Lending Without Access to Collateral
A Theory of Micro-Loan Borrowing Rates*

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Abstract

We develop a model of lending and borrowing in markets where the lender has no access to physical collateral and where the borrower is heavily capital constrained. Our model of micro loans, which incorporates a) the absence of access to physical collateral, b) peer monitoring, c) threat of punishment upon default, and d) costly monitoring by lenders is used to determine the equilibrium borrowing rates. Monitoring by lenders is shown to be critical for an equilibrium to exist in our model if the maturity of the loan is too long. On the other hand, with short maturity loans, excessive monitoring is shown to be counterproductive. Monitoring plays a dual role: on the one hand, monitoring by lenders lowers the borrowing group's ability to divert the loan for non-productive uses, but it increases the administrative costs of the loan; this increases the borrowing rate and consequently the probability of default. The manner in which the loan rates and the range of equilibria depend on the monitoring costs, joint-liability provisions and punishment technology is characterized when the borrowing group optimally chooses the timing of default to maximize the group's value. Increases in the cost of funding of lenders is shown to result in disproportionately larger increases in the borrowing rates, at high rates of interest. Finally, cetaris paribus, an increase in the size of the loan typically leads to higher default probability.

Contents

1 Introduction

There are very large sections of society, especially in poor and developing parts of the world who do not have access to even rudimentary financial services such as bank savings accounts, credit facilities, or insurance. Households in these sections of the society are typically poor and tend to access credit in *informal credit markets.* Such informal credit markets include: a) local money-lenders, b) local shop-keepers, who provide trade credit, c) pawn-brokers, d) payday lenders, and e) **RO**tating **S**avings and **C**redit **AS**sociations (ROSCAS). A number of economists have examined these informal credit markets, and their potential linkages to more formal credit markets. A partial list of such research includes Besley, Coate,and Loury (1993), Braverman, and Guasch (1986), Varghese (2000, 2002), and Caskey (2005). It is well understood that the interest rates in such informal markets tend to be much higher than the borrowing rates that prevail in formal credit markets. Economists have also recognized the possibility that a large fraction of poor households who do not participate in credit markets may actually be credit-worthy. Excluding such borrowers from access to credit could lead to such households being trapped in poverty and such state of affairs would lead to underdevelopment of the economy as a whole. Arguments of this nature can be found in Bannerjee (2003), and Aghion and Bolton (1997), for example.[1]

Micro-loan markets represent one of the more recent developments, which enable poor households to access credit. These are markets where very small (hence micro) loans are extended to poor households. Often, such loans are given only to women, and in groups. Borrowers in these market have no meaningful physical collateral and are heavily credit constrained. Micro-loans are characterized by three essential features: a) the loans are short-term in nature, are relatively small amounts and consummated without physical collateral, but structured with social collateral, b) the loans are extended typically to a group, whose size can range from 5 (in the Grameen

[1] A complete survey of research in informal credit markets and micro-finance is well beyond the scope of our paper. We refer to two excellent sources: 1) Armendariz, and Morduch (2005), and 2) Bolton and Rosenthal (2005).

model) to 20 (in the Self-Help-Groups or SHG), where the group members are *jointly-liable* for default by any member of the group, and c) Loans carry frequent interest payments (weekly in many cases) and carry significant administrative expenses that are incurred in order to ensure this delivery of loans to remote villages and for the collection of payments[2].

To our understanding no formal model has been developed for understanding the determination of borrowing rates in micro loans, and how they depend on these features of the micro-loan contracts[3]. Often, solvent borrowers who successfully pay off the loans in the earlier rounds are awarded additional loans in increasing amounts. This is another powerful incentive for borrowers not to default given their outside borrowing costs are prohibitive.

We motivate our work by first providing an overview of the micro-loan markets in the next section. We give a geographical breakdown of the market, as well as a breakdown across different organizational forms used in the delivery of loans. In addition, we provide estimates of loan rates charged by different lending organizations, their ex-ante assessment of risk, and ex-post default related write-offs.

1.1 Some Evidence On Microloan Markets, Interest Rates, and Defaults

Since 1976, micro-finance and micro-loans have emerged as a sector where poor households are able to accumulate savings and access credit. Additional financial services such as rainfall insurance, livestock insurance, and health insurance are also being provided increasingly through these channels[4]. The focus of our paper is on micro-loans, which are loans of very small amounts that are extended to financially and socially disadvantaged borrowers by institutions, which may be organized as a) banks, b) non-bank financial institutions, c) credit unions and cooperatives, d) rural banks, and e) Non-Government Organizations(NGOs). Table 1 illustrates the size of the

[2] Savita Subramanian.

[3] See *The Economics of Microfinance* (with Beatriz Armendariz), Forthcoming from the MIT Press, 2005, for a full discussion of this area.

[4] See Ananth, Barooah, Ruchismita, and Bhatnagar (2004) for an illuminating discussion on designing a framework for delivering financial services to the poor in India.

micro-loan market as of 2003, based on voluntary reporting of lending institutions to a centralized database maintained by MIX[5].

Table 1: **Micro-Loan Market**

REGION	Gross Loan Portfolio Size in US $	Active Borrowers Number	Per Capita Loan Size	Loan Rates Percent	Write-off Ratio Percent	PAR $\geq$ 30 Days Percent
Africa	1,010,088,380 (12%)	3,154,502 (11%)	320.21	39.84%	3.32%	9.15%
East Asia	1,983,635,418 (23%)	4,103,326 (15%)	483.42	39.10%	4.12%	7.83%
East Europe	1,449,653,047 (17%)	789,936 (3%)	1,835.15	28.86%	0.60%	1.78%
Latin America	2,740,536,803 (31%)	3,231,062 (11%)	848.18	35.82%	2.52%	6.62%
Middle East	208,032,901 (2%)	794,083 (3%)	261.98	34.61%	0.15%	2.60%
South Asia	1,327,858,980 (15%)	16,057,919 (57%)	82.69	20.29%	0.74%	10.44%
TOTAL:	8,719,805,529 (100%)	28,130,828 (100%)				
AVERAGE:				33.09%	1.91%	6.40%

The total size of the micro-loan market covering a little over 28 million borrowers as reported in Table 1 is about $8.7 billion; this is potentially a very serious underestimate of the actual size of the market since many lenders do not report their activities to MIX. Another estimate found in Microcredit Summit (2003) reports that nearly 2500 lending institutions covered a total of 67 million borrowers as of 2002. Note that Latin America and East Asia are the regions that account for more than 50% of the loans, but South Asia accounts for more than 50% of active borrowers. The total number of active borrowers based on voluntarily reported data is in excess of 28 million. Regardless of the estimates, what is clear is that the number of households in need of rudimentary financial services such as loans, savings, and insurance is considerably higher. For example, in India alone, the estimated number of people in need of rudimentary financial services is over 200 million.

[5]MIX was incorporated in June 2002 as a not-for-profit private organization. The MIX (Microfinance Information eXchange) aims to promote information exchange in the microfinance industry.

No systematic evidence of defaults and borrowing rates are available to our knowledge. In this section, we use the MIX data to estimate interest rates on micro-loans, ex-ante assessment of default exposure and ex-post write-offs on loans.

Table 1 also provides estimates of borrowing rates by assuming that the net revenue reported by the institutions as comprising exclusively of loan interest income. This is likely to be a somewhat noisy estimate to the extent that the income may include both principal and interest payments, as well as income from other sources such as investments made by the lending institutions. Hence the estimates of interest rates reported in Table 1 should be interpreted with caution. The ex-ante assessment of default is captured by the Portfolio At Risk (PAR) measure reported by each lending institution. The ex-post measure is captured by the write-offs reported in the data set.

The average figures indicate that the micro-loan rates are in excess of 30%. The rates are significantly lower in South Asia, which is characterized by many borrowers who borrow very small amounts. Morduch (1999) has provided estimates of borrowing rates ranging from 20% in Grameen Bank in India to 55% in Indonesia. Our estimates, which reflect the bias of self-reporting institutions range from 20% to 40%. The rates are much higher in Africa. The rates charged in micro-loans appear to be still well below the rates that local money lenders tend to charge. If we think of local money lenders as the outside option to the borrowers, then micro-loan rates may not look usurious. The portfolio at risk (PAR) is highest in South Asia, but the write-offs are higher in Africa and Latin America.

It is interesting to note that the PAR estimates are systematically higher than the actual write-offs. This suggests that, ex-ante, the lender believes that the default probability is much higher than ex-post observed default probability.

To get an appreciation of the composition of lenders, Table 2 gives a breakdown of the micro-loans across different lenders. Of the 613 lending institutions voluntarily reporting, banks account for more than 50% of the dollar value of loans, accounting for more than 31% of all active borrowers. In sharp contrast, NGOs account for just 17.3% of the dollar value of the loans, but spans more than 46% of all active borrowers. NGOs and non-bank financial institutions have a roughly similar overall coverage globally with very important cross-sectional variations.

Table 2: **Lender Composition**

Lender	Gross Loan Portfolio[6]	Active Borrowers[7]	Per Capita Loan Size	Loan Rates	Write-off Ratio	PAR ≥ 30 Days
	Size in US $	Number		Percent	Percent	Percent
Banks (47)	4,502,900,041 (51.6%)	8,853,649 (31.5%)	509	24.39%	0.51%	9.61%
Cooperatives and Credit (98) Unions	701,205,569 (8.0%)	805,018 (2.9%)	871	30.92%	3.29%	8.58%
Non-bank financial (49) Institutions	1,627,919,395 (18.7%)	1,831,316 (17.2%)	132	37.80%	0.83%	4.71%
NGO (286)	1,509,926,890 (17.3%)	13,074,367 (46.5%)	115	35.75%	2.48%	7.52%
Others (32)	346,857,110 (4.0%)	425,679 (1.5%)	815	37.54%	1.51%	4.26%
Rural bank (15)	30,996,524 (0.4%)	144,947 (0.5%)	214			7.58%
TOTAL:	8,719,805,529 (100%)	28,130,828 (100%)				
AVERAGE:				33.28%	1.72%	7.04%

While banks do form a significant percentage of the dollar value of the loans extended, this sector of borrowers has not been a major part of loan portfolio of commercial banks for the following reasons: first, borrowers in this sector are very poor and are often unable to post physical collateral of any consequence; this implies that borrowers are unable to signal their credit-worthiness to potential lenders. Second, given the economic opportunity set faced by these borrowers, the loan sizes that are demanded by these borrowers are typically very small. The small size of the loan and the presence of numerous borrowers make investment, in additional screening and monitoring efforts, an expensive proposition. This makes micro-loan portfolio unattractive for many big commercial banks. Due to these factors, new micro-lending approaches have developed, which focus on a) contractual arrangements, b) punishment conditional on default, c) peer group efforts to in effect substitute social collateral for physical collateral, and d) partnering with local financial institutions, which may possess informational advantages, and thus may be able to monitor the loans better.

In Table 2, we also provide a breakdown of interest rates and default measures across the different organizational forms. Banks appear to charge the lowest interest rates. The rates charged by others vary, ranging from 30% to 38%.

To summarize, the following facts emerge based on our analysis: first, the micro-loan interest rates are rather high ranging from 30% to nearly 40%. Second, the actual losses as conveyed by the write-off ratios are relatively small ranging from 0.50% to 3.50%. The lenders tend to estimate the portfolio at risk at a much higher level, averaging at around 7%. There is a cross-sectional variation in the borrower size, and default rates depending on the organizational arrangement of the lender.

In our numerical illustrations, we will attempt to calibrate the model so that loan rates roughly match these numbers. Then, we will ask how various terms of the loan contract can be used to bring the loan rates down without significantly increasing the default probabilities. In particular, we find that decreasing monitoring has the most significant impact.

1.2 Contractual Features:

The borrowing rates in micro-loan contracts must depend on the following important factors:

1. Administrative and monitoring expenses: these are needed to deliver the loans at the doorsteps of poor borrowers in rural areas. High administrative expenses, while arguably keep the default rates low, render the borrowing costs very high. Note that these costs are

borne by the lender, and are eventually passed on to the micro-borrower. CGAP(2003)[8] reports that the administrative expenses range from 18.9% in Asia to 38.2% in Africa on the loans. Consequently, micro-loan interest rates are rather high; this should, however, be put in the context of the fact that the other "outside options" for the micro-borrowers are even more expensive. For example, CGAP (2003) reports money-lenders charging anywhere from 10% per month. In Philippines, the estimated daily interest rates on loans made by local money lenders is 20% per day.

2. Joint-Liability Arrangements: when properly structured, groups of borrowers are formed through an assortative matching procedure to provide sufficient peer pressure and monitoring in order to keep the default rates of group members low. This should reduce the costs of borrowing. Note that the peer monitoring costs are borne by the borrowers. Thus, there is a trade-off for the group: active peer monitoring helps to reduce defaults and delinquencies, but increases the efforts required. As noted by Stiglitz (1990) peer monitoring can help to mitigate if not solve the ex-ante moral hazard problem: it prevents any member of the group to take risky projects as others in the group who are jointly-liable will attempt to prevent that from happening. Considerable research has focussed on joint-liability contracting (see Ghatak (1999), and Ghatak and Guinnane (1999), for example.), where borrowers form groups and members of the groups agree to take responsibility for delinquencies and defaults by individual group members. By requiring each member of the group to be jointly-liable for the entire group's liabilities, peer monitoring efforts are greatly improved. In addition, the group's ability to obtain additional loans will be predicated on the entire group fulfulling the contractual obligations on existing loans.

3. Credible Threat of Punishment Upon default: the lender must be able to communicate credibly, ex-ante, that default by the group will lead to significant costs to the group including the inability to access further loans. This can be done in two ways: first, the target group is chosen so that they have little or no access to formal credit markets, and second,

[8]CGAP stands for Consultative Group to Assist the Poor.

institutional mechanisms such as credit bureaus be put in place so that lenders are able to share information about defaulters and collectively enforce punishment. Furthermore, the group lending mechanism places a very high social cost on individual defaulters. Often, borrowers are promised additional loans if they successfully pay of existing loans. Rational borrowers know that their access to micro-loans in future is conditional on not defaulting existing loans. Moreover, unreported data that we have examined from two villages in India suggests that the average loan size as well as the minimum loan size increase with every successive round of borrowing. For example, in one village, the data reveals that the average loan size in the seventh round of borrowing was nearly five times the average loan size in the first round of borrowing. In the other village, the average loan size more than doubled by the time the borrower reached the seventh round of borrowing.[9] The potentially increasing size of the loan in future rounds provide strong incentives for the micro-loan borrowers not to default in earlier rounds.

4. Informational Asymmetry: An aspect of the micro-loans is the fact that banks that have depth in lending ability do not necessarily possess informational advantages about numerous borrowers located in different corners of the country. They also lack the monitoring technology that is needed to enforce payment should the joint-liability contracting in and of itself fails to deliver acceptable recovery rates on loans. The informational advantages and monitoring tools in such a market are usually the domain of local lenders and MFIs. Increasingly, micro-loans are structured with complementary participation by local lenders and bigger financial institutions (See, Nini (2004), in the context of emerging market lending): big banks will lend to local MFIs at a certain rate of interest requisite amount of capital. Local Micro Finance Institutions (MFIs) will then re-lend the capital to groups of local borrowers under a joint liability scheme. The local MFI will assume the "first loss tranche" (say, the initial 10% of losses experienced by the loan portfolio), and the rest will pass through to the bank. In models used by certain banks in India, the loans remain on

[9]The data were collected from two villages in India: Islampur, and Raipally.

the books of the bank and not with the local MFI. This so-called partnership model is discussed in Ananth (2005) and Harper and Kirsten (2006).

1.3 Goals of the Paper

The primary goal of the paper is to construct a model of borrowing and lending where the lender has no access to collateral and the borrower is severely credit constrained. The model relates equilibrium borrowing rates to some of the salient features of the loan contracts, which we described earlier. While the model is applied to micro-loans, it is easy to explore other markets in which borrowers obtain loans from lenders who do not have access to collateral upon default. We wish to understand the relative importance of a) lender monitoring, b) punishment upon default, c) maturity structure of the loan, and d) peer monitoring on the loan rates. In particular, we explore whether a particular combination of these variables can keep the default risk low, and at the same time provide lower interest rates to the borrower. Such an outcome should be of great interest in the design of micro-loan contracts, and should be of policy interest. To this end, we wish to characterize the borrower's value function as a function of these levers that the lender has at her disposal.

The paper is organized as follows: Section 2 formulates the basic model. In section 3, we characterize the equilibrium and its properties. In particular, we show monitoring does not work in equilibrium and that the debt maturity can be used to control default probability. We also characterize a) default probabilities b) loan rates, and c) borrower's welfare for varying levels of monitoring, joint-liability efforts, and punishment technology. Section 5 concludes. We also include a technical appendix including the details of the derivations for interested readers.

2 Model Specification

Our model is directly specified at the level of the borrowing group. In doing so, we abstract from several interesting questions about how the group gets formed, and the role that joint liability plays in the choice of the members of the group as well as the choice of the riskiness of projects by the members of the group. See Stiglitz (1991) and Ghatak (1999) for a treatment of issues of this nature. The salient features of the model are that a) the group cannot undertake any productive investment in the absence of the loan (they are capital constrained) and b) the group is simply unable to post any meaningful collateral. Our goal is then to determine the equilibrium interest rates in such a model, where the lender must resort to a different approach for attempting to enforce the loan as there is no physical collateral or a bankruptcy code of any relevance.

The following are key variables in our model. The aggregate loan for the entire group is denoted by L. A key assumption we make is that the group cannot engage in production in the absence of the loan. Hence, the loan is extremely attractive to the borrowers. Administrative expenses incurred by the lenders for monitoring the group is denoted by x per unit time. The members of the group are subject to joint-liability, with y denoting the associated expenses. This tends to reduce the value of the loan to the borrowers because of the efforts expended by the members of the group, but may improve the value of the loan due to improved performance from peer-monitoring efforts induced by joint-liability. We capture this trade-off explicitly in our model. These features have a powerful ex-ante effects on how the group selects its members, and the riskiness of the projects chosen. By operating at the level of the group, our model will not be able to shed any insight on these ex-ante effects.

We denote by $\delta(x, y)$, the proportion of wealth diverted away for consumption by the group. This is a decreasing function of the administrative expenses, x, incurred by the lender and the group efforts, y, to monitor each other. In practice, loans have a short maturity, which we denote by T. The lender is assumed to employ a punishment technology to deal with ex-post defaults. In reality, such costs take the following form. The lender might be able to make the cost of entry into credit markets in future very high for borrowers who default. This is certainly a relevant cost with credibility if the borrowers need repeated access to credit markets. In our model, we do not consider dynamic borrowing. Hence, we may think of the cost as the present value of the difference between the rates at which the defaulting borrower should borrow in informal credit markets and the (lower) rates that would have prevailed in micro-loan markets. Also, defaults in the context of group borrowing can have a significant social cost to the defaulting borrower. Anecdotal evidence suggests that exiting borrowers often pay the loans rather than defaulting to avoid such social costs. Punishment technology or the credible costs associated with default, takes the form a lump sum cost of K. If K is too small, the borrowers will rationally take the loan and default promptly in an endogenous model of default. If it is too high, they will rather not borrow. It has to be high enough to induce payments of contractual obligations, which will increase the value of loans to the lender but not so large as to adversely reduce the participation of borrowers in the micro-loan programs. We will find these limits in our set up. The endogenously determined equilibrium loan rate R is one of the key objects our study.

The dynamics of wealth, C_t, generated by the investment for borrowing group is given by the equation shown below[10]:

$$dC_t = ((\mu - \delta(x,y) - y)C_t - L(R+y))dt + \sigma C_t dW(t) \tag{1}$$

It is assumed that $C(0) = L$ is the initial wealth of the borrowing group. The loan amount L is specified exogenously at $t = 0$. The equilibrium borrowing rate, R, is determined endogenously at $t = 0$. Group-specific risks are characterized by the constant drift and diffusive coefficients.

It is useful to motivate the technology in the context of micro-loans. Micro-borrowers tend to invest their borrowing into activities such as a) livestock, b) kiosks, c) repair shops, d) paying high-interest loans to local money lenders, etc. These are typically small investments on which the rates of returns can be very high. CGAP reports estimates of rates of returns on micro-loans ranging from 40% to as high as 600%. But returns must decrease with scale, and this raises questions about the existence of a threshold scale level of micro-loans beyond which they may be less effective as a development tool unless ways are found to reduce the interest rates charged on the loans.

By requiring that $L = C(0)$ we are making the loan extremely valuable to the borrower, for

[10] We assume that μ is the expected growth rate of wealth, and $\{W_t, t \geq 0\}$ is a Brownian motion process.

in the absence of the loan the borrower cannot access the technology and will have a utility of zero. It is easy to introduce an endowment of $x(0)$, which will lead to the requirement that $C(0) = x(0) + L$. We have noted in our data that some borrowers have prior indebtedness when they enter the first round of borrowing in the micro-loan markets, implying that $x(0) \leq 0$. This makes the loan even more attractive to the borrowers.

Note that joint-liability has several interesting effects on budget dynamics of the borrowing group. First, the peer monitoring activity is costly to the group, and this increases with the number of members in the group. Second, when the wealth level is low, the peer monitoring, which leads to a low $\delta(x, y)$ helps to overcome potential liquidity shortages facing the group. Third, the overall risk of the project portfolio of the group represented by σ is much lower than the risks of the projects of individual borrowers, when the group members ensure that only low risk borrowers join their group, recognizing the joint-liability feature of the loan.

Note that $\delta(x, y)$ is the "payout" or the amount diverted by the borrowing group for consumption purposes. The excess of wealth over $\delta(x, y)$, peer monitoring expenses and contractual payments is consumed in good states of the world. In bad states of the world, when the payout is less than the required payments, wealth must be liquidated to make the contractual payments.

3 Borrower's Problem and Endogenous Default

The objective of the borrowing group is to maximize the discounted payoffs from the loan and select the optimal default strategy as follows. Let $\tau = inf\{t \geq 0 : C_t \geq c^*\}$ be the first passage time of the wealth process, where c^* is the borrower's endogenously chosen optimal default trigger.

Borrower's Problem:

$$\begin{aligned} B(C_0) &= supE[\int_0^{\tau \wedge T} e^{-rs}[\delta(x, y)C_s - L(R + y)]ds] \\ &\quad + E[e^{-r\tau} J_B(C_\tau - K)1_{\{\tau \geq T\}}] - Le^{-rT} P(\tau > T) + E[J_B(C_T)]e^{-rT} P(\tau > T) \end{aligned} \tag{2}$$

where $J_B(\cdot)$ denotes the payoffs to the borrowing group upon optimally choosing to default and r denotes the risk-free rates. Here we consider two possibilities. First, default leads to a lump sum punishment K but the group continues to have access to technology in which the residual wealth can be invested. This is akin to saying that the tools of trade of borrowers may not be seized when default occurs. Certainly, with micro-loans the political costs of such an action by lenders are rather high. One interpretation of this punishment is the following: when the borrowing group defaults, it is precluded from entering the micro-loan markets again and is forced to borrow from local money lenders at a prohibitive cost to continue to run their business. This way, the group is able to continue to have access to the technology but suffers a lump sum cost upon default. In this case, the payoff function upon default is[11]:

$$J_B(C_\tau - K) = \frac{\delta(x=0,y)(C_\tau - K)}{r + \delta(x=0,y) + y - \mu}$$

Alternatively, we can assume that default leads to a lack of access to the technology itself. This is a more severe punishment. The borrowing group gets a certain amount of wealth, which may be thought of as the liquidation value of their business net of punishment costs K and they must consume out of that for the rest of their lives. Given their lack of access to savings, this will constitute a more severe punishment. In this case, the payoff upon default is simply the cash flow at time of default minus the punishment cost, namely:

$$J_B(C_\tau - K) = C_\tau - K$$

The maximization problem of the borrower leads to the following HJB equation:

$$0 = \max\left[-rB + \delta(x,y)C - L(R+y) + B_C(\mu - \delta(x,y) - y)C + B_{CC}\frac{1}{2}\sigma^2 C^2\right] \tag{3}$$

When the wealth level of the borrowing group reaches a threshold low level c^*, the group collectively defaults and receives a payoff as follows:

$$B(C \downarrow c^*) = J_B(c^* - K) \tag{4}$$

[11]We assume that the group operates as a unit even after default and enjoy the benefits of peer monitoring after default. This can be relaxed to consider the case where default eliminates the benefits of peer monitoring.

In order for the expected payoffs of the borrowing group to be finite, we need to impose a transversality condition.[12] We proceed to characterize the borrower's value from taking a micro-loan of size L and defaulting optimally. The finite maturity loan is a very complicated problem and does not have a closed form solution. We seek an accurate approximation of the finite maturity problem, which is given in the appendix. In the appendix, we detail for the approximation procedure. The idea of the approximation is to decompose the finite maturity loan contract into two parts: 1. the no early exercise loan contract (i.e. the "European" counterpart of the loan contract) and 2. the value of the early exercise option to the borrower. This approximation is particularly good for very short dated maturity and very long dated maturity. This approximation is particular attractive in the context of non-collateral lending as micro-loans is usually short maturity, whereas sovereign loans are usually long maturity.

3.1 Equilibrium & Endogenous Borrowing Rates

The lender will then take into account the optimal default strategy of the borrower in valuing the loan as follows. If the loan were to have a finite maturity date T, the lender's problem is:

$$D(L) = E[\int_0^{\tau \wedge T} e^{-rs} L(R - x) ds] + E[e^{-rT} L 1_{\{\tau \geq T\}}] \tag{6}$$

An equilibrium in this economy is an endogenously determined borrowing rate R for a loan of size L to the borrowing group, such that:

1. Borrowing group's value is maximized.
2. Lender's required rate of return satisfies the fixed point requirement that $D(L) = L$.

We can solve the lender's break even condition for the equilibrium interest rate on the micro-loan, which can be expressed as follows: Carrying out the integration and applying the lender's

[12] We will maintain the following assumption throughout our analysis:

$$r + \delta(x, y) + y - \mu > 0. \tag{5}$$

break even condition yields :

$$R = x + r\frac{1 - e^{-rT}P(\tau > T)}{1 - E[e^{-r\tau}1_{\{\tau \leq T\}}] - e^{-rT}P(\tau > T)} \tag{7}$$

Note that the equilibrium interest rate must compensate for a) administrative expenses x borne by the lenders, b) the funding costs, which is assumed to be the risk-free rate, and c) the possibility of default by the borrowing group. Note that the probability of default will be influenced by the factors x, y and K, which we will characterize in the next section.

It is easy to verify that for a perpetual loan, the equilibrium interest rate will be as follows:

$$R = x + r\frac{1}{1 - E[e^{-r\tau}]} \tag{8}$$

where

$$E[e^{-r\tau}] = (\frac{c^*}{c})^{\beta 1} \tag{9}$$

and $\beta_1 = \frac{1}{2} - \frac{\mu - \delta(x,y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x,y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}$.

The intuition of the loan rate is straight forward. The first term is the monitoring cost expended by the lender. The second term is a risk premium demanded by the lender in order to take on the risky position. The proportionality factor for risk premium demanded is $1 - E[e^{-r\tau}]$, where $E[e^{-r\tau}]$ can be interpreted as the price of an Arrow-Debreu security which pays \$1 in the event of default. Note that for a 1% increase in the funding cost or the interest in the micro loan, borrowing rate is magnified by the risk premium, which is greater than 1.

The equilibrium interest rates on micro-loans is now completely characterized for given default boundary c^* and maturity T, and a risk structure of default premium can be obtained in our model. The borrowing rates under finite maturity differs from the perpetual loan rates. The key difference (in additional to different default boundaries) is in the proportionality factor for risk premium demanded. We first ignore the denominator and note that the numerator is now $1 - e^{-rT}P(\tau > T)$, which suggests that loan rates can be reduced. Economically, this implies the possibility of an increase in the range of equilibrium, suggesting that lower monitoring cost may be admissible in finite maturity. We explore this further later formally and demonstrate that this is in fact the case.

Our model for the determination of loan rates differ sharply from how practitioner sets the interest rate. A CGAP report indicates that the following is a standard formula for the interest rate charged in micro-finance markets:

$$R = \frac{AE + LL + CF + K - II}{1 - LL} \tag{10}$$

where, AE denotes the administrative expense/monitoring cost - namely, x in our model, LL denotes the loan Losses, CF is the cost of funds - namely r in our model, K is the capitalization rate, and II denotes the Investment Income. This equation presents *linear relationship* among monitoring effort, interest rate, and joint-liability (implicit in AE). However, our model suggests that this relationship should be highly nonlinear. The difference is mainly due to the fact that the practitioner's loan pricing equation ignores the fact that the loan loss rate (or defaults) depend on the borrowing rate. Furthermore, a given increase in AE, which is x in our model will lead to a higher loan loss rate due to the increased probability of default. The intricate dependence of these variables and the necessary risk premium are not properly reflected in the loan pricing equation of practitioners.

3.2 Role of Lender Monitoring & Defaults

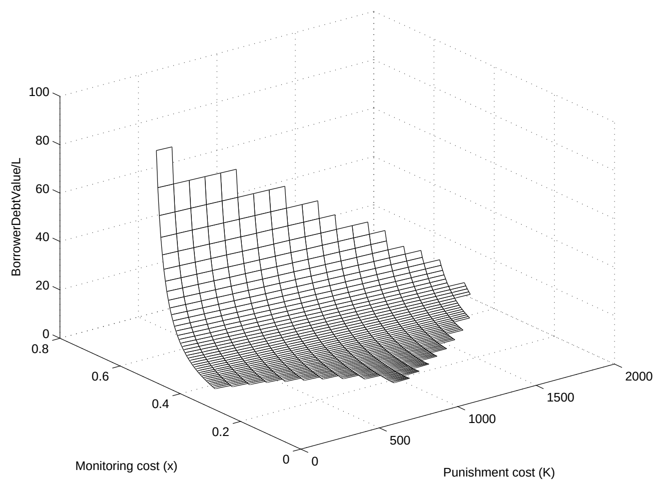
The model delivers several implications for the role of monitoring by lenders. We now state one of the main results of our paper.

Proposition 1 *In the absence of any collateral, when the borrowing group has access to technology upon default, there must be monitoring by the lender in order for there to exist an equilibrium for a perpetual debt.*[13].

The intuition behind proposition 1 is the following. When there is no monitoring by lenders, the consumption rate by the borrowing group is the same before or after default. But during

[13]Proof is in the appendix.

the period that the loan is solvent, borrowers are forced to pay contractual interest payments, which are costly. Immediate default allows the borrowing group to maintain their consumption and avoid paying the interest rates, which already reflect the ex-post costs of punishment. Using the parameters in the numerical illustration section, we present here the equilibrium behavior of the borrower's valuation function as a function of monitoring cost x and punishment cost K for perpetual debt. In deriving this proposition, we assume that the group still operates as a unit after default. Were this not the case, the peer monitoring benefits may still induce the group not to default even in the absence of lender monitoring.



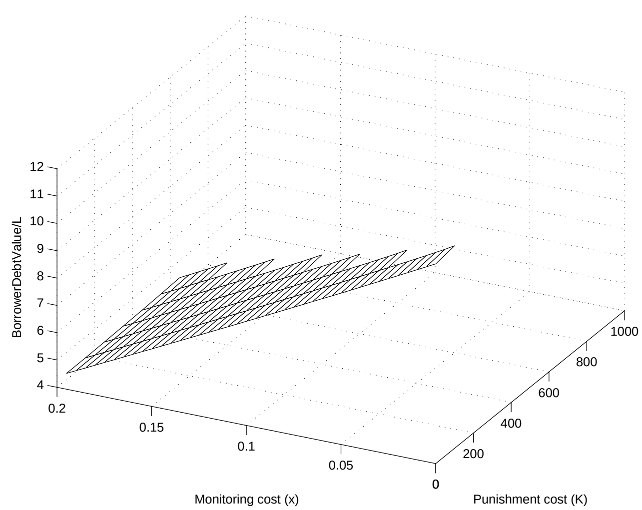
Borrower's value as a function of monitoring cost (x) and punishment cost (K).

The above graph illustrates our proposition. Equilibrium only exists after monitoring increases up to a certain threshold level. This is due to the continuity of our problem. As monitoring decreases too much (i.e. say, below $x = 0.2$), then there does not exist an equilibrium anymore. This suggests how a minimum level of lender monitoring is essential to sustain an equilibrium in micro-loan markets. Monitoring, however, can potentially be a very inefficient method for the lender to enforce the loan. We note that for each monitoring cost, there exists a K_{max} such that after this point there will not be an equilibrium anymore. To the right of this region the borrowers realize that the costs of default are too excessive for them to take the loan. We further note that $K = 0$ can be sustained as an equilibrium in our model. This is due to the fact that the borrowers' consumption is specified as an exogenous process. As long as there is monitoring, this will increase the drift of the borrower's production process. The increase in the drift is big enough to stop the borrowers from immediate default.

We now examine the situation when the maturity of the loan is very short (3 months).

Proposition 2 *In the absence of any collateral, when the borrowing group has access to technology upon default, an equilibrium exists for a finite maturity debt even in the absence of monitoring.*[14].

The intuition for this result is simple: the impending balloon payment of the principal accelerates the punishment costs K, and hence the borrower does not immediately default even in the absence of monitoring by lenders.



Borrower's value as a function of monitoring cost (x) and punishment cost (K) for T = 3 months.

This suggests the use of the debt maturity as a substitute for monitoring by lenders. We will later show that debt maturity indeed outperforms monitoring in reducing defaults in equilibrium.

[14]Proof is in the appendix.

3.3 Distinction from Corporate Debt

Our formulation differs in a number of respects from the structural models of corporate loans. First, the lender in our model is unable to seize the collateral and obtain recoveries on the loans extended. In this sense, the problem that we formulate is similar the one faced in the sovereign loan markets. Thus, the lender is not the residual claimant to the value generated by the technology. The lender in determining the loan rate must ensure that the value of the loan conditional on the default strategy of the borrower is equal to the present value of the future payments. The loan rate or the coupon rate is determined *endogenously* in our model for a given loan size. Moreover, the enforcement of the loan is not through the threat of liquidation and the seizure of assets of the borrower. It is through the subtle interplay of a) monitoring by lenders, b) threat of punishment conditional on default, and c) peer monitoring by members of the borrowing group. finally, a corporate borrower can issue equity to postpone default. This is not a serious option for micro-loan borrowers.

In corporate debt literature the total value of equity and debt is simply equal to the value of the underlying firm that borrows. The additional value created by borrowing is a tradeoff between the tax benefits created by debt net of the costs associated with financial distress induced by debt. In contrast, in our model, borrowing creates value as follows. In the absence of borrowing in the micro-loan markets we assume that the borrower has a utility of $\hat{U}$, which we assume to be zero for simplicity. This is the utility associated with borrowing from local money lenders, without the benefits of assortatively matched groups and peer monitoring. Once the group has access to micro-loans it is able to access the technology and create value for the borrowers. The value of the loan to the lenders is set equal to the present value of the payments promised by the borrowers, leaving the lenders with no surplus; namely $D(L) = L$. The value to the borrower is determined by the optimization problem described above. The value created by the loan to the borrower is summarized by the ratio $\frac{B(C)}{L}$. The lender can extract some of this surplus by requiring a higher rate of return on the loan. This is easily accommodated in our framework.

To make a direct comparison between our model of lending without collateral and the corporate bond models, we will focus at the Leland (1994) model, which assumes that the lender receives a fraction $(1-\alpha)$ of the firm value conditional on default. Specifically, we will compare and contrast the credit spreads implied by both model. To do so, we have to first modify the lender's break even condition to accommodate the existence of collateral upon default in our model. Let c^* be the borrower's default strategy associated with a given loan size L, then the lender's valuation is[15]:

$$D(L) = E[\int_0^\tau e^{-rs} L(R-x)ds] + E[e^{-r\tau}(1-\alpha)c^*] \tag{11}$$

As before, we now invoke the break even condition $D(L) = L$ to get the equilibrium loan rate, as given by:

$$R = x + r\frac{1-(1-\alpha)\frac{c^*}{L}(\frac{c^*}{c})^{\beta_1}}{1-(\frac{c^*}{c})^{\beta_1}} \tag{12}$$

where $\beta_1 = \frac{1}{2} - \frac{\mu-\delta(x,y)-y}{\sigma^2} + \sqrt{(\frac{\mu-\delta(x,y)-y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}$.

Note that in the above expression, the term $1-(1-\alpha)\frac{c^*}{L}(\frac{c^*}{c})^{\beta_1}$ takes into the account of the existence of collateral. If we set $\alpha = 1$, we will recover our previous equation.

We now recall the credit spreads in Leland (1994)'s model:

$$R_{corp} = r\frac{1}{1-(\frac{c^*_{corp}}{c})^{\beta_{corp}}(1-\frac{r}{C}(1-\alpha)c^*)} \tag{13}$$

where c is the current firm value, $C = LR$ is the coupon paid by the firm to the debt holders and β_{corp} is the root of the characteristic function in Leland's model.

For any given c^*, we can now compare between Leland (1994) model and our model. Note that as α approaches 1, the credit spreads in both models have the same formula. The numerical value may be different for both model since c^* will be different in both models. The driving force of corporate bond models is the associated tax benefit, whereas in our model an initial loan is necessary for investing. Unlike a corporate borrower, in our model there is no possibility for

[15] We explore the case of perpetuity here for simplicity.

borrowers to issue equity. These factors make the loan much more valuable to borrowers in the micro loan market.

An important qualitative difference is the effect of funding costs r on the credit spreads. In Structural models of default, such as Leland (1994) as the risk-free rate increases, the spreads decline. This is due to the fact that an increase in r increases the drift rate of the risk-neutral process describing the evolution of the wealth of the borrower. In our model, an increase in r typically increases the credit spreads. Also, our model would predict that in the absence of sufficient controls (as modeled by y, K and T), the defaults in micro-loans will occur sooner and spreads would be higher in micro-loans as compared to corporate loans.

The following relations hold between corporate loan rates and micro-loan rates.

For $x = 0$, $y = 0$, $K = 0$ in our model and zero tax in Leland(1994)'s model:

1. For fixed loan rates $R = R^*_{corp}$, we have $c^* \geq c^*_{corp}$ for all $\alpha \leq 1$. Namely, micro-loan rates are higher than corporate loan rates.

2. For fixed default boundary $c^* = c^*_{corp}$, we have $R^* \geq R^*_{corp}$ for all $\alpha \leq 1$. Namely defaults in micro-loans will occur sooner than in corporate loans, in the absence of peer monitoring, lender monitoring and punishments upon default.

3.4 Numerical Results

In order to obtain additional results, we impose the following structure on the $\delta(x, y)$ function.

A threshold joint-liability contracting effort y^* is defined as a steady state level at which there is no excess diversion of output from lenders, and the borrowing group consumes an amount that is denoted by $\underline{\delta}$[16]. This may be thought of as the best outcome of an "assortative matching" process in which low risk borrowers with no informational disadvantages identify other low risk borrowers (and thereby exclude higher risk borrowers) in forming the group so that the pool in

[16]This level, $\underline{\delta}$, can be thought of as the subsistence level of consumption for the borrowing group.

the group has low risk both in payment behavior and the riskiness of the projects undertaken by group members[17]. At any effort level y which is below the threshold contracting effort y^*, there is additional diversion of output for private consumption.

With a monitoring effectiveness or efficiency parameter $\beta < 0$, lenders can in the limit approach this ideal benchmark level. In the absence of any monitoring, the existence of a punishment technology and peer monitoring through joint-liability contracting is assumed to lead to a diversion rate of $\hat{\delta} > \underline{\delta}$. Our specification assumes that a minimum level of monitoring x^* is needed even when the group is formed under ideal conditions. In the absence of any monitoring, the existence of a punishment technology and peer monitoring through joint-liability contracting is assumed to lead to a diversion rate of $\hat{\delta} > \underline{\delta}$.

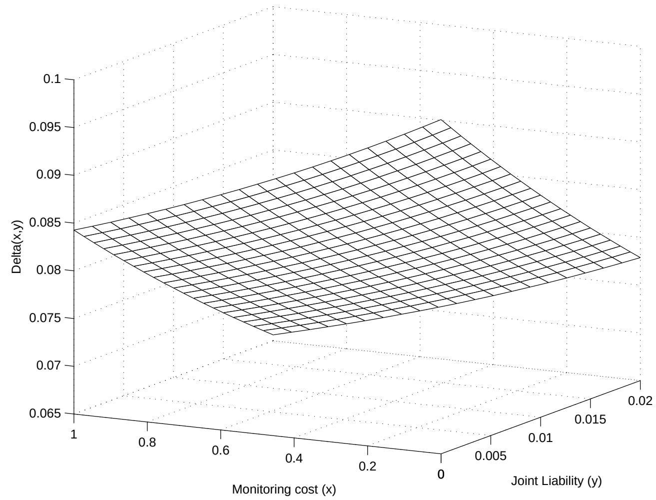
We need to ensure that the consumption function $\delta(x, y)$ is decreasing in monitoring and joint-liability. That is, more monitoring and peer-monitoring lead to less consumption for the borrowers. We also require that the cross derivative $\frac{\partial^2 \delta(x,y)}{\partial x \partial y} > 0$. This ensures no effect dominates each other. A particular example of such function, which we will employ throughout our analysis, is[18]:

$$
\begin{aligned}
\delta(x > 0, y < y^*) &= [(1 - e^{-\beta x})\underline{\delta} + e^{-\beta x}\hat{\delta}] \times e^{-b\frac{y}{y^*}} \\
\delta(x = \infty; y = y^*) &= \underline{\delta} \times e^{-b} \\
\delta(x = 0; y = 0) &= \hat{\delta}
\end{aligned}
$$

The delta function used in this numerical illustration is plotted as follows:

[17] See the work of Bannerjee, A; Besley T; and Guinnane, (1994)

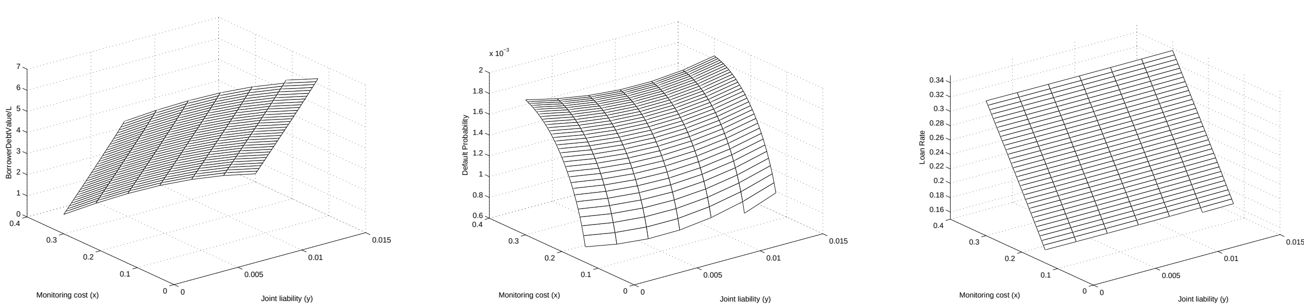
[18] Specifically, we choose $\beta = -0.5$, $\underline{\delta} = 0.06$, $\hat{\delta} = 0.10$, $b = 0.25$, and $y^* = 0.02$.



Delta as a function of monitoring cost (x) and joint liability (y).

Throughout this section we have assumed the following parameters. We set the interest rate, r, at 3%, the drift parameter, μ at 12%, the Volatility parameter, σ^2 at 20%, the Initial Lending Amount, L at 1000, the Punishment Cost, K, at 500 and the maturity, T at 3 months. These parameters are chosen to reflect the contractual parameters observed in practice. The average size of micro-loans are 3 months and are relatively small.

We investigate below the range of equilibrium and how different amount of monitoring cost and joint liability affects equilibrium.



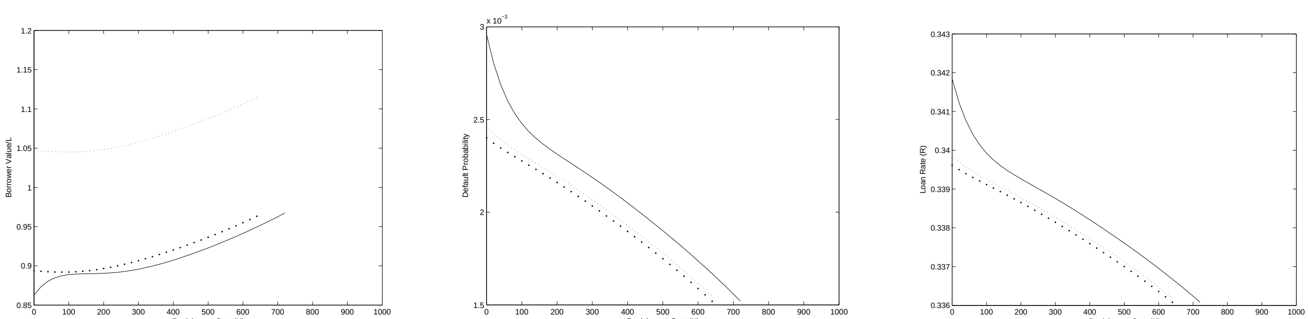
Equilibrium behavior as a function of x and y when there is access to technology upon default.

The left panel displays the borrower's value normalized by initial borrowing amount. The middle panel displays default probability. The right panel displays equilibrium loan rates.

We note that with short maturity, excessive monitoring reduces the welfare of the borrower. First, default probability is increasing in monitoring. This is a consequence of the high loan rates charged by the lenders to compensate for the increased monitoring costs. With short maturity, the borrowers are now faced with higher repayment rates, and since the technology may not be able to produce enough cash flow for repayments in such short maturity, the borrowers are forced to default. Although the lender can transfer all the cost incurred to the borrower, the lender still has to suffer from lower repayment probabilities. This is due to the feedback effect of increased probability of default induced by the higher borrowing rates in our equilibrium analysis, which is absent in the practitioner model. Hence, practitioners believe lender monitoring can lead to lower defaults, whereas it is not necessarily the case in our model. This suggests that finite maturity is an useful substitute for monitoring. It reduces default probability while keeping loan rates feasible for the borrowers.

Joint liability, on the other hand, serves as another substitute for monitoring. Default probability is concave in joint liability, suggesting that when joint liability is low, default probability increases. However, once the borrowers exert too much joint liability, they do not concentrate in the production process enough and causing more defaults in equilibrium. This effect translates to a concave borrower's valuation function. Loan rates are decreasing in joint liability as a very slow rate.

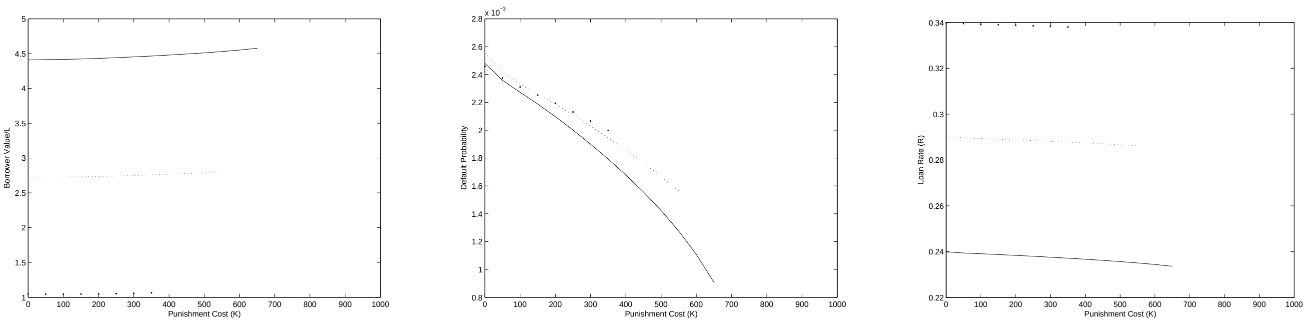
Let us first investigate the dynamics of the equilibrium quantities in terms of punishment cost K and joint liability y. We will analyze values joint-liability for $y = 0\%$ (solid line), $y = 0.5\%$ (dashed line), and $y = 1\%$ (dotted line). We will fix $x = 30\%$.



Equilibrium behavior as y increases..

Several aspects of these pictures are worthy of additional discussion: first, note that a higher level of peer monitoring leads to a lower probability of default at all levels of K. Second, at higher levels of peer monitoring as proxied by the variable y, the range of punishment costs K conditional on default is much higher: in other words, the default probability is a much flatter function of K at higher levels of y. Although the loan rates are lower with higher levels of y, the borrower's value function is declining in y as the borrowing group is forced to put in the peer monitoring effort.

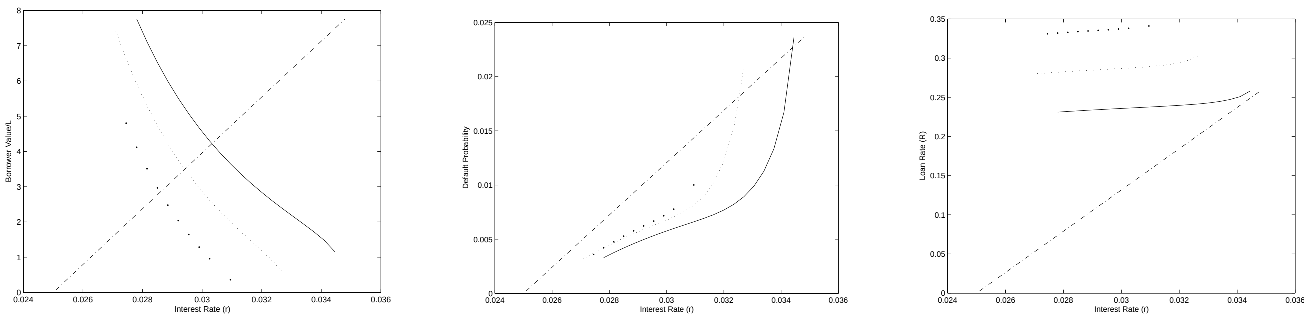
We explore below how equilibrium changes as a function of punishment cost as we increase monitoring cost for $x = 20\%$ (solid line), $x = 25\%$ (dashed line), and $x = 30\%$ (dotted line). We will fix $y = 0.5\%$.



Equilibrium behavior as x increases..

As the monitoring effort x by the lenders increases, the borrowing costs increase as well. We note that monitoring increases default probability while raising loan rates. It also lowers borrower's value due to the increased defaults. Hence, in short maturity debt contracts, monitoring do not play an effective role.

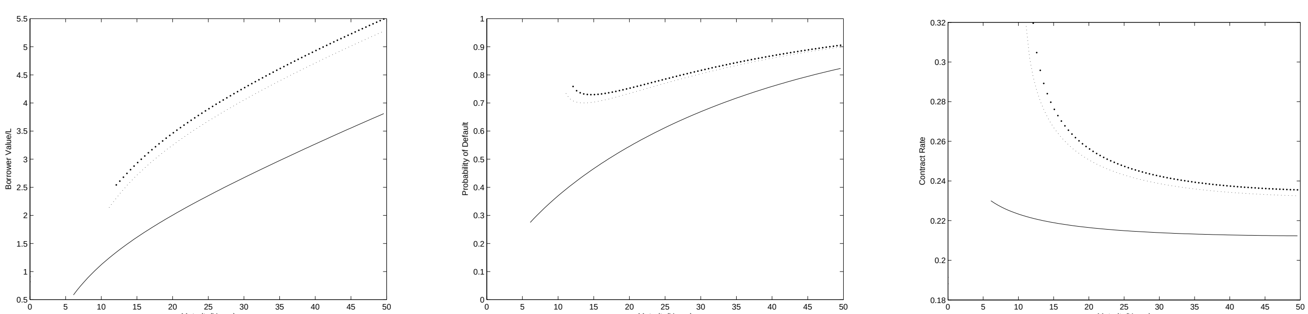
We now explore the relationship between the borrowers and the lender's actual cost of lending. As before, we focus on the cases where there exist an equilibrium. Namely, we have x=20% (solid line), x=25% (dashed line), and x=30% (dotted line).



Equilibrium behavior as r increases..

Default probability and loan rates are increasing in both the monitoring expense and the interest rate. Loan rate is increasing in interest rate, as the lender demands a higher risk premium to compensate for his increasing opportunity cost. Furthermore, higher administrative cost induces a higher equilibrium loan rate. As a result of the higher loan rates demanded by the lender, default probability is increasing in loan rates. These effects force the borrower's value to be decreasing in monitoring expense and interest rate. The intuition behind these results is that as interest rate goes up, loan rates increases much more significantly. This suggests that the cost of funding for the lender may be a key to the determination of micro-loan interest rates.

We now explore the term-structure implied by our model. We present the cases with $K = 500$ (upper panel) and $K = 750$ (lower panel) and show that increasing punishment cost can increase the range of admissible outcomes.

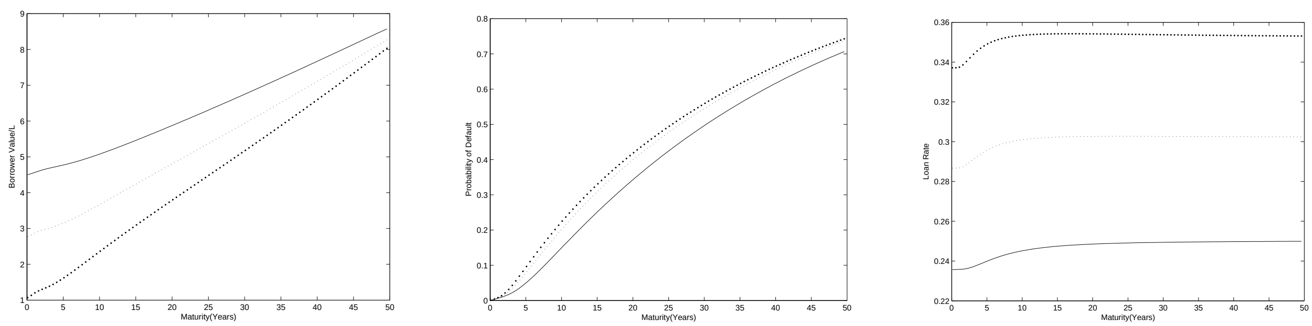


Equilibrium behavior as T increases..

The lines plot the term structure of normalized borrower's value, default probability, and equilibrium loan rates for $L = 1000$ (solid line), $L = 5000$ (dashed line), and $L = 10000$ (dotted line).

Note that for fixed punishment cost K, our model requires that large loans and long maturities. Small loans on the other hand require short maturities. Note that as K increases from 500 to 750, the short maturities become admissible for loan size of 1000. This suggests the use of punishment cost as a very powerful device for enlarging the range of equilibriums. This is due to the nature of a credit punishment cost being ex-post effective, in contrast to administrative cost being ex-ante effective. Finally, as the loan size increases, the probability of default increases and the loan rates dramatically increase unless the maturity of the loans are increased.

We now present the trade off between monitoring and debt maturity and show the choice of debt maturity can be used as a substitute to monitoring. We focus on x=20% (solid line), x=25% (dashed line), and x=30% (dotted line):



Equilibrium behavior as T increases..

The above pictures clearly show that debt maturity can be used to control defaults in equilibrium. Short maturity loans almost never default while long maturity loans are more inclined to default. On the other hand, increasing monitoring simply increases loan rates and default probability. This suggests that use of debt maturity as an contracting device for the lenders over the use of monitoring.

In summary, a prevalent effect in our model is that default probability and loan rates are increasing in monitoring. This suggests that when practitioner decides their micro-lending business strategy, monitoring costs should be given extra concern. If the group is properly formed (ie. for a reasonable joint liability y), and a fixed opportunity cost r, micro-finance institutions may achieve a lower default rate by reducing monitoring. Furthermore, the micro-finance institution may substitute monitoring using maturity T of debt as a tool. Finally, punishment cost K can also be used to alter equilibrium risk structure.

4 Conclusion

We have presented a simple model of lending without collateral. The lender attempts to enforce the contract by relying on three things: a) monitoring to reduce the diversion of resources by the borrower from productive uses, b) peer monitoring by lending to a group, which is jointly-liable for the fulfillment of the contractual provisions, and c) a punishment technology that imposes a finite cost on defaulting group of borrowers. We show that peer monitoring combined with a limited amount of monitoring by lenders is sufficient to reduce default probability to acceptable levels, so long as there is a credible punishment cost. Excessive monitoring by lenders increases the cost of borrowing and this might lead to non-participation by borrowers. As the loan size increases, we show that the probability of default increases and the loan rates dramatically increase unless the maturity of the loans are increased.

We have extended our analysis to examine situations where the borrowers face low frequency jump risks. Such episodes as heavy monsoons, or health epidemics could have dire consequences for borrowers in this market. Predictably, we found initial loan rates to be too prohibitive and the range of rates over which lenders and borrowers could agree on becomes much smaller. An important limitation of our work is that we do not examine repeated borrowings and the discipline that may impose on the borrowing group.

5 Appendix

In all our derivations, let $\tau = inf\{t > 0 : C_t > c^*\}$ be the first passage time of the cash flow process.

5.1 GBM Perpetual Loan Contract

Since most structural corporate debt models assume perpetual debt, we present here the borrower's valuation with perpetual loans:

$$B(c, T) = B(\alpha c - K) \tag{14}$$

for $c \geq c^*$:

$$B(c, T) = A_1(\frac{c^*}{c})^{\beta_1} + A_3 c + A_4 \tag{15}$$

where

$$\begin{aligned}
A_1 &= (\alpha B - A_3)c^* - (BK + A_4) \\
A_3 &= \frac{\delta(x, y)}{r + \delta(x, y) + y - \mu - \lambda E[e^z - 1]} \\
A_4 &= -\frac{L(R + y)}{r} \\
c^* &= \frac{(BK + A4)\beta_1}{(\alpha B - A3)(1 + \beta_1)} \\
B &= \frac{\delta(x = 0, y)}{r + \delta(x = 0, y) + y - \mu - \lambda E[e^z - 1]} \\
\beta_1 &= \frac{1}{2} - \frac{\mu - \delta(x, y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x, y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}
\end{aligned}$$

and B is defined above, which can take on two values depending whether we allow for access to technology or not upon default. A complete solution is in the technical appendix. The interpretation of the above formula straight forward. The first term is the risk neutral expectation of the investment technology net of punishment cost and default risk. The second term is the value of the technology up on default. The last term is the present value of the total cost exerted by the borrower.

Proof Let $x = log(C)$ and assume:

$$B(C) = A_1 e^{-x\beta_1} + A_3 e^x + A_4$$

We will suppress the dependencies of the δ for convenience. Substituting this into the HJB equation yields:

$$\begin{aligned} 0 &= A_1 e^{-x\beta_1}(-r + (\mu - \delta - y - \frac{1}{2}\sigma^2)(-\beta_1) + \frac{1}{2}\sigma^2\beta_1^2) \\ &\quad + e^x((-r + \mu - \delta - y)A_3 + \delta) \\ &\quad - L(R + y) - rA_4 \end{aligned}$$

Now, by the technique of matching the coefficient, we can solve for β_1, A_3 and A_4 in closed form. A_1 is obtained via the *Principal of Continuity*:

$$B\alpha e^{x_0} - BK = A_1 e^{-\beta_1 x_0} + A_3 e^{x_0} + A_4$$

where B is defined in the paper, referring to different values according to whether the borrower has access to technology or not after default.

Finally, the *Principal of Smoothing Pasting* gives the optimal default boundary:

$$B\alpha e^{x_0} = -A_1\beta_1 e^{-\beta_1 x_0} + A_3 e^{x_0}$$

We have 5 equations and 5 unknowns, giving us an identified system to solve for: β_1, A_1, A_3, A_4 and x_0. Now recognizing that $c^* = e^{x_0}$, the proof is complete.

5.2 GBM Finite Maturity Loan

The borrower's value function is given by: For $C \leq c^*$:

$$B(e^x, T) = B(e^x - K) \tag{16}$$

for $C \geq c^*$:

$$B(C_0, T) = EuB(C_0, T) + A_1(\frac{c^*}{c})^{\beta_1} + A_3 c + A_4 \tag{17}$$

where

$$
\begin{aligned}
A_1 &= (B - A_3 - Q)c^* - (BK + A_4) \\
A_3 &= \frac{\delta(x,y)/z}{r/z + \delta(x,y) + y - \mu} \\
A_4 &= -\frac{L(R+y)}{r} \\
c^*_{approx} &= \frac{(BK + A4)\beta_1}{(B - A3 - Q)(1+\beta_1)} \\
Q &= Be^{(\mu - r - \delta(x,y) - y)T} \\
z &= 1 - e^{-rT} \\
\beta_1 &= \frac{1}{2} - \frac{\mu - \delta(x,y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x,y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r/z}{\sigma^2}}
\end{aligned}
$$

The constant B is again chosen according to whether there is access to technology or not up on default. $EuB(C_0, T)$ represents the European version of the same debt contract, and is given by:

$$
\begin{aligned}
EuB(C_0) &= -Le^{-rT} + e^{-rT}E[J_B(C_T)] \\
&= -Le^{-rT} + BC_0 e^{(\mu - r - \delta(x,y) - y + \lambda(E[e^z] - 1))T}
\end{aligned}
$$

where $B = \frac{\delta(x=0,y)}{r + \delta(x=0,y) + y - \mu - \lambda E[e^z - 1]}$.

Note that c^*_{approx} converges to c^* and the finite maturity approximation of the borrower's value converges to the perpetuity function as T goes to infinity, verifying the accuracy of our approximation scheme.

From the finite maturity approximation, we see that even with $x = 0$, there can be an equilibrium - a main difference between the finite maturity and the perpetual conclusion. The intuition is that finite maturity itself is also an additional tool both the lender and the borrower can use to screen out unwanted loan contracts. Although the consumption rate is still the same before and after default, with the additional constraint of finite maturity, the borrowers will choose not to immediate default as long as the amount of interest paid is less than the punishment cost of defaulting.

Proof In the following, we will suppress the dependencies of δ for convenience. Let $x = log(C)$.

The borrower's value has to satisfy the following HJB equation: For $C \geq c^*$:

$$0 = \max\left[-B_t - rB + \delta C - L(R+y) + B_C(\mu - \delta - y)C + B_{CC}\frac{1}{2}\sigma^2 C^2\right] \tag{18}$$

and

$$B(e^x, T) = J_B(e^x - K) \tag{19}$$

By Feymann-Kac, we know that $EuB(e^x, T)$ solves the following partial differential equation for all c:

$$0 = \max\left[-EuB_t - rEuB + \delta C - L(R+y) + EuB_C(\mu - \delta - y)C + EuB_{CC}\frac{1}{2}\sigma^2 C^2\right] \tag{20}$$

Hence, the early exercise premium $\epsilon(x, T)$ must satisfy: For $C \geq c^*$:

$$-\epsilon_t - r\epsilon + (\mu - \delta - y)\epsilon_x + \frac{1}{2}\sigma^2\epsilon_{xx} + \delta(x, y)e^x - L(R+y) = 0 \tag{21}$$

and for $C \leq c^*$:

$$\epsilon(e^x, T) = J_B(e^x - K) - EuB(e^x, T) \tag{22}$$

Now by letting $z = 1 - e^{-rt}$ and $g(x, z) = \frac{\epsilon(e^x, T)}{z}$, it is easy to to see the HJB becomes:

$$-r(1-z)g_z - \frac{r}{z}g + \frac{1}{2}\sigma^2 g_{xx} + (\mu - \delta - y - \frac{1}{2}\sigma^2)g_x + \delta e^x - L(R+y) = 0 \tag{23}$$

We will assume that $(1-z)g_z = 0$ for the approximation. This approximation becomes very accurate for very short and very long maturity. Applying this and substituting out g yields: For $C \geq c^*$:

$$-\frac{r}{z}\epsilon + (\mu - \delta - y - \frac{1}{2}\sigma^2)\frac{\epsilon_x}{z} + \frac{1}{2}\sigma^2\frac{\epsilon_{xx}}{z} + \delta\frac{e^x}{z} - \frac{L(R+y)}{z} = 0 \tag{24}$$

and for $C \leq c^*$:

$$B(e^x, T) = J_B(e^x - K) - EuB(e^x, T) \tag{25}$$

We are now in the position to solve this equation. We recognize this as the Euler's equation and hence assume:

$$\epsilon(C) \quad = \quad A_1 e^{-x\beta_1} + A_3 e^x + A_4$$

By the method of matching coefficients, we get the following equations:

$$\begin{aligned}
-r/z + (\mu - \delta - y - \frac{1}{2}\sigma^2)(-\beta_1) + \frac{1}{2}\sigma^2\beta_1^2 &= 0 \\
(-\frac{r}{z} + \mu - \delta(x,y) - y)A_3 + \frac{\delta}{z} &= 0 \\
rA_4 + L(R+y) &= 0
\end{aligned}$$

Imposing *continuity* at the boundary:

$$\epsilon(e^{x_0}, T) = B(e^{x_0} - K) - EuB(e^{x_0}, T) \tag{26}$$

allows us to solve for the coefficient A_1 as a function of optimal default boundary c^*. Imposing the *principal of smooth fit*:

$$\epsilon'(e^{x_0}) = Be^{x_0} - \frac{\partial}{\partial x} EuB(e^x, T)|_{x=x_0} \tag{27}$$

gives the optimal default boundary.

We have a system of 5 equations and 5 unknowns and hence all the variables are identified. This completes the proof.

5.3 Proof of Proposition 1

Proof Step 1.

First, we will show that the value of continuation is always less than value of defaulting immediately, when $x = 0$.

Case 1. $c^* > L$

There is no equilibrium by definition.

Case 2. $c^* < L$

Suppose $c^* = cbar < L$, then by transversality condition, the borrower's value function is well defined and finite. Hence, $c*$ must satisfy the equation for the optimal default boundary:

$$c^* = \frac{(BK + A4)^{\frac{\beta_1\beta_2}{\eta_2}}}{(B - A3)^{\frac{(1+\beta_1)(1+\beta_2)}{(\eta_2+1)}}}$$

Note that the LHS is finite by assumption. The numerator of RHS depends on R, which we can calculate given default boundary using (8). However, the denominator is 0 and hence $c*$ is undefined, contradicting the fact that transversality condition (5) guarantees a well-defined value's function.

Case 3. $c^* = L$

When $c^* = L$, for any triplet (K, x, y) the value of continuation equals the value of immediate default by the *Principle of Smooth Fit* (i.e. the value function is continuous at c^*).

Step 2.

Let us now show that $c^* = L$ cannot be an equilibrium for any punishment cost K.

Case 1. $K < L$

If the punishment cost is less than initial loan amount, the borrower's optimal strategy is to default immediately. However, the lender knows it and hence she won't lend.

Case 2. $K > L$

If the punishment cost is higher than initial loan amount, the borrower's value function will always be negative since $J_B(L) < 0$. Hence, the borrower is better off not borrowing.

Case 3. $K = L$

The borrower has a value function exactly 0. Hence, she is indifferent between lending and borrowing.

5.4 Corporate versus Micro-loan Rates

Proof We note that in Leland(1994)'s model, we can rewrite the default boundary as:

$$\begin{aligned} c^*_{corp} &= \frac{C}{r}\frac{\beta_{corp}}{1+\beta_{corp}} \\ &= \frac{LR_{corp}}{r}\frac{\beta_{corp}}{1+\beta_{corp}} \end{aligned}$$

Similarly, in the absence of access to technology upon default and the drift of the technology process restricted to r for the existence of a martingale measure, we have $A_3 = 1$ and $B = 1$. This gives:

$$\begin{aligned} c^* &= \frac{\frac{LR}{r}\frac{\beta_1}{1+\beta_1}}{1-(1-\alpha)} \\ &= \frac{\frac{LR}{r}\frac{\beta_1}{1+\beta_1}}{\alpha} \end{aligned}$$

The first part of the proposition follows immediately once we note that $\alpha \geq 1$ is a natural assumption. After rearranging for R, the same analysis leads to the second result.

5.5 Numerical Procedure

This section documents the numerical procedure in solving for the equilibrium (R, c^*). We solve for our equilibrium as follows:

1. In the equation for c^*, we plug in the equation for equilibrium loan rate R, which depends on c^* as well. Note that the equations for R in the finite maturity case involves the terms $P(\tau > T)$ and $E[e^{-r\tau} 1_{\{\tau \leq T\}}]$, which we use the method of Laplace transforms to obtain. Specifically, we apply the Gaver–Stehfest inversion algorithm to the Laplace transforms of $P(\tau > T)$ and $E[e^{-r\tau} 1_{\{\tau \leq T\}}]$.
2. We numerically vary c^* until the fixed point equation for c^* is satisfied.
3. We then use the solution for c^* to get our equilibrium loan rate R.
4. If c^* is within the admissible range $(0, L)$, then we check whether the borrower's valuation function is positive. If so, we have an equilibrium. Otherwise, there is no equilibrium.

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