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Research Note

On Managerially Efficient Experimental Designs

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In most marketing experiments, managerial decisions are not based directly on the estimates of the parameters but rather on functions of these estimates. For example, many managerial decisions are driven by whether or not a feature is valued more than the price the consumer will be asked to pay. In other cases, some managerial decisions are weighed more heavily than others. The standard measures used to evaluate experimental designs (e.g., A-efficiency or D-efficiency) do not accommodate these phenomena. We propose alternative "managerial efficiency" criteria (M-errors) that are relatively easy to implement. We explore their properties, suggest practical algorithms to decrease errors, and provide illustrative examples. Realistic examples suggest improvements of as much as 30% in managerial efficiency. We close by considering approximations for nonlinear criteria and extensions to choice-based experiments.

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1. Motivation

Consider the following three stylized examples.[1]

(1) A manufacturer of consumer electronic devices is considering five features that might be added to the device and is planning a conjoint analysis to determine whether the value of each feature to the consumer is greater than the price that must be charged based on the marginal cost of providing each feature.

(2) A retailer is considering seven changes in its store layout and wants to design an experiment to evaluate the potential changes. This experiment would be carried out across 50 stores in an experimental region. Three of those changes are fundamental and would change completely the layout of the store, while four are less critical and are easily reversible. Furthermore, store-layout constraints are such that the fundamental changes can be implemented only in pairs.

(3) A manager of a chain of fast-food restaurants would like to test a new sales training program and a focus on product bundles in display (both binary variables). The manager controls two additional binary variables (sign types and interior décor). The restaurants are spread around four independent districts that are homogeneous within but heterogeneous between with respect to sales and response to managerial actions. The manager is focused primarily on the effects of the first two variables and their interaction. The other variables are of minor interest, unless they interact with the first two. The manager is faced with the task of deciding how to design an efficient experiment to determine which initiatives to implement and where. Each observation is costly, and the stakes are high: there is not only the cost of the initiatives but the potential loss in sales if an initiative fails.

These examples illustrate two related issues. Some managerial decisions are based on combinations of the partworth estimates (feature value versus price; layout changes occurring in pairs) and some managerial decisions are more critical than others (focus on sales training and display bundles). The electronic devices manufacturer is concerned with the partworths of the features *relative* to the partworths of price, and the retailer is most concerned with three

[1] The first two examples were inspired by recent applications. The third was suggested by the area editor.

pairs of partworths representing the three critical features taken two at a time. Precision on individual partworths matters less to these managers. The fast-food manager is focused on individual partworths (and their interactions), but this manager wants more precision on some partworths (and interactions) than other partworths. Other practical examples include magazine cover design (as in *Conde Nast's* Web-based system), services packages (as offered by Comcast, DirecTV, the Dish Network, or Time-Warner Cable), feature packages (as offered by General Motors, Ford, and others), advertising copy design, and the design of panels for sales forecasting (Sinha et al. 2005).

In this paper we illustrate that the standard measures of efficiency, used to evaluate conjoint analysis designs and other experimental designs in marketing, can be modified to take both issues of managerial relevance into account. We introduce revised error criteria, which we call managerial efficiency or *M*-efficiency, and suggest that researchers should seek designs that are *M*-efficient rather than efficient by standard measures (*D*-efficiency, *A*-efficiency, or *G*-efficiency—defined below). We examine properties of various *M*-efficiency criteria and suggest algorithms (some existing) that can be used to improve *M*-efficiency. We begin with a brief review of efficiency criteria.

For the purpose of this paper we consider designs that are not adapted for each individual as in adaptive conjoint analysis (ACA) or polyhedral methods, although *M*-efficiency criteria can be used, post hoc, to evaluate the adapted experimental designs. We begin with metric data and generalize our analysis to choice data in §7.

2. Classical and Bayesian Definitions of Efficiency

Researchers (and managers) usually seek experimental designs such that the estimates of the magnitude of the experimental treatments, e.g., partworths in conjoint analysis, have the smallest possible variance. This goal is achieved by the use of designs that are orthogonal (i.e., the parameter estimates are uncorrelated) and balanced (i.e., each level occurs equally often within each factor). However, such designs are not always feasible or cost effective (Addelman 1962, Kuhfeld et al. 1994). In response, researchers have defined several measures of efficiency by which to evaluate designs. These measures are known as *A*-efficiency, *D*-efficiency, and *G*-efficiency, with the two most common being *A*- and *D*-efficiency (Bunch et al. 1994, Kuhfeld et al. 1994, Huber and Zwerina 1996, Arora and Huber 2001). These measures have three characteristics in common. First, if a balanced and orthogonal design exists, it has maximum efficiency. Second, efficiency is maximized if the corresponding type of error is minimized (*A*-, *D*-, or *G*-error). Third, that error is proportional to a norm defined on the covariance matrix of the estimates of the partworths (or experimental treatments). *A*-errors are monotonically increasing in the trace of the covariance matrix, *D*-errors in the determinant of the matrix, and *G*-errors in the maximum diagonal element. For metric data, *A*- and *D*-errors are defined as

$$A\text{-error} = q\,\text{trace}((X'X)^{-1})/n,$$
$$D\text{-error} = q\,\det((X'X)^{-1})^{1/n}.$$

In these definitions, X is the (suitably coded) experimental design matrix, q is the number of questions, and n is the number of parameters.

As we move to managerial relevance it is important to note that each of the so-called "alphabetical optimality" criteria can be justified with Bayesian loss functions (Chaloner and Verdinelli 1995). For example, if priors are uninformative, *A*-efficiency minimizes expected posterior squared-error loss. *D*-efficiency maximizes the expected gain in Shannon's information measure from the prior to the posterior—the same criterion used in the U^2 statistic that is common in logit analyses. If priors are informative, we replace $X'X$ with $X'X+R$, where $\sigma^2 R^{-1}$ is the prior covariance matrix and σ^2 is the measurement error variance. For ease of exposition we assume uninformative priors, commenting on the impact of R where relevant.

3. A Numerical Example

Consider a metric conjoint analysis study, as in the electronic devices example (first example in §1), with five binary attributes plus price. The appendix provides an example of a balanced and orthogonal experimental design (the first column corresponds to the intercept) and the corresponding covariance matrix of the parameters. This design minimizes *A*-errors (as well as *D*-errors and *G*-errors).

Now consider the managerial decisions that must be made. Assume that the cost of each feature is \$16.50 and that the difference between the low and high levels of price is \$50. In this numerical example the manufacturer will include a feature in the device if the consumer values a feature more than a price reduction of \$16.50. Let $u_1, u_2, \ldots, u_5$ represent the partworths (utilities) for the five features, let u_p represent the partworth of a \$50 price reduction, and let C be an intercept in the estimation.

For suitably scaled partworths, the managerially relevant willingness-to-pay (WTP) criterion is equivalent to a focus on linear combinations of the partworths: $m_1 = u_1 - 0.33u_p$, $m_2 = u_2 - 0.33u_p, \ldots, m_5 =$

$u_5 - 0.33u_p$.[2] We rewrite these goals in matrix form as follows:

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 1 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0.33 \\ 0 & 0 & 0 & 0 & 1 & 0 & -0.33 \\ 0 & 0 & 0 & 0 & 0 & 1 & -0.33 \end{bmatrix} \cdot \begin{bmatrix} C \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_p \end{bmatrix}$$

$$= M_{\text{WTP}} \cdot \begin{bmatrix} C \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_p \end{bmatrix}.$$

The covariance matrix of the estimates of managerial interest is proportional to $\Sigma^M = M_{\text{WTP}}(X'X)^{-1}M'_{\text{WTP}}$ (Judge et al. 1985, p. 57).[3] As shown in the appendix, the average variance of the estimates of the managerial quantities is approximately 10% higher than the average variance of the estimates of the partworths. Moreover, even though the estimates of the parameters are uncorrelated, the estimates of managerial interest are not. Minimizing *A*-errors (or *D*-errors) ensures that the estimates of the partworths have the highest possible precision; it does not ensure that the estimates of managerial interest have the highest possible precision.

In some managerial situations a 10% change in managerial precision is minor and traditional "alphabetical optimization" is sufficient. In other situations where the value of the correct decision is high or sampling costs are high, this change in precision can be important. For example, we have been involved in specialized pharmaceutical studies where respondent incentives are \$250 per respondent and more than 50 respondents must be screened to find a respondent in the target category. The total cost per respondent is close to \$450.[4] With a typical target sample of 300 respondents, a 10% change in efficiency is worth approximately \$13,500; a 30% gain, as in one of our illustrative examples, is \$40,500. Over many studies, such costs are substantial.

4. Defining *M*-Efficiency

We begin by generalizing standard efficiency measures and then examining their relationship to Bayesian loss functions. By defining errors with respect to a norm on Σ^M, we obtain the following measures of managerial efficiency for suitably coded X matrices (we denote the number of estimates of managerial interest by n_M):[5]

$$M_A\text{-error} = q\,\text{trace}(M(X'X)^{-1}M')/n_M,$$
$$M_D\text{-error} = q\,\det(M(X'X)^{-1}M')^{1/n_M}.$$

M_A- and M_D-errors shift focus by considering the covariance matrix of the estimates of managerial interest rather than that of the partworths. The concepts of orthogonality and balance may be generalized as well: We say that a design is *M*-orthogonal if $M(X'X)^{-1}M'$ is diagonal, and *M*-balanced if all the diagonal elements of this matrix are equal. Intuitively, a design is *M*-orthogonal (respectively, *M*-orthogonal and *M*-balanced) if it is orthogonal (respectively, orthogonal and balanced) *in the managerial quantities*.

It is important to note that, unlike *A*- and *D*-errors, M_A- and M_D-errors are not minimized by designs that are *M*-orthogonal and *M*-balanced. Recall that for *A*- and *D*-errors, "if a balanced and orthogonal design exists, then it has optimum efficiency" (Kuhfeld et al. 1994, p. 546). This does not generalize to M_A-errors or M_D-errors. In particular, a design can be *M*-orthogonal and *M*-balanced but not maximally M_A-efficient or M_D-efficient. We provide an example in a technical appendix at http://mktsci.pubs.informs.org (on the *Marketing Science* website). Kuhfeld et al.'s (1994,

[2] WTP is the amount the consumer is willing to pay for the feature, calculated as the price change that is needed to "buy" the utility of the feature. If the utilities of no features and base price are scaled to zero, the criterion becomes $u_j(\$50/u_p)$. WTP > \$16.50 requires that $u_j(\$50/u_p) > \$16.50$, which is equivalent to $u_j - 0.33u_p > 0$. Working with the difference of two random variables is easier than their ratio—the ratio of two noncentral normal variates. Using differences also likely reduces variance. Empirical evidence suggests that the two measures (ratio and difference) are highly correlated and that differences fit consumer choices slightly better than ratios (Hauser and Urban 1986).

[3] We restrict ourselves to full rank designs, X, where all partworths are estimable. Efficiency measures can be defined for singular X matrices, but many of the difficulties of such definitions are beyond the scope of this paper. See Sibson (1972).

[4] Cost estimates from market research professionals. Because efficiency is proportional to a norm on variance, a 10% change in efficiency implies a reciprocal change in sample size necessary to obtain the same precision.

[5] The statistics literature has examined linear weightings of the covariance matrix. It is easy to demonstrate that M_D-errors are a version of the general D_A-optimality criterion introduced by Sibson (1972), who examines a variety of duality theorems with respect to D_A-optimality. We are unaware of any subsequent applications. See also summary in Silvey (1980). Chaloner and Verdinelli (1995) allow covariance weighting for Bayes *A*-optimality but do not tie their *A* matrix to the managerial focus implied by *M*.

pp. 548–549) caution for A- and D-errors is even more relevant for M_A- and M_D-errors: "Preserving orthogonality at all costs can lead to decreased efficiency. Orthogonality was extremely important in the days before general linear model software became widely available. Today, it is more important to consider efficiency when choosing a design." Despite this caveat, M-orthogonality and M-balance remain attractive properties and usually tend to decrease M_A- and M_D-errors.

Defining efficiency with respect to M addresses our first goal, enabling managers to focus on combinations of partworths rather than the partworths themselves. However, M_A- and M_D-errors, like A- and D-errors, do not explicitly enable managers to place different weights on different decisions. As defined, M_A-error implicitly weighs the precision of each combination equally.[6] M_D-error weighs the precisions equally and, because the determinant is the product of the eigenvalues of the managerial covariance matrix, tends to favor equal precisions among the combinations. We allow differential weighting by defining M_1-errors, a generalization of M_A-errors. For brevity and focus, for the remainder of this paper, we focus on M_A- and M_1-errors, leaving generalizations to M_D-errors to other researchers.[7,8]

We define weights, $\{w_i\}_{i\in\{1,\ldots,n_M\}}$, that allow the manager to accept less precision on the estimates of some managerial quantities in exchange for more precision on others. Let W be the diagonal matrix such that $W_{ii}=w_i$. We define M_1-errors as follows:

$$M_1\text{-error} = q\frac{\text{trace}(WM(X'X)^{-1}M')}{\text{trace}(W)} = q\frac{\sum_i^{n_M} w_i\cdot\sigma_{ii}^M}{\sum_i^{n_M} w_i},$$

where σ_{ii}^M is the variance of the estimate of the ith managerial partworth combination. M_A-error is a special case of M_1-error where all the weights are equal.

We leave the choice of the weights $\{w_i\}_{i\in\{1,\ldots,n_M\}}$ to the manager or researcher, potentially adding a layer of complexity to the application of M-efficiency. Simulations (below) suggest that changing the focus from individual partworths to combinations of partworths has a larger impact on efficiency than allowing differential weights for the precisions of these combinations. Thus, the selection of weights might be less critical managerially than the selection of combinations of partworths on which to focus.

Like A- and D-errors, the M_A- and M_1-error criteria might be justified with Bayesian loss functions (Chaloner and Verdinelli 1995). Let us assume that our objective is to minimize a measure of estimation error captured by the following quadratic loss function:

$$\begin{aligned} E\left(\sum_{i=1}^{n_M} w_i[(M\vec{u})_i-(M\hat{u})_i]^2\right) &= E[(M\vec{u}-M\hat{u})'W(M\vec{u}-M\hat{u})]\\ &= E[(\vec{u}-\hat{u})'M'WM(\vec{u}-\hat{u})], \end{aligned}$$

where $\vec{u}$ is the vector of partworths, $\hat{u}$ is an estimator for $\vec{u}$, and the expectation is over a prior distribution on $\vec{u}$ and the respondent's possible answers. If we assume a diffuse prior on $\vec{u}$, this loss function is proportional to $\text{trace}(M'WM(X'X)^{-1}) = \text{trace}(WM(X'X)^{-1}M')$, i.e., it is proportional to M_1-error (M_A-error is obtained if $W=I$). For informative priors, we replace $X'X$ with $X'X+R$.

5. Algorithms to Generate M-Efficient Designs

Most existing algorithms that minimize A- or D-errors are discrete optimization methods that can be adapted readily to the managerial criteria. Researchers can develop M-efficient designs with Dykstra's (1971) sequential search method, Mitchell and Miller's (1970) simple exchange algorithm, Mitchell's (1974) DETMAX algorithm, or Cook and Nachtsheim's (1980) modified Federov algorithm (see Kuhfeld et al. 1994 or Kuhfeld 2005 for a review). We illustrate the generation of M-efficient designs using the modified Federov algorithm. This algorithm starts with a design X^0, and, for each row i in sequence (starting with $i=1$), computes whether efficiency can be improved by replacing X_i with each possible row not currently in the design. If more than one exchange increases efficiency, the exchange that leads to the greatest improvement is performed. The algorithm iterates until no further improvement is possible.

M_A-Errors

We first examine the improvements possible for M matrices that focus precision on managerial issues (M_A-errors). We applied the algorithm to generate M_A-efficient designs corresponding to 1,000 randomly

[6] However, one can implicitly achieve differential focus by rescaling the rows of M. Our definition of M_1-errors, below, makes differential weighting explicit and should make it easier for managers to specify differential weighting.

[7] For example, M_{1D}-errors might be based on a weighted sum of the logarithms of the eigenvalues of $M(X'X)^{-1}M'$. The algorithms we explore for M_1-errors are extendable to such definitions of M_{1D}-errors. However, it might be difficult to tie this definition to an easy-to-define loss function.

[8] We note also that, if M is square and full rank, a design X that minimizes D-errors also minimizes M_D-errors because $\det(\Sigma_M) = \det(M(X'X)^{-1}M') = \det(M)\det((X'X)^{-1})\det(M')$. Because M is fixed, this last expression is minimized if $\det((X'X)^{-1})$ is minimized. Fortunately, the relationship between M_A-errors (or M_1-errors) and A-errors is more interesting, even if M is square and full-rank.

generated square M matrices.[9] The M_A-error of the final design was, on average, 9.1% lower than the M_A-error of the initial orthogonal design (85.7% of the improvements were between 5% and 15%).

We improve the algorithm's performance slightly by modifying its starting point. Note that any design proportional to XM is M-orthogonal and M-balanced if X is balanced and orthogonal ($X'X = qI$). Specifically, $M(M'X'XM)^{-1}M' = MM^{-1}qI(M')^{-1}M' = qI$. Intuitively, right multiplying X by M makes the design orthogonal and balanced in the estimates of managerial interest, rather than the initial parameters. If XM has noninteger elements, we round the elements to the closest "admissible" integers or combinations of integers. Using XM as a starting solution and applying the modified Federov algorithm to 1,000 randomly generated M matrices as above, the average improvement is 9.2% relative to the orthogonal design X—small but significant improvements relative to starting with X ($p < 0.05$).

It is interesting that the design XM alone (without applying the modified Federov algorithm) might decrease M_A-errors relative to X, as illustrated in the appendix. Let X and M_{WTP} be as in §3 (see also the appendix). Because in this case there are fewer managerial issues than partworths, we obtain a full-rank square matrix M^+_{WTP} by augmenting M_{WTP} (we select the extra rows such that all entries of XM^+_{WTP} are between -1 and 1 and all integrality constraints are satisfied).[10] The new design X_{WTP} has an M_A-error equal to the (optimal) A-error of the original design X ($M_{\text{WTP}}(X'_{\text{WTP}}X_{\text{WTP}})^{-1}M'_{\text{WTP}}$ versus $(X'X)^{-1}$—see the appendix). This M_A-error is 9.8% lower than for X ($M_{\text{WTP}}(X'_{\text{WTP}}X_{\text{WTP}})^{-1}M'_{\text{WTP}}$ versus $M_{\text{WTP}}(X'X)^{-1}M'_{\text{WTP}}$—see the appendix). Qualitatively, relative to the original design X, the improved design X_{WTP} does not set price to its extreme levels, but rather closer to the "cost" of providing features (as embedded in M_{WTP}). Doing so increases the variance of the price partworth, but lowers the variance of the estimates of managerial interest. This *managerial* focus contradicts common wisdom of setting continuous factors to their extreme levels to minimize A- (or D-) errors.[11]

M_1-errors

We next explore the improvements possible for M matrices that focus on managerial issues *and* W matrices that imply that some managerial issues are more important than others. We randomly generated 1,000 square M matrices as above and, in addition, generated a W matrix for each M. The w_is were drawn from an i.i.d. uniform distribution on $[0, 1]$. We applied the modified Federov algorithm (using XM to generate a starting point). To compare the impact of focus (M) versus importance (W) we also generated equally weighted designs (generated under the assumption that $W = I$). We evaluate M_1-error using the "correct" weights, W.

Using M_1-error and the "correct" W, the algorithm produced a design that was, on average, 14.4% better than the orthogonal design (A-efficient design). Using the algorithm with an "incorrect" set of managerial weights, $W = I$, but evaluating the design using the "correct" weights W, the algorithm produced a design that was still better than the orthogonal design in 95.6% of the cases, the average improvement being 8.9%.[12] These simulations suggest that focusing precision on the managerial quantities has a greater impact than weighting these managerial quantities. It appears that M_1-errors might be moderately robust with respect to the manager's choice of weights.

Illustrative Example—Fast-Food Manager

We further illustrate the use of M-efficiency for a manager of a chain of fast-food restaurants, who is interested in four factors and four districts (third example from §1):

- x_1 (binary): existence of a training program
- x_2 (binary): focus on product bundles in display ("display bundles" for short)
- x_3 (binary): sign type
- x_4 (binary): interior décor
- x_5 (four levels): district fixed effect

We consider two related problems. In the first problem the manager is more interested in some effects than others (W). In the second, the manager also prefers to focus on combinations of factors (M and W). We highlight the approach and results here and refer the reader to the technical appendix for details.

[9] The row and column of M corresponding to the intercept were set to 0, except for the first element of the first row, which was set to 1. All other elements of M were independent and identically distributed (i.i.d.) as uniform variates between -1 and $+1$. We re-drew the matrix M until its conditioning number was less than 30. (If we impose no constraint on the conditioning number, the average improvement is 15.6%.) The starting point of the algorithm was the orthogonal design X from the appendix.

[10] Although in the previous simulations integrality constraints applied to all columns of all design matrices (and were enforced by rounding the elements of XM), in this example the last column corresponds to price and might take continuous values between -1 and $+1$.

[11] One exception to common wisdom is Kanninen (2002), who reduced D-errors for *choice-based* designs by dropping an integrality constraint and setting one feature to a nonextreme level.

[12] Recall that this 8.9% measures improvements in M_1-errors. The earlier 9.2% was improvements in M_A-errors.

Different W. Suppose the manager wishes to examine the effects of x_1 and x_2 and their interactions with the other factors and the districts: $x_1 * x_2$, $x_2 * x_3$, $x_2 * x_4$, $x_1 * x_5$, $x_2 * x_5$, and $x_2 * x_3 * x_4$. This represents the manager's judgment that the training program might interact with display bundles, that display bundles might interact with sign type and interior décor (with a potential three-way interaction between these three variables), and that both the training program and display bundles might interact with districts. In that case, the effect of the M matrix is only to select a subset of all possible effects. We represent differential focus with weights on the precision of the more important managerial quantities: the parameters associated with x_1, x_2, $x_1 * x_2$ are three times the weights of the other nine interactions.

Let us assume that the number of restaurants to be included in the experiment is 24, a number that favors traditional measures of efficiency. We used the %Mktex macro in SAS to generate a D-efficient design that allows the estimation of the interactions of interest (Kuhfeld 2005).[13] We then used the modified Federov algorithm to generate an M_1-efficient design. The algorithm achieved a 5.1% improvement in M_1-errors (compared to the D-efficient design).[14]

Different M and W. For the second problem we assume that, due to budget constraints, the manager can implement at most one of the main factors (training program or display bundles) and that he or she plans to set the sign type and interior décor to their (known) more desirable levels. In this case, the managerial quantities of interest are the effects on sales of three alternative feature combinations (no change, training program only, display bundles only). Because of potential interactions with the four districts we obtain 12 managerial quantities. As a further illustration we assume that more restaurants are located in the first district, such that the precisions of the corresponding estimates are three times more important to the manager. These assumptions imply a different focus *and* a different set of weights. Applying the modified Federov algorithm now provides a design that reduces M_1-errors by 30.5% (relative to the D-efficient design). These reductions are consistent with our simulations; that is, a larger improvement if we change focus as well as weight. The larger decrease is larger than average but within the range of the simulations.

[13] D-efficiency in this example includes main effects as well as the interactions of interest. We compare the M-efficient design to a D-efficient and not an A-efficient design because the %Mktex macro does not work with A-efficiency. Note however that M_D-errors are also lower for the M_1-efficient design compared to the D-efficient design.

[14] We ran the algorithm with five different random starting points and retained the most M_1-efficient design. This was necessary because the D-efficient design was locally optimal.

6. Generalization to Nonlinear Managerial Quantities

Linear combinations of partworths (including interactions) can be used to represent many important managerial decisions, but some managers might wish to model nonlinear functions directly, such as profitability or market share.[15] Suppose that the managerial quantities are given by $m_1 = f_1(u_1, \ldots, u_n)$, $m_2 = f_2(u_1, \ldots, u_n), \ldots, m_{n_M} = f_{n_M}(u_1, \ldots, u_n)$ and assume that the manager has prior beliefs on the value of $\vec{u}$. Such priors, used frequently in choice-based conjoint analysis, can be obtained through pretests (Huber and Zwerina 1996, Arora and Huber 2001) or interviews with managers (Sandor and Wedel 2001). They may be captured by point estimates (Huber and Zwerina 1996, Arora and Huber 2001) or by continuous subjective probability distributions (Sandor and Wedel 2001). In theory, one can define loss functions with respect to the m_is, specify full prior distributions, model the data likelihood, and obtain a design to minimize posterior loss. However, "approximations must typically be used because the exact expected utility is often a complicated integral" (Chaloner and Verdinelli 1995, p. 284). We suggest a practical way to approximate M-efficiency for such functions.[16]

Consider first a point prior estimate $\vec{u}^0$ on $\vec{u}$. Using a first-order Taylor's series expansion, we have

$$\begin{bmatrix} m_1 \\ m_2 \\ \cdots \\ m_{n_M} \end{bmatrix} \sim \begin{bmatrix} f_1(u_1^0, u_2^0, \ldots, u_n^0) \\ f_2(u_1^0, u_2^0, \ldots, u_n^0) \\ \cdots \\ f_{n_M}(u_1^0, u_2^0, \ldots, u_n^0) \end{bmatrix} + M^{u^0} \begin{bmatrix} (u_1 - u_1^0) \\ (u_2 - u_2^0) \\ \cdots \\ (u_n - u_n^0) \end{bmatrix}$$
$$+ \text{higher-order terms},$$

where the ijth element of M^{u^0} is $(\partial f_i / \partial u_j)(\vec{u}^0)$. The approach used in this paper can then be applied using $M_A^{u^0}$- or $M_1^{u^0}$-errors. In the case of a prior described by a continuous distribution g, M_A- and M_1-errors may be defined as

$$M_A\text{-error} = \int_{\Re^n} (M_A^{u^0}\text{-error}) g(u^0)\, du^0$$

$$M_1\text{-error} = \int_{\Re^n} (M_1^{u^0}\text{-error}) g(u^0)\, du^0.$$

7. Generalization to Choice-Based Conjoint Designs

In choice-based conjoint designs the covariance matrix, Ω^{-1}, is given by: $\Omega^{-1} = (Z'PZ)^{-1}$, where Z is a probability-centered design matrix and P is a

[15] We thank the area editor for suggesting this extension.

[16] In contrast, the Bayesian experimental design literature typically approaches nonlinear problems by maximizing the expected Fisher information matrix.

diagonal matrix of choice probabilities. A-, D-, and G-errors are then defined with respect to Ω^{-1} (Huber and Zwerina 1996, p. 308; Arora and Huber 2001, p. 274). For example, we have[17]

$$D\text{-error(choice)} = \det((Z'PZ)^{-1})^{1/n},$$
$$A\text{-error(choice)} = \text{trace}((Z'PZ)^{-1})/n.$$

Following the arguments in this paper, the asymptotic covariance matrix of the managerial quantities is $\Sigma^M_{\text{choice}} = M(Z'PZ)^{-1}M'$, and we define M_A- and M_1-errors analogously. The algorithms employed to derive M-efficient designs generalize readily to choice data. We provide an example in the technical appendix, again using WTP as the estimates of managerial interest. We obtain a D-efficient design from the balanced and orthogonal design X in the appendix by cyclically generating alternatives as in Arora and Huber (2001) and Huber and Zwerina (1996). The M-efficient design obtained from the design X_{WTP} (also in the appendix) improves M_A-errors by 9.8% and M_D-errors by 8.4%.

8. Summary and Future Research

We suggest easy-to-implement criteria to focus experimental designs on estimates of managerial interest when (1) managerial decisions are based on combinations of conjoint partworths or experimental treatments, and/or (2) some managerial decisions are more crucial than others. We extend (and improve slightly) existing algorithms to develop experimental designs that minimize M_A- or M_1-errors and obtain improvements of as much as 30%. Our simulations suggest that improvements are greater when the manager seeks to change his or her focus than when the manager seeks to weight criteria differentially. M-efficiency can be generalized to choice-based experiments and can approximate nonlinear managerial criteria.

Future research might further extend, adapt, and generalize traditional measures of efficiency to take into account recent developments in the conjoint analysis literature. For example, efficiency measures could be developed for recently proposed optimization-based estimation methods (Cui and Curry 2005, Evgeniou et al. 2005). Measures of efficiency could be developed to evaluate adaptive design algorithms or to account for the endogeneity of adaptive designs (Hauser and Toubia 2005, Toubia et al. 2003). Finally, efficiency measures could be generalized to account for parameter dynamics (Liechty et al. 2005).

[17] In this definition we follow the literature and drop the constant, q, which does not affect any optimization.

Appendix

Orthogonal and balanced design, X:

$$X = \begin{bmatrix} +1 & +1 & -1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & +1 & +1 & +1 \\ +1 & +1 & +1 & +1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 & +1 & +1 & -1 \\ +1 & +1 & -1 & +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & -1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & -1 & -1 & -1 \\ +1 & +1 & +1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & +1 & -1 \end{bmatrix}$$

Managerial quantities:

$$M_{\text{WTP}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 1 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0.33 \\ 0 & 0 & 0 & 0 & 1 & 0 & -0.33 \\ 0 & 0 & 0 & 0 & 0 & 1 & -0.33 \end{bmatrix}$$

Covariance matrix of the parameter estimates:

$$\Sigma = (X' \cdot X)^{-1} = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0833 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0833 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0833 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0833 \end{bmatrix}$$

Covariance matrix of the managerial estimates under X:

$$\Sigma^M = M_{\text{WTP}}(X' \cdot X)^{-1} \cdot M'_{\text{WTP}} = \begin{bmatrix} 0.0924 & 0.0091 & 0.0091 & 0.0091 & 0.0091 \\ 0.0091 & 0.0924 & 0.0091 & 0.0091 & 0.0091 \\ 0.0091 & 0.0091 & 0.0924 & 0.0091 & 0.0091 \\ 0.0091 & 0.0091 & 0.0091 & 0.0924 & 0.0091 \\ 0.0091 & 0.0091 & 0.0091 & 0.0091 & 0.0924 \end{bmatrix}$$

$$M_A\text{-error}(X) = 1.1089$$

$$M_D\text{-error}(X) = 1.0908$$

Augmented matrix of managerial quantities:

$$M^{+}_{\mathrm{WTP}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 1 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0.33 \\ 0 & 0 & 0 & 0 & 1 & 0 & -0.33 \\ 0 & 0 & 0 & 0 & 0 & 1 & -0.33 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}$$

Managerial design, X_{WTP}:

$$X_{\mathrm{WTP}} = \begin{bmatrix} +1 & +1 & -1 & -1 & -1 & -1 & 0.98 \\ +1 & -1 & +1 & -1 & +1 & +1 & -0.32 \\ +1 & +1 & +1 & +1 & -1 & +1 & -0.98 \\ +1 & -1 & -1 & +1 & +1 & -1 & 0.34 \\ +1 & +1 & +1 & -1 & +1 & +1 & -1.00 \\ +1 & +1 & -1 & +1 & -1 & +1 & -0.32 \\ +1 & +1 & -1 & -1 & +1 & -1 & 0.34 \\ +1 & -1 & -1 & -1 & -1 & +1 & 0.98 \\ +1 & -1 & +1 & -1 & -1 & -1 & 1.00 \\ +1 & -1 & +1 & +1 & -1 & -1 & 0.32 \\ +1 & +1 & +1 & +1 & +1 & -1 & -1.00 \\ +1 & -1 & -1 & +1 & +1 & +1 & -0.34 \end{bmatrix}$$

Covariance matrix of the managerial estimates under X_{WTP}:

$$M_{\mathrm{WTP}} \cdot (X'_{\mathrm{WTP}} \cdot X_{\mathrm{WTP}})^{-1} \cdot M'_{\mathrm{WTP}} = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 \\ 0 & 0.0833 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0 & 0 \\ 0 & 0 & 0 & 0.0833 & 0 \\ 0 & 0 & 0 & 0 & 0.0833 \end{bmatrix}$$

$$M_A\text{-error}(X_{\mathrm{WTP}}) = 1.00$$

$$M_D\text{-error}(X_{\mathrm{WTP}}) = 1.00$$

References

Addelman, S. 1962. Symmetrical and asymmetrical fractional factorial plans. *Technometrics* **4**(February) 47–58.

Arora, N., J. Huber. 2001. Improving parameter estimates and model prediction by aggregate customization in choice experiments. *J. Consumer Res.* **28**(September) 273–283.

Bunch, D. S., J. J. Louviere, D. Anderson. 1994. A comparison of experimental design strategies for multinomial logit models: The case of generic attributes. Working paper, Graduate School of Management, University of California at Davis, Davis, CA.

Chakravarti, I. M. 1956. Fractional replication in asymmetrical factorial designs and partially balanced arrays. *Sankhya* **17** 143–164.

Chaloner, K., I. Verdinelli. 1995. Bayesian experimental design: A review. *Statist. Sci.* **10**(3) 273–304.

Cook, D. R., C. J. Nachtsheim. 1980. A comparison of algorithms for constructing exact *D*-optimal designs. *Technometrics* **22**(August) 315–324.

Cui, D., D. Curry. 2005. Prediction in marketing using the support vector machine. *Marketing Sci.* **24**(4) 595–615.

Dykstra, O. 1971. The augmentation of experimental data to maximize $(X'X)^{-1}$. *Technometrics* **13**(August) 682–688.

Evgeniou, T., C. Boussios, G. Zacharia. 2005. Generalized robust conjoint estimation. *Marketing Sci.* **24**(3) 415–429.

Hauser, J. R., O. Toubia. 2005. The impact of utility balance and endogeneity in conjoint analysis. *Marketing Sci.* **24**(3) 498–507.

Hauser, J. R., G. L. Urban. 1986. Value priority hypotheses for consumer budget plans. *J. Consumer Res.* **12**(4) 446–462.

Huber, J., K. Zwerina. 1996. The importance of utility balance in efficient choice designs. *J. Marketing Res.* **33**(August) 307–317.

Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lutkepohl, T. C. Lee. 1985. *The Theory and Practice of Econometrics*. John Wiley and Sons, New York.

Kanninen, B. J. 2002. Optimal design for multinomial choice experiments. *J. Marketing Res.* **39**(May) 214–227.

Kuhfeld, W. F. 2005. *Marketing Research Methods in SAS*. SAS Institute Inc., Cary, NC. Available at http://support.sas.com/techsup/technote/ts722.pdf.

Kuhfeld, W. F., R. D. Tobias, M. Garratt. 1994. Efficient experimental design with marketing applications. *J. Marketing Res.* **31**(4, November) 545–557.

Liechty, J. C., D. K. H. Fong, W. S. DeSarbo. 2005. Dynamic models incorporating individual heterogeneity: Utility evolution in conjoint analysis. *Marketing Sci.* **24**(2) 285–293.

Mitchell, T. J. 1974. An algorithm for the construction of *D*-optimal experimental designs. *Technometrics* **16**(May) 203–210.

Mitchell, T. J., F. L. Miller, Jr. 1970. Use of design repair to construct designs for special linear models. *Mathematics Division Annual Progress Report* (ORNL–4661). Oak Ridge National Laboratory, Oak Ridge, TN, 130–131.

Sándor, Z., M. Wedel. 2001. Designing conjoint choice experiments using managers' prior beliefs. *J. Marketing Res.* **38**(4) 440–443.

Sibson, R. 1972. D_A-optimality and duality. *Progress in Statistics, Colloquia Mathematica Societatis János Bolyai* **9** 677–692.

Silvey, S. D. 1980. *Optimal Design: An Introduction to the Theory for Parameter Estimation*. Chapman and Hall, New York.

Sinha, A., J. Inman, Y. Wang, J. Park. 2005. Practice prize report: Attribute drivers: A factor analytic choice map approach for understanding choices among SKUs. *Marketing Sci.* **24**(3) 351–359.

Toubia, O., D. I. Simester, J. R. Hauser, E. Dahan. 2003. Fast polyhedral adaptive conjoint estimation. *Marketing Sci.* **22**(3, Summer) 273–303.