WAREHOUSE BANKING*

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Abstract

We develop a theory of banking that explains why banks started out as commodities warehouses. We show that warehouses become banks because their superior storage technology allows them to enforce the repayment of loans most effectively. Further, interbank markets emerge endogenously to support this enforcement mechanism. Even though warehouses store deposits of real goods, they make loans by writing new “fake” warehouse receipts, rather than by taking deposits out of storage. Our theory helps to explain how modern banks create funding liquidity and why they combine warehousing (custody and deposit-taking), lending, and private money creation within the same institutions. It also casts light on a number of contemporary regulatory policies.

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The banks in their lending business are not only not limited by their own capital; they are not, at least immediately, limited by any capital whatever; by concentrating in their hands almost all payments, they themselves create the money required....

Wicksell (1907)

1 Introduction

Banking is an old business. The invention of banking preceded the invention of coinage by several thousand years. Banks evolved from ancient warehouses, where grain and precious metals were deposited for storage. For example, in ancient Egypt, grain harvests were deposited for storage in centralized warehouses and depositors could write orders for the withdrawal of grain as means of payment. These orders constituted some of the earliest paper money. Later, London’s goldsmiths took deposits of “money or plate” for storage in their safes, and they operated a payment system based on these deposits. Throughout history, such warehouses for the storage of commodities began making loans, thereby evolving into banks. However, current banking theories are not linked to this evolution of banks from warehouses. This raises the questions we address in this paper. Why did banks start out as warehouses? And what are the implications of banks’ warehousing function for contemporary bank regulation?

To address these questions, we develop a theory of banking that is linked to these historical roots. It explains why banks offer deposit-taking, account-keeping, and custodial services—i.e. warehousing services—within the same institutions that provides

1 The earliest known coins were minted in the kingdom of Lydia in the 7th Century BC (British Museum (2016)). Banking is much older. It seems to have originated in ancient Mesopotamia c. 3000 BC. Early laws pertaining to banks (banking regulation) appeared in the Code of Hammurabi and the Laws of Eshnunna (Davies (1994), Geva (2011)).

2 The connection between banking and warehousing is fundamental. Throughout history, banks have evolved systematically from warehouses, specifically from warehouses whose deposit receipts served as private money. For example, depositories of barley and silver evolved into banks in ancient Mesopotamia (Geva (2011)); grain silos developed into banks in ancient Egypt (Westermann (1930)); goldsmith bankers came to be in Early Modern Europe due to their superior safes for storing “money and plate in trust” (Richards (1934) p. 35, Lawson (1855)); rice storage facilities began the practice of fractional reserve banking in 17th century Japan (Crawcour (1961)); tobacco warehouses were instrumental in the creation of banking and payments in 18th century Virginia, where warehouse receipts were ultimately made legal tender (Davies (1994)); still in the 19th century, granaries were doing banking in Chicago (Williams (1950)); and even today grain silos in Brazil perform banking activities (Skrastins (2015)).

3 These goldsmiths owned safes that gave them an advantage in safe-keeping. This interpretation is emphasized in He, Huang and Wright (2003, 2008) as well as in many historical accounts of banking, including, for example, the Encyclopedia Britannica, which states that “The direct ancestors of modern banks were the goldsmiths. At first the goldsmiths accepted deposits merely for safe keeping; but early in the 17th century their deposit receipts were circulating in place of money” (1954, vol. 3, p. 41).
lending services. The theory sheds new light on the importance of interbank markets and banks’ private money creation. It also offers a new perspective on regulatory policies such as narrow banking and liquidity requirements, capital requirements, and monetary policy.

**Model preview.** In the model, an entrepreneur has a productive investment project. He needs to hire a worker to do the project, but his endowment is limited. Further, the output of his project is not pledgeable so he cannot pay the worker on credit. After his project pays off, the entrepreneur needs to store his output before he consumes. He can store it privately, in which case it depreciates, or he can store it in a warehouse, in which case it does not depreciate. This superior storage technology of the warehouse could reflect the fact that the warehouse prevents spoilage like grain silos in ancient Egypt or protects against theft like safes in Early Modern Europe. Further, warehouse deposits are publicly observable, and hence pledgeable.

**Results preview.** Our first main result is that the entrepreneur is able to borrow from the warehouse to finance his project, even though his output is non-pledgeable. The warehouse can overcome the non-pledgeability problem because the entrepreneur wants to store his deposits in the warehouse and the warehouse has the right to seize the deposits of a defaulting borrower as repayment—banks still have this right today, called “banker’s setoff.” Thus, warehouses’ superior storage technology makes it incentive compatible for the entrepreneur to repay his debt in order to access warehouse storage. This mechanism explains why the same institutions should provide both the warehousing and lending services in the economy.

Our second main result is that interbank markets, i.e. “inter-warehouse markets,” for the entrepreneur’s debt are sufficient to support this enforcement mechanism even if the entrepreneur can store his output in another warehouse. This is because this new warehouse can buy the entrepreneur’s debt in the interbank market, thereby obtaining the right to seize the entrepreneur’s deposits. As a result, the entrepreneur ends up repaying in full no matter which warehouse he deposits in. This finding may cast light on why successful banking systems throughout history, such as those operated by Egyptian granaries and London goldsmiths, as well as those in existence today, have indeed developed interbank clearing arrangements.

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4See Holmström and Tirole (2011) for a list of “...several reasons why this [non-pledgeability] is by and large reality” (p. 3).

5Allen and Gale (1998) also assume that the storage technology available to banks is strictly more productive than the storage technology available to consumers.

6We relax the assumption that warehouse deposits are pledgeable in Subsection 4.4.

7Empirical evidence that seems to support this result appears in Skrastins (2015). Using a differences-in-differences research design, Skrastins (2015) documents that agricultural lenders in Brazil extend more credit when they merge with grain silos, i.e. banks lend more when they are also warehouses.

8See Geva (2011) p. 141 for a description of how warehouse-banks in Greco-Roman Egypt relied on
Our third main result is that warehouse-banks make loans even if they have no initial deposits to lend out. In fact, they do the constrained-efficient amount of lending by making loans in new “fake” warehouse receipts—i.e. receipts to redeem deposits that are not backed by current deposits. Thus, when a warehouse-bank makes a loan, it is not reallocating assets—it is not reallocating cash deposits into loans on the left-hand side of its balance sheet. Rather, it is creating a new liability—it is lending out fake deposit receipts, expanding its balance sheet. The entrepreneur uses these receipts to pay the worker, who accepts them to access the warehouse’s superior storage technology. In this way, warehouse receipts emerge as a medium of exchange because they are a store of value. Thus, the warehouse’s superior storage technology allows it not only to enforce the repayment of loans, but also to create circulating private money to make loans, i.e. to make loans by creating deposits. This is reminiscent of Keynes:

> It is not unnatural to think of deposits of a bank as being created by the public through the deposits of cash representing either savings or amounts which are not for the time being required to meet expenditures. But the bulk of the deposits arise out of the action of the banks themselves, for by granting loans, allowing money to be drawn on an overdraft or purchasing securities, a bank creates a credit in its books which is the equivalent of a deposit (Keynes in his contribution to the Macmillan Committee, 1931, p. 34).

We refer to these new receipts as “fake receipts” due to their lack of deposit-backing, although we emphasize that they are good-value IOUs. Note that other authors have used this term before; however, they have suggested that when banks create money they are performing a kind of swindle. For example, Rothbard (2008) says banks have habitually created warehouse receipts (originally bank notes and now deposits) out of thin air. Essentially, they are counterfeiters of fake warehouse-receipts to cash or standard money, which circulate as if they were genuine, fully backed notes or checking accounts.... This sort of swindling or counterfeiting is dignified by the term “fractional-reserve banking.

Our findings contrast with this perspective. For us, the creation of such private money is essential for banks to extend the efficient level of credit. However, the quantity of receipts the warehouse can issue is limited by the entrepreneur’s ability to repay his debt and hence the worker’s willingness to accept them as a store of value. This finding connects Tobin’s (1963) work to the origins of banks as warehouses, as we discuss further below.

A related expansion of bank balance sheets occurs in the textbook relending model of bank money creation—the so called “money multiplier” associated with fractional reserves banking (see, e.g., Samuelson (1980), ch. 16). In this model, a bank takes deposits and lends them out. Then, later, the deposits are deposited back in the bank, expanding the balance sheet. That is, the money multiplier is created when the bank keeps only a fraction of its deposits on reserve, lending out the rest. By contrast, warehouse-banks in our model make loans even with no deposited goods, and this expansion of economic activity occurs with a single transaction—a loan. Borrowers use warehouse receipts as working capital to make productive investments. As a result, making a loan by issuing new receipts creates an intertemporal transfer of liquidity, improving efficiency.
As in this description, a loan is just an exchange of IOUs in our model—the entrepreneur gives the warehouse a promise to repay (the loan) and, in exchange, the warehouse gives the entrepreneur a promise to repay (the deposit receipt). However, this seemingly zero-net transaction circumvents the entrepreneur’s non-pledgeability problem and thus has a positive effect on aggregate output. Indeed, banks’ money creation only improves efficiency in the baseline model. However, we show in an extension that it can lead to financial fragility, since the worker may wish to withdraw in order to take up an unexpected private investment opportunity. Since fake receipts are not actually backed by real deposits, this “disintermediation of savings” can cause bank failure.

**Application to modern banks.** Despite our focus on the origins of banking, our model casts light on some aspects of modern banking as well. Modern banks are complex institutions that perform many important functions outside of our model. However, the storage and payments services that warehouse-banks provide in our model remain fundamentally important for banks after thousands of years. Instead of providing safekeeping for grain or gold and issuing receipts that serve as a means of payment, modern banks create bank accounts for storing wealth and issue claims, checkbooks, and cards that serve as means of payment. In the model, loan repayment is ensured by the threat of excluding a delinquent borrower from warehousing services. Modern banks too benefit from similar advantages for storing money and financial securities.

Recently, exclusion from the banking system made the high costs of storing cash salient for some firms in Colorado. Specifically, marijuana businesses have had their bank accounts closed, forcing them to store cash privately. The costs of private storage are reflected in the following quote from the *New York Times*:

> [Marijuana entrepreneur] Dylan Donaldson...knows the hidden costs of a bank-challenged business. He has nine 1,000-pound safes bolted to the floor in...his dispensary [and] he pays $100,000 a year for armed guards (Richtel (2015)).

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11 This is consistent with Quinn and Roberds’s (2014) empirical finding that the Bank of Amsterdam’s ability to create unbacked private money allowed it to finance its loans and resulted in the Bank florin becoming the dominant international currency throughout Europe.

12 Important bank functions include risk-sharing (Diamond and Dybvig (1983)), delegated monitoring (Diamond (1984)), and screening (Ramakrishnan and Thakor (1984)). Historically, after warehouses began lending due to their superior storage technology, they would have had incentive to develop further banking-specific expertise in these areas. This may suggest a historical rationale for why deposit-taking, lending, and payments continue to remain concentrated in the same institutions.

13 The costs of private storage of money are reflected in the negative bond yields that currently prevail in Japan, Switzerland, and around the Eurozone. Further, in 2011, even before sovereign rates became negative, the Bank of New York–Mellon, which is the largest depository institution in the world today, charged its depositors a fee to hold cash (Rappaport (2011)). This bank is usually classified as a custodian bank, i.e. an institution responsible for the safeguarding, or warehousing, of financial assets. Its negative deposit rates for cash reflected its own storage cost.
Policy. Our model is rooted in history, but gives a perspective on contemporary bank regulation which offers support for some current policy proposals. First, we examine narrow banking. Since the same institutions must do both deposit-taking and lending in our model, restricting banks to invest in a narrow set of securities like Treasuries undermines funding liquidity creation. Even though banks in our model endogenously hold some reserves, imposing liquidity requirements, such as the LCR or NSFR in Basel III, have the similar effect of constraining liquidity creation. We also extend the model to examine how liquidity requirements affect financial fragility. We show that, surprisingly, increasing liquidity requirements may increase the likelihood of bank runs.

Next, we ask how bank equity affects lending. To create a role for warehouse bank equity, we relax the assumption that warehouse deposits are perfectly pledgeable. We find that warehouse-banks need equity to extend credit, because it mitigates this non-pledgeability problem between banks and depositors. Our result echoes the skin-in-the-game argument for bank capital, which also appears in Holmström and Tirole (1997), Coval and Thakor (2005), Mehran and Thakor (2011), and Rampini and Viswanathan (2015). In contrast to this literature, we find that bank capital allows banks to take more deposits, rather than just strengthening monitoring/screening incentives and allowing banks to make more loans. Bank equity was historically important for banks to create circulating banknotes, i.e. receipts. Indeed, some banknotes in the Free Banking Era were embossed with the amount of equity held by the issuing bank.

We also analyze a reduced form of monetary policy. We model the policy rate as the return on warehouse-banks’ assets, since it is the rate of return on bank reserves in reality. We find that increasing the policy rate can increase the supply of credit—i.e. “tighter” monetary policy “loosens” credit in our model. This is because when warehouse-banks can store at a higher rate, borrowers have a stronger incentive to repay their loans in order to access this high savings rate. This mitigates the effects of non-pledgeability ex post, leading to more credit ex ante. Thus, our model points out a new effect of monetary policy.

Related literature. We make four main contributions relative to the literature.

First, we point out two distinguishing features of warehouses that make them the natural banks: (i) they prevent depreciation or theft so that goods stored in the warehouse earn a better rate of return than goods stored privately and (ii) goods stored in[14] To be fair to the narrow-banking proposal, our model does not have deposit-insurance-related distortions that provide one of the justifications for narrow banking.

[15] Our result that higher bank capital leads to more liquidity creation by the bank is consistent with Berger and Bouwman’s (2009) evidence for the bulk of the U.S. banking system. But it is in contrast to liquidity creation theories. In Bryant (1980) and Diamond and Dybvig (1983) bank capital plays no role, whereas in DeAngelo and Stulz (2013) and Diamond and Rajan (2001) higher bank capital leads to lower liquidity creation.
the warehouse are pledgeable, unlike the entrepreneur’s output. This second feature is essential for deposit-taking, as Gu, Mattesini, Monnet and Wright (2013) emphasize. Our contribution relative to this paper is to show that the storage technology gives warehouses an advantage not only in taking deposits, but also in enforcing loans. Thus, our model provides a new rationale for why deposit-taking and lending should be done in the same institution. This complements the analysis in Kashyap, Rajan and Stein (2002) who argue that these two functions of a bank should go together since combining them provides insurance against drawdowns of both demand deposits and credit lines (loans). We abstract from this liquidity-insurance channel in our baseline model, since we focus on banks’ creating private money (warehouse receipts), which could allow them to meet drawdowns by issuing new receipts.  

Second, we show that the secondary market for defaulted debt prevents a borrower from avoiding repayment by depositing his output in a warehouse different from the one he initially borrowed from. This is related to Broner, Martin and Ventura’s (2010) finding that secondary markets for sovereign debt reduce strategic default. In that model, a sovereign defaults if its debt is held by foreign investors. However, these foreign investors are still willing to lend to the sovereign because they anticipate being able to sell their debt to domestic investors. We add to this result in three ways. (i) We analyze how secondary markets deter borrowers from diverting output and storing “abroad” in “foreign” warehouses. (ii) We show that secondary markets support the private enforcement of repayments via seizure. (iii) We point out the special importance of interbank markets: banks have the ability to enforce debts via seizure because they hold borrowers deposits in storage.

Third, in our model, banks must lend in fake receipts to extend the efficient level of credit. Even though these receipts are not backed by real deposits, the worker accepts them in order to access warehouses’ superior storage technology. Thus, in our model, bank money creation expands the supply of credit as in the verbal descriptions of Hahn (1920) and Wicksell (1907) and the reduced-form models of Bianchi and Bigio (2015) and Jakab and Kumhof (2015). However, our model is consistent with Tobin’s (1963) critique that banks cannot create money beyond the demand for savings, i.e. storage in warehouse-bank deposits. But, contrary to Tobin’s view, bank money creation can itself

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16 We consider an extension that does include liquidity risk in Subsection 5.2

17 Our paper is also related to papers in which debt serves as inside money generally. For example, Kahn and Roberds (2007) develop a model that shows the advantage of circulating liabilities (transferable debt) over simple chains of credit. Townsend and Wallace (1987) develop a model of pure intertemporal exchange with informationally-separated markets to explain the role of circulating liabilities in exchange. We also provide a framework to study private money creation in a relatively classical model (a Walrasian equilibrium subject to appropriate constraints). Brunnermeier and Sannikov (2010), Kiyotaki and Moore (2001), Hart and Zingales (2015a, 2015b), and Wang (2016) provide complementary Walrasian models in which bank money-creation is valuable because it creates safe and/or resaleable liabilities.
increase the equilibrium amount of deposits in our model, since it increases aggregate output by mitigating the non-pledgeability friction. However, this is not a “widow’s curse” in which banks can create an arbitrary amount of money, since money creation is limited by the entrepreneur’s ability to repay his debt and the worker’s willingness to save in fake receipts.

Fourth, warehouses are effectively able to enforce exclusion from financial markets, which allow for efficient savings/storage in our model. We show that they can implement exclusion in a finite horizon model, even if they are not able to make long-term commitments. In other words, exclusion from storage at the final date is subgame perfect. This adds to the results in Bolton and Scharfstein (1990), who show that the threat of exclusion from credit markets can mitigate incentive problems in corporate finance with commitment, and Bulow and Rogoff (1989), who analyze how the threat of exclusion can mitigate incentive problems in sovereign debt markets with an infinite horizon.


Layout. In Section 2 we describe the environment and present two benchmarks, the first-best allocation and the allocation with no credit. In Section 3 we characterize the equilibrium. In Section 4 we study liquidity creation and policy implications. In Section 5 we show that our conclusions are robust to different utility specifications and to the possibility of bank runs. In Section 6 we conclude. The appendix contains all proofs and a glossary of notations.

2 Environment and Benchmarks

In this section, we present the environment and two benchmark allocations. Given our historical motivation, we frame the model in terms of farmers who hire laborers to plant grain. Farmers want to deposit output in warehouses, i.e. grain silos, which prevent grain from depreciating.

\[ Bond \text{ and Krishnamurthy (2004) study the problem of credit market exclusion with competing banks. They find that in this setting a regulatory intervention is required to implement exclusion. They do not have an interbank market. In our model the interbank market facilitates the enforcing of borrower repayment, thereby obviating the need for regulation.} \]
2.1 Timeline, Production Technology, and Warehouses

There are three dates, Date 0, Date 1, and Date 2, and three groups of players, farmers, warehouses, and laborers. There is a unit continuum of each type of player. There is one real good, called grain, which serves as the numeraire. There are also receipts issued by warehouses, which entail the right to withdraw grain from a warehouse.

All players are risk neutral and consume only at Date 2. Denote farmers’ consumption by $c^f$, laborers’ consumption by $c^l$, and warehouses’ consumption by $c^b$ (the index $b$ stands for “bank”). Farmers have an endowment $e$ of grain at Date 0. No other player has a grain endowment. Laborers have labor at Date 0. They can provide labor $\ell$ at the constant marginal cost of one. So their utility is $c^l - \ell$. Farmers have access to the following technology. At Date 0, a farmer invests $i$ units of grain and $\ell$ units of labor. At Date 1, this investment yields

$$y = A \min \{\alpha i, \ell\},$$

i.e. the production function is Leontief.

We make two main assumptions on technologies. First, farmers’ output $y$ is not pledgeable and, second, warehouses have a superior storage technology. Specifically, if grain is stored privately, it depreciates at rate $\delta \in [0, 1)$: if player $j$ stores $s^j_t$ units of grain privately from Date $t$ to Date $t + 1$, he has $(1 - \delta)s^j_t$ units of grain at Date $t + 1$. In contrast, if grain is stored in a warehouse, it does not depreciate.

Note that only the output of the farmers’ technology is not pledgeable. Warehoused grain is pledgeable and, as a result, warehouses can issue receipts as “proof” of their deposits. These receipts are enforceable against the issuing warehouse and payable to the “bearer upon demand,” so the bearers of receipts can trade them among themselves. Warehouses can issue these “proof-of-deposits” receipts even when there is no deposit. These receipts, which we refer to as “fake receipts,” still entail the right to withdraw grain from a warehouse, and thus they are warehouses’ liabilities that are not backed by the grain they hold.

\[^{19}\text{We relax these assumptions in Subsection 5.1 where we consider farmers with logarithmic utility over consumption at Date 1 and Date 2.}\]

\[^{20}\text{None of our main results depends on the functional form of the production function. For example, if labor were the only input to the farmers’ production function all the results would go through.}\]

\[^{21}\text{Historically, warehouses had a physical advantage in storing goods or safeguarding valuables. And, even today, physical safes play an important role in banking, even in developed economies. For example, custodians like Clearstream hold physical certificates for all publicly-listed companies in Germany.}\]

\[^{22}\text{We partially relax the assumption that warehouse deposits are pledgeable in Subsection 4.4.}\]
2.2 Parameter Restrictions

In this subsection, we impose two restrictions on the deep parameters of the model. The first ensures that farmers’ production technology generates sufficiently high output that the investment has positive NPV in equilibrium. The second ensures that the incentive problem that results from the non-pledgeability of farmers’ output is sufficiently severe to generate a binding borrowing constraint in equilibrium. Note that since the model is linear, if a farmer’s IC does not bind, he will scale his production infinitely.

PARAMETER RESTRICTION 1. *The farmers’ technology is sufficiently productive,*

\[ A > 1 + \frac{1}{\alpha}. \]  
(2)

PARAMETER RESTRICTION 2. *Depreciation from private storage is not too high,*

\[ \delta A < 1. \]  
(3)

2.3 Benchmark: First Best

We now consider the first-best allocation, i.e. the allocation that maximizes utilitarian welfare subject only to the aggregate resource constraint. Since the utility, cost, and production functions are all linear, in the first-best allocation all resources are allocated to the most productive players at each date. At Date 0 the farmers are the most productive and at Date 1 the warehouses are the most productive. Thus, all grain is held by farmers at Date 0 and by warehouses at Date 1. Laborers exert labor in proportion \( \frac{1}{\alpha} \) of the total grain invested to maximize production.

**Proposition 1. (First-best Allocation)** The first-best labor and investment allocations are given by \( \ell_{fb} = \alpha e \) and \( i_{fb} = e \). All grain is stored in warehouses at Date 1.

2.4 Benchmark: No Credit

Consider a benchmark model in which there is no lending. Recall that only farmers have an endowment at Date 0, warehouses and laborers have no endowments. Thus, the benchmark with no lending is tantamount to a model in which warehouses cannot lend in fake receipts (we discuss this more formally in Subsection 4.3). Here, farmers simply divide their endowment between their capital investment \( i \) and their labor investment \( \ell \); their budget constraint reads

\[ i + w\ell = e, \]  
(4)
where \( w \) is the wage paid to laborers. The Leontief production function implies that they will always make capital investments equal to the fraction \( \alpha \) of their labor investments, or

\[
\alpha \ell = \ell. 
\]  

(5)

We summarize the solution to this benchmark model in Proposition 2 below.

**Proposition 2. (Benchmark Case with No Credit)** With no credit, the equilibrium is as follows:

\[
\ell_{nc} = \frac{ae}{1 + \alpha}, 
\]

(6)

\[
i_{nc} = \frac{e}{1 + \alpha}. 
\]  

(7)

Note that even though warehouses do not improve efficiency by extending credit to farmers, they still provide a useful service in the economy by taking grain deposits and providing efficient storage of grain from Date 1 to Date 2. Below we will see that farmers’ incentive to access this storage technology is what makes them repay their debt, making them not only take deposits but also make loans—making them banks.

### 3 Equilibrium with Incentive Constraints

In this section, we consider the equilibrium of the model in which farmers’ repayments must be incentive compatible.

#### 3.1 Financial Contracts

There are three types of contracts in the economy: labor contracts, deposit contracts, and lending contracts. We restrict attention to bilateral contracts, although liabilities are tradeable: farmers use warehouse receipts to pay laborers and warehouses trade farmers’ debt in an interbank market.

Labor contracts are between farmers and laborers. Farmers pay laborers \( w\ell \) in exchange for laborers’ investing \( \ell \) in the production technology \( y \).

Deposit contracts are between warehouses and the other players, i.e., laborers, farmers, and (potentially) other warehouses.Warehouses accept grain deposits with gross rate \( R_t^D \) over one period, i.e. if player \( j \) makes a deposit of \( d_t^j \) units of grain at Date \( t \) he has the right to withdraw \( R_t^D d_t^j \) units of grain at Date \( t + 1 \). When a warehouse accepts a deposit of one unit of grain, it issues a receipt in exchange as “proof” of the deposit.
Figure 1: A Timeline Representation of Sequence of Moves

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Date 1</th>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Warehouses</strong></td>
<td><strong>Warehouses</strong></td>
<td><strong>Warehouses</strong></td>
</tr>
<tr>
<td>accept deposits $D_0$</td>
<td>accept deposits $D_1$</td>
<td>repay $R^D_1 D_1$ to depositors</td>
</tr>
<tr>
<td>lend $L$ to farmers</td>
<td>repay $R^D_0 D_0$ to depositors</td>
<td>consume $c^b = s^b_1 - R^D_1 D_1$</td>
</tr>
<tr>
<td>store $s^b_0$</td>
<td>store $s^b_1$</td>
<td></td>
</tr>
<tr>
<td><strong>Farmers</strong></td>
<td><strong>Farmers</strong></td>
<td><strong>Farmers</strong></td>
</tr>
<tr>
<td>borrow $B$ from warehouses</td>
<td>receive cash flow $y(i, \ell)$</td>
<td>receive $R^D_1 d^f_1$ from warehouses</td>
</tr>
<tr>
<td>invest $i$ and $\ell$ in technology $y$</td>
<td>receive $R^D_0 d^f_0$ from warehouses</td>
<td>have total grain holding $g^f_1$</td>
</tr>
<tr>
<td>pay laborers $w\ell$</td>
<td>deposit $d^f_1$ in warehouses</td>
<td>deposit $d^f_1$ in warehouses</td>
</tr>
<tr>
<td>store $s^f_0$</td>
<td>store $s^f_1$</td>
<td>store $s^f_1$</td>
</tr>
<tr>
<td><strong>Laborers</strong></td>
<td><strong>Laborers</strong></td>
<td><strong>Laborers</strong></td>
</tr>
<tr>
<td>exert labor $\ell$</td>
<td>receive $R^D_0 d^f_0$ from warehouses</td>
<td>receive $R^D_1 d^f_1$ from warehouses</td>
</tr>
<tr>
<td>accept wage $w\ell$</td>
<td>deposit $d^f_1$ in warehouses</td>
<td>consume $c^l = R^D_1 d^f_1 + (1 - \delta)s^f_1$</td>
</tr>
<tr>
<td>deposit $d^f_0$ in warehouses</td>
<td>store $s^f_1$</td>
<td></td>
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<tr>
<td>store $s^f_0$</td>
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</table>

Lending contracts are between warehouses and farmers. Warehouses lend $L$ to farmers at Date 0 in exchange for farmers’ promise to repay $R^L L$ at Date 1, where $R^L$ is the lending rate. Warehouses can trade farmers’ debt at Date 1 in the interbank market.

When warehouses make loans, they can either lend grain or issue new receipts. A loan made in receipts is tantamount to a warehouse offering a farmer a deposit at Date 0 in exchange for the farmer’s promise to repay grain at Date 1. When a warehouse makes a loan in receipts, we say that it is “issuing fake receipts.” We refer to a warehouse’s total deposits at Date $t$ as $D_t$. These deposits include both those deposits backed by grain and those granted as fake receipts.

The timeline of moves for each player and their contractual relationships are illustrated in Figure 1.
3.2 Incentive Constraint

Lending contracts are subject to a form of limited commitment on the farmers’ side. Because farmers’ Date 1 output is not pledgeable, they are free to divert their output and store it privately. If they deposit their output in a warehouse, the warehouse can seize the grain that the farmer owes it. The next proposition gives a condition for the farmer to prefer to deposit in a warehouse and repay his debt than to store privately.

**Proposition 3. (Incentive Constraint)** If a farmer has grain $g$ and debt $R_L B$ at Date 1, he prefers to repay his debt than to store privately if and only if

$$R^D_1 (g - R_L B) \geq (1 - \delta)g.$$  

(IC)

The incentive constraint above says that in order to repay his debt the return $R^D_1$ on warehouse deposits must be sufficiently high relative to the return $1 - \delta$ on private storage.

3.3 Interbank Market

The next result says that the incentive constraint in Proposition 3 is not only necessary but also sufficient for a farmer to repay his debt. This is a result of the fact that warehouses can trade farmers’ debt in the interbank market.

**Proposition 4. (Interbank Markets Enforce Repayment)** Given the interbank market for farmers’ debt, a farmer cannot avoid repayment by depositing his output in a warehouse different from the warehouse he originally borrowed from. A farmer’s global incentive constraint is as in Proposition 3.

To see the mechanism behind this result, consider the case in which there are two warehouses, called Warehouse 0 and Warehouse 1. Suppose that a farmer borrows from Warehouse 0, but diverts his output at Date 1 and deposits with Warehouse 1. Can the farmer thus both avoid repayment and take advantage of the warehouses’ superior storage technology? The answer is no. This is because Warehouse 0 now sells the farmer’s debt in the interbank market. Warehouse 1 buys it and seizes the entire amount that the farmer borrowed from Warehouse 0, with interest. In summary, the farmer’s repayment does not depend on the warehouse in which he deposits.

We have now established that a farmer repays in full, as long as his debt is not too high relative to his output. In other words, even though his output is not pledgeable.

---

23In the proof of the proposition, we show that farmers’ debt trades in the market at par, since it is effectively a riskless bond. As a result, a lending warehouse does not risk selling the farmers’ debt at a discount. Thus, the repayment the original warehouse receives is independent of the warehouse the farmer deposits in—it is repaid in full as long as it is incentive compatible for the farmer to deposit.
by assumption, a fraction of it is “effectively pledgeable,” since he has incentive to make
the repayment in order to access the warehouses’ superior storage technology.

3.4 Individual Maximization Problems and Equilibrium Definition

We now turn to the definition of the market equilibrium. All players take prices as given
and maximize their Date 2 consumption subject to their budget constraints. Farmers’
maximization problems are also subject to their incentive compatibility constraint (IC).

The warehouse’s maximization problem is

\[
\text{maximize } \ c^b = s^b_1 - R^b_1 D_1
\]

over \( s^b_1, s^b_0, D_0, D_1, \) and \( L \) subject to

\[
s^b_1 = R^L L + s^b_0 - R^D_0 D_0 + D_1, \\
s^b_0 + L = D_0,
\]

and the non-negativity constraints \( D_t \geq 0, s^b_t \geq 0, L \geq 0 \). To understand this max-
imization program, note that equation (8) says that the warehouse maximizes its con-
sumption \( c^b \), which consists of the difference between what is stored in the warehouse
at Date 1, \( s^b_1 \), and what is paid to depositors, \( R_1 D_1 \). Equation (BC\(_b^1\)) is the warehouse’s
budget constraint at Date 1, which says that what is stored in the warehouse at Date 1,
\( s^b_1 \), is given by the sum of the interest on the loan to the farmer \( R^L L \), the warehouse’s
savings at Date 0, and the deposits at Date 1 \( D_1 \) minus the interest the warehouse must
pay on its time 0 deposits \( R^D_0 D_0 \). Similarly, equation (BC\(_b^0\)) is the warehouse’s budget
constraint at Date 0, which says that the sum of the warehouse’s savings at Date 0 \( s^b_0 \)
and its loans \( L \) must equal the sum of the Date 0 deposits \( D_0 \).

The farmer’s maximization problem is

\[
\text{maximize } \ c^f = R^D_1 d^f_1 + (1 - \delta) s^f_1
\]

over \( s^f_1, s^f_0, d^f_0, i, \ell^f, \) and \( B \) subject to

\[
(R^D_1 - 1 + \delta) \left( y(i, \ell^f) + R^D_0 d^f_0 + (1 - \delta) s^f_0 \right) \geq R^D_1 R^L B, \\
d^f_1 + s^f_1 - R^L B = y(i, \ell^f) + R^D_0 d^f_0 + (1 - \delta) s^f_0, \\
d^f_0 + s^f_0 + i + w\ell^f = e + B,
\]

and the non-negativity constraints \( s^f_t \geq 0, d^f_t \geq 0, B \geq 0, i \geq 0, \ell^f \geq 0 \). The farmer’s
maximization program can be understood as follows. In equation (9) the farmer maxi-
mizes his Date 2 consumption $c^f$, which consists of his Date 1 deposits gross of interest $R_1^D d_1^f$, and his depreciated private savings, $(1 - \delta)s_1^f$. Equation (IC) is the incentive compatibility constraint. Although the incentive compatibility constraint looks different from the expression in equation (IC) in Subsection 3.1, it follows directly from substitution: the farmer’s Date 1 grain holding $g$ comprises his Date 1 output $y$, his Date 0 deposits gross of interest $R_0^D d_0^f$, and his depreciated savings $(1 - \delta)s_0^f$. Equation (BC$f^1_1$) is the farmer’s budget constraint at Date 1 which says that the sum of his Date 1 deposits and his Date 1 savings $s_1^f$ minus his repayment $R^L B$ must equal the sum of his output $y$, his Date 0 deposits gross of interest $R_0^D d_0^f$, and his depreciated savings $(1 - \delta)s_0^f$. Equation (BC$f^0_0$) is the farmer’s budget constraint at Date 0 which says that the sum of his Date 0 deposits $d_0^f$, his Date 0 savings $s_0^f$, his investment in grain $i$, and his investment in labor $w^f \ell$ must equal the sum of his initial endowment $e$ and the amount he borrows $B$.

The laborer’s maximization problem is

$$\text{maximize } c^l = R_1^D d_1^l + (1 - \delta)s_1^l - \ell^l$$

over $s_1^l, s_0^l, d_1^l, d_0^l$, and $\ell^l$ subject to

$$d_1^l + s_1^l = R_0^D d_0^l + (1 - \delta)s_0^l, \quad (BC^l_1)$$
$$d_0^l + s_0^l = w^f \ell^l, \quad (BC^l_0)$$

and the non-negativity constraints $s_i^l \geq 0, d_i^l \geq 0, \ell^l \geq 0$. The laborer’s maximization program can be understood as follows. In equation (10), the laborer maximizes his Date 2 consumption $c^l$, which consists of his Date 1 deposits gross of interest $R_1^D d_1^l$ and his depreciated private savings $(1 - \delta)s_1^l$ minus his cost of labor $\ell^l$. Equation (BC$l^1_1$) is the laborer’s budget constraint that says that the sum of his Date 1 savings $s_1^l$ and his Date 1 deposits $d_1^l$ must equal the sum of his Date 0 deposits gross of interest $R_0^D d_0^l$ and his depreciated savings $(1 - \delta)s_0^l$. Equation (BC$l^0_0$) is the laborer’s budget constraint at Date 0 which says that the sum of his Date 0 deposits $d_0^l$ and his Date 0 savings $s_0^l$ must equal his labor income $w^f \ell^l$.

The equilibrium is a profile of prices $\langle R_t^D, R^L, w \rangle$ for $t \in \{0, 1\}$ and a profile of allocations $\langle s_j^t, d_j^t, d_f^t, D_t, L, B, \ell^f, \ell^l \rangle$ for $t \in \{0, 1\}$ and $j \in \{b, f, l\}$ that solves the warehouses’ problem, the farmers’ problem, and the laborers’ problem defined above and satisfies the market clearing conditions for the labor market, the lending market,
the grain market, and deposit market at each date:

\[\ell f = \ell l, \quad (MC^f)\]

\[B = L, \quad (MC^L)\]

\[i + s_0^f + s_0^b + s_0^b = e, \quad (MC^g)\]

\[s_1^f + s_1^f + s_1^b = (1 - \delta)s_0^f + (1 - \delta)s_0^l + s_0^b + y, \quad (MC^g_1)\]

\[D_0 = d_0^f + d_0^l, \quad (MC^D_0)\]

\[D_1 = d_1^f + d_1^l, \quad (MC^D_1)\]

3.5 Preliminary Results for the Equilibrium

Here we state three results to characterize the prices, namely the two deposit rates \(R_0^D\) and \(R_1^D\), the lending rate \(R_L\), and the wage \(w\). We then show that, given the equilibrium prices, farmers and laborers will never store grain privately. The results all follow from the definition of competitive equilibrium with risk-neutral agents.

The first two results say that the risk-free rate in the economy is one. This is natural, since the warehouses have a scalable storage technology with return one.

**Lemma 1.** (Deposit Rates at \(t = 0\) and \(t = 1\)) \(R_0^D = R_1^D = 1\).

Now we turn to the lending rate. Since warehouses are competitive and the farmers’ incentive compatibility constraint ensures that loans are riskless, warehouses also lend to farmers at rate one.

**Lemma 2.** (Lending Rate) \(R_L = 1\).

Finally, since laborers have a constant marginal cost of labor, the equilibrium wage must be equal to this cost; this says that \(w = 1\), as summarized in Lemma 3 below.

**Lemma 3.** (Wages) \(w = 1\).

These results establish that the risk-free rate offered by warehouses exceeds the rate of return from private storage, or \(R_0^D = R_1^D = 1 > 1 - \delta\). Thus, farmers and laborers do not wish to make use of their private storage technologies. The only time a player may choose to store grain outside a warehouse is if a farmer diverts his output; however, the farmer’s incentive compatibility constraint ensures he will not do this. Corollary 1 below summarizes this reasoning.

**Corollary 1.** (Grain Storage) Farmers and laborers do not store grain privately, i.e., \(s_0^f = s_0^l = s_1^f = s_1^l = 0\).

\(^{24}\)Note that we have omitted the effect of discounting in the preceding argument—laborers work at Date 0 and consume at Date 2; discounting is safely forgotten, though, since the laborers have access to a riskless storage technology with return one via the warehouses, as established above.
3.6 Equilibrium Characterization

Now we characterize the equilibrium of the model. We begin by showing that, given the equilibrium prices established in Subsection 3.5 above, the solution to the farmers’ maximization problem is a solution to the model.

**Lemma 4. (Equilibrium Program)** The equilibrium allocation solves the program to

\[
\text{maximize } d_1^f \tag{11}
\]

subject to

\[
\delta \left( y(i, \ell^f) + d_0^f \right) \geq B, \quad \text{(IC)}
\]

\[
d_1^f + B = y(i, \ell^f) + d_0^f, \quad \text{(BC}_1^f\text{)}
\]

\[
d_0^f + i + \ell^f = e + B, \quad \text{(BC}_0^f\text{)}
\]

and \(i \geq 0, \ell^f \geq 0, B \geq 0, d_0^f \geq 0, \text{ and } d_1^f \geq 0\).

Solving the program above allows us to characterize the equilibrium allocations.

**Proposition 5. (Equilibrium Values of Debt, Labor, and Investment)** The equilibrium allocation is as follows:

\[
B = \frac{\delta A \alpha e}{1 + \alpha (1 - \delta A)}, \quad \text{(12)}
\]

\[
\ell = \frac{\alpha e}{1 + \alpha (1 - \delta A)}, \quad \text{(13)}
\]

\[
i = \frac{e}{1 + \alpha (1 - \delta A)}. \quad \text{(14)}
\]

The equilibrium above is the solution of a system of linear equations, from the binding budget constraints and the farmers’ binding incentive constraints.

3.7 The Equilibrium Is Constrained Efficient (Second Best)

We now show that the equilibrium in our model is constrained efficient, in the sense that it maximizes welfare among all individually rational incentive-compatible allocations.

**Proposition 6. (The Equilibrium Is Constrained Efficient)** If the worst feasible punishment for farmers is autarky, then the equilibrium summarized in Proposition 5 is optimal in the sense that it maximizes output and utilitarian welfare among all incentive-feasible allocations.
The efficiency of the equilibrium in our model suggests a rationale for the development of banks from warehouses. Exclusion from warehouse storage implements the worst feasible default penalty in our model since exclusion from storage is effectively autarky. The interbank market allows warehouse-banks to implement this punishment even in our finite-horizon setting.

4 Liquidity Creation, Welfare, and Policy

In this section, we present the analysis of the equilibrium in the context of liquidity creation. We then consider the implications of several policies, all of which have been debated by policy makers after the financial crisis of 2007–2009, namely liquidity requirements and narrow banking, equity capital for banks, and monetary policy.

4.1 Liquidity Creation

In this subsection, we turn to the funding liquidity warehouses create. We begin with the definition of a “liquidity multiplier,” which describes the total investment (grain investment plus labor investment) that farmers can undertake at Date 0 relative to the total endowment $e$.

**Definition 1.** The liquidity multiplier $\Lambda$ is the ratio of the equilibrium investment in production $i + w\ell$ to the total grain endowment in the economy $e$,

$$\Lambda := \frac{i + w\ell}{e}. \quad (15)$$

The liquidity multiplier $\Lambda$ reflects farmers' total investment at Date 0.

**Proposition 7. (Fake Receipts and Liquidity Creation)** If warehouses cannot issue fake receipts, no liquidity is created, $\Lambda_{nr} = 1$. With fake receipts, the equilibrium liquidity multiplier is

$$\Lambda = \frac{1 + \alpha}{1 + \alpha(1 - \delta A)} > 1. \quad (16)$$

Recall that warehouses have no initial endowment. Thus, if warehouses cannot issue fake receipts, they cannot lend at all. Indeed, the allocation with no fake receipts coincides with the benchmark allocation with no credit whatsoever (Proposition 2). Thus, this result implies that it is warehouses’ ability to make loans in fake receipts, not their ability to take deposits, that creates liquidity. Warehouses lubricate the economy because they lend in fake receipts rather than in grain. They can do this because of their dual function: they keep accounts (i.e. warehouse grain) and also make loans. This is the crux of farmers’ incentive constraints: because warehouses provide valuable
warehousing services, farmers go to these warehouse-banks and deposit their grain, which is then also the reason that they repay their debts.

We now analyze the effect of the private storage technology—i.e. the depreciation rate $\delta$—on warehouses’ liquidity creation. Differentiating the liquidity multiplier $\Lambda$ with respect to $\delta$:

$$ \frac{\partial \Lambda}{\partial \delta} = \frac{\alpha A}{(1 + \alpha(1 - \delta A))} > 0. $$

This leads to our next result.

**Corollary 2. (Warehouse Efficiency and Liquidity Creation)** The more efficiently warehouses can store grain relative to farmers (the higher is $\delta$), the more liquidity warehouses create by issuing fake receipts.

This result says that a decrease in the efficiency in private storage leads to an increase in overall efficiency. The reason is that $\delta$ measure the storage advantage that the warehouse has over private storage, so a higher $\delta$ weakens farmers’ incentive to divert capital, thereby allowing banks to create more liquidity. We return to this result when we discuss monetary policy below.

### 4.2 Fractional Reserves

We now proceed to analyze warehouses’ balance sheets. Absent reserve requirements, do they still store any grain or is everything lent out? Our next result addresses this question.

**Proposition 8. (Deposit Reserves Held by Warehouses)** Warehouses hold a positive fraction of grain at $t = 0$; in equilibrium,

$$ s_b^0 = e - i = \frac{\alpha(1 - \delta A)e}{1 + \alpha(1 - \delta A)} > 0. $$

Farmers have a constant-returns-to-scale technology and, therefore, they would prefer to invest all grain in the economy in their technology, leaving no grain for storage. However, we find that warehouses still store grain in equilibrium. This is because farmers’ incentive constraints put an endogenous limit on the amount that each farmer can borrow. Farmers cannot borrow enough from warehouses to pay laborers entirely in fake receipts as they would in the first best. Rather, they pay laborers in a combination of fake receipts and real grain. Laborers deposit the grain they receive in warehouses for storage.

Note that the storage of grain by warehouses at Date 0 is inefficient. Grain could be put to better use by farmers (in conjunction with labor paid for in fake receipts).
Thus, a policy maker in our model actually wishes to reduce warehouse holdings or bank (liquidity) reserves to have the economy operate more efficiently. We say more about this in the next section.

4.3 Liquidity Requirements, Liquidity Creation and Narrow Banking

Liquidity requirements. Basel III, the Basel Committee on Banking Supervision’s third accord, extends international financial regulation to include so-called liquidity requirements. Specifically, Basel III mandates that banks must hold a sufficient quantity of liquidity to ensure that a “liquidity ratio” called the Liquidity Coverage Ratio (LCR) is satisfied. The ratio effectively forces banks to invest a portion of their assets in cash and cash-proximate marketable securities. In our model, the LCR puts a limit on the ratio of loans that a bank (warehouse) can make relative to the deposits (grain) it stores, i.e. it puts a limit on the quantity of fake receipts a bank can issue. This inhibits liquidity creation, since warehouses create liquidity only by combining deposit-taking and lending within the same institution.

We now make this more formal. Consider a liquidity regulation that, like the LCR, mandates that a bank hold a proportion \( \theta \) of its assets in liquid assets, or

\[
\frac{\text{liquid assets}}{\text{total assets}} \geq \theta. \tag{19}
\]

In our model, the warehouses’ liquid assets are the grain they store and their total assets are the grain they store plus the loans they make. Thus, within the model, the liquidity regulation described above prescribes that, at Date 0,

\[
\frac{s_0^b}{B + s_0^b} \geq \theta. \tag{20}
\]

We see immediately by rewriting this inequality that this regulation imposes a cap on bank lending,

\[
B \leq \frac{1 - \theta}{\theta} s_0^b. \tag{21}
\]

This leads to the next result.

**Proposition 9. (Effect of Liquidity Requirements)** Whenever the required liquidity ratio \( \theta \) is such that

\[
\theta > 1 - \delta A, \tag{22}
\]

liquidity regulation inhibits liquidity creation. Farmers’ investment is thus below the equilibrium level.
Narrow banking. Advocates of so-called narrow banking have argued that banks should hold only liquid securities as assets, with some arguing for banks to invest only in Treasuries. In our model, such a restriction robs the bank of a fundamental economic service—expanding aggregate investment in real projects through the provision of funding liquidity. Thus far, we have focused on the effects of liquidity requirements on bank lending. However, advocates of liquidity requirements often focus on their effects on the bank’s ability to meet unexpected withdrawals and hence financial stability, not funding liquidity. In Subsection 5.2, we include the possibility of a bank run (or “warehouse run”) and explore the connection between liquidity requirements and financial stability. In this setting, higher liquidity requirements may reduce stability.

4.4 Bank Capital and Liquidity Creation

In this subsection, we extend the model to include a role for warehouse-bank capital. To do this, we assume that the warehouse has equity endowment $e_b$ at Date 1 and we add a pledgeability problem for the warehouse. Specifically, after a warehouse accepts deposits, it has the following choice: it can either divert grain and store it privately or not divert grain and store it in the warehouse. If it diverts the grain, the depositors will not be able to claim it, but it will depreciate at rate $\delta$. If the warehouse does not divert, depositors will be able to claim it, but it will not depreciate. We show that warehouse equity has an important function: it gives the warehouse the incentive not to divert deposits.

The results of this subsection follow from the analysis of the warehouse’s incentive constraint: depositors store in a warehouse at Date 1 only if the warehouse prefers not to divert deposits. Its payoff, if it diverts, is the depreciated value of its equity plus its deposits, or $(1 - \delta)(e_b + D_1)$. Its payoff, if it does not divert, is the value of its equity plus its deposits less its repayment to its depositors, or $e_b + D_1 - R_1 D_1$. Since $R_1 D_1 = 1$ by Lemma 1, the warehouses incentive compatibility constraint at Date 1 is

$$ (1 - \delta)(e_b + D_1) \leq e_b. \quad (23) $$

Thus, the equilibrium allocation is constrained efficient (second-best) as summarized

---

25 See Kay (2010), Kotlikoff (2010), and Pennacchi (2012), for example.
26 Because we do not have bank failures and crises in our baseline model, our analysis likely understates the value and role of bank capital. Berger and Bouwman (2013) document that higher-capital banks have an advantage during financial crises. Calomiris and Nissim (2014) document that the market is attaching a higher value to bank capital after the 2007–09 crisis.
27 Note that if the warehouse diverts, it must do so at Date 1 before depositing grain in the warehouse. If it deposits grain in the warehouse from Date 1 to Date 2, it is too late to divert, because warehoused grain is publicly observable and therefore pledgeable.
in Proposition 4 only if the incentive constraint above is satisfied, or

\[ \frac{e^b}{D_1} \geq \frac{1 - \delta}{\delta}. \]  

(24)

This constraint says that the second-best is attained only if the warehouse’s capital ratio is sufficiently high. Substituting the equilibrium value of \( D_1 \) from Proposition 5 gives the next result.

**Proposition 10. (Role of Warehouse Equity) The second-best allocation in Proposition 4 is attained only if warehouse equity is sufficiently high, or**

\[ e^b \geq \hat{e}^b := \frac{1 - \delta \alpha [1 + (1 - \delta)A]}{\delta [1 + \alpha(1 - \delta A)]} c. \]  

(25)

If warehouse equity is below \( \hat{e}^b \), the warehouse’s incentive constraint binds (and the farmer’s incentive constraint does not) and an increase in warehouse equity loosens the warehouse’s incentive constraint. This allows it to accept more deposits. Since accepting more deposits allows it to obtain a larger repayment from borrowers, this also allows the warehouse to make more loans, which leads to the next result.

**Proposition 11. (Liquidity Creation for Different Levels of Warehouse Capital) When warehouse equity \( e^b \) is below a threshold,**

\[ \hat{e}^b := \frac{\alpha(1 - \delta)(1 + A)e}{(1 + \alpha)\delta}, \]  

(26)

there is no lending and hence no liquidity creation. For \( e^b \in (\hat{e}^b, \hat{e}^b] \), liquidity creation is strictly increasing in warehouse equity \( e^b \). For \( e^b > \hat{e}^b \), warehouse equity has no effect on liquidity creation. Specifically,

\[ \Lambda = \begin{cases} 
1 & \text{if } e^b \leq \hat{e}^b, \\
\frac{1 + \alpha}{\alpha A - 1} \left( \frac{\delta}{1 - \delta} \frac{e^b}{c} - 1 \right) & \text{if } e^b \in (\hat{e}^b, \hat{e}^b], \\
\frac{1 + \alpha}{1 + \alpha(1 - \delta A)} & \text{if } e^b > \hat{e}^b.
\end{cases} \]  

(27)

The expression for \( \Lambda \) in this proposition, illustrated in Figure 2, says that when warehouse equity is very low, the incentive problem is so severe that warehouses do not lend at all. As equity increases, warehouses start lending and the amount they lend increases linearly until the farmers’ incentive constraints bind. Above this threshold, an increase in equity has no further effect, because the farmers’ incentive constraints bind.
4.5 Monetary Policy

We now extend the model to analyze how monetary policy affects liquidity creation. We define the central bank rate $R_{CB}$ as the (gross) rate at which warehouses can deposit with the central bank.\footnote{We are considering a rather limited aspect of central bank monetary policy here, thereby ignoring things like the role of the central bank in setting the interest rate on interbank lending, as in Freixas, Martin and Skeie (2011), for example.} This is analogous to the storage technology of the warehouse yielding return $R_{CB}$. In this interpretation of the model, grain is central bank money and warehouse receipts are private money.

We first state the necessary analogs of the parameter restrictions in Subsection 2.2. Note that they coincide with Parameter Restriction 1 and Parameter Restriction 2 when $R_{CB} = 1$, as in the baseline model.

**Parameter Restriction 1’.** The farmers’ technology is sufficiently productive,

$$A > \frac{1}{R_{CB}} + \frac{R_{CB}}{\alpha}.$$  \hfill (28)

**Parameter Restriction 2’.** Depreciation from private storage is not too fast,

$$A (R_{CB} - 1 + \delta) < 1.$$  \hfill (29)
The preliminary results of Subsection 3.5 lead to the natural modifications of the prices. In particular, due to competition in the deposit market, the deposit rates equal the central bank rate. Further, because laborers earn interest on their deposits, they accept lower wages. Thus we have:

**Lemma 5. (Interest Rates and Wages with a Central Bank)** When warehouses earn the central bank rate \( R^{CB} \) on deposits, in equilibrium, the deposit rates, lending rate, and wage are as follows:

\[
R_{0}^{D} = R_{1}^{D} = R^{L} = R^{CB} \tag{30}
\]

and

\[
w = \left( R^{CB} \right)^{-2}. \tag{31}\]

The crucial takeaway from the result is that the warehouse pays a higher deposit rate when the central bank rate is higher. This means that the farmer’s incentive constraint takes into account a higher return from depositing in a warehouse, but the same depreciation rate from private storage. Formally, with the central bank rate \( R^{CB} \), the farmer’s incentive constraint at Date 1 reads

\[
R^{CB} (y - R^{CB} B) \geq (1 - \delta) y \tag{32}
\]

or

\[
B \leq \frac{1}{R^{CB}} \left( \frac{1}{1 - \delta} \right) y. \tag{33}
\]

Observe that whenever farmers are not too highly levered—\( B < y \left( \frac{2R^{CB}}{R^{CB}} \right)^{-2} \)—increasing \( R^{CB} \) loosens the incentive constraint. The reason is that it makes warehouse storage relatively more attractive at Date 1, inducing farmers to repay their debt rather than to divert capital.

**Proposition 12. (Monetary Policy and Liquidity Creation)** A tightening of monetary policy (an increase in \( R^{CB} \)) increases liquidity creation \( \Lambda \) as long as \( \alpha + 2R^{CB} (1 - \delta) > (R^{CB})^{2} \) (otherwise it decreases liquidity creation).

This contrasts with the established idea that a lowering of the interest rates by the central bank stimulates bank lending. In our model, high interest rates allow banks to lend more. This result complements Corollary 2 which says that liquidity creation is increasing in the depreciation rate \( \delta \). Both results say that the better warehouses are at storing grain relative to farmers, the more warehouses can lend.

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29 The reason that increasing \( R^{CB} \) does not loosen the constraint when \( B \) is high, is that it also increases the lending rate between Date 0 and Date 1.

30 See, for example, Keeton (1993). Mishkin (2010) provides a broad assessment of monetary policy, bank lending, and the role of the central bank.
5 Robustness

In this section, we show that our mechanism is robust to two changes in the model specification. First, we show that the incentive mechanism is robust to the possibility of the farmer consuming at the interim date. Second, we show that our analysis of liquidity requirements in Subsection 4.3 is robust to the inclusion of bank runs—financial fragility concerns do not overturn our results that liquidity requirements inhibit liquidity creation.

5.1 Consumption at Date 1

In the baseline model, farmers produce at Date 1 and consume only at Date 2. This gives them the need to save between Date 1 and Date 2, generating the demand to deposit in a warehouse to access its superior storage technology. In this subsection, we consider a different specification of farmers’ preferences, in which farmers have log utility and consume at Date 1 and Date 2. We show that our mechanism is robust to the possibility of allowing farmers to consume at Date 1. The intuition for this is that with log utility the farmer has incentive to smooth consumption across dates. Thus, he always has incentive to save something for Date 2.

Here we denote consumption at Date $t$ by $c_t$, so that a farmer’s total payoff is given by

$$U(c_1, c_2) = \log c_1 + \log c_2.$$  \hfill (34)

If a farmer has grain $g$ at Date 1, it is incentive compatible for him to repay his debt if he prefers to deposit and repay than to divert, where now if he diverts he may either store privately, as before, or consume immediately. This generates an incentive constraint analogous to that in the baseline model (equation (IC)), except with a lower rate of depreciation, as summarized in the next proposition.

**Proposition 13. (Borrowing Constraint with Consumption at Date 1)** If farmers have logarithmic utility at Date 1 and Date 2, their borrowing constraint is given by

$$L \leq \frac{\sqrt{R_D} - \sqrt{1 - \delta}}{\sqrt{R_D} R_L} g$$  \hfill (35)

or, in equilibrium,

$$L \leq \left(1 - \sqrt{1 - \delta}\right) y \approx \frac{\delta g}{2},$$  \hfill (36)

where the approximation follows from the Taylor expansion.
5.2 Financial Fragility

In Subsection 4.3, we showed that higher liquidity requirements decrease bank liquidity creation by inhibiting lending. However, the oft-stated purpose of liquidity requirements is to enhance financial stability. The argument goes that a bank with more liquid reserves can withstand more withdrawals or a larger “run” from its depositors, creating stability. In this subsection, we address this argument by extending the model to include a bank run game among depositors. To do this, we suppose that laborers randomly gain access to a superior private storage technology, which creates the incentive to withdraw from the banking system, i.e. to “disintermediate” savings. We show that more liquid reserves may make a financial system more fragile in our setting. The reason is that while liquid reserves do indeed allow a bank to withstand a bigger run, they also make depositors more prone to running the bank.

We consider a warehouse with total deposits $\theta$ at Date 0 and add an additional stage immediately after Date 0, called Date $0^+$. At this stage, each depositor may demand to withdraw grain or leave it in the warehouse. Let $\lambda$ denote the total amount of grain demanded by all depositors at Date $0^+$. If $\lambda \leq \theta$, then the warehouse has sufficient reserves to pay all withdrawing depositors. It remains solvent. Withdrawing depositors take out their grain and store it privately, where the depreciation rate on private storage is the random rate $\delta$. Note that negative $\delta$ corresponds to the laborers having an investment opportunity with a higher return than warehouse storage. Non-withdrawing depositors do not take out their grain, but can claim it at Date 1. Thus, they avoid depreciation. If $\lambda > \theta$, then the warehouse does not have sufficient reserves to pay all withdrawing depositors. The warehouse is insolvent. It liquidates the reserves for a positive amount $h(\theta)$—where $h(\theta) < \theta$, reflecting the costs of early liquidation—and it distributes them among withdrawing depositors according to a pro rata rule, i.e. for each unit of grain demanded, a withdrawing depositor receives $h(\theta)/\lambda$ units of grain. Since the warehouse is insolvent, depositors who have not withdrawn cannot cash in their receipts at Date 1. They receive zero.

In order to analyze the effect of liquid reserves $\theta$ on financial stability, we select an equilibrium using global games techniques. We suppose that the rate of depreciation

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31 The Basel Committee on Banking Supervision states that “The LCR is one of the Basel Committee’s key reforms to strengthen global capital and liquidity regulations with the goal of promoting a more resilient banking sector. The LCR promotes the short-term resilience of a bank’s liquidity risk profile” (see http://www.bis.org/publ/bcbs238.htm).

32 In the case in which the warehouse is a granary or a goldsmith’s safe, the difference $\theta - h(\theta)$ could represent the transportation costs of making additional withdrawals. In the case in which the warehouse represents a modern bank or a custodian, $\theta - h(\theta)$ represents the costs of unexpected liquidation of liquid assets, for example due to price impact or transactions costs. Note that our results hold even if $h$ is very small.

33 See Goldstein and Pauzner (2003) for an application to bank runs and Morris and Shin (2003) for a
\(\delta\) is random and depositors observe it with a small amount of noise. In the limit as this noise vanishes, the game has a unique equilibrium, depending on the realization of \(\delta\). There is a number \(\delta^*\) such that if \(\delta > \delta^*\) no one withdraws and if \(\delta < \delta^*\) everyone withdraws—i.e. there is a bank run. Thus, \(\delta^*\) is a measure of financial fragility. In other words, there is a one-to-one correspondence between the attractiveness of disintermediated private investment and the fragility of the warehouse-banking system.

**Proposition 14. (Liquidity Reserves and Financial Fragility)** Whenever

\[
h'(\theta) > \frac{h(\theta) + h(\theta)|\log \theta|}{\theta|\log \theta|},
\]

(37)

an increase in reserves \(\theta\) causes an increase in financial fragility \(\delta^*\), or

\[
\frac{\partial \delta^*}{\partial \theta} > 0.
\]

(38)

The intuition is that an increase in reserves can increase the attractiveness of withdrawing early. To see this, consider the extreme case in which the warehouse holds no reserves or \(\theta = 0\). In this case, a depositor never wishes to withdraw early because he never receives any grain, regardless of whether there is a run. Thus, having more reserves has an “incentive effect,” in that it makes withdrawing more attractive for depositors. Note that, despite this incentive effect, increasing reserves does not increase financial fragility for all parameters (cf. equation (37) in the Appendix). This is because the usual “buffer effect,” by which more reserves make the bank able to withstand more withdrawals, is still present. Which effect dominates is determined by the slope of the function \(h\).

6 Conclusion

In this paper we have developed a new theory of banking that is tied to the origins of banks as commodity warehouses. The *raison d'être* for banks does not require asymmetric information, screening, monitoring, or risk aversion. Rather, we show that the institutions with the best storage (warehousing) technology have an advantage in enforcing contracts, and are therefore not only the natural deposit-takers but are also the natural lenders—i.e. they are the natural banks. The development of interbank markets supports this enforcement mechanism in the presence of competing banks. Further, we gloss over the necessary conditions for global games techniques to apply here. For example, it is critical that \(\delta < 0\) and \(\delta > 1\) with some probability, since to withdraw and not to withdraw must be dominant strategies for some parameters.
despite evolving from warehouses that store real goods, banks make loans by issuing “fake” warehouse receipts—in our model, it is not only the case that deposits create loans, as in much of the existing literature, but also that loans create deposits.

Our theory has regulatory implications. It shows that proposals like narrow banking and liquidity requirements may diminish bank liquidity creation. By contrast, higher levels of bank capital enhance bank liquidity creation. Moreover, we establish conditions under which a tighter monetary policy induces more liquidity creation.
Appendix

A Proofs

Proof of Proposition 1

As discussed in the text preceding the statement of the proposition, in the first best all grain is invested in its first-best use at Date 0. This corresponds to $i_{fb} = e$, since the farmer’s technology is the most productive. The production function requires $\ell_{fb} = \alpha i = \alpha e$ units of labor to be productive, and any more is unproductive. In summary, $i_{fb} = e$ and $\ell_{fb} = \alpha e$, as stated in the proposition.

Proof of Proposition 2

First observe that the wage $w = 1$ since laborers’ marginal cost of labor is one (cf. Lemma 3 below). Now, substituting this into the budget constraint in equation (4) and solving the system with equation (5) gives the result immediately.

Proof of Proposition 3

This result follows immediately from comparing a farmer’s payoff from repaying his debt $R^L B$ and storing $g$ in a warehouse at rate $R^D_1$ with his payoff from defaulting on his debt and storing $g$ privately at rate $1 - \delta$. Thus, he gets $R^D_1 (g - R^L B)$ if he repays and deposits and he gets $(1 - \delta) g$ if he diverts and stores privately. The comparison of these expression gives the statement in the proposition.

Proof of Proposition 4

A farmer has debt $R^L B$, which warehouse trades in the interbank market. Without loss of generality, suppose that the warehouses trade bonds with face value one. Thus, the supply of the farmer’s bonds is equal to its total outstanding debt $R^L B$. Denote the price of one bond by $p$.

We first show that if a farmer deposits his output in any warehouse then his bonds trade at par. The key to the argument is that, because the warehouses that hold the farmers’ grain can seize it at par, the debt cannot trade at a discount from par—if the price of debt is less than one, then the warehouses that hold it demand more than the total supply.

We assume that the incentive constraint in Proposition 3—that the farmer prefers to deposit and repay than to private privately—is satisfied. This is without loss of
generality, since if it does not hold the farmer will just store privately, i.e. he will not deposit in a warehouse at all and there will be no trade in the interbank market. It follows from the IC that the farmer’s total Date 1 deposits $g$ exceed his total debt $R^L B$.

**Lemma 6.** *The price of the farmer’s bonds in the interbank market is one, $p = 1$.*

*Proof.* For this proof, we augment our notation slightly and denote the grain that the farmer has deposited in warehouse $b$ by $d^b$ and warehouse $b$’s demand for the farmer’s debt by $x^b$. The proof is by contradiction.

First suppose (in anticipation of a contradiction) that $p < 1$. Each warehouse $b$ who holds the farmer’s grain has demand for the farmer’s debt equal to the deposits he has $x^b = d^b$. Thus, the total demand for the farmers’ debt equals the total of his deposits $g$, $\int_{b} x^b db = \int_{b} d^b db = g$. This is greater than the total supply of the farmer’s debt. Thus, the market cannot clear. We conclude that it must be that $p \geq 1$.

Now suppose (in anticipation of a contradiction) that $p > 1$. All warehouses sell the farmer’s debt, supplying $R^L B$, but no warehouse buys the farmer’s debt, since the price is greater than any warehouse’s private value (which is at most one). Thus, the market cannot clear. We conclude that it must be that $p \leq 1$.

Since $p \geq 1$ and $p \leq 1$, $p = 1$.

We now conclude the analysis of the farmer’s repayment. Trade in the interbank market results in warehouses that hold deposits buying all $R^L B$ units of the farmer’s debt at price $p = 1$. These warehouses seize the total $R^L B$ units of the farmer’s grain that they are owed—the farmer repays in full. Further, the warehouses that lend to the farmer either seize repayment from their deposits at par or sell his debt at par in the interbank market—the lending warehouses are repaid in full.

*Proof of Lemma 1*

We show the result by contradiction. If $R^D_t \neq 1$ in equilibrium, deposit markets cannot clear.

First suppose (in anticipation of a contradiction) that $R^D_t < 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s^b_t = D_t$ in the warehouse’s problem in Subsection 3.4. The warehouse’s objective function (equation (8)) goes to infinity as $D_t \to \infty$ without violating the constraints. The deposit markets therefore cannot clear if $R^D_t < 1$, a contradiction. We conclude that $R^D_t \geq 1$.

Now suppose (in anticipation of a contradiction) that $R^D_t > 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s^b_t = D_t$ in the warehouse’s problem. The warehouse’s objective function goes to infinity as $D_t \to -\infty$ without violating the budget constraints.

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35I.e. $g > R^L B$. This follows from the fact that the incentive constraint (equation (IC)) must be satisfied, as noted above.
Thus, if \( R_t^D > 1 \), it must be that \( D_t = 0 \). However, since the depreciation rate \( \delta > 0 \), the demand from laborers and farmers to store grain is strictly positive for \( R_t^D > 1 - \delta \). Thus, again, deposit markets cannot clear, a contradiction. We conclude that \( R_t^D \leq 1 \).

The two contradictions above taken together imply that \( R_t^D = 1 \) for \( t \in \{0, 1\} \). □

Proof of Lemma 2

We show the result by contradiction. If \( R^L \neq 1 \) in equilibrium, loan markets cannot clear.

First suppose (in anticipation of a contradiction) that \( R^L > 1 \) in equilibrium. Now set \( L = D_t \) in the warehouse’s problem in Subsection 3.4. Given that \( R_0^D = 1 \) from Lemma 1 above, the warehouse’s objective function (equation (8)) goes to infinity as \( L \to \infty \) without violating the constraints. The deposit markets therefore cannot clear if \( R^L > 0 \), a contradiction. We conclude that \( R^L \leq 1 \).

Now suppose (in anticipation of a contradiction) that \( R^L < 1 \) in equilibrium. Now set \( L = D_0 \) in the warehouse’s problem. Given that \( R_0^D = 1 \) from Lemma 1 above, the warehouse’s objective function goes to infinity as \( L \to -\infty \) without violating the budget constraints. Thus, if \( R^L < 1 \), it must be that \( D_0 = 0 \). However, since the depreciation rate \( \delta > 0 \), the demand from laborers and farmers to store grain is always strictly positive for \( R_t^D > 1 - \delta \). Thus, again, deposit markets cannot clear, a contradiction. We conclude that \( R^L \geq 1 \).

The two contradictions above taken together imply that \( R^L = 1 \). □

Proof of Lemma 3

We show the result by contradiction. If \( w \neq 1 \) in equilibrium, labor markets cannot clear.

First suppose (in anticipation of a contradiction) that \( w > 1 \) in equilibrium. From Corollary 1, \( d^l_0 = w\ell^l \) and \( d^l_1 = R^D_0 d^l_0 \) in the laborer’s problem in Subsection 3.3. The constraints collapse, and the laborer’s objective function (equation (10)) is \( R^D_1 R^D_0 w\ell^l - \ell^l = (w - 1)\ell^l \), having substituted \( R_0^D = R_1^D = 1 \) from Lemma 1 above. Since \( w > 1 \) by supposition, the objective function approaches infinity as \( \ell^l \to \infty \) without violating the constraints. The labor market therefore cannot clear if \( w > 1 \), a contradiction. We conclude that \( w \leq 1 \).

Now suppose (in anticipation of a contradiction) that \( w < 1 \) in equilibrium. As above, the laborer’s objective function is \((w - 1)\ell^l\). Since \( w < 1 \) by supposition, the laborer sets \( \ell^l = 0 \). The farmer, however, always has a strictly positive demand for labor if \( w < 1 \)—he produces nothing without labor and his productivity \( A > 1 + 1/\alpha \)

\[36\text{The proof of Corollary 1 is below; it does not depend on this result.} \]
by Parameter Restriction 1. The labor market therefore cannot clear if \( w < 1 \), a contradiction. We conclude that \( w \geq 1 \).

The two contradictions above taken together imply that \( w = 1 \). \( \square \)

**Proof of Corollary 1**

Given Lemma 1 above, the result is immediate from inspection of the farmer’s problem and the laborer’s problem in Subsection 3.3 given that \( R_0^D = R_1^D = 1 > 1 - \delta \). \( \square \)

**Proof of Lemma 4**

Before explaining the proof of the lemma, we state a preliminary result which says that, given the equilibrium prices established in Subsection 3.5 for any solution to the farmer’s individual maximization problem, laborers’ and warehouses’ demands are such that markets clear.

**Lemma 7. (Warehouse and Laborer Preferences)** Given the equilibrium prices, \( R_0^D = R_1^D = R^L = w = 1 \), warehouses are indifferent among all deposit and loan amounts and laborers are indifferent among all labor amounts.

**Proof.** The result follows immediately from the proofs of Lemma 1, Lemma 2 and Lemma 3 which pin down the prices in the model by demonstrating that if prices do not make these players indifferent, markets cannot clear, contradicting that the economy is in equilibrium. \( \square \)

Now, the result follows from Lemma 7 above and substituting in prices and demands from the preliminary results in Subsection 3.5. In short, since, given the equilibrium prices, laborers and warehouses are indifferent among allocations, they will take on the excess demand left by the farmers to clear the market. \( \square \)

**Proof of Proposition 5**

We begin by rewriting the farmer’s problem in Lemma 4 as

\[
\text{maximize} \quad d_1^f \\
\text{subject to} \quad \delta \left( A \min \{ \alpha_i, \ell^f \} + d_0^f \right) \geq B, \quad \text{(IC)}
\]

\[
d_1^f + B = A \min \{ \alpha_i, \ell^f \} + d_0^f, \quad \text{(BC}_1^f) \]

\[
d_0^f + i + \ell^f = e + B, \quad \text{(BC}_0^f) \]
and \( i \geq 0, \ell^f \geq 0, B \geq 0, d_0^f \geq 0, \) and \( d_1^f \geq 0. \)

Now observe that at the optimum, \( \min \{ \alpha i, \ell^f \} = \ell^f \) and \( \ell^f = \alpha i. \) Further, eliminate the \( d_1^f \) in the objective from the budget constraint. Now we can write the problem as

\[
\text{maximize } A\ell^f + d_0^f - B \\
\text{subject to } \\
\delta (A\ell^f + d_0^f) \geq B, \\
\ell^f = \alpha i
\]

and \( i \geq 0, \ell^f \geq 0, B \geq 0, \) and \( d_0^f \geq 0. \)

We see that the budget constraint and \( \ell^f = \alpha i \) imply that

\[
B = d_0^f + \frac{1 + \alpha}{\alpha} \ell^f - e
\]

and, thus, the objective is

\[
A\ell^f - \frac{1 + \alpha}{\alpha} \ell^f + e = \frac{\alpha(A - 1) - 1}{\alpha} \ell^f + e.
\]

This is increasing in \( \ell^f \) by Parameter Restriction \([1] \) so \( \ell^f \) is maximal at the optimum. Thus, the incentive constraint binds, or

\[
\delta (A\ell^f + d_0^f) = B = d_0^f + \frac{1 + \alpha}{\alpha} \ell^f - e. \\
\]

or

\[
e - (1 - \delta) d_0^f = \left(1 - \delta A + \frac{1}{\alpha}\right) \ell^f.
\]

Since, by Parameter Restriction \([2] \) \( \delta A < 1, \) setting \( d_0^f = 0 \) maximizes \( \ell^f. \) Hence,

\[
\ell^f = \frac{ae}{1 + \alpha(1 - \delta A)}.
\]

Combining this with the budget constraint and the equation \( i = \ell^f / \alpha \) gives the expressions in the proposition.

\[
\square
\]

Proof of Proposition \([6] \)

We divide the proof of the proposition into three steps. In Step 1, we explain that a mechanism that implements the most severe feasible punishments can implement the
(constrained) optimal outcome. In Step 2, we argue that the most severe punishments in our environment are the exclusion from warehousing. In Step 3, we show that our environment with Walrasian markets in which warehouses can seize their deposits implements these punishments.

**Step 1.** A mechanism can implement an outcome if the outcome is incentive compatible given the mechanism. Increasing the severity of punishments corresponds to loosening incentive constraints, which expands the set of implementable outcomes. Hence, increasing the severity of punishments expands the set of implementable outcomes.

**Step 2.** In our environment, punishments must be administered at Date 1 (at Date 2 agents consume, so we are effectively already in autarky and at Date 0 it is too early to punish them for anything). At Date 1, there are only two technologies, private storage and warehouse storage. Thus, the only benefit the environment provides beyond autarky is access to warehousing. In other words, the worst possible punishment is exclusion from warehousing.

**Step 3.** The only limit to commitment in our environment comes from the non-pledgeability of farmers’ output—the farmer is the only player who might not fulfill his promise. However, given the interbank market, anything the farmer deposits in the warehouse ultimately can be seized. Thus, the only way that a farmer can avoid repayment at Date 1 is by storing privately. This is equivalent to saying that if a farmer breaks his promise, he cannot store in a warehouse—he receives the autarky payoff. Thus, our model imposes the most severe feasible punishments on defecting players. As a result (from Step 1), our model implements the optimal incentive-feasible outcome.

**Proof of Proposition 7**

The result that $\Lambda_{nr} = 1$ follows from the fact that farmers have the entire initial endowment. With no fake receipts, warehouses cannot lend because they have no initial grain endowment. (Laborers never lend, i.e. provide labor on credit, because they cannot enforce repayment from farmers.) Thus, there is no credit extended and no liquidity created: the equilibrium allocation equals the allocation with no credit in Proposition 2.

The second part of the result follows immediately from comparison of the equilibrium expression for $i + wL$ given in Proposition 5 and Parameter Restriction 2.

**Proof of Corollary 2**

The result is immediate from differentiation, as expressed in equation (17).
Proof of Proposition 8

The expression given in the proposition is positive as long as \(1 - \delta A > 0\). This holds by Parameter Restriction 2. The result follows immediately.

Proof of Proposition 9

The liquidity ratio inhibits liquidity creation whenever warehouses’ equilibrium Date 0 grain holdings \(s_0^b\) are insufficient to satisfy their liquidity requirements. In other words, given equation (21), if

\[
B < \frac{1 - \theta}{\theta} s_0^b, \tag{A.9}
\]

then liquidity requirements inhibit liquidity creation. Given the equilibrium values of \(s_0^b\) and \(B\), this can be rewritten as

\[
\frac{\delta A \alpha e}{1 + \alpha (1 - \delta A)} < \frac{1 - \theta}{\theta} \frac{\alpha (1 - \delta A) e}{1 + \alpha (1 - \delta A)}. \tag{A.10}
\]

This holds only if

\[\theta < 1 - \delta A. \tag{A.11}\]

Whenever this inequality is violated, liquidity requirements inhibit liquidity creation. This is the negation of the above equality stated in the proposition.

Proof of Proposition 10

The proof comes from solving for the Date 1 deposits \(D_1\) in the equilibrium in Proposition 5 and checking when the warehouse’s incentive constraint (equation (24)) is violated. In the equilibrium in Proposition 5 we have that

\[
D_1 = y + s_0^b = A \alpha i + (e - i) = \frac{\alpha [1 + (1 - \delta) A] e}{1 + \alpha (1 - \delta A)}, \tag{A.12-14}
\]

having substituted for \(i\) from the expression in Proposition 5. Thus, the warehouse’s IC is violated for \(e^b < \hat{e}^b\), where \(\hat{e}^b\) solves

\[
\frac{1 - \delta}{\delta} \frac{\hat{e}^b}{D_1} = \frac{[1 + \alpha (1 - \delta A)] \hat{e}^b}{\alpha [1 + (1 - \delta) A] e}, \tag{A.15}
\]

or

\[
\hat{e}^b = \frac{1 - \delta \alpha [1 + (1 - \delta) A]}{\delta} \frac{e}{1 + \alpha (1 - \delta A)}. \tag{A.16}
\]
Proof of Proposition 11

This result follows from solving for the equilibrium with the warehouse’s incentive constraint binding. We proceed assuming that lending $L$ is positive. If it is negative, the formulae do not apply and $L = 0$.

Begin with the warehouses’ binding incentive constraint, which gives a formula for $D_1$, the total grain deposited at Date 1,

$$D_1 = \frac{\delta}{1 - \delta} e^b.$$  \hfill (A.17)

The Date 1 deposit market clearing condition implies that the total amount of deposits equals the total amount of grain at Date 1. This is the sum of the farmer’s output $y$ and the grain stored in the warehouse at Date 0, $s_0^b$,

$$D_1 = y + s_0^b = A \alpha i + e - i.$$  \hfill (A.18)

Combining this with the warehouses’ incentive constraint gives

$$i = \frac{1}{\alpha A - 1} \left( \frac{\delta}{1 - \delta} e^b - e \right).$$  \hfill (A.20)

(Note that $\alpha A - 1 > 0$ by Parameter Restriction 1.) Now, since the farmers’ technology is Leontief, $\ell = \alpha i$. This allows us to write the expression for the liquidity multiplier $\Lambda$:

$$\Lambda = \frac{i + w \ell}{e} = \frac{1 + \alpha}{\alpha A - 1} \left( \frac{\delta}{1 - \delta} e^b - e \right).$$  \hfill (A.21)

This expression applies when it is greater than one (and $e^b$ is below the threshold in Proposition 10) or

$$\frac{1 + \alpha}{\alpha A - 1} \left( \frac{\delta}{1 - \delta} e^b - e \right) \geq 1.$$  \hfill (A.23)

Which can be rewritten as

$$e^b \geq \frac{\alpha (1 - \delta)(1 + A)e}{(1 + \alpha)\delta} \equiv \hat{e}^b.$$  \hfill (A.24)

Otherwise, no liquidity is created and the liquidity multiplier is one. \hfill \Box
Proof of Lemma 5

The proofs that $R_D^0 = R_D^1 = R_L = R_{CB}$ are all identical to the proofs of the analogous results in Subsection 3.5 with the warehouses’ return on storage (which is one in the baseline model) replaced with the central bank rate $R_{CB}$. The result is simply that warehouses lend and borrow at their cost of storage, which is a result of warehouses being competitive.

The result that $w = (R_{CB})^{-2}$ is also nearly the same as the proof of the analogous result (Lemma 3) in Subsection 3.5. The modification is that the laborer’s objective function (equation (10)) reduces to $c^l = (R_{CB})^2 w \ell - \ell$, since the laborer invests its income in the warehouse for two periods at gross rate $R_{CB}$. In order for the laborer not to supply infinite (positive or negative) labor $\ell$, it must be that $w = (R_{CB})^{-2}$. \qed

Proof of Proposition 12

Solving for the equilibrium again reduces to solving the farmer’s problem with binding incentive and budget constraints. With the prices given in Lemma 5, these equations are

$$R_{CB} (y - R_{CB}) = (1 - \delta) y$$

(A.25)

and

$$i + (R_{CB})^{-2} \ell = e + B$$

(A.26)

where $y = A \min \{\alpha i, \ell\}$ and, in equilibrium, $i = \alpha \ell$. From the budget constraint we find that

$$\ell = \frac{\alpha (R_{CB})^2 (e + B)}{\alpha + (R_{CB})^2}$$

(A.27)

and, combining the above with the incentive constraint,

$$B = \frac{\alpha A (R_{CB} - 1 + \delta) e}{\alpha + (R_{CB})^2 - \alpha A (R_{CB} - 1 + \delta)}.$$  

(A.28)

This gives the following equilibrium allocation:

$$\ell = \frac{\alpha (R_{CB})^2 e}{\alpha + (R_{CB})^2 - \alpha A (R_{CB} - 1 + \delta)},$$

(A.29)

$$i = \frac{(R_{CB})^2 e}{\alpha + (R_{CB})^2 - \alpha A (R_{CB} - 1 + \delta)}.$$  

(A.30)
We use the allocation to write down the liquidity multiplier $\Lambda$ as
\[
\Lambda = \frac{i + w\ell}{e} = \frac{\alpha + (R_{CB})^2}{\alpha + (R_{CB})^2 - \alpha A (R_{CB} - 1 + \delta)}.
\]
(A.31)

We now compute the derivative of $\Lambda$ with respect to $R_{CB}$ to show when increasing $R_{CB}$ increases $\Lambda$:
\[
\frac{\partial \Lambda}{\partial R_{CB}} = \frac{2R_{CB} \left[ \alpha + (R_{CB})^2 - \alpha A (R_{CB} - 1 + \delta) \right] - \left( (R_{CB})^2 + \alpha \right) (2R_{CB} - \alpha A)}{\left[ \alpha + (R_{CB})^2 - \alpha A (R_{CB} - 1 + \delta) \right]^2}
\]
\[
= \frac{\alpha A \left[ \alpha + 2 (1 - \delta) R_{CB} - (R_{CB})^2 \right]}{\left[ \alpha + (R_{CB})^2 - \alpha A (R_{CB} - 1 + \delta) \right]^2}.
\]

This is positive exactly when $\alpha + 2R_{CB}(1 - \delta) > (R_{CB})^2$ as stated in the proposition.

Proof of Proposition 13

If a farmer has grain $g$ at Date 1, it is incentive compatible for him to repay his debt if he prefers to deposit and repay than to divert, where now if he diverts he may either store privately, as before, or consume immediately. His payoff if he does not divert is
\[
U_{\text{deposit}} = \max \left\{ u(c_1) + u(c_2) \mid c_2 = R_D (g - c_1 - R_LL) \right\}. 
\]
(A.33)

Solving the program with the first-order approach gives
\[
U_{\text{deposit}} = \log \left( \frac{g - R_LL}{2} \right) + \log \left( \frac{R_D (g - R_LL)}{2} \right). 
\]
(A.34)

Likewise, the payoff if the farmer does divert is
\[
U_{\text{divert}} = \max \left\{ u(c_1) + u(c_2) \mid c_2 = (1 - \delta) (g - c_1) \right\}. 
\]
(A.35)

Solving this program with the first-order approach gives
\[
U_{\text{divert}} = \log \left( \frac{g}{2} \right) + \log \left( \frac{(1 - \delta)g}{2} \right). 
\]
(A.36)

Now we can write the farmer’s incentive constraint with log utility. He prefers to deposit than to divert if $U_{\text{deposit}} > U_{\text{divert}}$ in equations (A.33) and (A.35) above. Thus
the borrowing constraint is given by

\[ L \leq \frac{\sqrt{R_D} - \sqrt{1 - \delta}}{\sqrt{R_D}R_L} g, \]  

(A.37)

as stated in the proposition. Above, we have used the fact that \( \log x + \log y = \log xy \) and simplified. If we substitute \( R_D = R_L = 1 \) and use the Taylor approximation, we can express the borrowing constraint as follows:

\[ L \leq \left( 1 - \sqrt{1 - \delta} \right) g \approx \frac{\delta g}{2}. \]  

(A.38)

This is exactly the incentive constraint in the model with linear utility and consumption only at Date 2 and rate of depreciation \( \delta/2 \). Thus, we conclude that our basic mechanism is not affected by consumption at the interim date, although it may attenuate the importance of savings. Specifically, it corresponds to lowering the rate of depreciation. 

Proof of Proposition 14

The payoffs from withdrawing or not withdrawing a unit of grain (described in Section 5.2) are summarized in Figure 3.

![Figure 3: Payoff Matrix of the Bank Run Game at Date 0⁺](image)

At Date 0⁺ depositors now play a coordination game. There are multiple Nash equilibria as long as \( \delta \in (0, 1) \). In particular, there are two pure strategy Nash equilibria, one in which all depositors withdraw (a bank run) and another in which no depositors withdraw.

In order to analyze the effect of liquid reserves \( \theta \) on financial stability, we select an equilibrium using global games techniques. By a standard result (see Morris and Shin)
\( \delta^* \) is the solution of the following equation:

\[
\int_0^1 \text{don’t withdraw payoff } (\delta) \, d\lambda = \int_0^1 \text{withdraw payoff } (\delta) \, d\lambda \tag{A.39}
\]

i.e.

\[
\int_0^1 \mathbb{1}_{\{\lambda \leq \theta\}} \, d\lambda = \int_0^1 \left[ \mathbb{1}_{\{\lambda \leq \theta\}} (1 - \delta) + \mathbb{1}_{\{\lambda > \theta\}} \frac{(1 - \delta) h(\theta)}{\lambda} \right] \, d\lambda. \tag{A.40}
\]

This implies that

\[
\delta^* = \frac{h(\theta) \log(\theta)}{h(\theta) \log(\theta) - \theta}, \tag{A.41}
\]

which can be increasing in \( \theta \), as summarized in the proposition.
# B Table of Notations

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References


Williams, Jeffrey, “Fractional Reserve Banking in Grain,” Journal of Money, Credit and Banking, 1986, 16, 488–496.