Project Assignment Rights and Incentives for Eliciting Ideas

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In this paper, we study an incentive problem that arises between a principal and two agents because they value a real option differently. The real option in our model is a timing option. The agents have limited capacity to undertake projects, and each agent’s capacity can be filled now or later. Because the principal cares about capacity in the aggregate but each agent cares only about his own capacity, the agents assign a higher value to the option to wait. As a result, agents sometimes withhold ideas from the principal. We show that decentralization can be a solution to this problem. Delegating assignment rights to an agent reduces the option value of waiting for the other agent sufficiently that he is willing to reveal his ideas.

(Real Options; Incentive Problems; Decentralization; Promotion)

1. Introduction

In this paper, we study an incentive problem that arises between a principal (central manager) and two agents (local managers) because they value a real option differently. The real option in our model is a timing option. The agents have limited capacity to undertake projects, and each agent’s capacity can be filled now or later. Because the principal cares about capacity in the aggregate while each agent cares only about his own capacity, the agents assign a higher value to the option to wait. As a result, agents sometimes withhold ideas from the principal.1 We show that a solution to this problem can be for the principal to delegate the decision rights for project assignment to one of the agents. That is, decentralized project assignment is sometimes preferred to centralized project assignment.

The theory of real options, which is primarily a theory of single-person decision making, has proven to be insightful in evaluating investments.2 The traditional static net present value (NPV) rule is expanded to include not only the passive NPV of expected cash flows, but also the value of options from active management. Active management allows a firm to defer, expand, contract, abandon, or otherwise alter a project at different stages during its operating life. Different projects provide a firm with differing degrees of flexibility. For example, a research project does not commit the firm to development or production. The decision to commercially develop a research project can be made contingent on the results of the research. Failing to account for the flexibility provided by a

1 Those who keep quiet at faculty meetings are not necessarily devoid of good ideas. They may instead be trying to avoid a response from the dean of “good idea, why don’t you run with it.” The faculty member may prefer keeping himself free for other projects.

2 According to Ross (1995, p. 101), “when evaluating investments, optionality is ubiquitous and unavoidable. If modern finance is to have a practical and salutary impact on investment-decision making, it is now obliged to treat all major investment decisions as option pricing problems.” For an overview of this literature, see Dixit and Pindyck (1994); Ross (1995); and Trigeorgis (1995, 1997).
project can result in undervaluation. This paper takes a multiperson perspective to the theory of real options and makes a connection between a timing option and the optimal assignment of decision rights within organizations.

We study a three-period model of a firm with a principal and two agents, A and B. At the beginning of the first two periods, each agent may or may not come up with a project idea. Projects take two periods to implement and can be implemented by either of the agents, but the agent who came up with the idea is better suited to implement it. The objective of each agent is to maximize the (expected) value of the projects he implements. The objective of the principal is to maximize the value of the firm, which is the sum of the values of the projects implemented by both agents. Managerial capacity is limited in that each agent can implement only one project during the life of the firm.

Limited managerial capacity leads to an option value of waiting, both for the principal and for the agents. In our setting there is a need to coordinate the agents’ capacity options because they interact. The capacity options interact because assigning an agent a project in Period 1 destroys his option value of waiting, but increases the other agent’s option value. This is because an agent’s Period-2 idea can be assigned to the other agent. Because of the interaction, each agent maximizing the value of the project he implements does not always result in firm-value maximization. What is important in obtaining this result is that the agents’ opportunity cost of implementing an idea differs from the principal’s opportunity cost. In our model, the source of such a difference is the real option associated with limited managerial capacity.

Under centralized project assignment, the principal coordinates the agents’ capacity options. The principal asks the agents to report their project ideas before she makes her assignment decisions. It is assumed that the principal cannot ex ante commit to a project assignment rule. For some Period-1 realizations, the principal wants to assign the best project to its proposer and keep the other agent free for the next period. It is this case that can lead to a goal incongruence. If the best project is only marginally profitable, the agent may prefer to conceal the project. Because the principal cares about the sum of the projects undertaken by the agents but each agent cares only about the project he undertakes, the agent’s option value of waiting is higher than the principal’s.

Consider hierarchically decentralized project assignment. Suppose an agent, say Agent A, is put in charge of project assignment. We can think of Agent A as a boss or a team leader because he both assigns projects and may implement a project himself. There are two benefits to decentralization. First, Agent A no longer has incentives to conceal his idea. When Agent A comes up with an idea, he does not have to worry about the possibility of being forced to implement it. Because he has the decision rights, he can assign his marginally good project to Agent B and keep himself free for a potentially better Period-2 project. Second, the option value of waiting for Agent B, who was not delegated the decision rights, decreases. Agent A has the authority to assign Agent B a project in Period 1 and to expropriate Agent B’s Period-2 idea. As a result, Agent B reports truthfully in Period 1.

The cost of decentralization is opportunistic project assignment. A project is sometimes assigned to an agent other than the one best suited to implement it. We show that decentralized project assignment is preferred to centralized assignment if the efficiency loss from assigning agent i’s idea to agent j is not too large.

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3 This insight led Merck to increase their research budget (Nichols 1994).
4 Trigeorgis (1995, p. 27) suggests that “[e]xtending real options in an agency context” is a research direction deserving attention. Antle et al. (2001) also study a multiperson model involving options. In their model, a principal rations resources both to exploit the option feature of an investment and limit the agent's informational rents (slack).
5 An option to wait (defer) arises when investments are irreversible. In our paper, the irreversibility stems from the fact that when new projects are realized, it is not optimal to abandon the initial project. For example, projects can have a plant and harvest phase. Once planting is done, it is optimal to harvest. Irreversibility can also arise due to physical constraints, such as with an oil well. Accepting a project now (drilling a well) implies foregoing a project in the future (a well can be drilled only once in the same location).
6 In a setting in which capacity is unconstrained, Prendergast (1995) highlights a similar cost of decentralized project (task) assignment.
Hierarchical decentralization is a common feature of many organizations. A frequently discussed advantage of hierarchies is that they result in fewer communication requirements than the alternative of centralizing all decision making. A frequently discussed disadvantage is the control loss associated with delegation. Melumad et al. (1995) provide a benchmark setting in which there is no such control loss. In our paper, the principal is actually better able to control the agents through a hierarchy than she could through centralized decision making. Putting an agent in charge whose preferences are different than the principal’s is a substitute for the principal’s commitment.

We also consider variants of our basic model. In particular, we extend the results to settings with continuous values and to settings with multiple (three or more) agents. Finally, we discuss ex ante differences in agents that affect which agent is optimally put in charge.

2. Model
Consider a firm that has a life of three periods and whose participants are a principal and two agents, A and B. At the beginning of the first and second periods, each agent may develop an idea for a project. There are no new ideas in the third period. Projects take two periods to implement, and, once started, it is not optimal to abandon them. If the period-t project is not undertaken in period t, it is not available in future periods. If agent i implements his period-t idea, its value is denoted by $V_t^i$, $i = A, B$, $t = 1, 2$. $V_t^i$ takes on a value from the set $\{V_0, V_1, \ldots, V_N\}$, $V_0 = 0 < V_1 < \cdots < V_N$. $V_t^i = V_0$ denotes the event that agent i does not come up with an idea. Agent i alone observes the realization of $V_t^i$ at the beginning of period t.

If agent i’s idea is implemented by agent j, $j \neq i$, the project’s value is $kV_t^i$, where $k \in [0, 1]$. (The superscript on V denotes the agent who came up with the idea.) $k = 0$ represents the setting in which ideas are individual specific; agent i’s idea can be undertaken only by agent i. $k = 1$ represents the setting in which each agent’s idea can be equally efficiently implemented by either agent. The intermediate case, $k \in (0, 1)$, characterizes the setting wherein an agent who identifies a project is better suited to implement it. He is more familiar with the project, and communicating all details is costly or impossible.

Arrival of project ideas and project values are independent across agents and time. Also, the agents are assumed to be ex ante identical. Denote the probability that $V_t^i = V_n$, by $p_n$, $n = 0, 1, \ldots, N$; $i = A, B; t = 1, 2$.

The principal and the agents have von Neumann-Morgenstern utility functions and maximize expected utility. The principal’s utility (payoff) is the value of the firm, which is the sum of the values of the projects implemented by both agents. Agent i’s utility is the value of the projects he implements. One interpretation of the agents’ preferences is that they reflect the slack (e.g., perquisites) he consumes. Each agent consumes a proportion of the value of the project he implements as slack. Alternatively, the agents’ preferences might reflect such considerations as the prestige, power, or external visibility associated with running important projects.

8 If $s$ is the fraction of the project value the agent consumes as slack, the agent’s payoff is $s$ times the value of the project he undertakes and the principal’s payoff is $(1 - s)$ times the sum of the value of the projects undertaken by both agents. For any $s$, $0 < s < 1$, this model yields the same conclusions as those obtained by ignoring $s$ in the participants’ utility functions (as in the text), since this is just an affine transformation.

9 We do not explicitly model compensation. Yet another interpretation of the agents’ preferences is that they arise from a compensation contract based at least in part on divisional performance. Under this interpretation of our preferences, one might question why the agents are not rewarded instead solely on the basis of firmwide profits or paid for ideas. The typical response given in the literature on incomplete contracts is that, because of other unmodeled sources of division-specific moral hazard, it is better to base agents’ compensation at least partly on divisional profits. As long as there is some division-specific component to pay, the agents will put their own division’s interests ahead of firmwide value maximization. See Ancil and Dutta (1999) for a discussion and an explicit analysis of divisional versus firmwide performance.
Each agent has a limited capacity for projects. He can implement only one project over the life of the firm. This capacity constraint creates an option to defer filling capacity. It can be optimal for the agent not to implement a profitable project in Period 1 in order to keep himself free for a potentially more profitable project in Period 2.

The sequence of events is as follows. First, the principal publicly either retains the decision rights for project assignment (the centralized regime) or delegates them to one of the agents (the decentralized regime). Second, at the beginning of Period 1, agent i learns \( V^i \) and either reports \( V^i \) to the in-charge individual or keeps quiet (reports \( V_0 \)). The only possibility of misrepresentation is that an agent may choose not to reveal his idea when he does have one.  

Third, the in-charge individual assigns projects to the agents. There is no precommitment to follow a particular rule in assigning projects; no contracts are written. Period 2 repeats the project arrival, reporting, and project assignment. The timeline in Figure 1 summarizes the sequence of events.

The principal and agents make decisions that are dynamically optimal at the time they are making those decisions, correctly anticipating any simultaneous decisions of other individuals and all future decisions. When indifferent among actions, the agents choose actions that maximize the principal’s expected utility.

Conditional on the Period-1 realizations of \( V^A_1 \) and \( V^B_1 \), the expected firm value under the centralized and decentralized regimes is denoted by \( \pi_C(V^A_1, V^B_1; k) \) and \( \pi_D(V^A_1, V^B_1; k) \), respectively. The corresponding unconditional (prior to Period-1 realizations) expected firm values are denoted by \( \Pi_C(k) \) and \( \Pi_D(k) \).

Assuming each agent will be assigned his own Period-2 idea, agent i’s option value of waiting is \( E[V^i_2] = \sum_{n=0}^{\infty} p_n V_n \), which we denote by \( \bar{V} \). If agent i is allowed to pick the best Period-2 project, his option value of waiting is \( E[\max\{V^i_2, kV^j_2\}] = \bar{V} + \delta(k) \), \( \delta(k) \geq 0 \). Note that \( \delta(0) = 0 \), and \( \delta(k) \) is a monotonically increasing, continuous function of \( k \). At \( k = 1 \), \( \delta(k) \) attains its highest value which, using standard results on order statistics, can be written as 

\[
\delta(1) = \sum_{m=0}^{N} \sum_{m+1}^{N} p_m p_n [V_m - V_n].
\]

3. Equilibrium

3.1. First-Best Setting

The first-best setting is one in which the principal also observes \( V^i_t \) at the beginning of period \( t \). In this setting, it is optimal for the principal to retain the decision rights for project assignment. The optimal assignment rule is determined using backward induction.

In Period 2, there is no reason to save capacity for the third period (there are no new ideas in Period 3). Suppose both agents are unavailable in Period 2 (i.e., they were assigned projects in Period 1). In this case, the principal has to forgo all Period-2 projects. Suppose only one agent, say agent i, is available in Period 2. In this case, agent i is assigned his own project if \( V^i_2 \geq kV^j_2 \); otherwise, he is assigned agent j’s project. Finally, if both agents are available in Period 2, each agent is assigned his own project idea.

The principal’s first-period assignment rule needs to account for the option value of keeping one or both agents free. Conditional on Period-1 realizations \( V^A_1 \) and \( V^B_1 \), the first-best firm value, denoted \( \pi_{FB}(V^A_1, V^B_1; k) \), is:

\[
\pi_{FB}(V^A_1, V^B_1; k) = \max\{V^A_1 + V^B_1, \max\{V^A_1, V^B_1\} + \bar{V} + \delta(k), 2\bar{V}\}.
\]

The first term \( (V^A_1 + V^B_1) \) corresponds to the principal assigning both Period-1 projects to their respective project proposers. The second term \( \max\{V^A_1, V^B_1\} + \bar{V} + \delta(k) \) corresponds to the principal assigning the best Period-1 project to its proposer while keeping the other agent free for the best Period-2 project. The third term \( 2\bar{V} \) corresponds to the principal assigning no
projects in Period 1 and keeping both agents free for their own Period-2 ideas. The ex ante expected firm value is $\Pi_{FB}(k) = \sum_{m=0}^{N} \sum_{n=0}^{N} p_m p_n \pi_{FB}(V_m, V_n; k)$. 

### 3.2. Second-Best Setting

Now suppose that the principal does not observe project values. In this setting, there is no reason for the principal to consider delegating project assignment decisions if the agents have incentives to reveal their ideas. If the agents reveal their ideas, the firm value will be first-best; i.e., $\Pi_C(k) = \Pi_{FB}(k)$.

Withholding ideas by the agents is, however, an issue in the second-best setting because the agents value the option to wait differently than the principal does. The principal cares about the projects implemented in aggregate, while each agent cares only about the project he implements. Each agent may not want to undertake a Period-1 project that the principal would like him to undertake.

When agents have incentives to withhold ideas, the firm value will not necessarily be first best under centralization. We say there is a goal incongruence between the principal and agents if there exists some $(V^A, V^B)$ for which $\pi_C(V^A, V^B; k) \neq \pi_{FB}(V^A, V^B; k)$. Otherwise, there is goal congruence.

Proposition 1 presents the range of Period-1 values for which there is a goal incongruence in the centralized regime. (Proofs of all propositions are provided in the Appendix.)

**Proposition 1. Assume centralized project assignment.**

(a) For $k = 0$ or $k = 1$, there is no goal incongruence.

(b) For $k \in (0, 1)$, a goal incongruence arises between the principal and the agents if there exists a $V_m$ such that $\overline{V} - \delta(k) < V_m < \overline{V}$.

The intuition for this result is as follows. For there to be a goal incongruence, the principal and agents must value the option to wait differently. A necessary condition for a difference in valuations is that the principal sometimes finds it optimal to assign ideas developed by one agent to the other agent for implementation. For $k = 0$, there is no reason to have one agent implement the other’s idea, so there is no goal incongruence. The divisions are run as if they were separate firms.

For $k = 1$, the principal can commit to any project assignment rule because there are no mismatching costs. Hence, an optimal assignment rule for the principal is to implement the same projects as under the first-best assignment rule, but to treat one of the agents, say Agent A, as the favorite. That is, Agent A is assigned either the best Period-1 idea or kept free for the best Period-2 idea, depending on which is better for Agent A. This ensures that Agent A has no incentives to withhold his ideas from the principal.

Agent B, the unfavored agent, also has no incentive to withhold his ideas. If Agent A is assigned a project in Period 1, Agent B’s option value of waiting is $E[\max(V^A, V^B)] = \overline{V} + \delta(1)$, which is the same as the principal’s option value. If Agent A is not assigned a project in Period 1, Agent B’s option value of waiting is $E[\min(V^A, V^B)] = \overline{V} - \delta(1)$, which is again the same as the principal’s option value.

Hence, both agents reveal their ideas and the same projects are implemented as under the first-best assignment rule. There is some mismatching, but with $k = 1$ the cost of mismatching is zero. Because the firm’s expected profit is as under the first-best setting, there is no goal incongruence.

Now consider the $k \in (0, 1)$ case. Assuming agent $j$ is free for a Period-2 project, agent $i$’s option value of waiting is $\overline{V}$ because the principal will assign each agent to implement his own idea to avoid mismatching costs. The principal’s option value of keeping a second agent free for a Period-2 project is lower than $\overline{V}$ because the second agent’s idea could have been undertaken by the other free agent if his own idea was not better. The principal’s option value is $2\overline{V} - [\overline{V} + \delta(k)] = \overline{V} - \delta(k)$.

The difference in option values does not imply goal incongruence. If the Period-1 realizations are such that the principal wants to assign zero or two projects in Period 1, then there is no goal incongruence. In the former case, the option value of waiting is large enough (relative to the realization of first-period values), that both the principal and the agents want to wait. In the latter case, the option value is small enough that no party wants to wait. It is only for intermediate option values, i.e., when the principal wants to assign one project in Period 1 and keep
one agent free for Period 2, that there can be a goal incongruence.

Comparing the three possible values for $\pi_{FB}(V^A_t, V^B_t; k)$ implies that the principal wants to undertake the best Period-1 project and keep one agent free for Period 2 if:

$$\min\{V^A_t, V^B_t\} < \bar{V} + \delta(k) \quad \text{and} \quad \max\{V^A_t, V^B_t\} > \bar{V} - \delta(k).$$

Both these inequalities are satisfied if Period-1 realizations are such that $\max\{V^A_t, V^B_t\} = V_m$, where $\bar{V} - \delta(k) < V_m < \bar{V}$. In this case, a goal incongruence arises because the agent with the best Period-1 project is not willing to reveal his Period-1 idea. By withholding his Period-1 idea, the agent receives at least $\bar{V}$, and $\bar{V} > V_m$.

If the condition in Proposition 1(b) is satisfied, $\Pi_C(k) < \Pi_{FB}(k)$. This allows for the possibility that decentralization may be the preferred organizational structure. A benefit of decentralized project assignment, relative to centralized assignment, is presented in Proposition 2.

**Proposition 2.** Under decentralized project assignment, the agent not in charge has no incentive to withhold ideas.

When the principal delegates the decision rights for project assignment to one of the agents, say Agent A, the option value of waiting for Agent B decreases sufficiently that he no longer has incentives to withhold his Period-1 idea. Agent B realizes that if both agents are available in Period 2, Agent A will pick the best project for himself and only assign the leftover project to Agent B. This reduces Agent B’s option value of waiting. Note that this Period-2 assignment rule is incentive compatible for Agent A, whose objective is to maximize his own division’s profits. It is not incentive compatible for the principal who cares for firmwide profits and, hence, wants to minimize mismatching costs.

Truthfully eliciting project ideas is the benefit of decentralization. There is also a cost. Decentralization increases the expected mismatching costs relative to first best (and relative to centralization). For example, in Period 2, even when both agents are free, the in-charge agent will undertake the other agent’s project if the in-charge agent does not have an idea of his own (or if his own idea is less valuable). Proposition 3 states that if the mismatching costs are sufficiently small, decentralized project assignment is optimal.

**Proposition 3.** Assume there exists a $V_m$ such that $\bar{V} - \delta(1) < V_m < \bar{V}$. Then there is a nonempty interval $[k^*, 1)$ such that, for all $k \in [k^*, 1)$, the expected firm value is strictly greater under decentralized project assignment than under centralized project assignment.

The basic idea in the proof of Proposition 3 is to show that as $k \to 1$, the expected firm value under decentralization approaches the first-best firm value while the firm value under centralization is strictly less than first-best. That is, as $k \to 1$, the mismatching costs of decentralization go to 0, but the cost of the goal incongruence under centralization does not go to 0. Continuity arguments are then used to prove the existence of a nonempty interval of $k$ for which decentralization is strictly preferred.

Proposition 3 suggests that maximizing firm value may require delegating the project assignment decisions to an individual whose objective is different than maximizing firm value, even when another individual whose objective is to maximize firm value is available. The in-charge agent’s preferences (which differ from the principal’s) allow him to credibly commit to assignment rules the principal cannot commit to (but would like to). This, in turn, changes how other individuals in the organization react. In our setting, there is also a more traditional advantage to delegating the assignment decision to an agent. The agent is better informed than the principal because he at least knows whether he himself has an idea. (In §5.1, we isolate this advantage.)

### 3.3. Example

In this section, we present an example to highlight the intuition for our results.

**Parameters:**\[ \{V_0, V_1, V_2\} = \{0, 11, 20\} \quad \text{and} \quad \{p_0, p_1, p_2\} = \{1/4, 1/4, 1/2\}. \]

We first check whether there is a goal incongruence under centralization. In the example, $\bar{V} = 12.75$ and $\delta(k) = 3.1875k$ if $0 \leq k \leq 0.55$, and $5.6875k - 1.375$ otherwise. Since $\bar{V} - \delta(1) < V_1 < \bar{V}$, for sufficiently large $k$, 

there is a goal incongruence problem under centralization. Hence, for large \( k \), decentralization is the preferred structure.

Clearly, there is no goal incongruence between the principal and agent \( i \) if \( V_1 = V_0 \) or \( V_2 \). In the former case, both parties want to wait; in the latter case, both parties want to undertake the Period-1 project. A goal incongruence can arise when \( V_1 = V_2 \). Since \( V_1 < \bar{V} \), agent \( i \) wants to withhold his \( V_1 \) idea. However, for sufficiently large large \( k \), when \((V_1, V_2) = (V_0, V_1)\), \((V_1, V_0)\), or \((V_1, V_1)\), the principal wants to undertake one \( V_1 \) project in Period 1 and keep one agent free for Period 2. The condition \( V_1 > \bar{V} - \delta(1) \) implies that for large \( k \) the principal prefers to assign a Period-1 \( V_1 \) project to the idea proposer if the other agent is being kept free for Period 2.

Conditioned on all Period 1 realizations, the firm value in the first best, the centralized setting (for \( k < 1 \)), and the decentralized setting, are presented in Table 1. The firm value in the decentralized setting is the sum of the last two columns, with Agent A assumed to be in charge. Assume Agent A is available for a Period-2 project. In this case, we denote Agent B’s value of waiting by \( W(k) \). In Table 1, \( W(k) = 12.75 \) if \( k = 0 \), \( W(k) = 9.5625 \) if \( 0 < k < 0.55 \), and \( W(k) = 7.0625 + 1.375k \), otherwise.

<table>
<thead>
<tr>
<th>((V_1, V_2))</th>
<th>(\pi_0(V_1, V_2; k))</th>
<th>(\pi_C(V_1, V_2; k))</th>
<th>(\pi_A)</th>
<th>(\pi_B)</th>
</tr>
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<tbody>
<tr>
<td>((V_0, V_0))</td>
<td>(2\bar{V})</td>
<td>(2\bar{V})</td>
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<td>((V_0, V_1))</td>
<td>Max ((V_1 + \bar{V} + \delta(k), 2\bar{V}))</td>
<td>(2\bar{V})</td>
<td>(\bar{V} + \delta(k))</td>
<td>Max ((V_1, W(k)))</td>
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<tr>
<td>((V_0, V_2))</td>
<td>(V_2 + \bar{V} + \delta(k))</td>
<td>(V_2 + \bar{V} + \delta(k))</td>
<td>Max ((kV_2, \bar{V} + \delta(k)))</td>
<td>If (kV_2 &gt; \bar{V} + \delta(k)), then (\bar{V} + \delta(k)); else (V_2)</td>
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<tr>
<td>((V_1, V_0))</td>
<td>Max ((V_1 + \bar{V} + \delta(k), 2\bar{V}))</td>
<td>(2\bar{V})</td>
<td>(\bar{V} + \delta(k))</td>
<td>Max ((kV_1, W(k)))</td>
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<td>((V_1, V_1))</td>
<td>Max ((V_1 + \bar{V} + \delta(k), 2\bar{V}))</td>
<td>(2\bar{V})</td>
<td>(\bar{V} + \delta(k))</td>
<td>Max ((V_1, W(k)))</td>
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<tr>
<td>((V_2, V_2))</td>
<td>(2V_2)</td>
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From Proposition 1, at \( k = 0 \) and \( k = 1 \), the expected firm value under centralization and decentralization is first best. At \( k = 0 \), the option to wait is valued at \( \bar{V} \) by the principal and each agent. At \( k = 1 \), the principal can commit to the same assignment rule as the agent. However, \( k = 0 \) and \( k = 1 \) are points of discontinuity under decentralization and centralization, respectively. These points of discontinuity and the unconditional expected firm value for all other \( k \)-values are detailed in Figure 2 and the subsequent discussion.

A few observations regarding Figure 2.

Observation 1. At \( k = 0 \), \( V_1 < \bar{V} \) implies that the principal’s preferred assignment rule is to assign a project to agent \( i \) in period 1 if and only if \( V_1 = V_2 \). Hence, the first-best expected firm value at \( k = 0 \) is \( 2[0.5(12.75) + 0.5(20)] = 32.75 \).

Observation 2. There is a nonempty interval of \( k \)-values, \( k \in [0.9333, 1) \), where decentralization is strictly preferred to centralization.

Observation 3. The condition presented in Proposition 3 for decentralization to be strictly preferred to centralization is sufficient but not necessary. For example, at \( k = 0.79 \), the firm value under centralization is 34.31 and under decentralization is 34.33. However, at \( k = 0.85 \), the firm value under centralization is higher; the firm value under centralization is 34.48.
and under decentralization is 33.88. The key is how the in-charge agent acts when the other agent has a V2 project in Period 1. At k = 0.79, the in-charge agent’s option value of waiting is greater than undertaking the other agent’s V2 project. At k = 0.85, the reverse is true. The mismatching costs at 0.85 (which are absent at 0.79) make centralization preferred at 0.85 but not at 0.79.

Observation 4. Finally, a comment on the discontinuities in $\Pi_D(k)$. The reason for the discontinuity at $k = 0$ is that the in-charge agent’s act changes abruptly at $k = 0$. Assume both agents are available in Period 2 and $(V^A_2, V^B_2) = (V_0, V_1)$. At $k = 0$, Agent B is assigned his own $V_1$ idea. The firm value under decentralization is $V_1$. If $k > 0$, Agent A undertakes Agent B’s idea. The firm value is $kV_1$. As $k \to 0^+$, $kV_1 \to 0$. Hence, $\lim_{k \to 0^+} \Pi_D(0) \neq \Pi_D(0)$.

The second point of discontinuity occurs at the value of $k$ that solves the equation $kV_2 = V_1$. In the example, this value is 0.55. Say both agents are available in Period 2 and $(V^A_2, V^B_2) = (V_1, V_2)$. At $k \leq 0.55$, each agent undertakes his own project. At $k > 0.55$, each agent undertakes the other agent’s project. Mismatching costs are absent when $k = 0.55$ and are strictly positive when $k > 0.55$. This results in an abrupt drop in the firm value at 0.55. (This is why $k = 0$ and $k = 0.55$ are discontinuities in $W(k)$.)

The third point of discontinuity occurs at the value of $k$ that solves the equation $kV_2 = 2V + \delta(k)$. In the example, this value is 0.795. If $(V^A_1, V^B_1) = (V_0, V_2)$, Agent A chooses to wait when $k \leq 0.795$, but chooses to undertake Agent B’s Period-1 project if $k > 0.795$. (See the last two columns in Table 1 corresponding to the event $(V^A_1, V^B_1) = (V_0, V_2)$.) Again, mismatching costs are absent when $k = 0.795$ and are strictly positive when $k > 0.795$. This discontinuity in Figure 2 suggests a somewhat counterintuitive result. The principal can sometimes be better off when agents are less versatile (i.e., have lower $k$ values). At $k = 0.794$, decentralization is optimal, and firm value under decentralization is 34.35. At $k = 0.796$, centralization is optimal, and firm value under centralization is 34.33.

4. Relaxing Assumptions\(^{12}\)

In this section we relax some of the assumptions in our model. Section 4.1 shows that the results hold in a continuous value setup. Section 4.2 shows that the results also hold in a setting with three or more agents when each agent can generate and implement ideas. Section 4.3 discusses a setting in which only Agents A and B can generate ideas, but the principal can hire more agents to implement ideas. In this relaxed capacity constraint setting, there is no goal incongruence in the centralized regime.

4.1. Continuous Values Setup

Suppose $V^*_i \in [0, V_u]$. The continuous probability distribution function $f$ over $V^*_i$, $f(V^*_i) > 0$, is as follows:

\[
f(V^*_i) = \begin{cases} 
  p_0 & \text{if } V^*_i = 0 \\
  (1 - p_0) \cdot g(V^*_i) & \text{if } 0 < V^*_i \leq V_u \\
  0, & \text{otherwise.}
\end{cases}
\]

That is, with probability $p_0$, agent $i$ is unsuccessful in coming up with an idea; with probability $1 - p_0$, the agent develops an idea. Given that agent $i$ comes

\(^{12}\) The authors thank two anonymous referees for suggesting several of the issues raised in §§ 4 and 5 of the paper.
up with an idea, the probability distribution function over $V_i^t$ is $g$.

Proposition 1(a) is unchanged. With a continuum of values, for all $k \in (0, 1)$, there is always some $V_i^A$ and $V_i^B$ that will satisfy $\tilde{V} - \delta(k) < \max\{V_i^A, V_i^B\} < \tilde{V}$. Hence, the condition in Proposition 1(b) is always satisfied. In other words, for all $k \in (0, 1)$, $\Pi_C(k) < \Pi_B(k)$. Proposition 2 is unchanged: Under decentralization, the agent who is not in charge has incentives to report truthfully. Under the continuous setup, Proposition 3's condition always holds: There exists a $V_m$ such that $\tilde{V} - \delta(1) < V_m < \tilde{V}$. Hence, there is a nonempty interval $[k^*, 1)$ such that, for all $k \in [k^*, 1)$, $\Pi_C(k) < \Pi_D(k)$.

### 4.2. Multi-Agent (Three or More) Setup

Assume the continuous values setup introduced in the previous subsection. Suppose there are three agents: A, B, and C. As before, under centralized project assignment, a goal incongruence arises because (i) in the second-best setting, each agent has incentives to withhold his Period-1 project idea if it is less than $\tilde{V}$ and (ii) in the first-best setting, the principal wants to undertake at least one Period-1 project if $V_i^1$ is smaller than, but sufficiently close to, $\tilde{V}$ for some $i$.

Under decentralization, agents do not have any incentives to withhold their Period-1 ideas. To see this, consider the following argument. Suppose the principal delegates the assignment rights to Agent A. Agent A can credibly commit to adhering to a project assignment policy under which, after choosing his own project, he delegates the assignment rights for the remaining projects to Agent B. Effectively, the principal creates a tiered hierarchy with the top position being occupied by her, the next lower position by A, followed by B, and then by C. Proposition 2 applies to the game starting at each node. Hence, under decentralization, each agent reports his Period-1 idea truthfully. (The same iterative argument applies if there are more than three agents.)

As in the two-agent setting, the expected firm value under decentralization (but not under centralization) approaches first best. Hence, Proposition 3 holds.

### 4.3. Availability of More Implementing Agents

Suppose there are four agents, Agents A, B, C, and D. Agents A and B are capable of generating and implementing ideas. On the other hand, Agents C and D can only implement ideas. As before, if agent $i$'s idea is implemented by agent $j$, the project's value is $kV_i^j$.

Note that since there are at most four ideas over the two periods, the availability of four agents guarantees that all ideas can be implemented. As alluded to earlier, relaxing the capacity constraint (from the principal's perspective) removes all friction in our setting. With the relaxed capacity constraint, agents do not have any incentives to withhold their Period-1 ideas. Hence, there is no need for the principal to delegate project assignment rights.

To see why agents do not withhold their Period-1 ideas, consider the following argument. It is credible for the principal to commit to a Period-2 assignment rule under which agent $i, i = A, B$, is assigned a project only if he is free and has a project idea. In this case, agent $i$ is assigned his own project idea. This assignment rule is credible (and optimal) because in Period 2, if agent $j$ is busy and has an idea, Agent C or D is available and it is equally efficient to assign the idea to C or D.

Under the above assignment rule, agent $i$'s option value of waiting is $\tilde{V}$. Hence, agent $i$ is willing to reveal his Period-1 project idea if it is optimal for the principal to assign him a project in Period 1 if and only if the project idea is valued at least $\tilde{V}$. As we next argue, this is indeed the case.

Suppose $V_i^1 = V_m$. The benefit to the principal of assigning this Period-1 project to agent $i$, rather than Agents C or D, is $(1 - k)V_m$. The cost of this assignment is that agent $i$'s Period-2 idea will have to be implemented by an agent other than agent $i$. The expected cost is $(1 - k)\tilde{V}$. From the principal's perspective, the benefit exceeds the cost if and only if $V_m > \tilde{V}$. The principal's criteria for assigning agent $i$ a Period-1 project matches agent $i$'s own criteria. Under centralization, there is no goal incongruence; hence, there is no demand for decentralization.

---

13 Of course, the agents' option value of waiting in the three-agent setup is different than in the two-agent setup, but qualitatively the same arguments apply.
5. Extensions

5.1. Boss Who Weights Subordinate-Run Project and Own Project Equally

Recall that decentralization helps in our model for two reasons. First, assignment decisions are made by an agent whose preferences are different than the principal’s preferences. Second, assignment decisions are made by an agent who is more informed (knows whether he himself has an idea or not) than the principal. To isolate this second, more traditional, advantage of decentralization we reconsider the example in §3.3, with the only change being that we assume the boss cares equally for his own project and his subordinate’s project; i.e., the boss has the same objective function as the principal.

Assume $k = 0.95$. The first-best and the centralized solution can be determined as before using Table 1. The decentralized solution is, however, different. This is because it is no longer credible for the boss (say Agent A) to commit to undertaking the best Period 2 project himself and leaving the leftover project to Agent B in the event both agents are available in Period 2. Agent B realizes that Agent A, just as is the case with the principal under centralization, will minimize mismatching costs in Period 2. This guarantees Agent B that he will be allowed to implement his own Period-2 project idea if he wants to. Hence, Agent B sometimes has incentives to withhold his Period-1 project idea.

In the example, under decentralized project assignment: (i) Agent B reveals his project idea in Period 1 if and only if its value is $V_2$ and, in which case, Agent B is assigned the project, and (ii) Agent A undertakes his own Period-1 idea. The expected firm value under each of the regimes is as follows.

<table>
<thead>
<tr>
<th></th>
<th>First-best expected firm value:</th>
<th>Second-best expected firm value under</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Centralization:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>34.764</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Decentralization:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>35.049</td>
</tr>
</tbody>
</table>

In the example, it is better to delegate the decision rights to an agent than to make centralized task assignment decisions. The reason is that while Agent A has the same objective function as the principal, Agent A is better informed than the principal—Agent A knows when he has a project idea. Decentralization allows the firm to make better use of local information.

5.2. The Promotion Question

Our main result, Proposition 3, identifies conditions under which the principal prefers to delegate the decision rights for project assignment to one of the agents (i.e., promote one agent to be the boss) than to make project assignment decisions herself. Because the agents were (ex ante) identical, the principal was indifferent between promoting either of the two agents. In fact, one purpose of Proposition 3 was to emphasize that it may be optimal to treat even identical agents differently. However, it is also interesting to consider differences in agents that influence the principal’s promotion decision.

The benefit of decentralization is that it allows the principal to elicit Period-1 ideas from the agents. This benefit arises because the in-charge agent’s incentive to expropriate the other agent’s Period-2 idea reduces the other agent’s option value of waiting. However, expropriation can be excessive under decentralization. Excessive expropriation manifests itself in the form of increased mismatching costs relative to first best. The principal’s promotion decision optimally trades off the benefits of truthful elicitation with the costs of mismatching.

Several agent characteristics might affect this trade-off, including differences in agents in their versatility ($k$-values), opportunity sets, level of optimism (“perceived” opportunity sets), and closeness to capacity constraint. In this subsection, we show that firm value is sometimes maximized by promoting the less versatile agent.

Let $k_j$ denote agent $j$’s versatility parameter. If agent $j$ implements agent $i$’s idea the project value is $k_jV_i$. The more versatile the agent is, the higher is his $k$ value.

Return to the numerical example in §3.3 (with each agent caring only about the project he implements). Assume Agent A, but not Agent B, is perfectly versatile: $k_A = 1$ and $0 < k_B < 1$. Using the same arguments as given in the existing proof, Proposition 2 continues to hold. Given truthful revelation, Table 2 presents the
agents’ payoffs, assuming Agent A and Agent B are put in charge of project assignments, respectively.

There is no clear ranking of firm value, $\pi_A + \pi_B$, in Panels A and B of Table 2. In fact, parameters can be chosen such that the unconditional firm value in any particular panel is higher. Further, to determine the optimal individual to put in charge, the panels have to be compared not only with each other but also with centralized project assignment. Such a comparison is provided in Table 3 for two different values of $k_B$. In Panel A, putting the more versatile Agent A in charge of project assignments is optimal. In Panel B, putting the less versatile Agent B in charge is optimal. (The asterisks in each panel of Table 3 denote the in-charge individual that maximizes firm value.)

In the example, it is advantageous to put B, the less versatile agent, in charge if $k_B$ is sufficiently low. A high $k_B$ Agent B and Agent A each have incentives to expropriate the other’s Period-1 idea if it has a value of $V_2$. For a high $k_B$, it is better to have the perfectly versatile agent, Agent A, in charge. This is true when $k_B = 0.9$: $0.9V_2 = 18 > \bar{V} + \delta(0.9) = 16.49$, which implies Agent B, if in charge, would expropriate Agent A’s $V_2$ idea. At $k_B = 0.7$, this is not true: $0.4V_2 = 14 < \bar{V} + \delta(0.7) = 15.36$.

6. Conclusion

In this paper, we study the effect of a real option created by limited managerial capacity on organizational design. For intermediate values of the option, the principal’s interests are best served if one of the two agents (but not both) is kept free to take on a Period-2 project. Each agent’s interests are best served if he is the one kept free to take on a Period-2 project. We provide conditions under which decentralization mitigates this goal incongruence. Delegating assignment rights to one agent reduces the option value of waiting for the other agent sufficiently that he is willing to reveal his Period-1 idea.
 project assignment rights.

Appendix

Proof of Proposition 1. Consider the $k = 1$ case. If agent $j$ is available for Period 2, the principal’s option value of keeping agent $i$ free for Period 2 is $2V - [V + \delta(1)] = V - \delta(1)$. This is also agent $i$’s option value of waiting since the principal can credibly commit to the following assignment rule.

In Period 2, the best project is assigned to agent $j$ and any leftover project is assigned to agent $i$ if both agent $i$ and agent $j$ are available. In Period 1, agent $j$ is given the choice of picking the best Period-1 project or waiting for a Period-2 project. This assignment rule is self-enforcing after ideas are revealed because the principal bears no cost if any agent’s idea is implemented by the other agent.

Under this assignment rule, if agent $j$ is available in Period 2, agent $i$’s option value of waiting is $E[\min(V_i, V_j)]$. The identity $E[\max(V_i, V_j)] + E[\min(V_i, V_j)] = E[V_i] + E[V_j]$ implies $E[\min(V_i, V_j)] = 2V - [V + \delta(1)] = V - \delta(1)$. Agent $i$’s option value of waiting is the same as the principal’s. Hence, at $k = 1$, there is no goal incongruence.

Consider the case $k \in (0, 1)$. If agent $j$ is available in period two, the principal and agent $i$’s option value of waiting is $V - \delta(k)$ and $V$, respectively. Since $\delta(0) = 0$, the principal and agent $i$ value the option identically at $k = 0$. At $k = 0$, there is no goal incongruence. If $k \in (0, 1)$, agent $i$’s option value of waiting is strictly greater than the principal’s since $\delta(k) > 0$ for $k > 0$.

In the remainder of the proof, consider the $k \in (0, 1)$ case and show that there is no goal incongruence if the principal wants to assign zero or two projects in Period 1 (referred to as Cases 1 and 2, respectively). However, if the principal wants to assign only one project in Period 1 and keep one agent free for Period 2, goal incongruence can arise (Case 3).

Case 1. $\pi_{FB}(V_i^A, V_j^B; k) = VA + VB$. Conditional on $VA$ and $VB$, the first-best firm value $\pi_{FB}(V_i^A, V_j^B; k)$ is:

$$\pi_{FB}(V_i^A, V_j^B; k) = \max(V_i^A + V_j^B, 2V)$$

(A1)

Case 1 corresponds to the principal’s preferred choice being to undertake both Period-1 projects. Comparing the first and the second terms in (A1) implies $\min(V_i^A, V_j^B) \geq V - \delta(k)$. The right-hand side is the option value of waiting for an agent if he is the only one kept free. Since even that is less than taking on their respective Period-1 projects, both agents prefer undertaking their projects in Period 1.

Case 2. $\pi_{FB}(V_i^A, V_j^B; k) = 2V$. Case 2 corresponds to the principal’s preferred choice being to undertake no Period-1 project. Comparing the second and the third terms in (A1) implies $\max(V_i^A, V_j^B) \geq V - \delta(k)$. Since the agent’s option value of waiting is greater than the best Period-1 project ($V > \max(V_i^A, V_j^B)$), each agent prefers not to undertake any project in Period 1.

Case 3. $\pi_{FB}(V_i^A, V_j^B; k) = \max(V_i^A + V_j^B + V + \delta(k))$. Case 3 corresponds to the principal’s (strictly) preferred choice being to assign the best Period-1 project to the project proposer and to keep the
other agent free for Period 2. Comparing the second term with the first and third terms in (A1) implies:

\[
\min(V^a_1, V^a_2) < V + \delta(k) \quad \text{and} \quad \max(V^a_1, V^a_2) > V - \delta(k).
\]  

(A2)

Goal congruence is achieved as long as the agent with the best Period-1 project, say Agent A, is willing to reveal his Period-1 idea. Suppose \(\max(V^a_1, V^a_2) = V^a_1 = V^a_2\), where \(V - \delta(k) < V^a_1 < V\). This implies that (A2) is satisfied. However, when \(V^a_1 = V^a_2 = V\), Agent A withholds his Period-1 idea because by withholding he receives at least \(V > V^a_1\). Hence, for all \(k \in (0, 1)\) and \((V^a_1, V^a_2) = (V^a_1, V^a_2)\), \(V - \delta(k) < V^a_1 < V^a_2 < V\), \(\pi_c(V^a_1, V^a_2, k) \neq \pi_B(V^a_1, V^a_2, k)\). There is goal incongruence.

Proof of Proposition 2. Suppose Agent A is in charge. In Period 2, Agent B does not withhold his idea because there is no reason to save capacity for any future period. Denote by \(\pi_i\) agent i’s payoff when Agent B withholds his Period-1 idea; when Agent B reveals his idea, agent i’s payoff is denoted by \(\pi_{ir}\).

Case 1. Assume Agent B withholds his Period-1 idea. Agent A can undertake his own idea or the best Period-2 project: \(\pi_A = \max(V^a_1, V + \delta(k))\). If \(\pi_A = V^a_1\), \(\pi_A = V^a_1 + \delta(k)\) because the B is the only available agent in Period 2.

If \(\pi_A = V^a_1 + \delta(k)\), Agent B is assigned the better of Agent A’s Period-1 project or the second-best Period-2 project (the project that is leftover after Agent A has had his pick). Denote the expected value of the second-best Period-2 project by \(W(k)\). If \(k = 0\), Agent A never undertakes Agent B’s Period-2 project: \(W(k) = E[V^a_2] = V\).

If \(k > 0\), Agent A undertakes the best Period-2 project. If Agent B is available in Period 2, Agent B is assigned Agent A’s Period-2 project if \(V^a_1 < kV^a_2\) and is assigned his own Period-2 project otherwise. In the former case, his payoff is \(kV^a_2\); in the latter, his payoff is \(V^a_2\). Hence, \(W(k) = E[k\min(V^a_1, kV^a_2)]\), where \(q = k\) if \(V^a_1 < kV^a_2\) and \(q = 1/k\) otherwise. Hence, if \(\pi_A = V + \delta(k)\), \(\pi_B = \max(kV^a_1, W(k))\).

Case 2. Assume Agent B reveals his Period-1 idea. If \(kV^a_1 < \pi_B\), Agent A does not undertake his B’s idea. Agent A undertakes the same project he did in the previous case: \(\pi_A = \pi_B\). He will assign Agent B’s first-period project to B only if it improves B’s payoff. That is, \(\pi_B = \max(V^a_1, \pi_B)\).

If \(kV^a_1 < \pi_B\), Agent B undertakes B’s Period-1 idea. In this case, \(\pi_B = \max(kV^a_1, V + \delta(k))\). The expected value of the second-best Period-2 project is surely less than the expected value of the best Period-2 project; i.e., \(W(k) = \min(V^a_1, kV^a_2)\). Also, \(\max(V^a_1, V + \delta(k)) < V^a_1 + V\).

Case 3. If \(k\pi_B < \pi_A\), Agent B’s Period-1 idea is the second-best Period-2 project. From (A3), \(\pi_B = \max(V^a_1, kV^a_2)\). If \(\pi_B = V^a_1\), \(\pi_B = \max(kV^a_1, V + \delta(k))\). Also, \(\max(V^a_1, V + \delta(k)) < V^a_1 + V\).

Hence, \(\pi_B < \pi_A\).

Agent B is better off revealing his idea. □

Proof of Proposition 3. In Step 1, we derive a lower bound \(L(V^a_1, V^a_2; k)\) for \(\pi_a(V^a_1, V^a_2; k)\). In Step 2, we show that \(\Pi_a(k)\) is a continuous function of \(k\) on \([0, 1]\) and \(\lim_{k \to 0^+} \Pi_a(k)\) is strictly less than \(\Pi_a(1)\). In Step 3, we argue that \(\Pi_a(k)\) and \(E[L(V^a_1, V^a_2; k)]\) are continuous functions of \(k\) on the interval \([0, 1]\) and \(\lim_{k \to 0^+} E[L(V^a_1, V^a_2; k)] = \Pi_a(1)\).

The proof then follows from the following argument. From Steps 2 and 3, \(\Pi_a(k) < \Pi_a(1)\) while \(\lim_{k \to 0^+} E[L(V^a_1, V^a_2; k)] = \Pi_a(1)\). Hence, from the continuity arguments in Steps 2 and 3, there exists a nonempty interval \([k^*, 1]\) such that for all \(k \in [k^*, 1]\), \(E[L(V^a_1, V^a_2; k)] > \Pi_a(k)\). Combining this with the result in Step 1 that \(\pi_a(V^a_1, V^a_2; k) \geq L(V^a_1, V^a_2; k)\) implies that for all \(k \in [k^*, 1]\), \(\Pi_a(k) > \Pi_a(k)\).

The remainder of the proof is devoted to proving Steps 1, 2, and 3.
from (A1), it must be the case that:

$$\min\{V^A_1, V^B_1\} \leq \bar{V} + \delta(k) \text{ and } \max\{V^A_1, V^B_1\} \geq \bar{V} - \delta(k).$$  \hfill (A7)

There are two possibilities. Say \( \pi_A = \max\{V^A_1, kV^B_1\} \). In this case, Agent B is either asked to wait and obtains \( \bar{V} + \delta(k) \) or is given the leftover Period-1 project if it improves Agent B's payoff; i.e., \( \pi_B \geq \bar{V} + \delta(k) \). Hence, \( \pi_B(V^A_1, V^B_1; k) \geq \max\{kV^A_1, kV^B_1\} + \bar{V} + \delta(k) \geq \max\{kV^A_1, kV^B_1\} + \bar{V} + \delta(k) \geq \max\{kV^A_1, kV^B_1\} + \bar{V} + \delta(k) \) = \( k\pi_B(V^A_1, V^B_1; k) \geq L(V^A_1, V^B_1; k) \).

Say \( \pi_A = \bar{V} + \delta(k) \). From (A6), \( \pi_B = \max\{kV^A_1, V^B_1, W(k)\} \). Hence, \( \pi_B(V^A_1, V^B_1; k) \geq \bar{V} + \delta(k) + \max\{kV^A_1, V^B_1, W(k)\} \geq \bar{V} + \delta(k) + \max\{kV^A_1, kV^B_1\} \geq k[\bar{V} + \delta(k) + \max\{V^A_1, V^B_1\}] = k\pi_B(V^A_1, V^B_1; k) \geq L(V^A_1, V^B_1; k) \).

Step 2. Consider centralized assignment. From Proposition 1, for \( k \in [0, 1] \), \( \Pi_c(k) \leq [1 - p(k)]\Pi_{th}(k) + p(k)2\bar{V} \), where \( p(k) \) denotes the probability that \( \bar{V} - \delta(k) < \max\{V^A_1, V^B_1\} < \bar{V} \), and \( \Pi_c(1) = \Pi_{th}(1) \). Note that \( \Pi_c(0) = \Pi_{th}(0) \), since \( p(0) = 0 \); also, \( k = 1 \) is the only point of discontinuity in \( \Pi_c(k) \).

The existence of a \( V_m \) such that \( \bar{V} - \delta(1) < V_m < \bar{V} \) implies (i) \( \lim_{k \to 1} p(k) = p(1) > 0 \) and (ii) \( \lim_{k \to 1} \pi_B(V^A_1, V^B_1; k) = \pi_B(V^A_1, V^B_1; 1) > 2\bar{V} \). Hence, \( \lim_{k \to 1} \Pi_{th}(k) = \Pi_{th}(1) > 2\bar{V} \) and \( \lim_{k \to 1} \Pi_c(k) = \Pi_c(1) \).

Step 3. \( \Pi_c(k) \) is continuous on \([0, 1] \), since \( \pi(k) \) is continuous. \( E[L(V^A_1, V^B_1; k)] = E[k\pi_B(V^A_1, V^B_1; k) - k(\delta(k) - \delta(k))] \) is also continuous on \([0, 1] \). Hence, \( \lim_{k \to 1} E[L(V^A_1, V^B_1; k)] = \Pi_{th}(1) \).

References


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