Optimal Fiscal Policy without Commitment: Revisiting Lucas-Stokey

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January 12, 2020

Abstract

According to the Lucas-Stokey result, a government can structure its debt maturity to guarantee commitment to optimal fiscal policy by future governments. In this paper, we overturn this conclusion, showing that it does not generally hold in the same model and under the same definition of time-consistency as in Lucas-Stokey. Our argument rests on the existence of an overlooked commitment problem that cannot be remedied with debt maturity: a government in the future will not tax on the downward sloping side of the Laffer curve, even if it is ex-ante optimal to do so.

Keywords: Public debt, optimal taxation, fiscal policy

JEL Classification: H63, H21, E62

*We would like to thank Fernando Alvarez, Manuel Amador, Albert Marcet, Chris Moser, Rigas Oikonomou, Juan Pablo Nicolini, Facundo Piguillem, Jesse Schreger, Kjetil Storesletten, Harald Uhlig, and seminar participants at Columbia, Federal Reserve Bank of Boston, Humboldt, Surrey, SciencesPo, University of Toronto, ASSA 2019 meetings, Royal Economic Society at Warwick, EEA-ESEM at Manchester, SED at St. Louis, and the Money Macro and Finance at LSE for comments. George Vojta provided excellent research assistance. Davide Debortoli acknowledges the financial support of the Spanish Ministry of Economy and Competitiveness through grants RyC-2016-20476 and ECO-2017-82596-P.

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1 Introduction

In a seminal paper, Lucas and Stokey (1983) consider a closed economy with no capital in which the government finances exogenous spending with taxes and debt. They argue that if the government can issue a sufficiently rich maturity of bonds, then the optimal policy is time-consistent. That is, if given the opportunity to reevaluate policy ex-post, the government would choose the ex-ante optimal policy. This result has led to a large literature that builds on this analysis and characterizes the optimal debt maturity structure, such as Alvarez et al. (2004), Persson et al. (2006), and Debortoli et al. (2017), among others.

In this paper, we overturn this result, showing that it does not generally hold in the same model and under the same definition of time-consistency as in Lucas-Stokey. Our argument rests on the existence of an overlooked commitment problem that cannot be remedied with debt maturity: a government in the future will not tax on the downward sloping side of the Laffer curve, even if it is ex-ante optimal to do so. More specifically, we construct an example in which the government wants to roll over some initial short-term debt. If initial debt is small enough, optimal policy under commitment requires future governments to choose low tax rates on the upward sloping side of the Laffer curve, and the policy is time-consistent. In contrast, and more interestingly, if initial debt is large enough, optimal policy under commitment requires future governments to choose high tax rates on the downward sloping side of the Laffer curve. This is optimal ex-ante since the reduction in future consumption (due to high future tax rates) decreases short-term interest rates today, allowing today’s government to roll over debt at a lower cost. However, a problem arises since the government tomorrow strictly prefers to repay any rolled over debt with a lower tax rate on the upward sloping side of the Laffer curve, as this maximizes consumption and welfare ex-post. Therefore, the optimal policy under commitment cannot be sustained under lack of commitment: the government in the future would never choose the preferred future tax rate of the government today, independently of the inherited government debt maturity.

Our argument does not rely on the presence of non-concavities in the government’s program and multiplicity of solutions at any date. Our example uses commonly applied isoelastic preferences in which the program is concave and the constraint set is convex at all dates. We show that under these preferences, the Lucas-Stokey procedure for guaranteeing time-consistency need not always work. More specifically, the procedure takes the optimal commitment allocation and then selects a sequence of debt portfolios and Lagrange multipliers (on future governments’ budget constraints) to satisfy future governments’ first order conditions under this allocation. We illustrate that the sign of the
Lagrange multiplier is a key part of the argument. Assuming future debt portfolios are positive at all maturities, this procedure guarantees time-consistency if the constructed future Lagrange multipliers are all positive. However, the procedure is invalid if some constructed multipliers are negative, since the shadow cost of debt cannot be negative along the equilibrium path. When the constructed multiplier is negative, today’s government and the future government disagree as to which tax rate should be chosen to satisfy the future budget constraint, and optimal policy is not time-consistent. From a practical viewpoint, this observation means that implementation of the Lucas-Stokey procedure to guarantee time-consistency may be valid, but it must be checked quantitatively. In some economies, the procedure works, whereas in others—like in our example—it does not.1

The main contribution of this paper is to highlight the limitations of the Lucas-Stokey analysis. Our work relates broadly to a literature on optimal government debt maturity in the absence of government commitment. We depart from this literature in two ways. First, we consider the optimal maturity without imposing arbitrary constraints on the bonds available to the government.2 Second, our model is most applicable to economies where the risk of default and surprise in inflation are not salient, but the government is still not committed to a path of taxes and debt maturity issuance.3 In this regard, our paper is related to the quantitative analysis of Debortoli et al. (2017), though in contrast to that work, we follow Lucas-Stokey and do not arbitrarily confine the set of bonds available to the government, and we consider a deterministic economy and ignore the presence of shocks.4

We review the Lucas-Stokey model in Section 2. In Section 3, we solve for the optimal policy under commitment, and we present conditions under which taxing on the downward sloping side of the Laffer curve is optimal. In Section 4, we present and discuss our main result. Section 5 concludes. The Online Appendix includes additional results not included in the text.

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1Our example suggests that validation should depend intuitively on the extent to which optimal taxes are on the downward sloping part of the Laffer curve. See Trabandt and Uhlig (2011) for quantitative work analyzing the shape of the Laffer curve in advanced economies.

2Krusell et al. (2006) and Debortoli and Nunes (2013) consider a similar environment to ours in the absence of commitment, but with only one-period bonds, for example.

3Other work considers optimal government debt maturity in the presence of default risk, for example, Aguiar et al. (2017), Arellano and Ramanarayan (2012), Dovis (2019), Fernandez and Martin (2015), and Niepelt (2014), among others. Bocola and Dovis (2016) additionally consider the presence of liquidity risk. Bigio et al. (2017) consider debt maturity in the presence of transactions costs. Arellano et al. (2013) consider lack of commitment when surprise inflation is possible.

4Angeletos (2002), Bhandari et al. (2017), Buera and Nicoli (2004), Faraglia et al. (2010, 2018), Guibaud et al. (2013), and Lustig et al. (2008) also consider optimal government debt maturity in the presence of shocks, but they assume full commitment.
2 Model

We consider an economy identical to the deterministic case of Lucas-Stokey, and we follow their primal approach to the evaluation of optimal policy.

2.1 Environment

There are discrete time periods \( t = \{0, 1, ..., \infty\} \). The resource constraint of the economy is

\[
c_t + g = n_t,
\]

where \( c_t \) is consumption, \( n_t \) is labor, and \( g > 0 \) is government spending, which is exogenous and constant over time.

There is a continuum of mass 1 of identical households that derive the following utility:

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \quad \beta \in (0, 1).
\]

\( u(\cdot) \) is strictly increasing in consumption, strictly decreasing in labor, globally concave, and continuously differentiable. As a benchmark, we define the first best consumption and labor \( \{c_{fb}, n_{fb}\} \) as the values of consumption and labor that maximize \( u(c_t, n_t) \) subject to the resource constraint (1).

Household wages equal the marginal product of labor (which is 1 unit of consumption), and are taxed at a linear tax rate \( \tau_t \). \( b_{t,k} \geq 0 \) represents government debt purchased by a representative household at \( t \), which is a promise to repay 1 unit of consumption at \( t + k > t \). \( q_{t,k} \) is the bond price at \( t \). At every \( t \), the household’s allocation and portfolio \( \{c_t, n_t, \{b_{t,k}\}_{k=1}^{\infty}\} \) must satisfy the household’s dynamic budget constraint:

\[
c_t + \sum_{k=1}^{\infty} q_{t,k} (b_{t,k} - b_{t-1,k+1}) = (1 - \tau_t) n_t + b_{t-1,1}. \tag{3}
\]

Moreover, the household’s transversality condition is

\[
\lim_{T \to \infty} q_{0,T} \sum_{k=1}^{\infty} q_{T,k} b_{T,k} = 0. \tag{4}
\]

\( B_{t,k} \geq 0 \) represents debt issued by the government at \( t \) with a promise to repay 1 unit of consumption at \( t + k > t \). At every \( t \), government policies \( \{\tau_t, g_t, \{B_{t,k}\}_{k=1}^{\infty}\} \) must
satisfy the government’s dynamic budget constraint:

\[ g_t + B_{t-1,1} = \tau_t n_t + \sum_{k=1}^{\infty} q_{t,k} (B_{t,k} - B_{t-1,k+1}). \tag{5} \]

The economy is closed, which means that the bonds issued by the government equal the bonds purchased by households:

\[ b_{t,k} = B_{t,k} \forall t, k. \tag{6} \]

Initial debt \( \{B_{-1,k}\}_{k=1}^{\infty} = \{b_{-1,k}\}_{k=1}^{\infty} \) is exogenous. The government is benevolent and shares the same preferences as the households in (2).

### 2.2 Primal Approach

We follow Lucas-Stokey by taking the primal approach to the characterization of competitive equilibria, since this allows us to abstract away from bond prices and taxes. Let

\[ \{c_t, n_t\}_{t=0}^{\infty} \tag{7} \]

represent a sequence of consumption and labor allocations. We can establish necessary and sufficient conditions for (7) to constitute a competitive equilibrium. The household’s optimization problem implies the following intratemporal and intertemporal conditions, respectively:

\[ 1 - \tau_t = -\frac{u_n (c_t, n_t)}{u_c (c_t, n_t)} \quad \text{and} \quad q_{t,k} = \frac{\beta^k u_c (c_{t+k}, n_{t+k})}{u_c (c_t, n_t)}. \tag{8} \]

Substitution of these conditions into the household’s dynamic budget constraint implies the following condition:

\[ u_c (c_t, n_t) c_t + u_n (c_t, n_t) n_t + \sum_{k=1}^{\infty} \beta^k u_c (c_{t+k}, n_{t+k}) b_{t,k} = \sum_{k=0}^{\infty} \beta^k u_c (c_{t+k}, n_{t+k}) b_{t-1,k+1}. \tag{9} \]

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\( ^5 \)We follow the same exposition as in Angeletos (2002) in which the government rebalances its debt in every period by buying back all outstanding debt and then issuing fresh debt at all maturities. This is without loss of generality. For example, if the government at \( t - k \) issues debt due at date \( t \) of size \( B_{t-k,k} \) which it then holds to maturity without issuing additional debt, then this can equivalently be implemented in our framework with all future governments at date \( t - k + l \) for \( l = 1, \ldots, k - 1 \) choosing \( B_{t-k+l,k-l} = B_{t-k,k} \), implying that \( B_{t-1,1} = B_{t-k,k} \).
Forward substitution into the above equation and taking into account (4) implies the following implementability condition:

$$
\sum_{k=0}^{\infty} \beta^k (u_c(c_{t+k}, n_{t+k}) c_{t+k} + u_n(c_{t+k}, n_{t+k}) n_{t+k}) = \sum_{k=0}^{\infty} \beta^k u_c(c_{t+k}, n_{t+k}) b_{t-1,k+1}.
$$

(10)

By this reasoning, if a sequence in (7) is generated by a competitive equilibrium, then it necessarily satisfies (1) and (10). Satisfaction of (1) and (10) is also sufficient for a competitive equilibrium, as we show in the below lemma.

**Lemma 1 (competitive equilibrium)** A sequence (7) is a competitive equilibrium if and only if it satisfies (1) \( \forall t \) and (10) at \( t = 0 \) given \( \{b_{-1,k}\}_{k=1}^{\infty} \).

**Proof.** The necessity of these conditions is proved in the previous paragraph. To prove sufficiency, suppose a sequence (7) satisfies (1) \( \forall t \) and (10) at \( t = 0 \) given \( \{b_{-1,k}\}_{k=1}^{\infty} \). Let the government choose the associated level of debt \( \{b_{t,k}\}_{t=0}^{\infty} \) which satisfies (9) and a tax sequence \( \{\tau_t\}_{t=0}^{\infty} \) which satisfies (8). Let bond prices satisfy (8). (9) given (1) implies that (3) and (5) are satisfied. Therefore household optimality holds and all dynamic budget constraints are satisfied along with market clearing, so the equilibrium is competitive. ■

### 3 Optimal Policy under Commitment

In this section, we solve for optimal policy in an example, and we show that, under some conditions, future tax rates should be on the downward sloping side of the Laffer curve. In the next section, we prove our main result: Applying the Lucas-Stokey definition of time-consistency, we show that in the cases where optimal tax rates are on the downward sloping side of the Laffer curve, optimal policy is not time-consistent, independently of the government’s choice of maturities. In contrast, if tax rates are on the upward sloping side of the Laffer curve, then optimal policy is time-consistent.

#### 3.1 Preferences

Consider an economy with isoelastic preferences over consumption \( c \) and labor \( n \), where

$$
u(c, n) = \log c - \eta \frac{n^\gamma}{\gamma}
$$

(11)
for \( \eta > 0 \) and \( \gamma \geq 1 \), which corresponds to a commonly used utility function for the evaluation of optimal fiscal policy (e.g., Werning, 2007).\(^6\)

Under these preferences, (1) and (8) imply that the primary surplus, \( \tau n - g \), is equal to \( c (1 - \eta (c + g)\gamma) \). To facilitate the discussion, define \( c_{laffer} \) as the level of consumption that maximizes the primary surplus:

\[
c_{laffer} = \arg \max_c c (1 - \eta (c + g)\gamma).
\]

\( c_{laffer} \) is the level of consumption associated with the maximal tax revenue at the peak of the Laffer curve under tax rate \( \tau_{laffer} \). We assume that \( g < \left( \frac{1}{\eta} \right)^{1/\gamma} \) to guarantee that \( c_{laffer} > 0 \). The primary surplus on the right hand side of (12) is depicted in Figure 1 for the quasilinear case with \( \eta = \gamma = 1 \) and \( g = 0.2 \).\(^7\) This is essentially the Laffer curve except that the x-axis refers to consumption instead of taxes which are substituted out in the primal approach. We refer to the upward and downward sloping sides of the Laffer curve in reference to the original representation of the curve as revenue \( \tau n \) as a function of the tax rate \( \tau \).

The primary surplus is strictly concave in \( c \) and equals 0 if \( c = 0 \) (100 percent labor income tax) and \(-g\) if \( c = c^{fb} \) (0 percent labor income tax). More broadly, if \( c > c_{laffer} \), then the tax rate is below the revenue-maximizing tax rate and the economy is on the upward sloping side of the Laffer curve. If \( c < c_{laffer} \), then the tax rate is above the revenue-maximizing tax rate and the economy is on the downward sloping side of the Laffer curve.

Observe that a primary surplus between 0 and \( c_{laffer} (1 - \eta (c_{laffer} + g)^\gamma) > 0 \) can be generated by the government in two ways: either with a tax rate on the upward sloping side of the Laffer curve \((c > c_{laffer})\) or with a tax rate on the downward sloping side of the Laffer curve \((c < c_{laffer})\). Importantly, the tax rate on the upward sloping side of the Laffer curve provides a strictly higher instantaneous welfare log \( c - \eta n_t/\gamma \), since consumption is higher in that case. This is an important observation to keep in mind when considering optimal policy under lack of commitment.

\(^6\)These preferences imply that the implementability condition and the primary surplus are globally concave in allocations, which provides us with analytical tractability. In the Online Appendix, we present several numerical examples under other utility functions, and we reach the same conclusion that the optimal policy is not always time-consistent.

\(^7\)This parametrization implies that \( \tau_{laffer} = 60\% \) in line with the values for the labor tax reported in Trabandt and Uhlig (2011).
Notes: This figure depicts the primary surplus, $\tau n - g$, as a function of consumption, $c$. We set $\eta = \gamma = 1$ and $g = 0.2$. The figure refers to the upward and downward sloping sides of the Laffer curve in reference to the common representation of the curve as revenue $\tau n$ as a function of the tax rate $\tau$. The upward sloping side corresponds to the case where $\tau < \tau^{laffer}$ and the downward sloping side to the case where $\tau > \tau^{laffer}$.

3.2 Optimal Policy at Date 0

Using Lemma 1, we can consider the date 0 government’s optimal policy under commitment, where we have substituted in for labor using the resource constraint (1):

$$\max_{\{c_t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \left( \log c_t - \eta \frac{(c_t + g)^\gamma}{\gamma} \right)$$

s.t.

$$\sum_{t=0}^\infty \beta^t \left( 1 - \eta (c_t + g)^\gamma - \frac{b_{-1,t+1}}{c_t} \right) = 0.$$ 

Equation (14) represents the date 0 implementability condition, which is the present value constraint of the government.

Suppose that $b_{-1,1} > 0$ and $b_{-1,k} = 0 \ \forall k \geq 2$. To guarantee the existence of a solution
that satisfies (14), let \( b_{-1,1} \leq \bar{b} \) for

\[
\bar{b} = \max_{\tilde{c}} \left\{ \tilde{c} \left( \frac{1}{1 - \beta} - \eta (\tilde{c} + g)\gamma - \frac{\beta}{1 - \beta} \eta g\gamma \right) \right\}. \tag{15}
\]

\( \bar{b} \) represents the highest value of \( b_{-1,1} \) for which (14) can be satisfied.\(^8\) We now characterize the solution to (13) – (14) given our assumption on preferences and initial debt.

**Lemma 2 (unique solution)** The solution to (13) – (14) is unique.

**Proof.** Consider the relaxed problem in which (14) is replaced with

\[
1 - \frac{b_{-1,1}}{c_0} - \eta (c_0 + g)\gamma + \sum_{t=1}^{\infty} \beta^t (1 - \eta (c_t + g)\gamma) \geq 0. \tag{16}
\]

We can establish that (16) holds as an equality in the relaxed problem, implying that the relaxed and constrained problems are equivalent. We prove this by contradiction. Suppose that (16) holds as an inequality in the relaxed problem. Then, the solution to the relaxed problem would admit \( c_t = c^{fb} \), which given (11) satisfies \( \eta c^{fb} (c^{fb} + g)\gamma^{-1} = 1 \). Substitution of \( c_t = c^{fb} \) into (16) yields

\[
\frac{1}{c^{fb}} \left( -b_{-1,1} - \frac{1}{1 - \beta} g \right) \geq 0
\]

which is a contradiction since \( b_{-1,1} > 0 \). Therefore, (16) holds as an equality in the solution to the relaxed problem and the solutions to the relaxed and constrained problems coincide.

Since the left hand side of (16) is concave in \( c_t \) for all \( t \geq 0 \) given that \( b_{-1,1} > 0 \) and since the objective (13) is strictly concave, it follows that the solution is unique. \( \blacksquare \)

Since the solution is unique, we can characterize the solution using first order conditions.

**Lemma 3 (optimal policy)** The unique solution to (13) – (14) satisfies the following properties:

1. \( c_t = c_1 \ \forall t \geq 1 \).

\(^8\)This follows since \( c_t \geq 0 \) for all \( t \), implying that (14) can be rewritten as

\[
b_{-1,1} = \sum_{t=0}^{\infty} \beta^t (c_0 (1 - \eta (c_t + g)\gamma)) \leq c_0 \left( \frac{1}{1 - \beta} - \eta (c_0 + g)\gamma - \frac{\beta}{1 - \beta} \eta g\gamma \right).
\]

Letting \( c^* \) correspond to the argument that solves (15), it follows that (14) can be satisfied for \( b_{-1,1} = \bar{b} \) and \( b_{-1,k} = 0 \ \forall k \geq 2 \) with \( c_0 = c^* \) and \( c_1 = 0 \).
2. $c_0$ and $c_1 < c_0$ are the unique solutions to the following system of equations for some $\mu_0 > 0$

\[
\frac{1}{c_0} - \eta (c_0 + g)^{\gamma - 1} + \mu_0 \left( \frac{b_{-1,1}}{c_0^2} - \eta \gamma (c_0 + g)^{\gamma - 1} \right) = 0, \quad \text{(17)}
\]

\[
\frac{1}{c_1} - \eta (c_1 + g)^{\gamma - 1} + \mu_0 \left( -\eta \gamma (c_1 + g)^{\gamma - 1} \right) = 0, \quad \text{and} \quad \text{(18)}
\]

\[
1 - \frac{b_{-1,1}}{c_0} - \eta (c_0 + g)^\gamma + \frac{\beta}{1 - \beta} (1 - \eta (c_1 + g)^\gamma) = 0. \quad \text{(19)}
\]

**Proof.** Given Lemma 2, we can consider the relaxed problem, letting $\mu_0 > 0$ correspond to the Lagrange multiplier on (16). The first order condition for $c_0$ is (17). The first order condition for $c_t$ for all $t \geq 1$ is

\[
\frac{1}{c_t} - \eta (c_t + g)^{\gamma - 1} + \mu_0 \left( -\eta \gamma (c_t + g)^{\gamma - 1} \right) = 0. \quad \text{(20)}
\]

Since the left hand side of (20) is strictly decreasing in $c_t$, it follows that the solution to (20) is unique with $c_t = c_1 \forall t \geq 1$, where (18) defines $c_1$. It follows from the fact that the program is strictly concave and constraint set convex that satisfaction of (17) – (19) is necessary and sufficient for optimality for a given $\mu_0 > 0$. We are left to verify that $c_0 > c_1$. Note that the left hand side of (17) is strictly increasing in $b_{-1,1}$ and strictly decreasing in $c_0$ for a given $\mu_0 > 0$. Therefore, $c_0$ is strictly increasing in $b_{-1,1}$ for a given $\mu_0 > 0$, where $c_0 = c_1$ if $b_{-1,1} = 0$. It follows then that since $b_{-1,1} > 0$, $c_0 > c_1$. 

The first part of the lemma states that consumption—and therefore the tax rate—is constant from date 1 onward. Since initial debt due from date 1 onward is constant (and equal to zero), tax smoothing and interest rate smoothing from date 1 onward is optimal. The optimal allocation is unique since the problem is concave.

The second part of the lemma characterizes the solution in terms of first order conditions for a positive Lagrange multiplier $\mu_0$ on the implementability constraint (14). These conditions are necessary and sufficient for optimality given the concavity of the problem. Implicit differentiation of (17) and (18), taking into account second order conditions, implies that initial consumption $c_0$ exceeds long-run consumption $c_1$, which means that the initial tax rate is below the future tax rate. Back-loading tax rates is optimal since the reduction in future consumption relative to present consumption allows the government to roll over its initial short-term debt at a lower interest rate.

We can now prove the main result of this section, which establishes that taxes from date 1 onward are on the downward sloping side of the Laffer curve—i.e., $c_1 < c_{\text{laffer}}$—if
and only if initial debt $b_{-1,1}$ is large enough. To prove this result, we first establish that $c_1$ is strictly decreasing in $b_{-1,1}$. We then show that there exists $b^* \in (0, \bar{b})$ that solves the problem with $c_1 = c^{laffer}$. We therefore obtain the result that if initial short-term debt $b_{-1,1}$ is above a threshold $b^*$, then future consumption $c_1$ is below $c^{laffer}$, implying that the future tax rate $\tau_1$ is above the revenue-maximizing tax rate at the peak of the Laffer curve $\tau^{laffer}$. Otherwise, $c_1$ is above $c^{laffer}$, and the future tax rate $\tau_1$ is below the revenue-maximizing tax rate at the peak of the Laffer curve.

**Proposition 1 (taxes relative to peak of Laffer curve)** There exists $b^* \in (0, \bar{b})$ such that the solution admits $c_1 > c^{laffer}$ if $b_{-1,1} < b^*$ and $c_1 < c^{laffer}$ if $b_{-1,1} > b^*$.

**Proof.** We prove this result in two steps.

**Step 1.** We establish that the solution to the system in (17) – (19) admits $c_1$ that is strictly decreasing in $b_{-1,1}$. Let $F^0(c_0, \mu_0, b_{-1,1})$ correspond to the function on the left hand side of (17), let $F^1(c_1, \mu_0)$ correspond to the function on the left hand side of (18), and let $I(c_0, c_1, b_{-1,1})$ correspond to the function on the left hand side of (19). Since the solution to this system of equations is unique, we can apply the Implicit Function Theorem. Implicit differentiation yields

$$\frac{dc_1}{db_{-1,1}} = \frac{-F^0_{c_0} I_{b_{-1,1}} + F^0_{b_{-1,1}} I_{c_0}}{F^0_{c_0} I_{c_1} + \frac{F^0_{\mu_0} F^1_{c_1} I_{c_0}}{F^1_{\mu_0}}}. \tag{21}$$

From the second order conditions for (17) and (18), $F^0_{c_0} < 0$ and $F^1_{c_1} < 0$. Moreover, by inspection, $I_{c_1} < 0$ and $F^1_{\mu_0} < 0$. Finally, note that $F^0_{\mu_0} I_{c_0} = [I_{c_0}]^2 > 0$. This establishes that the denominator in (21) is positive. To determine the sign of the numerator, let us expand the numerator by substituting in for the functions. By some algebra, the numerator is equal to

$$\frac{1}{c_0} \left( -\frac{1}{c_0^2} - \eta (\gamma - 1) (c_0 + g)^{\gamma - 2} \right) + \mu_0 \left[ -\frac{b_{-1,1}}{c_0^4} - \frac{1}{c_0} \eta \gamma (\gamma - 1) (c_0 + g)^{\gamma - 2} - \frac{1}{c_0^2} \eta \gamma (c_0 + g)^{\gamma - 1} \right] < 0.$$

This establishes that $c_1$ is strictly decreasing in $b_{-1,1}$.

**Step 2.** We complete the proof by establishing that there exists $b^* \in (0, \bar{b})$ for which the solution to (17) – (19) admits $c_1 = c^{laffer}$. We first establish that if $b^*$ exists, it exceeds 0. Note that if $b_{-1,1} = 0$ then the solution admits $c_1 > c^{laffer}$. This is because (17) – (19) imply that the solution admits $c_0 = c_1$. Substitution into (19) yields

$$\frac{c_1 (1 - \eta (c_1 + g)^\gamma)}{1 - \beta} = 0. \tag{22}$$
This equation admits two solutions: \( c_1 = 0 \) and \( c_1 = \eta^{-1/\gamma} - g \), and the optimal policy satisfies \( c_1 = \eta^{-1/\gamma} - g \), since welfare is arbitrarily low otherwise. Given the definition of \( \ell^{\text{affer}} \) in (12) and the strict concavity of the objective in (12), it follows that \( \ell^{\text{affer}} \) must strictly be between 0 and \( \eta^{-1/\gamma} - g \), which means that \( c_1 > \ell^{\text{affer}} \).

We now establish that \( b^* \) below \( \bar{b} \) exist, where \( b^* \) solves the system (17)–(19) for \( b_{-1,1} = b^* \) and \( c_1 = \ell^{\text{affer}} \). To see that such a solution exists, note that \( \frac{1}{\ell^{\text{affer}}} - \eta \left( \ell^{\text{affer}} + g \right)^{\gamma-1} \) is positive since \( \ell^{\text{affer}} < f^b \). Therefore, a value of \( \mu_0 > 0 \) which satisfies (18) under \( c_1 = \ell^{\text{affer}} \) exists. Multiply (17) by \( c_0 \) and substitute (19) into (17) to achieve

\[
1 - \eta c_0 (c_0 + g)^{\gamma-1} + \mu_0 \left( 1 - \eta (c_0 (1 + \gamma) + g) (c_0 + g)^{\gamma-1} + \frac{\beta}{1 - \beta} (1 - \eta (\ell^{\text{affer}} + g) ) \right) = 0.
\]

Note that given the value of \( \mu_0 > 0 \) satisfying (18) for \( c_1 = \ell^{\text{affer}} \), a solution to (23) which admits \( c_0 > 0 \) exists. This is because the left hand side of (23) goes to

\[
1 + \mu_0 \left( 1 - \eta g^{\gamma} + \frac{\beta}{1 - \beta} (1 - \eta (\ell^{\text{affer}} + g) ) \right) > 0
\]

as \( c_0 \) goes to 0, where we have used the fact that \( g < \left( \frac{1}{\eta} \right)^{1/\gamma} \). As \( c_0 \) goes to infinity, the left hand side of (23) becomes arbitrarily negative. Therefore a solution to (23) for \( c_0 > 0 \) exists. Given that \( b_{-1,1} \) enters linearly in (19), it follows that a value of \( b_{-1,1} \) which satisfies the system also exists. ■

### 3.3 Taxation on the Downward Sloping Side of the Laffer Curve

According to Proposition 1, it may be optimal to choose a constant tax rate from date 1 onward on the downward sloping side of the Laffer curve. This result may appear puzzling, and to better understand it, let’s consider the set of policies available to the government under commitment.

An implication of Lemma 1 is that there are multiple maturity structures of debt issuance that are consistent with optimal policy. For example, suppose that the government issues a flat maturity at date 1 and never rebalances its portfolio. Specifically, it chooses some debt \( \{b_{0,k}\}_{k=1}^\infty \), where \( b_{0,k} = b \) \( \forall k \geq 1 \) for some \( b > 0 \), while also choosing a smooth path of consumption from date 1 onward, with \( c_t = c_1 \) \( \forall t \geq 1 \) that satisfies its budget.
constraint (9), so that

\[ c_0 (1 + \eta (c_0 + g)^\gamma) + \frac{\beta}{1 - \beta} \frac{c_0 b}{c_1} = b_{-1,1} \text{ and } \]

\[ c_1 (1 - \eta (c_1 + g)^\gamma) = b. \tag{24} \]

According to (25), the primary surplus from date 1 onward is constant and equal to the issued debt at date 0, which is a consol of size \( b \). The optimal policy satisfying Lemma 3 can be implemented with this issued debt maturity structure. To understand what drives optimal policy, we can consider how the economy—characterized by (24) and (25)—changes under different values of \( b \), given some initial \( b_{-1,1} \).

Suppose that at date 0 the government issues a given amount of debt \( b \) at date 0. It then faces a trade-off at date 1 in its choice of how to repay the debt. On the one hand, it could set low tax rates (on the upward sloping side of the Laffer curve), associated with a high level of consumption and utility in date 1. On the other hand, it could collect the same tax revenues setting high tax rates (on the downward sloping side of the Laffer curve), thereby reducing consumption and utility at date 1. While the former strategy clearly maximizes flow utility at date 1, the latter strategy actually maximizes the flow utility at date 0, since it reduces the interest rate at date 0.

This policy trade-off is illustrated in Figure 2 for a case with quasilinear preferences with \( \eta = \gamma = 1, g = 0.2, \) and \( \beta = 0.96 \). Panel A displays utility from date 1 onward, \( \log c_1 - \eta (c_1 + g)^\gamma / \gamma \) and shows that for a given value of issued debt \( b \), there are two possible values of welfare: a higher value associated with a higher \( c_1 \) on the upward sloping side of the Laffer curve, and a lower value associated with a lower \( c_1 \) on the downward sloping side of the Laffer curve. Panel B in Figure 2 displays the gross interest rate at date 0, \( R \equiv \beta^{-1} c_1 / c_0 \), as a function of issued debt \( b \). For a given value of issued debt, there are two possible interest rates: a higher interest rate associated with a higher \( c_1 \) on the upward sloping side of the Laffer curve, and a lower interest rate associated with a lower \( c_1 \) on the downward sloping side of the Laffer curve. As such, while taxing on the upward sloping side of the Laffer curve increases welfare from date 1 onward, taxing on the downward sloping side of the Laffer curve increases welfare at date 0, since it increases the price of the issued debt.

As initial debt \( b_{-1,1} \) increases, the relative benefits of taxing on the upward sloping side of the Laffer curve at date 1 decrease, since increasing the price of the issued debt at date 0 becomes more important for the government. In other words, conditional on the total amount of government borrowing at date 0, taxing on the upward sloping side
Figure 2: Future Utility and Current Interest Rate as a Function of Issued Debt

Notes: The x-axis in both panels is the issued debt at \( t = 0, \) \( b \). The y-axis in Panel A is utility from \( t = 1 \) onward, \( \log(c_1) - \eta(c_1 + g)^\gamma/\gamma \). The y-axis in Panel B is the interest rate at \( t = 0 \). We set \( \eta = \gamma = 1, \beta = 0.96, g = 0.2, \) and \( b_{-1,1} = 0 \).

of the Laffer curve increases the interest rate in date 0 and, therefore, consumption \( c_0 \) declines. This effect through the interest rate is stronger the higher is \( b_{-1,1} \) because the amount of debt being rolled over in date 0 is higher (see equation (24)). This is the logic underlying Proposition 1. Once \( b_{-1,1} \) exceeds some value \( b^* \), it becomes optimal to tax on the downward sloping side of the Laffer curve at date 1.

Figure 3 displays optimal policy as a function of initial debt \( b_{-1,1} \), with Panel A displaying tax revenue at date 1, Panel B displaying consumption at date 1, and Panel C displaying the interest rate at date 0. As \( b_{-1,1} \) increases between 0 and \( b^* \), the government responds by issuing more debt at date 0, which it repays with higher taxes that generate higher tax revenues from date 1 onward, while remaining on the upward sloping side of the Laffer curve from date 1 onward. The result is a reduction in consumption and welfare from date 1 onward, as well as a reduction in the interest rate faced at date 0, as implied by the lower consumption at date 1. At \( b_{-1,1} = b^* \), the issued debt at date 0 is maximized and taxes from date 1 onward are chosen at the peak of the Laffer curve.

As \( b_{-1,1} \) increases beyond \( b^* \), the comparative statics with respect to changing initial
conditions become very different. The government is now taxing on the downward sloping side of the Laffer curve from date 1 onwards, as it prioritizes reducing the interest costs of rolling over initial debt. In this case, a higher initial level of debt $b_{-1,1}$ leads to a decrease, as opposed to an increase, in $b$, the issued debt at date 0. In fact, a (marginal) increase in $b$ would require higher tax revenues in the future and—being on the downward sloping side of the Laffer curve—a lower future tax rate. In turn, this would lead to higher future consumption, and thus to an increase in the interest rate at date 0. This is why if initial debt $b_{-1,1}$ increases, the government decreases the issued debt $b$ so as to increase future tax rates and decrease future consumption in an effort to reduce the date 0 interest rate.

A natural question regards what factors drive the value of $b^*$, since a higher $b^*$ implies a higher debt threshold for future taxes to be on the downward sloping side of the Laffer curve. We performed numerically these comparative statics around the benchmark conditions.

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Notes: The x-axis in all panels is the initial debt at $t = 0$, $b_{-1,1}$. The y-axis in Panel A is revenue from $t = 1$ onward, $\tau_1 n_1$. The y-axis in Panel B is consumption from $t = 1$ onward, $c_1$. The y-axis in Panel C is the interest rate at $t = 0$. We set $\eta = \gamma = 1$, $g = 0.2$, and $\beta = 0.96$.

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These effects can also be observed in Figure 2.
quasilinear example of Figures 2 and 3. We find that $b^\ast$ is increasing in $\beta$. That is, taxes from date 1 onward are more likely to be on the downward sloping side of the Laffer curve if the government is relatively impatient. This is intuitive, since a lower $\beta$ implies that the government places more weight on boosting utility at date 0 versus the future, and it therefore prioritizes reducing the cost of rolling over the initial debt. Moreover, $b^\ast$ is decreasing in $\eta$, because a higher value of $\eta$ implies a lower maximal tax revenue $c_{\text{laffer}} (1 - \eta (c_{\text{laffer}} + g)^\gamma)$, and therefore a lower tax capacity for the government. This diminished tax capacity at date 1 implies that the government at date 0 is more likely to accommodate an increase in initial liabilities $b_{-1,1}$ by reducing the cost of rolling over the initial debt versus increasing taxation at date 1.

4 Time-Consistency of Optimal Policy

We now show that the policy under commitment may not be time-consistent. We follow Lucas-Stokey and consider what happens if at date 1, policy is reevaluated and chosen by a government with full commitment from date 1 onward. As in Lucas-Stokey, we define an optimal policy as time-consistent if the government at date 1 chooses the same allocation as the government at date 0.

4.1 Optimal Policy at Date 1

Given an inherited portfolio of maturities, the government at date 1 solves the following problem:

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \left( \log c_t - \frac{\eta (c_t + g)^\gamma}{\gamma} \right)$$

s.t.

$$\sum_{t=1}^{\infty} \beta^{t-1} \left( 1 - \eta (c_t + g)^\gamma - \frac{b_{0,t}}{c_t} \right) = 0.$$  \hfill (27)

Letting $\mu_1$ represent the Lagrange multiplier on (27), first order conditions with respect to $c_t$ are:

$$\frac{1}{c_t} - \eta (c_t + g)^{\gamma - 1} + \mu_1 \left( \frac{b_{0,t}}{c_t^2} - \eta \gamma (c_t + g)^{\gamma - 1} \right) = 0 \ \forall t \geq 1.$$  \hfill (28)

An optimal policy is therefore time-consistent if the solution to (26) – (27) coincides with the solution to (13) – (14).
Proposition 2 (time-consistency of optimal policy) If \( b_{-1,1} < b^* \), then the optimal date 0 policy is time-consistent. If \( b_{-1,1} > b^* \), then the optimal date 0 policy is not time-consistent.

Proof. We consider each case separately.

**Case 1.** Suppose that \( b_{-1,1} < b^* \). From Proposition 1, the date 0 solution admits \( c_t = c_1 > c_{laffer} \forall t \geq 1 \). To show that this solution is time-consistent, suppose that the date 0 government chooses \( \{b_{0,k}\}_{k=1}^\infty \) satisfying

\[
b_{0,k} = c_1 (1 - \eta (c_1 + g)\gamma) > 0 \forall k \geq 1
\]  

(29)

for \( c_1 \) defined in (17) − (19). \( b_{0,k} > 0 \) since the highest value of \( c_1 > c_{laffer} \) is below that associated with \( b_{-1,1} = 0 \) which satisfies (22), given the arguments in the proof of Proposition 1. Now consider the solution to (26) − (27). Analogous arguments as those in the proofs of Lemmas 2 and 3 imply that the unique solution satisfies (27) and (28) for some \( \mu_1 > 0 \). Therefore, to check that the date 1 solution admits \( c_t = c_1 \forall t \geq 1 \) for \( c_1 \) which satisfies (18), it is sufficient to check that there exists some \( \mu_1 > 0 \) satisfying (28). Using (29) to substitute in for \( b_{0,k} \) in (28), we find that

\[
\mu_1 = -\frac{1 - \eta c_1 (c_1 + g)\gamma^{-1}}{1 - \eta (c_1 + g)\gamma - \eta \gamma c_1 (c_1 + g)\gamma^{-1}} > 0,
\]  

(30)

where we have appealed to the fact that \( c_1 < c_{fb} \) (from (18)) to assign a positive sign to the numerator in (30) and the fact that \( c_1 > c_{laffer} \) to assign a negative sign to the denominator in (30). This establishes that the date 0 solution is time-consistent.

**Case 2.** Suppose that \( b_{-1,1} > b^* \) and suppose by contradiction that the optimal date 0 policy is time-consistent. This would require (28) to hold for \( c_t = c_1 \forall t \geq 1 \) for \( c_1 < c_{laffer} \) which satisfies (18). For a given \( \mu_1 \), satisfaction of (28) thus requires that \( b_{0,k} = b_{0,1} \forall k \geq 1 \). Equation (27) thus implies that (29) for \( b_{0,k} > 0 \) holds, and substitution of (29) into (28) implies that

\[
\mu_1 = -\frac{1 - \eta c_1 (c_1 + g)\gamma^{-1}}{1 - \eta (c_1 + g)\gamma - \eta \gamma c_1 (c_1 + g)\gamma^{-1}} < 0,
\]  

(31)

where we have appealed to the fact that \( c_1 < c_{fb} \) (from (18)) to assign a positive sign to the numerator and the fact that \( c_1 < c_{laffer} \) to assign a positive sign to the denominator. However, conditional on \( \{b_{0,k}\}_{k=1}^\infty \) for \( b_{0,k} > b_{0,1} > 0 \forall k \geq 1 \), the solution to (26) − (27) must admit a positive multiplier \( \mu_1 > 0 \), and this follows by analogous arguments as those
in the proofs of Lemmas 2 and 3, which contradicts (31). Therefore, the date 1 solution does not coincide with the date 0 solution.

If $b_{-1,1} < b^*$, then the optimal date 0 policy can be sustained under lack of commitment with the government at date 0 issuing a flat maturity distribution with $b_{0,k} = b_{0,1} \forall k \geq 1$. Under such a flat distribution, the government at date 1 optimally chooses to smooth tax rates into the future.\textsuperscript{10} Moreover, given that date 1 tax rates under commitment are on the upward sloping side of the Laffer curve, the choice of such tax rates is time-consistent. The date 0 and date 1 governments agree about the optimal tax rate to repay this debt.

If instead $b_{-1,1} > b^*$, then the optimal date 0 policy cannot be sustained under lack of commitment. If the government at date 0 tried to induce the date 1 government into a smooth policy from date 1 onward by issuing a flat maturity distribution with $b_{0,k} = b_{0,1} \forall k \geq 1$, the date 1 government would never choose a value $c_1 < c_{laffer}$ and $\tau_1 > \tau_{laffer}$ and would instead repay the inherited debt with a value $c_1 > c_{laffer}$ and $\tau_1 < \tau_{laffer}$. Choosing a lower tax rate on the upward sloping side of the Laffer curve increases consumption and increases welfare ex-post. Thus, while the date 0 government can commit the date 1 government to a smooth path of revenue and interest rates, it cannot commit the date 1 government to a particular tax rate. As such, the optimal date 0 policy is not time-consistent.

\subsection*{4.2 Why the Lucas-Stokey Argument Fails}

It is instructive to consider why the original arguments of Lucas-Stokey can fail in our example. In developing their argument, Lucas-Stokey consider the optimal allocation under commitment from the perspective of date 0, which satisfies the following first order condition for $t \geq 1$ (the analog of (18) starting from any arbitrary initial maturity distribution, under general utility functions, after suppressing some notation):

\begin{equation}
 (u_{ct} + u_{nt})(1 + \mu_0) + \mu_0 \left( -\left( u_{cc,t} + u_{cn,t} \right) b_{-1,t+1} + \left( u_{cc,t} + 2u_{cn,t} + u_{nn,t} \right) c_t + \left( u_{cn,t} + u_{nn,t} \right) g \right) = 0 \ \forall t \geq 1. \quad (32)
 \end{equation}

Lucas-Stokey claim that the optimal policy under commitment at date 0 that satisfies (32) could be made time-consistent at date 1. They argue that this is possible with the appropriate choice of maturities that satisfy the date 1 implementability condition (27),

\textsuperscript{10}This flat maturity structure is equivalent to a consol. The use of consols has been pursued historically, most notably by the British government during the Industrial Revolution, when consols were the largest component of the British government’s debt (see Mokyr, 2011). Moreover, the introduction of consols has been discussed as a potential option in the management of U.S. government debt (e.g. Cochrane, 2015), an idea that is supported by the quantitative analysis of Debortoli et al. (2017).
which can be rewritten more generally as
\[ \sum_{t=1}^{\infty} \beta^{t-1}((u_{c,t} + u_{n,t})c_t + u_{n,t}g) = \sum_{t=1}^{\infty} \beta^{t-1}u_{c,t}b_{0,t} \] (33)
and the future government’s first order condition at date 1 (28), which can be rewritten more generally as
\[ (u_{c,t} + u_{n,t})(1+\mu_1) + \mu_1 \left( -u_{cc,t} + u_{cn,t}b_{0,t} \right) \\
+ (u_{cc,t} + 2u_{cn,t} + u_{nn,t})c_t + (u_{cn,t} + u_{nn,t})g \right) = 0 \ \forall t \geq 1. \] (34)
for some Lagrange multiplier \( \mu_1 \). Their procedure thus combines (32) and (34) to yield:
\[ b_{0,t} = b_{-1,t+1} + \frac{u_{c,t} + u_{n,t}}{u_{cc,t} + u_{cn,t}} \left( \frac{1 + \mu_1}{\mu_1} - \frac{1 + \mu_0}{\mu_0} \right) \ \forall t \geq 1, \] (35)
which determines the issued maturity distribution at date 0 as a function of four objects: the inherited maturity distribution, the optimal allocation, and the Lagrange multipliers \( \mu_0 \) and \( \mu_1 \).

According to Lucas-Stokey logic, given an optimal allocation and value of \( \mu_0 \) from the perspective of date 0, one can construct a value of \( \mu_1 \) and a portfolio of bonds \( \{b_{0,k}\}_{k=1}^{\infty} \) that satisfy (33) and (34), and accordingly, this implies that the policy is time-consistent. To see why this logic is flawed, suppose for illustration that the constructed values of \( \{b_{0,k}\}_{k=1}^{\infty} \) are all non-negative, so that the constraint represented by (33) must imply a positive shadow value of debt. Then if the constructed value of \( \mu_1 \) that satisfies (33) and (34) is negative, Lucas-Stokey logic fails and the optimal policy is not time-consistent. Intuitively, the solution to the date 1 problem under a positive debt portfolio \( \{b_{0,k}\}_{k=1}^{\infty} \) would never admit a negative multiplier—since the shadow cost of inherited debt is positive.\(^{11}\)

Our specific example illustrates a situation in which \( \mu_1 < 0 \) and the Lucas-Stokey construction fails. (33) and (35) in our example can be written as
\[ b_{0,1} = c_1 \left( 1 - \eta (c_1 + g)^\gamma \right), \] (36)
\[ b_{0,1} = \left( 1 - \frac{\mu_0}{\mu_1} \right) \eta \gamma c_1^2 (c_1 + g)^{\gamma-1}, \] (37)
respectively, for \( \mu_0 \) and \( c_1 \) that satisfy (17) – (19). If \( b_{-1,1} < b^* \), the solution to (36) – (37) admits \( \mu_1 > 0 \), and the optimal policy is time-consistent. If instead \( b_{-1,1} > b^* \), the
\(^{11}\)If the implied value of \( \mu_1 \) is positive, then Lucas-Stokey logic holds with the optimal policy being time-consistent, assuming that the date 0 and date 1 programs for the government are concave.
solution to (36) – (37) admits $\mu_1 < 0$, and the optimal policy is not time-consistent, since the shadow cost of debt cannot be negative.\footnote{It is also straightforward to see that our example would work using the same logic if $b_{-1,k} = \hat{b}, \forall k \geq 2$ for some $\hat{b} > 0$ (rather than $\hat{b} = 0$) as well as in an economy with state-contingent bonds with a similar decay structure.}

## 5 Concluding Remarks

An important literature on optimal fiscal policy without commitment has built on the Lucas-Stokey conclusion that a government can structure debt maturity issuance to guarantee commitment by future governments. In this paper, we overturn this result, using the same model and the same definition of time-consistency as Lucas-Stokey under standard assumptions on preferences. We show using an example that whether or not the Lucas-Stokey conclusion holds depends on the environment.

There are three important points to note regarding our example. First, our example does not rely on the presence of an infinite horizon, which we only choose here to be consistent with Lucas-Stokey. A $T$-period version of this example would yield the same conclusion, namely that in some cases, the optimal policy under commitment does not coincide with that under lack of commitment.

Second, our example does not rely on the presence of non-concavities in the government’s program and multiplicity of solutions at any date. Our isoelastic preferences imply that the government’s welfare is concave and the constraint set is convex, which guarantees that the solution to the government’s problem at dates 0 and 1 is unique. We conjecture that considering cases with multiplicity (for instance examples with negative debt positions, which may make the implementability condition no longer a convex constraint) could make it even more challenging for today’s government to induce commitment by future governments.

Finally, our paper provides a method of verifying whether or not the Lucas-Stokey procedure holds in other environments with a different utility function or initial maturity distribution of government debt. For example, take a model that satisfies standard dynamic programming properties with a globally concave program for the government at all future dates $t$ (so that first order conditions are necessary and sufficient to characterize the solution from the perspective of date $t$), where the shadow value of debt is positive at every date $t$. It then follows that if the Lagrange multipliers at all future dates $t$ constructed by the Lucas-Stokey procedure—that is, the analogs of $\mu_1$ in (37)—are positive, then the Lucas-Stokey procedure is valid. If instead some multipliers are negative, as is
the case in our constructed example, then the Lucas-Stokey procedure is not valid since the shadow cost of debt cannot be negative.

The Lucas-Stokey model has motivated an enormous literature that has extended their framework to environments with incomplete markets, financial frictions, liquidity frictions, and international flows. We have focused on a simple example to illustrate that their conclusions cannot always be directly applied. Our analysis implies that any study of optimal fiscal policy without commitment must move beyond the Lucas-Stokey definition of time-consistency, since the optimal policy may not be time-consistent. Instead, future work should consider the solution to a dynamic game between sequential governments, taking into account that the commitment and no-commitment solution may not coincide.

References


