CONTRACTING TO COMPETE FOR FLOWS*

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Abstract

We present a model in which asset managers design their contracts to attract flows of investor capital. We find that they make their contracts depend on public information, e.g. credit ratings or benchmark indices, as a way to attract flows, rather than as a way to mitigate incentive problems, as has been emphasized in the literature. Unfortunately, asset managers' competition for flows triggers a race to the bottom: asset managers use public information in their contracts even though it is socially inefficient. This inefficiency arises because contracting on public information prevents risk sharing.

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1 Introduction

Delegated asset managers compete to attract flows of investor capital.[1] This flow-motivation can create incentive problems between asset managers and investors. However, contracting can mitigate such incentive problems, for example by making contracts depend on verifiable public information such as credit ratings (He and Xiong (2013) and Parlour and Rajan (2016)) or benchmark indices (Buffa, Vayanos, and Woolley (2015), Holmstrom (1982), and van Binsbergen, Brandt, and Koijen (2008)). But these contracts themselves are written by asset managers who are competing to attract flows. Does asset managers' flow-motivation distort the contracts they offer?

In this paper, we present a model that suggests that the answer to this question is yes. We find that asset managers make their contracts depend on public information as a way to compete for flows, rather than as a way to mitigate incentive problems. Indeed, in our model, asset managers do away with incentive problems by contracting on final wealth alone. But they still contract on public information, because it allows them to offer lower fees to attract investors' capital flows, while still breaking even. Unfortunately, asset managers' competition for flows triggers a race to the bottom: asset managers use public information in their contracts even though it is socially inefficient. This inefficiency arises because contracting on public information prevents risk sharing.

Model preview. To understand how the incentive to attract flows affects asset management contracts, we model asset managers that offer contracts to compete for the capital of a single investor. The investor wants to delegate his investment to an asset manager because asset managers have better information about asset returns than he does. However, delegation comes with an incentive problem, because the asset managers have different degrees of risk aversion from the investor. We make no assumption as to whether asset managers or the investor is more risk averse; we assume only that they have utility functions in the same

[1] Papers that show the importance of delegated asset managers' competition for flows include Berk and Green (2004), Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997, 1999), Dasgupta and Piacentino (2015), and Dasgupta and Prat (2006, 2008).

HARA class.[2]

In the model, asset managers design contracts taking into account potential incentive problems. Thus, we allow their contracts to depend not only on the final wealth and the asset manager's action, i.e. the "portfolio allocation," but also on a public signal. In the baseline model, we assume that the public signal is released before the investor delegates his investment to an asset manager, which we argue applies well to contracting on credit ratings.[3] But we also analyze the case in which the public information is realized later, which we argue applies well to contracting on benchmark indices (Section 5).[4] Once the asset manager has been chosen, he gets his private information and makes an investment decision, i.e. he allocates the investor's capital to a portfolio of assets.[5] Finally, the investment pays off and the final wealth is divided according to the asset manager's contract.

Results preview. We begin our analysis by studying the constrained-efficient outcome, i.e. the delegation contract and investment decision that maximize social welfare subject to the constraint that the asset manager's portfolio choice is incentive compatible. We show that the first-best outcome can be implemented by an affine contract, i.e. a fixed fee and a constant proportion of final wealth. This follows from an application of a general result due to Wilson (1984): as long as utility functions are in the same HARA class, an affine contract *both* implements efficient risk sharing *and* aligns incentives.[6] This benchmark implies that

[2]See Subsection 2.1 for the precise definition of a class of HARA ("hyperbolic absolute risk-aversion") utility functions. This includes a relatively wide class of preferences; e.g., if everyone has CARA utility then our results apply for all risk-aversion parameters.

[3]Asset management contracts frequently depend on credit ratings in practice. For example, according to the Bank for International Settlements (2003), "it is common, for example, for fixed income investment mandates to restrict the manager's investment choices to investment grade credits"; that is to say that they restrict their portfolios to securities rated BBB − or higher by Standard & Poor's or Baa3 or higher by Moody's. E.g., in its European Corporate Bond Fund prospectus, Threadneedle Investments says, "The portfolio will not be more than 25% invested in securities rated AAA.... A maximum of 10% of the portfolio can be invested in below investment grade securities." Ashcraft and Schuermann (2008) affirm the importance of contracting on ratings, saying that "As investment mandates typically involve credit ratings, they comprise another point where the CRAs play an important role."

[4]Asset management contracts also frequently depend on benchmarks in practice; see Ma, Tang, and Gómez (2016).

[5]Actually, we set up a general model that includes delegated portfolio choice as a special case. Since we are motivated by delegated portfolio management contracts, we restrict attention to this application in the Introduction for illustrative purposes.

[6]To be specific, the main theorem in Wilson (1984) is that "If the sharing rule is efficient and linear

the classic trade-off between incentives and risk sharing is fully resolved by contracting on final wealth alone, so contracting on the public signal is not necessary to mitigate the incentive conflict between an asset manager and the investor.[7] Thus, our analysis reflects the effect of contracting on public information beyond that of aligning incentives, which has been studied elsewhere (see the literature review below).

We then solve for the equilibrium of our model. Our first main result is that asset managers *do* make their contracts contingent on the public signal, even though it does not mitigate the incentive problem. This is because contracting on the signal allows asset managers to compete to attract flows, i.e. to be employed to manage the investor's capital. It allows them to offer the investor lower fees for "good" signals, undercutting the competition while still breaking even themselves. To see how this works, suppose there is an equilibrium in which all asset managers offer contracts that do *not* depend on the public signal. Because they are competitive, asset managers must break even in expectation across all realizations of the public signal. Since their contracts do not depend on the signal, they take losses for "bad" signals and make offsetting profits for "good" signals. Given these strictly positive profits for these good signals, there is room to offer a lower fee conditional on a good signal, i.e. to deviate to a contingent contract and undercut the competition. Extending this argument implies that asset managers must break even not only in expectation, but also for every realization of the public signal. They achieve this by writing the public signal into their contracts. Thus, our model suggests that flow competition, rather than incentive problems, may be the reason that asset management contracts frequently depend on public information such credit ratings and benchmark indices.

[or affine] then truthful revelation is a Nash equilibrium." The efficient sharing rule is affine if and only if everyone has preferences in the same HARA class. We thus apply this result to an optimal contracting setting. It is also worth noting that Ross (1974) finds related results for the principal-agent problem, which are sometimes grouped under the heading of "The Principle of Similarity." Related results are also in Amershi and Stoeckenius (1983), Pratt and Zeckhauser (1989), and Wilson (1968).

[7]Note that this contrasts with the common intuition about principal-agent problems that the agent (the asset manager here) must bear an inefficiently high fraction of the risk in order to give him a strong incentive to act in the interest of the principal (the investor here). To understand why this trade-off is absent if and only if utility functions are in the same HARA class see Subsection 3.2 below and Pratt (2000).

Our second main result is that competition among asset managers has a dark side: it prevents risk sharing and thus lowers welfare. Since asset managers contract on the public signal to compete, they must break even *for each realization of the signal* in equilibrium. In other words, they get the same payoff (of zero) no matter what the realization is. Thus, all of the risk in the public signal is borne by the investor—there is no risk sharing between him and his asset manager. This result is related to Hirshleifer's (1971) result that if public information is revealed before markets open, it inhibits risk sharing. We find that if public information is *contractable* before asset managers are employed, it inhibits risk sharing. Thus, unlike in Hirshleifer (1971), the presence of any contractable public signal inhibits risk sharing even if it contains no new information. Since contracting on the public signal is not necessary to align incentives in our model, prohibiting contracting on it is Pareto improving.

Policy. Even though our model is stylized, we think that it provides a relevant perspective for policy. In the context of the model, regulators can increase welfare by limiting the extent to which asset managers can contract on credit ratings, a leading example of public signals used in delegated asset management contracts. This policy is in line with advice from the Financial Stability Board, which said that "Investment managers and institutional investors must not mechanistically rely on CRA ratings...[and should limit] the proportion of a portfolio that is CRA ratings-reliant." This quote reflects the fact that regulators have already identified some potential risks of the mechanistic reliance on ratings, such as increased systemic risk; our analysis reveals another one: the mechanistic reliance on ratings may inhibit risk sharing. Thus, limiting contractual dependence on ratings may have the added benefit of preventing the race to the bottom which inhibits financial markets from performing one of their main functions: allowing investors to share risk. A regulator may implement this policy directly, by prohibiting asset managers from contracting on ratings, or indirectly, by encouraging ratings agencies to publicize information in a "soft" way that is difficult to contract on, e.g., they could release verbal reports rather than announce letter-based ratings.

Our finding that a regulator can improve risk sharing without changing the precision of ratings highlights the difference between our results and those in Hirshleifer (1971). In his setting, a regulator must decrease the precision of ratings to improve risk sharing; in our setting, in contrast, a regulator can limit contracting on ratings to improve risk sharing. However, if contracting on ratings is possible, then increasing their precision may have negative effects in our model, in line with Hirshleifer's findings. In particular, we show that under the assumption that asset managers do not get new information from ratings, coarser ratings Pareto dominate more precise ones. This is because a more precise public signal allows asset managers to compete more aggressively—i.e. asset managers break even for more realizations of the signal—which makes risk sharing more difficult.[8]

Layout. In the remainder of the Introduction, we discuss our model's empirical relevance and the related literature. In Section 2, we present the baseline model. In Section 3, we present the first-best and constrained efficient benchmarks. In Section 4, we solve the baseline model and analyze welfare. In Section 5, we study a modified version of the baseline model that applies to portfolio benchmarking. In Section 6, we analyze extensions. Section 7 is the Conclusion. The Appendix contains all proofs and a table of notations.

1.1 Applications, Realism, Magnitudes, and Limitations

Applications. In addition to our motivating examples of asset managers contracting on credit ratingsand benchmark indices, our analysis applies broadly to settings in which agents post contracts to compete for flows. Notably, life insurers, who hold upwards of fifteen percent of outstanding corporate and foreign bonds in the US, are another type of asset manager that makes contracts contingent on public information. The bulk of insurers' liabilities are annuities, many of which give investors the option to take "cash surrender value" before

[8] Of course, there are other reasons that regulators may wish to pursue policies to make ratings more precise. Indeed, in our setting, if asset managers do get new information from ratings, then increasing ratings' precision has the benefit of improved investment efficiency. However, our findings show that, if ratings are contractable, then this benefit comes with the cost of worse risk sharing. Thus, we suggest that if regulators work to increase ratings' precision, it is all the more important that they also work to minimize the negative effects of increased ratings' precision on risk sharing, ideally by limiting contractual dependence on ratings.

maturity (i.e. before the death of the policy holder).[9] This exposes investors to market risk, since they get the market value of the annuity, rather than a fixed amount. Our model suggests a reason for this: if an insurer offered a fixed surrender value, a competing insurer could offer a cheaper product in good times, attracting flows away from the initial insurer.[10] Thus, insurers' competition for flows prevents them from sharing market risk with their investors—it prevents insurers from providing insurance.

Realism of assumptions. Our first main result, that contracts depend on public signals, follows from our assumption that asset managers offer contracts to compete for flows. Wahal and Wang (2011) find evidence consistent with this assumption. They study the contracts offered by new entrant mutual funds, who, almost by definition, are competing to attract new investors. They find that "on the basis of prices over which fund managers have direct control (management fees), price competition is strong" (p. 42).

In the baseline model, this result also depends the timing, in which asset managers first post contracts, then the public signal is released, and then the investor chooses an asset manager (although we consider an alternative timing in Section 5 on portfolio benchmarks). This timing captures the idea that asset managers must post contracts in anticipation of changes in public information rather than post new contracts in response to any new information. Since public information is released continuously, posting new contracts in response to it would quickly become very costly (e.g. due to legal fees for contracting).[11] In reality, asset managers avoid these costs by contracting on public information and updating their contracts relatively rarely; according to Warner and Wu (2011), "the semi-annual contract change frequency is approximately 5% [for mutual funds], with contract changes often

[9]See Berends, McMenamin, Plestis, and Rosen (2013).

[10]In practice, some annuities' surrender values are indeed fixed. However, they typically have penalties for early withdrawal. This effectively adds a cost of switching away from an incumbent asset manager, curbing competition; we formalize this in Subsection 6.2.

[11]These dynamics, in which asset managers change their contracts in response to changes in public information, could be captured by reversing the timing in our model, so asset managers would post contracts after the public signal was released. With this timing, asset management contracts would still depend on the signal, in that asset managers would post different contracts for its different realizations. But asset managers would not need to write the public signal into their contracts explicitly, since each contract would already correspond to a particular realization of the signal.

shifting the percentage fee up or down by more than a fourth" (p. 273).

Given our first main result, our second main result, that contracting on ratings can decrease risk sharing, follows from only the assumption that people are risk averse.

Magnitude. Although our baseline setup is abstract, we consider a portfolio choice application that allows us to map most of the variables in our model to concrete real-world quantities such as the risk-free rate and the expected return and variance on the market (Subsection 6.2). This allows us to do a numerical example with "reasonable" numbers in which we find that an investor would sacrifice 27 basis points of his total wealth to ban contracting on ratings.

Discussion of results and limitations. Within the context of the numerical example in Subsection 6.2, we also discuss adding a cost of switching away from an "incumbent" asset manager. This allows us to address an unrealistic feature of our equilibrium contracts: they depend on *all* realizations of the public signal, rather than on a coarse partition of them. For example, if the public signal is a credit rating, the equilibrium contract in our model depends on the exact rating, AAA, AA+, AA, etc., rather than just on whether the rating is investment grade or not, as is typical in practice. Our numerical exercise suggests that switching costs are unlikely to be large enough to lead an incumbent asset manager to offer a contract that does not depend on the public signal at all, as in the constrained-efficient contract, but may lead him to leave out some contingencies, as in real-world contracts.

Our contracting environment includes non-linear contracts that depend on the final wealth, on the asset manager's portfolio decisions, and on public information. Despite the generality of our setup, we find that the equilibrium contract has a simple and realistic form. It is affine, as are many real-world asset management contracts,[12] and it depends on the public signal. Specifically, asset management fees are lower when the public signal indicates a high aggregate state. This is realistic if the signal represents a benchmark index: asset managers are often paid for performance in excess of a benchmark. However, this seems

[12]See Agarwal, Daniel, and Naik (2009) and Elton, Gruber, and Blake (2003).

less realistic if the signal represents a credit rating: although asset managers are often constrained by ratings-based investment mandates (see footnote 3), their fees do not seem to depend on ratings. However, we suggest that asset managers' fees actually do depend on ratings, given the following interpretation. There is a lot of heterogeneity in fund fees (Haslem (2015), Hortaçsu and Syverson (2004)). For example, fund families typically offer different fees for an investment-grade fund than for a sub-investment-grade fund. Although these are formally different fees for different funds, they are equivalent to different fees for different ratings in our model. If the rating is high, the investor invests in the investment-grade fund and pay its fee; if the rating is low, he invests in the sub-investment-grade fund and pay its fee.

We have assumed that all asset managers have the same skills and that they are competitive, so the investor gets the rent. However, some evidence suggests that asset managers have heterogeneous skills and leave little rent to investors (see, e.g, Berk and van Binsbergen (2015) and Carhart (1997)). In Subsection 6.1, we discuss how to embed our model in a directed search environment. We show how this framework would allow us to include different kinds of asset managers and competition among investors. Although this exercise is somewhat preliminary, it suggests that our results are robust to relaxing these assumptions.

1.2 Related Literature

We make two main contributions to the literature.

First, we show that competition for flows may lead asset managers to contract on public information even if it is inefficient to do so. In the context of benchmarks, this suggests a resolution to the puzzle in Admati and Pfleiderer (1997), who find that benchmarks "cannot be easily rationalized...[and] are generally inconsistent with optimal risk sharing and do not lead to the choice of an optimal portfolio" (p. 323). In the context of credit ratings, this provides an alternative view to He and Xiong (2013) and Parlour and Rajan (2016) who also take the "contracting view" of ratings, but do not study flow competition.

Second, we show that the ability to contract on public information may have adverse effects. This perspective contrasts with the contracting literature on public information in principal agent problems,[13] in which, absent flow competition, contracts refer to public information only when it is beneficial. However, it is in line with the findings in some other strands of the literature. Notably, in Kurlat and Veldkamp (2015) public information leads equilibrium prices to adjust, so it is too late for people to trade to share risk. And in Glosten (1989) private information leads to high bid ask spreads, so it is too costly for people to trade to share risk. Our contribution relative to these papers is to show that such inefficiencies arise even when agents write optimal contracts before information is released.

More generally, our work is related to theory papers on contracting in delegated asset management[14] and on information production by credit rating agencies.[15] Unlike these papers, we focus on flow competition in asset management and abstract entirely from information production by credit rating agencies (the "rating" in our model is just a public signal).

2 Baseline Model

In this section, we set up the baseline model. The model constitutes an extensive game of incomplete information in which asset managers first compete in contracts in the hope of being employed by a single investor and then the employed asset manager takes an action on behalf of the investor. Final wealth is divided according to the contract of the employed asset manager.

[13]See, e.g., Chaigneau, Edmans, and Gottlieb (2014a, 2014b), Cremer and McLean (1988), Holmstrom (1979), Kessler, Lülfesmann, and Schmitz (2005), Nalebuff and Scharfstein (1987), and Riordan and Sappington (1988).

[14]See, e.g., Bhattacharya and Pfleiderer (1985), Dybvig, Farnsworth, and Carpenter (2010), Palomino and Prat (2003), and Stoughton (1993).

[15]See, e.g., Bolton, Freixas, and Shapiro (2012), Bar-Isaac and Shapiro (2013), Boot, Milbourn, and Schmeits (2006), Donaldson and Piacentino (2012), Fulghieri, Strobl, and Xia (2013), Goel and Thakor (2015), Goldstein and Huang (2016), Kashyap and Kovrijnykh (2016), Doherty, Kartasheva, and Phillips (2012), Kartasheva and Yilmaz (2013), Kovbasyuk (2013), Manso (2014), Mathis, McAndrews, and Rochet (2009), Opp, Opp, and Harris (2013), Sangiorgi and Spatt (2016), and Skreta and Veldkamp (2009).

The motivating portfolio-choice application from the Introduction is a special case of our general model. We describe this special case explicitly in parallel to the general setup. This makes the model concrete and fixes ideas. It also allows us to provide closed-form solutions for the asset manager's action.

2.1 Players

There is a single investor with a unit of wealth and von Neumann–Morgenstern utility u_{I} and there are at least two competitive asset managers with von Neumann–Morgenstern utility u_{A} and outside option $\bar{u}$. All asset managers are identical. The investor and the asset managers differ in their risk aversion. We make no assumption as to whether the investor or the asset manager is more risk averse but we assume that both utility functions are in the same class of hyperbolic absolute risk-aversion (HARA). Specifically, their absolute risk tolerances are affine with the same slope,

$$-\frac{u_i'(w)}{u_i''(w)} = a_i + bw \tag{1}$$

for $a_i > -bw$ for all w and for $i \in \{\mathrm{I}, \mathrm{A}\}$.[16] Note that this assumption imposes no restriction on the magnitude of the difference between the investor's and asset manager's risk aversions. The HARA class is a relatively large class of utility functions. For example, it contains all utility functions in the CARA class (exponential utility), which is commonly used in the literature—if $b = 0$ condition (1) implies that the investor and the asset managers have CARA utility with coefficients of absolute risk aversion a_{I}^{-1} and a_{A}^{-1}. Further, if $b = -1$ condition (1) implies that utility functions are quadratic,

$$u_i(w) = -\frac{1}{2}\big(a_i - w\big)^2. \tag{2}$$

[16] This ensures that the coefficients of absolute risk tolerance in condition (1) are always strictly positive or, equivalently, that the utility functions are always strictly increasing and strictly concave.

In the application to portfolio choice that we explore below, we illustrate our results with quadratic utility, because it allows us to solve for the optimal contract and investment decisions in closed form.

Asset managers have private information, captured by a private signal σ, which is relevant for the investment decision. There is also a public signal ρ that the investor can observe as well. An asset manager's signal σ and the public signal ρ refer to the realizations of random variables $\tilde{\sigma}$ and $\tilde{\rho}$. We assume that asset managers' signal is better than the public signal, in the sense that the sigma-algebra generated by $\tilde{\sigma}$ is finer than the sigma-algebra generated by $\tilde{\rho}$, $\text{sigma}(\tilde{\rho}) \subset \text{sigma}(\tilde{\sigma})$. In other words, asset managers do not learn from the public signal. We make this assumption to switch off the public signal's role in information-provision and focus on its role in contracting. However, this assumption is not strictly necessary for most of our results or policy prescriptions.[17] We do not impose any other restrictions on the distributions of $\tilde{\sigma}$ and $\tilde{\rho}$.

2.2 Actions and Contracts

The investor wishes to delegate his investment to an asset manager because the asset manager is better informed about the optimal action to undertake. The asset manager will take an action x that affects the distribution of wealth that the investor and asset manager will divide ex post. Thus, for each action x that the asset manager takes, final wealth is a random variable which we denote by $\tilde{w}(x)$. We assume that $\tilde{w}$ is a concave function of x for every state of the world.[18] The asset manager's signal provides him with information about the distribution of this random variable, making delegation valuable. However, the investor

[17] We use this assumption only in the proof of Proposition 5, which says that coarser information structures Pareto dominate finer ones. If we assumed that asset managers learned from the public signal, the forces behind this result would not be affected, but the result would be attenuated due to a countervailing force: finer public information might provide asset managers with information that would lead them to make better investment decisions. However, this force would not affect our policy prescription that ratings-contingent asset management contracts should be limited. This is because asset managers could still use the information in ratings to guide their decisions without contracting on it.

[18] This technical assumption allows us to solve the general model using the first-order approach. Note that it is not strictly necessary for our main results. In particular, our results hold in the portfolio choice application in which this assumption does not hold ($\tilde{w}$ is an affine function of x).

anticipates a misalignment of investment incentives since his risk aversion differs from the asset manager's. Contracts attempt to align incentives to mitigate this downside of delegating investment. Each asset manager offers a contract Φ that specifies his compensation. This contract may depend on the final wealth w, the public signal ρ, and his action x, but not the private signal σ because it is not verifiable. In other words, the asset manager gets $\Phi(w, x, \rho)$ and the investor gets $w - \Phi(w, x, \rho)$.

Portfolio choice application. Our model is motivated by delegated asset management, in which case x represents an asset manager's portfolio choice decision. To fix ideas, consider the problem of allocating the initial unit of wealth between two assets, a risk-free asset with return R_f and a risky asset with return $\tilde{R}$. The final wealth is thus given by

$$\tilde{w}(x) = R_f + x\big(\tilde{R} - R_f\big). \tag{3}$$

In this case, we view asset managers' private information σ as the true standard deviation of $\tilde{R}$ and ρ as an imperfect public signal about σ.

Below, we use this portfolio choice application with quadratic utility (as in equation (2)) to provide an illustration of our general results. We make the following assumption to streamline this illustration: the mean return of the risky asset is known and is independent of the asset manager's private information σ and the rating ρ. This implies that

$$\mathbb{E}\left[\tilde{R} \,\middle|\, \tilde{\sigma} = \sigma\right] = \mathbb{E}\left[\tilde{R} \,\middle|\, \tilde{\rho} = \rho\right] = \mathbb{E}\big[\tilde{R}\big] =: \bar{R}.$$

With quadratic utility functions, we must restrict parameters to ensure that marginal utility is positive—i.e. that everyone always prefers more wealth to less. The following technical condition ensures this is the case in equilibrium:

$$\big(\bar{R} - R_f\big)\big(R - \bar{R}\big) \leq \sigma^2 \tag{4}$$

for all pairs (σ, R).[19]

2.3 Timing

Formally, the timing is as follows:

1. Each asset manager offers a contract Φ.
2. The public signal ρ is released.
3. The investor observes ρ and the profile of contracts and employs an asset manager.
4. The employed asset manager observes his private signal σ and takes an action x.
5. The final wealth is realized and it is distributed according to the contract Φ of the employed asset manager: the asset manager gets $\Phi(w, x, \rho)$ and the investor gets $w - \Phi(w, x, \rho)$.

Remark on timing. It is important for our results that the investor can condition his decision about which asset manager to employ on the asset managers' contracts and on the public signal. This allows asset managers to contract on the public signal as a way to compete for flows. We model this by assuming that (i) the investor observes the public signal after asset managers offer contracts and (ii) the investor observes the public signal before he chooses which asset manager to employ. These assumptions may seem stark, but we think that they have realistic interpretations: (i) captures the idea that investors are free to *switch* asset managers after observing the public signal and (ii) captures the idea that public information may *change*, leading investors to reallocate their capital. In other words, the assumptions that investors employ asset managers only once and that the public signal is released only once are not crucial to the mechanism.

[19] To ensure that marginal utility is positive, it must be that the investor's welath is always less than a_{I} and the asset managers' wealth is always less than a_{A}, as can be seen from the quadratic functional form. Given the equilibrium contract Φ, this implies that $w - \Phi(w) < a_{\mathrm{I}}$ and $\Phi(w) < a_{\mathrm{A}}$ for all possible realizations of w. Condition (4) ensures that these conditions are satisfied given the equilibrium contract Φ. Specifically, we solve the model assuming the conditions are satisfied and then find that they are satisfied as long as this condition (4) holds.

2.4 A Note on Notations

At times the contracting notation can be cumbersome, so we frequently suppress the arguments of some functions. In particular, the contract $\Phi = \Phi(w, x, \rho)$ is always a function of wealth w, the asset manager's action x, and the public signal ρ, but we frequently write just Φ or $\Phi(w)$. Φ' denotes the partial derivative of Φ with respect to w, $\Phi' := \partial\Phi/\partial w$. The asset manager chooses the action given his signal σ, but we usually write just x for $x(\sigma)$.

A table summarizing our notations is in Appendix B.

3 Benchmarks: First Best and Constrained Efficiency

In this section, we solve for the first-best and constrained-efficient outcomes of the model. The main result of this section is that these outcomes coincide, i.e. the asset manager's incentive constraints alone do not move the outcome away from first-best. This result is useful to solve the model below.

3.1 First Best

We define the first-best outcome as the contract Φ and action x that maximize a weighted sum of utilities[20] of the investor and a representative asset manager (since all asset managers are identical, the utility of this "representative asset manager" can also represent the utilities of all asset managers). We normalize the welfare weight on the investor to one and denote the welfare weight on the asset manager by λ so the social welfare function is $u_{\mathrm{I}} + \lambda u_{\mathrm{A}}$. Thus, we define the first-best outcome as the solution of the program to

$$\text{maximize} \quad \mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}(x) - \Phi\big) + \lambda u_{\mathrm{A}}\big(\Phi\big)\Big] \tag{5}$$

[20] We define any outcome on the Pareto frontier as first best for a given welfare weight. In contrast, it is common in the contract-theory literature to define the first-best outcome as the one that maximizes the payoff of the principal (the investor) subject to the participation constraint of the agent (the asset manager); see, e.g., Bolton and Dewatripont (2005). This is a special case of our definition (choose the welfare weight λ so that the asset manager's expected utility equals his reservation utility $\bar{u}$). We use the more general definition because the results we get here are useful below (cf. Proposition 5).

over all contracts $\Phi = \Phi(w, x, \rho)$ and all actions $x = x(\sigma)$. Note that, since x depends on σ and σ is a sufficient statistic for ρ, information frictions do not constrain this program. The next proposition characterizes the solution of the program.

PROPOSITION 1. (FIRST BEST.) *The first-best contract Φ_{fb} is affine and given by*

$$\Phi_{\text{fb}} = \begin{cases} \dfrac{a_{\text{I}} - \lambda^{-b} a_{\text{A}} + bw}{b\,(1+\lambda^{-b})} & \text{if } b \neq 0, \\[2ex] \dfrac{a_{\text{A}}}{a_{\text{A}} + a_{\text{I}}} \Big(a_{\text{I}} \log \lambda + w \Big) & \text{if } b = 0, \end{cases} \tag{6}$$

and the first-best action $x_{\text{fb}} = x_{\text{fb}}(\sigma)$ solves the first-order condition

$$\frac{\partial}{\partial x} \mathbb{E}\Big[u_{\text{I}}\big(\tilde{w}(x) - \Phi_{\text{fb}}\big) + \lambda u_{\text{A}}\big(\Phi_{\text{fb}}\big) \,\big|\, \tilde{\sigma} = \sigma \Big] = 0$$

for each realization of σ.

Proof. The proof is in Appendix A.1. □

The first-best contract is affine in wealth, like real-world asset management contracts that often constitute a fixed fee and a constant proportion of profits. Further, the contract depends only on the total final wealth w and the welfare weight λ. It does not depend on the public signal ρ. We cannot solve for the action x in closed form in the general model, but in the application to portfolio choice with quadratic utility, we can give a closed-form expression for the portfolio weight x. This is the next corollary.

COROLLARY 1. *In the portfolio-choice application with quadratic utility, the first best contract and investment are given by*

$$\Phi_{\text{fb}} = a_{\text{A}} + \frac{w - a_{\text{I}} - a_{\text{A}}}{1 + \lambda}$$

and

$$x_{\text{fb}} = \frac{\big(\bar{R} - R_f\big)\big(a_{\text{I}} + a_{\text{A}} - R_f\big)}{\sigma^2 + \big(\bar{R} - R_f\big)^2}. \tag{7}$$

Proof. The proof is in Appendix A.2. □

3.2 Constrained Efficiency

We now turn to the constrained-efficient outcome. This is the allocation that maximizes the expectation of the same social welfare function $u_{\text{I}} + \lambda u_{\text{A}}$ as the first-best outcome, but the asset manager's action x must be incentive-compatible given the contract Φ. Namely, the action x maximizes the asset manager's payoff, rather than the social welfare function, given the contract Φ. Thus, we define the constrained-efficient outcome as the solution to the program to

$$\begin{cases} \text{maximize} \;\; \mathbb{E}\left[u_{\text{I}}\big(\tilde{w}(x) - \Phi\big) + \lambda u_{\text{A}}\big(\Phi\big) \,\middle|\, \tilde{\rho} = \rho\right] \\ \\ \text{subject to} \;\; x \in \arg\max\Big\{\mathbb{E}\Big[u_{\text{A}}\big(\Phi\big) \,|\, \tilde{\sigma} = \sigma\Big]\Big\} \end{cases} \tag{8}$$

over all contracts $\Phi = \Phi(w, x, \rho)$. The next proposition characterizes the solution to this program.

Proposition 2. (Constrained-efficient outcome is first best.) *The constrained-efficient outcome coincides with the first-best outcome (as given in Proposition 1 and Corollary 1 above).*

Proof. The proof is in Appendix A.3. □

This proposition says that if the asset manager is compensated according to first-best contract, then the first-best action is incentive compatible. This result follows from the fact that the first-best contract is affine. In theory, the contract should balance two roles: to share risk and align incentives. However, when the contract that implements the first-best risk sharing is affine, it automatically aligns incentives perfectly.

To see why this is the case, you can compare the equations for efficient risk sharing and incentive alignment. The condition for efficient risk sharing is that $u_{\mathrm{I}}(w - \Phi) + \lambda u_{\mathrm{A}}(\Phi)$ is maximized for each w, or that the ratio of marginal utilities is

$$\frac{u_{\mathrm{I}}'(w - \Phi(w))}{u_{\mathrm{A}}'(\Phi(w))} = \lambda. \tag{9}$$

Now recall that two Neumann–Morgenstern utility functions induce the same choices—i.e. incentives are perfectly aligned—if one is an affine transformation of the other. In our context, this is the case if there are constants C_1 and C_2 such that $u_{\mathrm{I}}(w - \Phi(w)) = C_1 u_{\mathrm{A}}(\Phi(w)) + C_2$. Differentiating this condition with respect to wealth says that the ratio of marginal utilities must be

$$\frac{u_{\mathrm{I}}'(w - \Phi(w))}{u_{\mathrm{A}}'(\Phi(w))} = \frac{C_1 \Phi'(w)}{1 - \Phi'(w)}.$$

Equating the right-hand sides of the equation above and of equation (9) says that there is efficient risk sharing and incentive alignment only if Φ' is constant, i.e. Φ is affine.

4 Results

In this section, we solve the baseline model and prove our main results. We first show that asset managers offer contracts that depend on the public signal, even though contracting on it is not necessary to mitigate the incentive problem between the investor and an asset manager. Next we solve for the equilibrium contract. We do this by reformulating the model in a principal-agent framework and using the method of Lagrange multipliers. Finally, we show that increasing the precision of the public signal decreases welfare.

4.1 Competition Is "ρ by ρ"

We now turn to our first main result, that asset managers actively contract on the public signal ρ to compete for investor flows, namely to compete "ρ-by-ρ" and thus break even for every realization of $\tilde{\rho}$.

PROPOSITION 3. *The contract $\Phi = \Phi(w, x, \rho)$ of the employed asset manager depends on the public signal $\tilde{\rho}$. The asset manager breaks even for each realization ρ, or*

$$\mathbb{E}\left[u_{\mathrm{A}}\big(\Phi\big) \,\middle|\, \tilde{\rho} = \rho\right] = \bar{u}.$$

Proof. The proof is in Appendix A.4. □

Asset managers are competitive, so it should not be surprising that they receive their reservation utility in equilibrium. The takeaway from Proposition 3 above is that asset managers receive their reservation utility for *every realization of $\tilde{\rho}$.* In other words, there cannot be an equilibrium in which asset managers break even in expectation over all possible realizations of $\tilde{\rho}$ unless they break even for every realization of $\tilde{\rho}$. To see this, observe that if an asset manager did not break even for every ρ, but only in expectation, then an asset manager who receives less than his reservation utility $\bar{u}$ for some realization must receive more than $\bar{u}$ for another realization. But since the asset manager is getting strictly more than $\bar{u}$ for this realization, there is room for a competing asset manager to profitably undercut him by offering a contract *dependent on* $\tilde{\rho}$ that allocates more of the surplus to the investor.

The argument above glosses over one subtlety: when a competing asset manager offers a contract dependent on $\tilde{\rho}$ to attract the investor, this contract may not only reallocate surplus toward the investor for certain realization of $\tilde{\rho}$, but may also distort the manager's incentives and therefore change the action x. In the proof, we show that the competing asset manager can offer a "calibrated contract" that indeed undercuts the original asset manager's contracts while inducing him to choose the same action x. Specifically, if the original asset manager offers Φ, then for $\varepsilon > 0$ the calibrated contract $\Phi_\varepsilon(w) := u_{\mathrm{A}}^{-1}\big(u_{\mathrm{A}}\big(\Phi(w)\big) - \varepsilon\big)$ induces the same choice of x as Φ but allocates more of the surplus to the investor.

4.2 Equilibrium Contract

We now solve for the equilibrium contract by reformulating the model in a principal-agent framework. In this framework, the investor is the principal and the employed asset manager is the agent. The investor maximizes his utility over all contracts Φ subject to the asset manager's incentive constraint and participation constraint. The twist on the classical principal-agent setting is that the asset manager's participation constraint must bind for each realization ρ of the public signal, since, by Proposition 3, asset managers contract on the public signal to attract flows and thus must break even for each ρ. Thus, the contract of the employed asset manager solves the following principal-agent problem:

$$\begin{cases} \text{Maximize } \ \mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}-\Phi\big) \mid \tilde{\rho}=\rho\Big] \\ \\ \text{subject to } \ \mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big) \mid \tilde{\rho}=\rho\Big] = \bar{u} \text{ and} \\ \\ x \in \arg\max \Big\{\mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big) \mid \tilde{\sigma}=\sigma\Big]\Big\} \end{cases} \tag{10}$$

over all contracts $\Phi = \Phi(w, x, \rho)$ for each ρ.

Next we solve the principal-agent problem in the program (10) for each ρ. We eliminate the asset manager's participation constraint using the method of Lagrange multipliers, but do not eliminate his incentive constraint. Since the asset manager breaks even for each ρ, the participation constraint depends on ρ and thus so does the Lagrange multiplier on the constraint. We denote this Lagrange multiplier by λ_ρ and re-write the principal-agent problem as follows:

$$\begin{cases} \text{maximize } \ \mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi\big) + \lambda_\rho u_{\mathrm{A}}\big(\Phi\big) \,\Big|\, \tilde{\rho}=\rho\right] \\ \\ \text{subject to } \ x \in \arg\max \Big\{\mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big) \Big| \, \tilde{\sigma}=\sigma\Big]\Big\} \end{cases} \tag{11}$$

over all contracts $\Phi = \Phi(w, x, \rho)$, where the Lagrange multiplier λ_ρ makes the asset manager's participation constraint bind, i.e.

$$\mathbb{E}\Big[u_\mathrm{A}\big(\Phi\big) \mid \tilde{\rho} = \rho\Big] = \bar{u} \tag{12}$$

for all ρ. Now observe that the program (11) corresponds exactly to the program (5) above for the constrained-efficient outcome. The Lagrange multiplier λ_ρ on the asset manager's participation constraint corresponds to the welfare weight λ in the program for the constrained-efficient outcome. The twist is that the Lagrange multiplier depends on ρ, because the asset manager must break even for each ρ. Since we have already solved for the constrained-efficient outcome, we can apply our results above to express the equilibrium contract as a function of the Lagrange multiplier λ_ρ.

Proposition 4. (Equilibrium outcome in terms of Lagrange multiplier λ_ρ.) *The equilibrium contract is given by*

$$\Phi = \Phi_\mathrm{fb}\Big|_{\lambda=\lambda_\rho} = \frac{1}{b(1+\lambda_\rho^{-b})}\Big(a_\mathrm{I} - \lambda_\rho^{-b} a_\mathrm{A} + bw\Big).$$

The asset manager chooses the first-best action $x = x_\mathrm{fb}$. This corresponds to the first-best outcome from Proposition 1 with the social welfare weight λ replaced by the Lagrange multiplier λ_ρ. (Thus, since λ_ρ depends on ρ, the equilibrium contract depends on the ρ, whereas the first-best contract does not.)

Proof. The proof is in Appendix A.5. □

4.3 Coarser Ratings Are Pareto-superior

Having established that asset managers offer contracts that depend on the public signal, we now turn to the question of how this dependence affects welfare. We can focus on the investor's payoff alone because asset managers are competitive and so their payoff is always equal to their reservation utility $\bar{u}$. Now, for a given ρ, we have

$$\text{investor's expected payoff given } \rho \ = \mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}(x) - \Phi_\rho\big)\ \Big|\ \tilde{\rho} = \rho\Big].$$

Notice that we have modified our notation slightly and denoted the equilibrium contract given ρ by Φ_ρ.[21] Using the law of iterated expectations, we can write the investor's ex ante payoff as

$$\mathbb{E}\Bigg[\mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w} - \Phi_\rho\big)\ \Big|\ \tilde{\rho} = \rho\Big]\Bigg] = \mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w} - \Phi_{\tilde{\rho}}\big)\Big].$$

This expression reveals that, because the contract $\Phi_{\tilde{\rho}}$ depends on the random variable $\tilde{\rho}$, the investor's payoff is varying with the public signal. In other words, the investor bears the risk over the public signal. Because the investor is risk-averse, this decreases his welfare. Further, increasing the information contained in public signal—making it "finer"—only increases the risk that the investor bears over its outcome.[22] Indeed, coarser public signals Pareto dominate finer ones, as we formalize in the next proposition.

Proposition 5. (Coarser ratings Pareto-dominate finer ratings.) *Consider two public signals $\tilde{\rho}_c$ and $\tilde{\rho}_f$ such that $\tilde{\rho}_c$ is "coarser" than $\tilde{\rho}_f$—i.e.* $\text{sigma}(\tilde{\rho}_c) \subset \text{sigma}(\tilde{\rho}_f)$. *The ex ante utility of the investor and all asset managers is at least as high given $\tilde{\rho}_c$ as given $\tilde{\rho}_f$. (Typically the investor is strictly better off.)*

Proof. The proof is in Appendix A.6. In Appendix A.7 we provide a more direct alternative proof for the application to portfolio choice with quadratic utility. □

The mechanism behind this result hinges on Proposition 3. Because competition makes asset managers break even "ρ by ρ," there is one participation constraint for each ρ. Hence, with a finer structure there are more possible realizations of $\tilde{\rho}$, which correspond to more constraints

[21] Notice also that we have omitted the incentive constraint. This is without loss of generality since the asset manager takes the first-best action under the equilibrium contract (by Proposition 4).

[22] E.g. credit ratings have been made finer in reality. For example, in 1982 Moody's added numerical modifiers to its ratings, thereby refining its ratings partition. See Kliger and Sarig (2000) for analysis of this event. (Thanks to Joel Shapiro for drawing our attention to this.)

on the investor's objective. Because we know from Proposition 2 that the efficient action is always taken, these constraints only restrict risk sharing between the investor and the asset manager. Hence, a finer structure shuts down risk sharing and reduces welfare.

One way to get the intuition for this result is to contrast two situations: (i) ρ is complete noise versus (ii) ρ fully reveals σ. In (i), the asset manager's participation constraint must bind in expectation over σ, whereas in (ii) it must bind for every realization of σ. Given that the investor is risk-averse, optimal risk sharing entails that the asset manager's utility varies with σ (given that the investor's utility must vary with σ). Thus, forcing the asset manager to have the same utility for all realizations of σ leads to a sub-optimal outcome—contracting on the public signal is detrimental to risk sharing.

This result is closely related to the Hirshleifer (1971) effect, by which information destroys gains from risk sharing. Two differences between our finding and Hirshleifer's are (i) our result obtains only with competing asset managers, whereas Hirshleifer's would with a single asset manager and (ii) our result depends only on contracting on public information, even before it is released, whereas Hirshleifer's relies on trading after public information is released. This distinction also points to the importance of the sequencing of events in our model. Because asset managers offer contracts *before* the public signal is released, they have the potential to share risk with the investor. However, this is undermined by asset manager's contracting on the public signal to compete for flows. Further, the more precise the public signal is, the less risk sharing there is in equilibrium.

5 Ex Post Public Information and Benchmarking

In this section, we modify the model to apply it to portfolio benchmarking. We assume that the public signal $\tilde{\rho}$ is realized later, after the investor has employed an asset manager. Hence, $\tilde{\rho}$ can now represent a benchmark index, realized at the same time as portfolio returns. In this setup, the investor cannot make his choice of asset manager contingent on $\tilde{\rho}$ directly.

However, we find that if the investor observes the realization of a signal $\tilde{s}$ correlated with $\tilde{\rho}$ before choosing an asset manager, then the main result of the baseline model obtains: asset managers still contract on $\tilde{\rho}$. Just as in the baseline model, contracting on $\tilde{\rho}$ helps asset managers to compete for flows (cf. Proposition 3).

5.1 Timing with Ex Post Public Information

Formally, the modified timing is as follows:

1. Each asset manager offers a contract Φ.

2. A private signal $\tilde{s}$ is realized.

3. The investor observes s and the profile of contracts and employs an asset manager.

4. The employed asset manager observes his private signal σ, which is more informative than s, sigma$(\tilde{s}) \subset$ sigma$(\tilde{\sigma})$, and takes an action x.

5. The public signal $\tilde{\rho}$ and the final wealth w are realized. Wealth is distributed according to the contract Φ of the employed asset manager: the asset manager gets $\Phi(w, x, \rho)$ and the investor gets $w - \Phi(w, x, \rho)$.

Relative to the baseline timing (Subsection 2.3), the public signal $\tilde{\rho}$ is now released at stage 5, not at stage 2, but some information is still released at stage 2, in the form of a private signal $\tilde{s}$, which may be correlated with $\tilde{\rho}$.

5.2 Competition Is Benchmark by Benchmark

Here, contracting on $\tilde{\rho}$ allows asset managers to compete for flows for different realizations of $\tilde{s}$, just as it helps them to compete for flows for different realizations of $\tilde{\rho}$ in the baseline model.

PROPOSITION 6. *Suppose that the investor's private signal is binary,* $\tilde{s} \in \{s_0, s_1\}$, *and informative about the public signal in the sense that* $\mathbb{E}[\tilde{\rho} \mid s_0] \neq \mathbb{E}[\tilde{\rho} \mid s_1]$.

The contract $\Phi = \Phi(w, x, \rho)$ *of the employed asset manager depends on the public signal* ρ. *The asset manager breaks even for each realization* s *of the investor's private signal, or*

$$\mathbb{E}\left[u_{\mathrm{A}}\big(\Phi\big) \,\big|\, \tilde{s} = s\right] = \bar{u}$$

for each s.

Proof. The proof is in Appendix A.8. □

Intuitively, if the employed asset manager is getting more than his reservation utility for some realization of $\tilde{s}$, say $\hat{s}$, then a deviant asset manager would like to write a contract contingent on s to undercut him given $\hat{s}$. He cannot do this directly, since s is the investor's private information. But he can do it indirectly, by contracting on $\tilde{\rho}$, since $\tilde{\rho}$ is correlated with $\tilde{s}$. I.e. the deviant asset manager can sweeten the deal for the investor given $\hat{s}$ by offering a contract that increases the investor's payoff *on average* given $\hat{s}$.

In the context of real-world asset management, ρ represents the realization of a benchmark and s represents information about it. Thus, fees are lower when the signal s suggests a high realization of the aggregate state, or a high value of the benchmark $\tilde{\rho}$. This is broadly consistent with practice; asset managers are typically compensated for their performance in excess of a benchmark.

6 Extensions

In this section, we consider extensions of our baseline setup.

6.1 Competition among Investors

Empirically, it seems that investors in some asset managers, such as mutual funds, may get little rent after fees (Carhart (1997)). To reflect this, we modify our setup to include competition among investors. We model asset managers that are competitive but scarce, so that after asset managers post contracts, investors must search for them in a frictional market, rather than employ them directly. We assume that each asset manager can manage capital for only one investor, so if q investors search, asset managers only invest on behalf of a proportion $m(q)$ of them. m is a decreasing function of q, so that the more investors there are—or the more capital there is to manage—the less surplus each investor gets. This is a reduced-form way to capture decreasing returns to scale in asset management as in Berk and Green (2004). It is attractive in our setting since it allows us to preserve our bilateral contracting environment, whereas optimal contracts are hard to introduce explicitly in a setting like Berk and Green's in which each asset manager invests on behalf of a large number of investors. (Berk and Green restrict attention to flat fees proportional to assets under management, and do not model information or incentive problems.)

Specifically, suppose that, after asset managers offer their contracts, each investor decides whether to search for an asset manager in a competitive market. Investors that search find an asset manager with probability m and get zero with probability $1 - m$. Investors that do not search get the reservation utility $\bar{u}_{\mathrm{I}}$.[23]

Following the competitive-search literature, we have that asset managers must post contracts to attract investors subject to investors' entry condition as well as to asset managers' break-even and incentive constraints.[24] Thus, the principal-agent problem in Subsection 4.2

[23] Observe that although asset managers always get their reservation utility $\bar{u}_{\mathrm{A}}$, investors get their reservation utility only if they do not search. This loss of utility from searching is equivalent to including a search cost, which is standard in the literature.

[24] For the solution of a standard competitive search model, see, e.g., the survey by Rogerson, Shimer, and Wright (2005), page 973. Here, we intend our analysis to be slightly informal, and we apply this solution somewhat loosely. Notably, we gloss over the fact that the posted contracts Φ are infinite dimensional, unlike in the standard model.

is replaced by the following program for each ρ:

$$\begin{cases} \text{Maximize } \ \mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}-\Phi\big) \;\big|\; \tilde{\rho}=\rho\Big] \\ \\ \text{subject to } \ \mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big) \;\big|\; \tilde{\rho}=\rho\Big] = \bar{u} \ , \\ \\ x \in \arg\max \Big\{\mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big) \;\big|\; \tilde{\sigma}=\sigma\Big]\Big\}, \quad \text{and} \\ \\ m(q)\mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}-\Phi\big) \;\big|\; \tilde{\rho}=\rho\Big] = \bar{u}_{\mathrm{I}} \end{cases}$$

over all contracts $\Phi = \Phi(w, x, \rho)$. The only change from the baseline program is the addition of the last equation, which is investors' entry condition.

Observe that, like in the baseline setup, asset managers' contracts must depend on $\tilde{\rho}$—the argument in Proposition 3 still applies, so asset managers must break even for each realization of the public signal. However, unlike in the baseline setup, coarsening public signal alone does not affect welfare—everyone gets his reservation utility no matter what. This is because the risk-sharing benefits of coarser public signals are eaten up by search externalities. To see this, suppose that the public signal becomes coarser, so that an investor's payoff is higher *conditional on being matched.* This induces more investors to enter, hoping to find matches. This, in turn, decreases the payoff per investor due to decreasing returns to scale. And welfare is lowered back to the original level. This may suggest that, when investor competition and decreasing returns to scale are important, policies aimed to improve risk sharing should also limit the scale of asset management.

This framework could also be used to study heterogenous asset managers, e.g. with heterogenous private information or "skill." Namely, here, unlike in the baseline model, a submarket of unskilled asset managers could still attract investors; they would just operate at a smaller scale. In this setup, our baseline analysis would apply to each submarket of asset managers, suggesting our findings are not driven by our assumption that all asset managers are identical.

6.2 Numerical Example and Discussion of Partially Contracting on $\tilde{\rho}$

Although we intended our analysis to be mainly qualitative, we think it is useful to show that it delivers reasonable quantitive effects. Thus, we do a numerical example and discuss the potential economic magnitude of the inefficiency in Proposition 5. Within this context, we also comment on how relaxing perfect competition among asset managers might affect the equilibrium contract, something we found analytically intractable.

Consider the portfolio choice application in which the expected return on the market is $\bar{R} - 1 = 8\%$, the risk-free rate is $R_f - 1 = 0\%$, and there are two equally likely values for the market volatility, $\sigma_H = 50\%$ and $\sigma_L = 20\%$. These are realistic annual numbers; thus, the investor we have in mind invests with an asset manager for one year before considering switching.[25] We set the utility parameters $a_{\text{I}} = a_{\text{A}} = 1.5$ and $\bar{u} = -1$. This implies the investor's coefficient of relative risk aversion is two (recall his wealth is normalized to one) and that asset managers get the certainty equivalent of $1.5 - \sqrt{2}$, i.e. a small amount of wealth.[26]

Now suppose the public signal is perfectly informative, $\tilde{\rho} = \tilde{\sigma}$. How much of his total wealth is the investor willing to give up to ban contracting it? To answer this, we use equation (29) to compare the investor's expected utility given $\tilde{\rho} = \tilde{\sigma}$ to his expected utility given that

[25] According to the Financial Conduct authority, most investors have never switched funds, suggesting our estimates based on a one-year horizon are conservative (see FCA (2016), p. 64).

[26] This all follows from direct computation with quadratic utility. The coefficient of relative risk aversion is given by

$$\text{CRRA}_i = -w\frac{u_i''}{u_i'} = \frac{w}{a_i - w},$$

so $a_i = \left(1 + \text{CRRA}_i^{-1}\right)w$. The certainty equivalent solves

$$\bar{u} = u_i\left(\text{CE}_i\right) = -\frac{1}{2}\left(a_i - \text{CE}_i\right)^2,$$

so $\text{CE}_i = a_i - \sqrt{-2\bar{u}}$.

$\tilde{\rho}$ is completely uninformative. We find that the answer is 27 basis points. To get a sense of what this number means within our model, we also compare the costs of contracting on $\tilde{\rho}$ with the costs of a decrease in the expected market return. We find that the investor will tolerate a 15 basis point decrease in the expected market return to prohibit contracting on $\tilde{\rho}$. We think these numbers are reasonable: they suggest that for any given individual investor the effect of forgone risk sharing is non-negligible but unlikely to be salient. However, the effects added up over all the investors in the economy are likely to be important.

We also use this setup to discuss one potential concern about our baseline analysis: above, we found that asset management contracts should depend on every realization of $\tilde{\rho}$ (Proposition 3), but in reality these contracts depend on only a subset of public information. For example, whereas contracts are likely to depend on credit ratings and benchmark indices, they are unlikely to depend on equity analysts' reports. We suggest that this may be because asset managers are not perfectly competitive. As a result, they do not break even for each realization of $\tilde{\rho}$, but rather can share *some* of the risk with the investor. We capture this market power by assuming that there is an incumbent asset manager and that the investor can switch asset managers at a cost after he has observed the realization of $\tilde{\rho}$. Such switching costs are substantial in reality. Indeed, according to the Financial Conduct Authority,

> Investors can incur a range of costs if they switch between funds and asset managers. The costs include explicit charges, tax and the time and effort it takes to switch between funds. Investors may also be reluctant to switch if it would involve crystallising a loss or cutting short a recommended holding period (FCA (2016), p. 18).

Hence, we ask how large the switching costs have to be in order for the investor to stay with the incumbent if the incumbent asset manager offers the optimal non-contingent contract. Specifically, recall that the non-contingent contract is better for the investor in the low-surplus (high-volatility) state, $\tilde{\sigma} = \sigma_H$, but that the contingent contract is better for the investor in the high-surplus (low-volatility) state, $\tilde{\sigma} = \sigma_L$; we ask: what proportion of his wealth is the investor willing to give up to switch from the non-contingent contract to the contingent contract when $\sigma = \sigma_L$? In our example above, the answer is 7%. This is a large number, which implies that switching costs are unlikely to prevent contingent contracting on information associated with outcomes as disparate as $\sigma_L = 20\%$ and $\sigma_H = 50\%$. However, if we repeat the exercise with $\sigma_L = 25\%$ and $\sigma_H = 27.5\%$ we find that a switching cost of 90 basis points prevents the investor from switching and thus allows for non-contingent contracts to be offered in equilibrium, even in this extreme case of a perfectly informative public signal $\tilde{\rho} = \tilde{\sigma}$—adding noise would give a smaller number. Thus, we think it is likely within the range of reasonable switching costs and may explain why asset management contracts do not depend on every single realization of $\tilde{\rho}$ in reality.

7 Conclusion

We present a model of delegated asset management to understand why asset management contracts frequently depend on public information, such as credit ratings or benchmark indices. We show that asset managers contract on public information to attract investor flows, even when it is not necessary to mitigate incentive problems. Further, contracting on public information decreases welfare by preventing risk sharing. This finding gives some support to the regulatory proposal that contracting on credit ratings should be limited. Our equilibrium contracts share a number of features with real-world asset management contracts and our results are robust to a variety of modeling assumptions.

A Proofs

A.1 Proof of Proposition 1

We can compute the first-best contract directly by applying the first order approach[27] to the program (5):

$$\frac{\partial}{\partial \Phi}\Big(u_{\mathrm{I}}\big(w-\Phi\big)+\lambda u_{\mathrm{A}}\big(\Phi\big)\Big)=0,$$

or

$$u_{\mathrm{I}}'(w-\Phi)=\lambda u_{\mathrm{A}}'(\Phi). \tag{13}$$

By a standard result, assumption (1), that the risk tolerance is affine, implies that[28]

$$u_i'(w)=\begin{cases}(a_i+bw)^{-1/b} & \text{if } b\neq 0,\\ -e^{-w/a_i} & \text{if } b=0.\end{cases} \tag{14}$$

Thus, equation (13) becomes

$$\begin{cases}\Big(a_{\mathrm{I}}+b(w-\Phi)\Big)^{-1/b}=\lambda\Big(a_{\mathrm{A}}+b\Phi\Big)^{-1/b} & \text{if } b\neq 0,\\ -\exp\left(-\dfrac{w-\Phi}{a_{\mathrm{I}}}\right)=-\lambda\exp\left(-\dfrac{\Phi}{a_{\mathrm{A}}}\right) & \text{if } b=0.\end{cases} \tag{15}$$

[27] As is standard, we omit the expectation operator and maximize pointwise—if an outcome maximizes the objective at each point then it maximizes it on average.

[28] To derive this, write assumption (1) as

$$-\frac{1}{a_i+bw}=\frac{u''(w)}{u'(w)}=\Big(\log u'(w)\Big)'.$$

If $b\neq 0$, we can integrate to get $-b^{-1}\log(a_i+bw)=\log u'(w)$, which implies that $u'(w)=(a_i+bw)^{-1/b}$. If $b=0$, the condition says $a_i u''=-u'$. The solution of this differential equation is $u(w)=-e^{-w/a_i}$. (We have omitted the constants of integration; this is without loss of generality because affine transformations of a von Neumann–Morgenstern utility function are equivalent.)

This implies that

$$\Phi_{\mathrm{fb}} = \begin{cases} \dfrac{a_{\mathrm{I}} - \lambda^{-b} a_{\mathrm{A}} + bw}{b\,(1+\lambda^{-b})} & \text{if } b \neq 0, \\[2ex] \dfrac{a_{\mathrm{A}}}{a_{\mathrm{A}} + a_{\mathrm{I}}}\Big(a_{\mathrm{I}} \log\lambda + w\Big) & \text{if } b = 0, \end{cases} \tag{16}$$

which is affine in w.

A.2 Proof of Corollary 1

First, find the first-best contract using the first-order condition in equation (13),

$$u_{\mathrm{I}}'\big(w - \Phi\big) = \lambda\, u_{\mathrm{A}}'\big(\Phi\big),$$

or, for quadratic utility,

$$w - \Phi - a_{\mathrm{I}} = \lambda\big(\Phi - a_{\mathrm{A}}\big)$$

for all w. Thus the first-best contract is

$$\Phi_{\mathrm{fb}}(w) = a_{\mathrm{A}} + \frac{w - a_{\mathrm{I}} - a_{\mathrm{A}}}{1+\lambda} = A + Bw, \tag{17}$$

where

$$A = \frac{\lambda a_{\mathrm{A}} - a_{\mathrm{I}}}{1+\lambda} \quad \text{and} \quad B = \frac{1}{1+\lambda}. \tag{18}$$

Given the first-best contract, we now calculate the first-best investment in the risky security x_{fb} by computing the maximum of

$$\begin{aligned} &\mathbb{E}\left[u_{\mathrm{I}}\Bigg(R_f + x\big(\tilde{R} - R_f\big) - \Phi_{\mathrm{fb}}\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\Bigg) \,\Bigg|\, \tilde{\sigma} = \sigma\right] \\ &\quad + \lambda\mathbb{E}\left[u_{\mathrm{A}}\Bigg(\Phi_{\mathrm{fb}}\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\Bigg) \,\Bigg|\, \tilde{\sigma} = \sigma\right], \end{aligned} \tag{19}$$

over all x. That is, x_{fb} must maximize the expectation

$$
\begin{aligned}
-\frac{1}{2}\,\mathbb{E}\Bigg[& \left(R_f + x(\tilde{R} - R_f) - A - B\left(R_f + x(\tilde{R} - R_f)\right) - a_{\mathrm{I}}\right)^2 \\
& \qquad + \lambda\left(\left(A + B\left(R_f + x(\tilde{R} - R_f)\right) - a_{\mathrm{A}}\right)^2\right) \,\Big|\, \tilde{\sigma} = \sigma \Bigg]
\end{aligned}
$$

over all x. Thus the first-order condition says that for optimum x_{fb}

$$
\begin{aligned}
\mathbb{E}\Bigg[& (1 - B)(\tilde{R} - R_f)\left(R_f + x_{\text{fb}}(\tilde{R} - R_f) - A - B\left(R_f + x_{\text{fb}}(\tilde{R} - R_f)\right) - a_{\mathrm{I}}\right) \\
& \qquad + \lambda\, B(\tilde{R} - R_f)\left(A + B\left(R_f + x_{\text{fb}}(\tilde{R} - R_f)\right) - a_{\mathrm{A}}\right) \,\Big|\, \tilde{\sigma} = \sigma \Bigg] = 0,
\end{aligned}
$$

thus

$$
x_{\text{fb}} = \frac{\left(\bar{R} - R_f\right)}{\mathbb{E}\left[(\tilde{R} - R_f)^2 \,\middle|\, \tilde{\sigma} = \sigma\right]} \left(\frac{(1 - B)(A + a_{\mathrm{I}}) - \lambda\, B(A - a_{\mathrm{A}})}{(1 - B)^2 + B^2\lambda} - R_f\right).
$$

Substituting in for A and B from equation (18) in the numerator gives

$$
(1 - B)(A + a_{\mathrm{I}}) - \lambda B(A - a_{\mathrm{A}}) = \frac{\lambda\,(a_{\mathrm{A}} + a_{\mathrm{I}})}{1 + \lambda}
$$

and substituting in for A and B from equation (18) in the denominator gives

$$
(1 - B)^2 + B^2\lambda = \frac{\lambda}{1 + \lambda}.
$$

Therefore

$$
\begin{aligned}
x_{\text{fb}}(\sigma) &= \frac{\left(\bar{R} - R_f\right)\left(a_{\mathrm{I}} + a_{\mathrm{A}} - R_f\right)}{\mathbb{E}\left[\left(\tilde{R} - R_f\right)^2 \,\middle|\, \tilde{\sigma} = \sigma\right]} \\
&= \frac{\left(\bar{R} - R_f\right)\left(a_{\mathrm{I}} + a_{\mathrm{A}} - R_f\right)}{\sigma^2 + \left(\bar{R} - R_f\right)^2}.
\end{aligned}
$$

A.3 Proof of Proposition 2

To prove that the constrained-efficient outcome is the first-best outcome, we show that if the contract is the first best contract Φ_{fb}, then the incentive-compatible action is the first-best action. In other words, we show that

$$x \in \arg\max \left\{ \mathbb{E}\left[u_{\text{A}}\big(\Phi_{\text{fb}}\big) \,\middle|\, \tilde{\sigma} = \sigma \right] \right\} \tag{20}$$

implies

$$x \in \arg\max \left\{ \mathbb{E}\left[u_{\text{I}}\big(\tilde{w} - \Phi_{\text{fb}}\big) + \lambda u_{\text{A}}\big(\Phi_{\text{fb}}\big) \,\middle|\, \tilde{\sigma} = \sigma \right] \right\}. \tag{21}$$

We begin with the asset manager's incentive problem given the contract Φ_{fb} and show through a series of manipulations that the solution coincides with that of maximizing social welfare. Incentive compatibility implies the first-order condition

$$\frac{\partial}{\partial x} \mathbb{E}\left[u_{\text{A}}\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \,\middle|\, \tilde{\sigma} = \sigma \right] = 0$$

or

$$\mathbb{E}\left[u_{\text{A}}'\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \Phi_{\text{fb}}'\big(\tilde{w}(x)\big) \tilde{w}'(x) \,\middle|\, \tilde{\sigma} = \sigma \right] = 0.$$

By Proposition 1 Φ_{fb}' is a constant, thus we can pass it under the expectation operator. Further, since the right-hand side above is zero, we can remove Φ_{fb}' from the equation entirely to get

$$\mathbb{E}\left[u_{\text{A}}'\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \tilde{w}'(x) \,\middle|\, \tilde{\sigma} = \sigma \right] = 0.$$

Now recall from equation (13) that Φ_{fb} satisfies $u_{\text{I}}'(w - \Phi) = \lambda u_{\text{A}}'(\Phi)$. Thus, we can re-write the equation above as

$$\mathbb{E}\left[u_{\text{I}}'\Big(\tilde{w}(x) - \Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \tilde{w}'(x) \,\middle|\, \tilde{\sigma} = \sigma \right] = 0. \tag{22}$$

Next, we manipulate this equation to recover the first-order condition for the social optimum in equation (21). To do this, we subtract the following expression from equation (22)

$$\mathbb{E}\left[\Phi'_{\text{fb}}\big(\tilde{w}(x)\big)\tilde{w}'(x)\left[u'_{\text{I}}\Big(\tilde{w}(x)-\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)-\lambda u'_{\text{A}}\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\right]\,\Big|\,\tilde{\sigma}=\sigma\right]. \tag{23}$$

This expression equals zero, since, again by the definition of Φ_{fb} from equation (13), $u'_{\text{I}}(w-\Phi_{\text{fb}})-\lambda u'_{\text{A}}(\Phi_{\text{fb}})=0$. Now, factoring terms, we have

$$\mathbb{E}\left[\Big(\tilde{w}'(x)-\Phi'_{\text{fb}}\big(\tilde{w}(x)\big)\tilde{w}'(x)\Big)u'_{\text{I}}\Big(\tilde{w}(x)-\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]+$$
$$+\lambda\mathbb{E}\left[\Phi'_{\text{fb}}\big(\tilde{w}(x)\big)\tilde{w}'(x)u'_{\text{A}}\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]=0$$

or

$$\frac{\partial}{\partial x}\mathbb{E}\left[u_{\text{I}}\Big(\tilde{w}(x)-\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)+\lambda u_{\text{A}}\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]=0.$$

This is the first-order condition of the social welfare function for each σ. Since u_{I}, u_{A}, and $\tilde{w}$ are concave, the first order condition implies a global maximum, viz. the incentive compatible x is a social optimum.

A.4 Proof of Proposition 3

Suppose, in anticipation of a contradiction, an equilibrium in which the employed asset manager offers contract $\hat{\Phi}$ given $\hat{\rho}$ and receives strictly in excess of his reservation utility,

$$\mathbb{E}\left[u_{\text{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big)\,\Big|\,\tilde{\rho}=\hat{\rho}\right]>\bar{u}. \tag{24}$$

We now show that another asset manager $\hat{\text{A}}$ has a profitable deviation. In order for a contract $\hat{\Phi}_\varepsilon$ to be a profitable deviation for $\hat{\text{A}}$ it must (i) make the investor employ him given $\hat{\rho}$ and (ii) give him expected utility greater than his reservation utility $\bar{u}$ given $\hat{\rho}$. The subtlety in this proof is that $\hat{\text{A}}$'s contract determines not only the allocation of surplus, but also his action x. To circumvent the effect of changing actions on payoffs, we construct $\hat{\Phi}_\varepsilon$ to induce the asset manager to choose the same action that he would have chosen under $\hat{\Phi}$, but still to change the division of surplus. To summarize, $\hat{\Phi}_\varepsilon$ is a profitable deviation if given $\hat{\rho}$ (i) it gives the investor higher utility than does $\hat{\Phi}$,

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \hat{\Phi}_\varepsilon(\tilde{w})\Big) \mid \tilde{\rho} = \hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \hat{\Phi}(\tilde{w})\Big) \mid \tilde{\rho} = \hat{\rho}\right],$$

(ii) it gives the asset manager utility in excess of $\bar{u}$,

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}_\varepsilon(\tilde{w})\Big) \,\Big|\, \tilde{\rho} = \hat{\rho}\right] > \bar{u},$$

and (iii) the set of incentive compatible actions under $\hat{\Phi}$ and $\hat{\Phi}_\varepsilon$ coincide,

$$\arg\max_x \left\{\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}_\varepsilon(\tilde{w})\Big) \,\Big|\, \tilde{\sigma} = \sigma\right]\right\} = \arg\max_x \left\{\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big) \,\Big|\, \tilde{\sigma} = \sigma\right]\right\}.$$

One example of a contract that satisfies these three conditions is

$$\hat{\Phi}_\varepsilon(\tilde{w}) := u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big) - \varepsilon\Big) \tag{25}$$

given $\hat{\rho}$, so that

$$u_{\mathrm{A}}\big(\hat{\Phi}_\varepsilon\big) = u_{\mathrm{A}}\big(\hat{\Phi}\big) - \varepsilon. \tag{26}$$

Since $u_{\mathrm{I}}' > 0$, a sufficient condition for $\hat{\Phi}_\varepsilon$ to satisfy condition (i) is that

$$\tilde{w} - \hat{\Phi}_\varepsilon(\tilde{w}) > \tilde{w} - \hat{\Phi}(\tilde{w}),$$

or, substituting from equation (25),

$$\hat{\Phi}(\tilde{w}) > u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big) - \varepsilon\Big),$$

which is satisfied for $\varepsilon > 0$ by the inverse function theorem since $u'_{\mathrm{A}} > 0$.

Condition (ii) holds for $\varepsilon > 0$ and sufficiently small. This follows from equation (26) and inequality (24) with the continuity of u_{A}.

Finally, condition (iii) is immediate from equation (26) since affine transformations of utility do not affect choices.

Thus the investor will employ asset manager $\hat{\mathrm{A}}$ who will receive, given $\hat{\rho}$, utility greater than the utility that he would have received in the supposed equilibrium (in the supposed equilibrium he was unemployed and he was obtaining $\bar{u}$). Thus $\hat{\Phi}_\varepsilon$ is a profitable deviation for $\hat{\mathrm{A}}$ and Φ cannot be the contract of an asset manager employed at equilibrium given $\hat{\rho}$.

We have shown that the asset manager's expected utility given any ρ cannot exceed $\bar{u}$. To conclude the proof, note that his utility can never be strictly less than $\bar{u}$ because then his expected utility would be less than his reservation utility.

A.5 Proof of Proposition 4

This follows directly from the solution of the constrained efficient program in Proposition 2 and the expression for the first-best contract Φ_{fb} in Proposition 1.

A.6 Proof of Proposition 5

The main step of the proof below is to show that a contract that is feasible given a fine signal structure is also feasible given a coarse signal structure. This follows directly from the law of iterated expectations. Since coarsening the signal structure expands the set of feasible contracts, it can only increase the investor's objective (recall that the incentive constraints are not binding, which follows from Proposition 2). Since the asset manager always breaks even, increasing the investor's profits constitutes a Pareto improvement.

Below call $\Phi_{\lambda_{\rho_f}}$ and $\Phi_{\lambda_{\rho_c}}$ the efficient sharing rules associated with fine and coarse public signals respectively. First, the asset manager's participation constraint given $\tilde{\rho}_f$ is

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_f\right]=\bar{u}$$

from equation (12). Now, since sigma($\tilde{\rho}_c$) $\subset$ sigma($\tilde{\rho}_f$), use the law of iterated expectations and the condition above to observe that

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right]=\mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{A}}\left(\Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_f\right]\,\middle|\,\tilde{\rho}_c\right]=\mathbb{E}\left[\bar{u}\,\middle|\,\tilde{\rho}_c\right]=\bar{u}.$$

This says that $\Phi_{\lambda_{\rho_f}}$ satisfies the participation constraint given $\tilde{\rho}_c$. Since $\Phi_{\lambda_{\rho_c}}$ solves the principal-agent problem given ρ_c—viz. it maximizes the investor's utility given the asset manager's participation constraint—

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w}-\Phi_{\lambda_{\rho_c}}(\tilde{w})\Big)\,\middle|\,\tilde{\rho}_c\right]\geq\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w}-\Phi_{\lambda_{\rho_f}}(\tilde{w})\Big)\,\middle|\,\tilde{\rho}_c\right].$$

Now we use the inequality above and we apply the law of iterated expectations again to prove that the investor is better off given the coarser structure, namely

$$\begin{aligned}\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w}-\Phi_{\lambda_{\rho_c}}(\tilde{w})\Big)\right]&=\mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w}-\Phi_{\lambda_{\rho_c}}(\tilde{w})\Big)\,\middle|\,\tilde{\rho}_c\right]\right]\\&\geq\mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w}-\Phi_{\lambda_{\rho_f}}(\tilde{w})\Big)\,\middle|\,\tilde{\rho}_c\right]\right]=\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w}-\Phi_{\lambda_{\rho_f}}(\tilde{w})\Big)\right].\end{aligned}$$

Since asset managers always break even and the investor is better off with the coarser structure, $\tilde{\rho}_c$ Pareto dominates $\tilde{\rho}_f$. □

A.7 Proof of Proposition 5 in the Portfolio Choice Application

The proof of Proposition 5 in the portfolio choice example has two main steps. We summarize these steps briefly before giving the full proof. The first step is to show that the investor's ex ante expected utility is minus the expectation of a convex function,

$$\bar{u}\,\mathbb{E}\left[\lambda_{\tilde{\rho}}^2\right] = -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y \mid \tilde{\rho}\,\right]\Big)\right]$$

for (appropriately defined) $c > 0$, $f'' > 0$, and a random variable Y. The second step is to show that the expectation conditional on the coarse signal second-order stochastically dominates the expectation conditional on the fine signal,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right].$$

Whence utility is greater under the coarse signal because minus a convex function is a concave function, and, à la risk aversion, the expectation of a concave function of a stochastically dominated random variable is greater than the expectation of the function of the dominated random variable.

Before we proceed to these main steps, we derive an expression for the Lagrange multiplier λ_ρ and the investor's ex ante expected utility $\mathbb{E}\left[u_{\mathrm{I}}\right]$. These are routine calculations, although they are somewhat lengthy.

Calculation of λ_ρ expression. We give the following expression for the Lagrange multiplier λ_ρ:

$$\big(1+\lambda_\rho\big)^2 = \frac{\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)^2}{2\left|\bar{u}\right|}\,\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2}\;\middle|\;\tilde{\rho} = \rho\right]. \tag{27}$$

The expression follows from plugging in the expressions for u_{A}, Φ_ρ, and x_{fb} into the asset manager's participation constraint (12). This gives

$$
\begin{aligned}
2|\bar{u}|\big(1+\lambda_\rho\big)^2 &= \mathbb{E}\left[\left(R_f + \frac{\big(\bar{R}-R_f\big)\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\Big(\tilde{R}-R_f\Big) - a_{\mathrm{I}} - a_{\mathrm{A}}\right)^2 \,\middle|\, \tilde{\rho}=\rho\right] \\
&= \big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2\,\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} - 1\right)^2 \,\middle|\, \tilde{\rho}=\rho\right] \\
&= \big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2\,\left\{1 - 2\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right] + \right. \\
&\qquad \left. + \,\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\middle|\, \tilde{\rho}=\rho\right]\right\}.
\end{aligned}
\tag{28}
$$

Applying the law of iterated expectations gives

$$
\begin{aligned}
&1 - \frac{2|\bar{\lambda}|\big(1+\lambda_\rho\big)^2}{\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2} \\
&= 2\mathbb{E}\left[\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho}=\rho\right] - \mathbb{E}\left[\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho}=\rho\right] \\
&= 2\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\mathbb{E}\left[\big(\tilde{R}-R_f\big) \,\middle|\, \tilde{\sigma}\right]}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right] + \mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)^2\mathbb{E}\left[\big(\tilde{R}-R_f\big)^2 \,\middle|\, \tilde{\sigma}\right]}{\Big(\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2\Big)^2} \,\middle|\, \tilde{\rho}=\rho\right].
\end{aligned}
$$

Now since

$$
\mathbb{E}\left[\big(\tilde{R}-R_f\big)^2 \,\middle|\, \tilde{\sigma}\right] = \tilde{\sigma}^2 + \big(\bar{R}-R_f\big)^2,
$$

we have

$$
\begin{aligned}
&1 - \frac{2|\bar{\lambda}|\big(1+\lambda_\rho\big)^2}{\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2} \\
&= \big(\bar{R}-R_f\big)^2\left\{\mathbb{E}\left[\frac{2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right] - \mathbb{E}\left[\frac{1}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right]\right\} \\
&= \mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)^2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right].
\end{aligned}
$$

Finally, solve for $(1+\lambda_\rho)^2$ and cross multiply to recover equation (27).

Calculation of $\mathbb{E}[u_{\mathrm{I}}]$ expression. We show that the investor's ex ante expected utility can be expressed in terms of the Lagrange multiplier λ_ρ as follows:

$$\mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi_\rho\big) \,\big|\, \tilde{\rho}=\rho\right] = \bar{u}\,\lambda_\rho^2.$$

This follows from the following string of calculations.

$$\begin{aligned}
&\mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi_\rho\big) \,\big|\, \tilde{\rho}=\rho\right]\\
&= -\frac{1}{2}\mathbb{E}\left[\Big(a_{\mathrm{I}}-\tilde{w}+\Phi_\rho(\tilde{w})\Big)^2 \,\Bigg|\, \tilde{\rho}=\rho\right]\\
&= -\frac{1}{2}\mathbb{E}\left[\left(a_{\mathrm{I}}-\tilde{w}+a_{\mathrm{A}}+\frac{\tilde{w}-a_{\mathrm{I}}-a_{\mathrm{A}}}{1+\lambda_\rho}\right)^2 \,\Bigg|\, \tilde{\rho}=\rho\right]\\
&= -\frac{1}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\big(a_{\mathrm{I}}+a_{\mathrm{A}}-\tilde{w}\big)^2 \,\Big|\, \tilde{\rho}=\rho\right]\\
&= -\frac{1}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\Big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f-x\big(\tilde{R}-R_f\big)\Big)^2 \,\Bigg|\, \tilde{\rho}=\rho\right]\\
&= -\frac{1}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\left(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f-\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\Bigg|\, \tilde{\rho}=\rho\right]\\
&= -\frac{\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\left(1-\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\Bigg|\, \tilde{\rho}=\rho\right].
\end{aligned}$$

Now, from equation (28) above,

$$\mathbb{E}\left[\left(1-\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\Bigg|\, \tilde{\rho}=\rho\right] = 2|\bar{u}|\left(\frac{1+\lambda_\rho}{a_{\mathrm{I}}+a_{\mathrm{A}}-R_f}\right)^2,$$

so, finally,

$$\mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi_\rho\big) \,\big|\, \tilde{\rho}=\rho\right] = \bar{u}\,\lambda_\rho^2. \tag{29}$$

Main Step 1. Rewrite the investor's ex ante expected utility from the expression (29) above:

$$\begin{aligned}
\bar{u}\,\mathbb{E}\left[\lambda_{\tilde{\rho}}^2\right] &= \bar{u}\,\mathbb{E}\left[\left(\sqrt{\frac{(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f)^2}{2|\bar{u}|}\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\;\middle|\;\tilde{\rho}\right]}-1\right)^2\right] \\
&= \frac{\bar{u}(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f)^2}{\sqrt{2|\bar{u}|}}\mathbb{E}\left[\left[\sqrt{\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\;\middle|\;\tilde{\rho}\right]}-1\right]^2\right] \\
&= -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y|\tilde{\rho}\right]\Big)\right]
\end{aligned}$$

where

$$\begin{aligned}
c &:= \sqrt{|\bar{u}|/2}\,(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f)^2, \\
f(z) &:= \left(\sqrt{z}-1\right)^2,
\end{aligned}$$

and

$$Y := \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2}.$$

Note that $c > 0$ and $f''(z) = z^{3/2}/2 > 0$.

Main Step 2. By definition,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right]$$

if there exists a random variable $\tilde{\varepsilon}$ such that

$$\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] = \mathbb{E}\left[Y \mid \tilde{\rho}_c\right] + \tilde{\varepsilon}$$

and

$$\mathbb{E}\left[\tilde{\varepsilon}\,\middle|\,\mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] = 0.$$

For $\tilde{\varepsilon} = \mathbb{E}[Y \mid \tilde{\rho}_f] - \mathbb{E}[Y \mid \tilde{\rho}_c]$ from the above, the condition is

$$\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] - \mathbb{E}[Y \mid \tilde{\rho}_c] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] = 0$$

or

$$\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] = \mathbb{E}[Y \mid \tilde{\rho}_c].$$

Given the assumption sigma$(\tilde{\rho}_c) \subset$ sigma$(\tilde{\rho}_f)$ and since conditioning destroys information—sigma$\big(\mathbb{E}[Y \mid \tilde{\rho}_c]\big) \subset$ sigma$(\tilde{\rho}_c)$—apply the law of iterated expectations firstly to add and then to delete conditioning information to calculate that

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] &= \mathbb{E}\left[\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] \,\middle|\, \tilde{\rho}_c\right] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] \\
&= \mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_c] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] \\
&= \mathbb{E}\left[Y \,\middle|\, \tilde{\rho}_c\right],
\end{aligned}$$

as desired.

A.8 Proof of Proposition 6

The proof is analogous to the proof of Proposition 3, in which we show that the equilibrium contract depends on $\tilde{\rho}$ in the baseline model.

Here, suppose, in anticipation of a contradiction, an equilibrium in which the employed asset manager offers contract $\hat{\Phi}$ which does not depend on $\tilde{\rho}$ and suppose he receives greater than his reservation utility for some realization $\hat{s}$ of the investor's private signal, i.e.

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big) \,\middle|\, \tilde{s} = \hat{s}\right] > \bar{u}. \tag{30}$$

We now show that another asset manager $\hat{\mathrm{A}}$ has a profitable deviation. As in the proof of Proposition 3 we construct a profitable deviation $\hat{\Phi}_{\varepsilon}$ for $\hat{\mathrm{A}}$ such that three conditions are satisfied: (i) the investor employs $\hat{\mathrm{A}}$ when $\tilde{s} = \hat{s}$, (ii) $\hat{\mathrm{A}}$'s expected utility is greater than his reservation utility $\bar{u}$, and (iii) $\hat{\mathrm{A}}$ chooses the same action x that he would have chosen under $\hat{\Phi}$. Further, suppose that the expectation of $\tilde{\rho}$ given $\hat{s}$ is above its unconditional expection,

$$\mathbb{E}\left[\tilde{\rho} \mid \tilde{s} = \hat{s}\right] > \mathbb{E}\left[\tilde{\rho}\right]. \tag{31}$$

This is without loss of generality given that $\mathbb{E}\left[\tilde{\rho} \mid \tilde{s} = s_0\right] \neq \mathbb{E}\left[\tilde{\rho} \mid \tilde{s} = s_1\right]$ by the hypothesis in the proposition and the fact that we can always redefine the public signal as minus itself.

Now consider the following contract:

$$\hat{\Phi}_{\varepsilon}(\tilde{w}) := u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big) - \varepsilon\big(\tilde{\rho} - \mathbb{E}\left[\tilde{\rho}\right]\big)\Big), \tag{32}$$

so that

$$u_{\mathrm{A}}\big(\hat{\Phi}_{\varepsilon}\big) = u_{\mathrm{A}}\big(\hat{\Phi}\big) - \varepsilon\big(\tilde{\rho} - \mathbb{E}\left[\tilde{\rho}\right]\big). \tag{33}$$

We proceed to show that for $\varepsilon > 0$ sufficiently small, $\hat{\Phi}_{\varepsilon}$ satisfies the three conditions above.

Since $u_{\mathrm{I}}' > 0$, a sufficient condition for $\hat{\Phi}_{\varepsilon}$ to satisfy condition (i) is that

$$\mathbb{E}\left[\tilde{w} - \hat{\Phi}_{\varepsilon}\big(\tilde{w}\big) \;\Big|\; \tilde{s} = \hat{s}\right] > \mathbb{E}\left[\tilde{w} - \hat{\Phi}\big(\tilde{w}\big) \;\Big|\; \tilde{s} = \hat{s}\right]$$

or, substituting from equation (32),

$$\mathbb{E}\left[\hat{\Phi}\big(\tilde{w}\big) - u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\big(\hat{\Phi}(\tilde{w})\big) - \varepsilon\big(\tilde{\rho} - \mathbb{E}\left[\tilde{\rho}\right]\big)\Big) \;\Big|\; \tilde{s} = \hat{s}\right] > 0,$$

which is satisfied for $\varepsilon > 0$ by the inverse function theorem since $u_{\mathrm{A}}' > 0$ and $\mathbb{E}\left[\tilde{\rho} \mid \tilde{s} = \hat{s}\right] > \mathbb{E}\left[\tilde{\rho}\right]$.[29]

[29] To see this, recall that for ε small Euler's approximation implies that

$$u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\big(\hat{\Phi}(\tilde{w})\big) - \varepsilon\big(\tilde{\rho} - \mathbb{E}\left[\tilde{\rho}\right]\big)\Big) \approx \hat{\Phi}(\tilde{w}) - \big(u_{\mathrm{A}}^{-1}\big)'\,\varepsilon\big(\tilde{\rho} - \mathbb{E}\left[\tilde{\rho}\right]\big).$$

Condition (ii) holds for $\varepsilon > 0$ and sufficiently small. This follows from equations (31) and (33) and inequality (30) with the continuity of u_{A}.[30] Finally, condition (iii) is immediate from equation (33) since affine transformations of utility do not affect choices.

Thus the investor will employ asset manager $\hat{\mathrm{A}}$ who will receive, given $\hat{s}$, utility greater than the utility that he would have received in the supposed equilibrium (in the supposed equilibrium he was unemployed and he was obtaining $\bar{u}$). Thus $\hat{\Phi}_{\varepsilon}$ is a profitable deviation for $\hat{\mathrm{A}}$, and Φ cannot be the contract of an asset manager employed in equilibrium given $\hat{s}$. Thus, the employed agent must break even for both realizations of $\tilde{s}$. $\square$

[30]It is might be worth pointing out that this depends on our assumption that $\tilde{s}$ has binary realization, so equation (31) implies that

$$\mathbb{E}\left[\tilde{\rho} \mid \tilde{s} = \hat{s}\right] > \mathbb{E}\left[\rho\right] > \mathbb{E}\left[\tilde{\rho} \mid \tilde{s} \neq \hat{s}\right].$$

Thus the investor employs $\hat{\mathrm{A}}$ when $\tilde{s} = \hat{s}$ but not otherwise.

B Table of Notations

	Indices
I	index indicating the investor
A	index indicating an asset manager
	Utility Parameters
u_i	utility function of player $i \in \{\mathrm{I}, \mathrm{A}\}$
a_i	risk-aversion parameter of player $i \in \{\mathrm{I}, \mathrm{A}\}$ (see equation (1))
b	common utility parameter
	Signals/Information Structure
σ	asset managers' private signal
ρ	public signal
	Contracts, Actions, and Payoffs
Φ	contract an asset manager offers the investor
Φ_{fb}	first-best contract (Subsection 3.1)
Φ'	$\partial\Phi/\partial w$, the derivative of Φ with respect to the first argument
Φ_ρ	the contract Φ for a given ρ [31]
x	asset manager's action or "portfolio weight"
w	final wealth
	Application to Portfolio Choice
R_f	gross risk-free rate
$\tilde{R}$	gross rate of return on risky asset
$\bar{R}$	expected gross rate of return on the risky asset
	Other Quantities and Notations
λ	social planner's welfare weight on the asset manager (Subsection 3.1)
λ_ρ	Lagrange multiplier on the asset manager's participation constraint (Subsection 4.2)
sigma$(\cdot)$	the sigma-algebra generated by a random variable
	Notation in Extensions Section 6
$\tilde{s}$	investor's private signal
q	number of investors that enter
$m(q)$	probability that an investor is matched with an asset manager
$\bar{u}_{\mathrm{I}}$	investors' outside option

[31] This notation serves to emphasize that Φ actually depends on ρ, even though $\Phi = \Phi(w, x, \rho)$ is always a function of ρ.

References

Admati, Anat Ruth, and Paul Pfleiderer, 1997, Does it all add up? benchmarks and the compensation of active portfolio managers, *The Journal of Business* 70, 323–50.

Agarwal, Vikas, Naveen D. Daniel, and Narayan Y. Naik, 2009, Role of managerial incentives and discretion in hedge fund performance, *Journal of Finance* 64, 2221–2256.

Amershi, Amin H., and Jan H. W. Stoeckenius, 1983, The theory of syndicates and linear sharing rules, *Econometrica* 51, 1407–16.

Ashcraft, Adam B., and Til Schuermann, 2008, Understanding the Securitization of Subprime Mortgage Credit, *Federal Reserve Bank New York Staff Report 318*.

Bank for International Settlements, 2003, Incentive structures in institutional asset management and their implications for financial markets, Working paper, Report submitted by the Committee on the Global Financial System.

Bar-Isaac, Heski, and Joel D. Shapiro, 2013, Ratings quality over the business cycle, *Journal of Financial Economics* 108, 62–78.

Berends, Kyal, Robert McMenamin, Thanases Plestis, and Richard Rosen, 2013, The sensitivity of life insurance firms to interest rate changes, *Federal Reserve Bank of Chicago Economic Perspectives* 37.

Berk, Jonathan, and Jules van Binsbergen, 2015, Measuring skill in the mutual fund industry, *Journal of Financial Economics* 118, 1 – 20.

Berk, Jonathan B., and Richard Green, 2004, Mutual fund flows and performance in rational markets, *Journal of Political Economy* 112, 1269–1295.

Bhattacharya, Sudipto, and Paul Pfleiderer, 1985, Delegated portfolio management, *Journal of Economic Theory* 36, 1–25.

Bolton, P., and M. Dewatripont, 2005, *Contract Theory* (MIT Press).

Bolton, Patrick, Xavier Freixas, and Joel Shapiro, 2012, The credit ratings game, *The Journal of Finance* 67, 85–111.

Boot, Arnoud W. A., Todd T. Milbourn, and Anjolein Schmeits, 2006, Credit ratings as coordination mechanisms, *Review of Financial Studies* 19, 81–118.

Brown, Keith C., W. V. Harlow, and Laura T. Starks, 1996, Of tournaments and temptations: An analysis of managerial incentives in the mutual fund industry, *Journal of Finance* 51, 85–110.

Buffa, Andrea, Dimitri Vayanos, and Paul Woolley, 2015, Asset management contracts and equilibrium prices, Paul Woolley Centre Paper Series No 41.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *The Journal of Finance* 52, 57–82.

Chaigneau, Pierre, Alex Edmans, and Daniel Gottlieb, 2014a, The value of informativeness for contracting, Working paper.

———, 2014b, The informativeness principle under limited liability, Working paper.

Chevalier, Judith, and Glenn Ellison, 1997, Risk taking by mutual funds as a response to incentives, *Journal of Political Economy* 105, 1167–1200.

———, 1999, Career concerns of mutual fund managers, *Quarterly Journal of Economics* 114, 389–432.

Cremer, Jacques, and Richard P. McLean, 1988, Full extraction of the surplus in Bayesian and dominant strategy auctions, *Econometrica* 56, 1247–57.

Dasgupta, Amil, and Giorgia Piacentino, 2015, The wall street walk when blockholders compete for flows, *Journal of Finance* 70, 2853–2896.

Dasgupta, Amil, and Andrea Prat, 2006, Financial equilibrium with career concerns, *Theoretical Economics* 1, 67–93.

———, 2008, Information Aggregation in Financial Markets with Career Concerns, *Journal of Economic Theory* 143, 83–113.

Doherty, Neil A., Anastasia V. Kartasheva, and Richard D. Phillips, 2012, Information effect of entry into credit ratings market: The case of insurers' ratings, *Journal of Financial Economics* 106, 308–330.

Donaldson, Jason Roderick, and Giorgia Piacentino, 2012, Overrating agencies: Competition, collusion, and regulation, Working paper, The London School of Economics.

Dybvig, Philip H., Heber K. Farnsworth, and Jennifer N. Carpenter, 2010, Portfolio performance and agency, *Review of Financial Studies* 23, 1–23.

Elton, Edwin J., Martin J. Gruber, and Christopher R. Blake, 2003, Incentive fees and mutual funds, *Journal of Finance* 58, 779–804.

FCA, 2016, Asset management market study: Interim report, Discussion paper, Financial Conduct Authority https://www.fca.org.uk/publication/market-studies/ms15-2-2-interim-report.pdf.

Fulghieri, Paolo, Gunter Strobl, and Han Xia, 2013, The economics of solicited and unsolicited credit ratings, *Review of Financial Studies* 27, 484–518.

Glosten, Lawrence R, 1989, Insider trading, liquidity, and the role of the monopolist specialist, *The Journal of Business* 62, 211–35.

Goel, Anand M., and Anjan Thakor, 2015, Information reliability and welfare: A theory of coarse credit ratings, *Journal of Financial Economics* 115, 541–557.

Goldstein, Itay, and Chong Huang, 2016, Credit rating inflation and firm's investment behavior, Working paper,.

Haslem, John, 2015, Mutual fund heterogeneity and fee dispersion, *Journal of Wealth Management*.

He, Zhiguo, and Wei Xiong, 2013, Delegated asset management, investment mandates, and capital immobility, *Journal of Financial Economics* 107, 239–258.

Hirshleifer, Jack, 1971, The private and social value of information and the reward to inventive activity, *American Economic Review* 61, 561–74.

Holmstrom, Bengt, 1979, Moral hazard and observability, *Bell Journal of Economics* 10, 74–91.

———, 1982, Moral Hazard in Teams, *Bell Journal of Economics* 13, 324–340.

Hortaçsu, Ali, and Chad Syverson, 2004, Product differentiation, search costs and competition in the mutual fund industry: a case study of S&P 500 index funds, *Quarterly Journal of Economics* 119, 403–456.

Investment Company Act of 1940, Amended 2012, Section 18(f)(1) and Section 18(f)(2), http://www.sec.gov/about/laws/ica40.pdf.

Kartasheva, Anastasia, and Bilge Yilmaz, 2013, Precision of ratings, Working paper.

Kashyap, Anil K., and Natalia Kovrijnykh, 2016, Who should pay for credit ratings and how?, *Review of Financial Studies* 29, 420–456.

Kessler, Anke S., Christoph Lülfesmann, and Patrick W. Schmitz, 2005, Endogenous punishments in agency with verifiable ex post information, *International Economic Review* 46, 1207–1231.

Kliger, Doron, and Oded Sarig, 2000, The information value of bond ratings, *The Journal of Finance* 55, 2879–2902.

Kovbasyuk, Sergei, 2013, Seller-paid ratings, EIEF Working Papers Series 1330 Einaudi Institute for Economics and Finance (EIEF).

Kurlat, Pablo, and Laura Veldkamp, 2015, Should we regulate financial information?, *Journal of Economic Theory* 158, 697–720.

Ma, Linlin, Yuehua Tang, and Juan-Pedro Gómez, 2016, Portfolio manager compensation in the U.S. mutual fund industry, Working paper Northeastern University.

Manso, Gustavo, 2014, Feedback effects of credit ratings, *Journal of Financial Economics* 109, 535–548.

Mathis, Jérôme, James McAndrews, and Jean-Charles Rochet, 2009, Rating the raters: Are reputation concerns powerful enough to discipline rating agencies?, *Journal of Monetary Economics* 56, 657–674.

Nalebuff, Barry, and David Scharfstein, 1987, Testing in models of asymmetric information, *Review of Economic Studies* 54, 265–77.

Opp, Christian, Marcus Opp, and Milton Harris, 2013, Rating agencies in the face of regulation, *Journal of Financial Economics* 108, 46–61.

Palomino, Frederic, and Andrea Prat, 2003, Risk taking and optimal contracts for money managers, *RAND Journal of Economics* 34, 113–37.

Parlour, Christine, and Uday Rajan, 2016, Contracting on credit ratings: Adding value to public information, Working paper.

Pratt, John, and Richard Zeckhauser, 1989, The impact of risk sharing on efficient decision, *Journal of Risk and Uncertainty* pp. 219–234.

Pratt, John W., 2000, Efficient risk sharing: The last frontier, *Management Science* 46, 1545–1553.

Riordan, Michael H., and David E. M. Sappington, 1988, Optimal contracts with public ex post information, *Journal of Economic Theory* 45, 189–199.

Rogerson, Richard, Robert Shimer, and Randall Wright, 2005, Search-Theoretic Models of the Labor Market: A Survey, *Journal of Economic Literature* 43, 959–988.

Ross, Stephen, 1974, On the economic theory of agency and the principal of similarity, in *Essays on Economic Behavior Under Uncertainty* (Amsterdam: North-Holland Publishing Co.).

Sangiorgi, Francesco, and Chester Spatt, 2016, Opacity, credit rating shopping and bias, *Management Science* (forthcoming).

Skreta, Vasiliki, and Laura Veldkamp, 2009, Ratings shopping and asset complexity: A theory of ratings inflation, *Journal of Monetary Economics* 56, 678–695.

Stoughton, Neal M, 1993, Moral hazard and the portfolio management problem, *Journal of Finance* 48, 2009–28.

van Binsbergen, Jules H., Michael W. Brandt, and Ralph S. J. Koijen, 2008, Optimal Decentralized Investment Management, *Journal of Finance* 63, 1849–1895.

Wahal, Sunil, and Albert (Yan) Wang, 2011, Competition among mutual funds, *Journal of Financial Economics* 99, 40 – 59.

Warner, Jerold, and Joanna Shuang Wu, 2011, Why do mutual fund advisory contracts change? performance, growth, and spillover effects, *The Journal of Finance* 66, 271–306.

Wilson, R., 1968, The theory of syndicates, *Econometrica* 36, 119–132.

———, 1984, A note on revelation of information for joint production, *Social Choice and Welfare* 1, 69–73.