Team Incentives and Bonus Floors in Relational Contracts[1]

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Abstract

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A common means of incorporating non-verifiable performance measures in compensation contracts is via bonus pools. We study a principal-multi-agent relational contracting model in which the optimal contract resembles a bonus pool. It specifies a minimum joint bonus floor the principal is required to pay out to the agents and gives the principal discretion to use non-verifiable performance measures to both increase the size of the pool and to allocate the pool to the agents. The joint bonus floor is useful because of its role in motivating the agents to mutually monitor each other (team incentives). In an extension section, we introduce a verifiable team performance measure. The broader message that emerges is that "paying for poor performance" – either verifiable team performance or non-verifiable individual performance – can be optimal in a relational (self-enforcing) contracting setting because it creates the trust needed for the principal to tailor other promised payments to motivate mutual monitoring.

1 Introduction

This paper studies discretionary rewards based on non-verifiable performance measures. A concern about discretionary rewards is that the evaluator must be trusted by the evaluatees (Anthony and Govindarajan, 1998). In a single-period model, bonus pools are a natural economic solution to the "trust" problem (MacLeod, 2003; Baiman and Rajan, 1995; Rajan and Reichelstein, 2006; 2009; Ederhof, 2010). A bonus pool can be seen as a special case of a relational contract. While explicit contracts are enforced by the courts, a relational (implicit) contract must be self-enforcing. In a multi-period setting, relational contracts can be enforced by threats of retaliation by one party when the other(s) renege on their promises (MacLeod and Malcomson, 1989). In a single-period setting with multiple agents, the only self-enforcing contract is a bonus pool, which leaves the principal discretion only in allocating the fixed pool to the agents.

We study a multi-period relational contracting model in which the optimal contract resembles a bonus pool with added discretion to increase the size of the bonus pool. It specifies a minimum joint bonus (hereafter bonus floor) the principal is required to pay out to the agents and gives the principal discretion to use non-verifiable performance measures to both increase the size of the pool and to allocate the pool to the agents. Such discretion is fairly common in practice. As an example, Gibbons and Henderson (2013) analyze the bonus pay at Lincoln Electric. They find that the board of Lincoln Electric has complete discretion both in setting the size of the firm-wide bonus and in determining individual payouts based on its subjective evaluation of individual contributions such as ideas and cooperation. Similar discretion is common in Wall Street firms as well.[1] Empirically, Murphy and Oyer (2003) find that 42% of their sample of 262 firms gave the compensation committee discretion in determining the size of the executive bonus pool, while 70% had discretion in allocating the bonus pool to individual executives.[2] The related theoretical literature on bonus pools typically uses the

[1]See, for example, UBS Group AG 2015 Compensation Report and Eccles and Crane (1988).

[2]For evidence on discretion in individual bonus plans, see Bushman, Indjejikian, and Smith (1996).

term bonus pool for bonus pools with a non-discretionary total payout, while our focus is on repeated play and the role of trust in facilitating discretion in determining the size of the bonus pool.

Our multi-period relationship not only facilitates trust between the principal and the agents, it also creates the possibility of trust between the agents and, hence, opportunities for relational contracts between the agents. Understanding the nature of relational contracts between the agents is important because teamwork and team incentives are becoming increasingly prevalent in modern organizations (Deloitte, 2016). Hamilton Nickerson, and Owan (2003) study a manufacturing company and find that the introduction of team production and team-based incentive pay significantly increase productivity. They interpret their evidence as suggesting that mutual monitoring among the team members contributes to the increases in productivity. Similarly, Knez and Simester (2001) examine the introduction of team-based compensation system at Continental Airlines established in 1995. They find evidence consistent with the argument that the use of teams at the firm increased the extent of mutual monitoring, which improves the company's on-time performance. Knez and Simester also present anecdotal evidence of workers punishing each other for shirking as a way to sustain team incentives.

The demand for mutual monitoring using relational contracts in our model is similar to Arya, Fellingham, and Glover (1997) and Che and Yoo (2001).[3] The agents work closely enough that they observe each other's actions, while the principal observes only individual performance measures that imperfectly capture those actions. The key idea in those papers is to replace the agents' Nash incentive constraints with group incentive constraints. As Milgrom and Roberts (1992, p. 416) write: "[g]roups of workers often have much better information about their individual contributions than the employer is able to gather...[g]roup incentives

[3]There is an earlier related literature that assumes the agents can write explicit side-contracts with each other (e.g., Tirole, 1986; Itoh, 1993). Itoh's (1993) model of explicit side-contracting can be viewed as an abstraction of the implicit side-contracting that was later modeled by Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). As Tirole (1992), writes: "[i]f, as is often the case, repeated interaction is indeed what enforces side contracts, the second approach [of modeling repeated interactions] is clearly preferable because it is more fundamentalist."

then motivate the employees to monitor one another and to encourage effort provision."

When organizations focus on team performance, they evaluate employees using different metrics. For instance, Cisco, General Electric, and Google have invested substantially to transform their performance evaluation systems from evaluating an individual employee's performance to gauging how effectively employees contribute to their teams (Deloitte, 2017). Individual's contributions to the team are often non-verifiable. (Gibbons and Henderson 2013 present examples of non-verifiable performance measures in both blue-color and white-color settings.) By definition, non-verifiable measures cannot be explicated contracted on.

This paper studies relational contracts between a principal and two agents based on non-verifiable performance measures in an infinitely repeated relationship. The non-verifiable performance measures in our model can be interpreted as the principal's imperfect assessment of each agent's contribution to the team. To analyze the relational contracts in the most tractable setting, our main model does not include any verifiable performance measures (e.g., group, divisional, or firm-wide earnings). (We introduce such a verifiable measure as an extension in Section 5.)

All players in our model share the same expected contracting horizon (discount rate). Nevertheless, the players may differ in their relative credibility because of other features of the model such as the loss to the principal of forgone productivity. In determining the optimal incentive arrangement, both the common discount rate and the relative credibility of the principal and the agents are important. In particular, when the principal's ability to commit is strong, the optimal contract emphasizes team incentives. Joint performance evaluation (*JPE*), which rewards the agents when both are judged to have supplied high effort, emerges as an optimal means of setting the stage for the agents to mutually monitor each other. *JPE* provides the agents with incentives to monitor each other and a means of disciplining each other by creating a punishing equilibrium – a stage game (one shot) equilibrium with lower payoffs than the agents obtain on the equilibrium path. As the principal's reneging constraint becomes a binding constraint, rewarding joint poor performance via a positive bonus floor can be optimal because it helps the principal maintain a strategic complementarity in the agents' payoffs, which is efficient in providing incentives for mutual monitoring.[4] The alternative is to use relative performance evaluation (*RPE)* to partially replace mutual monitoring/team incentives with individual incentives.[5] The problem with *RPE* is that it undermines mutual monitoring by reducing the strategic complementarity in the agents' payoffs. Therefore, if the principal attempts to replace team incentives with individual incentives using *RPE* when her credibility is limited, even more individual incentives are needed to makeup for the reduced team incentives.

Discretionary increases to bonus pools are considerably more common than discretionary decreases (Murphy and Oyer, 2006). A common criticism of this asymmetric treatment is that managers get their bonuses too often – "pay for pulse" rather than "pay for performance." Our results suggest a different interpretation. Using a floor and allowing for discretionary increases but not discretionary decreases can be optimal as part of an incentive structure that fosters mutual monitoring/team incentives.

Mutual monitoring/team incentives are not always optimal (or even feasible). When individual rather than team incentives are optimal, the principal would use *RPE* if she did not have to prevent tacit collusion between the agents. The unappealing feature of *RPE* is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has them alternating between (*work, shirk*) and (*shirk, work*).[6] (We use the term mutual monitoring to refer to relational contracts that benefit the principal and collusion to refer to the relational contracts between the agents that harm the principal.) The collusion threat is severe if the agents' ability to commit is strong, in which case paying for poor performance via a bonus floor is again optimal because it creases a

[4]Strategic complementarities, which mean that each agent's marginal return to his action is increasing in the other agent's action, have been widely studied in economics (e.g., Milgrom and Roberts, 1990).

[5]Here, the potential benefit of *RPE* is not in removing noise from the agents' performance evaluation as in Holmstrom (1979) but rather in relaxing the principal's reneging constraint, since *RPE* has the principal making payments to only one of the two agents.

[6]Even in one-shot principal-multi-agent contracting relationships, the agents may have incentives to collude on an equilibrium that is harmful to the principal (Demski and Sappington, 1984; Mookherjee, 1984).

strategic independence in payoffs, which is a desirable property of collusion-proof incentives.

We then introduce a verifiable joint performance measure in an extension section. In order to keep the analysis tractable, we assume that each agent's marginal contribution to the verifiable joint performance measure does not depend on the other agent's effort choice. Here, it is optimal to condition the bonus floor on the realized verifiable measure. When the binary verifiable measure is low, the bonus floor is always zero. When the verifiable measure is high, a positive bonus floor can arise to foster mutual monitoring or prevent collusion, demonstrating the robustness of the role for bonus floors developed in our main model (without the verifiable joint performance measure). It can also be optimal for the principal to reward the agents even if the verifiable team measure is low in order to foster the agents' mutual monitoring. The broader message that emerges is that "paying for poor performance" – either verifiable team performance or non-verifiable individual performance – can be optimal in a relational (self-enforcing) contracting setting because it creates the trust needed for the principal to tailor other promised payments to motivate mutual monitoring.

The existing relational contracting literature has explored the role repeated interactions can have in facilitating trust and discretionary rewards based on non-verifiable performance measures (e.g., Baker, Gibbons, and Murphy, 1994; Ederhof, Rajan and Reichelstein, 2011), but this literature has mostly confined attention to single-agent settings.[7] In this paper, we explore optimal discretionary rewards based on non-verifiable individual performance measures in a multi-period, principal-multi-agent model. Like our paper, Che and Yoo (2001) study an infinitely repeated relationship in which two agents can mutually monitor each other's effort. The difference between our paper and Che and Yoo (2001) is that, since the performance measures are *non-verifiable* in our model, the principal too has to rely on a relational contract.

Kvaløy and Olsen (2006) also study team incentives in a multi-agent relational contracting setting. The most important difference between our paper and theirs is that they do not allow

[7]One exception is Levin (2002), which examines the role multilateral contracting can have in bolstering the principal's ability to commit if the principal's reneging on a promise to any one agent means she will loose the trust of both agents. We make the same assumption. Levin does not study relational contracting between the agents.

for a commitment to a joint bonus floor, which is the focus of our paper. While the principal cannot write a formal contract on the non-verifiable performance measures, it seems difficult to rule out contracts that specify a joint bonus floor. After all, this idea is at the heart of bonus pools.[8] In a single period version of our model, the principal's ability to make promises is so limited that she would renege on any promise to pay more than the minimum, so the joint bonus floor becomes a non-discretionary total payout bonus pool (as in Macleod, 2003; Rajan and Reichelstein 2006; 2009). Under repeated play, relational contracts allow the principal to use discretion not only in allocating the bonus pool between the agents but also to increase its size above the joint bonus floor.

Our paper is also closely related to Baldenius, Glover, and Xue (2016). In their model, the principal perfectly observes the agents' actions, while our non-verifiable performance measures are imperfect. Because the principal observes the agents' actions, there is no role for mutual monitoring in their model. In contrast, mutual monitoring/team incentives is the primary focus of our paper. Their focus is on the agents' productive interdependency through a verifiable joint performance measure, while we have assumed away such productive interdependency for tractability. In particular, they show that the optimal contract often converts any productive interdependencies into overall strategic independency in payoffs to combat agents' collusion. When mutual monitoring is not optimal in our model, our results complement theirs in that strategic independence emerges as an optimal response to the agents' collusion threat even under imperfect non-verifiable performance measures (but without the productive interdependency). We view this a secondary contribution of our study.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 studies implicit side-contracting between the agents. Section 4 specifies the principal's optimization problem and characterizes the optimal contracts. Section 5 introduces a verifiable joint performance measure and demonstrates the robustness of the main results. Section 6

[8]Umbrella plans or "inside/outside" bonus plans are often used in practice to ensure the incentive compensation is tax deductible per IRS Rule 162(m). Effectively, the inside plan can be used to specify a bonus floor, while the board of a company has the discretion to raise the total bonus utill it reaches the outside plan – a bonus cap.

concludes.

2 Model

A principal contracts with two identical agents, $i = A, B$, to perform two independent and ex ante identical projects (one project for each agent) in an infinitely repeated relationship, where t is used to denote the period, $t = 1, 2, 3, \ldots$. All parties are risk neutral. Each agent chooses a personally costly effort $e^i \in \{0, 1\}$ in period t: the agent chooses either "*work*" ($e_t^i = 1$) or "*shirk*" ($e_t^i = 0$). Each agent's personal cost of *shirk* is normalized to be zero and of *work* is normalized to one. Each agent's effort e_t^i generates a stochastic output $x_t^i \in \{H, L\}$ in the current period, with $q_1 = \Pr(x_t^i = H|e_t^i = 1), q_0 = \Pr(x_t^i = H|e_t^i = 0)$, and $0 < q_0 < q_1 < 1$. (Whenever it does not cause confusion, we drop sub- and superscripts.) The outputs (x_t^i, x_t^j), which are also the performance measures, are individual rather than joint measures in the sense that agent i's effort does not affect agent j's probability of producing a high output. Throughout the paper, we assume each agent's effort is so valuable that the principal wants to induce both agents to *work* ($e_t^i = 1$) in every period. (Sufficient conditions are provided in the appendix.) The principal's problem is to design a contract that motives both agents to work in every period at the minimum cost.

Because of their close interactions, the agents observe each other's effort in each period. As in Che and Yoo (2001), we assume that the communication from the agents to the principal is blocked and, therefore, the outcome pair (x^i, x^j) is the only signal on which the agents' wage payment can depend. As in Kvaløy and Olsen (2006), we assume the performance measures (x^i, x^j) are unverifiable. For example, the performance measures can be client relations and business development for sales person or the balance between (long-term) research and (short-term) drug hunting for researchers at a pharmaceutical company. The principal cannot directly incorporate the performance measures into an explicit contract. Instead, the performance measures can be used in determining compensation only via a self-enforcing *relational* (implicit) contract. We will introduce a verifiable joint measure in Section 5.

The relational contract governs the parties' entire relationship, specifying the agents' actions in each period, the equilibrium payments the principal makes to the agents, and what the parties will do in retaliation if any of them reneges on their promises. At the beginning of each period, the principal promises the agents a wage scheme that depends on the non-verifiable performance measures and, hence, must be self-enforcing. For tractability, we assume the ex ante identical agents are offered the same wage schemes, i.e., symmetric contracts.[9] In addition, because the production technology is stationary and the principal induces high efforts in each period (a stationary effort policy), we know from Levin (2003) that the same wages will be offered in each period on the equilibrium path.[10] Since a stationary wage scheme is optimal, we omit the time subscript from wages.

Denote by $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$ the relational contract the principal promises the agents, where w_{mn} is the wage agent i expects to receive according to the principal's promise if his (performance) outcome is $x^i = m$ and his peer's outcome is $x^j = n$, with $m, n \in \{H, L\}$. (Recall that we can drop the agent superscript for symmetric contracts.) The agents are protected by limited liability—the wage transfer from the principal to each agent must be nonnegative:

$$w_{mn} \geq 0, \forall m, n \in \{H, L\}. \qquad \text{(Non-negativity)}$$

[9]Confining attention to symmetric contracts is a restrictive assumption in that asymmetric contracts can be preferred by the principal, as in Demski and Sappington's (1984) single period model. As we will show in Section 4, restricting attention to symmetric contracts greatly simplifies our infinitely repeated contracting problem by reducing the infinite set of possibly binding collusion constraints to two. Without the restriction to symmetric contracts, we know of no way to simplify the set of collusion constraints into a tractable programming problem.

[10]The basic idea is that the agent's incentives come from two sources: performance measure contingent payments in the current period and a (possible) change in continuation payoffs that depend on the current period's performance measures. Because of the limited liability constraints, even the lowest of these continuation payoffs must be nonnegative. Since all parties are risk neutral and have the same discount rate, any credible promise the principal makes to condition continuation payoffs on current period performance measures can be converted into a change in current payments that replicates the variation in continuation payoffs without affecting the principal's incentives to renege on the promise or violating the limited liability constraints on payments. The new continuation payoffs are functions of future actions and outcomes only, removing the history dependence. The new contract is a short-term one. If instead the effort level to be motivated is not stationary, then, of course, non-stationary contracts can be optimal.

We assume each agent's best outside opportunity provides him with a payoff of 0 in each period. Therefore, any contract that satisfies the limited liability constraints will also satisfy the agents' individual rationality constraints, since the cost of low effort is zero. The individual rationality constraints are suppressed throughout our analysis.

In addition to the implicit promises $\mathbf{w}$, we assume the parties can write an *explicit* contract as long as the explicit contract does not depend on the non-verifiable performance measures (x^i, x^j). This effectively limits the explicit contract to a *bonus floor* $\underline{w} \geq 0$ – a minimum *total* bonus to be paid to both agents in the current period independent of the non-verifiable performance measures. Given the multilateral nature of the contracting relationship, it seems difficult to rule out such explicit contracts, since they require only that a court be able to verify whether or not the contractual minimum $\underline{w}$ was paid out. It is easy to see that the minimum joint bonus $\underline{w}$ satisfies $\underline{w} \leq \min_{m,n \in H,L}\{w_{m,n} + w_{n,m}\}$ in any optimal contract.[11] That is, the bonus floor/explicit contract is enforced by the courts only off the equilibrium path.

Denote by $\pi(k, l; \mathbf{w})$ the expected wage payment of agent i if he chooses an effort level $k \in \{1, 0\}$ while the other agent j chooses effort $l \in \{1, 0\}$, assuming the principal honors the relational contract $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$. The discussion above implies that

$$\pi(k, l; \mathbf{w}) = q_k q_l w_{HH} + q_k(1 - q_l)w_{HL} + (1 - q_k)q_l w_{LH} + (1 - q_k)(1 - q_l)w_{LL}. \tag{1}$$

To motivate the principal to honor her implicit promises $\mathbf{w}$ with the agents, we consider the following (grim) trigger strategy played by the agents: both agents behave as if the principal will honor the implicit promises $\mathbf{w}$ until the principal reneges, after which the employment relationship reverts to punishment phase – the agents shirk in all future periods, and the principal offers them a fixed salary of zero. Such a grim trigger strategy is without loss of

[11] Suppose by contradiction that $\underline{w} > w_{m,n} + w_{n,m}$ when the agents' outcome pair is (m, n). Since the court will enforce $\underline{w}$, it will allocate the difference $\Delta = \underline{\mathrm{w}} - (w_{m,n} + w_{n,m})$ between the two agents according to a certain allocation rule. The principal can directly give the agents the same payments the courts would impose by increasing the payments so that $(w'_{m,n} + w'_{n,m}) = \underline{w}$. The principal can do better by optimizing over possible $(w'_{m,n} + w'_{n,m}) = \underline{w}$.

generality, since this is the harshest punishment the agents can impose on the principal. The principal will not renege if

$$\frac{2\left[q_1 H - \pi(1,1;\mathbf{w})\right] - 2q_0 H}{r} \geq \max_{m,n \in \{H,L\}} \{w_{mn} + w_{nm} - \underline{w}\}. \qquad \text{(Principal's IC)}$$

The Principal's IC constraint ensures the principal will abide by the promised wage scheme $\mathbf{w}$ rather than renege and payout the minimum joint bonus floor $\underline{w}$. The left hand side is the cost of reneging, which is the present value of the production loss in all future periods net of wage payment. The right hand side of this constraint is the principal's benefit of paying out only the minimum joint bonus. If instead a bonus floor is not allowed, the right hand of the principal's incentive constraint becomes $\max_{m,n}\{w_{mn} + w_{nm} - 0\}$, i.e., the principal can always breach her promise and pay zero to both agents in the current period.

All parties in the model share a common discount rate r, capturing the time value of money or the probability $\frac{1}{1+r}$ the relationship will continue at the end of each period (the contracting horizon). Denote by H_t the history of all actions and outcomes before period t, including whether the principle has ever reneged her implicit promises. Denote by P_t the public profile before period t, i.e., the history without the agents' actions. The principal's strategy is a mapping from the public profile P_t to period t wages. Each agent's strategy maps the entire history H_t to his period t effort choice. The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Among the large set of PBE's, we choose the one that is best for the principal subject to collusion-proofness. To be collusion proof, there can be no self-enforcing deviation the agents could adopt that improves their payoffs (at least strictly for one of the agents) from the (*work, work*) equilibrium.

3 Relational Contracting between the Agents

The fact that agents observe each other's effort choice, together with their multi-period relationship, gives rise to the possibility that they would use relational contracts to motivate each other to work (mutual monitoring) as in Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). Consider the following trigger strategy used to enforce (*work, work*): both agents play work until one agent i deviates by choosing shirk; thereafter, the agents play (*shirk, shirk*):

$$\frac{1+r}{r}\left[\pi(1,1;\mathbf{w})-1\right] \geq \pi(0,1;\mathbf{w})+\frac{1}{r}\pi(0,0;\mathbf{w}). \tag{Mutual Monitoring}$$

Such mutual monitoring requires two conditions. First, each agent's expected payoff from playing (*work, work*) must be at least as high as from playing the punishment strategy (*shirk, shirk*). In other words, both working must Pareto dominate both shirking from the agents' point of view in order for (*shirk, shirk*) to be perceived as a punishment. That is,

$$\pi(1,1;\mathbf{w})-1 \geq \pi(0,0;\mathbf{w}). \tag{Pareto Dominance}$$

Second, the off-equilibrium punishment strategy (*shirk, shirk*) must be self-enforcing in that playing (*shirk, shirk*) is a stage game Nash equilibrium. That is,

$$\pi(0,0;\mathbf{w}) \geq \pi(1,0;\mathbf{w})-1. \tag{Self-Enforcing Shirk}$$

As we will show in Section 4, contracts that motivate agents to mutually monitor each other are already collusion proof. Despite its desirable properties, mutual monitoring is not always optimal because of the Principal's IC constraint. In this case, the principal must instead ensure (*work, work*) is a stage game Nash equilibrium, i.e.,

$$\pi(1,1;\mathbf{w})-1 \geq \pi(0,1;\mathbf{w}). \tag{Static NE}$$

However, the Nash constraint may not be sufficient to motivate the agents to act as the principal intends because they may find tacit collusion more desirable. To sustain a collusive strategy, the agents would adopt a grim trigger strategy in which agent i punishes agent j for deviating from the collusive strategy by playing the stage game equilibrium that gives j the lowest payoff. As we show in the proof of Lemma 1, for any contract that satisfies (Static NE), the grim trigger strategy calls for the agents to revert to the stage game equilibrium (*work, work*) in all future periods if any agent deviates from the collusive strategy. That is, when the principal chooses to provide individual incentives rather than using mutual monitoring, playing *(work,work)* is the most severe punishment the agents can impose on each other for deviating a collusive strategy.

The principal needs to prevent all collusive strategies. Given the infinitely repeated relationship, the space of potential collusions between the two agents is also infinite. Nonetheless, Lemma 1 below shows that we can confine attention to two intuitive collusive strategies when constructing collusion-proof contracts. First, the contract has to satisfy the following condition to prevent the "*Joint Shirking*" strategy in which agents collude on playing (*shirk, shirk*) in all periods:

$$\pi(1,0;\mathbf{w}) - 1 + \frac{\pi(1,1;\mathbf{w}) - 1}{r} \geq \frac{1+r}{r}\pi(0,0;\mathbf{w}). \qquad \text{(No Joint Shirking)}$$

In addition, the contract has to satisfy the following condition to prevent "*Cycling*" collusive strategy in which agents collude on alternating between (*work, shirk*) and (*shirk, work*):

$$\frac{1+r}{r}\left[\pi(1,1;\mathbf{w}) - 1\right] \geq \frac{(1+r)^2}{r(2+r)}\pi(0,1;\mathbf{w}) + \frac{(1+r)}{r(2+r)}\left[\pi(1,0;\mathbf{w}) - 1\right]. \qquad \text{(No Cycling)}$$

The left hand side of the two constraints is the agent's expected payoff if he unilaterally deviates by choosing work when he is supposed to shirk in some period t and is then punished indefinitely with the stage game equilibrium of (*work, work*). The right hand side is the expected payoff the agent derives from the collusive strategy – either *Joint Shirking* or *Cycling.* Lemma 1 provides necessary and sufficient conditions for any contract that provides individual Nash incentives to be collusion proof. (Lemma 1 is borrowed from Baldenius, Glover, and Xue (2016) but is repeated here for completeness.)

Lemma 1 *For contracts that provide individual incentives and hence satisfy* (Static NE)*, the* *necessary and sufficient condition to be collusion proof is that: either both* (No Joint Shirking) *and* (No Cycling) *are satisfied, or both* (No Cycling) *and* (Pareto Dominance) *are satisfied.*

Proof. All proofs are provided in Appendix A. ■

The intuition for the lemma is that all other potential collusive strategies can only provide some period t' shirker with a higher continuation payoff than under *Joint Shirking* or *Cycling* if some other period t'' shirker has a lower continuation payoff than under *Joint Shirking* or *Cycling*. Hence, if the contract motivates all potential shirkers under *Joint Shirking* and *Cycling* to instead deviate to work, then so will the period t'' shirker under the alternative strategy.

4 The Principal's Problem and Its Solution

4.1 The Principal's Problem

The principal designs an *explicit* joint bonus floor $\underline{w}$ and an *implicit* wage scheme $\mathbf{w} = \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$ to ensure (*work, work*) in every period is a collusion-proof equilibrium. When designing the optimal contract, the principal can choose to motivate mutual monitoring between agents if it is worthwhile (and feasible). Alternatively, she can implement a static Nash equilibrium subject to collusion-proof constraints (which is always feasible). The following integer program summarizes the principal's problem. It is an integer program because the variable T (short for team incentives) takes a value of either zero or one: $T = 1$ means the principal designs the contract to induce mutual monitoring, while $T = 0$ represents individual incentives.

$$\textbf{Program P:} \qquad \min_{T\in\{0,1\},\underline{w}\geq 0,w_{mn}\geq 0} \pi(1,1)$$

$$\begin{aligned}
&s.t. \\
&\text{Principal's IC} \\
&T \times \text{Mutual Monitoring} \\
&T \times \text{Pareto Dominance} \\
&T \times \text{Self-Enforcing Shirk} \\
&(1-T) \times \text{Static NE} \\
&(1-T) \times \text{No Joint Shirking} \\
&(1-T) \times \text{No Cycling}
\end{aligned} \tag{2}$$

Two features of the program deserve further discussion. First, the team incentives case ($T = 1$) does not include any collusion-proof constraints, which will be shown to be without loss. The idea is, even without collusion-proof constraints, the solution under $T = 1$ is such that playing (*work, work*) in the repeated game is Pareto optimal for the agents and, hence, is collusion proof by definition. Second, while Lemma 1 specifies the necessary and sufficient condition for contracts providing individual incentives ($T = 0$) to be collusion proof, once we endogenize the choice of T, it is without loss of generality to incorporate only the sufficient condition that both (No Joint Shirking) and (No Cycling) are satisfied. This is because, under the alternative collusion proof condition shown in Lemma 1, that is, (Pareto Dominance) and (No Cycling), the individual-incentive case $T = 0$ results in a strictly smaller feasible set than the team-incentive case.[12] Intuitively, if the contract already satisfies (Pareto Dominance), the principal will be better-off by providing team incentives ($T = 1$) than by providing individual incentives that requires additional collusion-proof conditions.

[12] To see this, note that (Pareto Dominance) and (Static NE) together imply (Mutual Monitoring). This, however, means the $T = 0$ case has more constraints than the $T = 1$ case. (Note that the (Self-Enforcing Shirk) constraint never binds in the $T = 1$ case.)

The following lemma links the explicit joint bonus floor $\underline{w}$ and the relational contract $\{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$ and, therefore, further simplifies the program **P** by reducing the number of control variables.

Lemma 2 *It is optimal to set* $\underline{w} = \min_{m,n \in H,L}\{w_{m,n} + w_{n,m}\}$.

That is, the principal optimally sets the contractible joint bonus floor equal to the minimum total compensation specified by the principal's implicit contract with the agents. The result is fairly obvious. If the bonus floor is set lower than the minimum promised total compensation, then the principal can relax her *IC* constraint by increasing the bonus floor. If the bonus floor is greater than the minimum promised compensation, then the agents will use the courts to enforce the bonus floor. That is, the actual minimum compensation will not be the promised minimum but instead the bonus floor. Lemma 2 allows us to rewrite the (Principal's IC) constraint as follows:

$$\frac{2\left[q_1 H - \pi(1,1;\mathbf{w})\right] - 2q_0 H}{r} \geq \max_{m,n,m',n' \in \{H,L\}} \{w_{mn} + w_{nm} - (w_{m',n'} + w_{n',m'})\}.$$

As the reformulated Principal's IC constraint suggests, the explicit bonus floor in our model is equivalent to an explicit contract that specifies the range of possible payments and asks the principal to *self-report* the non-verifiable performance measures.[13] The courts are used to enforce payments that are consistent with some realizations of the performance measures but not verify the actual realizations, since they are non-verifiable by assumption.

4.2 Optimal contract

We solve Program **P** by the method of enumeration and complete the analysis in two steps. In the first step, we solve Program P while setting $T = 1$ and then $T = 0$, respectively. We then compare the solutions for each parameter region and optimize over the choice of T.

[13] Deb, Li, and Mukherjee (2016) uses a similar interpretation in studying relational contracts with subjective measures.

The following proposition presents the overall optimal contract as a function of the common discount rate r. That is, we express the results as a function of discount rate cutoffs, which are themselves functions of other exogenous parameters. Following the literature, we label a wage scheme as joint performance evaluation (*JPE*) if $w_{HH} \geq w_{HL}$ and $w_{LH} \geq w_{LL}$ and relative performance evaluation (*RPE*) if $w_{HL} \geq w_{HH}$ and $w_{LL} \geq w_{LH}$ (with one $>$). We say that a contract has a bonus pool (*BP*) feature if it specifies a positive total bonus floor, i.e., $\underline{w} > 0$. Our *BP*-type contracts can be thought of as discretionary bonus pools that allow the principal to pay the agents more than the contractually agreed upon minimum $\underline{w}$.

Proposition 1 *The overall optimal contract is:*

(i) Pure JPE for $r \in (0, r^A]$*: T=1 and* w_{HH} *is the only positive payment;*

(ii) BPC for $r \in (r^A, \min\{r^L, r^C\}]$*: T=1 and* $w_{HL} = 2w_{LL} = \underline{w} > 0$*;*

(iii) JPE for $r \in (\max\{r^L, r^A\}, r^B]$*: T=1,* $w_{HH} > w_{HL} > 0$ *and* $w_{LL} = \underline{w} = 0$*;*

(iv) BPI for $r \in (r^B, r^H]$*: T=0,* $2w_{LL} = \underline{w} > 0$*,* $w_{HL} = w_{HH} + w_{LL}$*;*

(v) RPE for $r \in (\{r^B, r^H\}, r^D]$*: T=0,* $w_{HL} > w_{HH} > 0$*,* $2w_{LL} = \underline{w} = 0$*;*

(vi) BPS for $r > \max\{r^H, r^D\}$*: T=0,* $w_{HH} > 0$*,* $w_{HL} = 2w_{HH} > 2w_{LL} = \underline{w} > 0$*.*

All contracts satisfy $2w_{HH} > 2w_{LL} = \underline{w}$ *and* $w_{LH} = 0$*. We provide closed-form expressions for all solutions and discount rate cutoffs in Appendix A.*

Team incentives ($T = 1$) are optimal when players are patient, i.e., r is not too large. Denote by $U_{k,l} = \pi(k, l; \mathbf{w}) - C(e^i = k)$ agent i's expected utility if he chooses an effort $k \in \{1, 0\}$ while agent j chooses effort $l \in \{1, 0\}$. It is easy to verify that all contracts that induce team incentives create strategic payoff complementarity, i.e., $U_{1,1} - U_{0,1} > U_{1,0} - U_{0,0}$, denoted by the "$C$" in the *BP*-type contract BPC. In contrast, the *BP*-type contracts under individual incentives (Part iv and Part vi of Proposition 1) generate either strategic payoff independence or substitutability (denoted by "I" and "S", respectively).

It is not surprising that a bonus pool type contract BPS is optimal for large discount rates, since variation in the payments must be limited when the principal is sufficiently impatient.

In fact, BPS converges to a bonus pool with a fixed size as the discount rates $r \to \infty$. This coincides with the traditional view that bonus pools without discretion in determining the total payout is the only self-enforcing compensation in a one-shot game. The repeated relationship in our model gives the principal discretion to raise the size of the bonus pool from the bonus floor. We next provide necessary and sufficient conditions for such *BP*-type contracts to arise for intermediate discount rates.

Corollary 1 *(i) The interval $r^A < r \leq \min\{r^L, r^C\}$ over which BPC is optimal is non-empty if and only if $H \leq H^*$ and $q_0 > 1 - q_1$. (ii) The interval $r^B < r \leq r^H$ over which BPI is optimal is non-empty if and only if $q_1 > \frac{1}{2}$ and $H \leq H^{**}$, where H^* and H^{**} are characterized in Appendix A.*

To understand Part (i) of the corollary, it is helpful to analyze what the principal can do upon loosing her credibility to promise *Pure JPE*. She can either set a positive bonus floor $\underline{w} = 2w_{LL}$ as in *BPC*, which allows her to foster team incentives by further increasing w_{HH}. Alternatively, the principal could substitute individual incentives for team incentives by paying $w_{HL} > 0$ and holding the bonus floor $\underline{w} = w_{LL}$ at 0 (as in *JPE*), i.e., she could increase her reliance on relative performance evaluation. While rewarding joint poor performance w_{LL} through the positive bonus floor keeps the focus on team incentives, it has the cost of rewarding for performance that has the lowest likelihood ratio $\frac{q_0}{q_1}$. The condition $q_0 > 1 - q_1$ in the corollary above limits the opportunity cost of rewarding for poor performances, while the condition $H \leq H^*$ implies that the principal's credibility is limited, creating a demand for a *BP*-type contract. The discount rate region over which the principal can commit to the ideal *Pure JPE* contract expands for higher H. If H is sufficiently large ($H > H^*$), the principal's credibility is so strong that by the time she looses her credibility at r^A, the agents have already lost their patience to mutually monitor each other. Without the benefit of mutual monitoring, rewarding for poor performances w_{LL} via a *BP*-type contract is not optimal in the team incentive ($T = 1$) region of discount rates.

In the individual incentive region ($T = 0$) of discount rates, Corollary 1 - Part (ii) states that using the bonus floor to create strategy payoff independence is optimal when the agents' ability to enforce collusion is strong. The condition $q_1 > \frac{1}{2}$ in Corollary 1 is intuitive: the agents' ability to collude on the *Cycling* collusive strategy is stronger for higher q_1, because a higher q_1 increases the probability that the (only) working agent will collet the payment w_{HL}. As discussed previously, a higher output H strengthens the principal's credibility relative to the agents. If the condition $H \leq H^{**}$ is violated, the principal's credibility is so strong that the agents' ability to collude on the side-contract is weak by the time the principal looses her credibility at r^B, making RPE optimal instead.

The following numerical examples are intended to provide an intuitive interpretation of the conditions given in Proposition 1 and Corollary 1. Fix $q_0 = 0.53$ and $q_1 = 0.75$ for all examples to ease comparison. In the first example shown in Figure 1, set $H = 200$. The principal's ability to commit is high because the expected production she forgoes after reneging is large. In comparison, the agents' ability to enforce their relational contract – whether it is mutual monitoring or collusion – is limited, so relational contracting between the two agents is never the driving determinant of the form of the optimal compensation arrangement. Once the discount rate becomes large enough ($r > 7.3$) that *Pure JPE* is not feasible, the agents ability to make commitments is so limited that the principal optimally substitutes individual incentives for team incentives, making JPE optimal. As the discount rate becomes even larger ($r > 8.9$), the principal switches entirely to individual incentives (i.e., $T = 0$) and RPE rather than BPI is optimal because collusion is not costly to deal with. Once the discount rate is large enough ($r > 10.5$), the optimal contract BPS converges to a fixed bonus pool as r becomes arbitrarily large.

In the second example shown in Figure 2, $H = 100$. The principal's ability to commit is low relative to the agents', so the benefit of having a positive bonus floor (i.e., $2w_{LL} > 0$) in dealing with the agents' relational contracting (either mutual monitoring or collusion) can outweigh its cost of paying for joint bad performance. In particular, as the principal looses

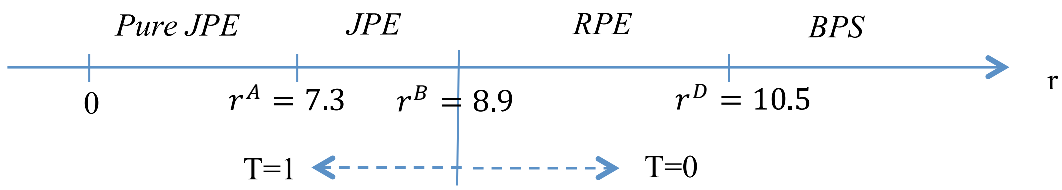

Figure 1: Optimal contract when the principal's relative credibility is *high* ($H = 200$).

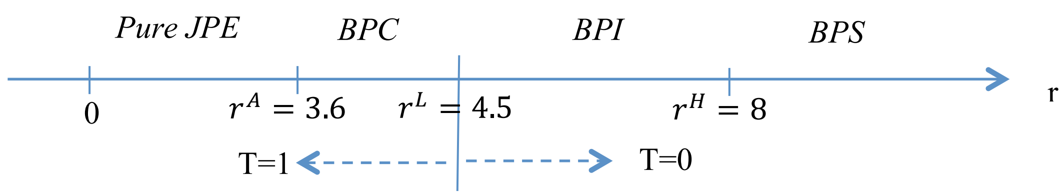

Figure 2: Optimal contract when the principal's relative credibility is *low* ($H = 100$).

her credibility in honoring *Pure JPE* at $r = 3.6$, the agents' ability to make commitments is still strong, making mutual monitoring valuable and *BPC* optimal. Individual incentives ($T = 0$) are optimal for $r > 4.5$, and *BPI* that creates strategic payoff independence is optimal because its benefit in upsetting collusion. For $r > 8$, the principal's ability to commit is so limited that *BPS* is the only feasible solution.

To further illustrate the second case (Figure 2), we present the optimal contract and the payoff matrix of the stage game of the two bonus pool type contract for the intermediate discount rate r. For $r = 4$, *JPE* with $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\} = (0, 0, 3.84, 4.66)$ is feasible but is not optimal. Instead, *BPC* is the optimal wage scheme with $\mathbf{w} = (0.68, 0, 1.36, 5.35)$. The stage game payoff matrix follows.

BPC ($r = 4$)

A/B	0	1
0	(1.99,1.99)	(2.39,1.69)
1	(1.69,2.39)	(2.31,2.31)

The benefit of free-riding of 2.39 - 2.31 = 0.08 is exactly equal to the punishment of reverting to the stage game Nash equilibrium of (2.31 - 1.99)/4 = 0.08.

For $r = 5$, team incentives are no longer optimal. The optimal means of preventing collusion is *BPI* with $\mathbf{w} = (1.09, 0, 5.82, 4.73)$. The payoff matrix follows.

BPI $(r = 5)$

A/B	0	1
0	(3.02,3.02)	(2.78,3.06)
1	(3.06,2.78)	(2.82,2.82)

The principal provides any shirking agent with a benefit of 0.04 for instead working and upsetting collusion on either *Joint Shirking* or *Cycling.* It is easy to verify that the shirking agent's continuation payoff under *Joint Shirking* of $\frac{3.02}{5} = 0.604$ is the same as his continuation payoff under *Cycling.* Under the equilibrium strategy (*work,work*), each agent's continuation payoff is $\frac{2.82}{5} = 0.564$. Therefore, the difference in continuation payoffs of $0.604 - 0.564 = 0.04$ is exactly equal to the benefit an agent receives for upsetting collusion by playing *work* instead of *shirk* in the current period. That is, both (No Joint Shirking) and (No Cycling) hold as equality – a feature of the contract that creates payoff strategic independence.

4.3 Optimal contract without bonus floor

To better understand the importance of the bonus floor in our model, we also derive the optimal contract assuming, as in Kvaløy and Olsen (2006), the principal cannot commit to a joint bonus floor. That is, $\underline{w} = 0$ by assumption. We present optimal contracts under this alternative assumption in Proposition 2 and compare it to the optimal contracts in Proposition 1 in Corollary 2.

Proposition 2 *When the principal cannot commit to a joint bonus floor, the optimal contract is (i) Pure JPE for $r \in (0, r^A]$, (ii) JPE for $r \in (r^A, r^B]$, (iii) RPE for $r \in (r^B, r^D]$, and infeasible otherwise.*

Corollary 2 *The ability to commit to a positive bonus floor $\underline{w} > 0$ results in*

(i) more relational contract in general:

- *when* $\underline{w} = 0$ *by assumption, there is no feasible solution for* $r > r^D$*;*
- *when* $\underline{w} \geq 0$*, relational contract is feasible for all* r *otherwise.*

(ii) more team incentives:

- *when* $\underline{w} = 0$ *by assumption, team incentives are optimal if and only if* $r < r^B$;
- *when* $\underline{w} \geq 0$*, team incentives are optimal if and only if (1)* $r < r^B$ *or (2)* $r < \min\{r^L, r^C\}$ *in the case of* $r^L \geq r^B$*.*

(iii) more strategic payoff independence $\Delta U = 0$*:*

- *when* $\underline{w} = 0$ *by assumption,* $\Delta U = 0$ *holds at the singular discount rate* r^B*;*
- *when* $\underline{w} \geq 0$*,* $\Delta U = 0$ *is optimal for a range of discount rates, all with* $\underline{w} > 0$*.*

The optimal contract essentially reproduces the results of Kvaløy and Olsen (2006).[14] Part (i) of Corollary 2 is straightforward, as the ability to commit to a joint bonus floor strengthens the principal's credibility. Inspecting Program **P** shows that the joint bonus floor relaxes the (Principal's IC) constraint in both the team incentive case and individual incentive case, it is unclear, a priori, whether such a commitment results in more or less team incentives once team incentives is a choice variable. Part (ii) of Corollary 2 shows that the region in which team incentives are optimal expands once the bonus floor is introduced. Part (iii) highlights the role of the bonus floor in combating the agents' possible collusion. When the principal cannot commit to such a bonus floor, JPE and RPE of Proposition 2 converge to independent performance evaluation (i.e., $w_{HH} = w_{HL} > 0$, and $w_{LH} = w_{LL} = 0$) at the singular discount rate r^B, creating strategic independence $\Delta U = 0$. As we have shown in Proposition 1 and Corollary 1, when the principal can commit to a joint bonus floor, BPI (with a bonus floor) creates $\Delta U = 0$ and is optimal for a range of discount rates r even though contracts without positive bonus floors could be used to prevent tacit collusion.

[14] We say "essentially" because they restrict attention to stationary strategies, while we do not.

In Appendix B, we study the other difference between our model and Kvaløy and Olsen (2006) – their restriction to stationary collusion strategies. In sum, Appendix B and Proposition 2 (and its corollary) demonstrate that the important qualitative differences between our results and theirs are due to the principal's ability to commit to a bonus floor in our model.

5 Incorporating a Verifiable Joint Performance Measure

In a typical bonus pool arrangement, the size of the bonus pool is based, at least in part, on a verifiable joint performance measure such as group, divisional, or firm-wide earnings (Eccles and Crane, 1988). Suppose that such an objective measure $y \in \{H, L\}$ exists and define $p_1 = \Pr(y = H | e^A = e^B = 1)$, $p = \Pr(y = H | e^A \neq e^B)$, and $p_0 = \Pr(y = H | e^A = e^B = 0)$ with $p_1 > p > p_0$. As before, agent j's non-verifiable performance measure is $x_j \in \{0, 1\}$, and $q_1 = \Pr(x^j = 1 | e^j = 1)$ and $q_0 = \Pr(x^j = 1 | e^j = 0)$. Denote by $\mathbf{w} \equiv \{w^y_{mn}\}$ the combined explicit/implicit contract the principal offers agent i, if the verifiable team measure is $y \in \{H, L\}$ and agent i and j's non-verifiable individual measures are m and n, respectively. The only possibility of the principal reneging is on the implicit contract that depends on the non-verifiable individual measures. Assuming the principal honors her promise truthfully, we can rewrite agent i's expected wage payment if he chooses effort k and agent j chooses l as:

$$\pi(k, l; \mathbf{w}) = \mathbb{E}_{y \in \{H,L\}} \left[q_k q_l w^y_{HH} + q_k(1 - q_l) w^y_{HL} + (1 - q_k) q_l w^y_{LH} + (1 - q_k)(1 - q_l) w^y_{LL} \right], \quad (3)$$

where the expectation is taken over the team measure y given effort k, l.

The principal's problem is same as Program **P** in the main model after substituting the expected payment $\pi(k, l; \mathbf{w})$ everywhere by (3) shown above. The other change to Program

$\mathbf{P}$ is to redefine the *Principal's IC* for each realized team measure y as follows:

$$\frac{2\left[\frac{p_1}{\min\{p_1-p,\,p-p_0\}}-\pi(1,1;\mathbf{w})\right]}{r}\geq \max_{m,n\in\{H,L\}}\{w^y_{mn}+w^y_{nm}-w^y_{m'n'}-w^y_{n'm'}\},\ \forall y\in\{H,L\}. \tag{Principal's new IC}$$

Introducing the objective team measure y qualitatively changes the fallback contract triggered by the principal's reneging. In the main model, the principal looses the agents' trust upon reneging and expects the agents to shirk in perpetuity off the equilibrium. With a verifiable team measure, the reneging principal can continue to induce (*work, work*) off the equilibrium by replying solely on the verifiable team measure. That is, the reneging principal can ensure (*work, work*) as the *unique* stage game equilibrium, which requires paying $\frac{p_1}{\min\{p_1-p,\,p-p_0\}}$ to each agent.[15]

Before we characterize the solution, we first demonstrate the robustness of our results in the main model via a numerical example. We use the same parameters as in Figures 1 and 2 to ease comparison: $q_0 = 0.53$ and $q_1 = 0.75$. The objective measure y satisfies $p_1 = 0.6, p = 0.5$, and $p_0 = 0.4$ in Figure 3. In this example, the optimal contract satisfies $w^L_{mn} = 0, \forall m, n$, i.e., the payment is zero whenever the team measure is low. Given the high team measure $y = H$, the way the optimal contact varies with the discount rate r is qualitatively the same as in Figure 2. The moment the principal cannot credibly commit to *Pure JPE* (only $w^H_{11} > 0$) at $r = 0.8$, she starts to pay for *poor individual performances* w^H_{00} in order to continue raising w^H_{11}, which has the benefit of fostering the agents' mutual monitoring. Individual incentives are optimal for higher discount rates $r > 3.6$, and BPI is optimal for $r < 8$ because it has the benefit of creating strategic independence, a desirable feature in combating agents' collusion. We use the names developed in the main model to label contracts in Figure 3, not only to facilitate comparison with Figure 2, but also because the optimal payments w^H_{mn} (i.e., given the team output $y = H$) are qualitatively similar in terms of having a positive bonus floor

[15]We assume in this section that the agent's effort is valuable enough so that, following reneging (out-of-equilibrium), the principal chooses to induce *(work, work)* by using the verifiable measure. Denote by X the incremental value that a working agent brings to the principal than a shirking agent. Our out-of-equilibrium specification requires $(q_1 - q_0)X \geq \frac{p_1}{\min\{p_1-p,\,p-p_0\}}$.

and their strategic payoff interdependencies.

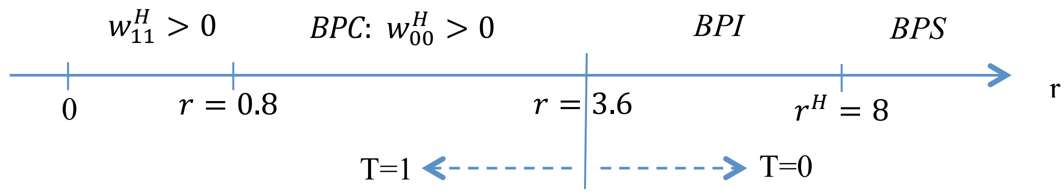

Figure 3: Optimal contract with a team measure ($p_0 = 0.4, p = 0.5, p_1 = 0.6$).

Continuing with the same example but lowering the informativeness of the verifiable joint performance measure so that $p_1 = 0.55, p = 0.5$, and $p_0 = 0.45$. Figure 4 demonstrates a new type of "pay for poor performance." When the principal cannot credibly commit to relying solely on w_{11}^H at $r = 1.3$, she starts to pay for *poor team performance* w_{11}^L as long as both agents' non-verifiable measures are high. For even higher discount rates $r > 1.7$, the principal rewards both for poor team measure w_{11}^L and for poor individual measures w_{00}^H to foster the agents' incentive to mutually monitor each other. For $r > 3.3$, the optimal contract instead provides individual incentives ($T = 0$), in which case a low team measure is never rewarded, i.e., $w_{mn}^L = 0$, as would be true in a static model.

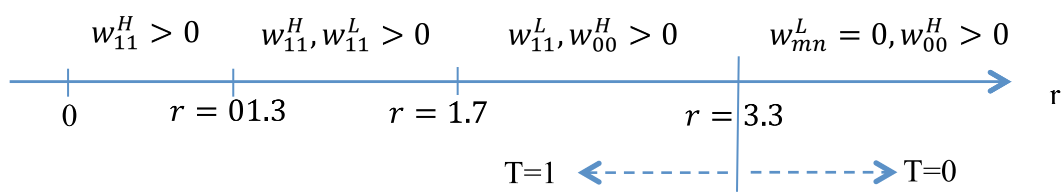

Figure 4: Optimal contract with a less informative team measure ($p_0 = 0.45, p = 0.5, p_1 = 0.55$).

The reminder of the extension characterizes the optimal solution analytically. To maintain tractability, we further assume that the verifiable team measure satisfies $p_1 = \frac{1}{2} + P$, $p = \frac{1}{2}$, and $p_0 = \frac{1}{2} - P$ for $P \in [0, 0.5]$, while each agent's subjective performance measure satisfies $q_1 = \frac{1}{2} + Q$ and $q_0 = \frac{1}{2} - Q$ for $Q \in [0, 0.5]$. That is, the informativeness of the team measure y (the individual measure x_j) are captured by the single parameter P (Q). Lemma 3 states that paying for poor team measure can be part of the optimal contract under team incentives.

Lemma 3 *Given team incentives T=1, the optimal contract for* $r \leq r1$ *has* w_{11}^H *as the only positive payment. As* r *increases from* $r1$*, there exists a unique* P^* *such that the principal*

- *pays for "poor team performance"* ($w_{11}^L > 0$) *while keeping* $w_{10}^H = 0$ *for* $P \leq P^*$;
- *increases* w_{10}^H *without paying for poor team performance (i.e.,* $w_{mn}^L = 0$*) for* $P > P^*$.

Part (ii) is at odds with the insights derived from standard single-period models of individual incentives (e.g., Holmstrom, 1979). The standard informativeness argument suggests that paying for w_{10}^H provides stronger (individual) incentive than rewarding w_{11}^L for *any* $P > 0$. However, compared to paying $w_{10}^H > 0$ (a form of RPE), paying for "poor" team performance w_{11}^L has the benefit of fostering mutual monitoring. The principal trades off the benefit of fostering mutual monitoring (via w_{11}^L) against the benefit of rewarding a measure with a higher likelihood ratio (w_{10}^H). Since the advantage of w_{10}^H in likelihood ratios is stronger when the team measure y is more informative (higher P), Part (ii) states that the benefit of focusing on team incentives dominates if the likelihood ratio loss is not particularly strong. Given Lemma 3 and our focus on mutual monitoring, we confine attention to $P < \bar{\Delta} \doteq \frac{Q}{2(1+Q)}$, which is sufficient to ensure that paying for poor team performance (i.e., Part ii of the lemma) is always part of the optimal contract.[16] Lemma 4 summarizes features of optimal contracts given individual incentives $T = 0$.

Lemma 4 *Given individual incentive T=0, the optimal contract*

i. has neither the principal's IC nor the agents' collusion constraints binding for low r.

ii. creates strategic payoff independence $\Delta U = 0$ *for intermediate discount rates* $r \in [\underline{r}, \bar{r}]$*, which is a non-empty interval if and only if the fall-back objective measure is not too noisy (hence the principal's credibility is not too strong), i.e.,* $P \geq \underline{\Delta}$.[17]

[16] We impose this sufficient condition to maintain tractability when we endogenize the choice between team and individual incentives. Each program has 15 constraints (excluding 8 non-negativity constraints).

[17] The threshold $\underline{\Delta}$ is implicitly characterized in the Appendix. A sufficient condition is $P > 0.025$.

iii. *creates payoff substitutability $\Delta U < 0$ for large discount rates. As $r \to \infty$, the contract converges to a conditional symmetric bonus pool: $w^L_{mn} = 0$ and $w^H_{11} = w^H_{00} = \frac{1}{2} w^H_{10}$.*

The condition $P \geq \underline{\Delta}$ in Part (ii) of the lemma reinforces another finding in the main model: the optimal contract depends on the principal's credibility *relative to* agents'. To see this, note that the principal's credibility is stronger the lower P is because, upon reneging, the principal's off-equilibrium option of relying solely on the objective measure is costlier for smaller P. The worse off-equilibrium contract, in turn, strengthens the principal's credibility to honor the subjective measures on equilibrium, echoing a result in Baker, Gibbons, and Murphy (1994). If $P \geq \underline{\Delta}$ is violated, the principal's credibility will be so strong that by the time she looses her credibility, the agents are already so impatient that their ability to collude is weak; in this case, creating $\Delta U = 0$ to combat collusion is unnecessary.

Proposition 3 endogenizes the choice of team incentives $T = \{0, 1\}$ and characterizes the optimal contract.

Proposition 3 *Assuming $\underline{\Delta} \leq P \leq \bar{\Delta}$, the overall optimal contract depends on r as follows:*

(i) $T = 1$ for $r \leq r1$, $w^H_{11} > 0$ is the only positive payment, i.e., Pure JPE.

(ii) $T = 1$ for $r \in (r1, r2]$. As r increases, w^H_{11} decreases while the payment for poor team performance w^L_{11} increases untill $w^L_{11} = w^H_{11}$ at $r = r2$. All other payments are zero.

(iii) $T = 1$ for $r \in (r2, \tilde{r}]$. As r increases, w^H_{10} increases from zero and w^L_{11} decreases.

(iv) $T = 0$ for $r \in (\tilde{r}, r3]$, the contract creates strategic payoff independence, and $w^H_{00} > 0$.

(v) $T = 0$ for $r > r3$, the contract creates payoff strategic substitute $\Delta U < 0$ and converges to a conditional symmetric bonus pool as $r \to \infty$.

We characterize the r-cutoffs in Appendix A.

Comparing Proposition 3 to Proposition 1 confirms the insights derived without the team measure. In both cases, team incentives are optimal if and only if the discount rate r is not

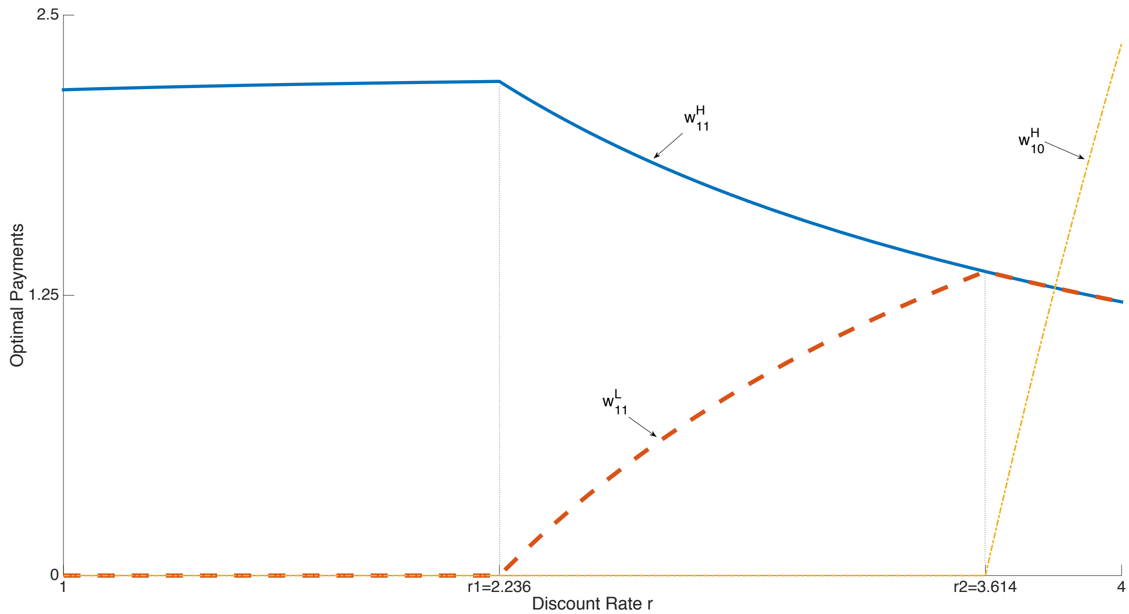

Figure 3: Optimal contract with a verifiable team measure $y = \{H, L\}$. ($Q = 0.4, P = 0.1$)

too large, and the limiting contract is either *Pure JPE* (for $r \to 0$) or the stage game bonus pools (for $r \to \infty$). More central to our main message, Part (ii) of both propositions share the feature that paying for poor performance is used to foster mutual monitoring even though it is feasible to increase other payments with higher likelihood ratios, which would be optimal in standard static models. Part (iv) of the two propositions are similar in that both pay for poor individual measures (either w_{LL} or w_{LL}^H) to facilitate strategic independence, which is desirable in combating collusion.

Figure 3 illustrates the provision of *team* incentives – the focus of the paper – via an example in which $Q = 0.4$ and $P = 0.1$. (Given $Q = 0.4$, the assumption $P \in [\underline{\Delta}, \bar{\Delta}]$ in Proposition 3 is $P \in [0.0048, 0.143]$.) *Pure JPE* (only $w_{11}^H > 0$) is optimal if the principal's credibility is strong enough, i.e., $r < 2.236$. As the principal cannot honor *Pure JPE* for higher r, she reduces her reliance on the verifiable measure, while increasing her reliance on the non-verifiable individual measures. In fact, the optimal contract completely ignores the team measure (i.e., $w_{mn}^H = w_{mn}^L$) and provides team incentives solely through the non-verifiable measures at $r = 3.614$. For $r > 3.614$, the principal starts to increase w_{10}^H because the agents' ability to enforce mutual monitoring is weak when the discount rate is high. The principal eventually tunes down team incentives for $r \in [3.614, 4.13]$ by increasing w_{10}^H and lowering w_{11}^L and switches to individual incentives ($T = 0$) for $r > \tilde{r} = 4.13$.

In summary, "paying for poor performance" – either a discretionary increase above the zero floor when the verifiable measure is low but both non-verifiable measures are high or a non-discretionary adherence to the positive bonus floor when the verifiable measure is high – can be part of the optimal relational contract used to foster mutual monitoring. Again, there is no role for mutual monitoring in Baldenius et al., so neither of these results arise in their model.

6 Conclusion

Our model makes a number of simplifying assumptions: binary actions/performance measures, risk neutral agents, symmetric contracts, and communication from the agents to the principal is blocked. Although these assumptions are standard in the relational contracting literature, they are nonetheless limiting. Of these assumptions, we view the restriction to symmetric contracts and blocked communication as most limiting. These limitations provide opportunities for future research.

A natural starting point is to relax these assumptions in a model of explicit contracting between the principal and the agents. With agents who have different roles such as top executives, static models predict the agents would be offered qualitatively different compensation contracts. Yet, in practice, the compensation of executives are often similar. As an extreme example, Apple pays the same base salary, annual cash incentive, and long-term equity award to each of its executive officers other than its CEO.[18] [19] We conjecture that compensating productively different agents similarly can be rationalized by a team-based model of dynamic incentives (with a low discount rate/long expected tenure).

Another natural avenue for future research is peer evaluation (or 360-degree evaluation) under relational side-contracting. We assumed communication from the agents to the principal is blocked, as in Arya, Fellingham, and Glover (1997), Che and Yoo (2001), and Kvaløy and Olsen (2006). Ma (1988) studies a single-period, multi-agent model of moral hazard without side-contracting under unblocked communication (peer reports) from the agents to the principal. In Ma (1988), if the agents perfectly observe each other's actions and the principal contracts on the agents' peer reports, the principal can implement the first-best if every action pair induces a unique distribution over performance measures.[20] Baliga and Sjostrom (1998) study a similar setting but allow for explicit side-contracting between the agents and show that side-contracting greatly limits the role of peer reports. Deb, Li, and Mookerherjee (2016) study peer evaluation under relational contracting between the principal and the agents but without (relational or explicit) side-contracts between the agents, showing that peer evaluations are sparingly incorporated in determining compensation. As far as we are aware, there have been no studies of peer evaluation under implicit side-contracting/collusion. Collusion between the principal and one of the agents may also arise.

If we apply the team-based model to thinking about screening, we might expect to see compensation contracts that screen agents for their potential productive complementarity, since productive complementarities reduce the cost of motivating mutual monitoring. A pro-

[18]According to Apple Inc.'s 2016 Proxy Statement: "Our executive officers are expected to operate as a team, and accordingly, we apply a team-based approach to our executive compensation program, with internal pay equity as a primary consideration. This approach is intended to promote and maintain stability within a high performing executive team, which we believe is achieved by generally awarding the same base salary, annual cash incentive, and long-term equity awards to each of our executive officers, except [CEO] Mr. Cook."

[19]For related observations about cash bonuses paid to CEOs that are similar to bonuses paid to other executives, see Core, Guay, and Verrecchia (2003) and Guay, Kepler, and Tsui (2016), which finds that, for approximately 75% of their sample, the CEO and the fifth-highest-paid executive have an identical set of performance targets for their cash bonuses.

[20]Towry (2003) initiated an important experimental literature that contrasted the Arya, Fellingham, and Glover (horizontal) and Ma (vertical) views of mutual monitoring, emphasizing the important role of team identity.

ductive substitutability (e.g., hiring an agent similar to existing ones when there are overall decreasing returns to effort) is particularly unattractive, since the substitutability makes it appealing for the agents to tacitly collude on taking turns working. We might also expect to see agents screened for their discount rates. Patient agents would be more attractive, since they are the ones best equipped to provide and receive mutual monitoring incentives. Are existing incentive arrangements such as employee stock options with time-based rather than performance-based vesting conditions designed, in part, to achieve such screening?

Even if screening is not necessary because productive complementarities or substitutabilities are driven by observable characteristics of agents (e.g., their education or work experience), optimal team composition is an interesting problem. For example, is it better to have one team with a large productive complementarity and another with a small substitutability or to have two teams each with a small productive complementarity?

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Appendix

Proof of Lemma 1. We start with a preliminary result that specifies the harshest punishment supporting the collusion.

Claim: It is without loss of generality to restrict attention to collusion constraints that have the agent punishing each other by playing $(1, 1)$ (i.e., *work, work*) forever after a deviation from the collusion strategy.

Proof. We first argue that the off-diagonal action profiles $(0, 1)$ and $(1, 0)$ cannot be punishment strategies harsher than $(1, 1)$. We illustrate the argument for $(0, 1)$; similar logic applies to $(1, 0)$:

		Agent B	
		L	H
Agent A	L	U_{00}, U_{00}	U_{01}, U_{10}
	H	U_{10}, U_{01}	U_{11}, U_{11}

If playing $(0, 1)$ were a harsher punishment for Agent A (i.e., $U_{01} < U_{11}$), he would be able to profitably deviate to $(1, 1)$. That is, playing $(0, 1)$ is not a stage-game equilibrium and thus cannot be used as a punishment strategy. If $(0, 1)$ were a harsher punishment for Agent B (i.e., $U_{10} < U_{11}$), we would need $U_{10} \geq U_{00}$ to prevent Agent B from deviating from $(0, 1)$ to $(0, 0)$. However, $U_{10} < U_{11}$, $U_{10} \geq U_{00}$, and (Static NE) together imply $U_{11} \geq \max\{U_{10}, U_{00}, U_{01}\}$, which means there is no scope for collusion because at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy.

To establish that $(1, 1)$ is the (weakly) harshest punishment, it remains to show that $U_{11} \leq U_{00}$. Suppose, by contradiction, $U_{11} > U_{00}$. If the wage scheme $\mathbf{w} \equiv \{w^x_{mn}\}$, $x = \{0, 1\}$, $m, n = \{L, H\}$, creates strategic payoff substitutability, i.e., $U_{10} - U_{00} > U_{11} - U_{01}$, (Static NE) again implies that $(0, 0)$ is not a stage-game equilibrium and thus cannot be used as a punishment strategy in the first place. If instead $\mathbf{w}$ creates (weak) strategic payoff complementarity, i.e., $U_{11} - U_{01} \geq U_{10} - U_{00}$, we have $U_{10} + U_{01} \leq U_{11} + U_{00} < 2U_{11}$, where the last inequality is due to the assumption $U_{11} > U_{00}$. But $U_{00} < U_{11}$ and $U_{10} + U_{01} < 2U_{11}$ together mean that at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy, meaning there is no scope for any collusion.

■

Let

$$V_t^i(\sigma) \equiv \sum_{\tau=1}^{\infty} \frac{U_{t+\tau}^i(a_\tau^i, a_\tau^j)}{(1+r)^\tau}$$

be agent i's continuation payoff from $t+1$ and forwards, discounted to period t, from playing $\{a_{t+\tau}^A, a_{t+\tau}^B\}_{\tau=1}^{\infty}$ specified in an action profile $\sigma = \{a_t^A, a_t^B\}_{t=0}^{\infty}$, for $a_t^A, a_t^B \in \{0,1\}$. We allow any σ satisfying the following condition to be a potential collusive strategy:

$$\sum_{t=0}^{\infty} \frac{U_t^i(a_t^i, a_t^j)}{(1+r)^t} \geq \frac{1+r}{r} U_{1,1}, \ \forall i \in \{A, B\}, \tag{4}$$

where $U_t^i(a_t^i, a_t^j)$ is Agent i's stage-game payoff at t given the action pair (a_t^i, a_t^j) specified in σ, and $\frac{1+r}{r} U_{1,1}$ is the agent's payoff from always working.

The outline of the sufficiency part of the Lemma is as follows:

Step 1: Any collusive strategy that contains only $(1,1), (1,0)$ and $(0,1)$ (i.e., without $(0,0)$ in any period) is easier for the principal to upset than *Cycling*.

Step 2: Any collusive strategy that ever contains $(0,0)$ at some period t would be easier for the principal to upset than either *Joint Shirking* or *Cycling*.

Step 1: The basic idea here is that, compared with *Cycling*, any reshuffling of $(0,1)$ and $(1,0)$ effort pairs across periods and/or introducing $(1,1)$, can only leave some shirking agent better off in some period if it also leaves another shirking agent worse off in another period, in terms of their respective continuation payoffs.

In order for the agents to be better-off under the collusive strategy σ that contains only $(1,1)$, $(1,0)$ and $(0,1)$ than under jointly work $(1,1)^\infty$, condition (4) requires $U_{1,0}+U_{0,1} > 2U_{1,1}$.

Therefore, we know

$$\overline{V}^{CYC} + \underline{V}^{CYC} \geq V_t^i(\sigma) + V_t^j(\sigma),\ \forall t, \tag{5}$$

where $\overline{V}^{CYC} = \sum\limits_{t=1,3,5,...} \frac{U_{1,0}}{(1+r)^t} + \sum\limits_{t=2,4,6,...} \frac{U_{0,1}}{(1+r)^t}$ and $\underline{V}^{CYC} = \sum\limits_{t=1,3,5,...} \frac{U_{0,1}}{(1+r)^t} + \sum\limits_{t=2,4,6,...} \frac{U_{1,0}}{(1+r)^t}$ are the continuation payoffs (under *Cycling*) of the shirking agent and the working agent, respectively. We drop the time index in $\overline{V}^{CYC}$ and $\underline{V}^{CYC}$ because they're time independent. Since (Static NE) and $U_{1,0} + U_{0,1} > 2U_{1,1}$ together imply $U_{1,0} > U_{1,1} \geq U_{0,1}$, simple algebra shows

$$\overline{V}^{CYC} > \max\{\underline{V}^{CYC}, V^*\}, \tag{6}$$

where $V^* \doteq \frac{U_{1,1}}{r}$ is the continuation payoff from playing $(1,1)^\infty$.

To prove the claim that the collusive strategy σ is easier for the principal to upset than *Cycling*, it is sufficient to show the following:

$$\exists\, t|\{(a_t^i = 0, a_t^j = 1) \wedge V_t^i(\sigma) \leq \overline{V}^{CYC}\}. \tag{7}$$

That is, there will be some period when the agents i is supposed to be the *only* "shirker" in that period faces a weakly lower continuation payoff (hence stronger incentives to deviate from shirking) under the collusive strategy σ than under *Cycling*. Suppose by contradiction that (7) fails. That is,

$$V_t^i(\sigma) > \overline{V}^{CYC},\ \forall\, t|\{(a_t^i = 0, a_t^j = 1)\}. \tag{8}$$

We know from (5) that (8) implies the following for the other agent j:

$$V_t^j(\sigma) < \underline{V}^{CYC},\ \forall\, t|\{(a_t^i = 0, a_t^j = 1)\}. \tag{9}$$

Since one agent always playing 0 is not a sub-game perfect equilibrium, there must be a switch from $(0,1)$ to $(1,0)$, possibly sandwiched by one or more $(1,1)$, in any strategy σ. We pick any such block in σ, and denote τ and $\tau + 1 + n$ ($n \in \{0, 1, 2, 3, ...\}$) as the time $(0,1)$

and $(1,0)$ are sandwiched by n period(s) of $(1,1)$.

We show below that, for all n, (8) leads to a contradiction, which then verifies (7) and proves the claim. We name the first agent as Agent A throughout the analysis.

- If $n = 0$, i.e., $(0,1)$ is followed immediately by a $(1,0)$ at $\tau + 1$. One can show the following for Agent A's continuation payoff at τ

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,0} + V_{\tau+1}^A(\sigma)}{1+r} \\ &< \frac{U_{1,0} + \underline{V}^{CYC}}{1+r} \\ &= \overline{V}^{CYC}, \end{aligned}$$

which contradicts (8). The inequality above applies (9) to $t = \tau + 1$.

- If n is an even number ($n = 2, 4, 6, ...$), i.e., there are even numbers of $(1,1)$ sandwiched between $(0,1)$ and $(1,0)$. We prove the case for $n = 2$ and same argument applies for all even n.

	...	$t=\tau$	$t=\tau+1$	$t=\tau+2$	$t=\tau+3$	...
σ	...	$(0,1)$	$(1,1)$	$(1,1)$	$(1,0)$	...

We can show the following for Agent A's continuation payoff at τ:

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2} + \frac{U_{1,0} + V_{\tau+3}^A(\sigma)}{(1+r)^3} \\ &< \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^3} \\ &< \frac{U_{1,0}}{1+r} + \frac{U_{0,1}}{(1+r)^2} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^3} \\ &= \overline{V}^{CYC}, \end{aligned}$$

which again contradicts (8). The first inequality applies (9) to $t = \tau + 3$. The second inequality applies (6) and the fact that $\overline{V}^{CYC} > V^*$ if and only if $\frac{U_{1,0}}{1+r} + \frac{U_{0,1}}{(1+r)^2} > \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2}$.

- If n is an odd number ($n = 1, 3, 5, ...$). Consider, without loss of generality, the following case for $n = 1$, i.e., there is one $(1,1)$ sandwiched between $(0,1)$ and $(1,0)$.

$$\begin{array}{cccccc} & ... & t=\tau & t=\tau+1 & t=\tau+2 & ... \\ \sigma & ... & (0,1) & (1,1) & (1,0) & ... \end{array}$$

We can show the following for Agent A's continuation payoff at τ

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,1}}{1+r} + \frac{U_{1,0} + V_{\tau+2}^A(\sigma)}{(1+r)^2} \\ &< \frac{U_{1,1}}{1+r} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^2} \\ &= \frac{rV^*}{1+r} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^2} \\ &= \frac{rV^*}{1+r} + \frac{\overline{V}^{CYC}}{1+r} \\ &< \overline{V}^{CYC}, \end{aligned}$$

which contradicts (8). The first inequality applies (9) to $t = \tau + 2$, the second equalities is from the definition of $V^* = \frac{U_{1,1}}{r}$, and the last equality is by the definition of $\underline{V}^{CYC}$ and $\overline{V}^{CYC}$ so that $\overline{V}^{CYC} = \frac{U_{1,0}+\underline{V}^{CYC}}{1+r}$.

Step 2: Given any contract offered by the principal, one of the following must be true:

$$\max\{2U_{1,1}, 2U_{0,0}, U_{1,0} + U_{0,1}\} = \begin{cases} 2U_{1,1} & \text{case 1} \\ U_{1,0} + U_{0,1} & \text{case 2} \\ 2U_{0,0} & \text{case 3.} \end{cases}$$

Case 1 is trivially collusion proof as no other action profile Pareto-dominates the equilibrium strategy $\{1,1\}_{t=0}^{\infty}$.

Case 2 has two sub-cases: sub-case 2.1 where $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{1,1} \geq U_{0,0}$ and sub-case 2.2 in which $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$. In sub-case 2.1, first note that we can ignore collusive strategy σ that contains $(0,0)$ at some period without loss of generality. The reason is that we can construct a new strategy σ' by replacing $(0,0)$ in σ by $(1,1)$, and σ' is more difficult to upset than σ because (a) both agents' continuation payoffs are higher under σ' and (b) σ' does not have the possibility of upsetting the collusive strategy at $(0,0)$. After ruling out collusive strategies containing $(0,0)$, we can refer to Step 1 and show that any such collusive strategy is deterred by the (No Cycling) constraint.

In sub-case 2.2 (i.e., $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$), we argue that any collusive strategy σ that contains $(0,0)$ at some point is easier to be upset than (thus implied by) the *Cycling* strategy. The reason is clear by comparing the action profiles between σ and *Cycling* from any $\tilde{t}$ such that $a_{\tilde{t}}(\sigma) = (0,0)$:

	$t = \tilde{t}$	$t = \tilde{t}+1, \tilde{t}+2, \cdots$
σ	$(0,0)$	$\{a^A(\sigma), a^B(\sigma)\}_{\tilde{t}+1}^{\infty}$
Cycling	$(0,1)$	$\{Cycle\}_{\tilde{t}+1}^{\infty}$

$\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$ implies $U_{1,0} - U_{0,0} > U_{1,1} - U_{0,1}$. That is, the benefit for either Agent A or B to unilaterally deviate from $(0,0)$ at $\tilde{t}$ is higher than the benefit for A to deviate from $(0,1)$ to $(1,1)$ at $\tilde{t}$ under the *Cycling* strategy. In addition, we know $V_{\tilde{t}}^A(CYC) + V_{\tilde{t}}^B(CYC) \geq V_{\tilde{t}}^A(\sigma) + V_{\tilde{t}}^B(\sigma)$ holds under Case 2, and therefore either $V_{\tilde{t}}^A(\sigma) \leq V_{\tilde{t}}^A(CYC)$ or $V_{\tilde{t}}^B(\sigma) \leq V_{\tilde{t}}^B(CYC)$. If $V_{\tilde{t}}^A(\sigma) \leq V_{\tilde{t}}^A(CYC)$ holds, then A has stronger incentive to deviate at $\tilde{t}$ under σ than he would have under *Cycling*. If it is $V_{\tilde{t}}^B(\sigma) \leq V_{\tilde{t}}^B(CYC)$, we make use of the observation that $V_{\tilde{t}}^B(CYC) < V_{\tilde{t}}^A(CYC)$ to conclude $V_{\tilde{t}}^B(\sigma) < V_{\tilde{t}}^A(CYC)$, which means that B has stronger incentive to deviate at $\tilde{t}$ under σ than A would have under *Cycling* at $\tilde{t}$. Again, once we rule out collusive strategies containing $(0,0)$, we can refer to Step 1 and show that any such collusive strategy is deterred by the (No Cycling) constraint.

Case 3 implies $V_t^A(SHK) + V_t^B(SHK) = \max_\sigma V_t^A(\sigma) + V_t^B(\sigma)$, $\forall t$. If a collusive strategy σ contains $a_{t'}^A(\sigma) = a_{t'}^B(\sigma) = 0$ at some period t', then either $V_{t'}^A(\sigma) \leq V_{t'}^A(SHK)$ or $V_{t'}^B(\sigma) \leq V_{t'}^B(SHK)$, which means at least one of the agents who is supposed to (jointly) shirk at t' will have a weakly stronger incentive to deviate than he would have under *Joint Shirking* strategy. If the collusive strategy does not contain $a_t^A = a_t^B = 0$ in any period, we can refer to Step 1 and show that any such collusive strategy is deterred by the (No Cycling) constraint.

Having shown the sufficiency of the lemma, it remains to prove necessity. (No Joint Shirking) is necessary for a contract to be collusion-proof unless playing $(1,1)$ indefinitely Pareto-dominates *Shirking* in the sense of (Pareto Dominance) defined in the text (i.e., $U_{11} \geq U_{00}$). Likewise, (No Cycling) is necessary unless $(1,1)^\infty$ Pareto-dominates *Cycling*, which, according to (Pareto Dominance) in the text, requires $\frac{(1+r)U_{11}}{r} \geq \sum\limits_{t=0,2,4,\ldots} \frac{U_{01}}{(1+r)^t} + \sum\limits_{t=1,3,5,\ldots} \frac{U_{10}}{(1+r)^t}$. Note that this condition is identical to (No Cycling). ■

Proof of Lemma 2. Given any wage scheme $\mathbf{w}$, denote $M = \min_{m,n \in H,L}\{w_{m,n} + w_{n,m}\}$. We first show $\underline{w} \leq M$. Suppose by contradiction that $\underline{w} > M$, which means $\underline{w} > w_{m,n} + w_{n,m}$ for some outcome pair (m,n). Since the court will enforce $\underline{w}$, it will allocate the difference $\Delta = \underline{\mathrm{w}} - (w_{m,n} + w_{n,m})$ between the two agents according to a certain allocation rule. The principal can directly give the agents the same payment as what they would have received through the court's enforcement by increasing the payments so that $(w'_{m,n} + w'_{n,m}) = \underline{w}$ and allocating it appropriately. Label the new wage scheme as $\mathbf{w}'$. It is easy to see that (i) $\mathbf{w}'$ costs the same as $\mathbf{w}$, and (ii) $\mathbf{w}'$ is feasible as long as the original $\mathbf{w}$ is. Since the principal can do better by optimizing over all possible $(w'_{m,n} + w'_{n,m}) = \underline{w}$ (as opposed to replicating the court's allocation rule), $\mathbf{w}'$ is at least weakly better than $\mathbf{w}$.

To further prove $\underline{w} = M$, suppose instead $\underline{w} < M$ in a wage scheme $\mathbf{w}$. We can construct a new wage scheme $\mathbf{w}'$ by increasing $\underline{w}$ (while keep the payment $w_{m,n}$ unchanged). It is easy to see that (i) $\mathbf{w}'$ and $\mathbf{w}$ yield the same objective value, and (ii) $\mathbf{w}'$ is feasible as long as $\mathbf{w}$ is. Since Principal's IC is more relaxed in $\mathbf{w}'$ than in $\mathbf{w}$, it is optimal to set $\underline{w} \geq M$. We know $\underline{w} = M$ as we already show $\underline{w} \leq M$ above. ■

Proof of the optimal contracts: We solve optimal contract by the method of enumeration and complete the analysis in two steps. In Lemmas 5 and 6, we solve Program P while setting $T = 1$ and then $T = 0$, respectively. We then compare the solutions for each parameter region and optimize over the choice of T in Proposition 1. The following parameters will be useful in the remainder of the appendix.

$$
\begin{aligned}
r^A &= \frac{1}{2}\left[\begin{array}{c} (q_0-q_1)^2 q_1 H - 1 - q_1^2 + \\ \sqrt{((q_0-q_1)^2 q_1 H - 1 - q_1^2)^2 + 4(q_0-q_1)^2(q_0+q_1)H - 4q_1^2} \end{array}\right], \\
r^B &= (q_0-q_1)^2 H - q_1, \\
r^C &= (q_0-q_1)^2(q_0+q_1)H - q_0 - q_1^2, \\
r^D &= \frac{1}{2}\left[\begin{array}{c} (q_0-q_1)^2(2-q_1)H - 1 - 2q_1 + q_1^2 + \\ \sqrt{((q_0-q_1)^2(2-q_1)H - 1 - 2q_1 + q_1^2)^2 + 4(q_1-2)q_1 - 8(q_0-q_1)^2(q_1-1)H} \end{array}\right], \\
r^L &= \frac{q_1+q_0-1}{(1-q_1)^2}, \\
r^H &= \frac{2q_1-1}{(1-q_1)^2}.
\end{aligned}
$$

Note that r^A,r^B,r^C,and r^D increase in H, and we assume throughout the paper that H is larger enough to assure that (i) $r^C > r^B$ whenever $q_0 + q_1 > 1$, and (ii) $r^A < r^B < r^D$ and $r^A < r^C$. In addition, the agent's effort is assumed to be valuable enough ($q_1 - q_0$ is not too small) such that $r^A > \sqrt{2}$ and $(q_0 - q_1)^2 H \geq \max\{1 + \frac{1}{q_1-q_0}, \frac{q_1^2}{2q_1-1}, \frac{q_1(2-q_1)}{2(1-q_1)}, \frac{q_0-(1-q_1)q_1}{q_0+q_1-1}\}$.

Lemma 5 *Given $T = 1$, the solution to Program P is one of the following (with $w_{LH} = 0$ and $w_{HH} > 0$ in all cases):*

(i) Pure JPE: w_{HH} is the only positive payment for $r \in (0, r^A]$ and it increases in r;

(ii) BPC: $w_{HL} = 2w_{LL} = \underline{w} > 0$ for $r \in (r^A, \min\{r^L, r^C\}]$;

(iii) JPE: $w_{HH} > w_{HL} > 0, w_{LL} = \underline{w} = 0$ for $r \in (\max\{r^L, r^A\}, r^B]$;

(iv) BPC2: $w_{HL} > 2w_{LL} = \underline{w} > 0$ for $r \in (\max\{r^L, r^B\}, r^C]$;

(v) infeasible otherwise.

Proof of Lemma 5. Given Lemma 2, we can simplify the program as follows:

$\min(1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$

$s.t$

$\frac{2[q_1 H - \pi(1,1;\mathbf{w})] - 2q_0 H}{r} \geq \max_{m,n,m',n'}\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}$, (Principal's IC, $\lambda_{mn\succ m'n'}$)

$(2-q_0-q_1+r(1-q_1))w_{LL}+(q_0+q_1+q_1 r-1)w_{LH}+(q_0+(q_1-1)(1+r))w_{HL}-(q_0+q_1+q_1 r)w_{HH} \leq \frac{-(1+r)}{q_1-q_0}$ (Mutual Monitoring, $\lambda_{Monitor}$)

$(2-q_0-q_1)w_{LL}+(q_0+q_1-1)w_{LH}+(q_0+q_1-1)w_{HL}-(q_0+q_1)w_{HH} \leq \frac{-1}{q_1-q_0}$ (Pareto Dominance, λ_{Pareto})

$(-1+q_0)w_{LL} - q_0 w_{LH} + (1-q_0)w_{HL} + q_0 w_{HH} \leq \frac{1}{q_1-q_0}$ (Self-Enforcing Shirk)

$-w_{LL} \leq 0\,(\mu_{LL})$; $-w_{HL} \leq 0(\lambda_{11})$; $-w_{HH} \leq 0(\mu_{HH})$; $-w_{LH} \leq 0(\mu_{LH})$;

Sketch of the proof: we first solve a *relaxed* program without the (Self-Enforcing Shirk) constraint and then verify that solutions of the relaxed program satisfy (Self-Enforcing Shirk).

Claim: Setting $w_{LH} = 0$ is optimal in the *relaxed program* (without Self-Enforcing Shirk constraint.)

Proof of the Claim. Suppose the optimal solution is $\mathbf{w} = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ with $w_{LH} > 0$. Construct a new solution $\mathbf{w}' = \left\{w'_{HH}, w'_{HL}, w'_{LH}, w'_{LL}\right\}$, with $w'_{LH} = 0$, $w'_{HL} = w_{HL} + w_{LH}$, $w'_{LL} = w_{LL}$ and $w'_{HH} = w_{HH}$. It is easy to see that $\mathbf{w}$ and $\mathbf{w}'$ generate the same objective function value, and that $\mathbf{w}'$ satisfies all the (Principal's IC) constraints $ICP_{mn\succ m'n'}$ as long as $\mathbf{w}$ does. Finally, we claim that, compared to $\mathbf{w}$, $\mathbf{w}'$ relaxes all the other constraints in the relaxed program. We illustrate the argument for (Pareto Dominance) only since the same argument applies to (Mutual Monitoring) and (No Cycling). Denote by C_{LH} (and C_{HL}) the coefficient of the payment w_{LH} (and w_{HL}, respectively). It is easy to verify that $C_{LH} - C_{HL} \geq 0$, and, therefore, $\mathbf{w}'$ relaxex (Pareto Dominance) relative to $\mathbf{w}$. ■

The Lagrangian for the relaxed problem is

$$L(\mathbf{w}, \lambda, \mu) = f_0(\mathbf{w}) - \sum_i \lambda_i f_i(\mathbf{w}) - \sum_s \mu_s w_{mn}.$$

A contract $\mathbf{w} = \{w_{mn}\}$ for $m, n \in \{H, L\}$ is optimal if and only if one can find a pair $(\mathbf{w}, \lambda, \mu)$ that satisfies the following four conditions: (i) Primal feasible, i.e., constraints $f_i(w) \geq 0$ and $w_{mn} \geq 0$, (ii) Dual feasible, i.e., Lagrangian multiplier vectors $\lambda \geq 0$ and $\mu \geq 0$, (iii) Stationary condition, i.e., $\nabla_{\mathbf{w}} L(\mathbf{w}, \lambda, \mu) = 0$, and (iv) Complementary slackness, i.e. $\lambda_i f_i(\mathbf{w}) = \mu_s w_{mn} = 0$. The proof lists the optimal contract, in particular the pair $(\mathbf{w}, \lambda, \mu)$, as a function of the discount rate r.

For $r < r^A$, the solution, denoted as as *Pure JPE*, is:

$w_{LL} = 0, w_{HL} = 0, w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}$;

$\lambda_{Pareto} = q_1, \lambda_{Monitor} = \frac{q_1^2}{q_0+q_1+q_1 r}, \lambda_{CYC} = 0, \lambda_{HH\succ LL} = 0,$

$\lambda_{HL\succ LL} = 0, \lambda_{HH\succ HL} = 0, \lambda_{HH\succ HH} = 0, \lambda_{LL\succ HH} = 0,$

$\lambda_{LL\succ HL} = 0, \mu_{LL} = \frac{q_0-2q_0q_1+q_1(1+r-q_1 r)}{q_0+q_1+q_1 r}, \mu_{HL} = \frac{q_0 q_1}{q_0+q_1+q_1 r}, \mu_{HH} = 0.$

The $ICP_{HH\succ LL}$ constraint yields the upper bound on r under *Pure JPE*.

For $r^A < r \leq \min(r^L, r^C)$, the solution, denoted as *BPC*, is:

$w_{LL} = \frac{(q_1-q_0)^2(q_0+q_1+q_1 r)H-(1+r)(q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1 r-r^2)}, w_{HL} = 2 * w_{LL},$

$w_{HH} = \frac{(q_1-q_0)^2(q_0+q_1+(-1+q_1)r)H-(1+r)(-1+q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1 r-r^2)}$;

$\lambda_{Pareto} = 0, \lambda_{Monitor} = \frac{-r}{q_0+q_1+q_1 r-q_1^2 r-r^2}, \lambda_{CYC} = 0, \lambda_{HH\succ LL} = \frac{q_0-(q_1-2)((q_1-q)r-1)}{q_0+q_1+q_1 r-q_1^2 r-r^2},$

$\lambda_{HL\succ LL} = 0, \lambda_{HH\succ HL} = -\frac{2(q_0-(q_1-1)((q_1-1)r-1))}{q_0+q_1+q_1 r-q_1^2 r-r^2}, \lambda_{HH\succ HH} = 0, \lambda_{LL\succ HH} = 0,$

$\lambda_{LL\succ HL} = 0, \mu_{LL} = 0, \mu_{HL} = 0, \mu_{HH} = 0.$

Under *BPC* , the non-negativity of w_{LL} and w_{HL} requires $r > r^A$, while (Pareto Dominance) and $\lambda_{HH\succ HL} \geq 0$ impose the upper bounds r^C and r^L, respectively.

For $\min\{r^L, r^A\} < r \leq r^B$, the solution, denoted as *JPE*, is:

$w_{LL} = 0, w_{HL} = \frac{\frac{(1+r)(q_1^2+r)}{q_1-q_0}-(q_1-q_0)(q_0+q_1+q_1 r)H}{(1-q_1)r(1+r)-q_0(q_1+r)},$

$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(q_0+(-1+q_1)(q+r))H}{(q_1-q_0)((-1+q_1)r(1+r)+q_0(q_1+r))}$;

$\lambda_{Pareto} = 0, \lambda_{Monitor} = \frac{(q_1-1)q_1 r}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_{CYC} = 0, \lambda_{HH\succ LL} = \frac{-q_0 q_1}{(q_1-1)r(1+r)+q_0(q_1+r)},$

$\lambda_{HL\succ LL} = 0, \lambda_{HH\succ HL} = 0, \lambda_{HH\succ HH} = 0, \lambda_{LL\succ HH} = 0,$

$\lambda_{LL\succ HL} = 0, \mu_{LL} = \frac{r(q_0-(q_1-1)((q_1-q)r-1))}{(q_1-1)r(1+r)+q_0(q_1+r)}, \mu_{HL} = 0, \mu_{HH} = 0.$

Under *JPE*, the non-negativity of w_{HL} requires $r > r^A$ and $\mu_{LL} \geq 0$ yields another lower bound r^L on r. The non-negativity of w_{HH} and w_{HL} also requires $r > s' \equiv \frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)}$. In addition, (Pareto Dominance) requires $r < r^B$. We claim $(\max\{s', r^A, r^L\}, r^B] = (\max\{r^A, r^L\}, r^B]$. The claim is trivial if $s' \leq r^A$ and therefore consider the case where $s' > r^A$. Since r^A increases in H while s' is independent of H, one can show $s' > r^A$ is equivalent to $H < H'$ for a unique positive H'. Meanwhile, algebra shows that $r^B < r^A$ for $H < H'$. Therefore, $s' > r^A$ implies $r^B < r^A$, in which case $(\max\{s', r^A, r^L\}, r^B] = (\max\{r^A, r^L\}, r^B] = \emptyset$.

For $\max\{r^L, r^B\} < r \leq r^C$ and $q_1 + q_0 \geq 1$, the optimal solution, denoted as *BPC2*, is:

$w_{LL} = \frac{q_0(q_1-q_0)^2H - q_0(q_1+r)}{(q_1-q_0)((1-q_1)r - q_0(-1+q_1+r))}$, $w_{HL} = \frac{(q_1-q_0)^2+r-2q_0r-(q_1-q_0)^3H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))}$,

$w_{HH} = \frac{(q_0+q_1)(q_1-1)+q_0r+(q_1-q_0)^2(-1+q_1)H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))}$;

$\lambda_{Pareto} = \frac{-q_0+(-1+q_1)(-1+(-1+q_1)r)}{(q_1-1)r+q_0(-1+q_1+r)}$, $\lambda_{Monitor} = \frac{q_0+q_1-1}{(q_1-1)r+q_0(-1+q_1+r)}$, $\lambda_{CYC} = 0$, $\lambda_{HH \succ LL} = \frac{q_0(1-q_1)}{(q_1-1)r+q_0(-1+q_1+r)}$,

$\lambda_{HL \succ LL} = 0, \lambda_{HH \succ HL} = 0, \lambda_{HH \succ HH} = 0, \lambda_{LL \succ HH} = 0,$

$\lambda_{LL \succ HL} = 0, \mu_{LL} = 0, \mu_{HL} = 0, \mu_{HH} = 0.$

Under *BPC2* , the non-negativity of λ_{Pareto} requires $q_1 + q_0 \geq 1$. Given $q_1 + q_0 \geq 1$, the non-negativity of w_{HH} and w_{HL} together yield $r > r^B$ and $r > s'' \equiv \frac{(1-q_1)q_0}{q_0+q_1-1}$. The other lower bound r^L on r is generated by intersecting requirements for $\lambda_{Pareto} \geq 0$ and for the non-negativity of w_{HH} and w_{HL}. The $ICP_{HH \succ HL}$ constraint yields the upper bound on r, i.e. $r \leq r^C$. We claim $(\max\{s'', r^B, r^L\}, r^C] = (\max\{r^L, r^B\}, r^C]$. To see this, note that a necessary condition for $(\max\{s'', r^B, r^L\}, r^C]$ to be non-empty is that $r^B < r^C$, which can be rewritten as $\frac{q_0-q_1(1-q_1)}{q_0+q_1-1} < (q_1 - q_0)^2H$. Subtracting q_1 from both sides of the inequality and collecting terms, one can derive $s'' < r^B$. That is, $r^B < r^C$ implies $s'' < r^B$, and, therefore, $(\max\{s'', r^B, r^L\}, r^C] = (\max\{r^L, r^B\}, r^C]$ is verified.

As r becomes even larger, the problem $T = 1$ becomes infeasible because the intersection of the (Mutual Monitoring) and (Principal's IC) is an empty set. Finally, tedious algebra verifies that the solutions characterized above satisfy the (Self-Enforcing Shirk) constraint that we left out in solving the *relaxed* program. Therefore adding this constraint back does not affect the optimal contract. This completes the proof of Lemma 5. ∎

Lemma 6 *Given $T = 0$, the solution to the Program P is one of the following ($w_{LH} = 0$ and $w_{HH} > 0$ in all cases):*

(i) IPE: $w_{HH} = w_{HL} = \frac{1}{q_1 - q_0}, w_{LL} = \underline{w} = 0$ for $r \in (0, r^B]$;

(ii) BPI: $2w_{LL} = \underline{w} > 0$, $w_{HL} = w_{HH} + w_{LL}$.;

(iii) RPE: $w_{HL} > w_{HH} > 0$, $w_{LL} = \underline{w} = 0$ for $r \in (\max\{r^B, r^H\}, r^D]$;

(iv) BPS: $w_{HH} > 0$, $w_{HL} = 2w_{HH} > 2w_{LL} = \underline{w} > 0$ for $r > \max\{r^H, r^D\}$.

Proof of Lemma 6. Similar argument as in the Proof of Lemma 5 shows that setting $w_{LH} = 0$ is optimal. Given $w_{LH} = 0$, one can rewrite the program as follows.

$\min(1 - q_1)^2 w_{LL} + (1 - q_1)q_1 w_{HL} + q_1^2 w_{HH}$

s.t

$\frac{2[q_1 H - \pi(1,1;\mathbf{w})] - 2q_0 H}{r} \geq \max\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}$, (Principal's IC, $\lambda_{mn \succ m'n'}$)

$(1 - q_1)w_{LL} - (1 - q_1)w_{HL} - q_1 w_{HH} \leq \frac{-1}{q_1 - q_0}$ (*Static NE*, λ_{SNE})

$((2 - q_1 - q_0) + (1 - q_0)r)w_{LL} + ((q_0 - 1)(1 + r) + q_1)w_{HL} - (rq_0 + q_0 + q_1)w_{HH} \leq \frac{-(1+r)}{q_1 - q_0}$ (*No Joint Shirking*, λ_{SHK})

$(1 - q_1)(2 + r)w_{LL} + ((q_1 - 1)(1 + r) + q_1)w_{HL} - q_1(2 + r)w_{HH} \leq -\frac{1+r}{q_1 - q_0}$ (*No Cycling*, λ_{CYC})

$-w_{LL} \leq 0(\mu_{00})$; $\quad -w_{HL} \leq 0(\mu_{10})$; $\quad -w_{HH} \leq 0(\mu_{11})$.

For $r \leq r^B$, the solution, denoted as *IPE*, is:

$w_{LL} = 0$, $w_{HL} = w_{HH} = \frac{1}{q_1 - q_0}$

$\lambda_{SNE} = q_1$, $\lambda_{SHK} = 0$, $\lambda_{CYC} = 0$, $\lambda_{HH \succ LL} = 0$,

$\lambda_{HL \succ LL} = 0$, $\lambda_{HH \succ HL} = 0$, $\lambda_{HL \succ HH} = 0$, $\lambda_{LL \succ HH} = 0$,

$\lambda_{LL \succ HL} = 0$, $\mu_{LL} = 1 - q_1$, $\mu_{HL} = 0$, $\lambda_{12} = 0$.

Under IPE, the $ICP_{HH \succ LL}$ constraint imposes the upper bound r^B on r.

For $r^B < r < r^H$, the solution, denoted as *BPI*, is:

$w_{LL} = \frac{(q_1 - q_0)^2(1+r)H - (1+r)(q_1+r)}{(q_1 - q_0)(q - r(-1+q_1+r))}$, $w_{HL} = w_{HH} + w_{LL}$,

$w_{HH} = \frac{(q_1 - q_0)^2 H - (1+r)(-1+q_1+r)}{(q_1 - q_0)(q - r(-1+q_1+r))}$;

$\lambda_{SNE} = 0$, $\lambda_{SHK} = \frac{r(1+r+q_1^2r-2q_1(q+r))}{(q_1-q_0)(1+r)(-1+(-1+q_1)r+r^2)}$, $\lambda_{CYC} = \frac{r(-1+q_0+q_1-r+q_0r-q_1^2r)}{(q_0-q_1)(1+r)(-q+(-1+q_1)r+r^2)}$, $\lambda_{HH\succ LL} = \frac{1+r-q_1r}{-1-(1-q_1)r+r^2}$,

$\lambda_{HL\succ LL} = 0$, $\lambda_{HH\succ HL} = 0$, $\lambda_{HL\succ HH} = 0$, $\lambda_{LL\succ HH} = 0$,

$\lambda_{LL\succ HL} = 0$, $\mu_{LL} = 0$, $\mu_{HL} = 0$, $\mu_{HH} = 0$.

Under *BPI*, both the non-negativity of w_{LL} and the (Static NE) constraints require $r > r^B$, and $\lambda_{SHK} \geq 0$ requires $r < r^H$.

For $\max\{r^H, r^B\} < r \leq r^D$, the optimal solution, denoted as *RPE*, is:

$w_{LL} = 0$, $w_{HL} = \frac{(q_1-q_0)q_1(2+r)H-\frac{(1+r)(q_1^2+r)}{q_1-q_0}}{q_1^2-r(1+r)+q_1r(2+r)}$,

$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(-1-r+q_1r(2+r))H}{(q_1-q_0)(q_1^2-r(1+r)+q_1r(2+r))}$,

$\lambda_{SNE} = 0$, $\lambda_{SHK} = 0$, $\lambda_{CYC} = \frac{(1-q_1)q_1r}{r(1+r)-q_1^2-q_1r(2+r)}$, $\lambda_{HH\succ LL} = \frac{q_1^2}{r(1+r)-q_1^2-q_1r(2+r)}$,

$\lambda_{HL\succ LL} = 0$, $\lambda_{HH\succ HL} = 0$, $\lambda_{HL\succ HH} = 0$, $\lambda_{LL\succ HH} = 0$,

$\lambda_{LL\succ HL} = 0$, $\mu_{LL} = \frac{r(1+r+q_1^2r-2q_1(1+r))}{r(1+r)-q_1^2-q_1r(2+r)}$, $\mu_{HL} = 0$, $\mu_{HH} = 0$.

Under *RPE*, the (Static NE) constraint and $\mu_{LL} \geq 0$ yields two lower bounds r^B and r^H on r. $ICP_{HL\succ LL}$ and the non-negativity of w_{HH} and w_{HL} together require $r \leq r^D$. $w_{HH} \geq 0$ also requires $r > s \equiv \frac{2q_1-1+\sqrt{(2q_1-1)^2+4(1-q_1)q_1^2}}{2(1-q_1)}$, and we claim $(\max\{s, r^B, r^H\}, r^D] = (\max\{r^B, r^H\}, r^D]$. Consider the case where $s > r^B$ (as the claim is trivial if instead $s \leq r^B$). Since r^B increases in H while s is independent of H, one can show that $s > r^B$ is equivalent to $H < H^*$ for a unique positive H^*. Algebra shows that $r^D < r^B$ for $H < H^*$. Therefore, in the case of $s > r^B$, we know $r^D < r^B$ and hence $(\max\{s, r^B, r^H\}, r^D] = (\max\{r^B, r^H\}, r^D] = \emptyset$.

For $r > \max\{r^D, r^H\}$, the optimal solution, denoted as *BPS*, is:

$w_{LL} = \frac{(1+r)((2-q_1)q_1+r)+(q_1-q_0)^2(-2(1+r)+q_1(2+r))H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}, w_{HL} = 2w_{HH}$,

$w_{HH} = \frac{(1+r)(-(1-q_1)^2+r)-(q_1-q_0)^2(1-q_1)(2+r)H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}$,

$\lambda_{SNE} = q_1, \lambda_{SHK} = 0, \lambda_{CYC} = \frac{r}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_{HH\succ LL} = \frac{q_1(-2+(-1+q_1)r)}{2+2r+q_1^2r-r^2-q_1(2+3r)}$,

$\lambda_{HL\succ LL} = \frac{2(1+r+q_1^2r-2q_1(1+r))}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_{HH\succ HL} = 0, \lambda_{HL\succ HH} = 0, \lambda_8 = 0$,

$\lambda_{LL\succ HL} = 0, \mu_{LL} = 1-q_1, \mu_{HL} = 0, \mu_{HH} = 0$.

The two lower bounds r^D and r^H are derived from the non-negativity constraint of w_{LL} and

$\lambda_{HL\succ LL}$, respectively. Collecting conditions verifies Lemma 6. ■

Proof of Proposition 1. We prove the proposition by showing a sequence of claims.

Claim 1: If *Pure JPE* is optimal given $T = 1$, it is the overall optimal contract.

Claim 2: If *JPE* is optimal given $T = 1$, it is the overall optimal contract.

Claim 3: If *BPC* is optimal given $T = 1$, it is the overall optimal contract.

Claim 4: *BPC2* of $T = 1$ is never the overall optimal contract.

Proof of Claim 1: We know from Lemma 5 that *Pure JPE* is the optimal solution of $T = 1$ for $r \in (0, r^A]$, over which the optimal solution of $T = 0$ is *IPE* (Lemma 6). Substituting the two solutions into the principal's objective function, we obtain $obj_{PureJPE} = \frac{q_1^2(1+r)}{(q_1-q_0)(q_0+q_1+q_1 r)}$ and $obj_{IPE} = \frac{q_1}{q_1-q_0}$. Algebra shows $obj_{IPE} - obj_{PureJPE} = \frac{q_0 q_1}{(q_1-q_0)(q_0+q_1+q_1 r)} > 0$, which verifies the claim.

Proof of Claim 2: *JPE* is the solution of $T = 1$ for $r \in (\max\{r^L, r^A\}, r^B]$, over which *IPE* is the corresponding solution of $T = 0$. Algebra shows that $obj_{JPE} = \frac{(q_1-1)q_1 r(1+r)+q_0(q_0-q_1)^2 q_1 H}{(q_0-q_1)((q_1-1)r(1+r)+q_0(q_1+r))}$, $obj_{IPE} = \frac{q_1}{q_1-q_0}$, and $obj_{JPE} - obj_{IPE} \leq 0$ if and only if $\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)} \leq r \leq r^B$ (with strict inequality except at the boundary of the support). The claim is true if $\max\{\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)}, r^L, r^A\} \leq r \leq r^B$, which is shown to be equivalent to $r \in (\max\{r^L, r^A\}, r^B]$ in the proof of Lemma 6. Therefore, JPE is the overall optimal contract whenever it is feasible.

Proof of Claim 3: We know that *BPC* is the solution of $T = 1$ if $r \in (r^A, \min\{r^L, r^C\}]$. In this region, *IPE* and *BPI* are potential solutions in $T = 0$ because the other two solutions (*RPE* and *BPS*) require $r \geq r^H > r^L$. Let us compare first *BPC* of $T = 1$ and *BPI* of $T = 0$. It is easy to show $obj_{BPC} = \frac{r(1+r)-(q_0-q_1)^2(q_0+q_1(1+r-q_1 r))H}{(q_0-q_1)(q_0+q_1-(-1+q_1)q_1 r-r^2)}$ and $obj_{BPI} = \frac{r(1+r)+(q_0-q_1)^2(r(q_1-1)-1)H}{(q_0-q_1)(r(q_1+r-1)-1)}$. Tedious algebra verifies $obj_{BPC} < obj_{BPI}$ for $r^B < r \leq \min\{r^L, r^C\}$ where both solutions are feasible.

Comparing *BPC* and *IPE* solution is more involved and we present the analysis in two steps. We first derive a sufficient condition for *BPC* to outperform *IPE* (i.e., $obj_{BPC} < obj_{IPE}$) and then show that the sufficient condition holds in the relevant region where the two solutions are optimal in their corresponding program, i.e., $r^A < r \leq \min\{r^L, r^B, r^C\}$. Given obj_{BPC} and obj_{IPE} derived above, one can show that $obj_{BPC} < obj_{IPE}$ if and only if $r < \delta$, where

$$\delta = \frac{1}{2(1-q_1)}[((q_1-q_0)^2H - q_1)q_1(1-q_1) - 1 + \sqrt{(((q_1-q_0)^2H - q_1)q_1(1-q_1) - 1)^2 + 4(1-q_1)((q_1-q_0)^2H - q_1)(q_1+q_0)}].$$

We will show $r < \delta$ (hence $obj_{BPC} < obj_{IPE}$) satisfies over the relevant region $r^A < r \leq \min\{r^L, r^B, r^C\}$ by considering two cases. First consider the case of $\delta \geq r^B$, in which $r < \delta$ satisfies trivially for $r^A < r \leq \min\{r^L, r^B, r^C\}$. Consider the second case in which $\delta < r^B$. The remainder shows that $\delta < r^B$ over the relevant region implies $r^L < \delta$, and, therefore, $r < \delta$ again satisfies given $r^A < r \leq \min\{r^L, r^B, r^C\}$. One can show that $\delta < r^B$ corresponds to either $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$ or $q_1 - \sqrt{\frac{q_1}{H}} < r < q_1$. We need only consider $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$ as the latter condition $r < q_1 \leq 1$ is outside the relevant region. Given $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$, algebra shows $r^L < \delta$ for any $q_0 \in [0, q_1)$.

Proof of Claim 4: Recall that *BPC2* of $T = 1$ is obtained by solving the following three binding constraints: (Mutual Monitoring), (Pareto Dominance), and $ICP_{HH \succ LL}$. When both (Mutual Monitoring) and (Pareto Dominance) are biding, it is easy to prove $U_{11} = U_{00} = U_{01}$. Since *BPC2* also creates strategic payoff complementarity (i.e., $U_{11} - U_{01} > U_{10} - U_{00}$), we can further show that the contract satisfies $U_{11} = U_{00} = U_{01} > U_{10}$, which means that *BPC2* would have satisfied the (No Cycling) had the constraint been part of $T = 1$. In other words, *BPC2* satisfies all the potentially binding constraints in the $T = 0$ program when the parameters are such that either *RPE* or *BPS* is optimal given $T = 0$ (recall that (No Joint Shirking) is not binding in these two solutions). Therefore, the optimality of *RPE* and *BPS* means that $obj_{BPC2} \geq obj_{RPE}$ and $obj_{BPC2} \geq obj_{BPS}$ over the corresponding parameter spaces. To see that *BPI* is more cost efficient than *BPC2* (i.e., $obj_{BPC2} \geq obj_{BPI}$), note that *RPE* is feasible over the region where *BPI* is optimal, and we know by revealed preference that $obj_{RPE} \geq obj_{BPI}$, and $obj_{BPC2} \geq obj_{RPE} \geq obj_{BPI}$ by transitivity.

BPS is optimal for sufficiently large r as team incentives $T = 1$ is infeasible in the region. ■

Proof of Corollary 1. For Part (i), it is easy to verify from Lemma 5 that the interval over which BPC is optimal is non-empty if and only if $r^A < r^L$, which requires $H \leq H^* \doteq \frac{(q_1+(q_1-1)q_1)\left(q_0+(q_1-1)\left((q_1-1)q_1^2+1\right)\right)}{(q_1-1)^2(q_0-q_1)^2\left((q_0-1)q_1^2-q_0q_1+q_0+q_1^3\right)}$ and $q_0 > 1 - q_1$. Similarly, following Lemma 6, we know the interval $r^B < r \leq r^H$ is non-empty if and only if $q_1 > \frac{1}{2}$ and $H \leq H^{**} \equiv \frac{q_1(3-(2-q_1)q_1)-1}{(1-q_1)^2(q_1-q_0)^2}$. ■

Proof of Proposition 2. Given Lemma 5, it is easy to verify that, fixing $T = 1$ and $\underline{w} = 0$, the solution to Program P is: (i) *Pure JPE* for $r \in (0, r^A]$; (ii) *JPE* for $r \in (r^A, r^B]$; and (iii) infeasible otherwise. Similarly, given Lemma 6, one can verify that fixing $T = 0$ and $\underline{w} = 0$, the solution to Program P is: (i) *IPE* for $r \in (0, r^B]$; (ii) *RPE* for $r \in (r^B, r^D]$; and (iii) infeasible otherwise. Endogenizing the choice of T follows similar steps shown in Proposition 1 and is hence suppressed. Collecting the conditions yields the overall optimal contract stated in the proposition. ■

Proof of Corollary 2. The corollary follows from comparing Propositions 1 and 2. ■

Proof of Lemma 3. Following Lemmas 5 and 6, we list positive payments and Lagrangian multipliers of the binding constraints (μ for non-negativity constraints and λ for other constraints).

For $r < r1 \doteq \frac{2P\left(2Q^2+Q-2\right)+\sqrt{(2(P(2Q^2+Q-2)+Q^2)+Q)^2+8P(2P+1)(P(1-2Q)^2+4Q)}+Q(2Q+1)}{8P}$, the solution is: $w_{11}^H = \frac{4(r+1)}{P(4Q(Q(r+2)+r)+r+2)+2Q(2Qr+r+2)}$, $\lambda_{Monitor} = \frac{4\left(P+\frac{1}{2}\right)\left(Q+\frac{1}{2}\right)^2}{P(4Q(Q(r+2)+r)+r+2)+2Q(2Qr+r+2)}$,

$\mu_{H00} = -\frac{(2P+1)Q\left(4P\left(4Q^2+1\right)+4Q^2(r-2)-r-2\right)}{2P(4Q(Q(r+2)+r)+r+2)+4Q(2Qr+r+2)}$, $\mu_{H10} = \frac{\left(4P^2-1\right)Q\left(4Q^2-1\right)}{2P(4Q(Q(r+2)+r)+r+2)+4Q(2Qr+r+2)}$,

$\mu_{L00} = \frac{\left(4PQ^2+P+2Q\right)\left(16PQ-4Q^2(r-2)+r+2\right)}{4P(4Q(Q(r+2)+r)+r+2)+8Q(2Qr+r+2)}$, $\mu_{L10} = \frac{\left(4Q^2-1\right)\left(P\left(4Q^2-1\right)r-2(P+Q)(4PQ+1)\right)}{4P(4Q(Q(r+2)+r)+r+2)+8Q(2Qr+r+2)}$,

$\mu_{L11} = -\frac{P(2Q-1)(2Q+1)^2(2Q(r-2)+r+2)}{4P(4Q(Q(r+2)+r)+r+2)+8Q(2Qr+r+2)}$.

$ICP_{H11 \succ H00}$ imposes the upper bound $r < r1$.

To prove Part (ii), note that for $r1 < r \leq \min\{r2, -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}\}$, the contract is

$w_{11}^H = \frac{2\left(-2P^2(1-2Q)^2-P(4Q(3Q+r-1)+2r+3)+2Q(2Qr+r+2)\right)}{P\left(P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)\right)}$,

$w_{11}^L = \frac{2\left(2P^2(1-2Q)^2+P\left(Q^2(8r+4)+4Q(r+1)-8r(r+1)+1\right)+2Q(2Qr+r+2)\right)}{P\left(P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)\right)}$.

$\lambda_{Monitor} = -\frac{2(2P-1)(2Q+1)^2 r}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,

$\lambda_{H11\succ H00} = -\frac{P(2Q-1)(2Q+1)^2(2Q(r-2)+r+2)}{2P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+16Qr(2Qr+r+2)}$,

$\mu_{H00} = \frac{2r\left(\left(4Q^2-1\right)r\left(4PQ^2+2PQ+P-Q\right)+2\left(4Q^2+1\right)(P-Q)(4PQ-1)\right)}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,

$\mu_{H10} = -\frac{\left(4Q^2-1\right)r\left(P\left(8PQ+4Q^2(r-2)-r-2\right)+2Q\right)}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,

$\mu_{L00} = -\frac{2(2P-1)Qr\left(4P\left(4Q^2+1\right)-4Q^2(r-2)+r+2\right)}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,

$\mu_{L10} = \frac{2\left(4P^2-1\right)Q\left(4Q^2-1\right)r}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$.

$w_{L11} > 0$ requires $r > r1$, $ICP_{L11\succ L00}$ requires

$$r < r2 \doteq \frac{4PQ^2 - 5P + 4Q^2 + 2Q + \sqrt{(4(P+1)Q^2 - 5P + 2Q)^2 - 16P\left(P(1-2Q)^2 - 4Q\right)}}{8P},$$

and $\mu_{H10} > 0$ requires $r < -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$. One can show that the interval $r1 < r \leq \min\{r2, -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}\}$ is non-empty if and only if $-\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)} \geq r1$, which is equivalent to $P \leq P^*$ for a unique threshold P^* implicitly characterized by $-\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)} = r1$.

The previous solution does not exist if, in contrast, $P > P^*$ (hence $r1 > -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$). As r increases from $r1$, the solution is

$w_{10}^H = \frac{\left(2P^2(1-2Q)^2+P\left(Q^2(8r+4)+4Q(r+1)-8r(r+1)+1\right)+2Q(2Qr+r+2)\right)}{P((4P^2-1)Q(4Q^2-1)+4P(4Q^2-1)r+2(2Q-1)r^2(2(P+1)Q+P))}$,

$w_{11}^H = \frac{(2P+1)(2PQ+P+2Qr)}{P((4P^2-1)Q(2Q+1)+2r^2(2(P+1)Q+P)+4P(2Q+1)r)}$; $\lambda_{Monitor} = \frac{(2P+1)(2Q+1)r}{(4P^2-1)Q(2Q+1)+2r^2(2(P+1)Q+P)+4P(2Q+1)r}$,

$\lambda_{H11\succ H00} = -\frac{\left(4P^2-1\right)Q(2Q+1)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r}$, $\mu_{H00} = -\frac{(2P+1)Q(2Q-1)r^2}{(4P^2-1)Q(2Q+1)+2r^2(2(P+1)Q+P)+4P(2Q+1)r}$,

$\mu_{L00} = \frac{r\left(2(2Q+1)(P+Q)(4PQ+1)-(2Q-1)r\left(4PQ^2+P+2Q\right)\right)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r}$, $\mu_{L10} = \frac{P\left(4Q^2-1\right)r(2Q(r-2)-r-2)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r}$,

$\mu_{L11} = -\frac{(2Q+1)r\left(P\left(8PQ+4Q^2(r-2)-r-2\right)+2Q\right)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r}$.

The non-negativity of w_{10}^H and μ_{L11} requires $r > r_1$ and $r > -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$, respectively; while $ICP_{H11\succ H00}$ and *Self-Shirking* impose an upper bound on r.

For Part (iii), note that the principal can always ignore the individual subjective measures and set $w_{mn}^H = \frac{(r+1)}{P(r+2)}$ and $w_{mn}^L = 0$ for all $m, n \in \{0, 1\}$, which is feasible for all r. ■

Proof of Lemma 4. For $r < rA \doteq \min\{\frac{(2P+1)Q^2}{P(2(P+1)Q-P)}, \frac{(2P+1)Q(3-2Q)}{4P}\}$, the optimal solution satisfies $w_{11}^H = \frac{\frac{4}{2(P+1)Q+P}+(2Q-1)w_{10}^H}{2Q+1}$. The principal can transfer between w_{11}^H and w_{10}^H, subject to the constraints. For instance, setting $w_{10}^H = 0$ is optimal for $r \le \frac{(2P+1)Q(2Q+1)}{4P}$, and setting $w_{10}^H = \frac{(2P+1)Q(2Q+1)-4rP}{P(2Q-1)r(2(P+1)Q+P)}$ is optimal for $\frac{(2P+1)Q(2Q+1)}{4P} < r < rA$. The two upper bounds of r is required by *No Cycling* and $ICP_{H10 \succ H00}$, respectively.

To prove Part (ii), we will show that the statement holds for $\underline{r} \doteq \frac{(2(P+1)Q-P)\left(4PQ^2+P-2Q\right)}{4P(P-Q)} < r < \bar{r} \doteq -\frac{4\left(P+2Q^2\right)}{(1-2Q)^2(2P(Q+1)-Q)}$. The optimal contract is

$$w_{00}^H = \frac{\left(P^2\left(2-8Q^2\right)-P\left(4Q^2(r+2)-4Q(r+1)+r(4r+5)\right)+4Q(r+1)\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))},$$
$$w_{10}^H = -\frac{2\left(r\left(P\left(4Q^2+5\right)-2Q\right)+2(P+1)\left(4PQ^2+P-2Q\right)+4Pr^2\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))},$$
$$w_{11}^H = -\frac{\left(r\left(P\left(4Q^2+5\right)-2Q\right)+2(P+1)\left(4PQ^2+P-2Q\right)+4Pr^2\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))},$$
$$w_{11}^L = -\frac{2(2P+1)(P+Qr)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}.$$

Substituting the payments verifies $\Delta U = 0$. Given the maintaining assumption $P < \frac{Q}{2(1+Q)}$, one can verify that $\bar{r} > \underline{r}$ is equivalent to $P > \underline{\Delta}$ for a uniquely determined $\underline{\Delta}$.

To prove Part (iii), note that for $r > -\frac{4\left(P+2Q^2\right)}{(1-2Q)^2(2P(Q+1)-Q)}$, the solution is

$$w_{00}^H = \frac{P\left(4\left((Q-2)Qr+Q(3Q-1)+r^2\right)+7r+3\right)+2Q(2Q(r+2)-3r-2)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$w_{10}^H = \frac{2\left(4P^2+P\left(4Q^2(r+3)-4Q(r+1)+r(4r+7)+5\right)+4Q(Q(r+2)-r-1)\right)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$w_{11}^H = \frac{4P^2+P\left(4Q^2(r+3)-4Q(r+1)+r(4r+7)+5\right)+4Q(Q(r+2)-r-1)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$w_{10}^L = \frac{4(2P+1)(P+Qr)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$w_{11}^L = \frac{2(2P+1)(P+Qr)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))}.$$
$$\lambda_{CYC} = \frac{2(r+2)(2Pr+r)}{P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4)},$$
$$\lambda_{H11 \succ H00} = -\frac{(2P+1)Q(2Q+1)((2Q-1)r-4)}{4P(4Q^2(r+2)-8Qr+(r+2)(4r+3))-4Q(4Q^2r-8Q(r+1)+r(3-4r)+4)},$$
$$\lambda_{L11 \succ L00} = \frac{\left(4Q^2-1\right)r(2P(Q+1)-Q)+4(2Q+1)(Q-P)}{4P(4Q^2(r+2)-8Qr+(r+2)(4r+3))-4Q(4Q^2r-8Q(r+1)+r(3-4r)+4)},$$
$$\lambda_{H10 \succ H00} = \frac{(2P+1)Q(4Q((Q-1)r-2)+r)}{2(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$\lambda_{L10 \succ L00} = -\frac{(1-2Q)^2r(2P(Q+1)-Q)+4P+8Q^2}{2(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$\mu_{L00} = \frac{4Pr(-2Qr+r+2)}{P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4)}.$$

Taking the limit shows that, as $r \to \infty$, $w_{mn}^L = 0, w_{11}^H = w_{00}^H = \frac{1}{2}w_{10}^H = \frac{1}{P+Q}$. ■

Proof of Proposition 3. We list the complete optimal solutions, taking as given team incentives ($T = 1$) or individual incentives ($T = 0$), separately. Endogenizing $T = \{0, 1\}$ then follows from comparing the expected payments across the two programs.

Solutions given $T = 0$:

SolA: the optimal contract for $r \leq \frac{(2P+1)Q^2}{P(2(P+1)Q-P)}$ is shown in Part (i) of Lemma 4. The maintaining assumption $P < \frac{Q}{2(1+Q)}$ helps simplify the upper bound on r.

SolB: for $\frac{(2P+1)Q^2}{P(2(P+1)Q-P)} < r \leq \frac{P^2\left(2Q\left(-4Q^2+6Q+1\right)-3\right)+8PQ-8Q^2}{4P(2(P-1)Q+P)}$, the solution is $w_{10}^H = \frac{1}{Q}$, $w_{11}^H = \frac{P^2(2Q+1)(1-2Q)^2-4PQ+4Q^2}{PQ(P(2Q+1)(4Q^2-4r-1)+8Qr)}$, $w_{11}^L = -\frac{4\left(P^2r+(2P+1)Q^2-2(P+1)PQr\right)}{PQ(P(2Q+1)(4Q^2-4r-1)+8Qr)}$. The binding constraints are $w_{00}^H = w_{00}^L = w_{10}^L = 0$, *Static NE*, and $ICP_{H11\succ H00}$. The non-negativity $w_{11}^L > 0$ and the constraint $ICP_{H10\succ H00}$ impose the lower and upper bound on r, respectively. Note that we identify the binding constrains (rather than listing each Lagrangian multipliers) in this proposition for space considerations.

SolC: for $\frac{P^2\left(2Q\left(-4Q^2+6Q+1\right)-3\right)+8PQ-8Q^2}{4P(2(P-1)Q+P)} < r \leq \frac{-P^2(1-2Q)^2(2Q-3)+4P(2Q-1)Q-8Q^2}{8(P-1)PQ-4P^2}$, the solution is $w_{10}^H = \frac{8(P-Q)}{P(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)}$, $w_{11}^H = \frac{4(P-Q)}{P(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)}$, and $w_{10}^L = \left[\frac{4((2P+1)Q(2Q-3)+4Pr)}{P(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)} + 2Qw_{11}^L + w_{11}^L\right]/(2Q-1)$. The solution is not unique: the principal can transfer between w_{11}^L and w_{10}^L subject to the constraints. For instance, setting $w_{10}^L = 0$ is optimal for $r \leq Q\left(\frac{1}{P} - Q + 1\right) - \frac{1}{4}$; while, for higher r, setting $w_{10}^L = \frac{4\left(P\left((1-2Q)^2+4r\right)-4Q\right)}{P(2Q-1)(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)}$ is optimal. The binding constraints are $w_{00}^H = w_{00}^L = 0$, *Static NE*, $ICP_{H11\succ H00}$, and $ICP_{H10\succ H00}$. The upper bound of r is imposed by *No Cycling* constraint.

SolD: for $\frac{-P^2(1-2Q)^2(2Q-3)+4P(2Q-1)Q-8Q^2}{8(P-1)PQ-4P^2} < r \leq \tilde{r} \doteq \frac{(2(P+1)Q-P)\left(4PQ^2+P-2Q\right)}{4P(P-Q)}$, the solution is $w_{00}^H = \frac{(2Q+1)\left(P^2(2Q-1)(4(Q-2)Q+4r+3)-4PQ(2Q+2r-1)+8Q^2\right)}{2PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}$,

$$w_{10}^H = \frac{\left(P^2(2Q-1)\left(2Q\left(4Q^2-2Q+4r+1\right)+4r-1\right)-2PQ(4Q(3Q+2r-1)+4r+3)+8Q^2\right)}{PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))},$$

$$w_{11}^H = \frac{\left(P^2(2Q-1)\left(2Q\left(4Q^2-2Q+4r+1\right)+4r-1\right)-2PQ(4Q(3Q+2r-1)+4r+3)+8Q^2\right)}{2PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))},$$

$$w_{10}^L = \frac{2\left(P^2\left(8Q^3-4Q^2+2Q-4r-1\right)+4PQ(Q(2Q-1)+r+1)-4Q^2\right)}{PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}, w_{11}^L = \frac{\left(P^2\left(8Q^3-6Q+2\right)+P(-4(Q-1)Q-5)Q-8Q^3\right)}{PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}.$$

The binding constraints are $w_{00}^L = 0$, *Static NE*, *No Cycling*, $ICP_{H11\succ H00}$, $ICP_{H10\succ H00}$, and $ICP_{L11\succ L00}$. The non-negativity constraints $w_{00}^H > 0$ and $w_{10}^L > 0$ imposes the lower and upper bound of r, respectively.

SolE: for $\tilde{r} < r \leq r3 \doteq -\frac{4\left(P+2Q^2\right)}{(1-2Q)^2(2P(Q+1)-Q)}$, the solution is the one shown in Lemma 4 - Part (ii).

SolF: for $r > r3$, the solution is the one shown in Lemma 4 - Part (iii).

Solutions given $T = 1$:

Sol1: The optimal contract for $r \leq r1$ is *Pure JPE* shown in Part (i) of Lemma 3.

Sol2: Part (ii) of Lemma 3 (the one with $w_{11}^L > 0$) specifies the contract for $r1 < r \leq r2 \doteq \frac{4PQ^2+\sqrt{(4(P+1)Q^2-5P+2Q)^2-16P(P(1-2Q)^2-4Q)}-5P+4Q^2+2Q}{8P}$. The maintaining assumption $P < \frac{Q}{2(1+Q)}$ assures the existence of the interval $(r1, r2]$.

Sol3: for $r2 < r \leq -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$, the solution is
$w_{10}^H = \frac{4\left(P\left(-4Q^2(r-1)-4Q+(r+1)(4r+1)\right)-2Q(2Qr+r+2)\right)}{P(P(4Q^2-1)(4Q^2(r-2)-(r+2)(4r+1))+4Q(2Q-1)(2Q-2r^2+1))}$,
$w_{11}^H = -\frac{2(2P+1)(2PQ+P+2Qr)}{P(P(2Q+1)(4Q^2(r-2)-(r+2)(4r+1))+4Q(2Q-2r^2+1))}$,
$w_{11}^L = -\frac{2(2P+1)(2PQ+P+2Qr)}{P(P(2Q+1)(4Q^2(r-2)-(r+2)(4r+1))+4Q(2Q-2r^2+1))}$. The binding constraints are $w_{00}^H = w_{00}^L = w_{10}^L = 0$, *Mutual Monitoring*, $ICP_{H11 \succ H00}$, and $ICP_{L11 \succ L00}$. The non-negativity $w_{10}^H > 0$ and $ICP_{H10 \succ H00}$ impose the lower and upper bound of r, respectively.

Sol4: for $-\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)} < r \leq \tilde{r}$, a feasible solution is
$w_{00}^H = \frac{P^2\left(2-8Q^2\right)-P\left(4Q^2(r+2)-4Q(r+1)+r(4r+5)\right)+4Q(r+1)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$,
$w_{10}^H = -\frac{2\left(r\left(P\left(4Q^2+5\right)-2Q\right)+2(P+1)\left(4PQ^2+P-2Q\right)+4Pr^2\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$,
$w_{11}^H = \frac{r\left(2Q-P\left(4Q^2+5\right)\right)-2(P+1)\left(4PQ^2+P-2Q\right)-4Pr^2}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$,
$w_{11}^L = -\frac{2(2P+1)(P+Qr)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$.
$w_{00}^H > 0$ and *Pareto* require $r > r3$ and $r < \tilde{r}$, respectively. The solution is optimal if the parameters satisfy $\tilde{r} \leq \frac{4P(2Q+1)}{(2Q-1)(2P(Q+1)-Q)}$ (the binding constraints are $w_{00}^L = w_{10}^L = 0$, *Mutual Monitoring*, $ICP_{H11 \succ H00}$, $ICP_{L11 \succ L00}$, and $ICP_{H10 \succ H00}$.) Setting $w_{10}^L = 0$ is no longer optimal if the condition is violated (i.e., $\tilde{r} > \frac{4P(2Q+1)}{(2Q-1)(2P(Q+1)-Q)}$), in which case, depending the value of r, either w_{00}^H or *Self-Shirking* replaces $w_{10}^L = 0$ as the binding constraint in the optimal solution. We verify that Part (iii) of the proposition holds for both cases as well.

Sol 5: for $\tilde{r} < r \leq -\frac{P^2-4PQ^3+4(P(3P+2)+1)Q^2-3PQ}{4P(P-Q)}$, the optimal solution is
$w_{00}^H = \frac{P^2(2Q-1)\left(12Q^2+4r+1\right)+4PQ\left((1-2Q)^2+2r\right)-8Q^2}{2PQ(2Q+1)(4PQ^2+4Pr+P-2Q)}$, $w_{11}^H = \frac{2P+1}{4PQ^2+4Pr+P-2Q} + w_{00}^H$, $w_{11}^L = \frac{2P+1}{4PQ^2+4Pr+P-2Q}$,
$w_{10}^H = \frac{P^2\left(4Q^2+1\right)\left(12Q^2+4r+1\right)+2PQ(4Q(Q(4Q-3)+2r+1)-1)-16Q^3}{PQ(2Q+1)^2(4PQ^2+4Pr+P-2Q)}$. The binding constraints are $w_{00}^L = w_{10}^L = 0$, *Self-Shirking*, $ICP_{H11 \succ H00}$, $ICP_{L11 \succ L00}$, and *Pareto*. $ICP_{H11 \succ H10}$ and *Mutual Mon-*

itoring require the upper and lower bound on r, respectively.

Sol 6: for $r > -\frac{P^2-4PQ^3+4(P(3P+2)+1)Q^2-3PQ}{4P(P-Q)}$, the solution is $w_{00}^H = \frac{1-\frac{2(2P+1)Q}{4PQ^2+4Pr+P-2Q}}{2P}$, $w_{01}^H = \frac{\frac{\left(8P^2-2\right)Q}{4PQ^2+4Pr+P-2Q}-\frac{P}{Q}+1}{2P}$, $w_{10}^H = \frac{-\frac{2(2PQ+Q)^2}{4PQ^2+4Pr+P-2Q}+P+Q}{2PQ}$, $w_{11}^H = \frac{P(4P+4(Q-1)Q+4r+3)-4Q}{2P(4PQ^2+4Pr+P-2Q)}$, $w_{11}^L = \frac{2P+1}{4PQ^2+4Pr+P-2Q}$. The binding constraints are $w_{00}^L = w_{10}^L = w_{01}^L = 0$, *Mutual Monitoring*, $ICP_{H11\succ H00}$, $ICP_{L11\succ L00}$, $ICP_{H11\succ H10}$, and *Pareto*. The non-negativity of w_{01}^H imposes the lower bound on r.

Endogenizing $T = \{0,1\}$: Having solved the optimal solution given $T = \{0,1\}$, endogenizing the choice of T is conceptually straightforward: comparing the expected payments between the two program. We sketch the three main steps of the comparison and skip the tedious algebra. First, the relevant solutions for $r \leq \tilde{r}$ are SolA to SolD (given $T = 0$) and Sol1 to Sol4 (given $T = 1$). Second, tedious but straightforward algebra shows that Sol1 to Sol4 are overall optimal as long as they are optimal given $T = 1$. Finally, Sol5 and Sol6 of $T = 1$ are never the overall optimal contract. The argument is similar to that in Claim 4 of the proof of Proposition 1: for parameters over which the two solutions are optimal given $T = 1$, they satisfy all the potentially binding constraints of program with $T = 0$. ■

Proof of Corollary 3. Part (i) follows from the fact that the program of $T = 1$ does not include *No Cycling* – the only difference from Lemma 5. One can verify that solutions in Lemma 5 satisfy strategic complements, i.e., $U(1,1) - U(0,1) > U(1,0) - U(0,0)$; and that strategic complements and the *Pareto Dominance* together imply $2U(1,1) > U(0,1)+U(1,0)$. Therefore, agents will not collude on playing any strategies that involve only *work* and *shirk*.

For $T = 0$, it is straightforward to plug in the new cycling constraint, investigate the Lagrangian, and verify the first part of Corollary 3 - (ii). The second part of (ii) follows from the observation that the *No Cycling* constraint is more restrictive than the restated cycling constraint. Note that the two agents are willing to collude on playing *work* and *shirk* only if their *joint* payoff satisfies $U(1,0) + U(0,1) > 2U(1,1)$. Given $T = 0$, $U(1,0) + U(0,1) > 2U(1,1)$ implies $U(1,0) - U(1,1) > U(1,1) - U(0,1) \geq 0$, where the last inequality follows from the *Static NE* constraint that must hold if $T = 0$. The observation $U(1,0) - U(1,1) >$

$U(1,1)-U(0,1) \geq 0$ further implies $U(1,0) > U(1,1) > U(0,1)$, which together with the time value argument, suggests that our *No Cycling* constraint provides strictly higher continuation payoff for the shirking agent in (0,1) (hence more costly for the principal to break) than any collusive strategy having agents randomizes between *work* and *shirk*.

Part (iii) follows directly form parts (i) and (ii) of the Corollary. ■

Appendix B: Stationary Collusive Strategies

In addition to allowing for a joint bonus floor, another difference between our paper and Kvaløy and Olsen (2006) is that they confine attention to stationary collusive strategies using a correlation device, while we allow for non-stationary collusive strategies but in pure strategies. In particular, stationary collusive strategies are characterized in Kvaløy and Olsen (2006) as probabilities $(a, b, b, 1 - a - 2b)$ on effort combinations of $(1,1), (1,0), (0,1)$, and $(0,0)$, respectively. If we allow for such correlated strategies (while keeping the ability to commit to a bonus floor), Program **P** will be intact except that (No Cycling) would be restated as follows:

$$\frac{1+r}{r}\left[\pi(1,1;w) - 1\right] \geq \pi(0,1;w) + \frac{[\pi(1,0;w) - 1 + \pi(0,1;w)]/2}{r}.$$

The second term of the right hand side of the equation is the continuation payoff for the shirking agent from indefinitely playing either $(1,0)$ or $(0,1)$ with equal probability in each period. Similar argument as Lemma 1 shows that a contract is collision proof if it satisfies both the (No Joint Shirking) and the modified cycling constraint above. We also solve Program **P** with the restated cycling constraint. The corollary below shows that the restated cycling constraint does not qualitatively change the nature of the optimal contract studied in the main model.

Corollary 3 *If we confine attention to stationary collusive strategies,*

(i) Given $T = 1$, the wage contract and the cutoffs are same as in Lemma 5.

(ii) Given $T = 0$, the wage contract and the cutoffs are characterized by the same binding constraints as those in Lemma 6. The expected wage payment, $\pi(1,1)$, is weakly lower than that in Lemma 6.

(iii) The overall optimal contract is same as Proposition 1 (the closed-form of cutoffs r^D and r^H are different).

Part (i) shows that the way we formulate cycling constraint is irrelevant for the team incentive case ($T = 1$). This is straightforward if we recall from Program **P** that team incentive case does not include any collusion-proof constraint. For individual incentives ($T = 0$), the restated *No Cycling* constraint only causes changes of a few cutoffs of the optimal contract. As Corollary 3 - Part (iii) shows, the optimal contract is qualitatively unaffected by the alternative way to formulate the cycling collusive strategies. In particular, the main insight is that that having a positive bonus floor ($w_{LL} > 0$) is still useful for the same reason as in the main model, either to encourage mutual monitoring or to mitigate agents' collusion problem.

Contrasting Corollary 3 with Proposition 2 makes it clear that the qualitative difference of our paper and Kvaløy and Olsen (2006) is due the the principal's ability to commit to a positive bonus floor. Nonetheless, we find it more appealing to formulate the collusive strategy as we did in the main model (without the correlated device) because the agents earns more rent by colluding on it than on the stationary collusive strategy studied in Kvaløy and Olsen (2006). This is shown in the second part of Corollary 3 - Part (ii), which suggests the *No Cycling* constraint formulated in our main model is more restrictive than the correlated cycling strategy above. Intuitively, because of the time value, the *Cycle* strategy formatted in the text gives the shirking agent a higher continuation payoff (hence more difficult for the principal to upset) than what he would receive under the correlated collusive strategy.