than 8%, for all possible c. On the other hand, workers' utility increases by up to 31%, and average welfare increases by up to 10%; both maxima being achieved at $c = 2\bar{c}^2$.

[6]We average over agents arriving to the system. In this case, since all agents on the same side use the same strategy, the average welfare at a given equilibrium is $\left(\lambda U_w(s_w; \bar{L}) + U_e(s_e; \bar{L})\right) / (\lambda + 1)$.

4 Vertical differentiation

We now augment the model from Section 3.2 to study the impact of vertical differentiation by considering the simplest case, in which we have ex-ante homogeneous employers, and two quality tiers of workers: top (high-quality) workers, and bottom workers. We consider a setting where employers arrive faster than top workers but slower than workers overall, and find that our dynamic model allows us to uncover many interesting new features under vertical differentiation. Again, we identify suitable platform interventions to boost agent welfare. We briefly describe our main findings here (see Appendix E for a full description including formal results and proofs).

Match utility u_{ij} is now the sum of the quality q_j of agent j and an idiosyncratic (i, j)-specific i.i.d. Uniform$(0, 1)$ term privately discoverable by i upon spending a screening cost. Agent quality, assumed to be $a \in (0, 1)$ for top workers and 0 for bottom workers and for employers (thus an employer has a positive probability of preferring a particular bottom worker over a particular top worker), is known to the platform, and to agent herself; further, in the no intervention setting the platform reveals quality information of potential matches to agents (including allowing an agent who has an opportunity to request a candidate, to indicate a preference ranking over quality tiers on the other side as part of the request). The main change to the model described in Section 2 is that employers must now (1) decide what to do with incoming proposals from agents from *each quality tier* and, (2) decide which tiers of workers they are interested in and in what order, and for each such tier, whether they will screen or propose without screening. We formally describe the augmented model with tiers, as well as the main changes to the equilibrium concept in Appendix E, Section E.1.

Under no platform intervention, we find a unique equilibrium (under reasonable assumptions) where (i) bottom workers propose without screening, (ii) top workers do not propose, and screen and accept/reject incoming proposals, (iii) employers do not propose to bottom workers but, given the opportunity, they screen an available top worker, and propose to him with threshold $\theta_e = 1 - \sqrt{2c}$. Employers split into two types based on how they respond to proposals from bottom workers: *reachers* –who ignore proposals from bottom workers, and instead wait in the hope of matching with a top worker at risk of dying without matching– and *settlers*, who screen and accept/reject incoming proposals from bottom workers with a threshold of θ_e, and consequently match with bottom workers in all but a vanishing fraction of cases (as $\mu \to 0$). Bottom workers have low welfare in this equilibrium, due to two inefficiencies which persist even as screening costs go to zero. First, a positive fraction of reachers end up *dying unmatched* because they wait for an ideal partner (top worker) while *ignoring* all proposals from bottom workers. This is wasteful since it reduces the number of matches formed, hurting bottom workers (top workers all get matched). Second, bottom workers have most of their proposals completely ignored (as opposed to being rejected after screening, which also occurs). This is due to an inspection paradox: reachers, who wait for dream matches, stay longer in the system and, as a result, at any given time, a large majority of employers in the system are reachers, causing most proposals from bottom men to be ignored. This prevents the bottom workers from being selective even for small values of c. We characterize the equilibrium when no intervention is

Intervention	θ_w^{t}	θ_e	θ_w^{b}	Employers die or not	Bottom workers can choose or not
No intervention	1	1	$1/(2\lambda_\delta)$	Die!	Can't choose!
Workers can't propose	1	1	$1/(2\lambda_\delta - 1)$	Die!	Can choose
Identify employers who consider b-workers	1	1	$1/(2\lambda_\delta - 1)$	Die!	Can choose
Hide quality of workers, and workers can't propose	1	1	$1/(2\lambda - 1)$	Don't die	Can choose

Table 1: Welfare under different interventions in markets with vertical differentiation, parameterized by $\lambda_w^{\mathrm{t}}, \lambda_w^{\mathrm{b}}, \lambda_e$ and $a \in (0,1)$ where $\lambda_e \in (\lambda_w^{\mathrm{t}}(1+a/2), \lambda_w^{\mathrm{t}} + \lambda_w^{\mathrm{b}})$; we consider $\mu \to 0$ and then $c \to 0$. We use $\lambda_\delta = \lambda_w^{\mathrm{b}}/(\lambda_e - \lambda_w^{\mathrm{t}}(1+a/2))$ and $\lambda = \lambda_w^{\mathrm{b}}/(\lambda_e - \lambda_w^{\mathrm{t}})$. Note that $\lambda_\delta > \lambda > 1$.

present in Appendix E, Section E.2.

We can relate these results with the ones obtained in Section 3.2. The behavior of agents under equilibrium in the top submarket (reachers and top workers) resembles that in Theorem 2 equilibrium 1, where the long side screens and proposes.[7] In the bottom submarket (settlers and bottom workers), the behavior resembles that in Theorem 2 equilibrium 2, where the long side proposes without screening. However, there is a major difference between the present case and an unbalanced market without vertical differentiation. Here, bottom workers *do not know* who the settler employers are when proposing. Further, since $\mu \to 0$ (and $\mu/\sqrt{c} \to 0$), an inspection paradox is created: most of the employers present in the market are reachers who ignore proposals from bottom workers. This makes it highly unattractive for workers to screen before proposing even when $c \to 0$, resulting in the welfare being low for bottom workers even in this limit.

Furthermore, the inefficiencies we find appear related to phenomena observed in real online platforms (we do not pursue a detailed mapping between our model and reality at this point). [12] finds that searchers on Airbnb often leave the market although they could have found a suitable partner. A similar effect has been uncovered in the context of O-Desk [18]. Consider also some empirical findings from Tinder [33]: a third of men on Tinder report that they casually "like" most profiles, cf. the equilibrium behavior of bottom workers. Women are much more selective, and 59% of women (as compared to just 9% of men) report that they like fewer than 10% of all profiles that they encounter, cf. the equilibrium behavior of reacher employers. Less than 1% of likes by men result in match (a match occurs when two users like each other), whereas the corresponding number is over 10% for women, cf. our equilibrium finding that most proposals by bottom workers are ignored.

To mitigate these inefficiencies, we propose the following interventions, see Table 1 (we focus on $\mu \to 0$ and then $c \to 0$ for simplicity):

- *Block workers from proposing, regardless of their tier*: As in the case without intervention, there is a unique equilibrium with two types of employers arising in equilibrium: reachers

[7] The small modification is that here, an employer's utility for a worker is uniformly distributed in $(a, a+1)$.

and settlers. Reachers seek to match with a top worker; if none is available, they just wait in the system. Settlers first request a top worker and, if none is available, request a bottom worker (since bottom workers are not permitted to propose). All workers screen and accept/reject. In particular, *bottom workers are now able to be selective*, because they receive proposals instead of having to actively reach out to workers. Although for $c > 0$ the expected utility of employers slightly decreases, as $c \to 0$, blocking workers from proposing *helps bottom workers without affecting other agents.* However, this intervention does not fix the inefficiency that some employers (reachers) are dying unmatched.

- *Identifying employers who will consider bottom workers:* Recall that, under no intervention, bottom workers cannot afford to screen as most of their proposals go to reachers in steady state, who ignore them. To alleviate this, suppose the platform is able to identify settlers, and direct bottom workers' search efforts towards such employers exclusively. Bottom workers' proposals are no longer ignored, and they can hence afford to screen before proposing, improving their expected utility somewhat (employers and top workers are unaffected). Again, the problem of some employers dying without matching persists.

- *Hiding quality information and blocking men from proposing:* The platform now not only blocks workers from proposing, but also does not reveal to employers whether a worker is a top worker or a bottom worker. In equilibrium, employers screen and propose, and top and bottom workers screen and accept/reject. The limiting utilities for both top workers and employers remain unchanged, but bottom workers' utility exceeds that of the other interventions (and that under no intervention). Bottom workers are able to be selective, but also *almost all employers match in equilibrium* as they are able to either find a top worker, or a bottom worker they like, increasing the fraction of bottom workers who match.

  We remark that the intervention of hiding information fits well with what many dating platforms do already: for instance, Tinder learns the attractiveness of a user's profile, and encodes this internally –but does not publicly reveal– in a vertical "Elo" rating (that it uses to guide its recommendations), but does not reveal this rating to its users. We further remark that completely hiding information quality information may not always be the best approach. When there are multiple quality levels, the platform may maximize welfare by providing *partial* quality information, allowing users to prune the consideration set somewhat, but not to differentiate between those whom she can realistically "reach" and those whom she can settle for. The platform may simulate this partial revelation of quality information by a combination of recommendation engine design and keeping quality information invisible.

5 Discussion

We focus on average welfare across both sides of the market, and suggest platform interventions to improve this metric. One argument in favor of optimizing overall welfare is that platforms often have tools as their disposal to transfer welfare roughly via "charging" one group of agents while subsidizing another group [11]. However, this may not be possible in certain settings, and a potential concern may be that for a platform to compete successfully with other platforms, it may need to focus on the welfare of a subset of agents (e.g., scarce agents). A piece of suggestive evidence here is provided by the dating app Bumble, which requires women to send the first message, may actually benefit men and hurt women slightly (see Section 3). One might think that their design would attract more men while inducing women to prefer other dating platforms, yet Bumble has a more balanced set of users (about 50-50) relative to other platforms where the majority of users (60-70%) are male, suggesting pitfalls in natural approaches that consider platform competition (nevertheless this is an interesting aspect to study [16]). Further, user welfare is a primary objective for any platform in terms of attracting and retaining users, even if the ultimate goal is maximizing revenues (but again, this is of interest to study).

One may ask if we can obtain similar insights in a simpler setup such as synchronous matching game [16] or a flow economy [22], instead of taking on the challenge of studying a dynamic steady state. It would appear that such alternate approaches would yield some of our insights but not others. The cause of this is an inspection paradox, which leads to (i) the steady state mix of agents being very different from the arriving mix of agents in many of the settings considered, allowing us to identify properties of equilibria that we would miss otherwise (e.g., the long side proposes in all equilibria in Section 3.2); and (ii) the steady state mix of agents being heavily dependent on platform design (despite no change in the arriving mix), which allows us to identify good platform designs (e.g., we find benefits from hiding information when there is vertical differentiation in Section 4).

In the interest of simplicity and tractability, we assumed idiosyncratic values are drawn i.i.d from a Uniform(0,1) distribution, and that a worker i's match value for an employer j is independent of j's match value for i. We expect our insights to be reasonably robust to these assumptions. If values are strongly correlated within pairs, some equilibrium features may be modified, but we expect that the welfare maximizing designs in the case of ex ante homogeneous agents on each side will still be: (i) to have the side with lower screening cost go first/choose (Section 3.1), and (ii) to have the short side go first (Section 3.2). The benefits from intervention should be exacerbated when the distribution of valuations has negative values in the support, as now search effort might be invested in candidates that provide negative match utility. Also, one may ask what happens if proposal responses are not immediate. Such delays may *increase* the welfare gains from intervention. The intuition is that our recommended interventions replace proposals with low likelihood of being accepted with proposals with high likelihood of being accepted (in particular in unbalanced markets, most proposals by agents on the long side will be ignored if there is a delay in responding to proposals, causing an even lower likelihood of acceptance under no intervention). Finally, it is natural to ask whether further improvements are possible using other interventions, such as limiting the rate at which one or both sides of the market are able to propose. We leave this question for future work, remarking that this relates with the literature on signaling in matching markets (see Section 1.1).

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A Equilibrium concept and steady state characterization

A.1 Full description of the matching formation rates (Section 2.2)

When defining the matching formation rates for the case of $N_e = 0$, we only described the matching formation rates for the case in which N_e remains zero for a non-zero interval of time. For completeness, we now fully characterize the matching formation rates for the case where $N_e = 0$.

There are two possibilities here. The first one is that N_e remains zero for a non-zero interval of time, which was briefly sketched in the main text. The other possibility is that

$$\frac{\mathrm{d}N_e}{\mathrm{d}t} > 0 \quad \Rightarrow \quad \frac{\mathrm{d}N_e(s_e)}{\mathrm{d}t} > 0 \;\forall s_e \in \mathcal{S}_e \text{ with } f_e(s_e) > 0\,. \tag{12}$$

For simplicity, we first analyze these two cases for the a setting in which all employers are following the same strategy s. First, consider the case where the second possibility arises (Eq. (12) is satisfied). Then, there are always employers in the system and we can calculate the flow rate of matching as before. We have

$$\rho_w(s_w, s; \bar{N}) = N_w(s_w)\eta_w(s_w, s)\,. \tag{13}$$

The total rate of matching is then

$$\rho_e(s; \bar{N}) = \sum_{s_w \in \mathcal{S}_w} \rho_w(s_w, s; \bar{N}) = \sum_{s_w \in \mathcal{S}_w} N_w(s_w)\eta_w(s_w, s)$$

and must satisfy

$$\begin{aligned} &\rho_e(s; \bar{N}) < \lambda_e \\ \Leftrightarrow &\sum_{s_w \in \mathcal{S}_w} N_w(s_w)\eta_w(s_w, s) < \lambda_e\,. \end{aligned} \tag{14}$$

It is easy to see that this is the necessary and sufficient condition for the case in Eq. (12) to arise. In this case, we simply have

$$\frac{\mathrm{d}N_e}{\mathrm{d}t} = \lambda_e - \sum_{s_w \in \mathcal{S}_w} N_w(s_w)\eta_w(s_w, s)\,. \tag{15}$$

The complementary case arises when

$$\sum_{s_w \in \mathcal{S}_w} N_w(s_w)\eta_w(s_w, s) \geq \lambda_e\,. \tag{16}$$

In this case, whenever there are employers in the system, they form matches with workers following strategy s_w at a rate proportional to $N_w\eta(s_w, s)$. It follows that the flow rates of match formation are given by

$$\rho_w(s_w, s; \bar{N}) = \lambda_e \frac{N_w(s_w)\eta_w(s_w, s)}{\sum_{s_w \in \mathcal{S}_w} N_w(s_w)\eta_w(s_w, s)}\,. \tag{17}$$

We now generalize the discussion to the case where employers possibly use different strategies.Suppose N_e does not remain 0 for any interval of time, i.e., Eq. (12) holds. Let

$$\frac{\mathrm{d}N_e(s_e)}{\mathrm{d}t} = \alpha R(s_e) > 0\,, \tag{18}$$

where $\alpha > 0$ be such that

$$\sum_{s_e} R(s_e) = 1\,. \tag{19}$$

Then, after a very short time, the relative masses of different s_e's in the system will be proportional to $R(s_e)$, and hence employers following s_e will be shown as a potential option to a worker with probability $R(s_e)$. So we have

$$\rho_w(s_w, s_e; \bar{N}) = N_w(s_w)\eta_w(s_w, s_e)\frac{R(s_e)}{\sum_{s_e'} R(s_e')}\,.$$

Using Eq. (1) and (18) and $N_e = 0$, we have

$$\begin{aligned} \alpha R(s_e) &= \lambda_e f_e(s_e) - \rho(s_e; \bar{N}) \\ \Rightarrow \qquad R(s_e) &= \frac{\lambda_e f_e(s_e)}{\alpha + \sum_{s_w} N_w(s_w)\eta_w(s_w, s_e)}\,. \end{aligned} \tag{20}$$

Substituting in Eq. (19), we have

$$\sum_{s_e} \frac{\lambda_e f_e(s_e)}{\alpha + \sum_{s_w} N_w(s_w)\eta_w(s_w, s_e)} = 1\,. \tag{21}$$

Note that the left-hand side is decreasing in α for $\alpha > 0$, and tends to zero as α tends to infinity. In order for the equation to have a solution, it must be that

$$\sum_{s_e} \frac{\lambda_e f_e(s_e)}{\sum_{s_w} N_w(s_w)\eta_w(s_w, s_e)} > 1\,.$$

One can check that this condition is also sufficient to produce the case $\frac{\mathrm{d}N_e}{\mathrm{d}t} > 0$. Suppose this condition holds, then Eq. (21) implicitly specifies a unique value of α, which, in turn, determines $R(s_e)$'s, ρ_w's and $\rho_e(s_e; \bar{N})$'s.

The complementary case of N_e remaining zero arises if and only if

$$\sum_{s_e} \frac{\lambda_e f_e(s_e)}{\sum_{s_w} N_w(s_w)\eta_w(s_w, s_e)} \leq 1\,.$$

In this case, similarly to our prior argument, we have

$$\rho_w(s_w, s_e; \bar{N}) = \lambda_e f_e(s_e)\, \frac{N_w(s_w)\eta_w(s_w, s_e)}{\sum_{s_w \in \mathcal{S}_w} N_w(s_w)\eta_w(s_w, s_e)}\,.$$

and $\rho_e(s_e; \bar{N}) = \lambda_e f_e(s_e)$ as expected.

A.2 Evolutionarily stable equilibrium

As defined in Section 2, let

$$N_w(s_w) = \text{Mass of workers in the system following strategy } s_w. \tag{22}$$

Define $N_e(s_e)$ similarly. Further, let $\bar{N}_w = (N_w(s))_{s\in\mathcal{S}_w}$ and $N_w = \sum_{s\in\mathcal{S}_w} N_w(s)$, and similarly for employers. Let $\bar{N} = (\bar{N}_w, \bar{N}_e)$.

When a new worker enters, he considers the continuation value $V_w(s_w; \bar{N})$ that would result from using strategy s_w assuming $\bar{N}$ will remain unchanged over time. The worker chooses strategy $s^*_w(\bar{N}) = \arg\max_{s_w\in\mathcal{S}_w} V_w(s_w; \bar{N})$. (When there are ties, we will allow them to be broken arbitrarily, including possible mixing. This will ensure that all stationary equilibria will be captured as fixed points of the differential equations below.) We think of the thresholds θ_w, θ_e as being fixed.[8] Agents do not change their strategy during their lifetime. This leads to the following coupled ODEs capturing system evolution in the continuum limit

$$\begin{aligned}
\frac{\mathrm{d}N_w(s)}{\mathrm{d}t} &= \mathbb{I}\big(s = s^*_w(\bar{N})\big)\lambda_w - N_w(s)\mu - \rho_m(s; \bar{N}) \qquad \forall s \in \mathcal{S}\,, \\
\frac{\mathrm{d}N_e(s)}{\mathrm{d}t} &= \mathbb{I}\big(s = s^*_e(\bar{N})\big)\lambda_e - N_e(s)\mu - \rho_w(s; \bar{N}) \qquad \forall s \in \mathcal{S}
\end{aligned} \tag{23}$$

We now characterize the matching rates and continuation values in the hard case of interest. Suppose $N_w = 0$. (The case $N_e = 0$ is analogous.)

Then we have that the likelihood of an incoming worker almost immediately matching with an employer following s_e is proportional to $N_e(s_e)\eta_e(s_e, s^*_w(\bar{N}))$, leading to

$$\rho_e(s; \bar{N}) = N_e(s_e)\eta_e(s_e, s^*_w(\bar{N})) \min\left(\frac{\lambda_w}{\sum_{s_e\in\mathcal{S}_e} N_e(s_e)\eta_e(s_e, s^*_w(\bar{N}))}, 1\right)\,,$$

and $V_e(s; \bar{N})$ is the utility for an employer which results from being offered a potential match

[8] These thresholds are chosen to match the continuation value at the equilibrium/fixed point. We expect that holding these thresholds fixed generally should not impact whether an equilibrium classifies as stable or not, since the utility loss due to error in the choice of threshold should grow only quadratically with distance from the equilibrium, whereas the difference between utilities of different strategies in $\mathcal{S}$ should grow linearly with the distance from the equilibrium.

(following strategy $s_w^*(\bar{N})$) at a rate equal to the last term $\min(\,\cdot\,,1)$ above. For the workers, $V_w(s_w;\bar{N})$ is the payoff from receiving proposals at rate ∞ with the proposer strategy being s_e with likelihood proportional to $N_e(s_e)\big(\mathbb{I}(s_e \text{ involves P}) - G(\theta_e)\mathbb{I}(s_e \text{ involves S+P})\big)$, and (relevant only if s_w ignores incoming proposals) always being offered a potential match, the strategy of the potential match being s_e with likelihood proportional to $N_e(s_e)$. The rate of matching is given by

$$\rho_w(s_w^*(\bar{N});\bar{N}) = \min\left(\lambda_w, \sum_{s_e\in\mathcal{S}_e} N_e(s_e)\eta_e(s_e, s_w^*(\bar{N}))\right).$$

(The rates of matching for $s_w \neq s_w^*(\bar{N})$ are irrelevant.) When the min is the second term, we see that $\frac{\mathrm{d}N_w}{\mathrm{d}t} > 0$ leading to $N_w > 0$ in future.

As mentioned in Section 2.2, all stationary equilibria correspond to fixed points of our dynamical system (23). We focus on the subset of stationary equilibria that are plausible from an evolutionary/dynamical standpoint.

Definition 3. *Each stationary equilibrium corresponds to a fixed point of the dynamical system* (23) *when the threshold θ_w (and θ_e) is equal to the continuation value of the best response for workers (employers) at the fixed point, and conversely. A stationary equilibrium is* evolutionarily stable *if the corresponding fixed point is attractive. (An attractive/stable fixed point of a dynamical system is a point such that if the state starts sufficiently close to the fixed point, it remains close to the fixed point and converges to it.) We sometimes refer to this simply as a* stable equilibrium*.*

A.3 Dynamics when agents on each side follow a single strategy

We now analyze the system dynamics when all agents on the same side use the same strategy. We will find that the corresponding dynamical system always has a *unique* steady state/fixed point L, that is always stable. When the fixed strategies employed are the unique best responses on each side of the market to L, they are also best responses in a neighborhood of L, hence the system dynamics precisely matches the dynamics under best responses (Eq. (23)) in a neighborhood of L, implying that L corresponds to a stable equilibrium. As we will argue later, in all the settings consider in Section 3, in each stable equilibrium, all agents on the same side of the market do, in fact, use the same strategy. In other words, there is no mixed stable equilibrium that is stable.

Suppose all workers employ strategy s_w and all employers use strategy s_e. Let η_w and η_e be defined as:

$$\eta_w = \big(\mathbb{I}(s_w \text{ involves P}) - F(\theta_w)\mathbb{I}(s_w \text{ involves S+P})\big)\big(\mathbb{I}(s_e \text{ involves A}) - G(\theta_e)\mathbb{I}(s_e \text{ involves S+A/R})\big)$$

i.e., the fraction of options shown to workers that result in matches, and similarly

$$\eta_e = \big(\mathbb{I}(s_e \text{ involves P}) - G(\theta_e)\mathbb{I}(s_e \text{ involves S+P})\big)\big(\mathbb{I}(s_w \text{ involves A}) - F(\theta_w)\mathbb{I}(s_w \text{ involves S+A/R})\big). \tag{24}$$

Note that the expressions in Eq. (24) are special cases of the expressions defined in Eq. (2).

We will show convergence to a limiting mass of workers and employers (i.e. L_w and L_e respectively[9]) and calculate the limits assuming that:

$$\lambda_w \neq \tfrac{\eta_e \lambda_e}{\mu+\eta_e} \tag{25}$$
$$\lambda_e \neq \tfrac{\eta_w \lambda_w}{\mu+\eta_w} \tag{26}$$

If $\eta_e = 0$ because the employers do not propose under s_e, then condition (25) holds automatically. Suppose $\eta_e > 0$. We will find that the limiting values of L_w and L_e resulting from $\lambda_w \to \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)_+$ and $\lambda_w \to \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)_-$, holding everything else fixed, are identical. Though we omit the details, a coupling argument can be used to establish that this pair of values matches the L_w and L_e that arise from $\lambda_w = \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)$.

Note that the mass of workers in the system in steady state is bounded above by λ_w/μ, since agents die at rate $\mu > 0$ (even if they don't leave by matching), and similarly for employers. Also, note that the only way agents can have a vanishing expected lifetime in the system is if they receive incoming proposals at a rate tending to ∞. All other agents have a positive expected lifetime in the system. We will argue that:

(i) All agents have a positive expected lifetime in the system if the left-hand side is greater than the right in both conditions (25) and (26), and

(ii) If the left-hand side is smaller in (25), then workers will have a vanishing lifetime in the system. Similarly, if the left-hand side is smaller in (26), then employers will have a vanishing lifetime in the system.

To that end, suppose that the left-hand side is greater than the right-hand side in both conditions (25) and (26). Note that, even if employers see potential options a rate equal to their arrival rate, their likelihood of matching before dying is only $\eta_e/(\mu + \eta_e)$. Hence, the maximum rate at which workers match due to proposals by employers is $\lambda_e \eta_e/(\mu + \eta_e)$. If the left-hand side is larger in condition (25), then a positive fraction of workers do not match as a result of proposals by employers; thus, the mass of workers in the system must be positive. Analogously, if the left-hand side is larger in condition (25), then the mass of employers in the system is positive. Therefore, agents have a positive expected lifetime in the system if the left-hand side is more than the right in both conditions (25) and (26).

Next, suppose the left-hand side is smaller than the right-hand side in condition (25). Then, we know that $\lambda_e > \lambda_w$, so a simple argument can be used to show that at any time, the mass of employers in the system is positive (since employers die at rate at least $(\lambda_e - \lambda_w)$ for all $t \geq t_0$, for some t_0). Also, employers must issue proposals at a slower rate than their arrival rate, which means that limiting mass of workers in the systems is 0. Moreover, the number of workers will stay at that level —if it starts to build up, then employers will be able to issue proposals at a faster rate, and form matches at rate $\lambda_e \eta_e/(\mu + \eta_e) > \lambda_w$, reducing the mass of workers. Hence, in steady state, the mass of workers remains 0. Therefore, if the left-hand side is smaller than

[9] Here, we abused notation and suppressed the dependence on the strategy, as it is the same for all agents on the same side.

the right-hand side in condition (25), workers will never build up in the system. Note that the analogous argument can be applied in the case in which the left-hand side is smaller than the right-hand side in condition (26).

Limiting steady state when left-hand side is smaller than the right-hand side in condition (25)**.** In fact, we can precisely characterize the steady state as follows. Since the mass of workers is 0, the rate at which workers die is 0, meaning that workers form matches at rate λ_w (moreover, these matches occur at a steady pace). Hence, employers match with workers at a rate λ_w, meaning that employers die at a rate of $\lambda_e - \lambda_w$, hence the mass of employers in the system is $(\lambda_e - \lambda_w)/\mu$, and in fact the mass of employers remains steady near this value since matches occur in a steady fashion. It follows that the mass of employers concentrates around the limiting value of $L_e = \frac{\lambda_e - \lambda_w}{\mu}$, whereas the mass of workers in the system is 0. Note that $\lambda_w < L_e \eta_e$, consistent with the mass of workers remaining 0.

Note that as $\lambda_w \to \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)_-$ we have $L_w = 0$ (in fact, this holds everywhere in this case) and $L_e \to \frac{\lambda_e}{\mu+\eta_e}$.

Limiting steady state when the left-hand side is larger in both conditions (25) **and** (26)**.** Let N_w be the mass of workers in the system at time t. (Recall that all workers are using the same strategy s_w.) Define N_e similarly. The limiting dynamical system when the left-hand side is larger in both conditions (25) and (26), is given by (refer to the definitions of the η's in Eq. (24))

$$\frac{\mathrm{d}N_w}{\mathrm{d}t} = A\bar{N} + b\,, \text{ for}$$
$$\bar{N} = \begin{bmatrix} N_e \\ N_w \end{bmatrix}, \; b = \begin{bmatrix} \lambda_e \\ \lambda_w \end{bmatrix}, \; A = \begin{bmatrix} -\mu - \eta_e & -\eta_w \\ -\eta_e & -\mu - \eta_w \end{bmatrix}$$

This is a pair of coupled linear differential equations in N_w and N_e, and note that is a special case of those defined in Eq. (23). Matches resulting from options shown to employers form at rate $N_e \eta_e$ and matches resulting from options shown to workers form at rate $N_w \eta_w$. In addition, individual agents die at rate μ, leading to the form of the equations.

The eigenvalues of A are $-\mu$ and $-\mu - \eta_e - \eta_w$. Since the eigenvalues are negative, we deduce [32] that

$$L = \begin{bmatrix} \frac{\lambda_e(\mu+\eta_w) - \lambda_w \eta_w}{\mu(\mu+\eta_e+\eta_w)} \\ \frac{\lambda_w(\mu+\eta_e) - \lambda_e \eta_e}{\mu(\mu+\eta_e+\eta_w)} \end{bmatrix},$$

which solves $A\bar{N} + b = 0$, is a stable fixed point of the dynamical system with a global basin of attraction. Hence, the dynamical system converges globally to L.

Proposition 1. *When agents on each side follow a single strategy the steady-state of the system $L = [L_e, L_w]$ can be characterized as follows:*

1. *When the left-hand side is smaller than the right-hand side in condition* (25), *the system converges to a steady state of*

$$L = \begin{bmatrix} \frac{\lambda_e - \lambda_w}{\mu} \\ 0 \end{bmatrix}. \tag{27}$$

2. *When the left-hand side is larger in both conditions* (25) *and* (26), *the system converges to a steady state of*

$$L = \begin{bmatrix} \frac{\lambda_e(\mu+\eta_w) - \lambda_w \eta_w}{\mu(\mu+\eta_e+\eta_w)} \\ \frac{\lambda_w(\mu+\eta_e) - \lambda_e \eta_e}{\mu(\mu+\eta_e+\eta_w)} \end{bmatrix}.$$

Note that as $\lambda_w \to \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)_+$ we have $L_w \to 0$ and $L_e \to \frac{\lambda_e}{\mu+\eta_e}$. These limiting values of L_w and L_e match the limiting values that arise when $\lambda_w \to \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)_-$.

B Appendix to Section 3: symmetric markets

The simplest case under our model occurs where arrival rates, screening costs, and the valuation distributions are identical on both sides. While this case is a rather standard setting, it is still useful to illustrate some of the main aspects of our model.

Given this market, the equilibria under no intervention can be described as follows. For small or medium-sized search costs, one side takes the role of proposer and the other side waits for proposals. Both sides screen when the search cost is small, whereas only the side receiving proposals screens for intermediate search costs. When screening costs are large, both sides propose without screening and also accept incoming proposals without screening. Formally:

Proposition 2. *Consider a market with $\lambda_w = \lambda_e = \lambda$ and $c_w = c_e = c$. For agents on both sides, their valuations for potential partners are drawn i.i.d. from a U(0,1) distribution. Consider the limit $\mu \to 0$ for each fixed c. Then, the following are the stable stationary equilibria as a function of c_w:*

1. *(workers screen + propose, employers screen + accept/reject) with thresholds $\theta_w = 1 - (2c)^{1/4}$ and $\theta_e = 1 - \sqrt{2c}$. This is an equilibrium for $c \in \left(0, \frac{1}{32}\right)$. In steady state, we have $L_w = L_m = \lambda/((1-\theta_w)(1-\theta_e)) = 1/(2c)^{3/4}$.*

2. *(workers propose w.o. screening, employers screen + accept/reject) with threshold $\theta_e = 1 - \sqrt{2c}$. Workers get an expected utility of $1/2$. This is an equilibrium for $c \in \left[\frac{1}{32}, \frac{1}{8}\right)$. In steady state, we have $L_w = L_m = \lambda/(1-\theta_e) = 1/\sqrt{2c}$.*

3. *Agents on both sides propose without screening, and accept all incoming proposals without screening. This happens when[10] $c \geq \frac{1}{8}$. Both sides earn expected utility $1/2$. In steady state, we have $L_w = L_m = \lambda/(1-\theta_e) = 1/2$.*

In addition to these equilibria, Equilibria 1 and 2 have symmetric counterparts where the roles of workers and employers are reversed, and these represent all the stable stationary equilibria in this setting. Agents who screen have expected utility identical to the threshold they employ.

Proposition 2 is a special case of Theorem 1 (hence it does not need a separate proof). However, we include a sketch of proof for $c \in \left(0, \frac{1}{32}\right)$ (Equilibrium 1), so as to familiarize the reader with the reasoning behind the equilibria characterization.

Before proceeding to the sketch of proof, note that the utility of the side proposing (say workers) is always (weakly) lower than the utility of the side waiting for proposals (employers). The reason is that, when proposing, there is always a risk that the proposal gets rejected if the other side is screening (in fact, it will get rejected with probability $F(\theta_e)$), i.e., there is a *negative externality of screening by recipients on proposers.* As a result, the workers must account for the cost of rejection as part of their search costs. (This also explains why the proposing side

[10]Since in the limit when $\mu \to 0$ all agents will match, this equilibrium achieves the same welfare as one where only workers (or only employers) propose, and neither side screens. We omit these equilibria just to simplify the discussion.

stops screening at a smaller value of c than the side receiving proposals.) In fact, the workers can internalize the cost imposed by the other side rejecting them by making decisions based on what we call the *effective screening cost.* The following simple lemma (proved in Appendix B) formalizes the idea of effective screening costs:

Lemma 1 (Effective screening cost)**.** *Consider the following two systems. In each case, the death rate is μ, and the value of an item to an agent is drawn i.i.d. from some distribution F. Any incoming option is screened (at some cost) revealing the true value of the option, and then accepted/requested if this value exceeds a threshold θ.*

- *System 1: "Potential opportunities" arise according to a Poisson point process of rate η. Each potential option is screened at a cost c, to reveal its value, and requested if the value exceeds θ. The request is approved i.i.d. with probability q, in which case the agent obtains the item and leaves. If there is no request or the request is denied, the agent remains active.*
- *System 2: Options arise according to a point process of rate ηq. Each option is screened at a cost c/q, to reveal its value. The agent chooses to obtain the item if the value exceeds θ.*

Then, the two systems produce the same expected value.

Proof. Consider the first system. Let q' be the likelihood that a value exceeds θ. The probability that a potential option is both requested and approved is qq'. Hence:

- The expected screening cost spent per obtained item is $c/(qq')$.
- The likelihood of obtaining an item before death is the likelihood that a Poisson clock of rate $\eta qq'$ rings before the death Poisson clock of rate μ.

It is easy to check that the two parts of this description each apply also to the second system, since the probability of a value exceeding θ is again q'. Finally, the expected value of an obtained item is just $\mathbb{E}_{X\sim F}[X|X > \theta]$ in each system. Combining, we obtain the claim. ☐

Lemma 1 allows us to relate the threshold strategies at the equilibria as follows. For a given c, consider an equilibrium where (workers screen and propose, employers screen and accept/reject) with strategies $\theta_w(c)$ and $\theta_e(c)$ respectively. Let $\theta^*_w(c)$ be the threshold of workers at an equilibrium when the proposing side is reversed. Then, we have that $\theta_w(c) = \theta^*_w\left(\frac{c}{1-\theta_e(c)}\right)$. The reader can verify that this is indeed the case by considering, e.g., Equilibrium 1 in Proposition 2 and its symmetric counterpart.

We are now in a position to present the sketch of proof of Proposition 2.

Proof sketch of Equilibrium 1 in Proposition 2. We first take the agent strategies as given and compute the steady state masses present on each side. These masses are always equal ($N_w = N_e$) since arrival rates are equal, matching flows affect both sides equally, and the death rate is the same on both sides. In the case described in Equilibrium 1, the number of workers N_w must be such that the flow rate at which workers see potential options (this is again N_w), times the likelihood that a potential option leads to a match, is equal to the match formation rate ρ, which is, in turn, equal to $\lambda - \mu N_w$. Now a potential option leads to a match only if the worker screens and finds that the option has match value exceeding θ_w, which causes the worker to apply, and then the employer screens and finds that the worker has match value exceeding θ_e. This occurs with probability $(1-\theta_w)(1-\theta_e)$. Combining, in steady state we must have $L_w = L_e$ and

$$\begin{aligned} L_w(1-\theta_w)(1-\theta_e) &= \rho = \lambda - \mu L_w \\ \Rightarrow \quad L_w &= \frac{\lambda}{\mu + (1-\theta_w)(1-\theta_e)} \xrightarrow{\mu \to 0} \frac{\lambda}{(1-\theta_w)(1-\theta_e)}\,. \end{aligned} \tag{28}$$

We notice also that the equilibrium has a vanishing flow of agents dying (on both sides) as $\mu \to 0$, consistent with each worker spending $L_w/\lambda_w = O(1)$ expected time in the system, so the likelihood that an agent dies without matching is vanishing, on both sides of the market.

Now, we argue that the strategies described induce an equilibrium. To that end, note that if workers are proposing, for employers is optimal to use the threshold $\theta_e(c) = 1 - \sqrt{2c}$. This follows from the fact that θ_e must be equal to the continuation value of the employers. As $\mu \to 0$ the fraction of employers that do not match vanishes, we then have that in the limit $\mu \to 0$ the continuation value for a fixed c if screening is given by

$$-c + \Pr(u_{ew} \geq \theta_e)\mathrm{E}[u_{ew}|u_{ew} > \theta_e] + (1 - \Pr(u_{ew} \geq \theta_e))\theta_e = -c + (1-\theta_e)\frac{1+\theta_e}{2} + \theta_e^2,$$

that is, the cost of screening an opportunity (first term), plus the probability that a match occurs times the expected match value if a match is formed (second term), plus the probability that a match does not occur times the continuation value θ_e (third term). Therefore, θ_e must satisfy

$$\theta_e = -c + (1-\theta_e)\frac{1+\theta_e}{2} + \theta_e^2.$$

Solving yields $\theta_e = 1 - \sqrt{2c} > 1/2$. This also means that accepting or proposing without screening, which would produce a utility at most $\mathrm{E}[u_{ew}] = 1/2$, is not attractive. Furthermore, employers do not have an incentive to propose (after screening) themselves —if they propose, they will be screened by workers, and thus they risk a costly rejection whereas the distribution of match values for potential options is the same as that for incoming proposals. Thus, we have showed that employers are playing a best response. Now consider workers. Having fixed the threshold of the employers, using Lemma 1 we can think of workers facing an effective screening cost of $c_{\text{eff}} = c/\sqrt{2c} = \sqrt{c/2}$, and hence by symmetry, their threshold will be $\theta_w(c) = \theta_e(\sqrt{c/2}) = 1 - (2c)^{1/4}$. Note that $\theta_w(c) > 1/2$ for $c < 1/32$, which also means that workers do not have any incentive to propose without screening, since that would produce a limiting utility $\mathrm{E}[u_{we}] = 1/2$. (Note how this changes for $c \geq 1/32$, causing workers to propose without screening.)

Finally, we note that the steady state described in Eq. (28) also evolutionarily stable. (It corresponds to attractive fixed points of the best response dynamics Eq. (23), which follows from Proposition 1 since agents on each side are playing their unique best response at $\bar{L}$, meaning that the best responses remain unchanged in a neighborhood of $\bar{L}$.) $\square$

Next, note that in all equilibria, all agents on the same side follow the same strategy and, further, in each equilibrium, one side proposes and the other side waits for incoming proposals (except for the no screening equilibrium where both sides propose). This feature is induced by our equilibrium concept (evolutionarily stable stationary equilibrium), and will continue to arise throughout the rest of the section. To see why, note that if one side was mixing between issuing proposals and only waiting for incoming proposals, the other side must also be mixing (for generic values of problem primitives). However, as soon as the fraction of agents who are proposing is slightly changed, say, increased, agents on the other side will receive proposals at a faster rate and thus have incentive to react by not proposing, which makes it beneficial for agents on the first side to propose. Therefore, such an equilibrium cannot be stable.

To conclude, we note that in this setting, the platform *cannot increase average welfare* by implementing one of the interventions described in Section 2. Intuitively, for a given fixed c, if the platform blocks a side from issuing proposals, the equilibrium that arises is the one under which the other side proposes, and due to market symmetry, this is welfare equivalent (in terms of average welfare) to the no-intervention equilibrium under which the first side was proposing. Similarly, if the platform blocks one side from screening and $c < 1/32$, there is a unique resulting equilibrium where the blocked side proposes (without screening) and the other side screens and accepts/rejects, whose welfare is smaller than equilibrium 1. On the other hand, the welfare at equilibrium will be unaltered if $c \geq 1/32$ as (at least) one side is not screening anyway. Though welfare improvement is via intervention is thus impossible, the equilibria suggest the following search designs: if $c < 1/32$, the platform should allow both sides to screen; if $1/32 \leq c < 1/8$, the platform can implement a one-sided search where only one side screens (saving the other side the effort of proposing, which one may argue is positive instead of zero in real settings); finally, if $c \geq 1/8$, the platform can opt for a centralized matching.

We will see in the next subsections that, as soon as some asymmetry is introduced in the market, interventions can be useful to either *select* the highest welfare equilibria, or *create* equilibria with higher welfare.

C Appendix to Section 3.1

Proof of Theorem 1. First, note that in all equilibria listed in the statement of Theorem 1, agents on each side of the market are using a unique strategy. Therefore, as argued in Section A.3, the steady state will exist in each case, and can be characterized using Proposition 1. In particular, for every equilibria we have $L_w(c_e) = L_e(c_e) = \frac{\lambda}{\mu+\eta_w(c_e)+\eta_e(c_e)} = \Theta(1)$, when $\mu \to 0$, where η_w and η_e are as defined by Eq. (24). We next argue that each of these is an equilibrium in the proposed regime.

1. (employers screen + propose, workers screen + accept/reject). The limiting steady state is $L_e = L_w = \frac{\lambda}{\mu+(2c_e/\alpha)^{1/4}(2\alpha c_e)^{1/2}}$. To argue that this is an equilibrium, note that, if employers are proposing, is optimal for workers to use the threshold $\theta_w(c_e) = 1 - \sqrt{2\alpha c_e}$. This follows from the fact that θ_w must be equal to the continuation value of the workers. As $\mu \to 0$ the fraction of workers who does not match vanishes, we have that in the limit $\mu \to 0$ the continuation value for a fixed c_e if screening is given by

   $$-\alpha c_e + \Pr(u_{we} \geq \theta_w)\mathrm{E}[u_{we}|u_{we} > \theta_w] + (1 - \Pr(u_{we} \geq \theta_w))\theta_w = (1-\theta_w)\frac{1+\theta_w}{2} - \alpha c_e + \theta_w^2$$

   that is, the cost of screening an opportunity (first term), plus the probability that a match occurs times the expected value of an opportunity that results in a match (second term), plus the probability that a match does not occur times the continuation value (third term). Therefore, θ_w must satisfy

   $$\theta_w = (1-\theta_w)\frac{1+\theta_w}{2} - \alpha c_e + \theta_w^2.$$

   Solving for θ_w yields the desired expression. Furthermore, they do not have an incentive to propose themselves –they will be screened by employers, and thus they risk a costly rejection. Therefore, having fixed the threshold of the workers, we can use Lemma 1 and think of employers facing an effective screening cost of $c_{\text{eff}} = c_e/\sqrt{2\alpha c_e} = \sqrt{c_e/(2\alpha)}$, and hence their threshold will be $\theta_e(c)$. In addition, note that workers don't want to screen beyond $c_e = \frac{1}{8\alpha}$; at this point, they would rather accept without screening. Also, employers don't want to screen beyond $c_e = \frac{\alpha}{32}$; at this point, it also becomes profitable for them to stop screening.

2. (employers propose w.o. screening, workers screen + accept/reject), with steady state given by $L_e = L_w = \frac{\lambda}{\mu+(2\alpha c_e)^{1/2}}$. Note that this equilibrium occurs if employers stop screening before workers in Equilibrium 1, that is, if $\frac{1}{8\alpha} \geq \frac{\alpha}{32}$. This is the only equilibrium whose existence depends on the value of α, and it occurs for some c_e's if $\alpha < 2$. Note that, again, if employers are proposing (regardless as to whether they are not screening), for workers is still optimal to use the threshold $\theta_w = 1 - \sqrt{2\alpha c_e}$.

3. (employers screen + propose, workers accept w.o. screening), with the corresponding steady state given by $L_e = L_w = \frac{\lambda}{\mu+(2c_e)^{1/2}}$. As argued in Equilibrium 1, if employers are

proposing, workers will give up on screening only for $c_e \geq \frac{1}{8\alpha}$. Note that, once workers decide not to screen, the effective screening cost for employers becomes c_e, and using the same arguments we used when describing Equilibrium 1, employers will use a threshold of $\theta_e(c_e) = 1 - \sqrt{2c_e}$. Noting that $\theta_e(c_e) \geq \frac{1}{2}$ (1/2 is the expected value that an employer can get by not screening) as long as $c_e \leq \frac{1}{8}$, we conclude that this will be an equilibrium for $c_e \in \left[\frac{1}{8\alpha}, \frac{1}{8}\right)$.

4. (workers screen + propose, employers screen + accept/reject), with the corresponding steady state given by $L_e = L_w = \frac{\lambda}{\mu+(2c_e)^{1/2}(2\alpha^2 c_e)^{1/4}}$. Here, if workers are proposing, it is optimal for a employers to screen and accept/reject with threshold $\theta_e = 1 - \sqrt{2c_e}$. Therefore, workers face an effective screening cost of $c_{\text{eff}} = \alpha c_e/\sqrt{2c_e} = \sqrt{\alpha^2 c_e/2}$ (see Lemma 1), which gives us θ_w. Note that, in this setting, employers will never stop screening before workers, as they have a lower screening cost plus they are not facing rejection (i.e. they are not proposing). Therefore, this will be an equilibrium as long as workers continue to screen, which happens if $c_e < \frac{1}{32\alpha^2}$.

5. (workers propose w.o. screening, employers screen + accept/reject), with the corresponding steady state given by $L_e = L_w = \frac{\lambda}{\mu+(2c_e)^{1/2}}$. This equilibrium occurs when workers no longer want to screen, that is, when $c_e \geq \frac{1}{32\alpha^2}$. In addition, employers will have an incentive to screen as long as $c_e \leq \frac{1}{8}$ (see Equilibrium 3), which defines the range for which this is an equilibrium.

6. Agents on both sides propose without screening, and accept all incoming proposals without screening when $c_e \geq 1/8$. By our previous arguments, it can easily be seen that agents will have an incentive to deviate and screen if $c_e < 1/8$. The corresponding steady state is given by $L_e = L_w = \frac{\lambda}{\mu+1}$.

To conclude the proof, we note that there cannot be any mixed equilibria that is evolutionary stable. If one side, say employers, mixes between proposing and not, then at least a fraction of workers must be proposing; otherwise, employers who are not proposing will never get matched and thus proposing is a profitable deviation. However, once the number agents on the other side who are proposing is slightly perturbed, it will affect the number of employers who want to propose, and thus this cannot be stable. Therefore, in this setting, we have that either all agents on one side propose, or none agent does. The difference can then be in whether they screen or not; however, at an equilibrium, all agents on the same side will have the same continuation value and thus they must use the same θ's if they screen, or none of them must screen.[11] Finally, the difference between the strategies of the agents on the same side can also be on how they handle incoming proposals. However, as we argued before, one side takes the role of proposer and the other one just receives proposals. For the proposers, the decision as to what to do with incoming proposals does not play a role, so we can ignore differences in this. On the other hand, those receiving proposals must either accept without screening or screen and accept/reject; ignoring

[11] Here, we assume that an agent only screens if $\theta > 0$.

proposals can never be an equilibrium strategy. However, as we argued before, all agents must have the same utility, and thus must follow the same strategy. □

Proof of Corollary 1. Recall that $\alpha \geq 1$. Equilibrium 1 exists whenever Equilibrium 4 exists and has (weakly) higher welfare. Similarly, Equilibrium 5 exists whenever Equilibrium 2 exists and has (weakly) higher welfare. Also, Equilibrium 5 exists whenever Equilibrium 3 exists and has identical welfare. We deduce that, for all $c_e < 1/8$, one of Equilibria 1 and 5 is a highest welfare equilibrium.

Now, the difference between the average welfares of Equilibrium 5 and Equilibrium 1 is

$$\frac{1}{2}\left(-\frac{1}{2}+\left(\frac{2c_e}{\alpha}\right)^{1/4}+\sqrt{2c_e}(\sqrt{\alpha}-1)\right),$$

which is strictly increasing in c_e, negative at $c_e = 1/(32\alpha^2)$ and positive at $c_e = \alpha/32$. Hence, c_* is the unique value of c_e such that the two equilibria have identical welfare. We also deduce that Equilibrium 5 maximizes welfare for $c_e > c_*$, and Equilibrium 1 maximizes welfare for $c_e < c_*$. □

D Appendix to Section 3.2

We now prove the results which are stated in Section 3.2. Note that in all equilibria listed in the statements of Theorems 2 and 3, agents on each side of the market are using a unique strategy. Therefore, as argued in Section A.3, the steady state exists in each case, and can be characterized using Proposition 1.

Semantic definition of $\bar{c}$. Consider a setting where employers screen and propose, and workers are not permitted to propose. When a proposal arrives, a worker must decide between screening or accepting it without screening. We define $\bar{c}$ to be the largest screening cost (as $\mu \to 0$) such that there exists a symmetric equilibrium between workers where they screen and accept/reject based on a threshold of $\theta_w = \xi(\lambda, c)$ as given by Eq. (9).

Now, the expected value of a worker who uses strategy θ_w is identical to θ_w, since the process of arrival of proposals/death as seen by a worker is memoryless when the system is steady state. Let V be the expected value from participation, just after a worker w has received a proposal. Let p be the likelihood that a worker receives a proposal before he dies.

Lemma 2. *In the limit $\mu \to 0$, we have*

$$p = \frac{1}{\theta_w + \lambda(1-\theta_w)} = \frac{1}{1 + (\lambda - 1)(1-\theta_w)}\,.$$

Proof. Employers make $1/(1-\theta_w)$ proposals per unit time as $\mu \to 0$, since a vanishing mass of employers die without being matched. In comparison, a mass of λ workers arrive per unit time. Hence, the expected number of proposals received by a worker (who uses strategy θ_w) during his lifetime is

$$n_w = 1/(\lambda(1-\theta_w))\,. \tag{29}$$

Let p be the likelihood that a worker receives a proposal from an employer before he dies. Checking for consistency when workers screen with threshold θ_w, we obtain

$$n_w = p(1 + \theta_w n_w)\,. \tag{30}$$

Combining Eqs. (29) and (30), we obtain

$$p = \frac{1}{\theta_w + \lambda(1-\theta_w)} = \frac{1}{1 + (\lambda - 1)(1-\theta_w)}\,. \tag{31}$$

(Notice that identical quantities appear in the analysis of the case where workers propose and employers screen and accept. Now, n_w is defined as the average number of opportunities that a worker receives to propose to an employer who will accept him, during his lifetime, if he adopts strategy θ_w. And p is the likelihood of receiving such an opportunity before he dies.) □

Remark 2. *Lemma 2 also applies to the likelihood p that a worker will get an opportunity to propose to an employer who will accept him, in the case where workers propose and employers screen and accept.*

Considering the possible cases —either a worker receives a proposal before he dies, or he does not—, we obtain

$$\theta_w = pV + (1-p)\cdot 0 = pV\,. \tag{32}$$

Note that if a worker simply accepts an incoming proposal, his expected value is 1/2. Hence, if the worker is indifferent between accepting without screening, and using strategy θ_w, we have $V = 1/2$. Using this together with Lemma 2 in Eq. (32), and making the dependence on c explicit, we obtain $1/2 = \theta_w(\bar{c})/p(\bar{c}) = \theta_w(\bar{c})\,(1+(\lambda-1)(1-\theta_w(\bar{c})))$. Solving for $\theta_w(\bar{c})$ we obtain that $\theta_w(\bar{c}) = \frac{\lambda-\sqrt{(\lambda-1)^2+1}}{2(\lambda-1)}$. Using the expression for $\xi(\cdot, c)$ in Eq. (9) we can solve for $\bar{c}$ to obtain Eq. (10).

Semantic definition of $\underline{c}$. We define $\underline{c}$ to be the smallest value of c (as $\mu \to 0$), such that, if the employers are proposing (and workers are not permitted to propose), there is a symmetric equilibrium between workers where they accept incoming proposals without screening. Suppose other workers are not screening (i.e., $\theta_w = 0$). Using Lemma 2, we know that the likelihood that a worker will receive a proposal before he dies, is $p = 1/\lambda$. Note that the value obtained by accepting without screening is $V' = p/2 = 1/(2\lambda)$. Now, suppose a worker receives a proposal. By accepting without screening, he can earn $V = 1/2$. This is a best response if and only if the worker cannot do better by screening the current proposal, accepting with a threshold of $1/(2\lambda)$ (this threshold is exceeded with likelihood $1 - 1/(2\lambda)$, and the expected value of the match, conditioned on the threshold being exceeded, is $(1/2)(1+1/(2\lambda))$), and if the value is below the threshold, accepting the next proposal, if any, without screening (this follows from the idea of a "rollout" in dynamic programming [6]). The value obtained from the latter strategy is

$$-c + (1/2)(1+1/(2\lambda))(1-1/(2\lambda)) + V'/(2\lambda)\,.$$

Comparing with $V = 1/2$ and using $V' = 1/(2\lambda)$, we find that the deviation does not increase welfare if and only if

$$c \geq 1/(8\lambda^2)\,, \tag{33}$$

leading to Eq. (10).

Semantic definition of $\hat{c}$. Again consider the setting where employers screen and propose and workers screen and accept/reject. Workers are not permitted to propose. We define $\hat{c}$ to be the screening cost (as $\mu \to 0$) at which employers are indifferent between screening, and proposing without screening, assuming workers are screening with threshold $\theta_w(c)$. (Employers do not have any externality on each other, being on the short side.)

Note that if workers are screening, then (using Lemma 1) the effective cost for employers is equal to $c_{\text{eff}} = c/(1-\theta_w(c))$. Now the value and threshold for employers when they screen before proposing is $\theta_e(c) = 1 - \sqrt{2c_{\text{eff}}}$. The value when employers don't screen is $1/2$. It follows that $1 - \sqrt{2c_{\text{eff}}} = 1/2$ for $c = \hat{c}$, since employers are indifferent between screening and not screening for $c = \hat{c}$. We deduce that $\theta_w(\hat{c}) - 1 + 8\hat{c} = 0$, which yields Eq. (11).

Remark 3. *It is easy to verify that $\bar{c} > \underline{c}$.*

One simply uses

$$\begin{aligned}\sqrt{1 + (\lambda - 1)^2} &= \lambda\sqrt{1 - \frac{2(\lambda - 1)}{\lambda^2}} \\ &> \lambda\left(1 - \frac{(\lambda - 1)}{\lambda^2}\right) \\ &= \lambda - 1 + 1/\lambda\,,\end{aligned}$$

to obtain

$$\bar{c} > \frac{1}{4(\lambda^2 + 1)} > \frac{1}{8\lambda^2} = \underline{c}\,.$$

Proof of Theorem 2. We first establish that the following is a subset of equilibria, as a function of c, taking $\mu \to 0$:

- (workers screen + propose, employers screen + accept/reject) with thresholds: $\theta_w = \theta_w(\sqrt{\frac{c}{2}})$ and $\theta_e = 1 - \sqrt{2c}$. This is an equilibrium for $c \in (0, 2\bar{c}^2)$. The limiting steady state $L = [L_e, L_w]$ is given by $L = [0, (\lambda - 1)/\mu]$.
- (workers propose w.o. screening, employers screen + accept/reject) with threshold: $\theta_e = 1 - \sqrt{2c}$. This is an equilibrium for $c \in \left(2\underline{c}^2, \frac{1}{8}\right)$. The limiting steady state $L = [L_e, L_w]$ is given by $L = [0, (\lambda - 1)/\mu]$.
- Agents on both sides propose without screening, and accept all incoming proposals without screening. This happens when $c \geq \frac{1}{8}$. Again, the steady state $L = [L_e, L_w]$ is given by $L = [0, (\lambda - 1)/\mu]$.

Suppose the workers are proposing. Then, it is clear that as $\mu \to 0$, the value of the employers is upper-bounded by $\max(1/2, 1 - \sqrt{2c})$ which is the value employers can get if they are guaranteed not to die. We show that if employers wait for incoming proposals, then the value they obtain approaches the upper bound as $\mu \to 0$. If $c \leq 1/8$, the employers screen and accept/reject, employing a threshold of $1 - \sqrt{2c}$, and producing a utility which tends to $1 - \sqrt{2c} \geq 1/2$ for employers, showing that this is a best response for employers. If $c \geq 1/8$, employers accept without screening. In this case, they will also propose if they are given the chance as, by symmetry, workers will not screen either. It remains to characterize the symmetric equilibria between workers in response to this behavior of employers. For $c \geq 1/8$, it is clearly a best response for workers to propose without screening, thus establishing the third bullet. Consider $c < 1/8$. The effective screening cost faced by workers is $c_{\text{eff}} = c/\sqrt{2c} = \sqrt{c/2}$, see Lemma 1.

Suppose other workers are not screening (i.e., $\theta_w = 0$). Using Lemma 2, we know that the likelihood that a worker will receive an opportunity to propose to an employer who will accept him before he dies, is $p = 1/\lambda$. Using Lemma 1, it suffices to analyze an alternate situation where a worker is receiving instead of making proposals, but screening costs are c_{eff} and the likelihood of getting a proposal before he dies is p. Consider this alternate situation, simultaneously for all workers. Then the condition for existence of a symmetric equilibrium where workers accept without screening is

$$c_{\text{eff}} \geq \underline{c} \quad \Rightarrow \quad c \geq 2\underline{c}^2.$$

Thus, we have established the second bullet.

For the first bullet, suppose there is a symmetric equilibrium between workers where they screen, with a threshold of θ_w. If workers have incentive to screen, then clearly so do employers, since the employers are not facing any possibility of rejection (with proposals being incoming) and the employers have unlimited opportunities to match, being on the short side of the market. Hence, we know that employers are screening, and using a threshold of $1 - \sqrt{2c}$. Thus, workers again face an effective screening cost of $c_{\text{eff}} = c/\sqrt{2c} = \sqrt{c/2}$. Using Lemma 2, we have $p = 1/(1 + (\lambda - 1)(1 - \theta_e))$. We again consider the alternate situation suggested by Lemma 1, with proposals guaranteed to be accepted, screening cost c_{eff}, and probability p of getting an opportunity before death. Then the value obtained a by a worker if he screens and uses the optimal threshold is[12] $\xi(c_{\text{eff}})$, cf. Eq. (9). In comparison, the value obtained by taking the first proposal opportunity without screening is $p(1/2) = p/2$. The best response condition is thus $\xi(c_{\text{eff}}) \geq p/2$, which yields $c_{\text{eff}} \leq \bar{c}$ by definition of $\bar{c}$. Plugging in $c_{\text{eff}} = \sqrt{c/2}$, we obtain $c \leq 2\bar{c}^2$, yielding the first bullet.

Finally, we argue that there are no other stable equilibria if $\lambda \geq 1.25$. We rule out the possibilities one by one. Suppose both sides mix between proposing and don't proposing. Then if a few more workers start proposing, this will make less employers propose, since proposing becomes relatively less attractive for employers. In turn, this will make more workers propose and so on. Therefore, an equilibrium where both sides mix between proposing and not cannot be stable.

Suppose one side mixes between proposing and don't proposing, but the other side does not propose. This is ruled out because all agents on the first side will want to propose. In addition, suppose one side mixes between proposing and don't proposing, but the other side proposes. Suppose the workers (long side) are mixing. Compared to the case where all workers are not proposing, some workers proposing makes things worse for the other workers. So the stable situation can only be that the long side is either all proposing or none are proposing. In addition, we can rule out the case in which employers are mixing, because if all workers are proposing, then employers don't want to propose. Therefore, we must have that either all workers propose or all workers don't propose, and similarly for employers. Furthermore, for $c < 1/8$, it also can't be that both sides propose. (For $c \geq 1/8$, both sides proposing and

[12] Here we suppress dependence on λ.

accepting without screening will be the unique equilibrium.) Hence, it must be that one side proposes and the other side does not. When $\lambda > 1.25$, we argued that all workers will want to propose as the unique best response, and thus employers will never propose. In addition, we can rule out that workers will mix between screening and not screening in any stable equilibrium. Therefore, workers' best response will be to propose if $\lambda \geq 1.25$. $\square$

Proof of Theorem 3. We want to prove that if workers are not allowed to propose, the following equilibria exist as a function of c, taking $\mu \to 0$:

- (employers screen + propose, workers screen + accept/reject) with thresholds: $\theta_w = \xi(\lambda, c)$ and $\theta_e = 1 - \sqrt{2c/(1-\theta_w)}$. This is an equilibrium for $c \in (0, \min(\bar{c}, \hat{c})]$. The steady state $L = [L_e, L_w]$ is given by $L = \left[\frac{1}{\mu+\sqrt{2c(1-\xi(\lambda,c))}}, \frac{\lambda}{\mu} - \frac{\sqrt{2c(1-\xi(\lambda,c))}}{\mu^2+\mu\sqrt{2c(1-\xi(\lambda,c))}}\right]$.
- (employers screen + propose, workers accept) with threshold: $\theta_e = 1 - \sqrt{2c}$. This is an equilibrium for $c \in [\frac{1}{8\lambda^2}, \frac{1}{8}]$. The corresponding steady state $L = [L_e, L_w]$ is given by $L = \left[\frac{1}{\mu+\sqrt{2c}}, \frac{\lambda}{\mu} - \frac{\sqrt{2c}}{\mu^2+\mu\sqrt{2c}}\right]$.
- (employers propose w.o. screening, workers screen + accept/reject) with threshold: $\theta_w = \xi(\lambda, c)$. This is an equilibrium for $c \in [\hat{c}, \bar{c}]$, and only exists if $\hat{c} < \bar{c}$ (might not exist at all). The corresponding steady state $L = [L_e, L_w]$ is given by $L = \left[\frac{1}{\mu+1-\xi(\lambda,c)}, \frac{\lambda}{\mu} - \frac{1-\xi(\lambda,c)}{\mu^2+\mu(1-\xi(\lambda,c))}\right]$.

The first bullet follows from the fact that it is a best response for workers to screen and accept/reject with threshold $\xi(\lambda, c)$ for $c \leq \bar{c}$, provided other workers are doing the same; and it is a best response for employers to screen and propose if $c \leq \hat{c}$.

The second bullet follows from the definition of $\underline{c}$ (hence accepting without screening is an equilibrium among workers), and the fact that when the workers are not screening and $c \leq 1/8$, it is a best response for the employers to screen and propose, with a threshold of $1 - \sqrt{2c}$.

It is easy to see that if workers are screening with a threshold of $\xi(\lambda, c)$, it is a best response for employers to propose without screening if $c > \hat{c}$. (Employers do not exert any externality on each other, hence fixing the way workers respond to proposals, exactly one of the two equilibria exist between employers.) Combining with the definition of $\bar{c}$ (implying that workers are playing a best response), we deduce the third bullet. $\square$

Proof of Corollary 2. Theorem 3 equilibrium 1 exists for all $c < \min(\bar{c}, \hat{c})$.

- It may coexist with Theorem 3 equilibrium 2 (but not with the other two equilibria in Theorem 3). If this is the case, Theorem 2 equilibrium 2 exists for the same market under no intervention, and has welfare identical to Theorem 3 equilibrium 2.
- For the same market under no intervention, the possible equilibria are Theorem 2 equilibria 1 and 2. One or both of them may exist for the market under consideration.

It follows that if the stated condition holds, then preventing workers from proposing can only increase (or leave unchanged) average welfare in equilibrium, relative to the case of no intervention. $\square$

Theorem 4. *Consider the market defined in the statement of Theorem 2. Then, in addition to those defined in Theorem 2, the following equilibria might also exist. (Here, the equilibria are characterized by their limiting description as a function of c, considering $\mu \to 0$ for each fixed c):*

1. *(employers screen + propose, workers screen + accept/reject) with thresholds: $\theta_w = \theta_w(c)$ and $\theta_e = 1 - \sqrt{2c/(1-\theta_w)}$. This is an equilibrium for $c \in (\bar{c}_2, \min(\bar{c}, \hat{c}, \bar{c}_4))$. Furthermore, this equilibrium exists if and only if $\lambda \leq 1.25$. The steady state $L = [L_e, L_w]$ is given by $L = \left[\frac{1}{\mu+\sqrt{2c(1-\xi(\lambda,c))}}, \frac{\lambda}{\mu} - \frac{\sqrt{2c(1-\xi(\lambda,c))}}{\mu^2+\mu\sqrt{2c(1-\xi(\lambda,c))}}\right]$.*
2. *(employers propose w.o. screening, workers screen + accept/reject) with threshold: $\theta_w = \xi(\lambda, c)$. This is an equilibrium for $c \in [\max(\hat{c}, \bar{c}_3), \min(\bar{c}, \bar{c}_4)]$. Furthermore, this equilibrium exists if and only if $\lambda \leq 1.25$. The corresponding steady state $L = [L_e, L_w]$ is given by $L = \left[\frac{1}{\mu+1-\xi(\lambda,c)}, \frac{\lambda}{\mu} - \frac{1-\xi(\lambda,c)}{\mu^2+\mu(1-\xi(\lambda,c))}\right]$.*

where $\xi(\cdot,\cdot)$ is as defined by Eq. (9), $\bar{c}$ and $\hat{c}$ are as defined by Eqs. (10) and (11) respectively, and

$$\bar{c}_2 = 4(\lambda - 1)^3 \,, \tag{34}$$

$$\bar{c}_3 = \frac{2(\lambda-1)^2}{(4\lambda-3)^2} \,. \tag{35}$$

$$\bar{c}_4 = \frac{(3-2\lambda)}{8} \,. \tag{36}$$

Furthermore, (employers screen + propose, workers accept) cannot be equilibrium unless there is a system intervention.

Proof of Theorem 4. To prove when (employers screen + propose, workers screen + accept/reject) is an equilibrium, recall that in Theorem 3 we showed that, when workers are not allowed to propose, (employers screen + propose, workers screen + accept/reject) with thresholds $\theta_w = \theta_w(c)$ and $\theta_e = 1 - \sqrt{2c/(1-\theta_w)}$ is an equilibrium for $c \in (0, \min(\bar{c}, \hat{c})]$. For this to be an equilibrium without any intervention, we must make sure that, given a chance to propose, a worker would prefer to ignore it.

To that end, suppose that all workers and employers follow the strategies described above, and a single worker deviates from this strategy by proposing if he gets the chance to do so. It is easy to see that an employer who receives this proposal will screen it with the same threshold θ_e as this maximizes her value. Using Lemma 1, a worker will face an effective cost of $c/(1-\theta_e) = \sqrt{\frac{c}{2}(1-\theta_w)}$ to screen that opportunity and decide whether to propose. Given this cost, if he decides to screen it, he will still do so with threshold θ_w. Then, he will only take the opportunity to screen and propose if:

$$-\sqrt{\frac{c}{2}(1-\theta_w)} + (1-\theta_w)\frac{1+\theta_w}{2} + \theta_w\theta_w \geq \theta_w.$$

The first term is the effective screening cost, the second term is the expected value if he likes the employer times the probability of liking her, the third term is the continuation value times the probability of not liking the proposed employer; this should exceed the continuation value obtained by doing nothing (θ_w). Rearranging the terms, we obtain that the deviation is profitable only if $\theta_w \leq 1-(2c)^{1/3}$. Therefore, for this to be an equilibrium we need to have $c \geq \bar{c}_2 = 4(\lambda-1)^3$ (and $c \leq 1/2$).

However, there is also the possibility that a worker would want to propose without screening. In this case, his proposal will be accepted with probability $1-\theta_e$, and if accepted he gets an expected utility of 1/2. Hence, this will be a profitable deviation if

$$\frac{1}{2}(1-\theta_e) + \theta_e\theta_w \geq \theta_w,$$

or equivalently, $\theta_w \leq 1/2$, which occurs only if $c \geq \bar{c}_4 = (3-2\lambda)/8$. Note that the interval $(\bar{c}_2, \min(\bar{c}, \hat{c}, \bar{c}_4))$ will be non-empty only if $\lambda \leq 1.25$. Furthermore, $\hat{c} = \min(\bar{c}, \hat{c}, \bar{c}_4)$ when $\lambda \in [1, 1.25]$, which completes the proof of the first claim.

To prove the second equilibria, again note that in Theorem 3 we showed that, when workers are not allowed to propose, (employers propose w.o. screening, workers screen + accept/reject) with threshold $\theta_w = \theta_w(c)$ is an equilibrium for $c \in [\hat{c}, \bar{c}]$. As before, for this to be an equilibrium in a no-intervention setting, it must be the case that a worker does not want to propose if he gets the chance. To that end, suppose that all workers and employers follow the strategies described above, and a single worker deviates from this strategy by proposing if he gets the chance to do so. It is easy to see that an employer who receives this proposal will now screen it with a threshold equal to 1/2. Hence, the worker will now face an effective cost of $2c$, if he wishes to screen such an opportunity. For him to choose not to screen and propose, and rather wait for a proposal, it must be that $\theta_w \geq 1-\sqrt{4c}$, which happens only if $c \geq \bar{c}_3 = \frac{2(\lambda-1)^2}{(4\lambda-3)^2}$. As before, we must also consider the possibility that a worker would rather propose without screening, which happens if $c \geq \bar{c}_4$. Therefore, (employers propose w.o. screening, workers screen + accept/reject) will be an equilibrium only if $c \in [\max(\hat{c}, \bar{c}_3), \min(\bar{c}, \bar{c}_4)]$. Noting that $\bar{c} \geq \bar{c}_4$ completes the proof.

Finally, note that (employers screen + propose, workers accept) cannot be equilibrium unless there is a system intervention. To see why, note that in the case a worker who gets an opportunity to propose, will do so. □

D.1 When does the intervention help

Numerics based on this Corollary 2 reveal (see Figure 3), that the platform should block workers from proposing for all c less than a threshold, where the threshold is equal to $\bar{c}$ for $\lambda > 1.67$ (including $\lambda = 2$, see Figure 2), and smaller than $\min(\hat{c}, \bar{c})$ for $\lambda < 1.67$. The intervention helps when the benefit to workers exceeds the cost to employers (note that employers face some risk

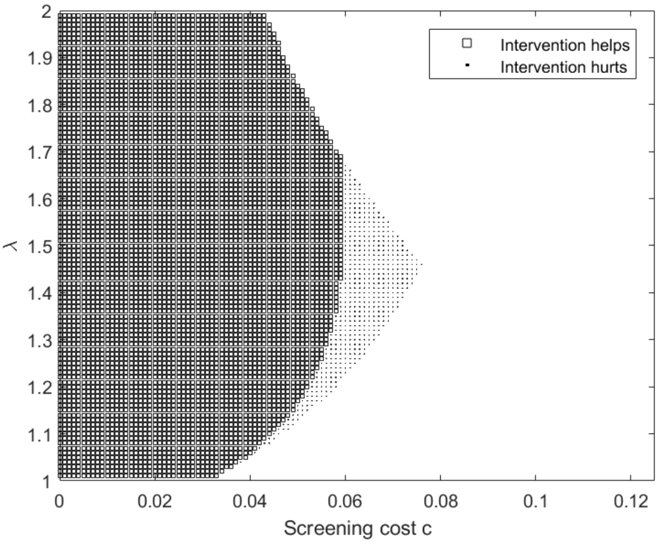

Figure 3: This figure shows the values of (λ, c) for which the average welfare increases/decreases by blocking workers from proposing (see Corollary 2). The intervention will help if the parameters fall in the dark gray region (shown via squares above), will hurt in the light gray region (shown via dots), and will not have an effect in the white area.

of rejection, and hence a raised effective screening cost).

E Appendix to Section 4

We formally define our model in Section E.1. We formally capture the equilibrium that arises under no intervention in Section E.2, and the equilibria that arise under the various interventions suggested (see Section 4) in Section E.3.

E.1 Augmented model: one tier of employers, two tiers of workers

Employers are ex ante homogeneous as before, but there are two *tiers* of workers: "top" (t) workers and "bottom" (b) workers. We now describe how the model introduced in Section 2 is extended to this new setting.

Match utility and informal agent-level dynamics. Employers, top workers and bottom workers arrive at exogenously specified rates λ_e, λ_w^{t}, λ_w^{b} respectively, and agents who are present have a hazard rate of μ of dying without matching. An employer's utility for matching with a worker of quality/tier τ is a^τ + Uniform$(0, 1)$, independent across pairs. We fix the "quality" terms $a^{\mathrm{t}} = a > 0$ and $a^{\mathrm{b}} = 0$, and these values are common knowledge. An employer must spend screening cost c to learn her "idiosyncratic" term (distributed as Uniform$(0, 1)$) for a particular worker. The utility of a worker (belonging to either tier) for matching with an employer is i.i.d. Uniform$(0, 1)$, and again can be learned by the worker by spending a screening cost c.

When an employer receives a proposal, she learns, at no cost, whether it is from a top worker or from a bottom worker. When an employer has an opportunity to request a potential match, she can specify what tiers of workers she is interested in, and in what order of priority. The platform will show her a uniformly random candidate from her most preferred tier of workers that currently has workers available, or do nothing if there is no available worker of the tier(s) desired by the employer. Again, the employer knows the tier of the potential partner she has been presented with, and can accordingly decide whether she wants to screen, etc. Workers who request a candidate when an opportunity arises are presented with a uniformly random employer as before. As before, each agent (of any kind) has a Poisson opportunity clock of rate 1.

Augmented strategy space. Strategies of both bottom and top workers are still as defined in Definition 1. However, the strategy space of employers is enlarged.

- Employers now decide what to do with incoming proposals by agents from *each tier*, and the choices are as before: ignore the proposals, accept without screening, or screen and accept/reject.
- Employer's strategy for proposing: Each employer has a (possibly incomplete) preference list over tiers of workers. If a tier is not listed, we assume that the employer is not interested in that tier. For each tier in the list, the employer specifies whether to do nothing, propose without screening, or to screen and propose if the match value is above the acceptability threshold.

- Each employer has a deterministic threshold θ_e used to screen participants. (Recall that, at equilibrium, θ_e will be equal for all employers *regardless* of their strategy.)

Augmented dynamics. As in Section 2.2, fix the thresholds $\theta_w^{\mathrm{t}}, \theta_w^{\mathrm{b}}$ and θ_e, and let $\mathcal{S}_w^{\mathrm{t}}$, $\mathcal{S}_w^{\mathrm{b}}$ and $\mathcal{S}_e$ be the sets of possible strategies available to top workers, bottom workers and employers respectively (again, these sets are finite). Fix the fractions $(f_w^{\mathrm{t}}(s))_{s\in\mathcal{S}_w^{\mathrm{t}}}$, $(f_w^{\mathrm{b}}(s))_{s\in\mathcal{S}_w^{\mathrm{t}}}$ and $(f_e(s))_{s\in\mathcal{S}_e}$. We want to study the evolution of the masses of employers and top and bottom workers following each of the possible strategies in the system. Denote these masses by $N_e(s)$, $N_w^{\mathrm{t}}(s)$, and $N_w^{\mathrm{b}}(s)$ respectively, $\bar{N}$ will denote the vector of such masses, and N_e, N_w^{t}, and N_w^{b} denote the total mass of agents of the given type in the system. The rate of evolution of these masses in the system is given by:

$$\begin{aligned}
\frac{\mathrm{d}N_w^{\mathrm{t}}(s_w)}{\mathrm{d}t} &= f_w^{\mathrm{t}}(s_w)\lambda_w^{\mathrm{t}} - N_w^{\mathrm{t}}(s_w)\mu - \rho_w^{\mathrm{t}}(s_w;\bar{N}) \qquad \forall s_w \in \mathcal{S}_w^{\mathrm{t}}\,, \\
\frac{\mathrm{d}N_w^{\mathrm{b}}(s_w)}{\mathrm{d}t} &= f_w^{\mathrm{b}}(s_w)\lambda_w^{\mathrm{b}} - N_w^{\mathrm{b}}(s_w)\mu - \rho_w^{\mathrm{b}}(s_w;\bar{N}) \qquad \forall s_w \in \mathcal{S}_w^{\mathrm{b}}\,, \\
\frac{\mathrm{d}N_e(s_e)}{\mathrm{d}t} &= f_e(s_e)\lambda_e - N_e(s_e)\mu - \rho_e^{\mathrm{t}}(s_w;\bar{N}) - \rho_e^{\mathrm{b}}(s_w;\bar{N}) \qquad \forall s_e \in \mathcal{S}_e
\end{aligned} \tag{37}$$

where $\rho_w^{\tau}(s_w;\bar{N})$, denotes the flow rate at which workers of tier $\tau \in \{t, b\}$ following strategy s_w are matched and, similarly, $\rho_e^{\tau}(s_e;\bar{N})$ denotes the flow rate at which employers following strategy s_e are matched to workers of tier $\tau \in \{t, b\}$.

Match formation rates. The reasoning and definitions in Section 2.2 remain mostly unchanged. However, now employers who want to propose specify *a preference order over tiers* when they request a candidate, which introduces some subtleties that need to be accounted for. We consider two cases that come up based on whether there is a positive mass of both worker types or of just one of the worker types (the third case of $N_w^{\mathrm{t}} = N_w^{\mathrm{b}} = 0$ cannot occur under our assumption $\lambda_e < \lambda_w^{\mathrm{t}} + \lambda_w^{\mathrm{b}}$).

$\mathbf{N_w^t > 0}$ **and** $\mathbf{N_w^b > 0}$. Employers will be able to issue proposals to workers of their preferred tier. Then analogs of Eq. (3) will capture the match flow resulting from proposals by employers. For instance, for a strategy s_e that prefers top worker candidates over bottom ones, we have

$$\rho_e^{\mathrm{t}}(s_e, s_w^{\mathrm{t}};\bar{N}) = N_e(s_e)\frac{N_w^{\mathrm{t}}(s_w^{\mathrm{t}})}{N_w^{\mathrm{t}}}\eta_e^{\mathrm{t}}(s_e, s_w^{\mathrm{t}}) \;\; \forall s_w^{\mathrm{t}} \in \mathcal{S}_w^{\mathrm{t}} \quad \text{and} \quad \rho_e^{\mathrm{b}}(s_e, s_w^{\mathrm{b}};\bar{N}) = 0 \;\; \forall s_w^{\mathrm{b}} \in \mathcal{S}_w^{\mathrm{b}}\,,$$

$$\begin{aligned}
\text{where} \quad \eta_e^{\mathrm{t}}(s_e, s_w^{\mathrm{t}}) = \big(&\mathbb{I}(s_e \text{ involves P}) - F(\theta_e - a^{\mathrm{t}})\mathbb{I}(s_e \text{ involves S+P to t})\big) \\
&\cdot \big(\mathbb{I}(s_w^{\mathrm{t}} \text{ involves A}) - G(\theta_w^{\mathrm{t}})\mathbb{I}(s_w^{\mathrm{t}} \text{ involves S+A/R})\big).
\end{aligned}$$

The match flows $\rho_w^{\tau}(s_w^{\tau}, s_e;\bar{N})$ arising due to proposals from workers (of each type) to employers are captured by Eq. (3) if $N_e > 0$. If $N_e = 0$ and remains zero for a non-zero interval of time (the complementary technical case of $N_e = 0$ for an instant will not come up), similar to Eq. (4), we have

$$\rho_w^\tau(s_w^\tau, s_e; \bar{N}) = \lambda_e f(s_e) \frac{N_w^\tau(s_w^\tau)\eta_w(s_w^\tau, s_e)}{\sum_{s_w^{\mathrm{t}} \in \mathcal{S}_w^{\mathrm{t}}} N_w^{\mathrm{t}}(s_w^{\mathrm{t}})\eta_w(s_w^{\mathrm{t}}, s_e) + \sum_{s_w^{\mathrm{b}} \in \mathcal{S}_w^{\mathrm{b}}} N_w^{\mathrm{b}}(s_w^{\mathrm{b}})\eta_w(s_w^{\mathrm{b}}, s_e)}$$

for $\tau \in \{\mathrm{t}, \mathrm{b}\}$.

$\mathbf{N_w^{\mathrm{t}} = 0}$ **and** $\mathbf{N_w^{\mathrm{b}} > 0}$. Further, suppose $N_w^{\mathrm{t}} = 0$ persists for a non-zero interval of time. (The complementary case of $N_w^{\mathrm{t}} = 0$ for an instant does not arise.) The flow of proposals to bottom workers now depends on both the mass of employers who list top workers as the first choice when they request a candidate, and also on the mass of the subset of these who ask to see a bottom worker candidate as their second choice.

The match flows $\rho_e^{\mathrm{t}}(s_e, s_w^{\mathrm{t}}; \bar{N})$ resulting from proposals by employers to top workers are given by Eq (4) (with the sum in the denominator being only over strategies in $\mathcal{S}_e$ that prefer to see a top tier candidate). For s_e such that a bottom worker candidate is requested as a second preference, the flow of bottom worker candidates seen by employers following s_e is then

$$\text{Flow of opportunities} - \text{Flow of top worker candidates} = N_e(s_e) - \sum_{s_w^{\mathrm{t}} \in \mathcal{S}_w^{\mathrm{t}}} \frac{\rho_e^{\mathrm{t}}(s_e, s_w^{\mathrm{t}}; \bar{N})}{\eta_e(s_e, s_w^{\mathrm{t}})}.$$

Multiplying this flow by $\frac{N_w^{\mathrm{b}}(s_w^{\mathrm{b}})\eta_e^{\mathrm{b}}(s_e, s_w^{\mathrm{b}})}{N_w^{\mathrm{b}}}$ yields the match flow $\rho_e^{\mathrm{b}}(s_e, s_w^{\mathrm{b}}; \bar{N})$. For strategies s_e that ask for a bottom worker candidate as the first preference, the match flows are given by $\rho_e^{\mathrm{b}}(s_e, s_w^{\mathrm{b}}; \bar{N}) = N_e(s_e)\frac{N_w^{\mathrm{b}}(s_w^{\mathrm{b}})}{N_w^{\mathrm{b}}}\eta_e^{\mathrm{b}}(s_e, s_w^{\mathrm{b}})$ as before.

The match flows due to proposals by top workers $\rho_w^{\mathrm{t}}(s_w^{\mathrm{t}}, s_e; \bar{N})$ are all 0 since there are no top workers in the system. The match flows $\rho_w^{\mathrm{b}}(s_w^{\mathrm{b}}, s_e; \bar{N})$ arising due to proposals by bottom workers are given by Eq. (3) if $N_e > 0$. (The complementary case $N_e = N_w^{\mathrm{t}} = 0$ does not arise.)

Expected utilities. As before, the expected utilities can be easily described in terms of the match flows and the steady state $\bar{L}$ (see Eq. (6)). In particular, the expected utilities for both bottom and top workers are again given by Eq. (6). The expected utilities for employers are similarly defined, with four terms capturing interactions *with each type of worker.*

Stationary equilibrium and stability. The definitions of stationary equilibrium (Definition 2) and evolutionary stability remain analogous to those for the case of no tiers.

E.2 Equilibrium under no platform intervention

We now describe the equilibrium when no intervention is present. We find a low welfare equilibrium where bottom workers propose without screening, and employers screen and propose only to top workers in all equilibria. Note that this is similar to the insights obtained in Section 3, in the sense that agents propose to those with more market power. However, here, we find that the equilibrium has poor welfare, in particular for bottom workers, even in the limit $c \to 0$.

Fix $a \in (0,1)$. Define $\delta^t = \lambda_w^t a/2 > 0$ and assume[13] $\lambda_e \in (\lambda_w^t + \delta^t, \lambda_w^t + \lambda_w^b + \delta^t)$. Then, there is a stable equilibrium —unique if $\lambda_w^b \geq 1.20(\lambda_e - \lambda_w^t - \delta^t)$— with the following description when we take $\mu \to 0$ and then $c \to 0$:

- Bottom workers propose without screening.
- Top workers do not propose, and screen and accept/reject incoming proposals using a threshold of $\theta_w^{\mathrm{t}} = 1 - \sqrt{2c}$.
- Employers do not propose to bottom workers. When the opportunity arises, employers screen an available top worker, and propose to him with the same threshold of[14] $\theta_e = 1 - \sqrt{2c}$. The employers split into two types based on how they respond to proposals from bottom workers.
  - **Reachers:** A fraction $\frac{\lambda_w^t + \delta^t}{\lambda_e}$ of employers ignore proposals from bottom workers, and instead wait in the hope of matching with a top worker.
  - **Settlers:** All other employers screen and accept/reject incoming proposals from bottom workers with a threshold of θ_e, as a result they typically match with bottom workers.

In steady state, there is a mass $\delta^t/\mu = \Theta(1/\mu)$ of reachers in the market (to leading order), a mass

$$\frac{(\lambda_e - \lambda_w^t - \delta^t)\delta^t}{(\lambda_w^b + \lambda_w^t + \delta^t - \lambda_e)\sqrt{2c}} = \Theta(1/\sqrt{c})$$

of settlers, and a mass $(\lambda_w^b + \lambda_w^t + \delta^t - \lambda_e)/\mu = \Theta(1/\mu)$ of bottom workers, whereas the mass of top workers in the system is always 0. In fact, there are effectively two (almost) independent submarkets:

- **"Top submarket":** Top workers very quickly match (typically to reacher employers) and leave, earning expected utility $\theta_w^{\mathrm{t}} \to 1$. A fraction $1/(1 + a/2) + o(1)$ of reacher employers match with top workers (earning a utility that is Uniform$(\theta_e, 1 + a)$ whereas the rest die without matching, consistent with their equilibrium utility of $\theta_e \to 1$).
- **"Bottom submarket":** Settler employers earn the same expected utility of θ_e by typically matching with a bottom worker (whom they like). A fraction $(\lambda_e - \lambda_w^t - \delta^t)/\lambda_w^b$ of bottom workers are lucky enough to match, earning expected match utility $1/2$ each (they like their partner no more than average), the rest die without matching. Thus, the overall expected utility of bottom workers is $(\lambda_e - \lambda_w^t - \delta^t)/(2\lambda_w^b)$.

[13]This is the "interesting" range for λ_e. If $\lambda_e < \lambda_w^t + \delta^t$, then employers do not match with bottom worker at all in equilibrium, and the interaction between top workers and employers is analogous to that captured in Section 3.2. On the other hand, if $\lambda_e > \lambda_w^t + \delta^t + \lambda_w^b$, then all bottom workers (and all top workers) match in equilibrium and the situation again becomes similar to that in Section 3.2 with workers being on the short side.

[14]Note that, at an equilibrium, all employers will be using the same threshold θ_e, which is also equal to their expected pay-off.

Note that the death-rate of reacher employers (δ^t) is determined endogenously: in equilibrium, an employer must be indifferent between being a reacher or a settler. For instance as a increases top workers become more attractive, and thus the fraction of reacher employers that die in equilibrium must increase to maintain this indifference.

We now state our formal result, which applies to $c \in (0, 1/32)$.

Theorem 5. *Fix $c \in (0, 1/32)$ and a satisfying*

$$a \in ((2c)^{1/4} - \sqrt{2c}, 1 - (2c)^{1/4} - \sqrt{2c})\,. \tag{38}$$

Define

$$\delta^{\mathrm{t}} = \lambda_w^{\mathrm{t}} \left(\frac{a - \sqrt{2c}\left(\frac{1}{a+\sqrt{2c}} - 1\right)}{2(1-\sqrt{2c})} \right) > 0 \tag{39}$$

(The lower bound on a ensures that $\delta^{\mathrm{t}} > 0$.) Assume $\lambda_e \in (\lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}}, \lambda_w^{\mathrm{t}} + \lambda_w^{\mathrm{b}} + \delta^{\mathrm{t}})$. Then, there is an equilibrium with the following description in the limit $\mu \to 0$: Bottom workers propose without screening. Top workers do not propose, screen incoming proposals from employers and accept/reject using a threshold of $\theta_w^{\mathrm{t}} = 1 - \sqrt{2c}$. Employers do not propose to bottom workers, and a fraction $1 - \frac{\lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}}}{\lambda_e}$ of employers (we call these "settlers") screen and accept/reject incoming proposals from bottom workers (the remaining, "reacher", employers ignore such proposals), with a threshold of $\theta_e = 1 - \sqrt{2c}$. When the opportunity arises, employers screen an available top worker, and propose to him with the same threshold of θ_e. There is a flow λ_w^{t} of reacher employers matching with top workers, a flow δ^{t} of reacher employers dying without matching, whereas all settler employers match with bottom workers, producing a flow $(\lambda_e - \lambda_w^{\mathrm{t}} - \delta^{\mathrm{t}})$ of such matches. Top workers earn utility θ_w^{t}, employers earn utility θ_e and bottom workers each earn utility $(\lambda_e - \lambda_w^{\mathrm{t}} - \delta^{\mathrm{t}})/(2\lambda_w^{\mathrm{b}})$. This equilibrium is dynamically stable under a fixed mix of agent strategies. It is also evolutionarily stable, i.e., it is a stable fixed point under dynamics where incoming agents choose a best response to the current masses of agents in the system (see Section A.2).[15]

If $\lambda_w^{\mathrm{b}} \geq \max(1.20(\lambda_e - \lambda_w^{\mathrm{t}} - \delta^{\mathrm{t}}), \lambda_e - \lambda_w^{\mathrm{t}})$ this is essentially[16] *the only equilibrium.*

Note that for small c the interval of allowed values for a approaches $(0, 1)$, and $\lim_{c \to 0} \delta^{\mathrm{t}} = a/2$. We can define an "effective arrival flow" of $\tilde{\lambda}^{\mathrm{t}} = \lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}}$ for top tier workers. Then, the equilibrium captured in the theorem has an incoming flow $\tilde{\lambda}_w^{\mathrm{t}}$ of reacher employers, who wait to propose and match to a top worker whom they like (or die waiting), and an incoming flow of settler employers arriving at the residual rate of $(\lambda_e - \tilde{\lambda}^{\mathrm{t}}) < \lambda_w^{\mathrm{b}}$, who consider incoming proposals from bottom workers, and typically match with a bottom worker whom they like.

[15] Though we had formally defined ESSE in a model with no tiers, the definition carries over to the case with tiers with no modification.

[16] There are always "equivalent" equilibria, where employers are not separated into the binary categories of "considers bottom workers" and "does not consider bottom workers" but may mix between these categories in some manner.

There is effectively a "top submarket", consisting of top workers and reacher employers with imbalance $\tilde{\lambda}^{\mathrm{t}}/\lambda_w^{\mathrm{t}}$ and employers being on the long side; and a "bottom submarket", consisting of settler employers and bottom workers with imbalance $\lambda_{\mathrm{b}}/(\lambda_w - \tilde{\lambda}_{\mathrm{t}}) > 1$ and bottom workers being on the long side. The behavior of agents under equilibrium in the top submarket resembles that in Theorem 2 equilibrium 1,[17] and in the bottom submarket resembles that in Theorem 2 equilibrium 2.[18]

There are four categories of agents – top workers, bottom workers, reacher employers and settler employers. We will first establish the existence of a fixed point with the given limiting description, of the dynamical system under a fixed mix of agent strategies (the mix of strategies will itself have the given limiting description) in Claim 1. This fixed point will have the additional property that reacher and settler employers obtain exactly the same utility. Next, we will show that the fixed point is stable in Claim 2. Third, we will show that each category of agent is playing a best response in Claim 3. This will establish that we have found a stationary equilibrium. Fourth, we will show that the equilibrium is also evolutionarily stable in Claim 5. Finally, we will show uniqueness of the equilibrium found in Claim 6.

Fixed point of dynamical system. Top workers arrive at rate λ_w^{t} and bottom workers arrive at rate λ_w^{b}. Employers arrive at rate λ_e. Let agent strategies be as per the theorem statement, including thresholds such that $\lim_{\mu\to 0}\theta_e = \theta_w^{\mathrm{t}} = 1-\sqrt{2c}$, where θ_e will be specified below.

Claim 1. *There exist $\epsilon > 0$, an arrival rate of reacher employers $\lambda_e^{\mathrm{r}} = \lambda_e^{\mathrm{r}}(\mu) \in (\lambda_w^{\mathrm{t}} + \epsilon, \lambda_e - \epsilon)$ such that $\lim_{\mu\to 0}\lambda_e^{\mathrm{r}} = \tilde{\lambda}^{\mathrm{t}}$, and a threshold $\theta_e = \theta_e(\mu) \in (\epsilon, 1-\epsilon)$ such that $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c}$, such that under this fixed mix of agent strategies, there is a steady state/fixed point for $\mu \in (0,\epsilon)$ with the following properties:*

- *Top workers match as soon as they arrive.*
- *The utility of reacher and settler employers is exactly the same, and equal to θ_e.*
- *The mass of reacher employers is δ^{t}/μ to leading order as $\mu \to 0$.*
- *The mass of settler employers is $\frac{\delta^{\mathrm{t}}(\lambda_e - \tilde{\lambda}^{\mathrm{t}})}{\sqrt{2c}(\lambda_w^{\mathrm{b}} + \tilde{\lambda}^{\mathrm{t}} - \lambda_e)}$ to leading order as $\mu \to 0$.*
- *The mass of bottom workers is $(\lambda_w^{\mathrm{b}} + \tilde{\lambda}^{\mathrm{t}} - \lambda_e)/\mu$ to leading order as $\mu \to 0$.*

The intuition for the claim is as follows. (We present a formal proof below.) For bullets 3 and 5, the numbers should be such that when multiplied by μ it gives the rate at which agents in that category die. In turn, this gives the rate at which these categories (want to) propose to the others. We expect that all top workers and settler employers match in the limit $\mu \to 0$, so this allows us to calculate their expected number in the system in order to achieve this rate, given

[17] The modification is that here, an employer's utility for a worker is uniformly distributed in $(a, a+1)$.

[18] The modification is that here, a bottom worker does not know who the settler employers are and in fact most of the employers available at any time are reachers and ignore proposals from bottom workers, making it highly unattractive for bottom workers to screen and propose.

the rate of incoming proposals. Next, we verify that for the given masses the rate of top workers and settler employers dying without matching is 0 as $\mu \to \infty$, and in fact the top workers match as soon as they arrive (bullet 1). Finally, we can choose λ_e^{r} such that incoming employers are indifferent between been reachers and settlers (bullet 2), and in the limit $\mu \to 0$, this condition implies that a fraction $\delta^{\mathrm{t}}/(\lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}})$ of reacher employers must die without matching, leading to bullet 3. The mass of settler employers is such as to ensure that the flow of matches is the same as the arrival flow as $\mu \to 0$, since there is an excess of bottom workers.

Proof of Claim 1. Let us write the DEs capturing system dynamics. Fix the incoming flow of reacher employers $\lambda_e^{\mathrm{r}} \in (\lambda_w^{\mathrm{t}} + \epsilon, \lambda_e - \epsilon)$ for any $\epsilon > 0$. Settler employers arrive at rate $\lambda_e^{\mathrm{s}} = \lambda_e - \lambda_e^{\mathrm{r}}$. Let N_e^{r} be the mass of reacher employers in the market, let N_e^{s} be the mass of settler employers, N_e be the total mass of employers, and let N_w^{b} be the mass of bottom workers in the market. If we begin with all three masses positive and a zero mass of top workers in the system, we have the following dynamical equations

$$\begin{aligned}
\frac{\mathrm{d}N_w^{\mathrm{b}}}{\mathrm{d}t} &= -\left(\frac{N_e^{\mathrm{s}}\eta_{\mathrm{b}}}{N_e} + \mu\right) N_w^{\mathrm{b}} + \lambda_w^{\mathrm{b}} \\
\frac{\mathrm{d}N_e^{\mathrm{r}}}{\mathrm{d}t} &= -\lambda_w^{\mathrm{t}} \frac{N_e^{\mathrm{r}}}{N_e} - \mu N_e^{\mathrm{r}} + \lambda_e^{\mathrm{r}} \\
\frac{\mathrm{d}N_e^{\mathrm{s}}}{\mathrm{d}t} &= -\lambda_w^{\mathrm{t}} \frac{N_e^{\mathrm{s}}}{N_e} - \frac{N_e^{\mathrm{s}}\eta_b N_w^{\mathrm{b}}}{N_e} - \mu N_e^{\mathrm{s}} + \lambda_e^{\mathrm{s}}
\end{aligned} \tag{40}$$

where $\eta_{\mathrm{b}} = 1 - \theta_e = \sqrt{2c} + O(\mu)$ is the fraction of bottom workers that employers find acceptable (we will choose the $O(\mu)$ term later, for now our analysis will work for an arbitrary term of this form). We used that matches form between top workers and employers at rate λ_w^t (these matches form with each type of employer in proportion to their respective mass present in the system). Matches form between settler employers and bottom workers (as a result of bottom workers proposing without screening) at rate

$$\begin{aligned}
&(\text{Rate at which bottom workers propose}) \times \\
&\ (\text{Fraction of proposals that go to settler employers}) \times (\text{Fraction of proposals accepted}) \\
&= N_w^{\mathrm{b}} \frac{N_e^{\mathrm{s}}}{N_e} \eta_{\mathrm{b}} \,.
\end{aligned}$$

We now show that there is a (stable) fixed point whose limiting description as $\mu \to 0$ is

$$\begin{aligned}
L_w^{\mathrm{b}} &= \frac{\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}}}{\mu} \\
L_e^{\mathrm{r}} &= \frac{\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}}{\mu} \\
L_e^{\mathrm{s}} &= \frac{\lambda_e^{\mathrm{s}}(\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}})}{(\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}})\sqrt{2c}} \,.
\end{aligned} \tag{41}$$

as claimed. Note that this fixed point has all three masses positive, and hence the dynamical equations (40) remain valid (including that top workers match as soon as they arrive, and hence the mass of top workers in the system remains zero).

Define

$$\begin{aligned} z_1 &= \mu(N_w^{\mathrm{b}} - L_w^{\mathrm{b}}) \\ z_2 &= \mu(N_e^{\mathrm{r}} - L_e^{\mathrm{r}}) \\ z_3 &= N_e^{\mathrm{s}} - L_e^{\mathrm{s}} \end{aligned} \tag{42}$$

Substituting Eqs. (42) and (41) in Eq. (40), and considering $\|z\| = O(1)$, we get

$$\begin{aligned} \frac{1}{\mu}\frac{\mathrm{d}z_1}{\mathrm{d}t} &= -\left(1 + \frac{\lambda_e^{\mathrm{s}}}{\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}}}\right) z_1 + \frac{\lambda_e^{\mathrm{s}}}{\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}} z_2 - \frac{\eta_{\mathrm{b}}(\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}})}{\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}} z_3 + O(\mu)\,, \\ \frac{1}{\mu}\frac{\mathrm{d}z_2}{\mathrm{d}t} &= -z_2 + O(\mu)\,, \\ \frac{\mathrm{d}z_3}{\mathrm{d}t} &= -\frac{\lambda_e^{\mathrm{s}}}{\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}}} z_1 + \frac{\lambda_e^{\mathrm{s}}}{\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}} z_2 - \frac{\eta_{\mathrm{b}}(\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}})}{\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}} z_3 + O(\mu)\,. \end{aligned} \tag{43}$$

Here the $O(\mu)$ terms are, in fact, Lipschitz continuous in z and λ_e^{s} with a Lipschitz constant that is $O(\mu)$. Let A be the coefficient matrix for the linear terms above. We have

$$A = \begin{bmatrix} -1-\xi & \beta & -\gamma \\ 0 & -1 & 0 \\ -\xi & \beta & -\gamma \end{bmatrix}, \\ \text{where} \quad \xi = \frac{\lambda_e^{\mathrm{s}}}{\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}}}\,, \quad \beta = \frac{\lambda_e^{\mathrm{s}}}{\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}}\,, \quad \gamma = \frac{\eta_{\mathrm{b}}(\lambda_w^{\mathrm{b}} - \lambda_e^{\mathrm{s}})}{\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}}\,. \tag{44}$$

It is easy to verify that A has full rank with singular values bounded away from 0. Hence, there is a fixed point z^* of this set of equations satisfying

$$z^* = O(\mu)\,.$$

Thus, Eq. (41) gives the correct steady state to within $O(\mu)$ in relative terms (also, the relative errors are Lipschitz with a Lipschitz constant that is $O(\mu)$).

The steady state utility of reacher employers is decreasing in L_e^{r} since more of them die as this value increases. In fact, this utility is a constant plus $(\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}})/\lambda_e^{\mathrm{r}} = \text{constant} - \lambda_w^{\mathrm{t}}/\lambda_e^{\mathrm{r}}$ up to $O(\mu)$, which is decreasing in λ_e^{r}. In comparison, the settler employers have a fixed utility in the limit that $\mu \to 0$. We deduce that there is a unique value of λ_e^{r} such that reacher and settler employers have the same utility. Using Eq. (41) and $\xi_{\mathrm{b}} = \sqrt{2c}$, yields that this uniquely determined rate of arrival of reacher employers is $\lambda_e^{\mathrm{r}} = \lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}} + O(\mu)$. We now need to set θ_e appropriately; we had left the $O(\mu)$-sized correction term ambiguous until now. Suppose we start with $\theta_e = 1 - \sqrt{2c}$ exactly and do the above process. Then we correct θ_e, for now holding λ_e^{r} fixed, so that it matches the utility employers were getting. This correction in θ_e will be

$O(\mu)$, because the utility of settler employers is $1 - \sqrt{2c} - O(\mu)$. This change in θ_e will change the fixed point by $O(\mu)$. As a result λ_e^{r} will need to be adjusted by $O(\mu)$ to make utilities equal again for reacher and settler employers. However, the change in (settler) employer utility is only

$$O\big(\mu|(\text{Change in steady state parameters})| + (\text{Change in } \theta_e)^2\big) = O(\mu^2) + O(\mu^2) = O(\mu^2)$$

So, the next time, when we again adjust θ_e, then λ_e^{r} and calculate the change in utility, all changes will be a factor $O(\mu)$ smaller than the previous iteration. In other words, this iterative process converges rapidly for small enough μ, and upon convergence, produces $\lambda_e^{\mathrm{r}} = \lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}} + O(\mu)$ and $\theta_e = 1 - \sqrt{2c} - O(\mu)$ such that the utilities of reacher and settler employers are both exactly equal to θ_e. As a quick check, note that the utility of top workers as $\mu \to 0$ is

$$\begin{aligned} &(\text{Fraction who match})\big((1+a+\theta_e)/2 - c\mathrm{E}[\text{Number of top workers screened per match}]\big) \\ &= \frac{\lambda_{\mathrm{t}}}{\lambda_{\mathrm{t}} + \delta_{\mathrm{t}}} \left(\frac{2 + a - \sqrt{2c}}{2} - \frac{c}{\sqrt{2c}(a + \sqrt{2c})} \right) = 1 - \sqrt{2c} \end{aligned}$$

Using Eq. (41) and $\xi_{\mathrm{b}} = \sqrt{2c}$, yields that this uniquely determined rate of arrival of reacher employers is $\lambda_e^{\mathrm{r}} = \lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}} + O(\mu)$. Substituting in Eq. (41) and using $\lambda_e^{\mathrm{s}} = \lambda_e - \lambda_e^{\mathrm{r}}$ yields the claim. □

The fixed point is stable.

Claim 2. *The fixed point found in Claim 1 is stable under a fixed mix of agent strategies, i.e., if we start close to the fixed point $|z(0) - z^*| < \epsilon^2$, we remain close, $|z(t) - z^*| < \epsilon$ for all t, and the dynamics converges to the fixed point, i.e., $\lim_{t\to\infty} z(t) = z^*$.*

To check dynamical stability (under a fixed mix of agent strategies), we need to investigate the eigenvalues of

$$A_1 = \begin{bmatrix} -\mu(1+\xi) & \mu\beta & -\mu\gamma \\ 0 & -\mu & 0 \\ -\xi & \beta & -\gamma \end{bmatrix},$$

since

$$\frac{\mathrm{d}z}{\mathrm{d}t} = A_1 z + \begin{bmatrix} \epsilon_1(z) \\ \epsilon_2(z) \\ \epsilon_3(z) \end{bmatrix}, \tag{45}$$

where ϵ_1 and ϵ_2 are $O(\mu^2)$ and Lipschitz continuous in z with constant that is $O(\mu^2)$, whereas ϵ_3 is $O(\mu)$ and Lipschitz continuous in z with constant that is $O(\mu)$. Let the Lipschitz constants by denoted by L_1, L_2, and L_3. The eigenvalues turn out to be

$$\begin{aligned}
\lambda_1 &= -\mu && \text{with eigenvector} \quad v_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \\
\lambda_2 &= \frac{-\phi + \sqrt{\phi^2 - 4\mu\gamma}}{2} = -\mu - O(\mu^2) && \text{with eigenvector} \quad v_2 = \begin{bmatrix} \gamma & 0 & -(\lambda_2 + \mu(1+\xi))/\mu \end{bmatrix}^T \\
& && \phantom{\text{with eigenvector} \quad v_2} = \begin{bmatrix} \gamma & 0 & -\xi + O(\mu) \end{bmatrix}^T \\
\lambda_3 &= \frac{-\phi - \sqrt{\phi^2 - 4\mu\gamma}}{2} = -\gamma + O(\mu) && \text{with eigenvector} \quad v_3 = \begin{bmatrix} \mu & 0 & -\big(\lambda_3 + \mu(1+\xi)\big)/\gamma \end{bmatrix}^T \\
& && \phantom{\text{with eigenvector} \quad v_3} = \begin{bmatrix} \mu & 0 & 1 - O(\mu) \end{bmatrix}^T \\
&\text{where } \phi = \mu(1+\xi) + \gamma\,.
\end{aligned} \tag{46}$$

(Note that all eigenvectors have been scaled to have magnitude of order 1.) Clearly, all eigenvalues have a negative real part. However, we still need to deal with the error terms in Eq. (48). Notice that each of the three error terms is only order μ times the size of the corresponding leading term (when the leading term is non-zero). Hence, we expect to obtain stability of the fixed point for small μ. We formally prove this below.

Remark 4. *The eigendecomposition of A_1 admits an elegant interpretation in the context of the dynamical system.*

- *The first eigenvalue corresponds to the mass of reacher employers in the system,*[19] *and if this number deviates, it returns to zero via the resulting imbalance between the arrival and departure flows. Since arrival and matching flows are fixed, this imbalance per unit mass of deviation is equal to μ (the death rate per unit mass), hence the value of λ_1.*

- *The third eigenvalue corresponds to a short-lived imbalance in the arrival and matching flow rate of settler employers (a vanishing flow of settler employers die without matching). If such an imbalance occurs, it quickly disappears, since the matching flow rate increases by nearly $L_w^{\mathrm{b}}/L_e^{\mathrm{s}} = \eta_{\mathrm{b}}(\lambda_w^b - \lambda_e^{bs})/(\lambda_e^{\mathrm{r}} - \lambda_w^{\mathrm{t}}) = \gamma$ per unit mass increase in z_3, due to more proposals by bottom workers going to reacher employers. There is a small effect on the scaled mass of bottom workers present, hence the μ valued first coordinate of the eigenvector.*

- *The second eigenvalue corresponds to a deviation in the mass of bottom workers, and how it disappears. As per the above bullet, the mass of settler employers adjusts and settles at a point such that the flow of matches is nearly identical to their arrival flow λ_e^{r}. Once the flow of matches has equilibrated, there is a slow adjustment of the mass of bottom workers in the system, similar to bullet 1, modulated by the death rate μ, since the arrival and matching flows are nearly fixed in time, hence we have $\lambda_2 \approx \mu$.*

[19] The scaling in the definition of z_2 does not affect the eigenvalue, since it affects both sides of the dynamical equation by the same factor.

Proof of Claim 2. Suppose we start at $z(t) = z = z^* + \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$, where z^* is a fixed point of Eq. (48). Allowing the system to evolve for a (small) time Δ, we have that

$$
\begin{aligned}
z(t+\Delta) &= z^* + \sum_{i=1}^{3} \alpha_i(t+\Delta) v_i\,, \\
&= z^* + \sum_{i=1}^{3} \alpha_i(t)(1-\lambda_i\Delta) v_i + \begin{bmatrix} \epsilon_1(z(t)) - \epsilon_1(z^*) \\ \epsilon_2(z(t)) - \epsilon_2(z^*) \\ \epsilon_3(z(t)) - \epsilon_3(z^*) \end{bmatrix} \Delta + O(\Delta^2)\,,
\end{aligned}
$$

Consider the expansion of the error term in terms of eigenvectors. The $\epsilon_3(z(t)) - \epsilon_3(z^*)$ term is of order $|z(t) - z^*|\Delta\mu = |\alpha(t)|\Delta\mu$, and when we express $[0\ 0\ \mu]^T$ in terms of eigenvectors, we obtain a coefficient of order μ for v_3 and a coefficient of order μ^2 for v_2. (Other error terms can be handled similarly.) For each of the eigenvectors, we see that the ratio of the magnitude of the coefficient (due to the error term) to the magnitude of the eigenvalue is order $\mu|\alpha(t)|\Delta$. We deduce that the coefficient of v_2 in the error term gets dominated by $\alpha_2(t)\lambda_2\Delta$ provided $\alpha_2(t)$ is large compared to $\mu|\alpha(t)|$. The coefficient of v_3 gets dominated by $\alpha_3(t)\lambda_3\Delta$ provided $\alpha_3(t)$ is large compared to $\mu|\alpha(t)|$. The coefficient of v_1 gets dominated by $\alpha_1(t)\lambda_1\Delta$ provided $\alpha_1(t)$ is large compared to $\mu|\alpha(t)|$. In other words, we have

$$
\begin{aligned}
\max(|\alpha_i(t+\Delta)|, \epsilon) \le \max(|\alpha_i(t)|, \epsilon) - \mathbb{I}(|\alpha_i(t)| > \epsilon)|\alpha_i(t)||\lambda_i|\Delta/2 \quad \text{for } i = 1, 2, 3 \\
\text{where } \epsilon = O(\mu|\alpha(t)|)\,.
\end{aligned}
$$

for Δ small enough. It follows that this condition remains valid for a fixed $\epsilon = O(\mu|\alpha(t)|)$ for all subsequent times. Hence, it must be that eventually all $|\alpha_i|$'s are no more than ϵ. At this point, we can reset the value of ϵ to a value that is $O(\mu)$ times the original value and repeat. It follows that the system returns to the fixed point z^*. This argument goes through for all μ smaller than some value μ_0. □

All agent categories are playing a best response.

Claim 3. *For the choice of $\lambda_e^{\mathrm{r}} = \lambda_e^{\mathrm{r}}(\mu) > 0$ and the fixed point/steady state in Claim 1, with $\theta_w^{\mathrm{t}} = 1 - \sqrt{2c}$ and a suitable choice of $\theta_e = \theta_e(\mu)$ satisfying $\lim_{\mu\to 0}\theta_e(\mu) = 1 - \sqrt{2c}$ the following holds for $\mu \in (0, \epsilon)$. Top workers, bottom workers and employers are all playing a best response. Top workers earn expected utility θ_w^{t}, employers earn expected utility θ_e and bottom workers earn limiting expected utility $(\lambda_e - \lambda_w^{\mathrm{t}} - \delta^{\mathrm{t}})/(2\lambda_w^{\mathrm{b}})$ as $\mu \to 0$.*

Proof of Claim 3. We consider each category of agent in turn.

- Top workers: They are earning the highest possible utility of $1 - \sqrt{2c}$ and cannot do any better by proposing or using a different threshold. The threshold is optimal for any μ without taking the limit, since top workers match instantly and depart, and hence a 0 fraction of them die.

- Employers: Employers will want to consider a top worker if such an option is available. Also, they will not want to propose to bottom workers if μ is small enough, since they are better off waiting for (frequent) incoming proposals from them and avoiding wasted screening effort. It remains to choose between being a reacher and a settler and to choose a threshold. By Claim 1, reachers and settlers earn exactly the same utility, and θ_e matches this utility, and is hence an optimal threshold. We deduce that employers, both reachers and settlers, are playing a best response and earning utility θ_e.
- Bottom workers: They have no hope of matching unless they propose. Only a vanishing fraction of employers who are present will even consider their proposal, hence the effective screening cost they face, cf. Lemma 1, is diverging and it is a best response for them to propose without screening. Bottom workers form matches only at a flow rate of $\lambda_w - \tilde{\lambda}_{\mathrm{t}}$ as $\mu \to 0$, since almost all arriving settler employers match with them. Hence, the fraction of bottom workers who match is $(\lambda_w - \tilde{\lambda}_{\mathrm{t}})/\lambda_{\mathrm{b}}$, and we compute the expected utility earned by bottom workers as $(\lambda_w - \tilde{\lambda}_{\mathrm{t}})/(2\lambda_{\mathrm{b}})$.

□

Evolutionary stability. Evolutionary stability requires that when entering agents choose a strategy that is a best response to the current mix of agents in the system (keeping the threshold θ fixed), the equilibrium is a stable fixed point under the resulting system dynamics.

When the system deviates slightly from the fixed point z^*, the best response of the top workers and the bottom workers remains unchanged, but for entering employers, being a reacher may be more or less attractive than being a settler, the main determinant of the relative attractiveness being the mass of reachers presently in the system. When there are more reachers present, this increases the likelihood of dying without matching for reachers, and hence makes being a reacher less attractive. This reasoning leads us to the following dynamical equations under the best response dynamics

$$\frac{\mathrm{d}z}{\mathrm{d}t} = A_1 z + \begin{bmatrix} 0 \\ \mu\big(\mathbb{I}(z_2 < \epsilon_6)\lambda_e - \lambda_e^{\mathrm{r}}\big) \\ -\mathbb{I}(z_2 < \epsilon_6)\lambda_e + \lambda_e^{\mathrm{r}} \end{bmatrix} + \begin{bmatrix} \epsilon_1(z) \\ \epsilon_4(z) \\ \epsilon_5(z) \end{bmatrix} ,$$

$$\text{where } \epsilon_6 = \epsilon_6(z_1, z_3) = O(\mu), \epsilon_5 = O(\mu), \epsilon_4 = O(\mu^2), \epsilon_1 = O(\mu^2)\,. \tag{47}$$

Again, each of the ϵ's is, in fact, Lipschitz continuous (for $|z| = O(1)$) with Lipschitz constant bounded as $O(\mu)$ for ϵ_6 and ϵ_5 and $O(\mu^2)$ for ϵ_4 and ϵ_1. Recall that $\lambda_e^{\mathrm{r}} = \tilde{\lambda}^{\mathrm{t}} + O(\mu)$. We call the three terms above, in order, the linear term (unchanged from before), the best response term and the error term.

To begin with, we show that for after an initial transient, the system will hit the boundary $z_2 = \epsilon_6(z_1, z_3)$ and stay there.

Claim 4. *Fix $\epsilon_9 > 0$. There exists $\mu_0 > 0$ such that for any $\mu \in (0, \mu_0)$ the following holds. There exists $\epsilon_7 > 0$ and $C > 0$ (that can depend on all model primitives), such that for any* *starting point $z(0)$ satisfying $|z(0) - z^*| < \epsilon_7$, after an initial transient of duration $C|z(0) - z^*|$, the system will hit the best-response boundary $z_2 = \epsilon_6(z_1, z_3)$ and stay on the boundary thereafter. Moreover, when it hits the boundary, $|z(0) - z^*| < \epsilon_9$.*

Proof. Consider the following dynamical system that captures (48) in the region $z_2 < \epsilon_6$.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = A_1 y + \begin{bmatrix} 0 \\ \mu\lambda_e^{\mathrm{s}} \\ -\lambda_e^{\mathrm{s}} \end{bmatrix} + \begin{bmatrix} \epsilon_1(y) \\ \epsilon_4(y) \\ \epsilon_5(y) \end{bmatrix} ,$$

$$\text{where } \epsilon_6 = \epsilon_6(y_1, y_3) = O(\mu), \epsilon_5 = O(\mu), \epsilon_4 = O(\mu^2), \epsilon_1 = O(\mu^2) . \tag{48}$$

Suppose the system starts at $y(0) = z(0)$ such that $y_2 < \epsilon_6$. (The complementary case can be handled via very similar argument.) Consider the evolution of y_2. It increases at a rate of

$$\mu\lambda_e^{\mathrm{s}} + \frac{\partial\epsilon_4}{\partial y_1}\frac{\mathrm{d}y_1}{\mathrm{d}t} + \frac{\partial\epsilon_4}{\partial y_3}\frac{\mathrm{d}y_3}{\mathrm{d}t} .$$

To hit the boundary, we need $y_2 = \epsilon_6$. The initial distance from the boundary along the y_2 coordinate is bounded by $2|y(0) - y^*|$ for small enough μ, using the fact that ϵ_6 is $O(\mu)$-Lipschitz continuous in y_1 and y_3, and that $z^* = y^*$ is on the boundary (in fact, it is a fixed point of Eq. (47). We claim that the time it will take to hit the boundary is bounded above by $\tau = 2(2|y(0) - y^*|)/(\mu\lambda_e^{\mathrm{s}}) = 4|y(0) - y^*|/(\mu\lambda_e^{\mathrm{s}})$. To establish this, it will suffice to show that

$$\frac{\mathrm{d}\epsilon_6}{\mathrm{d}t} - \frac{\mathrm{d}\epsilon_4}{\mathrm{d}t} = \sum_{j=1,3}\left(\frac{\partial\epsilon_6}{\partial y_j} - \frac{\partial\epsilon_4}{\partial y_j}\right)\frac{\mathrm{d}y_j}{\mathrm{d}t} \leq \mu\lambda_e^{\mathrm{s}}/2 , \tag{49}$$

holds for all $t \leq \tau$. Now, if ϵ_7 is sufficiently small, then τ is sufficiently small. There exists $\epsilon_8 > 0$ such that while $|y(t) - y^*| \leq \epsilon_8$ we have $\left|\frac{\mathrm{d}y}{\mathrm{d}t}\right| \leq \lambda_e$. It follows that $|y(t) - y^*| \leq \epsilon_7 + \lambda_e t$ for such times, and hence $|y(t) - y^*| \leq \epsilon_9 \leq \epsilon_8$ holds up to $\tau' = (\epsilon_9 - \epsilon_7)/\lambda_e$ for any $\epsilon_9 \in (0, \epsilon_8)$. By choosing ϵ_7 and hence τ sufficiently small, we can ensure that $\tau' \geq \tau$. It follows that $\left|\frac{\mathrm{d}y}{\mathrm{d}t}\right| \leq \lambda_e$ and $|y(t) - y^*| \leq \epsilon_9$ holds up to τ. We deduce that

$$\left|\frac{\partial\epsilon_i}{\partial y_j}\frac{\mathrm{d}y_j}{\mathrm{d}t}\right| = O(\mu^2) \leq \mu\lambda_e^{\mathrm{r}}/6 \quad \text{for } \mu < \mu_0 \text{ and } (i,j) \in \{(4,1),(4,3),(6,1)\} .$$

For $i = 4$ and $j = 1, 3$ we used that ϵ_4 is $O(\mu^2)$-Lipschitz continuous. For $i = 6, j = 1$, we used $\left|\frac{\mathrm{d}y_1}{\mathrm{d}t}\right| = O(\mu)$ and $|y_1 - y_1^*| = O(\mu)$ for $t \leq \tau$ and small enough ϵ_7. The remaining term is

$$\frac{\partial\epsilon_6}{\partial y_3}\frac{\mathrm{d}y_3}{\mathrm{d}t} .$$

Note that $\frac{\partial\epsilon_6}{\partial y_3} > 0$ since more settlers in the system makes it less attractive for an incoming employer to be a settler, and is justified only at a higher value of y_2. Also, observe from Eq. (48)

that $\dfrac{\mathrm{d}y_3}{\mathrm{d}t} < 0$ for $t \le \tau$. We deduce that this term is negative. It follows that Eq. (49) holds for all $t \le \tau$. We deduce that the system hits the boundary for the first time at $t = \tau'' \le \tau$, and until it hits the boundary, the distance between the y_2 coordinate and ϵ_6 is always decreasing. After τ'' the system does not leave the boundary, since if it "tries", the result we just derived implies that the dynamics immediately pushes the system back to the boundary. Also, we observe that $|y(\tau'') - y^*| < \epsilon_9$. □

We now write the equations for the two-dimensional dynamical evolution when the system is on the best response boundary, and employ an argument similar to that we used to prove Claim 2 (stability of the fixed point when there is a fixed mix of agent strategies), in order to complete the proof of evolutionary stability.

Claim 5. *The fixed point in Claim 1 is also a fixed point of the best response dynamics* (47)*, and is evolutionarily stable. In other words, it is an evolutionarily stable stationary equilibrium.*

Proof. Claim 3 establishes that all agent categories are, in fact, playing a best response in the fixed point in Claim 1. It follows that the fixed point is also a fixed point of the best response dynamics (47). We will now show evolutionary stability.

Suppose the best response dynamics (47) begin from $z(0)$ such that $|z(0) - z^*| < \epsilon_7$. Using Claim 4 we know that the dynamics hits the best response boundary at a point that is at most ϵ_9 from z^*, at some time τ''. Thereafter, we can follow the proof of Claim 2 to control system dynamics on the boundary. The dynamical system is now two-dimensional, in terms of z_1 and z_3, since $z_2 = \epsilon_6$ remains true. The dynamical equations are now

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \begin{bmatrix} -\mu(1+\xi) & -\mu\gamma \\ -\frac{\xi}{1+\rho} & -\frac{\gamma}{1+\rho} \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} + \begin{bmatrix} \epsilon_{10}(x) \\ \epsilon_{11}(x) \end{bmatrix} ,$$
$$\text{where } \rho = \frac{1}{\mu} \left.\frac{\partial \epsilon_6}{\partial x_3}\right|_{x^*} > 0 \tag{50}$$

Here, $x(t) = [x_1(t)\ x_3(t)]^T = [z_1(t+\tau'')\ z_3(t+\tau'')]^T$. We know $|x(0)| \le \epsilon_9$ and also have that $\epsilon_{10}(x)$ is $O(\mu^2)$-Lipschitz continuous and $\epsilon_{11}(x)$ is $O(\mu)$ at x^* and $O(\mu+\epsilon_9)$-Lipschitz continuous. The reason for this is as follows: Note that $\rho = \Theta(1)$. We expect $\left|\dfrac{\mathrm{d}x_3}{\mathrm{d}t}\right| = \Theta(|x(t) - x^*|)$. If this holds, using Taylor expansion to control $\dfrac{\partial \epsilon_6}{\partial x_3}$, we obtain that

$$\frac{\mathrm{d}z_2}{\mathrm{d}t} = \frac{\partial \epsilon_6}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}t} + O(\mu^2) = \mu\rho\frac{\mathrm{d}x_3}{\mathrm{d}t} + O(\mu(\mu + \epsilon_9^2))\,.$$

Since the best response term in (47) produces the leading order component of $\dfrac{\mathrm{d}z_2}{\mathrm{d}t}$, it must contribute an opposite push to $z_3 = x_3$ that is $1/\mu$ times larger in magnitude, to leading order, by definition. This push is then $-\rho\frac{\mathrm{d}x_3}{\mathrm{d}t} + O(\mu)$. It follows that

$$\frac{\mathrm{d}x_3}{\mathrm{d}t} = \begin{bmatrix} -\xi & -\gamma \end{bmatrix} x \;-\; \rho\frac{\mathrm{d}x_3}{\mathrm{d}t} \;+\; O(\mu + \epsilon_9^2)$$

Rearranging leads to the $\frac{\mathrm{d}x_3}{\mathrm{d}t}$ expression in Eq. (50). The best response of bottom workers remains unchanged, hence there is no similar leading order "correction" to $\frac{\mathrm{d}x_1}{\mathrm{d}t}$. The eigenvalues of the 2x2 matrix capturing the linear part are, defining $\tilde{\gamma} = \gamma/(1+\rho)$, the same as the eigenvalues λ_1 and λ_3 of A_1, see (46), when γ is replaced by $\tilde{\gamma}$. In particular, the eigenvalues are negative.

As in the proof of Claim 2, we can now show that for ϵ_9 and μ_0 small enough, this implies convergence to the fixed point x^* and hence to z^* for the overall system. □

The equilibrium is unique. We now show uniqueness of the equilibrium found.

Claim 6. *There is no other evolutionarily stable equilibrium besides the one described in Theorem 5, if* $\lambda_{\mathrm{b}} \geq 1.20(\lambda_w - \lambda_{\mathrm{t}} - \delta_{\mathrm{t}})$ *holds.*

Proof. One can rule out equilibria where the top workers propose, with or without screening, without imposing any additional conditions: The condition for employers wanting to screen and propose to top workers is $(1 + a + \theta_e)/2 - c/[(1 - \theta_w^{\mathrm{t}})(a + 1 - \theta_e)] > \theta_e$ which simplifies to $\theta_e < 1 + a - (2c)^{1/3}$, which always holds, irrespective of what is happening with bottom workers, since in any equilibrium $\theta_e \leq 1 - \sqrt{2c} < 1 + a - (2c)^{1/4} < 1 + a - (2c)^{1/3}$, using $a > (2c)^{1/4} - \sqrt{2c}$. Hence, *employers will want to screen and propose to top workers.* So top workers will not want to propose since proposals are coming in anyway.

We now consider the mix of strategies that employers and bottom workers may be employing, knowing that all employers screen and propose to top workers. There must be at least $(\lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}} - \epsilon)/\lambda_e$ fraction of reacher employers, who will look to screen and propose to top workers, and ignore bottom workers. (If not, there will be less than $(\delta^{\mathrm{t}} - \epsilon)/\mu$ mass of reacher employers in the system in steady state, and it will be a unique best response to be a reacher and earn utility of at least $1 - \sqrt{2c} + \epsilon'$, a contradiction.) On the other hand, there cannot be all reacher employers. (If this is the case, then bottom workers will want to propose w/o screening anyway, even if there is a very small mass of settler employers in the system, resulting from a small perturbation as per the notion of evolutionary stability we consider. Reachers will earn less than $1 - \sqrt{2c} - \epsilon''$, so the utility from being a settler will be strictly higher, a contradiction.) So the fraction of reachers must be in $((\lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}} + o(1))/\lambda_e, 1 - \epsilon)$. Now, we try to rule out different possibilities as in Theorem 2. Suppose bottom workers propose, then settler employers definitely don't want to propose. Suppose bottom workers mix between proposing and not proposing. As before this is not evolutionarily stable because bottom workers proposing has a negative externality on other bottom workers; as in Theorem 3 the stable situation can only be that the long side (bottom workers) is either all proposing or none are proposing. We already considered all proposing, suppose none of the bottom workers propose. Then it must be that settler employers all propose (else they would count as reachers). We rule out this case below.

It remains to consider whether employers will propose to bottom workers in some equilibrium. Note that the bottom submarket is always unbalanced in favor of employers, since $\lambda_e - \lambda_w^{\mathrm{t}} - \delta^{\mathrm{t}} < \lambda_{\mathrm{b}}$, and the rate of employers dying is at least $\delta_{\mathrm{t}} - o(1)$ as argued above (this rate can be higher if $\theta_w < 1 - \sqrt{2c}$, which occurs if employers propose to bottom workers, meaning that more reacher employers must die to make utilities equal), which means that the rate of employers participating in the bottom market is less than λ_w^{b}. As a result, employers will always want to screen before proposing since we have assumed $1 - (2c)^{1/4} > 1/2$, and $1/2$ is what an employers can earn by proposing without screening. Such an equilibrium is ruled out if $\lambda_w^{\mathrm{b}} \geq 1.20(\lambda_e - \lambda_w^{\mathrm{t}} - \delta^{\mathrm{t}})$, drawing on the analysis in Theorem 4: Note that in any such equilibrium $\theta_e < 1 - \sqrt{2c}$, hence all employers, including reacher employers will want to look at an incoming proposal from a bottom worker. In fact, all employers will react to such proposals by screening and accepting based on a threshold θ_w (with θ_w being a function of the imbalance in the bottom submarket, which is more than $\lambda_w^{\mathrm{b}}/(\lambda_e - \lambda_w^{\mathrm{t}} - \delta^{\mathrm{t}})$) which is the same as what happens in the analysis leading to Theorem 4 bullet 1. For any $c < \bar{c}_2$, cf. Eq. (34), it will be a best response for bottom workers to screen and propose; consequently employers will not want to propose to bottom workers (they will receive $\Omega(1/\mu) = \omega(1)$ proposals before they die, more than enough to find one they like before they die with probability approaching 1). So it suffices to have $\bar{c}_2 \geq 1/32$ to ensure that an equilibrium with employers proposing to bottom workers does not exist for any $c < 1/32$. In turn, this is ensured by the imbalance in the bottom submarket exceeding $1 + 1/2^{7/3} = 1.1984 < 1.20$, which is ensured by $\lambda_w^{\mathrm{b}} \geq 1.20(\lambda_w - \lambda_{\mathrm{t}} - \delta_{\mathrm{t}})$. It follows that bottom workers must be the only ones proposing in the bottom submarket. Hence, settlers earn a utility of $1 - \sqrt{2c} - o(1)$, and this then implies that the fraction of reacher employers is, in fact, $(\lambda_w^{\mathrm{t}} + \delta^{\mathrm{t}} - o(1))/\lambda_e$, in order for reachers and settlers to have the same utility. We obtain that the equilibrium in Theorem 5 is the unique evolutionarily stable equilibrium. □

Proof of Theorem 5. We can finally write a quick formal proof of the theorem.

Proof of Theorem 5. The existence and characterization part of the theorem follows combining Claims 1, 2, 3 and 4. Claim 6 yields uniqueness of the equilibrium under the condition $\lambda_w^{\mathrm{b}} \geq 1.20(\lambda_w - \lambda_{\mathrm{t}} - \delta_{\mathrm{t}})$. □

E.3 Interventions

Here we describe the equilibria obtained by the interventions discussed in Section 4.

E.3.1 Proposed intervention: block workers from proposing

Suppose all workers are disallowed from proposing, regardless of their tier. We start by describing the equilibrium, and informally argue its existence. Again, we will have two types of employers: reachers and settlers. Given the chance to see an opportunity, reachers will request a top worker; if none is available, they'll just wait in the system. Settlers will first request a top worker, and if non available, they'll request a bottom worker. As before, let λ_e^s λ_e^r denote the effective arrivals rates of the two types of employers, so that $\lambda_e^s + \lambda_e^r = \lambda_e$. Out of the λ_e^r employers, we have that λ_w^t will match to top workers, and δ^t will die. We need to determine the new values of $\theta_w^t, \theta_w^b, \theta_e$, and δ^t (note that δ^t immediately fixes the values of $\lambda_e^s = \lambda_e - \lambda_w^t - \delta^t$). As before, this will induce a bottom submarket and a top submarket. We start by focusing on the necessary conditions such an equilibrium must satisfy.

1. Top workers will still use the socially optimal strategies, i.e. $\theta_w^t(c) = 1 - \sqrt{2c}$, as their strategy is unaffected by the intervention.

2. Suppose δ^t is fixed. Then, the bottom submarket will have an imbalance of

$$\lambda = \frac{\lambda_w^b}{\lambda_e - \lambda_w^t - \delta^t}\,. \tag{51}$$

   Given this imbalance, we can use the insights in Theorem 2; in particular, provided that c is small and the imbalance is small enough, to determine that bottom workers will now screen with threshold

$$\theta_w^b = \xi(\lambda, c)\,. \tag{52}$$

   Furthermore, this implies that the threshold used by employers must satisfy:

$$\theta_e = 1 - \sqrt{\frac{2c}{1-\theta_w^b}}\,. \tag{53}$$

3. In addition, noting that at equilibrium both reachers and settlers should derive the same utility, we must then have:

$$\theta_e = \frac{\lambda_w^t}{\lambda_w^t + \delta^t}\left(\frac{a+1+\theta_e}{2} - \frac{c}{\sqrt{2c}(a+1-\theta_e)}\right)$$

   This can be re-expressed as:

$$\theta_e = \frac{(a+1)\frac{\lambda_w^t+\delta^t}{\lambda_w^r} - \sqrt{(a+1)^2\left(\frac{\lambda_w^t+\delta^t}{\lambda_w^t}\right)^2 + 4\left(\frac{\lambda_w^t+\delta^t}{\lambda_w^t} - \frac{1}{2}\right)\left(\sqrt{\frac{c}{2}} - \frac{(a+1)^2}{2}\right)}}{2\left(\frac{\lambda_w^t+\delta^t}{\lambda_w^t} - \frac{1}{2}\right)} \tag{54}$$

We can solve Eqs. (51), (52), (53) and (53) for $\delta^t, \lambda_e^r, \theta_e^s$ and θ_w^b.

We highlight that these are necessary conditions. In addition, for this to be an equilibrium, we must check that no agent has incentive to deviate:

1. Top workers will never have an incentive to deviate; this is easy to see, as they are screening using the socially optimal threshold and thus their outcome cannot be improved.

2. Bottom workers should continue to screen. Using Theorem 3, this will happen as long as $c \le \bar{c}$, i.e., it will happen in the limit $c \to 0$.

3. In addition, we must have that all employers screen and propose to top workers, and settlers also propose to bottom workers. The condition for settlers to screen and propose to bottom workers is $c \leq \hat{c}$. We can rule out the possibility that employers want to propose without screening to top workers. The reason is that there is a probability $(1-a)/2$ that the worker will have a value less than $(1+a)/2 < 1$, and the employer might as well screen to avoid such workers, since she faces an effective screening cost (see Lemma 1) of $\sqrt{c/2} \to 0$. A sufficient condition for employers to screen and propose to top workers is $\delta_{\mathrm{t}} \geq 0$ and $\theta_e \geq a + 1/2$.

Therefore, we have argued that the setting described above is indeed an equilibrium. We can now formally state the theorem, for which we just provided the sketch of proof.

Theorem 6. *Suppose $a \in (0,1)$ and $\lambda_e \in (\lambda_w^t(1+a/2), \lambda_w^t + \lambda_w^b)$. Define $\lambda = \lambda_w^b/(\lambda_e - \lambda_w^t(1+a/2))$. Then the following is an equilibrium along with the associated utilities earned by different agents, if we consider $\mu \to 0$ and then $c \to 0$:*

- *Top workers screen and accept/reject with threshold $\theta_w^t(c) = 1 - \sqrt{2c}$, and earn the same amount as expected utility.*
- *A fraction*
$$\frac{\lambda_w^t(1+a/2)}{\lambda_e}\left(1+o(1)\right)$$
of employers are reachers, meaning that they screen and propose to top workers, when they get a chance. The rest of the employers are settlers, meaning that they are willing to screen and propose to bottom workers but would first propose to a top worker if they get a chance. Both kinds of employers use a threshold of
$$\theta_e = 1 - \sqrt{\frac{c(2\lambda-1)}{\lambda-1}}\left(1+o(1)\right).$$
Reachers die at a rate of
$$\frac{\lambda_w^t a}{2}\left(1+o(1)\right).$$
- *Bottom workers screen and accept/reject incoming proposals with a threshold of*
$$\theta_w^b = \frac{2(\lambda-1)}{2\lambda-1}\left(1-o(1)\right),$$
and earn the same amount as expected utility.

E.3.2 Proposed intervention: Identifying settler employers who will consider bottom workers

Suppose the platform is able to identify employers who will consider proposals by bottom workers. The platform can then reveal this to bottom workers, who can direct their search efforts towards such settler employers exclusively. Whereas such a classification ("interested" or "not interested" in bottom workers) may not be binary, here we optimistically assume it is, and study the resulting welfare improvement for bottom workers. We remark that the welfare remains unchanged for top workers and employers, as their strategies should remain unaltered.

Theorem 7. *Let top workers arrive at rate λ_w^t, bottom workers arrive at rate λ_w^b and employers arrive at rate λ_e. Fix $c \in (0, 1/32)$ and a satisfying*

$$a \in \left((2c)^{1/4} - \sqrt{2c}, 1 - (2c)^{1/4} - \sqrt{2c} \right) . \tag{55}$$

Define

$$\delta^t = \lambda_w^t \left(\frac{a - \sqrt{2c}\left(\frac{1}{a+\sqrt{2c}} - 1\right)}{2(1-\sqrt{2c})} \right) > 0 \tag{56}$$

Assume $\lambda_e \in (\lambda_w^t + \delta^t, \lambda_w^t + \lambda_w^b + \delta^t)$. Furthermore, suppose $\lambda_w^b \geq 1.20(\lambda_e - \lambda_w^t - \delta^t)$.[20] *Also assume that each employer either always screens proposals from bottom workers (a settler), or never screens proposals from bottom worker (a reacher) —the fraction of settlers and reachers is part of the equilibrium description. Then, the following is a description of all equilibria in the limit $\mu \to 0$:*

- *Top workers do not propose; they screen incoming proposals from employers and accept/reject using a threshold of $\theta_w^t = 1 - \sqrt{2c}$.*
- *Employers do not propose to bottom workers. A fraction $1 - \frac{\lambda_w^t + \delta^t}{\lambda_e}$ of all employers will screen and accept/reject incoming proposals from bottom workers, with a threshold of $\theta_e = 1 - \sqrt{2c}$; all other employers ignore such proposals. When the opportunity arises, all employers screen an available top worker, and propose to him with the same threshold of θ_e.*
- *Bottom workers propose. For $c \in (0, \bar{c}^2)$, there is an equilibrium where they (all) screen and propose (considering only settler employers). For $c \in [\underline{c}, 1/32)$ there is an equilibrium where they propose without screening to settler employers (they may or may not propose to other employers, but such proposals are ignored anyway). We define $\bar{c}$ and $\underline{c}$ below.*

In any equilibrium, top workers each earn expected utility θ_w^t, employers each earn expected utility θ_e. If bottom workers screen and propose, they each earn expected utility $\theta_w^b = \xi(\lambda, \sqrt{c/2})$

[20] Actually, $\lambda_w^b \geq 1.20(\lambda_e - \lambda_w^t - \delta^t)$ is required solely for these equilibria to be the unique ones. If λ_w^b is less the proposed quantity, new equilibria can arise (see Theorem 4).

as explained below, whereas if bottom workers propose without screening, they each earn expected utility $(\lambda_e - \lambda_w^t - \delta^t)/(2\lambda_w^b)$.

Here $\bar{c}$ *and* $\underline{c}$ *are given by Eq.* (10)*, and* $\xi(\lambda, \cdot)$ *is given by Eq.* (9)*, in each case substituting*

$$\lambda = \frac{\lambda_w^b}{\lambda_e - \lambda_w^t - \delta^t} \, . \tag{57}$$

Sketch of proof. This result borrows heavily from Theorem 5. By making the assumption that $\lambda_w^b \geq 1.20(\lambda_e - \lambda_w^t - \delta^t)$ upfront, we can guarantee that the equilibrium behavior of top workers and employers is (essentially) uniquely determined. Thereafter, we borrow from Theorem 2 to deduce the equilibrium among bottom workers. We use the effective λ for the bottom, unbalanced, submarket. □

We notice that identifying the settler employers allows bottom workers to be selective for small values of c, but this benefit disappears if the value of c exceeds $2\bar{c}^2$. For $\lambda = 1.5$, we have $2\bar{c}^2 = 0.011$, whereas for $\lambda = 2$ we have $2\bar{c}^2 = 0.0037$. The above theorem placed an upper bound on the value that can be derived, in terms of gain in social welfare, if the platform is able to distinguish between employers who are settlers and reachers.

E.3.3 Hiding information regarding top and bottom workers

Suppose the platform prevents workers from proposing and, further, does not reveal to employers whether a worker is a top worker or a bottom one. In this case, we show that an equilibrium arises in which (as $c \to 0$) the social welfare per unit time accrues at a rate that is $\Omega(1)$ faster than the case of no platform intervention.

Theorem 8. *Suppose* $\lambda_e \in (\lambda_w^t, \lambda_w^t + \lambda_w^b)$*. Consider* $\mu \to 0$ *for fixed* c *and then consider* $c \to 0$*. We have the following equilibria and corresponding utilities for agents of each type:*

1. *Employers screen and propose with threshold*

$$\theta_e = 1 - K_e c(1 + o(1)) \, ,$$
$$\text{where } K_e = \frac{2(\lambda_e - \lambda_w^t)}{a\lambda_w^t} \cdot \frac{2\lambda - 1}{2(\lambda - 1)} \, .$$

2. *Top workers screen and accept/reject with threshold*

$$\theta_w^{\mathrm{t}} = 1 - K_{\mathrm{t}}\sqrt{c}\,(1 + o(1)) \, ,$$
$$\text{where } K_{\mathrm{t}} = \sqrt{\frac{4(\lambda_w^{\mathrm{t}} + \lambda_w^{\mathrm{b}} - \lambda_e)}{a^2\lambda_{\mathrm{t}}} + 2} \, ,$$

A fraction $(K_{\mathrm{t}}/2 - 1/K_{\mathrm{t}})\sqrt{c}\,(1 + o(1))$ *of them die without matching.*

3. Bottom workers will screen and accept/reject

$$\theta_w^{\mathrm{b}} = \xi(\lambda, 0)\,(1 + o(1)) = \frac{1}{2\lambda - 1}\,(1 + o(1))\,.$$

where $\lambda = \lambda_w^b/(\lambda_e - \lambda_w^t)$ *and* $\xi(\cdot, \cdot)$ *is as defined in Eq.* (9).

Note that the limiting utilities are 1 each for top workers and employers (as has been the case under all settings discussed so far), and $1/(2\lambda - 1)$ for bottom workers, which is an improves upon the utility they get under the other interventions.

Sketch of proof of Theorem 8. We first study the steady state of the system when agents follow actions as described. Let f_{t} be the fraction of workers in the system at any time, who are top workers. Then, when an employer is presented with an option (which she then proceeds to screen, not knowing whether it is a top or a bottom worker), it is a top worker with probability f_{t}. If it is a top worker, she proposes w.p. $a + o(1)$ and gets accepted w.p. $K_{\mathrm{t}}\sqrt{c}\,(1 + o(1))$, so the overall likelihood of the option resulting in a match with a top worker is

$$f_{\mathrm{t}} a K_{\mathrm{t}} \sqrt{c}\,(1 + o(1))\,.$$

If the option is a bottom worker, she proposes w.p. $cK_e\,(1 + o(1))$ and gets accepted w.p. $1 - 1/(2\lambda - 1) + o(1)$, so the overall likelihood of the option resulting in a match with a bottom worker is

$$(1 - f_{\mathrm{t}}) c K_e \frac{2(\lambda - 1)}{2\lambda - 1} + o(1)\,.$$

Suppose f_{t} is $\Omega(1)$. Then with likelihood $1 - O(\sqrt{c})$, an employer forms a match with a top worker, meaning that matches between employers and top workers are formed at a rate $\lambda_e - o(1) > \lambda_w^{\mathrm{t}}$, a contradiction. Hence, $f_{\mathrm{t}} = o(1)$. Now, almost all employers form matches (a vanishing fraction die without matching). It follows that workers die at a rate $\lambda_{\mathrm{b}}^w + \lambda_w^{\mathrm{t}} - \lambda_e + o(1)$, and since $f_{\mathrm{t}} = o(1)$, most of these are bottom workers. We deduce that the number of bottom workers in the system in steady state is $(\lambda_w^{\mathrm{b}} + \lambda_w^{\mathrm{t}} - \lambda_e + o(1))/\mu$. Also, employers match with bottom workers at rate $\lambda_e - \lambda_w^{\mathrm{t}} + o(1)$, and with top workers at rate $\lambda_w^{\mathrm{t}} - o(1)$. Hence,

$$\begin{aligned} \frac{f_{\mathrm{t}} a K_{\mathrm{t}} \sqrt{c}(2\lambda - 1)}{cK_e 2(\lambda - 1)} &= \frac{\lambda_w^{\mathrm{t}}}{\lambda_e - \lambda_w^{\mathrm{t}}} + o(1)\,, \\ \Rightarrow f_{\mathrm{t}} &= \frac{2\sqrt{c}}{a^2 K_{\mathrm{t}}}\,(1 + o(1)) = \frac{\lambda_w^{\mathrm{t}}(K_{\mathrm{t}}/2 - 1/K_{\mathrm{t}})}{\lambda_w^{\mathrm{t}} + \lambda_w^{\mathrm{b}} - \lambda_e} \sqrt{c}\,(1 + o(1))\,. \end{aligned}$$

This completes our understanding of the steady state of the system. Note that the fraction of top workers who die is $(K_{\mathrm{t}}/2 - 1/K_{\mathrm{t}})\sqrt{c}\,(1 + o(1))$ as stated in the theorem.

We now need to check that each type of agent is playing a best response. First consider employers. When they screen, they should use a threshold equal to their expected utility from playing a best response (under steady state). This expected utility is the same for all employers.

Call it θ_e. One can rule out values of θ_e that violate the estimate stated in the theorem because the result rate of match formation with top workers will not be $\lambda_w^{\mathrm{t}} - o(1)$. This rate cannot exceed λ_w^{t} since there are not enough top workers, and it cannot be $\lambda_w^{\mathrm{t}} - \Omega(1)$ because in that case, employers will be able to get utility exceeding 1 by matching only with top workers, a contradiction.

It remains to show that there exists an equilibrium with employers earning expected utility matching the stated estimate of θ_e. (We will provide a sketch of proof.) Suppose an employer is presented with an option (which she then proceeds to screen, not knowing whether it is a top worker or a bottom one). Based on the calculation above, the overall likelihood of the option resulting in a match with a top worker is

$$\frac{\lambda_w^{\mathrm{t}}(K_{\mathrm{t}}/2 - 1/K_{\mathrm{t}})}{\lambda_w^{\mathrm{t}} + \lambda_w^{\mathrm{b}} - \lambda_e} aK_{\mathrm{t}}c\,(1 + o(1)) = K_e c \frac{2(\lambda - 1)\lambda_w^{\mathrm{t}}}{(2\lambda - 1)(\lambda_e - \lambda_w^{\mathrm{t}})}\,(1 + o(1))\,,$$

and if such a match is formed, it yields an expected match utility of $1 + a/2 - o(1)$ for the employer.

The overall likelihood of the option resulting in a match with a bottom worker when an employer uses a threshold of θ_e is

$$cK_e \frac{2(\lambda - 1)}{2\lambda - 1} + o(1)\,,$$

and if such a match is formed, it yields an expected match utility of $1 - o(1)$ for the employer.

Combining, the likelihood of a presented option leading to a match is

$$\frac{2(\lambda - 1)cK_e}{2\lambda - 1}\left(1 + \frac{\lambda_w^{\mathrm{t}}}{\lambda_e - \lambda_w^{\mathrm{t}}}\right)(1 + o(1)) = \frac{2\lambda_e c}{a\lambda_w^{\mathrm{t}}}\,(1 + o(1))\,,$$

leading to lifetime expected screening cost of

$$\frac{a\lambda_w^{\mathrm{t}}}{2\lambda_e}\,(1 + o(1))\,.$$

There is likelihood ratio of $\frac{\lambda_w^{\mathrm{t}}}{\lambda_e - \lambda_w^{\mathrm{t}}} + o(1)$ of an employer matching with a top worker versus a bottom worker. Thus, the employer's expected utility is

$$\begin{aligned} &\text{Expected match utility} - \text{Expected screening cost} \\ &= \frac{\lambda_w^{\mathrm{t}}}{\lambda_e}(1 + a/2 - o(1)) + \left(1 - \frac{\lambda_w^{\mathrm{t}}}{\lambda_e}\right)(1 - o(1)) - \frac{a\lambda_w^{\mathrm{t}}}{2\lambda_e}\,(1 + o(1)) \\ &= 1 - o(1)\,. \end{aligned}$$

Though we did not do the full calculation, one can check that the second order term in the fraction f_{t} of workers in the system who are top workers adjusts to produce $\theta_e = 1 - K_e\, c(1+o(1))$ as needed. (It is easy to check that the expected utility of an employer following any fixed strategy is monotone increasing in f_{t}.) Finally, it is easy to see that employers have no incentive to propose without screening as their expected utility will be $1/2 < \theta_e$

Bottom workers screen and accept/reject since $c < \bar{c}$ (they would have accept without screening if $c > \underline{c}$, but here we are looking at c small).

It remains to show that top workers are playing a best response. Accepting without screening is ruled out since it produced expected utility only 1/2 or less. Hence, they must screen and accept/reject with threshold equal to their expected utility. It remains to show that when they use a threshold of θ_w^{t}, they earn the same amount as expected utility. Now, the expected utility of a worker is

$$
\begin{aligned}
&\text{Likelihood of matching} \cdot (\text{Expected match utility} - \text{Expected screening cost per match}) \\
&= \big(1 - (K_{\mathrm{t}}/2 - 1/K_{\mathrm{t}})\sqrt{c}\,(1 + o(1))\big)\Big(\frac{1+\theta_w^{\mathrm{t}}}{2} - \frac{c}{1-\theta_w^{\mathrm{t}}}\Big) \\
&= \big(1 - (K_{\mathrm{t}}/2 - 1/K_{\mathrm{t}})\sqrt{c}\,(1 + o(1))\big)\ \big(1 - (K_{\mathrm{t}}/2 + 1/K_{\mathrm{t}})\sqrt{c}\,(1 + o(1))\big) \\
&= 1 - K_{\mathrm{t}}\sqrt{c}\,(1 + o(1))\,,
\end{aligned}
$$

which matches the estimate for θ_w^{t}.

Finally, we expect that the described equilibrium to be evolutionary stable. For instance, suppose the fraction f_{t} of workers in the system who are top workers falls below its steady state value. Then employers' expected utility will fall, they will lower their threshold for accepting a match, form more matches with bottom workers, and as a result f_{t} will rise towards its steady state value. □