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Financial Fragility with SAM?

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Abstract

Shared Appreciation Mortgages (SAMs) feature mortgage payments that adjust with house prices. These mortgage contracts are designed to stave off home owner default by providing payment relief in the wake of a large house price shock. SAMs have been hailed as an innovative solution that could prevent the next foreclosure crisis, act as a work-out tool during a crisis, and alleviate fiscal pressure during a downturn. They have inspired fintech companies to offer home equity contracts. However, the home owner's gains are the mortgage lender's losses. A general equilibrium model with financial intermediaries who channel savings from saver households to borrower households shows that indexation of mortgage payments to aggregate house prices increases financial fragility, reduces risk sharing, and leads to expensive financial sector bailouts. In contrast, indexation to local house prices reduces financial fragility and improves risk-sharing. The two types of indexation have opposite implications for wealth inequality.

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1 Introduction

The \$10 trillion market in U.S. mortgage debt is the world's largest consumer debt market and its second largest fixed income market. Mortgages are not only the largest liability for U.S. households, they are also the largest asset of the U.S. financial sector. Banks and credit unions hold \$3 trillion in mortgage loans directly on their balance sheets in the form of whole loans, and an additional \$2.2 trillion in the form of mortgage-backed securities.¹ Given the exposure of the financial sector to mortgages, large house price declines and the default wave that accompanies them can severely hurt the solvency of the U.S. financial system. This became painfully clear during the Great Financial Crisis of 2008-2011. Moreover, exposure to interest rate risk could represent an important source of financial fragility going forward if mortgage rates rise from historic lows.

In this paper we study the allocation of house price and interest rate risk in the mortgage market between mortgage borrowers, financial intermediaries, and savers. The standard 30-year fixed-rate mortgage (FRM) dictates a particular distribution of these risks: borrower home equity absorbs the initial house price declines, until a sufficiently high loan-to-value ratio, perhaps coupled with an adverse income shock, leads the homeowner to default, inflicting losses on the lender. As a result, lenders only bear the risk of large house price declines.

During the recent housing crash, U.S. house prices fell 30% nationwide, and by much more in some regions. The financial sector had written out-of-the-money put options on aggregate house prices with more than \$5 trillion in face value, and the downside risk materialized. About 25% of U.S. home owners were underwater by 2010 and seven million forecloses ensued. Charge-off rates of residential real estate loans at U.S. banks went from 0.1% in mid-2006 to 2.8% in mid-2009, and remained above 1% until the end of 2012. Only by mid-2016 did they return to their level from a decade earlier. The stress on banks' balance sheets caused lenders to dramatically tighten mortgage lending standards, precluding many home owners from refinancing their mortgage and take advantage of the low interest rates. Homeowners' reduced ability to tap into their housing wealth short-circuited the stimulative consumption response from lower mortgage rates that policy makers hoped for.

This crisis led many to ask whether a fundamentally different mortgage finance sys-

¹Including insurance companies, money market mutual funds, broker-dealers, and mortgage REITs in the definition of the financial sector adds another \$1.5 trillion to the financial sector's agency MBS holdings. Adding the Federal Reserve Bank and the GSE portfolios adds a further \$2 trillion and increases the share of the financial sector's holdings of agency MBS to nearly 80%.

tem could lead to a better risk sharing arrangement between borrowers and lenders.² While contracts offering alternative allocations of interest rate risk are already widely available — most notably, the adjustable rate mortgage (ARM), which offers nearly perfect pass-through of interest rates — contracts offering alternative divisions of *house price* risk are essentially unavailable to the typical household. To fill this gap, researchers have begun to design and analyze such contracts.

The most well known proposal is the shared appreciation mortgage (SAM). The SAM indexes mortgage payments to house price changes. In the fully symmetric version, payments are linked to house prices — increasing when they rise and decreasing when they fall — making the contract more equity-like. Such a contract ensures that the borrower receives payment relief in bad states of the world, potentially reducing mortgage defaults and the associated deadweight losses to society. On the other hand, SAMs impose losses on mortgage lenders in these adverse aggregate states, which may increase financial fragility at inopportune times. We argue for a shift in focus in the mortgage design debate from a *household risk management* focus to a *system-wide risk management* focus. The main goal of this paper is to quantitatively assess whether SAMs present a better arrangement to the overall economy than FRMs.

We model the interplay between mortgage borrowers, mortgage lenders, and savers. All agents face aggregate labor income risk. Borrowers also face idiosyncratic house valuation shocks, which affect their optimal mortgage default decision. At lower frequencies, the economy transits between a normal state and a crisis state featuring high house price uncertainty (cross-sectional dispersion of the house valuation shocks) and a fall in aggregate home values. These crises strongly influence the economy-wide mortgage default rate and the key source of aggregate financial risk in this economy. Mortgage lenders make long-term, defaultable, prepayable mortgage loans to impatient borrowers, funded by deposits raised from patient savers. Borrowers face a maximum loan-to-value constraint, but only at loan origination, while banks face their own leverage constraint, capturing macro-prudential bank equity capital requirements.

We contrast this economy to an economy with SAMs. We study SAMs whose payments are indexed to aggregate house prices, as well as SAMs whose payments are partially indexed to idiosyncratic house price risk. We interpret the partial insurance against idiosyncratic house price risk as indexation to local price fluctuations, which is often used in place of direct indexation to individual house values to reduce moral hazard.

²The New York Federal Reserve Bank organized a two-day conference on this topic in May 2015 with participants from academia and policy circles.

Surprisingly, aggregate indexation reduces borrower welfare even though it (slightly) reduces mortgage defaults, because it amplifies financial fragility. Intermediary wealth falls substantially in crises as mortgage lenders absorb house price declines. The bank failure rate increases, triggering bailouts that must ultimately be funded by taxpayers, including the borrowers. Equilibrium house prices are lower and fall more in crises with aggregate indexation. Ironically, intermediary welfare increases as they reap the profits from selling foreclosed houses back to borrowers, as well as from the larger mortgage spreads lenders are able to charge in a riskier financial system.

In contrast, by partially indexing mortgage payments and principal to individual house valuation shocks, SAMs can eliminate most mortgage defaults. By extension, local indexation reduces bank failures and fluctuations in intermediary net worth substantially. Banking becomes safer, but also less profitable, due to a fall in mortgage spreads. Lower bank failure rates generate fewer deadweight costs and lower maintenance expenses from houses in foreclosure, so that more resources are available for consumption. Welfare of borrowers and savers rises, at the expense of that of bank owners.

Section 2 discusses the related literature. Section 3 presents the theoretical model. Section 4 characterizes the solution. Section 5 discusses its calibration. The main results are in section 6. Section 7 concludes. Model derivations are relegated to the appendix.

2 Related Literature

This paper contributes to the literature that studies innovative mortgage contracts. While an extensive body of work studies designs to mitigate an array of interest rate indexation and amortization schemes, we focus on mortgage contracts that are indexed to house prices.³

In early work, [Shiller and Weiss \(1999\)](#) discuss the idea of home equity insurance policies. The idea of SAMs was discussed in a series of papers by [Caplin, Chan, Freeman, and Tracy \(1997\)](#); [Caplin, Carr, Pollock, and Tong \(2007\)](#); [Caplin, Cunningham, Engler, and Pollock \(2008\)](#). They envision a SAM as a second mortgage in addition to a conventional FRM with a smaller principal balance. The SAM has no interest payments and its

³Related work on contract schemes other than house price indexation include [Piskorski and Tchisty \(2011\)](#), who study optimal mortgage contract design in a partial equilibrium model with stochastic house prices and show that option-ARM implements the optimal contract; [Kalotay \(2015\)](#), who considers automatically refinancing mortgages or ratchet mortgages (whose interest rate only adjusts down); and [Eberly and Krishnamurthy \(2014\)](#), who propose a mortgage contract that automatically refinances from a FRM into an ARM, even when the loan is underwater.

principal needs to be repaid upon termination (e.g., sale of the house). At that point the borrower shares a fraction of the house value appreciation with the lender, but only if the house has appreciated in value. The result is lower monthly mortgage payments throughout the life of the loan, which enhances affordability, and a better sharing of housing risk. They emphasize that SAMs are not only a valuable work-out tool after a default has taken place, but are also useful to prevent a mortgage crisis in the first place.⁴

Recently, Mian (2013) and Mian and Sufi (2014) introduced a version of the SAM, which they call the Shared Responsibility Mortgage (SRM). The SRM replaces a FRM rather than being an additional mortgage. It features mortgage payments that adjust down when the local house price index goes down, and back up when house prices bounce back, but never above the initial FRM payment. To compensate the lender for the lost payments upon house price declines, the lender receives 5% of the home value appreciation. They argue that foreclosure avoidance raises house prices in a SRM world and shares wealth losses more equitably between borrowers and lenders. When borrowers have higher marginal propensities to consume out of wealth than lenders, this more equitable sharing increases aggregate consumption and reduces job losses that would be associated with low aggregate demand. The authors argue that SRMs would reduce the need for counter-cyclical fiscal policy and give lenders an incentive to “lean against the wind” by charging higher mortgage rates when house price appreciation seems excessive.

Shared appreciation mortgages have graduated from the realm of the hypothetical. They have been offered to faculty at Stanford University for leasehold purchases for fifteen years (Landvoigt, Piazzesi, and Schneider, 2014). More recently, several fintech companies such as FirstREX and EquityKey have been offering home equity products where they offer cash today for a share in the future home value appreciation.⁵ These products

⁴Among the implementation challenges are (i) the uncertain holding period of SAMs, (ii) returns on investment that decline with the holding period, and (iii) the tax treatment of SAM lenders/investors. The first issue could be solved by a maximum maturity provision of say 15 years. The second issue can be solved by replacing the lender’s fixed appreciation share by a shared-equity rate. For example, instead of 40% of the total appreciation, the investor would have a 4% shared-equity rate. If the holding period of the SAM is 10 years and the original SAM principal represented 20% of the home value, the lender is entitled to the maximum of the SAM principal and $20\% \times (1.04)^{10} = 29.6\%$ of the terminal home value. This scheme delivers an annual rate of return to the lender that is constant rather than declining in the holding period. The authors refer to this variant as SAMANTHA, a SAM with A New Treatment of Housing Appreciation.

⁵EquityKey started issuing such shared equity contracts in the early 2000s. It was bought by a Belgian retail bank in 2006. The founders bought the business back from the Belgian bank after the housing crisis and resumed its activities. In 2016, the company closed its doors after the hedge fund that funded the operations lost interest. FirstREX changed its name to Unison Home Ownership Investors in December 2016. It has been making home ownership investments since March 2004. Its main product offers up to half of the down payment in exchange for a share of the future appreciation. The larger down payment eliminates the need for mortgage insurance. Its product is used alongside a traditional mortgage, just like

are presented as an alternative to home equity lines of credit, closed-end second mortgages, reverse mortgages for older home owners, or to help finance the borrower's down payment at the time of home purchase. They allow the home owner to tap into her home equity without taking on a new debt contract. Essentially, the home owner writes a call option on the local house price index (to avoid moral hazard issues) with strike price equal to the current house price value and receives the upfront option premium in exchange. Our work sheds new light on the equilibrium implications of introducing home equity products.

Kung (2015) studies the effect of the disappearance of non-agency mortgages for house prices, mortgage rates and default rates in an industrial organization model of the Los Angeles housing market. He also evaluates the hypothetical introduction of shared appreciation mortgages in the 2003-07 housing boom. He finds that symmetric SAMs would have enjoyed substantial uptake, partially supplanting non-agency loans, and would have further exacerbated the boom. They would not have mitigated the bust. Our model is an equilibrium model of the entire U.S. market with an endogenous risk-free rate rather than of a single city where households face an exogenously specified outside option of moving elsewhere and constant interest rates. Our lenders are not risk neutral, and charge an endogenously determined risk premium on mortgages. When lenders are risk neutral, they are assumed to be better able to bear house price risk than risk averse households. That seems like a fine assumption when all house price risk is idiosyncratic. However, banks may be severely negatively affected by aggregate house price declines and SAMs may exacerbate that financial fragility.

Hull (2015) studies house price-indexed mortgage contracts in a simple incomplete markets equilibrium model. He finds that such mortgages are associated with lower mortgage default rates and higher mortgage interest rates than standard mortgages. Our analysis features aggregate risk, long-term prepayable mortgage debt, and an intermediary sector that is risk averse.

Two contemporaneous papers also study mortgage design questions in general equilibrium. **Piskorski and Tchisty (2017)** study mortgage design from first principles in a tractable, risk neutral environment, emphasizing asymmetric information about home values between borrowers and unconstrained lenders. This setting yields closed-form solutions for the optimal contract, which takes the form of a Home Equity Insurance Mortgage that eliminates the strategic default option and insures borrower's home eq-

the original SAM contract. Unison is active in 13 U.S. states and plans to add 8 more states in 2017. It is funded by 8 lenders.

uity. They study the implications of this equilibrium contract for welfare relative to a fixed-rate mortgage benchmark. Our setup features risk averse borrowers and lenders, and focuses on the levered financial sector, bringing issues relating to risk sharing and financial fragility front and center.

Next, [Guren, Krishnamurthy, and McQuade \(2017\)](#) investigate the interaction of ARM and FRM contracts with monetary policy. They study an FRM that costlessly converts to an ARM in a crisis so as to provide concentrated payment relief in a crisis. These authors focus on interest rate risk, contrasting e.g., adjustable-rate and fixed-rate mortgages. Since interest rate risk is relatively easy for banks to hedge, these authors abstract from implications for financial sector fragility, instead emphasizing a rich borrower risk profile that includes a life cycle and uninsurable idiosyncratic income risk. In contrast, our framework considers house price risk that is difficult for banks to hedge, and emphasizes the role of the intermediation sector. We see both of these approaches as highly complementary to our own.

This study also connects to the macro-housing literature more generally. [Elenev, Landvoigt, and Van Nieuwerburgh \(2016\)](#) studies the role the default insurance provided by the government-sponsored enterprises, Fannie Mae and Freddie Mac. They consider an increase in the price of insurance that restores the absorption of mortgage default risk by the private sector and show it leads to an allocation that is a Pareto improvement. This paper introduces SAMs, REO housing stock dynamics, and long-term mortgages whose rate does not automatically readjusts every period. [Greenwald \(2016\)](#) studies the interaction between the payment-to-income and the loan-to-value constraint in a model of monetary shock transmission through the mortgage market but without default. [Favilukis, Ludvigson, and Van Nieuwerburgh \(2017\)](#) study the role of relaxed down payment constraints in explaining the house price boom. [Corbae and Quintin \(2014\)](#) investigate the effect of mortgage product innovation in a general equilibrium model with default. [Guren and McQuade \(2016\)](#) study the interaction of foreclosures and house prices in a model with search.

Our paper also relates to the literature that studies the amplification of business cycle shocks provided by credit frictions. E.g., [Bernanke and Gertler \(1989\)](#), [Bernanke, Gertler, and Gilchrist \(1996\)](#), [Kiyotaki and Moore \(1997\)](#), and [Gertler and Karadi \(2011\)](#). A second generation of models has added nonlinear dynamics and a richer financial sector. E.g., [Brunnermeier and Sannikov \(2014\)](#), [He and Krishnamurthy \(2012\)](#), [He and Krishnamurthy \(2013\)](#), [He and Krishnamurthy \(2014\)](#), [Gârleanu and Pedersen \(2011\)](#), [Adrian and](#)

Boyarchenko (2012), Maggiori (2013), Moreira and Savov (2016), and Elenev, Landvoigt, and Van Nieuwerburgh (2017). Our solution uses a state-of-the-art global non-linear solution technique of a problem with occasionally binding constraints.

Finally, we connect to recent empirical work that has found strong consumption responses and lower default rates (Fuster and Willen, 2015) to exogenously lowered mortgage interest rates Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017) and to higher house prices (Mian and Sufi, 2009; Mian, Rao, and Sufi, 2013).

3 Model

3.1 Demographics

The economy is populated by a continuum of agents of three types: borrowers (denoted B), depositors (denoted D), and intermediaries (denoted I). The measure of type j in the population is denoted χ_j , with $\chi_B + \chi_D + \chi_I = 1$.

3.2 Endowments

The two consumption goods in the economy — nondurable consumption and housing services — are provided by two Lucas trees. The overall endowment Y_t is equal to a stationary component \tilde{Y}_t scaled by a deterministic component that grows at a constant rate g :

$$Y_t = e^{gt} \tilde{Y}_t,$$

where $\mathbb{E}(\tilde{Y}_t) = 1$ and

$$\log \tilde{Y}_t = (1 - \rho_y) \mu_y + \rho_y \log \tilde{Y}_{t-1} + \sigma_y \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim N(0, 1). \quad (1)$$

The $\varepsilon_{y,t}$ can be interpreted as transitory shocks to the level of aggregate labor income. For nondurable consumption, each agent type j receives a fixed share s_j of the overall endowment Y_t , which cannot be traded.

Shares of the housing tree are in fixed supply. Shares of the tree produce housing services proportional to the stock, growing at the same rate g as the nondurable endowment. Housing also requires a maintenance cost proportional to its value, ν^K . Housing capital is divided among the three types of households in constant shares, $\bar{K} = \bar{K}^B + \bar{K}^I + \bar{K}^D$. Households can only trade housing capital with members of their own type.

3.3 Preferences

Each agent of type $j \in \{B, D, I\}$ has preferences following [Epstein and Zin \(1989\)](#), so that lifetime utility is given by

$$U_t^j = \left\{ (1 - \beta_j) (u_t^j)^{1-1/\psi} + \beta_j \left(\mathbb{E}_t \left[(U_{t+1}^j)^{1-\gamma_j} \right] \right)^{\frac{1-1/\psi}{1-\gamma_j}} \right\}^{\frac{1}{1-1/\psi}} \quad (2)$$

$$u_t^j = (C_t^j)^{1-\zeta_t} (H_t^j)^{\zeta_t} \quad (3)$$

where C_t^j is nondurable consumption and H_t^j is housing services, and the preference parameter ζ_t is allowed to vary with the state of the economy. Housing capital produces housing services with a linear technology. We denote by Λ^j the intratemporal marginal rate of substitution (or stochastic discount factor) of agent j .

3.4 Financial Technology

There are two financial assets in the economy: mortgages that can be traded between the borrower and the intermediary, and deposits that can be traded between the depositor and the intermediary.⁶

Mortgage Contracts. Mortgage contracts are modeled as nominal perpetuities with payments that decline geometrically, so that one unit of debt yields the payment stream $1, \delta, \delta^2, \dots$ until prepayment or default. The interest portion of mortgage payments can be deducted from taxes. New mortgages face a loan-to-value constraint (shown below in (11)) that is applied at origination only, so that borrowers do not have to delever if they violate the constraint later on.

Borrower Refinancing. Non-defaulting borrowers can choose at any time to obtain a new mortgage loan and simultaneously re-optimize their housing position. If a refinancing borrower previously held a mortgage, she must first prepay the principal balance on the existing loan before taking on a new loan.

The transaction cost of obtaining a new loan is proportional to the balance on the new loan M_t^* , given by $\kappa_{i,t} M_t^*$, where $\kappa_{i,t}$ is drawn i.i.d. across borrowers and time from a

⁶Equivalently, households are able to trade a complete set of state-dependent securities with households of their own type, providing perfect insurance against idiosyncratic consumption risk, but cannot trade these securities with members of the other types.

distribution with CDF Γ_κ . Since these costs largely stand in for non-monetary frictions such as inertia, these costs are rebated to borrowers and do not impose an aggregate resource cost. We assume that borrowers must commit in advance to a refinancing policy that can depend in an unrestricted way on $\kappa_{i,t}$ and all aggregate variables, but cannot depend on the borrower's individual loan characteristics. This setup keeps the problem tractable by removing the distribution of loans as a state variable while maintaining the realistic feature that a fraction of borrowers choose to refinance in each period and that this fraction responds endogenously to the state of the economy.

We guess and verify that the optimal plan for the borrower is to refinance whenever $\kappa_{i,t} \leq \bar{\kappa}_t$, where $\bar{\kappa}_t$ is a threshold cost that makes the borrower indifferent between refinancing and not refinancing. The fraction of non-defaulting borrowers who choose to refinance is therefore

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t).$$

Once the threshold cost (equivalently, refinancing rate) is known, the total transaction cost per unit of debt is defined by

$$\Psi_t(Z_{R,t}) = \int^{\bar{\kappa}_t} \kappa d\Gamma_\kappa = \int^{\Gamma_\kappa^{-1}(Z_{R,t})} \kappa d\Gamma_\kappa.$$

Borrower Default and Mortgage Indexation. Before deciding whether or not to refinance a loan, borrowers decide whether or not to default on the loan. Upon default, the housing collateral used to back the loan is seized by the intermediary. To allow for an aggregated model in which the default rate responds endogenously to macroeconomic conditions, we introduce stochastic processes $\omega_{i,t}$ for each borrower i that influence the quality of borrowers' houses. Because SAM contracts often propose indexing only to certain types of house price variation — most crucially to regional rather than individual house prices to avoid moral hazard issues — we decompose house quality into two components, $\omega_{i,t} = \omega_{i,t}^L \omega_{i,t}^U$, where $\omega_{i,t}^L$ is local component that shifts prices in an area relative to the national average, and can potentially be insured by mortgage contracts, while $\omega_{i,t}^U$ is an uninsurable component that we think of as shocks to an individual house price relative to its local area. These components follow

$$\log \omega_{i,t}^L = (1 - \rho_\omega) \mu_{\omega,t}^L + \rho_\omega \log \omega_{i,t-1}^L + \sigma_{\omega,t} e_{i,t}^L, \quad e_{i,t}^L \sim N(0, \alpha) \quad (4)$$

$$\log \omega_{i,t}^U = (1 - \rho_\omega) \mu_{\omega,t}^U + \rho_\omega \log \omega_{i,t-1}^U + \sigma_{\omega,t} e_{i,t}^U, \quad e_{i,t}^U \sim N(0, 1 - \alpha) \quad (5)$$

where the shocks $e_{i,t}^L$ and $e_{i,t}^U$ are uncorrelated and account for α and $1 - \alpha$ of the cross-sectional variance of $\omega_{i,t}$, respectively. The total standard deviation $\sigma_{\omega,t}$ is allowed to vary between normal times and financial recessions, while the means $\mu_{\omega,t}^I$ and $\mu_{\omega,t}^U$ are set so that the cross-sectional average of each component $\omega_{i,t}^L$ and $\omega_{i,t}^U$ is unity conditional on the $\sigma_{\omega,t}$ regime.⁷

In addition to the standard mortgage contracts defined above, we introduce Shared Appreciation Mortgages whose payments are indexed to house prices. We allow SAM contracts to insure households in two ways. First, mortgage payments can be indexed to the aggregate house price p_t . Specifically, each period, the principal balance and payment on each existing mortgage loan are multiplied by:

$$\zeta_{p,t} = \left(\frac{p_t}{p_{t-1}} \right)^{\iota_p}. \quad (6)$$

The special cases $\iota_p = 0$ and $\iota_p = 1$ correspond to the cases of no insurance and complete insurance against aggregate house price risk.

Second, mortgage contracts can be indexed against shocks to the individual house qualities $\omega_{i,t}$. We assume that the uninsurable component $\omega_{i,t}^U$ cannot be indexed due to moral hazard risk, but that the local component $\omega_{i,t}^L$ can be insured against. Specifically, each period, the principal balance and interest payment on the loan backed by a house that experiences regional house quality growth from $\omega_{i,t-1}^L$ to $\omega_{i,t}^L$ are multiplied by:

$$\zeta_{\omega}(\omega_{i,t-1}^L, \omega_{i,t}^L) = \left(\frac{\omega_{i,t}^L}{\omega_{i,t-1}^L} \right)^{\iota_{\omega}} \quad (7)$$

The special cases $\iota_{\omega} = 0$ and $\iota_{\omega} = 1$ correspond to zero insurance and complete insurance against cross-sectional local house price risk, respectively. We assume a stationary distribution (conditional on values for μ_{ω} and σ_{ω}) so that each borrower's debt and interest payments have been cumulatively scaled by $(\omega_{i,t}^L)^{\iota_{\omega}}$.

Borrowers must commit to a default plan that can depend in an unrestricted way on $\omega_{i,t}^L, \omega_{i,t}^U$, and the aggregate states, but not on a borrower's individual loan conditions. We guess and verify that the optimal plan for the borrower is to default whenever $\omega_{i,t}^U \leq$

⁷The required values are:

$$\mu_{\omega,t}^I = \frac{1}{2} \frac{\alpha \sigma_{\omega,t}^2}{1 - \rho_{\omega}}, \quad \mu_{\omega,t}^U = \frac{1}{2} \frac{(1 - \alpha) \sigma_{\omega,t}^2}{1 - \rho_{\omega}}.$$

$\bar{\omega}_t^U$, where $\bar{\omega}_t^U$ is the threshold value of uninsurable (individual-level) house quality that makes a borrower indifferent between defaulting and not defaulting. The level of the default threshold depends on the aggregate state, the insurable regional component $\omega_{i,t-1}^L$ and innovation $e_{i,t}^L$, and importantly, also on the level of mortgage payment indexation.

Given $\bar{\omega}_t^U$, the fraction of non-defaulting borrowers is

$$Z_{N,t} = \int \left(1 - \Gamma_{\omega,t}^U(\bar{\omega}_t^U)\right) d\Gamma_{\omega,t}^L$$

where $\Gamma_{\omega,t}^U$ and $\Gamma_{\omega,t}^L$ are the CDFs of $\omega_{i,t}^U$ and $\omega_{i,t}^L$, respectively, and where the outer integral is needed because $\bar{\omega}_t^U$ depends on $\omega_{i,t}^L$. Since non-defaulting borrowers are those who received relatively good uninsurable (individual) shocks, the share of borrower housing kept by non-defaulting households is:

$$Z_{K,t} = \int \left(\int_{\omega_{i,t}^U > \bar{\omega}_t^U} \omega_{i,t}^U d\Gamma_{\omega,t}^U \right) \omega_{i,t}^L d\Gamma_{\omega,t}^L. \quad (8)$$

The inner-most integral contains this selection effect — borrowers only keep their housing when their idiosyncratic quality shock was sufficiently good — while the outer integral again accounts for dependence of $\bar{\omega}_t^U$ on local house quality.

The fractions of principal and interest payments retained by the borrowers are defined by $Z_{M,t}$ and $Z_{A,t}$, respectively, and are given by

$$Z_{M,t} = Z_{A,t} = \int \underbrace{\left(1 - \Gamma_{\omega,t}^U(\bar{\omega}_t^U)\right)}_{\text{remove defaulters}} \underbrace{(\omega_{i,t}^L)^{\iota_\omega}}_{\text{indexation}} d\Gamma_{\omega,t}^L. \quad (9)$$

The first expression removes the fraction of debt that is defaulted on and is not repaid. The second component adjusts for the fact that debt is indexed based on the value of the local component. While $Z_{A,t}$ and $Z_{M,t}$ are identical in this baseline case, it is convenient to define them separately since they will diverge under separate indexation of interest and principal in Section 6.4.

It is straightforward to show that for the limiting case when all cross-sectional house price risk is insurable ($\alpha = 1$) and this risk is fully indexed ($\iota_\omega = 1$), we obtain $Z_{N,t} = Z_{M,t} = Z_{A,t} = Z_{K,t} = 1$, in which case borrowers' optimal policy is to never default on any payments. In contrast, under a standard mortgage contract with no indexation ($\iota_p = \iota_\omega = 0$) we would have $Z_{M,t} = Z_{A,t} = Z_{N,t}$, meaning that conditional on non-default, neither debt balances nor interest payments are directly influenced by local house

prices.

REO Sector. The housing collateral backing defaulted loans is seized by the intermediary and rented out as REO (“real estate owned”) housing to the borrower. Housing in this state incurs a larger maintenance cost than usual, $v^{REO} > v^K$, designed to capture losses from foreclosure. With probability S^{REO} per period, REO housing is sold back to borrowers as owner-occupied housing. The existing stock of REO housing is denoted by K_t^{REO} , and the value of a unit of REO-owned housing is denoted p_t^{REO} .

Deposit Technology. Deposits in the model take the form of risk-free one-period loans issued from the depositor to the intermediary, where the price of these loans is denoted q_t^f , implying the interest rate $1/q_t^f$. Intermediaries must satisfy a leverage constraint (defined below in (24)) stating that their promised deposit repayments must be collateralized by their existing loan portfolio.

3.5 Borrower’s Problem

Given this model setup, the individual borrower’s problem aggregates to that of a representative borrower. The endogenous state variables are the promised payment A_t^B , the face value of principal M_t^B , and the stock of borrower-owned housing K_t^B . The representative borrower’s control variables are nondurable consumption C_t^B , housing service consumption H_t^B , the amount of housing K_t^* and new loans M_t^* taken on by refinancers, the refinancing fraction $Z_{R,t}$, and the mortgage default rate $1 - Z_{N,t}$.

The borrower maximizes (2) subject to the budget constraint:

$$\begin{aligned}
C_t^B = & \underbrace{(1 - \tau)Y_t^B}_{\text{disp. income}} + \underbrace{Z_{R,t} \left(Z_{N,t}M_t^* - \delta Z_{M,t}M_t^B \right)}_{\text{net new borrowing}} - \underbrace{(1 - \delta)Z_{M,t}M_t^B}_{\text{principal payment}} - \underbrace{(1 - \tau)Z_{A,t}A_t^B}_{\text{interest payment}} \\
& - p_t \left[\underbrace{Z_{R,t}Z_{N,t}K_t^* + (v^K - Z_{R,t})Z_{K,t}K_t^B}_{\text{owned housing}} \right] - \underbrace{\rho_t (H_t^B - K_t^B)}_{\text{rental housing}} \\
& - \underbrace{(\Psi(Z_{R,t}) - \bar{\Psi}_t)Z_{N,t}M_t^*}_{\text{net transaction costs}} - \underbrace{T_t^B}_{\text{lump sum taxes}}
\end{aligned} \tag{10}$$

the loan-to-value constraint

$$M_t^* \leq \phi^K p_t K_t^* \tag{11}$$

and the laws of motion

$$M_{t+1}^B = \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{R,t} Z_{N,t} M_t^* + \delta(1 - Z_{R,t}) Z_{M,t} M_t^B \right] \quad (12)$$

$$A_{t+1}^B = \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{R,t} Z_{N,t} r_t^* M_t^* + \delta(1 - Z_{R,t}) Z_{A,t} A_t^B \right] \quad (13)$$

$$K_{t+1}^B = Z_{R,t} Z_{N,t} K_t^* + (1 - Z_{R,t}) Z_{K,t} K_t^B \quad (14)$$

where π_t is the inflation rate, r_t^* is the interest rate on new mortgages, τ is the income tax rate, which also applies to the mortgage interest deductibility, ρ_t is the rental rate for housing services, $\bar{\Psi}_t$ is a subsidy that rebates transaction costs back to borrowers, and T_t^B are taxes raised on borrowers to pay for intermediary bailouts (defined below in (28)).

3.6 Intermediary's Problem

The intermediation sector consists of intermediary households (bankers), mortgage lenders (banks), and REO firms. The bankers are the owners, the equity holders, of both the banks and the REO firms. Each period, the bankers receive income Y_t^I , the aggregate dividend D_t^I from banks, and the aggregate dividend D_t^{REO} from REO firms. The latter two are defined in equations (27) and (29) below. Bankers choose consumption C_t^I to maximize (2) subject to the budget constraint:

$$C_t^I \leq (1 - \tau) Y_t^I + D_t^I + D_t^{REO} - v^K p_t H_t^I - T_t^I, \quad (15)$$

where T_t^I are taxes raised on intermediary households to pay for bank bailouts (defined in (28) below). Intermediary households consume their fixed endowment of housing services each period, $H_t^I = \bar{K}^I$.

Banks and REO firms maximize shareholder value. Banks lend to borrowers, issue deposits, and trade in the secondary market for mortgage debt. They are subject to idiosyncratic profit shocks and have limited liability, i.e., they optimally decide whether to default at the beginning of each period. When a bank defaults, it is seized by the government, which guarantees its deposits. The equity of the defaulting bank is wiped out, and bankers set up a new bank in place of the bankrupt one.

REO firms buy foreclosed houses from banks, rent these REO houses to borrowers, and sell REO housing in the regular housing market after maintenance.

Bank Portfolio Choice. Each bank chooses a portfolio of mortgage loans and how many deposits to issue. Although each mortgage with a different interest rate has a different secondary market price, we show in the appendix that any portfolio of loans can be replicated using only two instruments: an interest-only (IO) strip, and a principal-only (PO) strip. In equilibrium, beginning-of-period holdings of the IO and PO strips will correspond to the total promised interest payments and principal balances that are the state variables of the borrower's problem, and will therefore be denoted A_t^I and M_t^I , respectively. Denote new lending by banks in terms of face value by L_t^* . Then the end-of-period *supply* of PO and IO strips is given by:

$$\hat{M}_t^I = L_t^* + \delta(1 - Z_{R,t})Z_{M,t}M_t^I \quad (16)$$

$$\hat{A}_t^I = r_t^*L_t^* + \delta(1 - Z_{R,t})Z_{A,t}A_t^I. \quad (17)$$

Denote bank *demand* for PO and IO strips, and therefore the end-of-period holdings of these claims, by \tilde{M}_t^I and \tilde{A}_t^I , respectively. In equilibrium, we will have that $\hat{M}_t^I = \tilde{M}_t^I$ and $\hat{A}_t^I = \tilde{A}_t^I$.

The laws of motion for these variables depend on the level of indexation. Since they are nominal contracts, they also need to be adjusted for inflation:

$$M_{t+1}^I = \pi_{t+1}^{-1}\zeta_{p,t+1}\tilde{M}_t^I \quad (18)$$

$$A_{t+1}^I = \pi_{t+1}^{-1}\zeta_{p,t+1}\tilde{A}_t^I. \quad (19)$$

Banks can sell new loans to other banks in the secondary PO and IO market. The PO and IO strips trade at market prices q_t^M and q_t^A , respectively. The market value of the portfolio held by banks at the end of each period is therefore:

$$J_t^I = \underbrace{(1 - r_t^*q_t^A - q_t^M)L_t^*}_{\text{net new debt}} + \underbrace{q_t^A\tilde{A}_t^I}_{\text{IO strips}} + \underbrace{q_t^M\tilde{M}_t^I}_{\text{PO strips}} - \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}}. \quad (20)$$

To calculate the payoff of this portfolio in period $t + 1$, we first define the recovery rate of housing from foreclosed borrowers, per unit of face value outstanding, as:⁸

$$X_t = \frac{(1 - Z_{K,t})K_t^B(p_t^{REO} - v^{REO}p_t)}{M_t^B}. \quad (21)$$

⁸Note that X_t is taken as given by each individual bank. A bank does not internalize the effect of its mortgage debt issuance on the overall recovery rate.

After paying maintenance on the REO housing for one period, the banks sell the seized houses to the REO sector at prices p^{REO} .

Then the portfolio payoff is:

$$W_{t+1}^I = \underbrace{\left[X_{t+1} + Z_{M,t+1} \left((1 - \delta) + \delta Z_{R,t+1} \right) \right] M_{t+1}^I + Z_{A,t+1} A_{t+1}^I}_{\text{payments on existing debt}} + \underbrace{\delta(1 - Z_{R,t+1}) \left(Z_{A,t+1} q_{t+1}^A A_{t+1}^I + Z_{M,t+1} q_{t+1}^M M_{t+1}^I \right)}_{\text{sales of IO and PO strips}} - \underbrace{\pi_{t+1}^{-1} B_t^I}_{\text{deposit redemptions}}. \quad (22)$$

This is also the net worth of banks at the beginning of period $t + 1$.

Bank's Problem. Denote by \mathcal{S}_t^I all state variables exogenous to banks. At the beginning of each period, before making their optimal default decision, banks receive an idiosyncratic profit shock $\epsilon_t^I \sim F_\epsilon^I$, with $E(\epsilon_t^I) = 0$. The value of banks that do not default can be expressed recursively as:

$$V_{ND}^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I - \epsilon_t^I + E_t \left[\Lambda_{t,t+1}^I \max \left\{ V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I), 0 \right\} \right], \quad (23)$$

subject to the bank leverage constraint:

$$B_{t+1}^I \leq \phi^I \left(q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I \right), \quad (24)$$

the definitions of J_t^I and W_t^I in (20) and (22), respectively, and the transition laws for the aggregate supply of IO and PO strips in (16) – (19). The value of defaulting banks to shareholders is zero. The value of the newly started bank that replaces a bank liquidated by the government after defaulting, is given by:

$$V_R^I(\mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} - J_t^I + E_t \left[\Lambda_{t,t+1}^I \max \left\{ V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I), 0 \right\} \right], \quad (25)$$

subject to the same set of constraints as the non-defaulting bank.

Clearly, beginning-of-period net worth W_t^I and the idiosyncratic profit shock ϵ_t^I are irrelevant for the portfolio choice of newly started banks. Inspecting equation (23), one can see that the optimization problem of non-defaulting banks is also independent of W_t^I and ϵ_t^I , since the value function is linear in those variables and they are determined before the portfolio decision. Taken together, this implies that all banks will choose identical

portfolios at the end of the period. In the appendix, we show that we can define a value function after the default decision to characterize the portfolio problem of all banks:⁹

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + E_t \left[\Lambda_{t,t+1}^I F_{\epsilon,t+1}^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right], \quad (26)$$

where

$$F_{\epsilon,t+1}^I \equiv F_{\epsilon}^I(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I))$$

is the probability of continuation, and $\epsilon_{t+1}^{I,-} = E[\epsilon_{t+1}^I | \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)]$ is the expectation of ϵ_{t+1}^I conditional on continuation. The objective in (26) is subject to the same set of constraints as (23).

Aggregation and Government Deposit Guarantee. By the law of large numbers, the fraction of defaulting banks each period is $1 - F_{\epsilon,t}^I$. The aggregate dividend paid by banks to their shareholders, the intermediary households, is:

$$\begin{aligned} D_t^I &= F_{\epsilon,t}^I \left(W_t^I - \epsilon_t^{I,-} - J_t^I \right) - \left(1 - F_{\epsilon,t}^I \right) J_t^I \\ &= F_{\epsilon,t}^I \left(W_t^I - \epsilon_t^{I,-} \right) - J_t^I. \end{aligned} \quad (27)$$

Bank shareholders bear the burden of replacing liquidated banks by an equal measure of new banks and seeding them with new capital equal to that of continuing banks (J_t^I).

The government bails out defaulted banks at a cost:

$$\text{bailout}_t = \left(1 - F_{\epsilon,t}^I \right) \left[\epsilon_t^{I,+} - W_t^I + \eta \delta (1 - Z_{R,t}) \left(Z_{A,t} q_t^A A_t^I + Z_{M,t} q_t^M M_t^I \right) \right],$$

where $\epsilon_t^{I,+} = E[\epsilon_t^I | \epsilon_t^I > V^I(W_t^I, \mathcal{S}_t^I)]$ is the expectation of ϵ_t^I conditional on bankruptcy. Thus, the government absorbs the negative net worth of the defaulting banks. The last term are additional losses from bank bankruptcies, which are a fraction η of the mortgage assets and represent deadweight losses to the economy. To finance the bailout, the government levies lump-sum taxes on all households, in proportion to their population share:

$$T_t^j = \chi_j \text{bailout}_t, \quad \forall j \in \{B, I, D\}. \quad (28)$$

The government bailout is what makes deposits risk-free, what creates deposit insurance.

⁹The value of the newly started bank with zero net worth is simply the value in (26) evaluated at $W_t^I = 0$.

REO Firm's Problem. There is a continuum of competitive REO firms that are fully owned and operated by intermediary households (bankers). Each period, REO firms choose how many foreclosed properties to buy from banks, I_t^{REO} , to maximize the NPV of dividends paid to intermediary households. The aggregate dividend in period t paid by the REO sector to the bankers is:

$$D_t^{REO} = \underbrace{\left[\rho_t + (S^{REO} - \nu^{REO}) p_t \right] K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO} I_t^{REO}}_{\text{REO investment}}. \quad (29)$$

The law of motion of the REO housing stock is:

$$K_{t+1}^{REO} = (1 - S^{REO}) K_t^{REO} + I_t^{REO}.$$

3.7 Depositor's Problem

The depositors' problem can also be aggregated, so that the representative depositor chooses nondurable consumption C_t^D and deposits B_t^D to maximize (2) subject to the budget constraint:

$$C_t^D \leq \underbrace{(1 - \tau) Y_t^D}_{\text{disp. income}} - \underbrace{\left(q_t^f B_{t+1}^D - \pi_t^{-1} B_t^D \right)}_{\text{net deposit iss.}} - \underbrace{\nu^K p_t H_t^D}_{\text{own housing maint.}} - \underbrace{T_t^D}_{\text{lump sum taxes}}. \quad (30)$$

and a restriction that deposits must be positive: $B_t^D \geq 0$. Depositors consume their fixed endowment of housing services each period, $H_t^D = \bar{K}^D$.

3.8 Financial Recessions

At any given point in time, the economy is either in a "normal" state, or a "crisis" state, the latter corresponding to a severe financial recession. This state evolves according to a Markov Chain with transition matrix Π . The financial recession state is associated with a higher value of $\sigma_{\omega,t}$, implying more idiosyncratic uncertainty; and a lower value of ξ_t , implying a fall in aggregate house prices. Our financial recession experiments will feature a transition from the normal state into the crisis state alongside a low realization of the aggregate income shock $\varepsilon_{y,t}$.

To maintain tractability, we assume that the cross-sectional distribution of $\omega_{i,t}^L$ and $\omega_{i,t}^U$ are always at their unconditional distributions given the state of the economy (normal or

financial recession). Specifically, if the economy is in state $j \in \{0, 1\}$ where a value of 0 indicates normal times, and a value of 1 indicates a financial recession, we assume that

$$\log \omega_{i,t}^L \sim N \left(\bar{\mu}_{\omega,j}, \alpha \frac{\bar{\sigma}_{\omega,j}^2}{1 - \rho_{\omega}^2} \right), \quad \log \omega_{i,t}^U \sim N \left(\bar{\mu}_{\omega,j}, (1 - \alpha) \frac{\bar{\sigma}_{\omega,j}^2}{1 - \rho_{\omega}^2} \right).$$

These distributions are stable within each Markov state, avoiding the need to carry around additional state variables.¹⁰

3.9 Equilibrium

Given a sequence of endowment and crisis shock realizations $[\varepsilon_{y,t}, (\sigma_{\omega,t}, \xi_t)]$, a competitive equilibrium is a sequence of depositor allocations (C_t^D, B_t^D) , borrower allocations $(M_t^B, A_t^B, K_t^B, C_t^B, H_t^B, K_t^*, M_t^*, Z_{R,t}, \bar{\omega}_t^U)$, intermediary allocations $(M_t^I, A_t^I, K_t^{REO}, W_t^I, C_t^I, L_t^*, I_t^{REO}, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I)$, and prices $(r_t^*, q_t^M, q_t^A, q_t^f, p_t, p_t^{REO}, \rho_t)$, such that borrowers, intermediaries, and depositors optimize, and markets clear:

$$\begin{aligned} \text{New mortgages:} & \quad Z_{R,t} Z_{N,t} M_t^* = L_t^* \\ \text{PO strips:} & \quad \tilde{M}_t^I = \hat{M}_t^I \\ \text{IO strips:} & \quad \tilde{A}_t^I = \hat{A}_t^I \\ \text{Deposits:} & \quad B_{t+1}^I = B_{t+1}^D \\ \text{Housing Purchases:} & \quad Z_{R,t} Z_{N,t} K_t^* = S^{REO} K_t^{REO} + Z_{R,t} Z_{K,t} K_t^B \\ \text{REO Purchases:} & \quad I_t^{REO} = (1 - Z_{K,t}) K_t^B \\ \text{Housing Services:} & \quad H_t^B = K_t^B + K_t^{REO} = \bar{K}^B \\ \text{Resources:} & \quad Y_t = C_t^B + C_t^I + C_t^D + G_t + \underbrace{\eta \delta (1 - Z_{R,t}) (Z_{A,t} q_t^A A_t^I + Z_{M,t} q_t^M M_t^I)}_{\text{DWL from bank failures}} \end{aligned}$$

¹⁰This specification is equivalent to assuming that when the economy enters the financial recession state, (4) and (5) receive special additional innovations $\tilde{e}_{i,t}^L$ and $\tilde{e}_{i,t}^U$, where

$$\tilde{e}_{i,t}^L \sim N \left(0, \alpha \frac{\bar{\sigma}_{\omega,1}^2 - \bar{\sigma}_{\omega,0}^2}{1 - \rho_{\omega}^2} \right) \quad \tilde{e}_{i,t}^U \sim N \left(0, (1 - \alpha) \frac{\bar{\sigma}_{\omega,1}^2 - \bar{\sigma}_{\omega,0}^2}{1 - \rho_{\omega}^2} \right).$$

These shocks are reversed at the end of the financial recession, and are re-drawn in each financial recession. Under these assumptions, the distribution immediately jumps to its stable unconditional form upon changing state. This specification is also consistent with the cross-sectional experience in the housing crash, which saw a rapid spike upward in dispersion at the start of the crisis, which then stabilized, rather than a gradual build-up of geographic dispersion.

$$+ \underbrace{\nu^K p_t (Z_{K,t} K_t^B + \bar{K}^I + \bar{K}^D) + \nu^{REO} p_t [K_t^{REO} + (1 - Z_{K,t}) K_t^B]}_{\text{housing maintenance expenditure}}$$

The resource constraint states that the endowment Y_t is spent on nondurable consumption, government consumption, deadweight losses from bank failures, and housing maintenance. Housing maintenance consists of payments for houses owned by borrowers, depositors, and intermediaries and for houses already owned by REO firms, K_t^{REO} , or newly bought by REO firms from foreclosed borrowers $(1 - Z_{K,t}) K_t^B$. Government consumption consists of income taxes net of the mortgage interest deduction:

$$G_t = \tau(Y_t - Z_{A,t} A_t^B).$$

4 Model Solution

4.1 Borrower Optimality

The optimality condition for new mortgage debt,

$$1 = \Omega_{M,t} + r_t^* \Omega_{A,t} + \lambda_t^{LTV},$$

equalizes the benefit of taking on additional debt — \$1 today — to the cost of carrying more debt in the future, both in terms of carrying more principal ($\Omega_{M,t}$) and higher interest payments ($\Omega_{A,t}$), plus the shadow cost of tightening the LTV constraint. The marginal continuation costs are defined recursively:

$$\begin{aligned} \Omega_{M,t} &= \mathbb{E}_t \left\{ \Lambda_{t+1}^B \pi_{t+1}^{-1} \zeta_{p,t+1} Z_{M,t+1} \left[(1 - \delta) + \delta Z_{R,t+1} + \delta (1 - Z_{R,t+1}) \Omega_{M,t+1} \right] \right\} \\ \Omega_{A,t} &= \mathbb{E}_t \left\{ \Lambda_{t+1}^B \pi_{t+1}^{-1} \zeta_{p,t+1} Z_{A,t+1} \left[(1 - \tau) + \delta (1 - Z_{R,t+1}) \Omega_{A,t+1} \right] \right\} \end{aligned}$$

where an extra unit of principal requires a payment of $(1 - \delta)$ in the case of non-default, plus payment of the face value of prepaid debt, plus the continuation cost of non-prepaid debt. An extra promised payment requires a tax-deductible payment on non-defaulted debt plus the continuation cost if the debt is not prepaid.

The optimality condition for housing services consumption sets the rental rate to be

the marginal rate of substitution between housing services and nondurables:

$$\rho_t = \frac{u_{H,t}}{u_{C,t}} = \left(\frac{\xi_t}{1 - \xi_t} \right) \left(\frac{C_t^B}{H_t^B} \right)$$

The borrower's optimality condition for new housing capital is:

$$p_t = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^B \left[\rho_{t+1} + Z_{K,t+1} p_{t+1} \left(1 - \nu^K - (1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K \right) \right] \right\}}{1 - \lambda_t^{LTV} \phi^K}.$$

The numerator represents the present value of holding an extra unit of housing next period: the rental service flow, plus the continuation value of the housing if the borrower chooses not to default, net of the maintenance cost. The continuation value needs to be adjusted by $(1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K$ because if the borrower does not choose to refinance, which occurs with probability $1 - Z_{R,t+1}$, then she does not use the unit of housing to collateralize a new loan, and therefore does not receive the collateral benefit.

The optimal refinancing rate is:

$$Z_{R,t} = \Gamma_\kappa \left\{ \underbrace{\left(1 - \Omega_{M,t} - \bar{r}_t \Omega_{A,t} \right) \left(1 - \frac{\delta Z_{M,t} M_t}{Z_{N,t} M_t^*} \right)}_{\text{equity extraction incentive}} + \underbrace{\Omega_{A,t} (\bar{r}_t - r_t^*)}_{\text{interest rate incentive}} \right. \\ \left. - \underbrace{p_t \lambda_t^{LTV} \phi^K \left(\frac{Z_{N,t} K_t^* - Z_{K,t} K_t^B}{Z_{N,t} M_t^*} \right)}_{\text{collateral expense}} \right\} \quad (31)$$

where $\bar{r}_t = A_t^B / M_t^B$ is the average interest rate on existing debt. The "equity extraction incentive" term represents the net gain from obtaining additional debt at the *existing* interest rate, while "interest rate incentive" term represents the gain from moving from the existing to new interest rate. The stronger these incentives, the higher the refinancing rate. The "collateral expense" term arises because housing trades at a premium relative to the present value of its housing service flow due to its collateral value. If the borrower intends to obtain new debt by buying more housing collateral, the cost of paying this premium must be taken into account.

The optimality condition for the default rate pins down the default threshold $\bar{\omega}_t^U$ as a

function of the aggregate state, as well as the value of the local component ($\omega_{i,t}^L$):

$$\bar{\omega}_t^U = \frac{(\omega_{i,t}^L)^{t\omega} (Q_{A,t}A_t + Q_{M,t}M_t)}{\omega_{i,t}^L Q_{K,t}K_t^B} \quad (32)$$

where $Q_{A,t}$ and $Q_{M,t}$ are the marginal benefits of discharging interest payments and principal, respectively, and $Q_{K,t}$ is the marginal continuation value of housing, defined by

$$\begin{aligned} Q_{A,t} &= \underbrace{(1 - \tau)}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{A,t}}_{\text{continuation cost}} \\ Q_{M,t} &= \underbrace{(\delta Z_{R,t} + (1 - \delta))}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{M,t}}_{\text{continuation cost}} \\ Q_{K,t} &= \left[\underbrace{Z_{R,t}}_{\text{refi case}} + \underbrace{(1 - Z_{R,t}) \left(1 - \lambda_t^{LTV} \phi^K\right)}_{\text{no refi case}} - \underbrace{v^K}_{\text{maint.}} \right] p_t. \end{aligned}$$

Equation (32) relates the benefit of defaulting on debt, which is eliminating both the current payment and continuation cost, potentially indexed by $\omega_{i,t}^L$, against the cost of losing a marginal unit of housing, which has been scaled by both $\omega_{i,t}^L$ and $\omega_{i,t}^U$. The marginal value of housing $Q_{K,t}$ is equal to the full market price p_t net of maintenance if used to collateralize a new loan (i.e., if the borrower refinances) but is worth less if the borrower does not refinance next period due to the loss of collateral services.

4.2 Intermediary Optimality

The optimality condition for new debt L^* is:

$$1 = q_t^M + r_t^* q_t^A,$$

which balances the cost of issuing new debt, \$1 today, against the value of the loan obtained, 1 unit of PO strip plus r_t^* units of the IO strip. The condition implies that the first term in (20) is zero.

The optimality condition for deposits is:

$$q_t^f = \mathbb{E}_t \left[\Lambda_{t+1}^I F_{\epsilon,t+1}^I \tau_{t+1}^{-1} \right] + \lambda_t^I$$

where λ_t^I is the multiplier on the intermediary's leverage constraint (24). The default

option, represented by the $F_{\epsilon,t+1}^I$ term in the expectation, drives a wedge between the valuation of risk free debt by intermediary households, $\mathbb{E}_t \left[\Lambda_{t+1}^I \pi_{t+1}^{-1} \right]$, and that of banks.

The optimality conditions for PO and IO strip holdings pin down their prices:

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{M,t+1} \left((1 - \delta) + \delta Z_{R,t+1} + \delta (1 - Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}$$

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{A,t+1} \left(1 + \delta (1 - Z_{R,t+1}) q_{t+1}^A \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}.$$

Both securities issue cash flows that are nominal (discounted by inflation) and indexed to house prices (discounted by $\zeta_{p,t+1}$). Both securities can also be used to collateralize deposits, leading to the collateral premia in the denominators. The IO strip's next-period payoff is equal to \$1 for loans that do not default, with a continuation value of q_{t+1}^A for loans that do not prepay or mature. The PO strip's next-period payoff is the recovery value for defaulting debt X_{t+1} plus the payoff from loans that do not default: the principal payment $1 - \delta$, plus the face value of prepaying debt, plus the continuation value q_{t+1}^M for loans that do not mature or prepay.

The optimality condition for REO housing is:

$$p_t^{REO} = \mathbb{E}_t \left\{ \Lambda_{t+1}^I \left[\rho_{t+1} - v^{REO} p_{t+1} + S^{REO} p_{t+1} + (1 - S^{REO}) p_{t+1}^{REO} \right] \right\}.$$

The right-hand side is the present discounted value of holding a unit of REO housing next period. This term is in turn made up of the rent charged to borrowers, the maintenance cost, and the value of the housing next period, both the portion sold back to the borrowers, and the portion kept in the REO state.

4.3 Depositor Optimality

The depositor's sole optimality condition for deposits, which are nominal contracts, ensures that the depositor's Euler equation is at an interior solution:

$$q_t^f = \mathbb{E}_t \left[\Lambda_{t+1}^D \pi_{t+1}^{-1} \right].$$

5 Calibration

This section describes the calibration procedure for key variables, and presents the full set of parameter values in Table 1. The model is calibrated at quarterly frequency and solved using global projection methods. Since the integrals (8) and (9) lack a closed form, we evaluate them using Gauss-Hermite quadrature with 11 nodes in each dimension.

Exogenous Shock Processes. Aggregate endowment shocks in (1) have quarterly persistence $\rho_y = .977$ and innovation volatility $\sigma_y = 0.81\%$. These are the observed persistence and innovation volatility of log real per capita labor income from 1991.Q1 until 2016.Q1.¹¹ In the numerical solution, this AR process is discretized as a five-state Markov Chain, following the Rouwenhorst (1995) method. Long-run endowment growth $g = 0$. The average level of aggregate income (GDP) is normalized to 1. The income tax rate is $\tau = 0.147$, as given by the observed ratio of personal income tax revenue to personal income.

The discrete state follows a two-state Markov Chain, with state 0 indicating normal times, and state 1 indicating crisis. The probability of staying in the normal state in the next quarter is 97.5% and the probability of staying in the crisis state in the next quarter is 92.5%. Under these parameters, the economy spends 3/4 of the time in the normal state and 1/4 in the crisis state. This matches the fraction of time between 1991.Q1 and 2016.Q4 that the U.S. economy was in the foreclosure crisis, and implies an average duration of the normal state of ten years, and an average duration of the crisis state of 3.33 years. These transition probabilities are independent of the aggregate endowment state. The low uncertainty state has $\bar{\sigma}_{\omega,0} = 0.200$ and the high uncertainty state has $\bar{\sigma}_{\omega,1} = 0.250$. These numbers allow the model to match an average mortgage default rate of 0.5% per quarter in expansions and of 2.05% per quarter in financial recessions, which are periods defined by low endowment growth and high uncertainty. The unconditional mortgage default rate in the model is 0.95%. In the data, the average mortgage delinquency rate is 1.05% per quarter; it is 0.7% in normal times and 2.3% during the foreclosure crisis.¹²

¹¹Labor income is defined as compensation of employees (line 2) plus proprietor's income (line 9) plus personal current transfer receipts (line 16) minus contributions to government social insurance (line 25), as given by Table 2.1 of the Bureau of Economic Analysis' National Income and Product Accounts. Deflation is by the personal income deflator and by population. Moments are computed in logs after removing a linear time trend.

¹²Data are for all residential mortgage loans held by all U.S. banks, quarterly data from the New York Federal Reserve Bank from 1991.Q1 until 2016.Q4. The delinquency rate averages 2.28% per quarter between 2008.Q1 and 2013.Q4 (high uncertainty period, 23% of quarters) and 0.69% per quarter in the rest of

Local House Price Process. We calibrate the persistence and variance of the local (insurable) housing quality process using FHFA house prices indices at the MSA level. Specifically, we run the annual panel regression

$$\log HPI_{i,t} = \delta_t + \phi_i + \rho_\omega^{ann} \log HPI_{i,t-1} + \varepsilon_{i,t} \quad (33)$$

where i indexes the MSA, and t indexes the year, and δ_t and ϕ_i are MSA and quarter fixed effects. The quarterly persistence is computed as $\rho_\omega = (\rho_\omega^{ann})^{1/4}$, which we estimate to be 0.977.¹³ Since this persistence parameter only matters for the indexation of local house price risk (in the asymmetric indexation case), it is appropriate to calibrate this parameter only to local house price data. To calibrate α , the share of house price variance at the local/regional level, we use (33) to compute the implied unconditional variance $\text{Var}(\omega_{i,t}^L) = \text{Var}(\varepsilon_{i,t}) / (1 - (\rho_\omega^{ann})^2)$, which delivers an unconditional standard deviation at the MSA level of 11.5%. We set $\alpha = 0.25$, which given our calibration for $\sigma_{\omega,t}$ implies that the standard deviation of regional house prices is 10% in the model in normal times, and 12.5% in financial recessions, consistent with our empirical estimates.

Demographics, Income, and Housing Shares. We split the population into mortgage borrowers, depositors, and intermediary households as follows. We use the 1998 Survey of Consumer Finances to define for every household a loan-to-value ratio. This ratio is zero for renters and for households who own their house free and clear. We define mortgage borrowers to be those households with an LTV ratio of at least 30%.¹⁴ Those households make up for 34.3% of households ($\chi_B = .343$). They earn 46.9% of labor income ($s_B = .469$). For parsimony, we set all housing shares equal to the corresponding income share. Since the aggregate housing stock \bar{K} is normalized to 1, $\bar{K}^B = .469$.

To split the remaining households into depositors and intermediary households (bankers), we set the share of labor income for bankers equal to 6.7%. To arrive at this number, we calculate the share of the financial sector (finance, insurance, and real estate) in overall stock market capitalization (16.4% in 1990-2017) and multiply that by the labor income share going to all equity holders in the SCF. We set the housing share again equal to the

the period.

¹³The annual estimate is $\rho_\omega^{ann} = 0.911$ with standard error 0.004 (clustered at the MSA level). The data source is the Federal Housing Finance Agency Quarterly All-Transactions House Price Index. The sample spans 1975.Q1 - 2017.Q1, and contains 13,649 observations drawn from 403 MSAs. The regression is run using an unbalanced panel as MSAs enter the sample over time, but results using a balanced panel limited to MSAs present since some given start date were nearly identical under a variety of start dates.

¹⁴Those households account for 88.2% of mortgage debt and 81.6% of mortgage payments.

Table 1: Parameter Values: Baseline Calibration (Quarterly)

Parameter	Name	Value	Target/Source
<i>Technology</i>			
Agg. income persistence	ρ_{TFP}	0.977	Real per capita labor income BEA
Agg. income st. dev.	σ_{TFP}	0.008	Real per capita labor income BEA
Profit shock st. dev.	σ_ϵ	0.070	FDIC bank failure rate
Transition: Normal \rightarrow Normal	Π_{00}	0.975	Avg. length = 10Y
Transition: Crisis \rightarrow Crisis	Π_{11}	0.925	25% of time in crisis state
<i>Demographics and Income</i>			
Fraction of borrowers	χ_B	0.343	SCF 1998 population share LTV > .30
Fraction of intermediaries	χ_I	0.020	Stock market cap. share of finance sector
Borr. inc. and housing share	s_B	0.470	SCF 1998 income share LTV > .30
Intermediary inc. and housing share	s_I	0.067	Employment share in finance
Tax rate	τ	0.147	Personal tax rate BEA
<i>Housing and Mortgages</i>			
Housing stock	\bar{K}	1	Normalization
Housing XS persistence	ρ_ω	0.977	FHFA MSA-level regression
Housing XS dispersion (Normal)	$\bar{\sigma}_{\omega,0}$	0.200	Mortg. delinq. rate US banks, no crisis
Housing XS dispersion (Crisis)	$\bar{\sigma}_{\omega,1}$	0.250	Mortg. delinq. rate US banks, crisis
Local share of XS dispersion	α	0.25	FHFA MSA-level regression
Inflation rate	$\bar{\pi}$	1.006	2.29% CPI inflation
Mortgage duration	δ	0.996	Duration of 30-yr FRM
Prepayment cost mean	μ_κ	0.370	Greenwald (2016)
Prepayment cost scale	s_κ	0.152	Greenwald (2016)
LTV limit	ϕ^K	0.850	LTV at origination
Maint. cost (owner)	v^K	0.616%	BEA Fixed Asset Tables
<i>Intermediaries</i>			
Bank regulatory capital limit	ϕ^I	0.940	Financial sector leverage limit
Deadweight cost of bank failures	η	0.085	Bank receivership expense rate
Maint. cost (REO)	v^{REO}	0.024	REO discount: $p_{ss}^{REO} / p_{ss} = 0.725$
REO sale rate	S^{REO}	0.167	Length of foreclosure crisis
<i>Preferences</i>			
Borr. discount factor	β_B	0.950	Borrower value/income, SCF
Intermediary discount factor	β_I	0.950	Equal to β_B
Depositor discount factor	β_D	0.998	3% nominal short rate (annual)
Risk aversion	γ	5.000	Standard value
EIS	ψ	1.000	Standard value
Housing preference (Normal)	$\bar{\zeta}_0$	0.220	Borrower hous. expend./income
Housing preference (Crisis)	$\bar{\zeta}_1$	0.160	HP growth volatility

income share. The population share of bankers is set to 2%, consistent with the observed employment share in the FIRE sector. The depositors make up the remaining $\chi_D = 63.7\%$ of the population, and receive the remaining $s_D = 46.4\%$ of labor income and of the housing stock.

Prepayment Costs. For the prepayment cost distribution, we assume a mixture distribution, so that with probability 3/4, the borrower draws an infinite prepayment cost, while with probability 1/4, the borrower draws from a logistic distribution, yielding

$$Z_{R,t} = \frac{1}{4} \cdot \frac{1}{1 + \exp\left(\frac{\bar{k}_t - \mu_\kappa}{\sigma_\kappa}\right)}$$

The calibration of the parameters follows [Greenwald \(2016\)](#), who fits an analogue of (31).¹⁵ The parameter σ_κ , determining the sensitivity of prepayment to equity extraction and interest rate incentives, is set to that paper's estimate (0.152), while the parameter μ_κ is set to match the average quarterly prepayment rate of 3.76% found in that exercise.

Mortgages. We set $\delta = .99565$ to match the fraction of principal US households amortize on mortgages.¹⁶ The maximum loan-to-value ratio at mortgage origination is $\phi^B = 0.85$, consistent with average standard mortgage underwriting norms.¹⁷ Inflation is set equal to the observed 0.57% per quarter (2.29% per year) for the 1991.Q1 - 2016.Q4 sample.

Banks. We set the maximum leverage that banks may take on at $\phi^I = 0.940$, following [Elenev et al. \(2017\)](#), to capture the historical average leverage ratio of the leveraged financial sector. The idiosyncratic profit shock that hits banks has standard deviation of $\sigma_\epsilon = 7.00\%$ per quarter. This delivers a bank failure rate of 0.33% per quarter, consistent

¹⁵See [Greenwald \(2016\)](#), Section 4.2. The parameters are fit to minimize the forecast error $LTV_t = Z_{R,t}LTV_t^* + (1 - Z_{R,t})\delta G_t^{-1}LTV_{t-1}$, where LTV_t is the ratio of total mortgage debt to housing wealth, LTV_t^* is LTV at origination, and G_t is growth in house values.

¹⁶The average duration of a 30-year fixed-rate mortgage is typically thought of as about 7 years. This low duration is mostly the result of early prepayments. The parameter δ captures amortization absent refinancing. Put differently, households are paying off a much smaller fraction of their mortgage principal than 1/7th each year in the absence of prepayment.

¹⁷The average LTV of purchase mortgages originated by Fannie and Freddie was in the 80-85% range during our sample period. However, that does not include second mortgages and home equity lines of credit. Our limit is a combined loan-to-value limit (CLTV). It also does not capture the lower down payments on non-conforming loans that became increasingly prevalent after 2000. [Keys, Piskorski, Seru, and Vig \(2012\)](#) document CLTVs on non-conforming loans that rose from 85% to 95% between 2000 and 2007.

with historical bank failure rate data from the FDIC.¹⁸ We assume a deadweight loss from bank bankruptcies equal to $\eta = 8.50\%$ of bank assets. This number falls in the interquartile range $[5.9\%, 15.9\%]$ of bank receivership expenses as a ratio of bank assets in a FDIC study of bank failures from 1986 until 2007 (Bennett and Unal, 2015). Deadweight losses from bank failures amount to 0.07% of GDP in equilibrium.

Housing Maintenance and REOs. We set the regular housing maintenance cost equal to $\nu^K = 0.616\%$ per quarter or 2.46% per year. This is the average over the 1991-2016 period of the ratio of current-cost depreciation of privately-owned residential fixed assets to the current-cost net stock of privately-owned residential fixed assets at the end of the previous year (source: BEA Fixed Asset Tables 5.1 and 5.4).

We calibrate the maintenance cost in the REO state to $\nu^{REO} = 2.40\%$ per quarter. It delivers REO housing prices that are 24.3% below regular housing prices on average. This is close to the observed fire-sale discounts reported by Fannie Mae and Freddie Mac during the foreclosure crisis. We assume that $S^{REO} = 0.167$ so that 1/6th of the REO stock is sold back to the borrower households each quarter. It takes eight quarters for 75% of the REO stock to roll off. This generates REO crises that take some time to resolve, as they did in the data.

Preferences. All agents have the same risk aversion coefficient of $\gamma_j = 5$ and intertemporal elasticity of substitution coefficient $\psi = 1$. These are standard values in the literature. We choose the value of the housing preference parameter in normal times $\bar{\zeta}_0 = 0.220$ to match a ratio of housing expenditure to income for borrowers of 18%, a common estimate in the housing literature.¹⁹ The model produces a ratio of 17.5%. To induce an additional house price drop, we set $\bar{\zeta}_1 = 0.16$ in the crisis states. This additional variation yields a volatility of quarterly log national house price growth of 1.41%, compared to 1.56% in the data (source: Case Shiller).

For the time discount factors, we set $\beta^B = \beta^I = 0.950$ to target the ratio of housing wealth to quarterly income for borrowers of 8.57, close to the same ratio for “borrowers” as defined above in the 1998 SCF (8.67). Finally, we set the discount rate of depositors

¹⁸Based on the FDIC database of all bank failure and assistance transaction from 1991-2016, we calculate the asset-weighted average annual failure rate to be 1.65%.

¹⁹Piazzesi, Schneider, and Tuzel (2007) obtain estimates between 18 and 20 percent based on national income account data (NIPA) and consumption micro data (CEX). Davis and Ortalo-Magné (2011) obtain a ratio of 18% after netting out 6% for utilities from the median value of 24% across MSAs using data on rents.

$\beta^D = 0.998$ to match the observed nominal short rate of 2.9% per year or 0.71% per quarter. With these parameters, the model generates average borrower mortgage debt to housing wealth (LTV) of 64.5%, close to the corresponding value 61.6% for the “borrower” population in the 1998 SCF.

6 Results on Mortgage Indexation

The main exercise is to compare the economy with regular mortgages to hypothetical economies with varying degrees and forms of mortgage indexation. Specifically, we solve models with: (i) no indexation corresponding to $\iota_p = \iota_\omega = 0$, which is the benchmark; (ii) only aggregate indexation, such that $\iota_p = 1$ and $\iota_\omega = 0$; (iii) only local indexation, such that $\iota_p = 0$ and $\iota_\omega = 1$ (iv) regional indexation, consisting of aggregate plus local indexation, which we parameterize as $\iota_p = 1$ and $\iota_\omega = 1$. We conduct a long simulation for each of the four models. Table 2 shows averages of key prices and quantities computed from the simulated time series.

These stylized experiments are designed to showcase the different properties of aggregate and local indexation. While the typical SAM proposal does not distinguish between the source of house price movements, any indexation scheme can be decomposed into these two types. Moreover, we will show that these forms of indexation yield sharply different economic implications, which should be considered when designing a mortgage product. For the aggregate indexation experiment, we choose the extreme case of full insurance ($\iota_p = 1$) to generate clear qualitative results. For the local indexation experiment, we choose partial (25%) indexation. This limited insurance, perhaps against MSA-level variation in house prices, is designed to avoid moral hazard problems from indexing to the value of an individual property, as well as asymmetric information problems from assets whose cash flows are tied to hyper-local price indexes, as analyzed in [Hartman-Glaser and Hebert \(2017\)](#).

6.1 Benchmark Model

Unconditional Moments. Before turning to the indexation results, it is useful to briefly discuss the benchmark model. On the borrower side, the model generates average mortgage debt to annual income of 64.9%, close to the observed value of 69%. It generates an aggregate LTV ratio among mortgage borrowers of 64.5%. The average mortgage default rate of 0.95% per quarter matches the data, and the loss-given-default rate of 38.61%

Table 2: Results: Quantities and Prices

	No Index	Aggregate	Local (25%)	Regional
Borrower				
1. Housing Capital	0.456	0.456	0.462	0.463
2. Refi rate	3.84%	3.81%	3.77%	3.74%
3. Default rate	0.95%	0.92%	0.49%	0.47%
4. Household leverage	64.41%	64.30%	65.79%	65.80%
5. Frac. LTV binds at orig	100.00%	100.00%	100.00%	100.00%
6. Mortgage debt to income	259.59%	252.53%	274.88%	267.74%
7. Loss-given-default rate	38.61%	37.54%	38.03%	37.21%
8. Loss Rate	0.40%	0.39%	0.20%	0.20%
Intermediary				
9. Mkt fin leverage	93.98%	93.87%	93.99%	93.92%
10. Frac. leverage binds	99.35%	90.16%	99.90%	90.92%
11. REO maint	0.34%	0.32%	0.19%	0.18%
12. REO return	5.32%	7.90%	5.23%	6.76%
13. Bank dividend	0.010	0.014	0.010	0.012
14. REO dividend	0.005	0.005	0.003	0.003
15. Bank equity capital	0.184	0.185	0.196	0.194
16. Bank equity ratio	7.09%	7.33%	7.13%	7.25%
17. Bank default rate	0.33%	0.84%	0.22%	0.50%
18. DWL of bank defaults	0.07%	0.16%	0.05%	0.10%
Depositor				
19. Deposits	2.454	2.381	2.599	2.526
Prices				
20. House Price	8.842	8.595	9.042	8.784
21. REO house price	6.692	6.599	6.811	6.693
22. Risk-free rate	0.71%	0.66%	0.74%	0.75%
23. Mortgage Rate	1.46%	1.54%	1.30%	1.35%
24. Credit spread	0.75%	0.87%	0.56%	0.60%
25. Mortgage Expec. Excess Ret	0.34%	0.49%	0.35%	0.40%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with full indexation of mortgage payments to aggregate house prices (second column), a model with partial indexation to relative local prices (third column), and a model with both aggregate and partial local indexation (fourth column). All flow variables are quarterly.

comes close to the data. The implied loss rate is 0.40% per quarter. The refinancing rate of 3.84% per quarter matches the implied average rate at which mortgages are replaced ex-

cluding rate refinances. The maximum LTV constraint, which only applies at origination and caps the LTV at 85% always binds in our simulations, consistent with the overwhelming majority of borrowers taking out loans up to the limit.

On the intermediary side, we match the leverage ratio of the levered financial sector, which is 93.98% in the model. Banks' regulatory capital constraints bind in 99.35% of the periods. Bank equity capital represents about 4.6% of annual GDP (18.4% of quarterly GDP) and 7.09% of bank assets in the model. Bank deposits (that go towards financing mortgage debt) represent just over 61.4% of annual GDP (245.4%/4). Bank dividends are 1.0% of GDP. The model generates a substantial amount of financial fragility. One measure thereof is the bank default rate. In the benchmark, it is 0.33% per quarter or 1.3% per year. Deadweight losses from bank bankruptcies are 0.07% of GDP on average.

The REO firms represent the other part of the intermediary sector. They spend 0.34% of GDP on housing maintenance on average, and pay 0.5% of GDP in dividends to their owners. REO firms earn very high returns from investing in foreclosed properties and selling them back to the borrowers: the return on equity is 5.3% per quarter (equal to the return on assets since the REO firms have no leverage).²⁰

The model somewhat overstates housing wealth, which represents about 220.3% of annual GDP in the model and 153% in the data. This discrepancy is an artifact of giving all agents the same housing to income ratio in the model, while the "borrower" type holds relatively more housing in the data than the other groups. At equilibrium, only borrower holdings of housing are relevant,²¹ so the quantitative effect of exaggerating total housing wealth is minimal. The mortgage rate exceeds the short rate by 75bps per quarter, which is close to the average spread between the 30-year fixed-rate mortgage rate and the 3-month T-bill rate of 89bps per quarter for 1991–2016. The model's expected excess return, or risk premium, earned by banks on mortgages is 34bps per quarter.

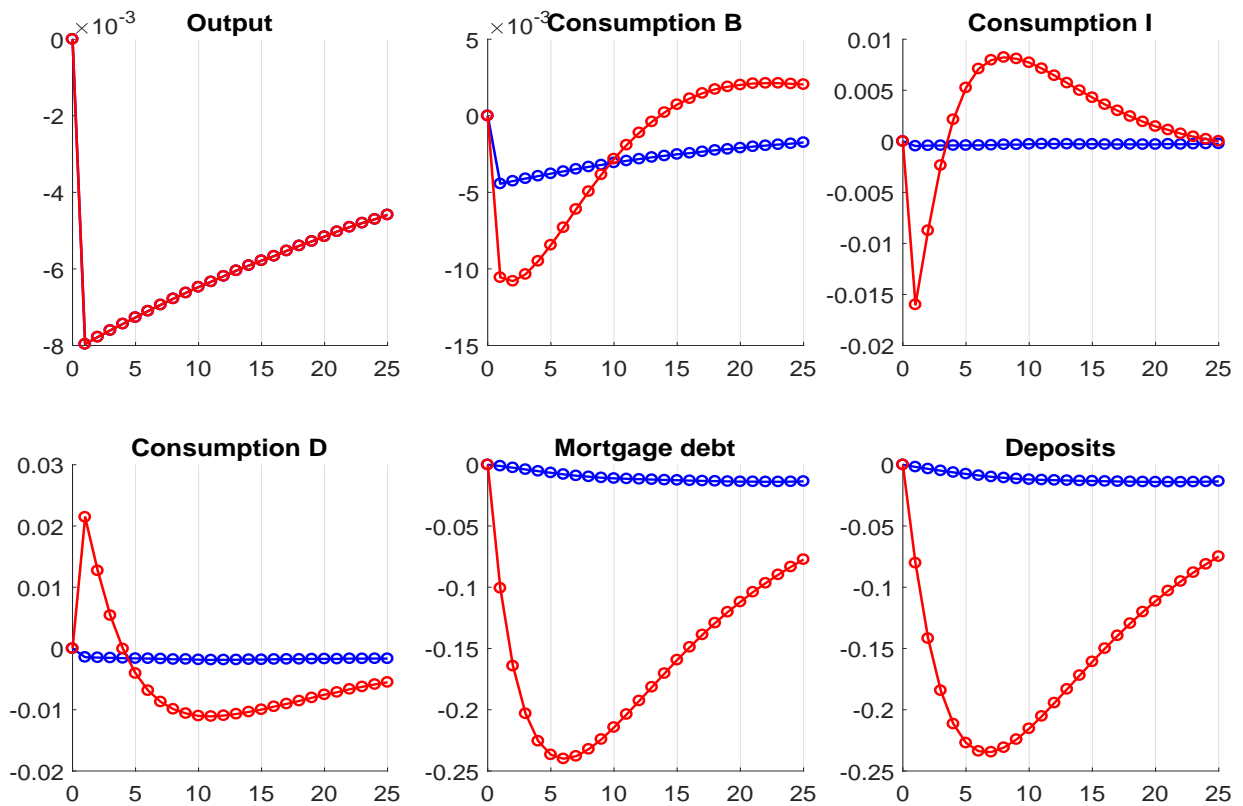
Financial Crises. To understand risk-sharing patterns in the benchmark economy, it is instructive to study how the economy behaves in a financial recession and a non-financial recession. We define a non-financial recession event as a one standard deviation drop in aggregate income while the economy is in the normal (non-crisis) state. In a financial

²⁰This return on equity in the model mimics the high returns earned by single-family rental firms like Blackstone's Invitation Homes over the past five years.

²¹We could set the housing to income ratios of intermediaries and depositors to match the overall housing to GDP ratio observed in the data. However, the constant housing capital of these two types of households only affects their equilibrium non-durable consumption levels through housing maintenance payments. The effect of such an adjustment on equilibrium outcomes is negligible.

recession, the economy experiences the same fall in income, but also transitions from the normal state into the crisis state, leading to an increase in house value uncertainty ($\bar{\sigma}_{\omega,0} \rightarrow \bar{\sigma}_{\omega,1}$) and a decrease in housing utility ($\bar{\xi}_0 \rightarrow \bar{\xi}_1$). We simulate many such recessions in order to average over the endogenous state variables (wealth distribution). Figures 1 and 2 plot the impulse-response functions as deviations in levels from the ergodic steady state, with financial recessions indicated by red circles and non-financial recessions in blue.²² By construction, the blue and red lines coincide in the top left panel of Figure 1.

Figure 1: Financial vs. Non-financial Recessions: Benchmark Model (part 1)



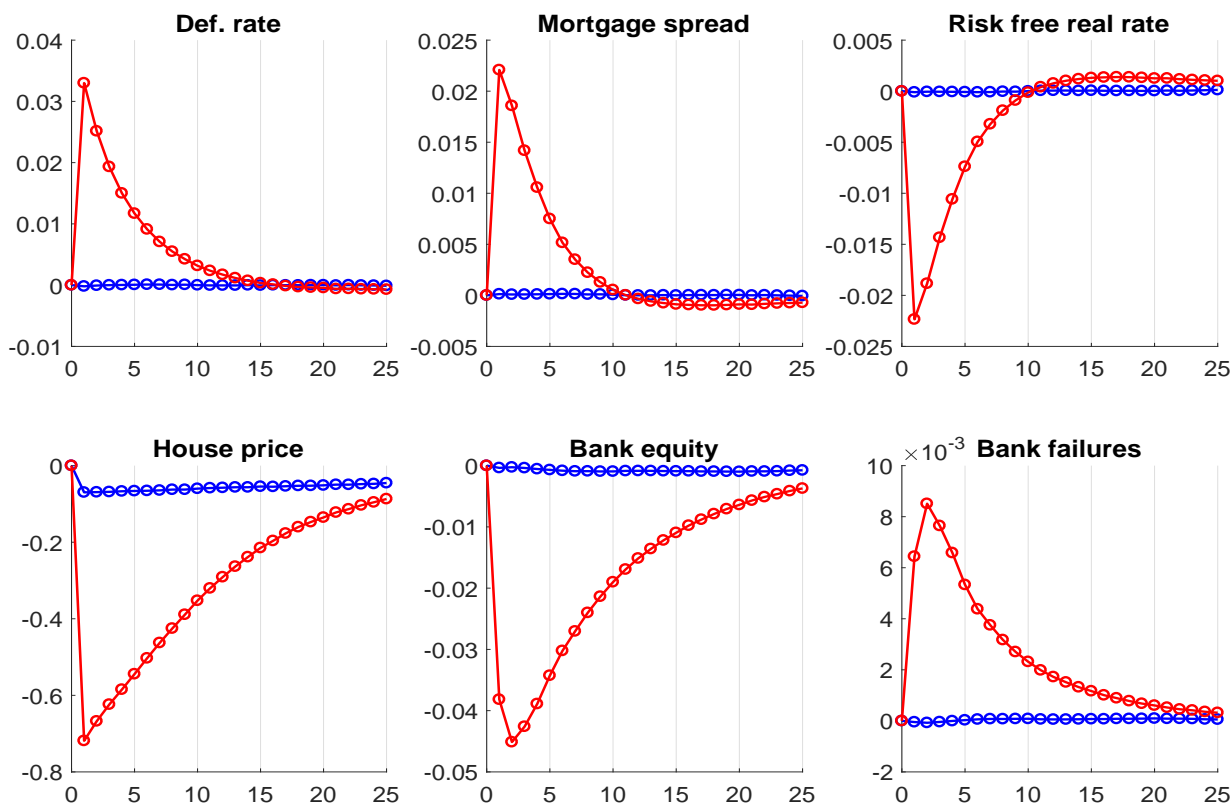
Blue line: non-financial recession, **Red line:** financial recession. Plots report deviations in levels from the ergodic steady state.

A financial crisis results in a significant increase in mortgage defaults as well as bank failures. Bank equity falls, forcing banks to delever in the wake of the losses they suffer. Banks shrink substantially, both in terms of their mortgage assets and their deposit liabilities. In order to induce depositor households to reduce deposits and increase consumption, the real interest rate falls sharply. Intermediary consumption falls heavily, as

²²The simulations underlying these generalized IRF plots are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the non-crisis state ($\bar{\sigma}_{\omega,0}, \bar{\xi}_0$).

the owners of the intermediary sector absorb losses from mortgage default, since fixed payments on existing loans do not adjust for newly increased default risk. Borrower consumption also falls as borrowers cut back on new mortgage borrowing, and must help pay for the bank bailouts by paying higher taxes. After the shock, the economy gradually recovers as high mortgage spreads (and expected returns on mortgages) eventually replenish the bank equity.

Figure 2: Financial vs. Non-financial Recessions: Benchmark Model (part 2)



Blue line: non-financial recession, **Red line:** financial recession. Plots report deviations in levels from the ergodic steady state.

6.2 Aggregate Indexation

The first experiment we consider is one where all mortgage payments are indexed to aggregate house prices. The conjecture in the literature is that this should reduce mortgage defaults and generally improve borrower's ability to smooth consumption. Perhaps surprisingly, we find that this conjecture does not hold up in general equilibrium. To the

contrary, Table 2 shows that by adding to financial fragility, aggregate indexation destabilizes borrower consumption while leaving mortgage default rates unchanged.

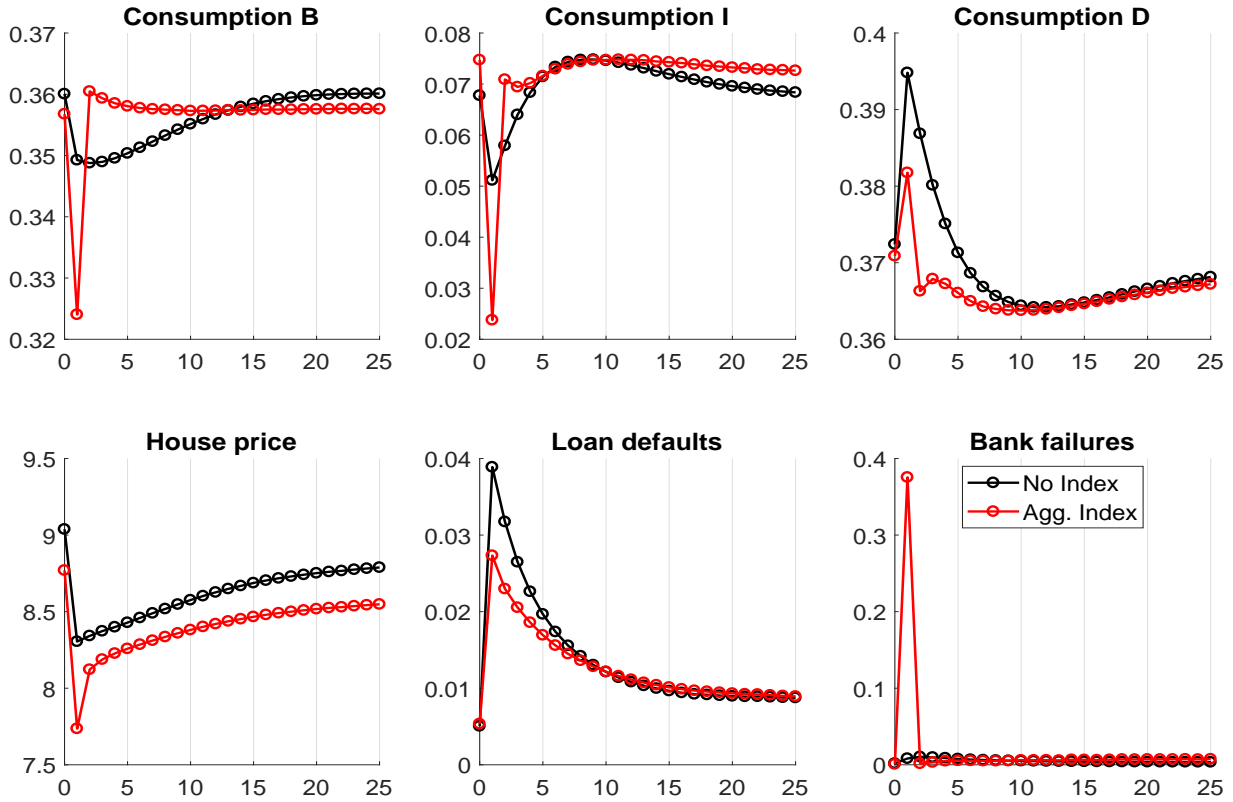
To understand this, we can turn to Figure 3, which compares financial recessions in the benchmark model to financial recessions in the model with aggregate indexation. Under aggregate indexation, banks find themselves exposed to increased risk through their loan portfolio. Although banks optimally choose to slightly decrease leverage and increase their capital buffer compared to the benchmark model, banks place their equity at much greater risk. Facing a trade-off between preserving charter value and taking advantage of limited liability, banks lean more toward their option to declare bankruptcy and saddle the government with the losses.

The combination of increased risk and the absence of precautionary capital means that the share of bank defaults upon entering a financial recession is three times larger in the aggregate indexation economy relative to the no indexation benchmark. This spike in bank failures necessitates a wave of government bailouts, placing a large tax burden of nearly 6% of GDP on the population. An increase in tax payments to fund bailouts squeezes the borrower budget constraint, causing consumption to fall. It also depresses borrower housing demand, leading to a larger drop in house prices under aggregate indexation.

Aggregate indexation provides a modest reduction in mortgage default in the financial recession. Although this indexation protects borrowers from the large fall in national house prices, it is unable to stave off the increase in defaults due to higher idiosyncratic dispersion $\sigma_{\omega,t}$. This occurs because aggregate indexation is indiscriminately targeted, providing equal relief to the hardest-hit and relatively unaffected regions/households alike, with limited effects on the number of foreclosures.

Next, Table 3 compares welfare and consumption outcomes across the different indexation regimes. The increased financial fragility results in incredibly volatile intermediary wealth (W^I growth volatility goes up 1366.8%) and intermediary consumption (C^I growth volatility goes up 392.9%), as well as a larger drop in that consumption in a financial crisis. Borrower consumption growth volatility increases by 351.3%, albeit from a much lower base. Depositor consumption growth volatility decreases slightly. These results point to a deterioration in risk sharing between borrowers and intermediaries in the economy with aggregate indexation, measured by the volatility of the log marginal utility ratio between this pairs of agents in row 39 of Table 3. This ratio increases by 145.7%, indicating that markets have become more “incomplete.”

Figure 3: Financial Recessions: Benchmark vs. Aggregate Indexation Model



Black line: benchmark financial recession, **Red line:** aggregate index. financial recession. Responses are plotted in levels.

To assess the gains from aggregate indexation, we aggregate agents' value functions to obtain measures of welfare.²³ Borrowers are made worse off (row 27), both because their consumption has become more volatile (row 35) and because their consumption is lower (row 32) for reasons explained above. Borrowers face lower house prices and higher mortgage rates. Depositors' welfare and risk exposure are roughly unchanged (rows 28 and 33). Their mean consumption is slightly lower mostly because they earn lower interest rates on their savings and accumulate less wealth as a result (row 19 in Table 2). However, their consumption also becomes less volatile (row 36), causing a neutral net effect on their overall welfare.²⁴

²³There are many ways of computing aggregate welfare in incomplete markets economies with heterogeneous agents. The measure we present calculates welfare per capita for each agent type, multiplies it by the population share of each type, and sums across types.

²⁴Depositor consumption becomes less volatile because it experiences a smaller spike in financial recessions, see also Figure 3. This smaller spike is due to higher taxes that need to be raised to cover losses from bank bailouts.

Table 3: Results: Welfare and Consumption

	No Index	Aggregate	Local (25%)	Regional
Welfare				
26. Aggregate welfare	0.821	+0.17%	+0.06%	+0.32%
27. Value function, B	0.379	-0.57%	+0.43%	+0.27%
28. Value function, D	0.374	-0.07%	+0.07%	+0.47%
29. Value function, I	0.068	+5.66%	-2.11%	-0.21%
30. Value function, Bank	0.195	+6.71%	+4.20%	+9.15%
Consumption and Risk-sharing				
31. Total housing maint	0.058	-2.97%	-0.28%	-3.27%
32. Consumption, B	0.359	-0.3%	+0.3%	+0.1%
33. Consumption, D	0.372	-0.6%	+0.1%	+0.3%
34. Consumption, I	0.068	+6.1%	-2.9%	-0.4%
35. Consumption gr vol, B	0.42%	+351.3%	+15.9%	+189.0%
36. Consumption gr vol, D	1.11%	-10.4%	-26.5%	-15.4%
37. Consumption gr vol, I	4.47%	+392.9%	-54.1%	+282.5%
38. WI gr vol	0.035	+1366.8%	-1.8%	+679.3%
39. log (MU B / MU D) vol	0.025	-4.6%	-10.4%	-21.5%
40. log (MU B / MU I) vol	0.061	+145.7%	-36.8%	+101.8%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with full indexation of mortgage payments to aggregate house prices (second column), a model with partial indexation to relative local prices (third column), and a model with both aggregate and partial local indexation (fourth column). All flow variables are quarterly. Columns 2-6 calculate percentage differences relative to the benchmark model.

Finally, and perhaps surprisingly, intermediary households are made better off. Intermediary consumption levels increase because of the higher risk premia they earn on mortgage assets from the banks they own, and because they earn higher returns on REOs from the REO firms they own. This positive effect on the average level of consumption outweighs the massive increase in the volatility of intermediary consumption caused by a deterioration in risk sharing. All told, we obtain the interesting distributional result that insuring borrower exposure to aggregate house price risk leads bankers to gain at the expense of both borrowers and savers.

6.3 Local Indexation

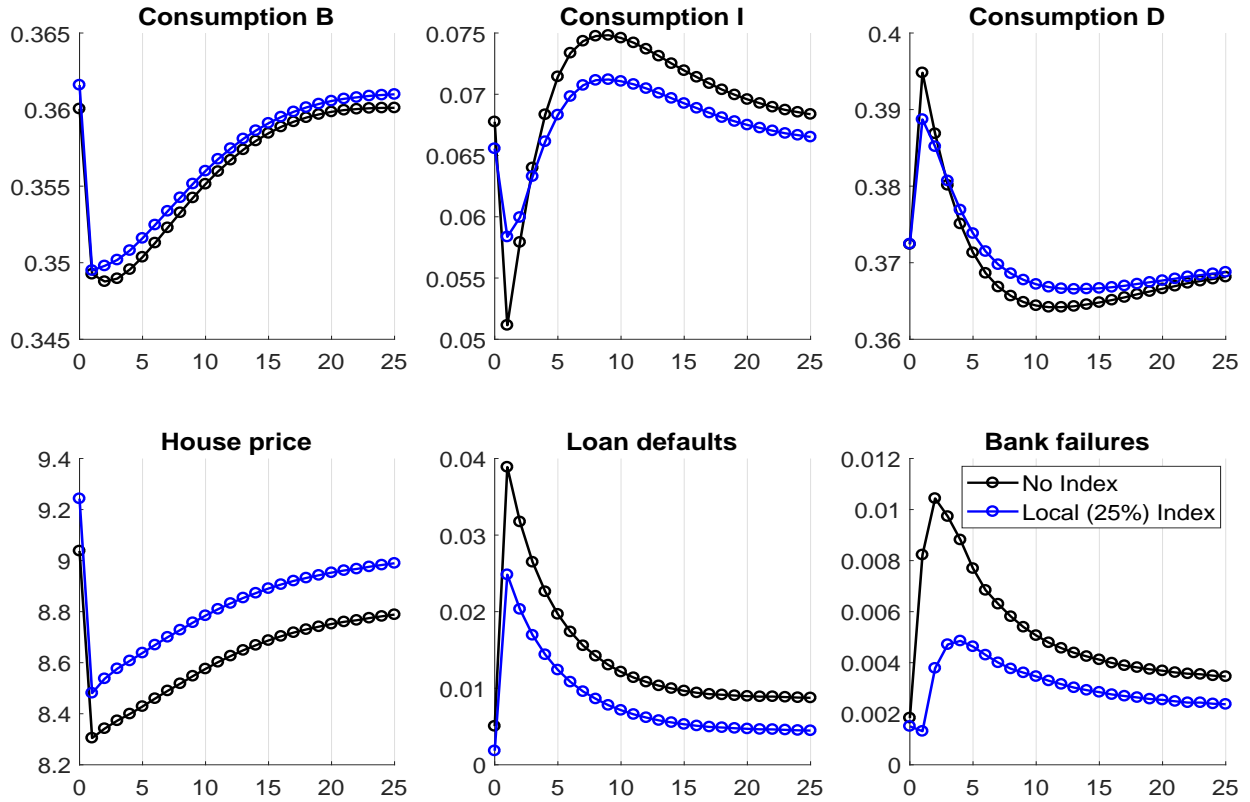
The third column of Table 2 reports simulation means for an economy with only local indexation ($\iota_p = 0, \iota_\omega = 1$), while the fourth column contains an economy with both types of indexation ($\iota_p = 1, \iota_\omega = 1$). To fix ideas how to interpret these two cases, we think of simultaneous aggregate and partial local indexation (column 4) as tethering mortgage debt to regional house price indexes, e.g. at the MSA or zip code level. The combined indexation co-moves one-for-one with the national house price index, and also adds a diversifiable local component. Purely local indexation (column 3) corresponds to indexation to only this local component. In practice, such a contract would have to be implemented by subtracting an aggregate house price index from regional indexes, and then indexing the debt of local borrowers to only the local residual. For example, during the Great Recession house prices fell substantially more in Las Vegas than in Boston. Purely local indexation would have implied a reduction in mortgage debt for Las Vegas borrowers, but an increase in debt for Boston borrowers.

Local Indexation Only. In sharp contrast to aggregate indexation, partially indexing mortgage debt only to relative local house prices causes the mortgage default rate to drop precipitously. This can also be seen in Figure 4, which compares crises in the benchmark model to crises in a model with only partial local indexation. Facing less default risk, banks lower mortgage interest rates, pushing up house values. These higher values support increased household borrowing, raising the average stock of mortgage debt, in turn financed with a larger deposit base.

While partial local indexation does not prevent the aggregate drop in house prices, it is highly successful at reducing foreclosures, sending debt relief to the households that experienced a larger drop in house prices. While banks react to this reduced risk by holding as little capital as allowed, the required minimum is sufficient to ensure a large decrease in risk overall. As a result, this economy does not suffer from financial fragility; bank failure rates fall to nearly zero as mortgage default risk dissipates, and bank wealth becomes dramatically less volatile. The risk-free interest rate rises slightly as the supply of deposits expands. At the same time, lower mortgage risk is reflected also in lower mortgage risk premia and mortgage spreads. Overall, the banking system is both safer and larger under this contract, but receives less compensation on a per-loan basis.

The welfare effects from local indexation are the reverse of those from aggregate indexation. Borrowers and depositors gain while intermediaries lose. Risk sharing in the econ-

Figure 4: Financial Recessions: Benchmark vs. Local Indexation Model



Black line: benchmark financial recession, **Blue line:** local index. financial recession. Responses are plotted in levels.

omy improves dramatically, as the volatility of marginal utility ratios between groups falls, especially between borrowers and intermediaries (rows 39, 40). Depositors and intermediaries also see large reductions in consumption growth volatility, while borrowers experience slightly increased volatility — albeit from a low level — due to larger housing and mortgage positions. The smaller changes in intermediary and depositor consumption during crises (top row of Figure 4) underscore this point. Depositors earn higher interest rates under this system, while borrowers pay lower rates on their mortgages, helping to boost the consumption of each group. In contrast, intermediary households’ mean consumption falls by 2.9% as dividends from REO firms and banks decline.

In sum, indexation to local house price shocks is highly effective at reducing the risk of foreclosures and financial fragility. More intermediation ensues, which makes both borrowers and savers richer. However, banking becomes less profitable.

Regional Indexation. The fourth column of Table 2 shows results for regional indexation, which indexes to both aggregate and local house price variation. Unsurprisingly, the simulation means in this column mostly lie between the aggregate-only and local-only cases in columns two and three. While adding pairing local and aggregate indexation decreases the bank default rate in the Regional model relative to the Aggregate model, the destabilizing effect of aggregate indexation is still enough to increase bank defaults relative to the No Index specification. The high degree of indexation in this economy strongly reduces the incentives to default, leading to the lowest borrower default rates among these four specifications. Nonetheless, signs of financial instability remain, particularly in the high consumption and wealth growth volatilities of the intermediary.

6.4 Interest vs. Principal Indexation

So far, our indexation applied both to interest payments and to principal. However, a number of the contract proposals mentioned in Section 2 envision indexing interest payments only, while leaving principal balances unchanged. These proposals are motivated by e.g., work by Fuster and Willen (2015) and Di Maggio et al. (2017) who suggest that households respond strongly to interest payment adjustments, as well as work by e.g., Ganong and Noel (2017) showing that households barely respond to principal adjustments, at least when the latter leave them underwater. To investigate these contracts, as well as the role of indexing each component of the mortgage payment more generally, we run a series of experiments in which either interest or principal payments, but not both, are indexed to house prices.

To this end, the first four columns of Tables 4 and 5 contrast the benchmark (no-indexation) and regional indexation models with Regional-IO and Regional-PO specifications that index only interest and principal payments, respectively. The default thresholds in these cases are given by:

$$\begin{aligned} \text{Interest Only : } \quad \bar{\omega}_t^U &= \frac{Q_{A,t}A_t + (\omega_{i,t}^L)^{\iota\omega} Q_{M,t}M_t}{\omega_{i,t}^L Q_{K,t}p_t K_t^B} \\ \text{Principal Only : } \quad \bar{\omega}_t^U &= \frac{(\omega_{i,t}^L)^{\iota\omega} Q_{A,t}A_t + Q_{M,t}M_t}{\omega_{i,t}^L Q_{K,t}p_t K_t^B} \end{aligned}$$

which are identical to (32) with the exception that only one component or the other in the numerator is indexed, but not both. Additionally, $Z_{M,t}$ is replaced by $Z_{N,t}$ and $\zeta_{p,t}$ is

replaced by 1 when scaling principal balances in the interest-only case. Symmetrically, $Z_{A,t}$ is replaced by $Z_{N,t}$ and $\zeta_{p,t}$ is replaced by 1 when scaling interest payments A_t in the interest-only case.

Interestingly, imposing *either* Regional-IO or Regional-PO indexation in isolation yields *lower* bank default rates (0.30% and 0.31%, respectively) relative to the No Index model (0.33%), even though applying both types of indexation simultaneously in the Regional model substantially increases bank failures (0.50%). This points to a potentially interesting nonlinearity, where a moderate amount of indexation improves financial stability, while too much indexation — particularly aggregate indexation — clearly harms it.

Turning to default, we see that indexing principal only delivers nearly all the default reduction in the Regional model, while indexing interest only gives up much of these gains. The effectiveness of the Regional-PO model in reducing default despite a lower level of indexation is a general equilibrium effect from reducing financial fragility. Specifically, a lower bailout burden in a financial recession dampens the drop in house prices, stabilizing household leverage. Mortgage credit spreads and expected excess returns are lower than in the regional model, consistent with the reduction in risk. Indexing principal only yields substantially higher levels of mortgage debt, deposits, and house prices relative to the Regional model, while indexing interest payments only delivers higher house prices, but no increase in debt, due to an increase in credit spreads.

Turning to welfare, we find that, as in the Regional model, borrowers benefit in both the Regional-IO and Regional-PO cases relative to the benchmark economy. These economies deliver higher average consumption to borrowers, who finance fewer bank bailouts, without generating the large increase in borrower consumption volatility found in the Regional model. The depositors' value function decreases modestly relative to the benchmark in both cases, as lower average consumption due to higher maintenance payments on more valuable houses outweighs the benefit of less volatile consumption.

In contrast, intermediaries suffer large welfare losses in the Regional-IO and Regional-PO economies. Under IO-indexation, a large drop in the volatility of intermediary wealth and consumption cannot make up for the loss of intermediation profits due to lower spreads and fewer bailouts, results akin to those we found with local indexation. In the case of PO-indexation, intermediary wealth and consumption are actually more volatile than in the benchmark model but still below the volatilities in the regional model. This additional intermediary instability in the Regional-PO relative to the Regional-IO model is intuitive, since in the former case banks adjust the entire mortgage principal when

Table 4: Results: Quantities and Prices (Alternative Indexation Schemes)

	No Index	Regional	Reg-IO	Reg-PO	Reg-Asym	Asym-IO
Borrower						
1. Housing Capital	0.456	0.463	0.458	0.463	0.468	0.461
2. Refi rate	3.84%	3.74%	3.84%	3.76%	4.42%	3.56%
3. Default rate	0.95%	0.47%	0.80%	0.49%	0.12%	0.55%
4. Household leverage	64.41%	65.80%	65.09%	65.63%	58.35%	62.85%
5. Frac. LTV binds at orig	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
6. Mortgage debt to income	259.59%	267.74%	261.60%	270.80%	231.85%	260.24%
7. Loss-given-default rate	38.61%	37.21%	39.98%	39.67%	33.83%	30.44%
8. Loss Rate	0.40%	0.20%	0.32%	0.20%	0.91%	0.35%
Intermediary						
9. Mkt fin leverage	93.98%	93.92%	93.99%	93.95%	93.99%	93.98%
10. Frac. leverage binds	99.35%	90.92%	99.04%	92.68%	100.00%	100.00%
11. REO maint	0.34%	0.18%	0.28%	0.18%	0.04%	0.21%
12. REO return	5.32%	6.76%	5.35%	5.76%	6.72%	5.25%
13. Bank dividend	0.010	0.012	0.010	0.011	0.013	0.010
14. REO dividend	0.005	0.003	0.004	0.003	0.001	0.003
15. Bank equity capital	0.184	0.194	0.187	0.194	0.170	0.180
16. Bank equity ratio	7.09%	7.25%	7.15%	7.16%	7.33%	6.92%
17. Bank default rate	0.33%	0.50%	0.30%	0.31%	0.94%	0.34%
18. DWL of bank defaults	0.07%	0.10%	0.06%	0.07%	0.16%	0.07%
Depositor						
19. Deposits	2.454	2.526	2.484	2.553	2.196	2.373
Prices						
20. House Price	8.842	8.784	8.806	8.900	8.488	8.663
21. REO house price	6.692	6.693	6.654	6.729	6.616	6.578
22. Risk-free rate	0.71%	0.75%	0.74%	0.74%	0.75%	0.77%
23. Mortgage Rate	1.46%	1.35%	1.41%	1.32%	2.37%	1.56%
24. Credit spread	0.75%	0.60%	0.70%	0.58%	1.62%	0.79%
25. Avg. Mort. Excess Ret.	0.34%	0.40%	0.35%	0.38%	0.49%	0.35%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with regional indexation (second column), a model with regional interest indexation only (third column), a model with regional principal indexation only (fourth column), a model with regional asymmetric indexation (fifth column), and a model with regional asymmetric interest indexation only (sixth column). All flow variables are quarterly.

Table 5: Results: Welfare and Consumption (Alternative Indexation Schemes)

	No Index	Regional	Reg-IO	Reg-PO	Reg-Asym	Asym-IO
Welfare						
26. Aggregate welfare	0.821	+0.32%	+0.20%	+0.18%	+0.73%	+0.25%
27. Value function, B	0.379	+0.27%	+0.30%	+0.33%	+1.85%	+0.53%
28. Value function, D	0.374	+0.47%	+0.25%	+0.21%	+0.07%	+0.37%
29. Value function, I	0.068	-0.21%	-0.61%	-0.75%	-1.91%	-2.02%
30. Value function, Bank	0.195	+9.15%	+1.16%	+6.20%	+2.78%	-1.32%
Consumption and Risk-sharing						
31. Total housing maint	0.058	-3.27%	-1.31%	-1.96%	-8.63%	-3.96%
32. Consumption, B	0.359	+0.1%	+0.1%	+0.3%	+1.9%	+0.5%
33. Consumption, D	0.372	+0.3%	+0.3%	+0.2%	+0.0%	+0.4%
34. Consumption, I	0.068	-0.4%	-1.1%	-1.7%	-1.6%	-2.9%
35. Consumption gr vol, B	0.42%	+189.0%	-14.4%	+14.0%	+393.6%	+0.5%
36. Consumption gr vol, D	1.11%	-15.4%	-6.6%	-14.6%	-25.2%	-14.2%
37. Consumption gr vol, I	4.47%	+282.5%	+2.1%	+130.8%	+332.9%	-18.7%
38. WI gr vol	0.035	+679.3%	-2.6%	+286.4%	+1435.6%	+12.2%
39. log (MU B / MU D) vol	0.025	-21.5%	-5.8%	-33.7%	+11.9%	-10.2%
40. log (MU B / MU I) vol	0.061	+101.8%	-7.7%	+51.0%	+104.2%	-20.5%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with regional indexation (second column), a model with regional interest indexation only (third column), a model with regional principal indexation only (fourth column), a model with regional asymmetric indexation (fifth column), and a model with regional asymmetric interest indexation only (sixth column).. All flow variables are quarterly. Columns 2-6 calculate percentage differences relative to the benchmark model.

prices move.

6.5 Asymmetric Indexation

Many real-world SAM proposals envision reducing mortgage payments when house prices fall but not increasing payments when prices rise. We now introduce such asymmetric contracts in the regional model. To accommodate these contracts, we assume full aggregate indexation and partial idiosyncratic (local) indexation, as in the Regional model, but cap the maximum upward indexation in each dimension. Specifically, we specify that aggregate indexation cannot increase debt balances or payments by more than $\bar{\zeta}_p$ per period, and that local indexation cannot increase debt balances or payments

by more than $\bar{\zeta}_\omega$ per period, implying that equations (6) and (7) become

$$\zeta_{p,t} = \min \left\{ \left(\frac{p_t}{p_{t-1}} \right)^{l_p}, \bar{\zeta}_p \right\}$$

$$\zeta_{\omega,t}(\omega_{i,t-1}^L, \omega_{i,t}^L) = \min \left\{ \left(\frac{\omega_{i,t}^L}{\omega_{i,t-1}^L} \right)^{l_\omega}, \bar{\zeta}_\omega \right\}$$

As a result, the default threshold is now defined by

$$\bar{\omega}_t^U = \frac{\min \left\{ \omega_{i,t}^L, \bar{\zeta}_\omega \omega_{i,t-1}^L \right\} (Q_{A,t} A_t + Q_{M,t} M_t)}{\omega_{i,t}^L Q_{K,t} K_t^B}$$

which, all else equal, weakly lowers the default threshold since local indexation terms become more generous when local house price growth is high.²⁵ For these experiments, we set $\bar{\zeta}_p = \bar{\zeta}_\omega = 1$, implying that mortgages are never indexed upward, but only downward. Column 5 of Tables 4 and 5 presents the results for the asymmetric indexation case. Column 6 presents results from a case that has asymmetric indexation but only of interest payments (Asymmetric IO). This case arguably comes closest to the real world proposals.

Asymmetric indexation substantially alters the mortgage landscape. To begin, banks now expect to take losses on average from indexation, since the debt relief they offer on the downside is not fully compensated by higher debt repayments on the upside. As a result, banks set much higher mortgage rates ex-ante, equal to 2.37% per quarter, to compensate them for these asymmetric transfers back to the households. At the aggregate, this has an effect very similar to shortening the amortization schedule of the bond (reducing δ), since borrowers make higher coupon payments in exchange for a much larger principal reduction each period — although in this case the paydown occurs largely through indexation rather than explicit principal payments. This has the consequence of shrinking the financial sector, with fewer deposits needed to back smaller mortgage balances. House prices also fall, as the collateral value of housing is lower under the effectively more frontloaded and therefore less desirable asymmetric indexation contracts.

Although borrowers compensate partially by increasing the average refinancing rate, the faster effective amortization of these loans leads to much lower household leverage, as the large increase in principal forgiveness overwhelms the higher rate of new borrowing. Lower household leverage in turn virtually wipes out default, since it now takes much larger shocks to push borrowers underwater. Nonetheless, financial fragility is massively

²⁵This update also requires changes to (8), (9), whose redefinitions can be found in Appendix A.3.

increased under this contract system, as the financial sector is exposed to a new source of potential losses in bad times that are no longer fully offset by increased payments when prices rise. The total loss rate for banks is 0.91%, several times larger than in the benchmark and in the regional indexation cases. Faced with this increase in losses, banks reduce leverage and increase their equity capital buffer modestly, but not by nearly enough to undo their additional risk. As a result, banks fail much more frequently, laying off the increased risk largely onto taxpayers. The deadweight losses from bank defaults increase by 70% relative to the symmetric indexation case, reducing resources available for consumption. The increase in financial fragility can also be seen in the severe increase in the volatility of intermediary wealth and consumption, on the order of the model with aggregate indexation only.

Turning to welfare, intermediaries suffer large losses, not only due to an increase in volatility, but also due to a drop in average consumption, as the much smaller financial system reduces intermediation income. This stands in contrast to our earlier finding with symmetric indexation that financial fragility tends to be good for intermediaries. Although borrower consumption becomes more volatile, borrowers are better off with asymmetric indexation due to an increase in average consumption. In part, this is due to the fact that under a lower level of steady state debt, borrowers make lower debt service payments. As a result, this welfare comparison between two steady states may overstate the benefits to borrowers, since it ignores the painful deleveraging period borrowers must undergo to get to the new steady state. Nonetheless, although Table 6 shows that borrower consumption initially falls along the transition path due to deleveraging, the welfare benefit for borrowers *including* the transition path, is a similar 1.76%, largely due to a fall in maintenance expenses under lower house values.

Column 6 of Tables 4 and 5 presents the asymmetric indexation of interest payments only, leaving the principal balance and payments unindexed. This case, featuring default threshold

$$\bar{\omega}_t^U = \frac{\min \left\{ \omega_{i,t}^L, \bar{\zeta} \omega_{i,t-1}^L \right\} Q_{A,t} A_t + Q_{M,t} M_t}{\omega_{i,t}^L Q_{K,t} p_t K_t^B}$$

lies in between the Regional-IO and Regional-Asymmetric models. Once again, banks anticipate substantial net forgiveness to borrowers, causing a rise in the mortgage rate, although not as extreme as in the Regional-Asymmetric case. Household leverage once again falls, in part for the same reasons as in the Regional-Asymmetric case, but strength-

ened by an additional and interesting force. Because interest payments are asymmetrically indexed, but principal payments are not, the average interest rate on loans as they age is falling. This tends to make older loans, many of which have experienced some interest forgiveness, favorable to new loans issued at the “full” interest rate. This pushes down the incentive to refinance, reducing the average refinancing rate. Less frequent refinancing of loans implies a longer period of repayment and forgiveness between renewals, leading to lower leverage. All told, this lower leverage reduces mortgage defaults to 0.55% per quarter, lower than the Regional model, but falling short of the Regional-Asymmetric case.

Reducing the financial sector exposure to indexation losses on principal leads to a lower bank failure rate relative to the Regional-Asymmetric case, but still a much higher one than in the Regional-IO case. Similarly, the mortgage market and banking sector do not shrink as much, nor do house prices fall as much. While borrowers gain by less than in the Regional-Asymmetric indexation regime, intermediaries find the Asymmetric-IO system to be even worse than the Regional-Asymmetric version, since it reduces intermediation profits without providing enough volatility for intermediaries to take full advantage of their limited liability option.

6.6 Robustness: Liquidity Defaults

A potential concern with our approach is that in reality, many mortgage defaults are triggered — at least in part — by household liquidity shocks, while our model only considers strategic default. Appendix A.3 studies a model of liquidity defaults and shows that it gives rise to a similar threshold rule for default that depends on the borrower’s loan-to-value ratio, generating default dynamics similar to those found in our setting. The reason is that households who cannot make their payments after a liquidity shock can sell their properties rather than entering into foreclosure if they have positive home equity. Generating substantially different results would require a large fraction of *above-water* foreclosures, an assumption that is not supported by the data. This suggests that our findings are robust to this source of borrower defaults.

7 Conclusion

Redesigning the mortgage market through product innovation may allow an economy to avoid a severe foreclosure crisis like the one that hit the U.S. economy in 2008-2010. We

study the welfare implication of indexing mortgage payments to aggregate or local house prices in a model with incomplete risk-sharing. A key finding is that indexation of mortgage payments to aggregate house prices may increase financial fragility. Inflicting large losses on highly-levered mortgage lenders in bad states of the world can cause systemic risk (high bank failure rates), costly tax-payer financed bailouts, meaningful house price declines, and higher risk premia on mortgages, all of which ultimately hurt the borrowers the indexation was trying to help. Moreover, aggregate indexation redistributes wealth from borrowers and depositors towards bankers, since a more fragile banking business also is a more profitable banking business.

In sharp contrast, indexation of cross-sectional local house price risk is highly effective at reducing mortgage defaults and financial fragility. It increases welfare for borrowers and depositors, but reduces intermediary welfare as mortgage banking becomes safer but less profitable.

Recent proposals have emphasized indexation of interest payments but not principal, and envision only downward payment adjustments. We find that implementing such contracts would result in modest aggregate welfare gains, with borrowers and depositors gaining and intermediaries losing. The switch would lead to a substantial reduction in mortgage defaults, but a small increase in financial fragility and bank defaults. Our results show that mortgage indexation in a world where intermediaries have limited liability can have important general equilibrium effects and must be designed carefully.

References

- ADRIAN, T. AND N. BOYARCHENKO (2012): "Intermediary Leverage Cycles and Financial Stability," Federal Reserve Bank of New York Staff Report No. 567.
- BENNETT, R. AND H. UNAL (2015): "Understanding the Components of Bank Failure Resolution Costs," *Financial Markets, Institutions and Instruments*, 24, 349–389.
- BERNANKE, B. S. AND M. GERTLER (1989): "Agency Costs, Net Worth and Business Cycle Fluctuations," *American Economic Review*, 79, 14–31.
- BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1996): "The Financial Accelerator and the Flight to Quality," *The Review of Economics and Statistics*, 78, 1–15.
- BRUNNERMEIER, M. K. AND Y. SANNIKOV (2014): "A Macroeconomic Model with a Financial Sector," *The American Economic Review*, 104, 379–421.
- CAPLIN, A., J. H. CARR, F. POLLOCK, AND Z. Y. TONG (2007): "Shared-Equity Mortgages, Housing Affordability, and Homeownership," *Fannie Mae Foundation Special Report*.
- CAPLIN, A., S. CHAN, C. FREEMAN, AND J. TRACY (1997): *Housing Partnerships: A New Approach to a Market at a Crossroads*, MIT Press.
- CAPLIN, A., N. CUNNINGHAM, M. ENGLER, AND F. POLLOCK (2008): "Facilitating Shared Appreciation Mortgages to Prevent Housing Crashes and Affordability Crises," The Hamilton Project at the Brookings Institution.
- CORBAE, D. AND E. QUINTIN (2014): "Leverage and the Foreclosure Crisis," *Journal of Political Economy*, forthcoming.
- DAVIS, M. A. AND F. ORTALO-MAGNÉ (2011): "Household Expenditures, Wages, Rents," *Review of Economic Dynamics*, 14, 248–261.
- DI MAGGIO, M., A. KERMANI, B. KEYS, T. PISKORSKI, R. RAMCHARAN, A. SERU, AND V. YAO (2017): "Monetary Policy Pass-Through: Mortgage Rates, Household Consumption and Voluntary Deleveraging," *American Economic Review*, forthcoming.
- EBERLY, J. AND A. KRISHNAMURTHY (2014): "Efficient Credit Policies in a Housing Debt Crisis," *Brookings Papers on Economic Activity*.
- ELENEV, V., T. LANDVOIGT, AND S. VAN NIEUWERBURGH (2016): "Phasing Out the GSEs," *Journal of Monetary Economics*, 81, 111–132.
- (2017): "A Macroeconomic Model with Financially Constrained Producers and Intermediaries," Working Paper, NYU Stern.
- EPSTEIN, L. AND S. ZIN (1989): "Substitution Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–968.
- FAVILUKIS, J., S. C. LUDVIGSON, AND S. VAN NIEUWERBURGH (2017): "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium," *Journal of Political Economy*, 125, 140–223.

- FUSTER, A. AND P. S. WILLEN (2015): "Payment Size, Negative Equity, and Mortgage Default," *Federal Reserve Bank of New York Staff Reports*, no. 582.
- GANONG, P. AND P. NOEL (2017): "The Effect of Debt on Default and Consumption: Evidence from Housing Policy in the Great Recession," .
- GÂRLEANU, N. AND L. H. PEDERSEN (2011): "Margin-based Asset Pricing and Deviations from the Law of One Price," *Review of Financial Studies*.
- GERTLER, M. AND P. KARADI (2011): "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58, 17–34.
- GREENWALD, D. L. (2016): "The Mortgage Credit Channel of Macroeconomic Transmission," Working Paper MIT Sloan School of Management.
- GUREN, A., A. KRISHNAMURTHY, AND T. MCQUADE (2017): "Mortgage Design in an Equilibrium Model of the Housing Market," Working Paper, Boston University and Stanford GSB.
- GUREN, A. AND T. MCQUADE (2016): "How Do Foreclosures Exacerbate Housing Downturns?" Working Paper Boston University and Stanford GSB.
- HARTMAN-GLASER, B. AND B. HEBERT (2017): "The Insurance is the Lemon: Failing to Index Contracts," Working Paper, UCLA Anderson.
- HE, Z. AND A. KRISHNAMURTHY (2012): "A Model of Capital and Crises," *The Review of Economic Studies*, 79, 735–777.
- (2014): "A Macroeconomic Framework for Quantifying Systematic Risk," Working Paper University of Chicago and Stanford University.
- HE, Z. AND A. KRISHNAMURTHY (2013): "Intermediary Asset Pricing," *American Economic Review*, 732–770.
- HULL, I. (2015): "The Macro-Financial Implications of House Price-Indexed Mortgage Contracts," *Economics Letters*, 127, 81–85.
- KALOTAY, A. (2015): "The Case for the Ratchet Mortgage," Presentation at the Mortgage Contract Design Conference, Federal Reserve Bank of New York.
- KEYS, B. J., T. PISKORSKI, A. SERU, AND V. VIG (2012): "Mortgage Financing in the Housing Boom and Bust," in *Housing and the Financial Crisis*, University of Chicago Press, 143–204.
- KIYOTAKI, N. AND J. MOORE (1997): "Credit cycles," *Journal of Political Economy*, 105, 211–248.
- KUNG, E. (2015): "Mortgage Market Institutions and Housing Market Outcomes," Working Paper, UCLA.
- LANDVOIGT, T., M. PIAZZESI, AND M. SCHNEIDER (2014): "Housing Assignment with Restrictions: Theory and Evidence from the Stanford Campus," *American Economic Review Papers and Proceedings*, 104, 67–72.
- MAGGIORI, M. (2013): "Financial Intermediation, International Risk Sharing, and Reserve Currencies," Working Paper, Harvard University.

- MIAN, A. (2013): "Financial Markets, The Macro Economy And The Middle Class," *Testimony On State of the American Dream: Economy Policy and the Future of the Middle Class*.
- MIAN, A., K. RAO, AND A. SUFI (2013): "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics*, 128, 1687–1726.
- MIAN, A. AND A. SUFI (2009): "The Consequences of Mortgage Expansion: Evidence from the U.S. Mortgage Default Crisis," *Quarterly Journal of Economics*, 124, 1449–1496.
- (2014): *House of Debt: How They (and You) Caused the Great Recession, and How We Can Prevent it from Happening Again*, University Of Chicago Press.
- MOREIRA, A. AND A. SAVOV (2016): "The Macroeconomics of Shadow Banking," *The Journal of Finance*.
- PIAZZESI, M., M. SCHNEIDER, AND S. TUZEL (2007): "Housing, Consumption and Asset Pricing," *Journal of Financial Economics*, 83, 531–569.
- PISKORSKI, T. AND A. TCHISTYI (2011): "Stochastic House Appreciation and Optimal Mortgage Lending," *Review of Financial Studies*, 24, 1407.
- (2017): "An Equilibrium Model of Housing and Mortgage Markets with State-Contingent Lending Contracts," NBER Working Paper No. 23452.
- ROUWENHORST, K. G. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, Princeton University Press.
- SHILLER, R. J. AND A. N. WEISS (1999): "Home Equity Insurance," *Journal of Real Estate Finance and Economics*, 19, 21–47.

A Appendix

A.1 Derivation of Bank FOCs

First, starting from the value function in (23), we can define a value function net of the idiosyncratic profit shock

$$V^I(W_t^I, \mathcal{S}_t^I) = V_{ND}^I(W_t^I, \mathcal{S}_t^I) + \epsilon_t^I$$

such that we can equivalently write the optimization problem of the non-defaulting bank after the default decision as

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + \mathbb{E}_t \left[\Lambda_{t,t+1}^I \max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0 \right\} \right], \quad (34)$$

subject to the same set of constraints as the original problem. We can now take the expectation with respect to ϵ_t^I of the term in the expectation operator

$$\begin{aligned} & \mathbb{E}_\epsilon \left[\max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0 \right\} \right] \\ &= \text{Prob}_\epsilon \left(\epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \mathbb{E}_\epsilon \left[V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I \mid \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right] \\ &= F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right), \end{aligned} \quad (35)$$

with $\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[\epsilon_{t+1}^I \mid \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right]$ as in the main text. Inserting (35) into (34) gives the value function in (26) in the main text.

To derive the first-order conditions for the bank problem, we formulate the Lagrangian

$$\begin{aligned} \mathcal{L}^I(W_t^I, \mathcal{S}_t^I) &= \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I, \lambda_t^I} W_t^I - J_t^I + \mathbb{E}_t \left[\Lambda_{t,t+1}^I F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right] \\ &\quad + \lambda_t^I \left(\phi^I \left(q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - B_{t+1}^I \right) \right), \end{aligned} \quad (36)$$

and further conjecture that

$$V^I(W_t^I, \mathcal{S}_t^I) = W_t^I + \mathcal{C}(\mathcal{S}_t^I), \quad (37)$$

where $\mathcal{C}(\mathcal{S}_t^I)$ is a function of the aggregate state variables but not bank net worth.

Before differentiating (36) to obtain first-order conditions, note that the derivative of the term in the expectation operator with respect to future wealth, after substituting in

this guess, is

$$\begin{aligned}
& \frac{\partial}{\partial W_{t+1}^I} F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \\
&= \frac{\partial}{\partial W_{t+1}^I} \left[F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) - \int_{-\infty}^{W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I)} \epsilon f_\epsilon^I(\epsilon) d\epsilon \right] \\
&= F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right).
\end{aligned}$$

Using this result, and differentiating with respect to L_t^* , \tilde{M}_t^I , \tilde{A}_t^I , B_{t+1}^I , and λ_t^I respectively, gives the first-order conditions

$$1 = q_t^M + r_t^* q_t^A, \quad (38)$$

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{M,t+1} \left((1-\delta) + \delta Z_{R,t+1} + \delta(1-Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}, \quad (39)$$

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{A,t+1} \left(1 + \delta(1-Z_{R,t+1}) q_{A,t+1}^A \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}, \quad (40)$$

$$q_t^f = \mathbb{E}_t \left[\Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \right] + \lambda_t^I, \quad (41)$$

and the usual complementary slackness condition for λ_t^I . Recalling the definition of J_t^I as

$$J_t^I = (1 - r_t^* q_t^A - q_t^M) L_t^* + q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - q_t^f B_{t+1}^I,$$

we note that the term in front of L_t^* is zero due to FOC (38), and we can substitute out prices q_t^M , q_t^A , and q_t^f from FOCs (39)-(41), both in J_t^I and in the constraint term in (36).

Further inserting our guess from (37) on the left-hand side of (36), and canceling and collecting terms, we get

$$\mathcal{C}(\mathcal{S}_t^I) = \mathbb{E}_t \left[\Lambda_{t,t+1}^I F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left(\mathcal{C}(\mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right], \quad (42)$$

which confirms the conjecture. $\mathcal{C}(\mathcal{S}_t^I)$ is the recursively defined value of the bankruptcy option to the bank. Note that without the option to default, one gets

$$\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[\epsilon_{t+1}^I \right] = 0.$$

Then the equation in (42) implies that $\mathcal{C}(\mathcal{S}_t^I) = 0$ and thus $V^I(W_t^I, \mathcal{S}_t^I) = W_t^I$. However, if the bank has the option to default, its value generally exceeds its financial wealth W_t^I by the bankruptcy option value $\mathcal{C}(\mathcal{S}_t^I)$.

A.2 Aggregation of Intermediary Problem

Before aggregating across loans, we must treat the distribution over $m_t(r)$, the start-of-period balance of a loan with interest rate r , as a state variable. In addition, the intermediary can freely choose her end-of-period holdings of these loans $\tilde{m}_t(r)$ by trading in the secondary market at price $q^m(r)$. In this case, the intermediary's problem is to choose nondurable consumption C_t^I , new debt issuance L_t^* , new deposits B_{t+1}^I , new REO investment I_t^{REO} , and end-of-period loan holdings $\tilde{m}_t(r)$ to maximize (2) subject to the budget constraint

$$\begin{aligned}
C_t^I = & \underbrace{(1 - \tau)Y_t^I}_{\text{disp. income}} + \underbrace{\int \left[X_t + Z_{A,t}r + Z_{M,t} \left((1 - \delta) + \delta Z_{R,t} \right) \right] m_t(r) dr}_{\text{payments on existing debt}} - \underbrace{(1 - q_t^m(r_t^*))L_t^*}_{\text{net new debt}} \\
& + \underbrace{q_t^f B_{t+1}^I - \pi_t^{-1} B_t^I}_{\text{net deposits}} - \underbrace{\int q_t^m(r) \left[\tilde{m}_t(r) - \delta(1 - Z_{R,t})Z_{M,t}m_t(r) \right] dr}_{\text{secondary market trades}} \\
& + \underbrace{\left[\rho_t + (S^{REO} - \nu^{REO}) p_t \right] K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO} \left[I_t^{REO} - X_t A_t^I \right]}_{\text{REO investment}}
\end{aligned} \tag{43}$$

and the leverage constraint

$$q_t^f B_t^* \leq \phi^M \int q_t^m(r) \tilde{m}_t(r) dr + \phi^{REO} p_t^{REO} \tilde{K}_t^{REO}$$

with the laws of motion

$$\begin{aligned}
m_{t+1}(r) &= \pi_{t+1}^{-1} \zeta_{p,t+1} \tilde{m}_t(r) \\
K_{t+1}^{REO} &= (1 - S^{REO}) K_t^{REO} + (1 - Z_{K,t}) K_t^B
\end{aligned}$$

and where the recovery rate X_t is defined as before. From the optimality condition for end-of-period holdings for loans with a given interest rate $\tilde{m}_t(r)$, we obtain

$$q_t^m(r) = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{A,t+1}r + Z_{M,t+1} \left((1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) q_{t+1}^m(r) \right) \right] \right\}}{1 - \lambda_t^I \phi^M}$$

where λ_t^I is the multiplier on the intermediary's leverage constraint. To obtain aggregation, we can split $q_t(r)$ into an interest-only strip with value q_t^M and a principal-only strip with value q_t^A , so that

$$q_t^m(r) = r q_t^A + q_t^M.$$

Substituting into the equilibrium condition for $q_t^m(r)$ verifies the conjecture and yields

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I Y_{t+1}^M \left[X_{t+1} + Z_{M,t+1} \left((1-\delta) + \delta Z_{R,t+1} + \delta(1-Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{1 - \lambda_t^I \phi^M}$$

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I Y_{t+1}^M Z_{A,t+1} [1 + \delta(1-Z_{R,t+1}) q_{t+1}^A] \right\}}{1 - \lambda_t^I \phi^M}.$$

Importantly, due to our assumption on the prepayment behavior of borrowers (ensuring a constant $Z_{R,t}$ across the r distribution), the prices q_t^A and q_t^M are independent of r . Substituting into the budget constraint, and applying the identities

$$M_t^I = \int m_t(r) dr$$

$$A_t^I = \int r m_t(r) dr$$

now yields the aggregated budget constraint (15) and leverage constraint (24).

A.3 Asymmetric Indexation: Additional Details

For the quantity of housing retained by the borrower, the only update needed is an extra integral to account for the dependence of $\bar{\omega}_t^U$ on both the lagged value of the local component and its innovation:

$$Z_{K,t} = \int \int \left(\int_{\omega_{i,t}^U > \bar{\omega}_t^U} \omega_{i,t}^U d\Gamma_{\omega,t}^U \right) \omega_{i,t}^L d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L$$

where $\Gamma_{e,t}^L$ is the CDF of the local component innovation $e_{i,t}^L$. Finally, the quantity of debt retained by the borrowers needs to be updated both for this change in the dependence of $\bar{\omega}_t^U$, as well as the cap on how much debt can be upwardly indexed:

$$Z_{M,t} = Z_{A,t} = \int \int \underbrace{\left(1 - \Gamma_{\omega,t}^U \left(\bar{\omega}_t^U(\omega_{i,t-1}^L, e_{i,t}^L) \right) \right)}_{\text{remove defaulters}} \underbrace{(\omega_{i,t}^L)^{t\omega}}_{\text{indexation}} d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L$$

In the case of interest-only asymmetric indexation, $Z_{K,t}$, $Z_{A,t}$, and $Z_{N,t}$ are computed as above, $Z_{M,t} = Z_{N,t}$, and $\zeta_{p,t}$ is replaced by 1 in the transition equation for principal balances M_t^B .

For all asymmetric cases, to maintain tractability, we account for updates to the average level of debt indexation through adjustments to $Z_{A,t}$ and $Z_{M,t}$ due to this asymmetry, but maintain the assumption — now only approximate — that the unconditional distribution of indexation still follows the same log-normal distribution around this mean.

A.4 Liquidity Defaults

This section considers the case where defaults are driven by liquidity concerns (the need to stop making mortgage payments) rather than the strategic motive of the baseline model. For simplicity we restrict attention to symmetric indexation ($\bar{\zeta}_p = \bar{\zeta}_\omega = \infty$) for this exercise. Assume that each period, fraction θ_t of borrowers are hit by a liquidity or turnover shock, so that they cannot make their mortgage payments this period. After being hit with the shock, borrowers have the choice of whether to sell the house or to default. Since the proceeds from a sale are:

$$\omega_{i,t} p_t K_t^B - (\omega_{i,t}^L)^{t_\omega} \delta \zeta_{p,t} M_t^B,$$

while the proceeds from a default are zero, the threshold uninsurable component level $\bar{\omega}_{i,t}^U$ at which the borrower defaults rather than sells (conditional on the local component and shock) is defined by

$$\bar{\omega}_t^U = \frac{(\omega_{i,t}^L)^{t_\omega} \delta \zeta_{p,t} M_t^B}{\omega_{i,t}^L p_t K_t^B}.$$

Given this threshold, the mortgage default rate is $\theta_t \Gamma_{\omega,t}^U(\bar{\omega}_t^U)$, and our other key default ratios are given by

$$\begin{aligned} Z_{N,t} &= \underbrace{(1 - \theta_t)}_{\text{no liq. shock}} + \underbrace{\theta_t (1 - \Gamma_{\omega,t}^U(\bar{\omega}_t^U))}_{\text{liq. shock}} \\ Z_{K,t} &= \underbrace{(1 - \theta_t)}_{\text{no liq. shock}} + \underbrace{\theta_t \int \int_{\omega_{i,t}^U > \bar{\omega}_t^U} \omega_{i,t}^U d\Gamma_{\omega,t}^U \omega_{i,t}^L d\Gamma_{\omega,t}^L}_{\text{liq. shock}} \end{aligned}$$

$$Z_{M,t} = Z_{A,t} = \underbrace{(1 - \theta_t)}_{\text{no liq. shock}} + \underbrace{\theta_t \int \int \left(1 - \Gamma_{\omega,t}^U(\bar{\omega}_t^U)\right) (\omega_{i,t}^L)^{t\omega} d\Gamma_{\omega,t}^L}_{\text{liq. shock}}.$$

This shows that the model with liquidity default generates very similar functional forms and implications as the model with strategic default, modulo the θ_t parameter. That θ_t could be endogenized to reflect the liquidity needs of consumers, or changed with economic conditions to reflect the hazard rate of falling into unemployment.

A.5 Transition Path Results

Table 6 shows the change in variables in the first period of transition on the path between the “No Index” steady state, and the steady state of an alternative model. This is particularly useful since the value functions will measure the total welfare change including the entire transition path to the new steady state.

Table 6: Transition Path Impacts (Alternative Indexation Schemes)

	No Index	Regional	Reg-IO	Reg-PO	Reg-Asym	Asym-IO
Welfare and Consumption						
Welfare	0.821	+0.61%	+0.36%	+0.51%	+0.90%	+0.28%
Value function, B	0.379	+0.68%	+0.61%	+0.83%	+1.76%	+0.36%
Value function, D	0.374	+0.54%	+0.34%	+0.28%	+0.11%	+0.47%
Value function, I	0.068	+0.53%	-0.95%	-0.03%	+0.51%	-1.25%
Consumption, B	0.359	+0.50%	+0.78%	+1.11%	-1.00%	-0.18%
Consumption, D	0.372	+0.82%	+1.49%	+0.32%	+0.47%	+2.42%
Consumption, I	0.068	+4.63%	-1.09%	+3.00%	+18.26%	+0.35%
Housing and Mortgage Market						
Deposits	2.454	+5.98%	+5.84%	+6.52%	-8.34%	+3.79%
House Price	8.842	+2.30%	+2.58%	+3.55%	-2.11%	+0.73%
Mortgage debt to income	2.596	+4.76%	+4.76%	+4.76%	+4.76%	+4.76%
Mortgage rate	1.46	-0.04%	-0.05%	-0.07%	+0.80%	+0.06%
Refi Rate	3.84	-0.00%	+0.07%	+0.10%	-0.82%	-0.15%
Loss Rate	0.40	-0.33%	-0.24%	-0.33%	+0.42%	-0.11%
Bank default rate	0.33	-0.24%	-0.19%	-0.21%	-0.29%	-0.20%

The table reports the initial change following a surprise switch from the baseline mortgage contract (“no index”) to an alternative contract. Each transition path is computed from a random starting point simulated from the stationary distribution of the benchmark model. All flow variables are quarterly.