Optimal Fiscal Policy without Commitment: Revisiting Lucas-Stokey*

Davide Debortoli† Ricardo Nunes‡ Pierre Yared§

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Abstract

According to the Lucas-Stokey result, a government can structure its debt maturity to guarantee commitment to optimal fiscal policy by future governments. In this paper, we overturn this conclusion, showing that it does not generally hold in the same model and under the same definition of time-consistency as in Lucas-Stokey. Our argument rests on the existence of an overlooked commitment problem that cannot be remedied with debt maturity: a government in the future will not necessarily tax above the peak of the Laffer curve, even if it is ex-ante optimal to do so.

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†Universitat Pompeu Fabra, CREI, Barcelona GSE and CEPR: davide.debortoli@upf.edu.

‡University of Surrey and CIMS: ricardo.nunes@surrey.ac.uk.

§Columbia University and NBER: pyared@columbia.edu.

1 Introduction

In a seminal paper, Lucas and Stokey (1983) consider a closed economy with no capital in which the government finances exogenous spending with taxes and debt. They argue that if the government can issue a sufficiently rich maturity of bonds, then the optimal policy is time-consistent. That is, if given the opportunity to reevaluate policy ex-post, the government would choose the ex-ante optimal policy. This result has led to a large literature that builds on this analysis and characterizes the optimal debt maturity structure, such as Alvarez et al. (2004), Persson et al. (2006), and Debortoli et al. (2017), among others.

In this paper, we overturn this result, showing that it does not generally hold in the same model and under the same definition of time-consistency as in Lucas-Stokey. Our argument rests on the existence of an overlooked commitment problem that cannot be remedied with debt maturity: a government in the future will not necessarily tax above the peak of the Laffer curve, even if it is ex-ante optimal to do so.

We consider an example in which the government at date 0 must finance some initial short-term debt. The optimal date 0 policy under commitment maximizes social welfare subject to the government budget constraint that guarantees that the present value of primary surpluses equal the initial debt. To assess the time-consistency of optimal policy, we consider whether the date 1 government would deviate from the date 0 optimal policy if given an opportunity to do so. We show that the policy is not time-consistent if the initial debt is high enough. The first part of our argument establishes that if initial debt is high enough, optimal policy under commitment implies that the date 1 government chooses high tax rates above the peak of the Laffer curve. The second part of our argument establishes that the date 1 government would never optimally choose to tax above the peak of the Laffer curve.

The first part of our argument rests on the fact that taxing above the peak of the Laffer curve in the future reduces future consumption and hence reduces current interest rates on any debt that is rolled over. When initial debt is sufficiently high, the immediate benefit of rolling over initial debt at a low cost outweighs the cost of taxing above the peak of the Laffer curve in the future.

To see how this works, let us start from the highest level of initial debt compatible with the date 0 government budget constraint. Such a level of debt must admit taxes from date 1 onward above the peak of the Laffer curve. Suppose this is not the case and that these taxes are at the peak of the Laffer curve, thus maximizing static surpluses from date 1 onward. Consider a perturbation that increases those future taxes by $\epsilon > 0$ arbitrarily small. The result is a second order loss in static primary surpluses from date 1 onward (since the perturbation is around the peak of the Laffer curve), but a first order increase in date 0 bond prices because of lower consumption from date 1 onward (due to higher taxes). Consequently, the perturbation increases the present value of primary surpluses and relaxes the date 0 government budget constraint, implying that initial debt could be even higher. By this logic, for extremely high values of initial debt, the government at date 0 must choose taxes above the peak of the Laffer curve in order to satisfy its budget constraint.

We extend this logic to cases where the date 0 government budget constraint can be satisfied with taxes below or above the peak of the Laffer curve. We show that in such cases if initial debt is high enough, optimal taxes from date 1 onward are above the peak of the Laffer curve. In our example, back-loading tax rates is optimal since the reduction in future consumption relative to present consumption allows the date 0 government to issue debt at a higher bond price. Therefore, an increase in taxes from date 1 onward loosens the date 0 government budget constraint, allowing a decrease in date 0 taxes. If initial debt is sufficiently high, the government budget constraint is sufficiently tight that the utility cost of increasing future taxes above the peak of the Laffer curve from date 1 onward is outweighed by the utility benefit at date 0. As such, even though taxing below the peak of the Laffer curve is feasible, it is optimal to increase future taxes and relax the current budget constraint.

The second part of our argument considers the time-consistency of optimal policy. Through its choice of debt, the date 0 government can commit the date 1 government to a level of revenue. However, the date 0 government cannot commit the date 1 government to a specific tax rate to raise this revenue. If two tax rates at date 1 are consistent with this revenue, the government at date 1 will choose the optimal one from its perspective. Thus, the date 1 optimal policy may not coincide with the date 0 optimal policy. If initial debt is high enough, the date 0 government would like to promise that taxes from date 1 onward will be above the peak of the Laffer curve in order to relax the date 0 budget constraint. However, this promise is not credible since the date 1 government facing the date 1 budget constraint can always be made strictly better off deviating from this promise and choosing taxes below the peak of the Laffer curve, as this will boost date 1 consumption and utility.

Time-inconsistency arises since the date 0 government internalizes the impact of date 1 policies on the date 0 government budget constraint through bond prices, whereas the date 1 government does not. If initial debt is low, this does not cause a problem, as both governments agree that taxes below the peak of the Laffer curve are optimal at date 1. If initial debt is high, there is a conflict of interest between the two governments: the date 0 government prefers date 1 taxes to exceed the peak of the Laffer curve, whereas the date 1 government does not.

The main contribution of this paper is to highlight the limitations of the Lucas-Stokey analysis. Our results do not rely on the presence of non-concavities in the government's program and multiplicity of solutions at any date. We use commonly applied isoelastic preferences in which the program is concave and the constraint set is convex at all dates. We show that under these preferences, the Lucas-Stokey procedure for guaranteeing time-consistency need not always work. More specifically, the procedure takes the optimal commitment allocation and then selects a sequence of debt portfolios and Lagrange multipliers (on future governments' budget constraints, also referred to as future implementability constraints) to satisfy future governments' first order conditions under this allocation. We illustrate that the sign of the constructed Lagrange multiplier is a key part of the argument. Assuming future debt portfolios are positive at all maturities, this procedure guarantees time-consistency if the constructed future Lagrange multipliers are all positive. However, the procedure is invalid if some constructed multipliers are negative, since the shadow cost of debt cannot be negative along the equilibrium path. When the constructed multiplier is negative, today's government and the future government disagree as to which tax rate should be chosen to satisfy the future budget constraint, and optimal policy is not time-consistent. From a practical viewpoint, this observation means that implementation of the Lucas-Stokey procedure to guarantee time-consistency may be valid, but it must be checked quantitatively. In some economies, the procedure works, whereas in others—like in our example—it does not.[1]

Our work relates broadly to a literature on optimal government debt maturity in the absence of government commitment.[2] We depart from this literature by focusing on economies where the risk of default and surprise in inflation are not salient, but the government is still not committed to a path of taxes and debt maturity issuance.[3] Our paper is related to the quantitative analysis of Debortoli et al. (2017), though in contrast to that work, we follow Lucas-Stokey and do not arbitrarily confine the set of bonds available to the government, and we consider a deterministic economy and ignore the presence of shocks.[4]

We review the Lucas-Stokey model in Section 2. In Section 3, we solve for the optimal policy under commitment, and present conditions under which taxing above the peak of the Laffer curve is optimal. In Section 4, we present and discuss our main result. Section 5 concludes. The Online Appendix includes additional results not included in the text.

[1] Our example suggests that validation should depend intuitively on the extent to which optimal taxes are on the downward sloping part of the Laffer curve. See Trabandt and Uhlig (2011) for quantitative work analyzing the shape of the Laffer curve in advanced economies.

[2] Krusell et al. (2006) and Debortoli and Nunes (2013) consider a similar environment to ours in the absence of commitment, but with only one-period bonds, for example.

[3] Other work considers optimal government debt maturity in the presence of default risk, for example, Aguiar et al. (2017), Arellano and Ramanarayanan (2012), Dovis (2019), Fernandez and Martin (2015), and Niepelt (2014), among others. Bocola and Dovis (2016) additionally consider the presence of liquidity risk. Bigio et al. (2017) consider debt maturity in the presence of transactions costs. Arellano et al. (2013) consider lack of commitment when surprise inflation is possible.

[4] Angeletos (2002), Bhandari et al. (2017), Buera and Nicolini (2004), Faraglia et al. (2010, 2018), Guibaud et al. (2013), and Lustig et al. (2008) also consider optimal government debt maturity in the presence of shocks, but they assume full commitment.

2 Model

We consider an economy identical to the deterministic case of Lucas-Stokey, and we follow their primal approach to the evaluation of optimal policy.

2.1 Environment

There are discrete time periods $t = \{0, 1, ..., \infty\}$. The resource constraint of the economy is

$$c_t + g = n_t, \tag{1}$$

where c_t is consumption, n_t is labor, and $g > 0$ is government spending, which is exogenous and constant over time.

There is a continuum of mass 1 of identical households that derive the following utility:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right), \ \beta \in (0, 1). \tag{2}$$

The function $u(\cdot)$ is strictly increasing in consumption, strictly decreasing in labor, globally concave, and continuously differentiable. As a benchmark, we define the first best consumption and labor $\left\{c^{fb}, n^{fb}\right\}$ as the values of consumption and labor that maximize $u\left(c_t, n_t\right)$ subject to the resource constraint (1).

Household wages equal the marginal product of labor (which is 1 unit of consumption), and are taxed at a linear tax rate τ_t. The value of $b_{t,t+k} \gtreqless 0$ represents government debt purchased by a representative household at t, which is a promise to repay 1 unit of consumption at $t+k > t$. The value of $q_{t,t+k}$ is the bond price at t. At every t, the household's allocation and portfolio $\left\{c_t, n_t, \{b_{t,t+k}\}_{k=1}^{\infty}\right\}$ must satisfy the household's dynamic budget constraint:

$$c_t + \sum_{k=1}^{\infty} q_{t,t+k}\left(b_{t,t+k} - b_{t-1,t+k}\right) = \left(1 - \tau_t\right) n_t + b_{t-1,t}. \tag{3}$$

Moreover, the household's transversality condition is

$$\lim_{T \to \infty} q_{0,T} \sum_{k=1}^{\infty} q_{T,T+k} b_{T,T+k} = 0. \tag{4}$$

$B_{t,t+k} \gtreqless 0$ represents debt issued by the government at t with a promise to repay 1 unit of consumption at $t + k > t$. At every t, government policies $\left\{\tau_t, g_t, \{B_{t,t+k}\}_{k=1}^{\infty}\right\}$ must satisfy the government's dynamic budget constraint:

$$g_t + B_{t-1,t} = \tau_t n_t + \sum_{k=1}^{\infty} q_{t,t+k} \left(B_{t,t+k} - B_{t-1,t+k}\right).^5 \quad (5)$$

The economy is closed, which means that the bonds issued by the government equal the bonds purchased by households:

$$b_{t,t+k} = B_{t,t+k} \ \forall t, k. \quad (6)$$

Initial debt $\{B_{-1,t}\}_{t=0}^{\infty} = \{b_{-1,t}\}_{t=0}^{\infty}$ is exogenous. The government is benevolent and shares the same preferences as the households in (2).

2.2 Primal Approach

We follow Lucas-Stokey by taking the primal approach to the characterization of competitive equilibria, since this allows us to abstract away from bond prices and taxes. Let

$$\{c_t, n_t\}_{t=0}^{\infty} \quad (7)$$

represent a sequence of consumption and labor allocations. We can establish necessary and sufficient conditions for (7) to constitute a competitive equilibrium. The household's optimization problem implies the following intratemporal and intertemporal conditions, respectively:

$$1 - \tau_t = -\frac{u_n\left(c_t, n_t\right)}{u_c\left(c_t, n_t\right)} \text{ and } q_{t,t+k} = \frac{\beta^k u_c\left(c_{t+k}, n_{t+k}\right)}{u_c\left(c_t, n_t\right)}. \quad (8)$$

Substitution of these conditions into the household's dynamic budget constraint implies the following condition:

$$u_c\left(c_t, n_t\right) c_t + u_n\left(c_t, n_t\right) n_t + \sum_{k=1}^{\infty} \beta^k u_c\left(c_{t+k}, n_{t+k}\right) b_{t,t+k} = \sum_{k=0}^{\infty} \beta^k u_c\left(c_{t+k}, n_{t+k}\right) b_{t-1,t+k}. \quad (9)$$

Forward substitution into the above equation and taking into account (4) implies the following implementability condition:

$$\sum_{k=0}^{\infty} \beta^k \left(u_c\left(c_{t+k}, n_{t+k}\right) c_{t+k} + u_n\left(c_{t+k}, n_{t+k}\right) n_{t+k}\right) = \sum_{k=0}^{\infty} \beta^k u_c\left(c_{t+k}, n_{t+k}\right) b_{t-1,t+k}. \quad (10)$$

[5] We follow the same exposition as in Angeletos (2002) in which the government rebalances its debt in every period by buying back all outstanding debt and then issuing fresh debt at all maturities. This is without loss of generality. For example, if the government at $t-k$ issues debt due at date t of size $B_{t-k,t}$ which it then holds to maturity without issuing additional debt, then this can equivalently be implemented in our framework with all future governments at date $t-k+l$ for $l = 1, ..., k-1$ choosing $B_{t-k+l,t} = B_{t-k,t}$, implying that $B_{t-1,t} = B_{t-k,t}$.

Equation (10) at $t = 0$ represents the government budget constraint at $t = 0$, with bond prices and tax rates substituted out. By our reasoning, if a sequence in (7) is generated by a competitive equilibrium, then it necessarily satisfies (1) and (10). Satisfaction of (1) and (10) is also sufficient for a competitive equilibrium, as we show in the below lemma.

Lemma 1 ***(competitive equilibrium)*** *A sequence* (7) *is a competitive equilibrium if and only if it satisfies* (1) $\forall t$ *and* (10) *at* $t = 0$ *given* $\{b_{-1,t}\}_{t=0}^{\infty}$.

Proof. The necessity of these conditions is proved in the previous paragraph. To prove sufficiency, suppose a sequence (7) satisfies (1) $\forall t$ and (10) at $t = 0$ given $\{b_{-1,t}\}_{t=0}^{\infty}$. Let the government choose the associated level of debt $\left\{\{b_{t,t+k}\}_{k=1}^{\infty}\right\}_{t=0}^{\infty}$ which satisfies (9) and a tax sequence $\{\tau_t\}_{t=0}^{\infty}$ which satisfies (8). Let bond prices satisfy (8). Then, (9) given (1) implies that (3) and (5) are satisfied. Therefore household optimality holds and all dynamic budget constraints are satisfied along with market clearing, so the equilibrium is competitive. ■

3 Optimal Policy under Commitment

In this section, we solve for optimal policy in an example, and we show that, under some conditions, future tax rates should be above the peak of the Laffer curve. In the next section, we prove our main result: Applying the Lucas-Stokey definition of time-consistency, we show that in the cases where optimal tax rates are above the peak of the Laffer curve, optimal policy is not time-consistent, independently of the government's choice of maturities. In contrast, if tax rates are below the peak of the Laffer curve, then optimal policy is time-consistent.

3.1 Preferences

Consider an economy with isoelastic preferences over consumption c and labor n, where

$$u(c,n) = \log c - \eta \frac{n^{\gamma}}{\gamma} \tag{11}$$

for $\eta > 0$ and $\gamma \geq 1$, which corresponds to a commonly used utility function for the evaluation of optimal fiscal policy (e.g., Werning, 2007).[6]

Under these preferences, (1) and (8) imply that the primary surplus, $\tau n - g$, is equal to $c\left(1 - \eta (c+g)^{\gamma}\right)$. To facilitate the discussion, define c^{laffer} as the level of consumption that

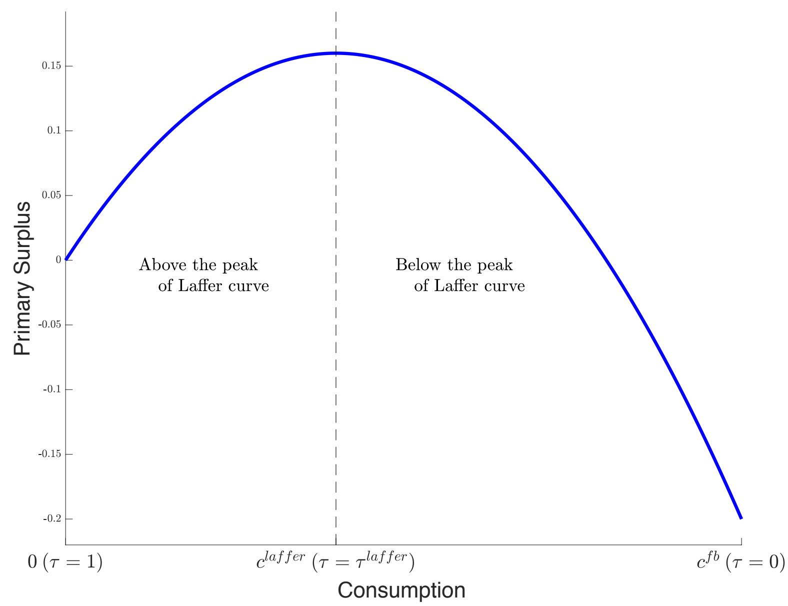
[6] These preferences imply that the implementability condition and the primary surplus are globally concave in allocations, which provides us with analytical tractability. In the Online Appendix, we present several numerical examples under other utility functions, and we reach the same conclusion that the optimal policy is not always time-consistent.

maximizes the primary surplus:

$$c^{laffer} = \arg\max_{c} c\left(1 - \eta\left(c+g\right)^{\gamma}\right). \tag{12}$$

Therefore, c^{laffer} is the level of consumption associated with the maximal tax revenue at the peak of the Laffer curve under tax rate τ^{laffer}. We assume that $g < \left(\frac{1}{\eta}\right)^{1/\gamma}$ to guarantee that $c^{laffer} > 0$. The primary surplus on the right hand side of (12) is depicted in Figure 1 for the quasilinear case with $\eta = \gamma = 1$ and $g = 0.2$.[7] This is essentially the Laffer curve except that the x-axis refers to consumption instead of tax rates which are substituted out using the primal approach.

Figure 1: Primary Surplus and Consumption



Notes: This figure depicts the primary surplus, $\tau n - g$, as a function of consumption, c. We set $\eta = \gamma = 1$ and $g = 0.2$. The figure refers to the common representation of the curve as revenue τn as a function of the tax rate τ. The values of τ^{laffer} and c^{laffer} are the tax rate and level of consumption associated with the peak of the Laffer curve, respectively. The value of c^{fb} is the level of consumption associated with the first best. The region below the peak of the Laffer curve corresponds to the case where $\tau < \tau^{laffer}$ and the region above the peak of the Laffer curve corresponds to the case where $\tau > \tau^{laffer}$.

The primary surplus is strictly concave in c and equals 0 if $c = 0$ (100 percent labor income tax) and $-g$ if $c = c^{fb}$ (0 percent labor income tax). More broadly, if $c > c^{laffer}$, then the tax rate is below the revenue-maximizing tax rate and the economy is below the peak of the

[7] This parametrization implies that $\tau^{laffer} = 60\%$ in line with the values for the labor tax reported in Trabandt and Uhlig (2011).

Laffer curve. If $c < c^{laffer}$, then the tax rate is above the revenue-maximizing tax rate and the economy is above the peak of the Laffer curve, that is, the “wrong side” of the Laffer curve.

Observe that a primary surplus between 0 and $c^{laffer}\left(1-\eta\left(c^{laffer}+g\right)^{\gamma}\right) > 0$ can be generated by the government in two ways: either with a tax rate below the peak of the Laffer curve ($c > c^{laffer}$) or with a tax rate above the peak of the Laffer curve ($c < c^{laffer}$). Importantly, the tax rate below the peak of the Laffer curve provides a strictly higher instantaneous welfare $\log c - \eta\frac{n^{\gamma}}{\gamma}$, since consumption is higher in that case. This is an important observation to keep in mind when considering optimal policy under lack of commitment.

3.2 Initial Debt

Using the resource constraint (1), we can rewrite the date 0 government budget constraint, or the implementability constraint (10) as

$$\sum_{t=0}^{\infty}\left(\beta^t\frac{c_0}{c_t}\right)\left\{c_t\left[1-\eta\left(c_t+g\right)^{\gamma}\right]\right\} = \sum_{t=0}^{\infty}\beta^t\frac{c_0}{c_t}b_{-1,t}. \tag{13}$$

For our analysis, we let $b_{-1,0} = b > 0$ and $b_{-1,t} = 0\ \forall t \geq 1$.

We will consider the optimal policy as we vary initial debt b. We let $b \leq \bar{b}$ for

$$\bar{b} = \max_{\widetilde{c}}\ \widetilde{c}\left\{\left[1-\eta\left(\widetilde{c}+g\right)^{\gamma}\right]+\frac{\beta}{1-\beta}\left(1-\eta g^{\gamma}\right)\right\}. \tag{14}$$

The value of $\bar{b}$ represents the highest value of b for which (13) can be satisfied under a feasible sequence $\{c_t\}_{t=0}^{\infty}$ associated with a sequence $\{\tau_t\}_{t=0}^{\infty}$. The level of debt $\bar{b}$ is implemented with $c_t = 0$ for all $t \geq 1$ and c_0 equal to the argument that maximizes the right hand side of (14).

To see how $\bar{b}$ is constructed, note that

$$\bar{b} > b^{laffer} = \frac{c^{laffer}\left[1-\eta\left(c^{laffer}+g\right)^{\gamma}\right]}{1-\beta}. \tag{15}$$

In other words, the value of initial debt b can exceed that associated with choosing $\tau_t = \tau^{laffer}$ for all dates t. While the tax rate τ^{laffer} maximizes the static primary surplus, choosing it forever does not maximize the present value of primary surpluses. More specifically, the date 0 present value of the primary surplus at date t is the product of the bond price $q_{0,t} = \beta^t c_0/c_t$ and the static primary surplus $c_t\left(1-\eta\left(c_t+g\right)^{\gamma}\right)$:

$$\left(\beta^t\frac{c_0}{c_t}\right)\left[c_t\left(1-\eta\left(c_t+g\right)^{\gamma}\right)\right] \tag{16}$$

Maximizing this present value requires taking advantage of the bond price, which is itself endogenous to taxes.

For example, starting from an economy where τ^{laffer} is chosen forever, the government can raise even more resources than b^{laffer} defined in (15). Consider a perturbation that keeps τ_t fixed for $t \geq 1$ and lets $\tau_0 = \tau^{laffer} - \epsilon$ for $\epsilon > 0$ arbitrarily small. This perturbation has a negative second order effect on the date 0 static primary surplus but a positive first order effect on the bond price $q_{0,t} = \beta^t c_0/c_t$, since date 0 consumption c_0 increases. Consequently, the perturbation increases the present value of primary surpluses.[8]

Using this observation, let us define $\widehat{b} \in \left(b^{laffer}, \overline{b}\right)$ as the solution to the following program:

$$\widehat{b} = \max_{\widetilde{c}} \ \widetilde{c} \left\{ \left[1 - \eta \left(\widetilde{c} + g\right)^{\gamma}\right] + \frac{\beta}{1-\beta} \left[1 - \eta \left(c^{laffer} + g\right)^{\gamma}\right] \right\}. \tag{17}$$

The value of $\widehat{b}$ corresponds to the highest value of debt that can be repaid while choosing $\tau_t = \tau^{laffer}$ for all $t \geq 1$. This value of debt exceeds b^{laffer} by our previous reasoning, since c_0 that maximizes the right hand side of (17) exceeds c^{laffer} (i.e., $\tau_0 < \tau^{laffer}$). Moreover, since the left hand side of (13) is strictly decreasing in c_t (increasing in τ_t) for all $t \geq 1$, it follows that $\widehat{b}$ corresponds to the highest value of debt that can be repaid while choosing $\tau_t \leq \tau^{laffer}$ for all $t \geq 0$.

Note that $\widehat{b}$ does not correspond to the highest feasible value of debt (i.e., $\widehat{b} < \overline{b}$). To see why, start from the allocation associated with the solution (17). Consider a perturbation that keeps τ_0 fixed and lets $\tau_t = \tau^{laffer} + \epsilon$ for $t \geq 1$ for $\epsilon > 0$ arbitrarily small. This perturbation has a negative second order effect on the date t static primary surpluses for $t \geq 1$ but a positive first order effect on the the bond price $q_{0,t} = \beta^t c_0/c_t$, since date t consumption c_t decreases for $t \geq 1$. Consequently, the perturbation increases the present value of primary surpluses.

In summary, the highest feasible level of debt $\overline{b}$ exceeds b^{laffer}, the value derived by choosing the tax rate τ^{laffer} at all dates. Moreover, $\overline{b}$ exceeds $\widehat{b}$, the highest value derived by choosing tax rates weakly below τ^{laffer} at all dates. These observations are useful to keep in mind when evaluating optimal policy, since we will vary the value of b and focus on when optimal policy admits $\tau_t > \tau^{laffer}$.

[8]Formally, since c_t for $t \geq 1$ enter symmetrically in (13), we consider perturbations where $c_t = c_1 \ \forall t \geq 1$. In that case, the present value of primary surpluses can be represented by the following object:

$$c_0 \left\{ \left[1 - \eta \left(c_0 + g\right)^{\gamma}\right] + \frac{\beta}{1-\beta} \left[1 - \eta \left(c_1 + g\right)^{\gamma}\right] \right\}.$$

The derivative of this object is positive with respect to c_0 at $c_0 = c_1 = c^{laffer}$ and negative with respect to c_1.

3.3 Optimal Policy at Date 0

We can consider the date 0 government's optimal policy under commitment, where we have substituted in for labor using the resource constraint (1):

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \eta \frac{(c_t + g)^{\gamma}}{\gamma} \right) \text{ s.t. } (13)\,. \tag{18}$$

Lemma 2 ***(unique solution)*** *The solution to* (18) *is unique.*

Proof. Consider the relaxed problem in which (13) is replaced with

$$1 - \frac{b}{c_0} - \eta (c_0 + g)^{\gamma} + \sum_{t=1}^{\infty} \beta^t \left(1 - \eta (c_t + g)^{\gamma}\right) \geq 0. \tag{19}$$

We can establish that (19) holds as an equality in the relaxed problem, implying that the relaxed and constrained problems are equivalent. We prove this by contradiction. Suppose that (19) holds as an inequality in the relaxed problem. Then, the solution to the relaxed problem would admit $c_t = c^{fb}$, which given (11) satisfies $\eta c^{fb} \left(c^{fb} + g\right)^{\gamma - 1} = 1$. Substitution of $c_t = c^{fb}$ into (19) yields

$$\frac{1}{c^{fb}} \left(-b - \frac{1}{1-\beta} g \right) \geq 0$$

which is a contradiction since $b > 0$. Therefore, (19) holds as an equality in the solution to the relaxed problem and the solutions to the relaxed and constrained problems coincide. Since the left hand side of (19) is concave in c_t for all $t \geq 0$ given that $b > 0$ and since the objective (18) is strictly concave, it follows that the solution is unique. ■

Since the solution is unique, we can characterize the solution using first order conditions.

Lemma 3 ***(optimal policy)*** *The unique solution to* (18) *satisfies the following properties:*

1. $c_t = c_1 \; \forall t \geq 1$,

2. c_0 *and* $c_1 < c_0$ *are the unique solutions to the following system of equations for some* $\lambda_0 > 0$

$$\frac{1}{c_0} - \eta (c_0 + g)^{\gamma - 1} + \lambda_0 \left(\frac{b}{c_0^2} - \eta\gamma (c_0 + g)^{\gamma - 1} \right) = 0, \tag{20}$$

$$\frac{1}{c_1} - \eta (c_1 + g)^{\gamma - 1} + \lambda_0 \left(-\eta\gamma (c_1 + g)^{\gamma - 1} \right) = 0, \text{ and} \tag{21}$$

$$1 - \frac{b}{c_0} - \eta (c_0 + g)^{\gamma} + \frac{\beta}{1-\beta} \left(1 - \eta (c_1 + g)^{\gamma}\right) = 0. \tag{22}$$

Proof. Given Lemma 2, we can consider the relaxed problem, letting $\lambda_0 > 0$ correspond to the Lagrange multiplier on (19). The first order condition for c_0 is (20). The first order condition for c_t for all $t \geq 1$ is

$$\frac{1}{c_t} - \eta \left(c_t + g\right)^{\gamma-1} + \lambda_0 \left(-\eta\gamma \left(c_t + g\right)^{\gamma-1}\right) = 0. \tag{23}$$

Since the left hand side of (23) is strictly decreasing in c_t, it follows that the solution to (23) is unique with $c_t = c_1$ $\forall t \geq 1$, where (21) defines c_1. It follows from the fact that the program is strictly concave and constraint set convex that satisfaction of (20) $-$ (22) is necessary and sufficient for optimality for a given $\lambda_0 > 0$. We are left to verify that $c_0 > c_1$. Note that the left hand side of (20) is strictly increasing in b and strictly decreasing in c_0 for a given $\lambda_0 > 0$. Therefore, c_0 is strictly increasing in b for a given $\lambda_0 > 0$, where $c_0 = c_1$ if $b = 0$. It follows then that since $b > 0$, $c_0 > c_1$. ■

The first part of the lemma states that consumption—and therefore the tax rate—is constant from date 1 onward. Since initial debt due from date 1 onward is constant (and equal to zero), tax smoothing and interest rate smoothing from date 1 onward is optimal. The optimal allocation is unique since the problem is concave.

The second part of the lemma characterizes the solution in terms of first order conditions for a positive Lagrange multiplier λ_0 on the implementability constraint (13). These conditions are necessary and sufficient for optimality given the concavity of the problem. In the optimum, c_0 exceeds long-run consumption c_1. Front-loading consumption (i.e. back-loading tax rates) is optimal since the reduction in future consumption relative to present consumption allows the government to issue debt at a higher bond price. For intuition, if it were the case that $c_0 = c_1$, then a perturbation that increases c_0 by $\epsilon > 0$ arbitrarily small and decreases c_1 by $\beta\epsilon/\left(1-\beta\right)$ has a second order effect on welfare but a first order effect on relaxing the implementability constraint (13). This is because the perturbation increases the bond price $q_{0,t}$ and the present value of primary surpluses.

We can now prove the main result of this section, which establishes that, under the optimal plan, taxes from date 1 onward are above the peak of the Laffer curve—i.e., $c_1 < c^{laffer}$—if and only if initial debt b is large enough. To prove this result, we first establish that c_1 is strictly decreasing in b. We then show that there exists $b^* \in (0, \bar{b})$ that solves the problem with $c_1 = c^{laffer}$. We therefore obtain the result that if initial short-term debt b is above a threshold b^*, then future consumption c_1 is below c^{laffer}, implying that the future tax rate τ_1 is above the revenue-maximizing tax rate at the peak of the Laffer curve τ^{laffer}. Otherwise, c_1 is above c^{laffer}, and the future tax rate τ_1 is below the revenue-maximizing tax rate at the peak of the Laffer curve.

Proposition 1 ***(taxes relative to peak of Laffer curve)*** *There exists $b^* \in (0, \bar{b})$ such that the solution admits $c_1 > c^{laffer}$ if $b < b^*$ and $c_1 < c^{laffer}$ if $b > b^*$.*

Proof. We prove this result in two steps.

Step 1. We establish that the solution to the system in $(20)-(22)$ admits c_1 that is strictly decreasing in b. Let $F^0(c_0, \lambda_0, b)$ correspond to the function on the left hand side of (20), let $F^1(c_1, \lambda_0)$ correspond to the function on the left hand side of (21), and let $I(c_0, c_1, b)$ correspond to the function on the left hand side of (22). Since the solution to this system of equations is unique, we can apply the Implicit Function Theorem. Implicit differentiation yields

$$\frac{dc_1}{db} = \frac{-F^0_{c_0} I_b + F^0_b I_{c_0}}{F^0_{c_0} I_{c_1} + \frac{F^0_{\lambda_0} F^1_{c_1} I_{c_0}}{F^1_{\lambda_0}}}. \tag{24}$$

From the second order conditions for (20) and (21), $F^0_{c_0} < 0$ and $F^1_{c_1} < 0$. Moreover, by inspection, $I_{c_1} < 0$ and $F^1_{\lambda_0} < 0$. Finally, note that $F^0_{\lambda_0} I_{c_0} = [I_{c_0}]^2 > 0$. This establishes that the denominator in (24) is positive. To determine the sign of the numerator, let us expand the numerator by substituting in for the functions. By some algebra, the numerator is equal to

$$\frac{1}{c_0}\left(-\frac{1}{c_0^2} - \eta(\gamma-1)(c_0+g)^{\gamma-2}\right) + \lambda_0\left[-\frac{b}{c_0^4} - \frac{1}{c_0}\eta\gamma(\gamma-1)(c_0+g)^{\gamma-2} - \frac{1}{c_0^2}\eta\gamma(c_0+g)^{\gamma-1}\right] < 0.$$

This establishes that c_1 is strictly decreasing in b.

Step 2. We complete the proof by establishing that there exists $b^* \in (0, \bar{b})$ for which the solution to $(20)-(22)$ admits $c_1 = c^{laffer}$. We first establish that if b^* exists, it exceeds 0. Note that if $b = 0$ then the solution admits $c_1 > c^{laffer}$. This is because $(20)-(22)$ imply that the solution admits $c_0 = c_1$. Substitution into (22) yields

$$\frac{c_1\left(1 - \eta(c_1+g)^\gamma\right)}{1-\beta} = 0. \tag{25}$$

This equation admits two solutions: $c_1 = 0$ and $c_1 = \eta^{-1/\gamma} - g$, and the optimal policy satisfies $c_1 = \eta^{-1/\gamma} - g$, since welfare is arbitrarily low otherwise. Given the definition of c^{laffer} in (12) and the strict concavity of the objective in (12), it follows that c^{laffer} must strictly be between 0 and $\eta^{-1/\gamma} - g$, which means that $c_1 > c^{laffer}$.

We now establish that b^* below $\bar{b}$ exist, where b^* solves the system $(20)-(22)$ for $b = b^*$ and $c_1 = c^{laffer}$. To see that such a solution exists, note that $\frac{1}{c^{laffer}} - \eta\left(c^{laffer}+g\right)^{\gamma-1} > 0$ since $c^{laffer} < c^{fb}$. Therefore, a value of $\lambda_0 > 0$ which satisfies (21) under $c_1 = c^{laffer}$ exists. Multiply (20) by c_0 and substitute (22) into (20) to achieve

$$1 - \eta c_0(c_0+g)^{\gamma-1} + \lambda_0\left(1 - \eta\left(c_0(1+\gamma)+g\right)(c_0+g)^{\gamma-1} + \frac{\beta}{1-\beta}\left(1 - \eta\left(c^{laffer}+g\right)^\gamma\right)\right) = 0. \tag{26}$$

Note that given the value of $\lambda_0 > 0$ satisfying (21) for $c_1 = c^{laffer}$, a solution to (26) which admits $c_0 > 0$ exists. This is because the left hand side of (26) goes to

$$1 + \lambda_0 \left(1 - \eta g^\gamma + \frac{\beta}{1-\beta} \left(1 - \eta \left(c^{laffer} + g\right)^\gamma\right) \right) > 0$$

as c_0 goes to 0, where we have used the fact that $g < \left(\frac{1}{\eta}\right)^{1/\gamma}$. As c_0 goes to infinity, the left hand side of (26) becomes arbitrarily negative. Therefore a solution to (26) for $c_0 > 0$ exists. Given that b enters linearly in (22), it follows that a value of b which satisfies the system also exists. ■

We next show that $b^* < \widehat{b}$, where recall that $\widehat{b}$ corresponds to the highest value of debt that can be repaid with $\tau_t \leq \tau^{laffer}$. In other words, this means that there exist initial levels of debt $b \in [b^*, \hat{b})$ such that it would be feasible to choose taxes below τ^{laffer} in every period, but it is not optimal to do so.

Corollary 1 *If the following generic condition holds,*

$$\beta \neq \frac{\gamma + g/c^{fb}}{\gamma + g/c^{fb} + \left[1 - \eta \left(c^{laffer} + g\right)^\gamma\right]}, \tag{27}$$

then $b^* < \widehat{b}$ for $\widehat{b}$ defined in (17).

Proof. Suppose by contradiction that $b^* \geq \widehat{b}$. It follows that $b^* = \widehat{b}$. This is because any solution to (18) for $b > \widehat{b}$ must admit $c_1 < c^{laffer}$, since an allocation with $c_1 > c^{laffer}$ cannot satisfy (22) for $b > \widehat{b}$ given the definition of $\widehat{b}$ in (17). We next show that $b^* \neq \widehat{b}$. Consider the unique allocation $\{c_0, c_1\}$ associated with $b = \widehat{b}$, where $c_1 = c^{laffer}$ and c_0 solves (17). The first order condition to the problem in (17) gives that c_0 must satisfy

$$c_0 \left[\eta\gamma \left(c_0 + g\right)^{\gamma - 1}\right] = 1 - \eta \left(c_0 + g\right)^\gamma + \frac{\beta}{1-\beta} \left[1 - \eta \left(c^{laffer} + g\right)^\gamma\right]. \tag{28}$$

Notice that the right hand side of (28) corresponds to the term in brackets in (17). Thus, substituting (28) and the fact that $b = \widehat{b}$ into (17) yields

$$\frac{b}{c_0^2} - \eta\gamma \left(c_0 + g\right)^{\gamma - 1} = 0. \tag{29}$$

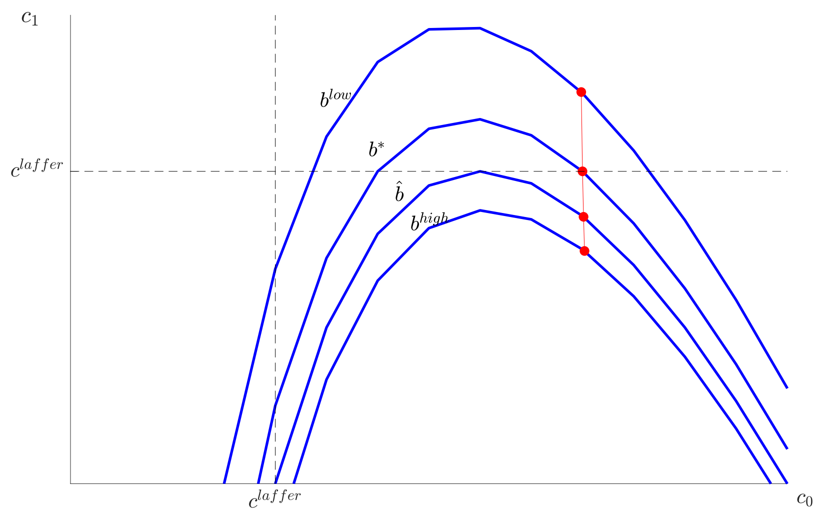
Substitution of (29) into (20) implies that $c_0 = c^{fb}$. However, for $c_0 = c^{fb}$ to satisfy (28) it must be that β equals the right hand side of (27), where use has been made of the fact that $c^{fb} \left[\eta \left(c^{fb} + g\right)^{\gamma - 1}\right] = 1$ and $1 - \eta \left(c^{fb} + g\right)^\gamma = -g/c^{fb}$. Note that c^{fb} and c^{laffer} are independent of β. Therefore, $b^* < \widehat{b}$ is a generic condition that is violated for a single unique value of β at which $b^* = \widehat{b}$. ■

3.4 Taxation above the Peak of the Laffer Curve

According to Proposition 1, if initial debt b exceeds an interior value b^*, then it is optimal for the government to choose a constant tax rate from date 1 onward above the peak of the Laffer curve. Corollary 1 highlights that the value of b^* is strictly below $\widehat{b}$, the highest value of debt that can be repaid with $\tau_t \leq \tau^{laffer}$ ($c_t \geq c^{laffer}$) for all t.

To understand this result, consider first the case for which initial debt b exceeds $\widehat{b} > b^*$. Since any debt in excess of $\widehat{b}$ cannot be sustained with taxes below τ^{laffer}, it follows that optimal taxes are necessarily set above the peak of the Laffer curve.

Figure 2: Implementable Allocations and Optimal Policy



Notes: The solid blue lines denote the combinations of $\{c_0, c_1\}$ that satisfy the implementability condition for different levels of initial debt $b^{low} < b^* < \hat{b} < b^{high}$. The red dots correspond to optimal policies. We set $\eta = \gamma = 1$, $g = 0.2$, and $\beta = 0.5$. The value of c^{laffer} is the level of consumption associated with the peak of the Laffer curve. The value of b^* is the level of debt above which optimal policy sets date 1 taxes above the peak of the Laffer curve, even though setting taxes below the peak may satisfy the implementability condition. The value of $\hat{b}$ is the level of debt above which optimal policy sets date 1 taxes above the peak of the Laffer curve because setting taxes below the peak of the Laffer curve cannot satisfy the implementability condition.

Figure 2 provides a visualization of this result by depicting the values of c_0 and c_1 that satisfy the implementability condition (22) for different values of b, for the quasi-linear case. For a given value of b, c_1 increases in c_0 for low values of c_0 and decreases in c_0 for high values of c_0. To see why, observe that the present value of primary surpluses in (22) always decreases in c_1. While an increase in c_1 has an ambiguous effect on the date $t \geq 1$ static primary surplus (depending on whether c_1 exceeds or is below c^{laffer}), it always decreases bond prices $q_{0,t} = \beta^t c_0/c_1$. This second force dominates, leading to an unambiguously negative effect of an increase in c_1 on the present value of primary surpluses. In contrast, while an increase in c_0 has an analogous ambiguous effect on the date $t = 0$ static primary surplus, it *increases* bond prices. For low values of c_0, the second force dominates, and an increase in c_0 increases the present value of primary surpluses. For high values of c_0 (which necessarily exceed c^{laffer}) the first force dominates, and an increase in c_0 decreases the present value of primary surpluses. This results in the inverted U-shaped curve in Figure 2.

As the value of b rises, this curve shifts lower, as it becomes less possible to satisfy the implementability condition with as high values of consumption in any date. Once b exceed $\widehat{b}$, the highest value of c_1 on the inverted U-shaped curve moves below c^{laffer}; in other words, satisfaction of the implementability condition (22) requires a choice of $c_1 < c^{laffer}$ with $\tau_1 > \tau^{laffer}$. As such, taxation above the peak of the Laffer curve is necessary for $b > \widehat{b}$.

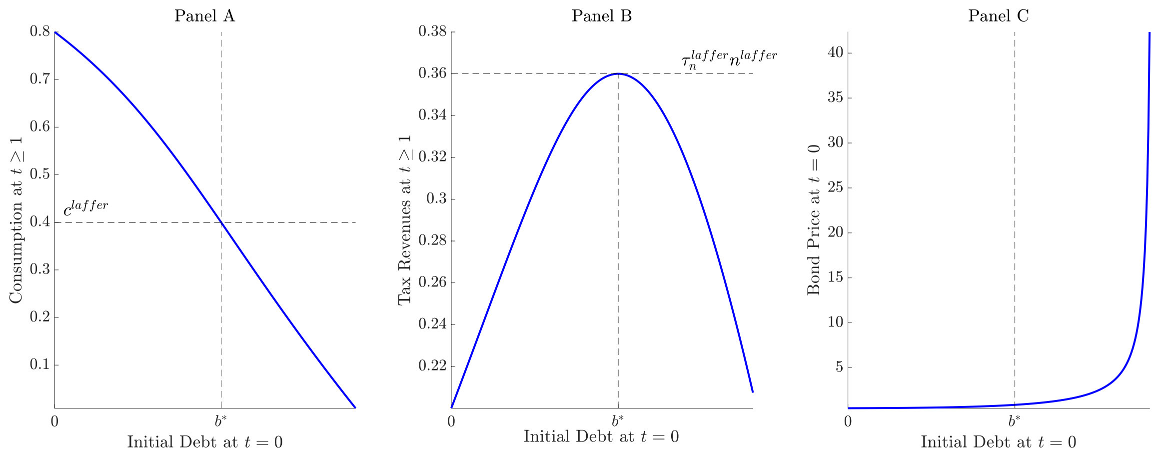
Now consider the case where $b \in (b^*, \widehat{b})$. In this circumstance, choosing $c_1 \geq c^{laffer}$ is feasible, that is, it is feasible to finance initial debt with taxes below the peak of the Laffer curve. However, taxing above the peak of the Laffer curve is optimal because of the beneficial effect of increasing date 0 bond prices. Figure 2 provides intuition by depicting an example where the optimal allocation at $b = b^*$ admits $c_0 < c^{fb}$ and $c_1 = c^{laffer}$. In this circumstance, the optimal allocation is on the downward sloping portion of the inverted U-shaped curve, and social welfare is rising in both c_0 and c_1 around the optimum. Consider a change in the allocation starting from the optimum that decreases c_0 (increases τ_0) and increases c_1 above c^{laffer} (decreases τ_1 below τ^{laffer}) while satisfying the implementability constraint (22). Such a change decreases static utility at date 0 but increases static utility from date 1 onward. However, it is suboptimal because the marginal utility benefit from lower taxation from date 1 onward does not exceed the marginal utility cost of lower bond prices and higher taxation at date 0.[9]

This visual representation of the optimal policy provides an explanation for why the optimal value of c_1 decreases in initial debt b. As the initial debt b rises, the inverted U-shape in Figure 2 shifts lower, making it infeasible for the government to sustain the same $\{c_0, c_1\}$ pair. The government responds by reducing the value of c_1, as this value becomes increasingly costly to sustain. Eventually, the optimal value of c_1 declines below c^{laffer}, at which point future taxes move to above the peak of the Laffer curve.

Figure 3 displays optimal policy as a function of initial debt b, with Panel A displaying consumption at date 1, Panel B displaying tax revenue at date 1, and Panel C displaying the bond price at date 0. As b increases, consumption at date 1 declines, attaining a value of c^{laffer} at b^*. As b rises towards b^*, tax revenue at date 1 rises, reaching a peak of $\tau^{laffer} n^{laffer}$ at

[9]An analogous argument applies starting from an optimum with $c_0 > c^{fb}$, in which case the optimum in Figure 2 is on the upward sloping portion of the inverted U-shaped curve. A change in allocation that increases c_1 above c^{laffer} in this case would also increase c_0 further above c^{fb}.

Figure 3: Initial Debt and Optimal Policy



Notes: The x-axis in all panels is the initial debt at $t = 0$, b. The y-axis in Panel A is consumption from $t = 1$ onward, c_1. The y-axis in Panel B is revenue from $t = 1$ onward, $\tau_1 n_1$. The y-axis in Panel C is the bond price at $t = 0$, $q_0 = \beta c_0/c_1$. We set $\eta = \gamma = 1$, $g = 0.2$, and $\beta = 0.5$. The value of c^{laffer} is the level of consumption associated with the peak of the Laffer curve. The value of $\tau^{laffer} n^{laffer}$ is the maximal tax revenue at the peak of the Laffer curve. The value of b^* is the level of debt above which optimal policy sets date 1 taxes above the peak of the Laffer curve.

$b = b^*$. As b rises beyond b^*, tax revenue at date 1 declines. While the static value of date 1 tax revenue is non-linear in b, the present value of date 1 tax revenue unambiguously rises as b rises. This is because the date 0 bond price rises in b, as depicted in Panel C.

A natural question regards what factors drive the value of b^*, since a higher b^* implies a higher debt threshold for future taxes to be above the peak of the Laffer curve. We performed numerically these comparative statics around the benchmark quasilinear example of Figures 2 and 3. We find that b^* is increasing in β.[10] That is, taxes from date 1 onward are more likely to be above the peak of the Laffer curve if the government is relatively impatient. This is intuitive, since a lower β implies that the government places more weight on boosting utility at date 0 versus the future, making it less costly to choose $c_1 \leq c^{laffer}$ in the future. Moreover, b^* is decreasing in η, because a higher value of η implies a lower maximal primary surplus $c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)^{\gamma}\right)$, and therefore a lower tax capacity for the government. This diminished tax capacity at date 1 implies that the government at date 0 is more likely to accommodate an increase in initial debt b by choosing $c_1 \leq c^{laffer}$.

[10] We also find that $b^*(1 - \beta)$ is increasing in β.

4 Time-Consistency of Optimal Policy

We now show that the policy under commitment may not be time-consistent. We follow Lucas-Stokey and consider what happens if at date 1, policy is reevaluated and chosen by a government with full commitment from date 1 onward. As in Lucas-Stokey, we define an optimal policy as time-consistent if the government at date 1 chooses the same allocation as the government at date 0.

4.1 Optimal Policy at Date 1

Given an inherited portfolio of maturities, the government at date 1 solves the following problem:

$$\max_{\{c_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \left(\log c_t - \eta \frac{(c_t + g)^{\gamma}}{\gamma} \right) \tag{30}$$

$$\text{s.t.} \quad \sum_{t=1}^{\infty} \beta^{t-1} \left(1 - \eta \left(c_t + g\right)^{\gamma} - \frac{b_{0,t}}{c_t} \right) = 0. \tag{31}$$

Letting λ_1 represent the Lagrange multiplier on (31), first order conditions with respect to c_t are:

$$\frac{1}{c_t} - \eta \left(c_t + g\right)^{\gamma-1} + \lambda_1 \left(\frac{b_{0,t}}{c_t^2} - \eta\gamma \left(c_t + g\right)^{\gamma-1} \right) = 0 \ \forall t \geq 1. \tag{32}$$

An optimal policy is therefore time-consistent if the solution to (30) − (31) coincides with the solution to (18).

Proposition 2 ***(time-consistency of optimal policy)*** *If $b < b^*$, then the optimal date 0 policy is time-consistent. If $b > b^*$, then the optimal date 0 policy is not time-consistent.*

Proof. We consider each case separately.

Case 1. Suppose that $b < b^*$. From Proposition 1, the date 0 solution admits $c_t = c_1 > c^{laffer}$ $\forall t \geq 1$. To show that this solution is time-consistent, suppose that the date 0 government chooses $\{b_{0,t}\}_{t=1}^{\infty}$ satisfying

$$b_{0,t} = c_1 \left(1 - \eta \left(c_1 + g\right)^{\gamma}\right) > 0 \ \forall t \geq 1 \tag{33}$$

for c_1 defined in (20) − (22). The level of issued debt $b_{0,t} > 0$ since the highest value of $c_1 > c^{laffer}$ is below that associated with $b = 0$ which satisfies (25), given the arguments in the proof of Proposition 1. Now consider the solution to (30) − (31). Analogous arguments as those in the proofs of Lemmas 2 and 3 imply that the unique solution satisfies (31) and (32) for some $\lambda_1 > 0$. Therefore, to check that the date 1 solution admits $c_t = c_1 \ \forall t \geq 1$ for c_1 which satisfies (21), it is sufficient to check that there exists some $\lambda_1 > 0$ satisfying (32). Using (33) to substitute in for $b_{0,t}$ in (32), we find that

$$\lambda_1 = -\frac{1 - \eta c_1 (c_1 + g)^{\gamma-1}}{1 - \eta (c_1 + g)^{\gamma} - \eta\gamma c_1 (c_1 + g)^{\gamma-1}} > 0, \tag{34}$$

where we have appealed to the fact that $c_1 < c^{fb}$ (from (21)) to assign a positive sign to the numerator in (34) and the fact that $c_1 > c^{laffer}$ to assign a negative sign to the denominator in (34). This establishes that the date 0 solution is time-consistent.

Case 2. Suppose that $b > b^*$ and suppose by contradiction that the optimal date 0 policy is time-consistent. This would require (32) to hold for $c_t = c_1 \ \forall t \geq 1$ for $c_1 < c^{laffer}$ which satisfies (21). For a given λ_1, satisfaction of (32) thus requires that $b_{0,t} = b_{0,1} \ \forall t \geq 1$. Equation (31) thus implies that (33) for $b_{0,t} > 0$ holds, and substitution of (33) into (32) implies that

$$\lambda_1 = -\frac{1 - \eta c_1 (c_1 + g)^{\gamma-1}}{1 - \eta (c_1 + g)^{\gamma} - \eta\gamma c_1 (c_1 + g)^{\gamma-1}} < 0, \tag{35}$$

where we have appealed to the fact that $c_1 < c^{fb}$ (from (21)) to assign a positive sign to the numerator and the fact that $c_1 < c^{laffer}$ to assign a positive sign to the denominator. However, conditional on $\{b_{0,t}\}_{t=1}^{\infty}$ for $b_{0,t} = b_{0,1} > 0 \ \forall t \geq 1$, the solution to (30) $-$ (31) must admit a positive multiplier $\lambda_1 > 0$, and this follows by analogous arguments as those in the proofs of Lemmas 2 and 3, which contradicts (35). Therefore, the date 1 solution does not coincide with the date 0 solution. ■

To understand this proposition, it is useful to consider the objectives of the date 0 government facing a date 1 government that will reevaluate policy. The date 0 government would like to commit the date 1 government to a constant policy from date 1 onward. Given the first order condition of the date 1 government (32), this goal can be achieved by leaving the date 1 government with a flat maturity with $b_{0,t} = b_{0,1} \ \forall t \geq 1$. Under such a flat distribution, the government at date 1 optimally chooses to smooth tax rates into the future.[11]

What the date 0 government cannot do, however, is to commit the date 1 government to the exact policy. The date 1 government may be able to choose from different tax rates to generate the same revenue to satisfy its budget constraint (31), and it has full discretion in doing so. Thus, the date 1 optimal policy may not coincide with the date 0 optimal policy.

[11] This flat maturity structure is equivalent to a consol. The use of consols has been pursued historically, most notably by the British government during the Industrial Revolution, when consols were the largest component of the British government's debt (see Mokyr, 2011). Moreover, the introduction of consols has been discussed as a potential option in the management of U.S. government debt (e.g. Cochrane, 2015), an idea that is supported by the quantitative analysis of Debortoli et al. (2017).

The proof of Proposition 2 appeals to the first order condition (32) of the date 1 government to establish when this is the case. If the date 1 government's first order condition (32) can be satisfied under the date 0 optimal policy with a positive Lagrange multiplier λ_1, then the date 0 and date 1 optimal policies coincide. Instead, if (32) can only be satisfied under the date 0 optimal policy with a *negative* Lagrange multiplier λ_1—as it is if $b > b^*$—, then the date 0 and date 1 policies do not coincide.

Intuitively, if $b > b^*$, the date 0 government would like to promise that taxes from date 1 onward be above the peak of the Laffer curve with $c_1 < c^{laffer}$ in order to satisfy its budget constraint (22). However, this promise is not credible since the date 1 government facing its budget constraint (31) can always make itself strictly better off deviating from this promise and choosing taxes below the peak of the Laffer curve with $c_1 > c^{laffer}$. The date 0 government internalizes the impact of consumption at date 1 on the date 0 budget constraint through the bond price, whereas the date 1 government does not. This commitment problem does not occur if $b < b^*$ since in that case both the date 0 and date 1 governments agree that $c_1 > c^{laffer}$ is optimal.

In sum, if $b > b^*$, optimal policy is not time-consistent, independently of the maturities available to the government, a result which stands in contrast with the arguments of Lucas-Stokey. Moreover, and even more starkly, if $b > \widehat{b} > b^*$, a time-consistent policy *does not exist*, as no date 1 government would ever choose $c_1 < c^{laffer}$, which is necessary for satisfaction of the date 0 budget contraint (22).

4.2 Why the Lucas-Stokey Argument Fails

It is instructive to consider why the original arguments of Lucas-Stokey can fail in our example. In developing their argument, Lucas-Stokey consider the optimal allocation under commitment from the perspective of date 0, which satisfies the following first order condition for $t \geq 1$ (the analog of (21) starting from any arbitrary initial maturity distribution, under general utility functions, after suppressing some notation):

$$(u_{c,t} + u_{n,t})(1 + \lambda_0) + \lambda_0 \left(\begin{array}{c} -(u_{cc,t} + u_{cn,t})b_{-1,t} \\ +(u_{cc,t} + 2u_{cn,t} + u_{nn,t})c_t + (u_{cn,t} + u_{nn,t})g \end{array} \right) = 0 \ \forall t \geq 1. \quad (36)$$

Lucas-Stokey claim that the optimal policy under commitment at date 0 that satisfies (36) could be made time-consistent at date 1. They argue that this is possible with the appropriate choice of maturities that satisfy the date 1 implementability condition (31), which can be rewritten more generally as

$$\sum_{t=1}^{\infty} \beta^{t-1}((u_{c,t} + u_{n,t})c_t + u_{n,t}g) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t} b_{0,t} \tag{37}$$

and the future government's date 1 first order condition (32), which can be rewritten more generally as

$$(u_{c,t} + u_{n,t})(1 + \lambda_1) + \lambda_1 \left(\begin{array}{c} -(u_{cc,t} + u_{cn,t})b_{0,t} \\ +(u_{cc,t} + 2u_{cn,t} + u_{nn,t})c_t + (u_{cn,t} + u_{nn,t})g \end{array} \right) = 0 \ \forall t \geq 1, \tag{38}$$

for some Lagrange multiplier λ_1. Their procedure thus combines (36) and (38) to yield:

$$b_{0,t} = b_{-1,t} + \frac{u_{c,t} + u_{n,t}}{u_{cc,t} + u_{cn,t}} \left(\frac{1+\lambda_1}{\lambda_1} - \frac{1+\lambda_0}{\lambda_0} \right) \ \forall t \geq 1, \tag{39}$$

which determines the issued maturity distribution at date 0 as a function of four objects: the inherited maturity distribution, the optimal allocation, and the Lagrange multipliers λ_0 and λ_1.

According to Lucas-Stokey logic, given an optimal allocation and value of λ_0 from the perspective of date 0, one can construct a value of λ_1 and a portfolio of bonds $\{b_{0,t}\}_{t=1}^{\infty}$ that satisfy (37) and (38), and accordingly, this implies that the policy is time-consistent. To see why this logic is flawed, suppose for illustration that the constructed values of $\{b_{0,t}\}_{t=1}^{\infty}$ are all non-negative, so that the constraint represented by (37) must imply a positive shadow value of debt. Then if the constructed value of λ_1 that satisfies (37) and (38) is negative, Lucas-Stokey logic fails and the optimal policy is not time-consistent. Intuitively, the solution to the date 1 problem under a positive debt portfolio $\{b_{0,t}\}_{t=1}^{\infty}$ would never admit a negative multiplier—since the shadow cost of inherited debt is positive.[12]

Our specific example illustrates a situation in which the constructed Lagrange multiplier $\lambda_1 < 0$ and the Lucas-Stokey construction fails. Figure 4 depicts the constructed Lagrange multiplier λ_1 as a function of initial debt b. It is constructed using (37) and (39), which in our example can be written as

$$b_{0,1} = c_1 \left(1 - \eta \left(c_1 + g\right)^{\gamma}\right), \text{ and} \tag{40}$$

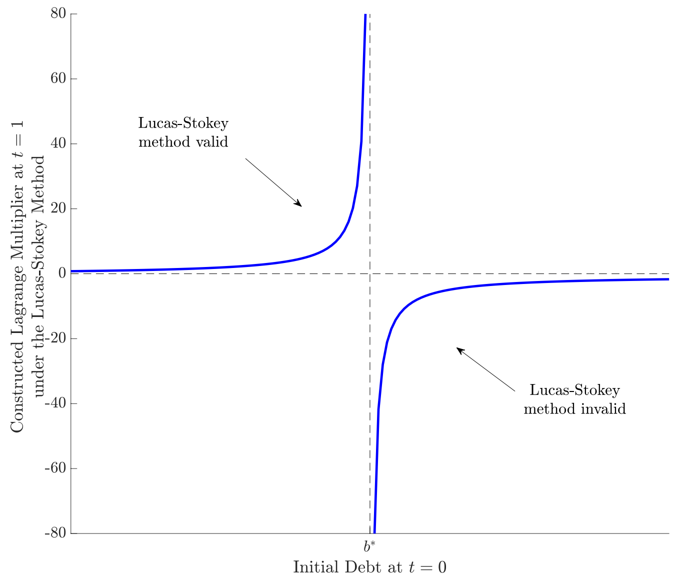
$$b_{0,1} = \left(1 - \frac{\lambda_0}{\lambda_1}\right) \eta \gamma c_1^2 \left(c_1 + g\right)^{\gamma - 1}, \tag{41}$$

respectively, for λ_0 and c_1 that satisfy (20) – (22). If $b < b^*$, the solution to (40) – (41) admits $\lambda_1 > 0$, the Lucas-Stokey method is valid, and the optimal policy is time-consistent. In this

[12] If the implied value of λ_1 is positive, then Lucas-Stokey logic holds with the optimal policy being time-consistent, assuming that the date 0 and date 1 programs for the government are concave.

case, the shadow cost of debt to the date 1 government is positive and approaches infinity as initial debt b approaches b^* from below. As b rises to b^*, inherited debt at date 1 rises, and taxes from date 1 rise towards the peak of the Laffer curve. In contrast, if $b > b^*$, the solution to (40) − (41) implies $\lambda_1 < 0$, and the Lucas-Stokey method is invalid; the date 1 government could never be facing a negative shadow cost of debt in the neighborhood of the optimum.[13]

Figure 4: Initial Debt and Constructed Lagrange Multiplier



Notes: The x-axis is the initial debt at $t = 0$, b. The y-axis is the constructed Lagrange multiplier under the Lucas-Stokey method, λ_1. We set $\eta = \gamma = 1$, $g = 0.2$, and $\beta = 0.5$. The value of b^* is the level of debt above which optimal policy sets date 1 taxes above the peak of the Laffer curve.

5 Concluding Remarks

An important literature on optimal fiscal policy without commitment has built on the Lucas-Stokey conclusion that a government can structure debt maturity issuance to guarantee commitment by future governments. In this paper, we overturn this result, using the same model and the same definition of time-consistency as Lucas-Stokey under standard assumptions on preferences. We show using an example that whether or not the Lucas-Stokey conclusion holds depends on the environment.

[13]It is also straightforward to see that our example would work using the same logic if $b_{-1,t} = \widetilde{b}\ \forall t \geq 1$ for some $\widetilde{b} \in (0, b)$ (rather than $\widetilde{b} = 0$). It would also work in an economy with state-contingent bonds with a similar decay structure. The example however fails if $\widetilde{b} > b$, since in that case consumption is backloaded.

There are three important points to note regarding our example. First, our example does not rely on the presence of an infinite horizon, which we only choose here to be consistent with Lucas-Stokey. A T-period version of this example would yield the same conclusion, namely that in some cases, the optimal policy under commitment does not coincide with that under lack of commitment.

Second, our example does not rely on the presence of non-concavities in the government's program and multiplicity of solutions at any date. Our isoelastic preferences imply that the government's welfare is concave and the constraint set is convex, which guarantees that the solution to the government's problem at dates 0 and 1 is unique. We conjecture that considering cases with multiplicity (for instance examples with negative debt positions, which may make the implementability condition no longer a convex constraint) could make it even more challenging for today's government to induce commitment by future governments.

Third, our paper provides a method of verifying whether or not the Lucas-Stokey procedure holds in other environments with a different utility function or initial maturity distribution of government debt. For example, take a model that satisfies standard dynamic programming properties with a globally concave program for the government at all future dates t (so that first order conditions are necessary and sufficient to characterize the solution from the perspective of date t), where the shadow value of debt is positive at every date t. It then follows that if the Lagrange multipliers at all future dates t constructed by the Lucas-Stokey procedure—that is, the analogs of λ_1 in (41)—are positive, then the Lucas-Stokey procedure is valid. If instead some multipliers are negative, as is the case in our constructed example, then the Lucas-Stokey procedure is not valid since the shadow cost of debt cannot be negative.

The Lucas-Stokey model has motivated an enormous literature that has extended their framework to environments with incomplete markets, financial frictions, liquidity frictions, and international flows. We have focused on a simple example to illustrate that their conclusions cannot always be directly applied. Our analysis implies that any study of optimal fiscal policy without commitment must move beyond the Lucas-Stokey definition of time-consistency, since the optimal policy may not be time-consistent. Instead, future work should consider the solution to a dynamic game between sequential governments, taking into account that the commitment and no-commitment solutions may not coincide.

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