Valuation of Projects with Minimum Revenue Guarantees:
A Gaussian Copula-Based Simulation Approach

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This article presents a numerical simulation technique to perform valuations of infrastructure projects with minimum revenue guarantees (MRG). It is assumed that the project cash flows--in the absence of the MRG--can be described in a probabilistic fashion by means of a very general multivariate distribution function. Then, the Gaussian copula (a numerical algorithm to generate vectors according to a pre-specified probabilistic characterization) is used in combination with the MRG condition to generate a set of plausible cash flow vectors. These vectors form the basis of a Monte Carlo simulation that offers two important advantages: it is easy to implement, and it makes no restrictive assumptions regarding the evolution of the cash flows over time or their correlation matrix. Thus, one can estimate the distribution of a broad set of metrics (net present value, internal rate of return, payback periods, etc.) Additionally, the method does not have any of the typical limitations of real options-based approaches, namely, cash flows that follow a Brownian motion or some specific diffusion process, or whose volatility needs to be constant. The usefulness of the proposed approach is demonstrated and validated using a simple, yet realistic, mining project.

Keywords: valuation, Monte Carlo simulation, infrastructure projects, net present value, probabilistic cash flows, project finance, Gaussian copula, minimum revenue guarantee

Introduction

Constructing and operating large infrastructure projects are risky endeavors. As a result, many governments have embraced the public-private partnership (PPP) model to give impetus to such undertakings (Walker and Smith, 1995; Hammami *et al*., 2006). Within this framework, the minimum revenue guarantee (MRG) approach-- where a government entity commits itself to cover the project revenue shortfall following some agreed-upon rule--has been a useful device to mitigate the concessionaire risks during the operation phase (Klein, 1997; Irwin; 2003).

Valuation problems are one of the most challenging in engineering management. Although everybody agrees on what a valuation should reflect –the present value of the future cash flows—in practice, many problems arise (see, for example, Damoradan, 2006). The main difficulty comes from the fact that cash flows are uncertain. Indeed, this is precisely the feature that makes valuation problems interesting. In the context of large infrastructure projects, valuation presents some additional challenges: the amounts of money involved are often significant, and the political implications of incorrectly valuing the economic merits of a project can be substantial. If we add to these complexities the effect of the MRG, the task can be daunting. The reason is that the MRG complicates the valuation exercise considerably since it alters the probabilistic behavior of the project cash flows. They typically switch from having a mathematically friendly distribution (for instance, normal) to more intractable distributions such as truncated or censored normal distributions (Hemming, 2006).

The most basic and still widely popular approach to handle valuation problems is the discounted cash flow (DCF) method. The idea is to estimate the project net present value (NPV) by discounting the expected cash flows with a risk-adjusted rate, also referred to, as the appropriate rate. This approach traces its origin to Dean (1951) who proposed an analogy between bond valuation and project valuation. A number of authors have discussed the determination of the appropriate discount rate (see, for instance, Brealey *et al*., 2007; Ross *et al*., 2009; Ehrhardt and Brigham, 2010). This method, despite its popularity, has some serious drawbacks.

First, it attempts to handle two different effects--the time-value of money and the uncertainty of the cash flows, with one single parameter: the discount rate. Leaving aside that this simplification is difficult to defend conceptually (see Fama, 1977) it renders the method ill-suited to describe the cash flows probabilistically, which is essential in any MRG project. A second limitation of the DCF method within the context of large construction projects is the difficulty in estimating the weighted-average cost of capital (WACC) which is often used as a proxy for the risk-adjusted discount rate. The reason is that in such projects the financing normally consists of a complex web of arrangements involving private equity, bank loans, different types of bonds, tax breaks, subsidies, preferred stock, etc. which, to make things worst, are all time-dependent. Espinoza and Morris (2013) and Espinoza (2014), in our view, give the best account of the limitations of the DCF approach as it applies to valuation of civil engineering projects. A chief concern discussed at length by these authors is the fair amount of subjectivity involved in estimating the correct discount rate. Other authors such as Halliwell (2000), Cheremushkin (2009), Creswell (1998), Ariel (1998), Black (1988), Berry and Dyson (1980), Dyson and Berry (1983) and Cifuentes and Valdivieso (2011) have advanced additional criticisms. To name a few: the systematic error introduced when discounting both negative and positive cash flows with the same rate, the fact that in general the method implicitly assumes that the cash flows become riskier with time, and more generally, the often unnoticed fact that the method relies on a rigid utility function structure to express the decision maker's preferences.

Consequently, many practitioners favor Monte Carlo simulations (Glasserman, 2003; Judd, 1998). Such simulations rely on generating many feasible cash flow paths and discounting them with the risk-free rate (see Brealey *et al*., 2007). Discounting with the risk-free rate is appropriate since the uncertainty in the cash flows is accounted for by randomly generating several plausible scenarios. This, eliminates the issue of estimating the risk-adjusted rate, which is far more challenging to estimate than the risk-free rate. Oftentimes these simulations assume that the revenue cash flows follow normal distributions (an assumption that is violated in the case of the projects that enjoy the benefits of a MRG) or assume that the cash flows at two different time-points are uncorrelated (a clearly naïve assumption). Alternatively, many Monte Carlo simulations assume that the cash flows can be generated with a recursive formula such as a discrete-time one-factor stochastic process (see Irwin, 2003; Chiara *et al.* 2007; Garvin and Cheah, 2004). Unfortunately, there is no solid evidence to support the view that such processes are adequate to describe well the time-evolution of realistic infrastructure-projects cash flows.

Finally, a number of practitioners have proposed the use of real options based-methods to analyze investments in projects (see, for instance, Ford *et al*., 2002; Ho and Liu, 2002; Pellegrino *et al.*, 2011; Liu and Cheah, 2009). More recently, such methods have been extended to projects with MRG. For example, Cheah and Liu (2006) as well as Huang and Chou (2006) have employed this approach assuming that the underlying stochastic process is a diffusion or Wiener process. Brandao and Saraiva (2008) have characterized the government support for a project using a sequence on independent European options. The validity of this assumption, which no doubt facilitates solving the problem, is questionable in the context of infrastructure projects. Clearly, what happens at a given time during the life of the project (especially during the construction phase) has an effect in future cash flows. Ashuri *et al*. (2012) have used the technique to analyze a toll road in which the traffic patterns are treated stochastically using a modified binomial lattice formulation to avoid unrealistic traffic patterns. Other authors who have applied real options-based methods to MRG projects are Wibowo (2004), Iyer and Sagheer (2011) and Shan *et al*. (2010). These methods have been inspired by advances in financial engineering following the Black-Scholes equation and its subsequent improvements by Merton (Black and Scholes, 1973; Merton, 1973).

Although the field of real options as it applies to valuation of infrastructure projects is still growing, it is fair to say that this approach has failed to be fully embraced by practitioners. Some academics think that the merits of this method have been overstated. Damoradan (2001) describes this point well. Carbonara *et al.* (2014) attribute this situation to the combined effect of the mathematical complexity behind this approach coupled with the fact that many assumptions that might be valid in the financial world are not necessary appropriate within the engineering and the construction world.

In summary, given this context, the need to develop a more appealing method to value infrastructure projects with MRG seems warranted. Specifically, there is a need for a more operationally friendly method that fully captures the salient features of such projects.

Problem Statement: without the MRG

We consider first the valuation problem without incorporating the MRG, which we do at a later stage.

A general infrastructure valuation problem can be described as follows. Let **X** be the vector of cash flows associated with the project, that is, $\mathbf{X}'=(x_0, x_1, \ldots, x_n)$ where the cash flows correspond to n equally spaced time intervals. We further assume that **X** follows a multivariate distribution, F, that is, $\mathbf{X} \sim F(\boldsymbol{\mu}, \mathbf{C})$ where $\boldsymbol{\mu}$ denotes the expected value of **X** ($\mu_i = E(x_i)$, for i=0, 1, …,n) and **C** represents the corresponding (n+1)x(n+1) variance-covariance matrix.

For convenience, most analysts prefer to work with the correlation matrix, $\mathbf{C\rho}$, rather than the variance-covariance matrix, which is less intuitive. Recall that $\mathbf{C\rho}$ contains 1's along the main diagonal and the off-diagonal terms ($\rho_{i,j}$) are given by

$\rho_{i,j} = \sigma_{i,j} / (\sigma_i \, \sigma_j)$, where $\sigma_{i,j}$ and σ_i represent the co-variances and standard deviations, respectively.

The valuation problem consists of estimating the expected value and the standard deviation of the usual metrics (net present value (NPV), internal rate of return (IRR), payback period (PBP), etc.) plus their probabilistic distributions. This, in turn, would permit the decision maker to estimate confidence intervals for these metrics. It would also allow to answer questions such as: what is the likelihood that the IRR will exceed a critical threshold value; or, what is the chance that the PBP will be less than a specific number of years. The idea is to tackle this problem using a Monte Carlo simulation technique since a closed-form solution is unfeasible.

A few practical considerations

Regarding the set-up described in the previous section, the following considerations apply:

[1] Estimating the expected value of the cash flow vector components (μ_i for i=0, 1, …,n) should be done by aggregating all the factors (inflows and outflows) that determine the free cash flows. It might seem that this is a very demanding task. However, any valuation method (even the simplistic DCF technique) starts with this assumption, which is, indeed, the basic input for any valuation exercise

[2] In all likelihood, the analyst would have an idea of the error (or degree of accuracy) associated with the cash flows estimates described in [1]. This provides the basis to estimate σ_i (for i=0, …,n).

[3] Estimating the covariances, or alternatively, the off-diagonal terms in the correlation matrix ($\rho_{i,j}$) can be seen as more challenging. (Recall that $\rho_{i,j}$ in this context captures the correlation between two cash flows separated by |i-j| time intervals). After all, empirical data regarding this matter is scant (Ranasinghe, 2000). However, a recent study by Hawas and Cifuentes (2014) provides some useful guidance. The authors show that some simple two-parameter correlation matrix structures (tri-diagonal or penta-diagonal) are sufficient in most cases. Additionally, Hudak and Maxwell (2007), provide some very useful guidelines to estimate correlation coefficients for construction projects. In any event, given the fact that correlation coefficients are constrained to take values in the (-1, 1) interval, in the absence of any guidance or experience, one can perform a valuation analysis with several choices and estimate a range for the project value.

This is better than failing to address the importance of correlation in the valuation.

[4] The structure of the F function could be, in principle, quite general. However, in most practical cases, simple functional forms will suffice (normal, lognormal, uniform, or, some analytical expression based on **μ** and **C**).

[5] Finally, it should be mentioned that within the context of a construction project, it is much easier for a practicing engineer to estimate the cash flow vector by aggregating the different components that go into it (and their errors and correlations) than attempting to capture such vector by means of a single stochastic process. In fact, we are not aware of any study that shows empirically that the peculiarities of construction projects cash flows can be satisfactorily described by the simple processes often used in Monte Carlo simulations (diffusion, multi-factor, Wiener, etc.)

Problem Statement: introducing the MRG

In reference to the formulation without MRG, introducing the MRG condition simply reduces to altering the definition of the F function described before. The MRG condition can be stated as follows: for any $j > q$ (where $0 \leq q \leq n$) the government makes up the difference if x_j drops below a critical threshold x_j*.

In essence, the effect of the government intervention is to change the distribution of the cash flows x_j ($j=q+1, \dots, n$) from F to a "censored" F distribution. In these instances, the corresponding probability density functions will exhibit a discontinuity at $x_j = x_j$* to account for the probability mass concentration on the left side of the distribution.

Thus, we denote as H the resulting "censored" F distribution. In short, $\mathbf{X} \sim H(\boldsymbol{\mu}, \mathbf{C})$. In the context of a numerical simulation redefining the multivariate distribution associated with **X** is rather trivial as it does not involve finding a new analytical expression. One can still rely on the F function (as defined before incorporating the MRG) with the caveat that any sampling resulting on a component below the MRG threshold is automatically adjusted to x_j*.

The Gaussian copula

Many simulations rely on the ability to generate vectors according to some pre-ordained correlation structure. A proven way to accomplish this task is by means of copula functions. That is, functions that can combine several univariate distributions to form a multivariate distribution. The technique owns its popularity to the pioneering work of Sklar (1959) who established its mathematical foundation. Monte Carlo simulations based on the Gaussian copula, by far the most popular copula, have been used successfully in a number of financial and engineering applications (see, for instance, Li, 2000; Schonbucher, 2003; Nelsen, 1999)**.** In the construction engineering arena, simulations based on the Gaussian copula have been used by Yang (2006) to model processes with repetitive tasks and have proven to offer important benefits compared to PERT-based approaches. However, to the best of our knowledge, this technique has not been employed to analyze valuation problems, that is, to generate cash flow vectors with specific distributions and correlations.

Ross (2013), Wang and Dyer (2012), and Cario and Nelson (1997) provide a good introduction to copula functions and their application to simulation problems. A practical way to implement the technique in Monte Carlo simulations is discussed by Haugh (2004), Schmidt (2006) and Embrechts *et al*. (2003).

Simulation algorithm

The idea is to employ a Monte Carlo simulation technique based on the following steps:

[0] Find the Cholesky decomposition (factorization) of the correlation matrix, that is, find a matrix **L** such that, $\mathbf{C\rho} = \mathbf{L}\,\mathbf{L}'$(this factorization can be performed easily with standard software packages such as *MATLAB* or *Mathematica*);

[1] Generate $\mathbf{Y}' = (y_0, y_1, \ldots, y_n)$ where the y_i's are random draws from iid N(0, 1);

[2] Compute $\mathbf{Z}=\mathbf{LY}$;

[3] Let Φ denote the cumulative distribution function of the standard normal distribution; determine the vector $\mathbf{U}' = (u_0, u_1, \ldots, u_n)$, with $u_i = \Phi(z_i)$ for i=0, 1, …, n (thus, $0 \leq u_i \leq 1$);

[4] Determine **X**, the desired sample vector, using $x_i = H_i^{-1}(u_i)$ for i=0, 1, …, n where the function H_i^{-1} represents the inverse cumulative distribution of the desired marginal distributions (of x_i).

Repeating this loop (steps [1] through [4]) several times, we can generate sufficient cash flow vectors (**X**), with the desired properties, to estimate whatever figures of merit (and their distributions) we need. This is accomplished by averaging the appropriate quantities across all samples.

Example of Application

We consider a mining project described in a Harvard case study, Harvard Business School (1983). We assume that the cash flows follow a normal distribution with estimated means (μ_i's) according to the data shown in Table 1. We further assume that in the construction phase (phase A) the uncertainty in the cash flows is given by a coefficient of variation, $\lambda_A = 10\%$. That is, we have $\sigma_i = \lambda_A \left| \mu_i \right|$ for i=0, 1, 2 and 3. We also make the assumption that during this phase the cash flows are uncorrelated. In short, the corresponding correlation sub-matrix (phase A) is the identity matrix.

For the operations phase (phase B) we assume the cash flows to be more volatile and thus we take λ_B, the coefficient of variation, to be equal to 40%. Thus, $\sigma_i = \lambda_B \left| \mu_i \right|$ for i=4, 5, …, 16. We also assume a correlation sub-matrix B with a tri-diagonal structure and with identical values ρ = 20% along the upper- and lower-sub-diagonals. Finally, at the time, the appropriate risk free rate, R, was 6.2 %. There is no MRG yet.

<u>Validation of method proposed</u>

In the case of the NPV, there are analytical expressions for both, its expected value and its variance, regardless of the distribution assumed for **X**. (For the other metrics, namely, the IRR, the PBP, etc., there are no analytical expressions). Thus, a good way to test the validity of the algorithm (which can be applied regardless the existence of a MRG) is the following: (i) estimate the E(NPV) and its standard deviation using the proposed algorithm; and compare these values with (ii) the E(NPV) and its standard deviation computed analytically.

After Carmichael and Balatbat (2008) we can write

and

where R designates the discount rate (the risk-free rate since the uncertainty in the cash flows is captured via its variance-covariance matrix).

Also, notice that in this case **X** ~ MN(**μ**, **C**), that is, H and F are both identical multivariate normal distributions; thus, the algorithm is simplified as follows, according to Haugh (2004): steps [3] and [4] can be collapsed in one step [3*], namely

[3*] Determine **X**, the desired sample vector, using $x_i = \mu_i + \sigma_i z_i$ (for i=0, 1, …, n).

Using Equations (1) and (2) we obtain that the E(NPV) and its standard deviation are $ 75.72 million and $ 24.84 million respectively. Both figures coincide with those estimated with the Monte Carlo simulation and 300,000 random samples. This fact also provides assurance that 300,000 samples of the vector **X** are appropriate to achieve a reasonable accuracy. Furthermore, the reliability of the estimates obtained with these numbers of draws is validated with a two-tail test for equality of means and variances at a 95% confidence.

Finally, the 95%-confidence interval for the NPV estimated with the simulation, renders [$ 26.89 million, $ 124.26 million], which is very close to the [$ 27.01 million, $ 124.42 million] interval determined analytically. This exercise provides comfort that the Gaussian copula–based simulation provides reliable estimates.

<u>Additional metrics: IRR and PBP</u>

Table 2 displays the results of the simulation to estimate the IRR distribution (top panel). The departure from normality is almost unnoticeable (skewness and kurtosis equal to 0.03 and 3.07 respectively; recall that in the case of a normal curve these values are 0 and 3, respectively). This is consistent with the fact that the IRR confidence interval estimated assuming a normal distribution—and relying on the mean and standard deviation provided by the simulation--is almost identical to the interval estimated directly from the simulation (Table 2, second panel).

Table 3 shows the results of the simulation to estimate the PBP. The departure from normality is a bit more noticeable, but not enough to have an important effect on the computation of the confidence interval: both approaches render similar values.

<u>Simulation incorporating a MRG</u>

We introduce now a modification, a government subsidy in the form of a MRG: $ 24 million, during the operation phase. Thus, if during the operation phase the cash flows, at any time, fall below $ 24 million, the government makes up the difference. In essence, the effect of the government intervention is to change the distribution of the cash flows x_i (i=4, …, 16) from normal to "censored" normal distributions. In these instances, the corresponding probability density functions exhibit a discontinuity at x_i = \$ 24 million, to account for the probability mass concentration on the left side of the distribution. Table 4 shows the project cash flows (mean and standard deviations) after taking into account the government support.

Therefore, we now carry out the simulation assuming that the cash flows follow: (i) during phase A, normal marginal distributions with means and standard deviations according to Table 4 (that is, identical to the previous--no MRG-- case); and (ii) during phase B, "censored" normal marginal distributions: that is, normal distributions specified by μ_i and σ_i (i=4, …, 16), from the previous case (Table 1) but cutoff at x_i= 24, which, in turn, results in distributions having the means and standard deviations shown in Table 4. Obviously, the effect of the MRG results in higher means and lower standard deviations for the Phase B-cash flows.

We employ, as before, 300,000 samples of the vector **X**.

NPV, IRR and PBP considering the MRG

The E(NPV) and St-Dev(NPV), as stated before, can be computed directly invoking Equations (1) and (2) and the data from Table 4. This yields, E(NPV) = \$ 101.86 million and St-Dev(NPV) = \$ 15.97 million. These figures are consistent with those obtained from the Monte Carlo simulation (another validation). Specifically, the Monte Carlo yields the same value for the E(NPV) and a very close approximation for its standard deviation (\$ 15.57 million). The reliability of the estimates is validated, again, with a two-tail test for equality of means and variances at a 95% confidence.

A 95%-confidence interval for the NPV estimated from the simulation gives [\$ 74.77 million, \$ 135.44 million]. Strictly speaking, the NPV does not follow a normal distribution; however, the central limit theorem suggests that in this case the normality assumption might render a good approximation. And this is indeed the case: a confidence interval obtained assuming normality yields [\$ 70.55 million, \$ 133.17 million].

Finally, a sensitivity analysis to investigate the response of the St-Dev(NPV) to variations in λ_B, ρ and R indicates that the St-Dev(NPV) is most sensitive to changes in λ_B, then R, and then ρ. Specifically, a 10% change in any of these parameters (holding the others constant) results in a 8.13%, -4.16 and 0.99% change in the value of St-Dev(NPV) respectively.

Tables 5 and 6 display the results of the simulation for the IRR and PBP distributions. As expected, the government subsidy improves all the figures of merit related to the project (higher NPV, higher IRR, and lower PBP) and reduces their variability.

Overall, the results also show that, (i) changes in ρ are less important than changes in λ_B ; and (ii) the departures from normality--all mild-- are not sufficient to impair the accuracy of confidence intervals estimates based on such assumption

Discussion of Results

It is not appropriate to derive sweeping conclusions from the example presented. Nevertheless, there are two encouraging observations. First, it seems that errors in estimating the correlation values are less influential than errors in the standard deviation of the cash flows in terms of the relevant metrics. This is important because in general analysts are much more likely to have a better grasp of the margin of error associated with estimating the project cash flows than the inter-temporal correlations. Hence, uncertainty about the "true" correlation values should not deter practitioners from using the probabilistic approach to valuation.

And second, even though the relevant metrics do not follow a normal distribution (the exception is the NPV in the case all cash flows are normal) confidence interval estimates assuming normality—that is, based on the mean and standard deviation estimated from the Monte Carlo simulation--- are satisfactory for all practical purposes. We hope future researchers will explore further the validity of these claims further.

Conclusions

We have presented an easy-to-implement algorithm to carry out valuations of projects with MRG. Our approach relies on two assumptions. First, we assume that we have a probabilistic characterization of the cash flows; and second, we assume that we have an estimate of the inter-temporal correlation structure of the cash flows. This approach makes no assumptions in terms of the cash flows marginal distributions and the structure of the correlation matrix. Hence, it is much more flexible than simulations techniques based on simple—but restrictive--recursive stochastic formulas such as Brownian motion or diffusion processes.

The reliability of the method has been validated by making reference to the expected value and standard deviation of the NPV (which can be obtained analytically). It has been shown that the values obtained with the Gaussian copula-based simulation and the analytical formulas are very close. On the other hand, since the distribution of the NPV as well as all the other relevant metrics (IRR, PBP, etc) resist a closed form solution this approach is a valuable tool.

Therefore, this method offers great potential to explore highly specialized problems where the cash flows might depart from normality, or follow other types of "censored" distributions. Some interesting cases that come to mind are projects where the cash flows are subject to a cap (due to a saturation or limit-capacity effect); cases where the MRG formula itself could have an upper bound; or cases in which the MRG only applies to certain time-periods. All these cases are impossible to handle analytically. And they are extremely difficult to cast in terms of real options techniques.

Finally, we think this approach should be taken as an invitation to explore in more depth, different ways to characterize probabilistically the cash flows that are likely to arise in different types of construction projects.

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Table 1 Project Cash Flows, no MRG

Year	Phase	Cash Flow (US$ Millions)
0	Construction	-7.5
1	Construction	-18.9
2	Construction	-42.5
3	Construction	-51.1
4	Operation	25.4
5	Operation	25.4
6	Operation	25.4
7	Operation	24.6
8	Operation	24.6
9	Operation	24.6
10	Operation	24.6
11	Operation	24.6
12	Operation	23.8
13	Operation	23.8
14	Operation	23.8
15	Operation	23.8
16	Operation	28.0

Table 2 Simulation Results (IRR), no MRG

	Metric	IRR (%)
	Mean	15.38
	Std-Dev	2.99
	Skewness	0.03
	Kurtosis	3.07
95% Confidence Interval		
	Lower Bound	**Upper Bound**
Normal	9.51	21.25
Empirical	9.52	21.30

Table 3 Simulation Results (PBP), no MRG

	Metric	PBP (years)
	Mean	8.89
	Std-Dev	1.07
	Skewness	0.65
	Kurtosis	3.742
95% Confidence Interval		
	Lower Bound	**Upper Bound**
Normal	6.78	11.00
Empirical	7.10	11.31

Table 4 Project Cash Flows with MRG

Year	Phase	Mean Cash Flow (US$ Millions)	St. Dev. of Cash Flow (US$ Millions)
0	Construction	-7.5	0.8
1	Construction	-18.9	1.9
2	Construction	-42.5	4.3
3	Construction	-51.1	5.1
4	Operation	28.8	6.4
5	Operation	28.8	6.4
6	Operation	28.8	6.4
7	Operation	28.2	5.9
8	Operation	28.2	5.9
9	Operation	28.2	5.9
10	Operation	28.2	5.9
11	Operation	28.2	5.9
12	Operation	27.7	5.5
13	Operation	27.7	5.5
14	Operation	27.7	5.5
15	Operation	27.7	5.5
16	Operation	30.8	7.9

Table 5 Simulation Results (IRR), with MRG

	Metric	IRR (%)		
	Mean	18.02		
	Std-Dev	1.97		
	Skewness	0.52		
	Kurtosis	3.42		
95% Confidence Interval				
	Lower Bound	**Upper Bound**		
Normal	14.14	21.89		
Empirical	14.65	22.36		
Sensitivity Analysis (% Variation)				
	E(IRR)	**Std-Dev(IRR)**	**Skewness**	**Kurtosis**
ρ (10% Up)	0.00%	0.75%	1.70%	0.39%
λ_B (10% Up)	1.56%	6.41%	5.66%	1.24%

Table 6 Simulation Results (PBP), with MRG

	Metric	PBP (years)		
	Mean	8.22		
	Std-Dev	0.54		
	Skewness	-0.31		
	Kurtosis	2.88		
95% Confidence Interval				
	Lower Bound	**Upper Bound**		
Normal	7.15	9.28		
Empirical	7.06	9.17		
Sensitivity Analysis (% Variation)				
	E(PBP)	**Std-Dev(PBP)**	**Skewness**	**Kurtosis**
ρ (10% Up)	0.01%	0.74%	2.80%|	-0.05%
λ_B (10% Up)	-0.65%	5.11%	1.77%	-1.24%

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