Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy

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Abstract

Local currency (LC) debt provides consumption-smoothing benefits if it gets inflated away during recessions. However, we document that countries with more procyclical inflation and countercyclical LC bond returns, where consumption-smoothing benefits are lowest, issue the most LC debt. Monetary policy credibility explains this pattern through its effect on bond risk premia. In our model, low-credibility governments are more likely to inflate during recessions, generating excessively countercyclical inflation beyond the standard inflationary bias. In the model, and the data, low-credibility governments pay higher risk premia on LC debt, leading them to borrow in foreign currency.

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1 Introduction

Over the past decade, the market for emerging market government debt has undergone a remarkable transformation. In the 1980s and 1990s, most emerging market sovereigns and several developed country governments relied heavily on foreign currency (FC) in their foreign borrowing. This left borrowers vulnerable to currency fluctuations and financial crises (Eichengreen and Hausmann, 2005). Since the Asian Financial Crisis, local currency (LC) government bond issuance has grown rapidly. It now constitutes an important asset class for international investors and more than half of external debt issued by major emerging market sovereigns (Du and Schreger, 2016b). However, the shift toward LC government bonds has been highly uneven across markets.

This paper takes an asset-pricing perspective to understand cross-country differences in sovereign debt portfolio choice. The standard approach to optimal government finance implies that governments should smooth the costs of taxation across states of the world (Barro, 1979). If the costs of taxation rise during recessions, due to high marginal consumption utility or distortionary taxes, governments should issue debt that requires lower repayments in recessions than in expansions. Applying this argument to nominal LC debt, a key benefit of LC debt is that it can provide debt relief at just the right time, provided that inflation reduces the real debt burden in recessions (Bohn, 1990a,b; Barro, 1997; Lustig et al., 2008). However, we find empirically that countries where nominal LC debt provides little or no flexibility during adverse states of the world issue the most nominal debt.

Our primary proxy for the hedging properties of LC debt is the regression beta of LC bond returns with respect to stock market returns. A positive bond-stock beta indicates that LC bonds’ expected real cash flows decline in stock market downturns and hence provide fiscal flexibility to the issuer. Figure 1 summarizes the key stylized fact that countries with the lowest LC bond betas have the highest LC debt shares. Even more puzzlingly, a substantial fraction of the most prolific LC debt issuers, including both developed and emerging markets, have negative bond-stock betas, so LC debt provides no hedging benefits or is even risky from these issuers’ perspective.\(^2\) This is the opposite of what we would expect if governments

\(^2\)We show average LC debt shares in central government debt and the estimated slope coefficient of LC
issue LC debt to take advantage of its fiscal hedging properties.

We show that positive bond-stock betas coincide with countercyclical inflation, or negative inflation-output betas. This finding is important, because it indicates that inflation expectations are a key driver of the hedging properties of LC bonds, which depreciate when inflation expectations increase. We also show that local equity excess returns have betas with respect to U.S. equity excess returns that are statistically indistinguishable from 1, making it plausible that global investors require a risk premium for holding bonds that depreciate during periods of high local marginal utility.

The key finding that countries with more countercyclical LC bond returns rely more on nominal LC debt is robust to controlling for the exchange rate regime, GDP, and the commodity share of exports. Results look similar for the LC debt share in all central government debt, which is most closely related to a central government’s active issuance decisions, or the LC debt share held by foreigners, which plausibly generates an especially strong ex post incentive to inflate. It is also robust to using the cyclicality of realized or expected inflation with respect to output and to using bond betas that control for default risk, control for real exchange rate cyclicality, or exclude the financial crisis.

What explains this apparently puzzling relation? We demonstrate that it is the equilibrium outcome when monetary policy credibility drives the cyclicality of inflation and risk-averse investors require a risk premium to hold LC bonds in countries with positive inflation cyclicality. In the model, the government communicates a contingent plan for future inflation, but with a given probability it may revert to a myopic policy (Kydland and Prescott, 1977; Barro and Gordon, 1983; Rogoff, 1985). When commitment fails, the government uses inflation to reduce the real burden of LC debt. The incentive to inflate is more pronounced during low-output states, when marginal utility is highest. Crucially, debt is priced by risk-averse lenders, whose stochastic discount factor (SDF) is correlated with domestic output.

The key insight of the model is that when governments with imperfect credibility borrow in nominal terms from risk-averse lenders, they not only have a classic inflationary bias but also lack the ability to commit to a degree of state contingency on the debt. With risk-averse lenders, a government’s temptation to generate excessively countercyclical inflation government bond returns against local stock market returns for the period 2005—2014 for a sample of 30 emerging and developed countries. For details, see Section 2.
leads lenders to charge an inflation risk premium. This lowers average borrower consumption. But a government with full commitment can lower the risk premium it pays on LC debt. It achieves this by committing to an inflation process that keeps LC bond payouts relatively stable during recessions, when investors’ marginal utility is high, thereby increasing the insurance value of its LC debt to international investors. In contrast, a government lacking commitment cannot credibly promise to restrict itself to such a limited amount of state contingency and therefore pays a higher-than-optimal risk premium. In equilibrium, governments that obtain little or no consumption smoothing from issuing nominal debt (those with more procyclical inflation) issue the most nominal debt, and those that could obtain the most consumption smoothing from issuing nominal debt (those with more countercyclical inflation) issue the least.

Significantly, in our model limited commitment alone (without risk premia) cannot resolve the positive relationship between LC debt shares and inflation cyclicality. The intuition is that, without risk premia, high-credibility issuers optimally commit to using inflation only in bad states of the world, thereby smoothing tax distortions over states of the world and generating countercyclical inflation. As a result, the relation between LC debt shares and inflation cyclicality is flat or downward-sloping, in contrast to the data. In our model, it is only the interaction of imperfect commitment and risk-averse lenders that can explain the empirical patterns.

Finally, we present empirical evidence on the connection between LC bond risk premia and bond return cyclicality, monetary policy credibility, and LC debt issuance. First, we show that higher LC bond-stock betas are associated with significantly higher LC bond risk premia, supporting the model mechanism, whereby investors require a premium for holding LC bonds that tend to depreciate during downturns. Second, we provide direct evidence for the model mechanism by relating LC bond-stock betas and LC bond risk premia to two de facto measures of monetary policy credibility, based on official central bank inflation targets and newspaper text analysis. Third, we show empirical evidence that LC debt shares are strongly negatively correlated with LC bond risk premia. Decomposing LC bond risk premia into a world capital asset pricing model (CAPM) component and a residual or alpha, we find that the world CAPM component accounts for the majority of the downward-sloping relation between LC debt shares and risk premia. Finally, we show that changes in inflation forecast cyclicality, proxying for bond risks during periods when many issuers did not have
LC bond price data, from the 1990s to the 2000s, have a positive relation with changes in LC debt issuance, providing time series evidence that the bond risks channel of monetary policy credibility can also help us understand the substantial changes in LC debt issuance since the 1990s.

We contribute to the international asset pricing literature along two dimensions. First, we argue that risk premia matter for sovereign debt portfolio choice. Second, we provide a channel for why LC debt of low-credibility countries co-moves with international investors’ stochastic discount factor and hence requires a risk premium. Similarly to Hassan (2016) and Hassan et al. (2016), we argue that international government bond yields reflect the insurance value for investors, even though the source of comovement that we focus on – monetary policy credibility – is different from the sources they emphasize. In our model, comovement with international fundamentals is priced, consistent with empirical evidence in Harvey (1991); Karolyi and Stulz (2003); Lewis (2011); Borri and Verdelhan (2011); Lustig et al. (2011); David et al. (2016); Della Corte et al. (2016) among others.

The notion that limited inflation commitment constrains nominal debt issuance has a long-standing tradition in economics, going back at least to Kydland and Prescott (1977) and Lucas and Stokey (1983). The continued relevance of this question is emphasized by Bolton (2016) and references therein, which analyzes sovereign debt finance within a corporate finance framework. By contrast, we study the asset pricing implications of limited monetary policy commitment, thereby contributing to the literature on optimal debt management with nominal and inflation-indexed debt (Bohn (1988); Calvo and Guidotti (1993); Barro (1997); Alfaro and Kanczuk (2010); Díaz-Giménez et al. (2008)), and to the contemporaneous and complementary work by Ottonello and Perez (2016) and Engel and Park (2016).3 We contribute both empirically (by documenting the relation between inflation cyclicality and LC debt shares in a cross-section of countries) and theoretically (by proposing that investor risk aversion interacting with limited monetary policy credibility can explain this new stylized fact). Broner et al. (2013) consider a sovereign’s optimal debt maturity choice for FC debt in the presence of risk-averse investors, but they take the correlation between bond returns and investors’ stochastic discount factor to be exogenous. We add to that by

3Engel and Park (2016) study the currency composition of debt with optimal contracts and endogenous default when investors are risk-neutral. Ottonello and Perez (2016) present a quantitative model that generates predictions for the business cycle properties of LC debt issuance.
explaining bond return cyclicality and risks as an endogenous outcome of monetary policy credibility and matching cross-country evidence of bond return cyclicality. This paper is also related to a recent literature on inflation commitment and debt limits when the debt denomination is exogenous (Jeanne, 2005; Araujo et al., 2013; Aguiar et al., 2014; Chernov et al., 2015; Sunder-Plassmann, 2014; Bacchetta et al., 2015; Du and Schreger, 2016b; Corsetti and Dedola, 2015) and to the large literature on government debt and inflation (Sargent and Wallace, 1981; Leeper, 1991; Sims, 1994; Woodford, 1995; Cochrane, 2001; Davig et al., 2011; Niemann et al., 2013), but it differs in that it considers the optimal portfolio choice between LC and FC debt issuance.

Finally, we contribute to a recent literature on time-varying bond risks (Baele et al. (2010); David and Veronesi (2013); Campbell et al. (2014); Ermolov (2015); Campbell et al. (2015)) that is primarily focused on the U.S. and the UK. In contrast to Campbell et al. (2015) and Guorio and Ngo (2016), we abstract from supply shocks as drivers of inflation and bond return cyclicality for two reasons. First, different from those papers, our main empirical fact is not just about understanding inflation cyclicality but about its relation with LC debt issuance in a cross-section of countries. Exogenously countercyclical inflation due to supply shocks should lead countries with the most countercyclical inflation to issue the most LC debt (Bohn (1988, 1990b)), whereas we see the opposite in the data. Second, we show that our empirical results are robust to controlling for the commodity share in exports as a proxy for countries’ exposure to supply shocks.

The structure of the paper is as follows: In Section 2, we present new stylized facts on the relation between the cyclicality of LC bond risk and shares of LC debt in sovereign portfolios. In Sections 3 and 4, we lay out the model, provide analytical intuition for the key mechanisms, and calibrate the model to demonstrate that it can replicate the observed patterns of the currency composition of sovereign debt and inflation cyclicality. Section 5 tests additional model implications for LC debt issuance and risk premia. Section 6 concludes.

4Vegh and Vuletin (2012) also emphasize the evolution and cross-country heterogeneity in the cyclicality of monetary policy but do not study implications for sovereign debt portfolios. Poterba and Rotemberg (1990) examine the correlation between taxes and inflation under both commitment and no commitment in five major developed countries but do not consider the interaction with the currency composition of government debt.
2 Empirical Evidence

In this section, we demonstrate the robust empirical evidence that countries with more countercyclical inflation have lower LC debt shares. Our evidence is based on as large a cross-section of countries as permitted by the availability of LC debt data, including 11 developed markets (Australia, Canada, Denmark, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, the United States, and the United Kingdom) and 19 emerging markets (Brazil, Chile, China, Colombia, the Czech Republic, Hungary, Indonesia, Israel, Malaysia, Mexico, Peru, Philippines, Poland, Russia, Singapore, South Africa, South Korea, Thailand, and Turkey).

2.1 Nominal Bond Risks: Bond-Stock Beta

Asset markets incorporate investors’ forward-looking information at much higher frequency than surveys and can therefore provide valuable proxies for inflation cyclicality that are potentially less subject to measurement error and more robust given the relatively short time series. LC bond-stock betas serve as an asset-market-based proxy of inflation cyclicality. We expect bond-stock betas to be inversely related to the cyclicality of inflation expectations.

We denote the log yield on a nominal LC $n$-year bond as $y_{nt}^{LC}$, where $y_{nt} = \log(1 + Y_{nt}^{LC})$. The log holding period return on the bond is given by

$$r_{n,t+\Delta t}^{LC} \approx \tau_n y_{nt}^{LC} - (\tau_n - \Delta t) y_{n-1,t+\Delta t}^{LC},$$

where $\tau_n = \frac{1-(1+Y_{nt}^{LC})^{-n}}{1-(1+Y_{n-1}^{LC})^{-1}}$ is the duration of a bond selling at par (Campbell et al. (1997)). We approximate $y_{n-\Delta t,t+\Delta t}^{LC}$ by $y_{n,t+\Delta t}^{LC}$ for the quarterly holding period. We let $y_{lt}^{LC}$ denote

\[\text{5}\text{For LC bond yields, we use primarily Bloomberg fair value (BFV) curves. BFV curves are estimated using individual LC sovereign bond prices traded in secondary markets. Since sufficient numbers of bonds spanning different maturities are needed for yield curve estimation, the availability of the BFV curve is a good indicator for the overall development of the LC nominal bond market. Countries such as Argentina, Uruguay, and Venezuela have only a handful of fixed-rate bonds and hence do not have a BFV curve. Because for most emerging markets in our sample BFV curves are available starting in the mid-2000s, we focus on the period 2005—2014 to maintain a balanced panel. To measure inflation risk and the perceived cyclicality of inflation, we use realized inflation from Haver and inflation forecasts from Consensus Economics, respectively. Finally, we measure the share of LC debt in total sovereign debt portfolios with data from BIS Debt Securities Statistics, OECD Central Government Debt Statistics, and several individual central banks. All results winsorize the highest and lowest observation to ensure that results are not driven by outliers.}\]
the three-month T-bill yield and then the excess return on LC bonds over the short rate is given by

\[ x_{t}^{LC} = r_{n,t+\Delta t}^{LC} - y_{1t}^{LC}. \]

From a dollar investor’s perspective, we can rewrite the excess return as

\[ x_{t}^{LC} = \left[ r_{n,t+\Delta t}^{LC} - (y_{1t}^{LC} - y_{1t}^{US}) \right] - y_{1t}^{US}. \]

The dollar investor can hedge away the currency risk of the holding period \( \Delta t \) by going long a U.S. T-bill and shorting an LC T-bill with the same market value as the LC bond. By doing so, any movement in the spot exchange rate of the LC has the same offsetting first-order impact on the bond position and the local T-bill position and hence cancels out. After hedging currency risk for the holding period, the dollar investor bears duration risk of the LC bond.

We define the local equity excess returns as the log return on local benchmark equity over the three-month LC T-bill:

\[ x_{t}^{m} = (p_{t+\Delta t}^{m} - p_{t}^{m}) - y_{1t}^{LC}, \]

where \( p_{t}^{m} \) denotes the log benchmark equity return index at time \( t \). Country subscripts are suppressed to keep the notation concise. We then compute the local bond-stock beta \( b(bond, stock) \) by regressing LC bond excess returns \( x_{t}^{LC} \) on local equity excess returns \( x_{t}^{m} \):

\[ x_{t}^{LC} = b_{0} + b(bond, stock) \times x_{t}^{m} + \epsilon_{t}. \quad (1) \]

Bond-stock betas measure the risk exposure of LC bond returns on local equity returns.

### 2.2 Cyclicality of Inflation Expectations: Inflation-Output Forecast Beta

We construct a new measure for the procyclicality of inflation expectations by regressing the change in the consumer price index (CPI) inflation rate predicted by forecasters on the change in their predicted real GDP growth rate. Each month, professional forecasters surveyed by Consensus Economics forecast inflation and GDP growth for the current and
next calendar year. We pool all revisions for 2006 through 2013 (so that the forecasts were all made post-2005) and run the country-by-country regression:

\[ \Delta \tilde{\pi}_t = b_0 + b(\tilde{\pi}_t, \tilde{gdp}_t) \times \Delta \tilde{gdp}_t + \epsilon_t, \]  

where \( t \) indicates the date of the forecast revision. The revisions to inflation forecasts (\( \Delta \tilde{\pi}_t \)) and GDP growth forecasts (\( \Delta \tilde{gdp}_t \)) are percentage changes of forecasts made three months before and proxy for shocks to investors’ inflation and output expectations. The coefficient \( b(\tilde{\pi}_t, \tilde{gdp}_t) \) measures the cyclicality of inflation expectations and is the coefficient of interest.

Because forecasts are made for calendar years, the forecast horizon can potentially vary. Consensus Economics has forecasts for the annual inflation rate up to two years in advance. This means that in January 2008, the forecast of calendar year 2008 inflation is effectively 11 months ahead and the forecast of calendar year 2009 is 23 months ahead. We focus on revisions to the two-year forecast (13—23 months ahead) to minimize variation in the forecast horizon.

2.3 Cyclicality of Realized Inflation: Realized Inflation-Output Beta

While investors’ beliefs about inflation cyclicality enter into government debt prices and hence sovereign debt portfolio choice, it is useful to verify that the composition of debt portfolios also lines up with the cyclicality of realized inflation and output. We compute the realized inflation-output beta by regressing the change in the inflation rate on the change in the industrial production growth rate:

\[ \Delta \pi_t = b_0 + b(\pi, IP) \Delta IP_t + \epsilon_t, \]  

where \( \Delta \pi_t \) is the 12-month change in the year-over-year inflation rate and \( \Delta IP_t \) is the 12-month change in the year-over-year industrial production growth rate. The coefficient \( b(\pi, IP) \) measures the realized inflation cyclicality with respect to output. We obtain the seasonally adjusted CPI and the industrial production index from Haver between 2005 and 2014.
2.4 Local Currency Debt Shares

For developed countries, we construct the share of LC debt based on the OECD Central Government Debt Statistics and supplement this data with hand-collected statistics from individual central banks.\footnote{The OECD Central Bank Debt Statistics database was discontinued in 2010. We collected the statistics between 2010 and 2014 from individual central banks.} Central banks typically directly report the instrument composition of debt securities outstanding issued by the central government.

For emerging markets, we measure the share of LC debt in sovereign debt portfolios using the BIS Debt Securities Statistics, supplemented with statistics from individual central banks. Table 16C of the BIS Debt Securities Statistics reports the instrument composition for outstanding domestic bonds and notes issued by the central government ($D_t^{\text{dom}}$) starting in 1995. Table 12E of the BIS Debt Securities Statistics reports total international debt securities outstanding issued by the general government ($D_t^{\text{int}}$). For emerging markets, as the vast majority of international sovereign debt is denominated in foreign currency, and local governments rarely tap international debt markets, $D_t^{\text{int}}$ offers a good proxy for central government FC debt outstanding. Data for developed countries are from individual central banks or the OECD. The share of LC debt is computed as the ratio of the fixed-coupon domestic sovereign debt outstanding ($D_t^{\text{dom,fix}}$) over the sum of domestic and international government debt:

$$s_t = \frac{D_t^{\text{dom,fix}}}{D_t^{\text{dom}} + D_t^{\text{int}}}.$$

Inflation-linked debt, floating-coupon debt, and FC debt are all treated as real liabilities. In our baseline results, we do not distinguish between foreign-owned and domestically owned debt, but we provide evidence in Appendix B that empirical results are similar for foreign-owned debt.

2.5 Summary Statistics

Table 1 reports summary statistics for inflation, inflation expectations, LC bond yields, bond-stock betas, inflation-output forecast betas, realized inflation-output betas, local equity—S&P betas, and LC debt shares. Emerging market realized inflation is 2.4 percentage points higher, and survey-based expected inflation is 2.0 percentage points higher than in
developed markets. In addition, expected inflation and realized inflation are less procyclical in emerging markets than in developed countries.

For LC bonds, five-year LC yields are 3.4 percentage points higher in emerging markets than in developed markets. Nominal bond returns are countercyclical in developed markets, as is evident from negative bond-stock betas. By contrast, LC bond returns are procyclical in emerging markets. Finally, developed markets borrow almost entirely with LC debt, while the LC debt share in emerging market averages only 60%.

Importantly, column (7) shows that the beta of local stock returns with respect to U.S. S&P 500 stock returns, estimated as the slope coefficient of regressing local log excess equity returns onto U.S. log equity excess returns, is 1 on average for both developed and emerging economies in our sample. If local stock return variation proxies for variation in the local stochastic discount factor (SDF) and U.S. equity returns reflect variation in international investors’ SDF, a local-U.S. stock beta close to 1 implies that assets that co-move with the local SDF also co-move with the international investor’s SDF and hence are risky for international investors. This evidence is also consistent with the evidence in David et al. (2016), who argue that emerging market stock returns have large betas with respect to the world equity portfolio and consequently comovement with local stock markets carries a risk premium in international markets. In particular, this implies that if the domestic government inflates away its LC debt in states of high local marginal utility of consumption, LC debt tends to depreciate in real terms in bad states of the world for international investors, making it a risky investment.

2.6 Relation between Nominal Risk Betas and Sovereign Debt Portfolios

Figure 2 adds to the evidence in Figure 1, showing that patterns are similar if we measure bond return cyclicality with respect to U.S. instead of local stock returns and if we replace bond return cyclicality with inflation cyclicality. The inverse of LC bond betas should proxy for the cyclicality of inflation expectations, if higher inflation expectations depress LC bond prices and stock returns fall during recessions. Panels A and B of Figure 2 confirm this intuition. Emerging markets tend to have lower LC debt shares and more negative realized and expected inflation betas, as would be the case if they inflate during recessions. This
finding is important, because it indicates that inflation and output dynamics are key to understanding the cross-country patterns of LC bond risks.

Panel C of Figure 2 shows LC bond betas with respect to U.S. S&P returns, which is constructed analogously using LC bond betas with respect to S&P excess returns. For instance, the Brazilian bond-stock beta is estimated as the slope coefficient of LC bond excess returns with respect to Brazilian stock excess returns as in (1), while the Brazilian bond-S&P beta is the estimated slope coefficient of LC bond excess returns with respect to U.S. S&P excess returns. Panel C shows a striking correlation between bond-stock betas and bond-S&P betas across countries. Given this result, it is unsurprising that the relation between LC debt shares and bond-S&P betas in panel D is downward sloping, similar to the results for bond-stock betas in Figure 1. Taken together, panels C and D indicate that LC bonds that provide the best hedge for the issuer are also riskiest for an international investor. This finding is important, because it gives us further indication that it is reasonable to think of international investors as risk-averse over LC bonds that lose value in real terms during bad states of the world for local consumers, consistent with our modeling assumption that domestic and international investors price bonds with correlated SDFs.

Table 2 shows cross-sectional regressions of LC debt shares on measures of inflation cyclicality. The first three columns show that all nominal risk betas are significantly correlated with LC debt shares. A 0.16 increase in the bond-stock beta, corresponding to the average difference between emerging and developed markets, is associated with an 18 percentage point reduction in the LC debt share. Column (4) shows that the relation is robust to controlling for mean log GDP per capita, the exchange rate regime, and the share of commodities in total exports.\footnote{We use the exchange rate regime developed by Reinhart and Rogoff (2004) and the “Commodity Share” is defined as the sum of “Ores and Metals” and “Fuel” exports as a percentage of total merchandise exports from World Bank World Development Indicators.}

The relationship between the LC debt share and nominal risk betas is robust to using long-term debt, excluding the financial crisis, adjusting for default risk, and using only externally held government debt. The robust result for the LC debt share in long-term debt is important, because Missale and Blanchard (1994) argue that shorter debt maturity reduces the incentive to inflate away debt. The recent increase in emerging market LC debt issuance has been accompanied by a surge in importance of this asset class among international
investors. Consistent with this, our findings are also robust to using a proxy of the LC debt share held in foreigners’ portfolios instead of the LC debt share in all government debt, thereby ruling out the possibility that high LC debt shares simply reflect high LC debt shares held domestically, which might lead to less inflationary incentives than LC debt held by foreigners. The finding that cross-sectional variation in LC debt shares is driven by LC debt held by foreigners is important, because it corresponds most closely to our modeling assumption that debt is held by foreign investors. It also further strengthens the puzzle, because a high LC debt share held by foreigners should be especially useful for smoothing domestic consumption with state-contingent inflation. Detailed results are available in Appendix A.

3 Model

This section describes the model and presents analytic solutions for optimal debt portfolio and inflation policies. The model has two periods, periods 1 and 2. The timing within periods is as follows. In period 1, the government chooses simultaneously the optimal share of LC government debt $s_1$ and the optimal commitment inflation policy. We require the commitment inflation policy to be a function of only the local period 2 output $x_2$, consistent with the notion that complex monetary policy rules may be hard to verify and enforce. The LC debt share and the commitment inflation policy are chosen to minimize a loss function that reflects expected inflation distortions and domestic agents’ consumption utility, while taking into account that the government may deviate in period 2 from the ex-ante optimal inflation policy. At the end of period 1, international investors with rational expectations determine asset prices and buy the debt. At the beginning of period 2, agents learn the exogenous level of second period output and whether the government maintains or loses its commitment. If the commitment state holds, which occurs with probability $p$, the government implements the previously announced commitment inflation policy. Otherwise (with probability $1 - p$), the government re-optimizes myopically while taking the LC debt share $s_1$ and bond prices as given, thereby generating an incentive to inflate away LC debt in bad states of the world. Finally, the real exchange rate shock is realized, the government repays all of the debt, consumption occurs and inflation costs are realized. The government and investors are assumed to have complete information at all times.
The government objective is standard, combining domestic agents’ power utility over consumption and a quadratic inflation cost. We assume that investor marginal utility is correlated with domestic marginal utility, consistent with the empirical evidence of high local-U.S. stock betas in Table 1 and the close correspondence between bond-stock and bond-S&P betas in Figure 2, panel C, generating risk premia on nominal bonds.

We solve the model both using an approximate analytical solution method following Campbell and Viceira (2002) and using numerical global projection methods. For the analytic solution, we first characterize the commitment and no-commitment inflation policy functions conditional on the LC debt share $s_1$ and then solve for $s_1$ by taking the first-order condition of the government’s expected loss. The analytic solution for the ex-ante optimal commitment inflation policy and LC debt share relies on a second-order expansion of the government loss function. We keep the analytic solution tractable by solving for the no-commitment inflation policy in period 2 using a first-order log-linear expansion of the relation between real debt values and inflation, similarly to the use of a first-order intertemporal budget constraint in the analytic portfolio choice solutions of Campbell and Viceira (2002) and Devereux and Sutherland (2011). Throughout the analytic solution, we keep only first- and second-order terms of $\bar{D}$ in the loss function to clarify the intuition of our results. This approximation is justified if the debt-to-GDP ratio is small and state-contingent debt does not eliminate consumption volatility completely, an empirically plausible assumption. Similar approximation methodologies have been found to be highly accurate in many standard portfolio choice applications, and we verify the numerical accuracy by comparing the analytic solution with a numerical global solution method in Appendix B.3.

3.1 Government Objective

We use lowercase letters to denote logs. The government’s loss function combines quadratic loss in log inflation $\pi_2$ and power utility over consumption:

$$L_2 = \alpha \pi_2^2 - C_2^{1-\gamma}. \quad (4)$$

We do not take a stand on the source of inflation costs. A quadratic inflation cost of the form (4) may arise from price-setting frictions leading to production misallocation, as in New
Keynesian models (see Woodford (2003)). Period 2 output is log-normally distributed:

\[ X_2 = \bar{X} \exp(x_2/\bar{X}), \quad x_2 \sim N(0, \sigma^2_x) \]  

(5)

Domestic consumption equals output minus real debt repayments to foreign bond holders \( D_2 \), generating an incentive for the government to reduce real debt repayments:

\[ C_2 = X_2 - D_2. \]  

(6)

One interpretation of (6) is that the government only cares about the consumption of non-bondholders, who could be either foreigners or domestic agents to whom the government attaches little weight for political economy reasons. We normalize steady-state period 2 consumption to 1, so \( \alpha \) captures the cost of inflation distortions in units of period 2 consumption.\(^8\) Formally, we require that \( \bar{X} = 1 + \bar{D} \).

### 3.2 Investors

Financial markets are integrated in the sense that all assets are priced by the same international investor. However, markets are segmented from the point of view of the domestic borrower, who has access only to LC and FC debt borrowing and cannot go long bonds. Inflation in the investor’s home currency is assumed to be zero for simplicity. International consumption and domestic consumption can differ if international agents prefer a different consumption bundle from domestic agents.

The international investor is risk-averse over world output \( x^*_2 \), which is log-normally distributed with standard deviation \( \sigma^* \). We model the international investor’s SDF in reduced form with risk-aversion coefficient \( \theta \), similarly to Arellano and Ramanarayanan (2012), with local output \( x_2 \) loading onto world output:

\[ m^*_2 = \log \beta - \theta x^*_2 - \frac{1}{2} \theta^2 (\sigma^*)^2, \]  

(7)

\[ x_2 = \lambda x^*_2 + \eta_2. \]  

(8)

\(^8\)Allowing period 2 steady-state consumption different from 1 would scale the loss function (6) by a constant, leaving the analysis unchanged.
Here, \( \eta_2 \sim N(0, \sigma^2_\eta) \) is an idiosyncratic shock uncorrelated with world output. The SDF (7) captures risk-neutral investors as a special case when \( \theta = 0 \). If investor risk aversion \( \theta \) is greater than zero and global and local output are positively correlated (\( \lambda > 0 \)), the SDF (7) implies that investors’ and the domestic consumer’s marginal utility of consumption are positively correlated, or that bad states of the world for the domestic consumer also tend to be bad states of the world for the investor.

We interpret the SDF (7) and (8) broadly, potentially reflecting several channels. First, if international investors are risk-averse over international consumption and output, and international output is correlated with domestic output, this may give rise to a correlation between international and domestic marginal utility. We document a high correlation in output growth across countries, lending credence to this channel. In our sample, the average correlation between emerging market output growth and U.S. output growth is equal to 58%. Second, it is crucial for the government’s trade-off that domestic and international stochastic discount factors are correlated, but it is not essential for our channel that stochastic discount factor correlations arise entirely from output correlations. It is plausible that output correlations are a lower bound for the degree of international comovement in SDFs (Brandt et al., 2006). We find that the average correlation between emerging market stock returns and U.S. stock returns is even higher, at 70%, as would be the case if SDFs co-vary more than output. Interpreting (7) more broadly, highly correlated consumption growth (Colacito and Croce, 2011; Lewis and Liu, 2015), correlated discount rate news (Borri and Verdelhan, 2011; Viceira et al., 2016), correlated risk premia (Longstaff et al., 2011), or increasing correlations during downturns (Ang and Bekaert, 2002) may further drive up the cross-country correlations between SDFs and hence the role of risk premia in the government’s debt portfolio choice. A different way to motivate an SDF of the form (7) and to generate the main channel in our model would be if bond investors are domestic and hence risk-averse over domestic output, but the government has an incentive to expropriate bondholders, because ex post it is more efficient to use the inflation tax rather than taxes that distort incentives and lead to deadweight costs, such as income or sales taxes.\(^9\)

The role of real exchange rate shocks in the model is to capture a principal cost of FC borrowing of Eichengreen and Hausmann (2005), namely that FC debt exposes issuers to

\(^9\)Appendix B.6 develops such a model extension, shows that the analytic solutions are of the same form as in our benchmark model, and illustrates that our channel remains quantitatively important.
sudden increases in the real cost of debt service. This cost incentivizes otherwise unconstrained borrowers, such as the U.S., to borrow in LC. We normalize the real exchange rate in period 1 to 1. The period 2 real exchange rate (in units of international goods per domestic goods) is given by

\[ exp \left( \varepsilon_2 - \frac{1}{2} \sigma_\varepsilon^2 \right), \quad \varepsilon_2 \sim N \left( 0, \sigma_\varepsilon^2 \right), \quad (9) \]

where \( \varepsilon_2 \) is uncorrelated with all other shocks and realized after the government has chosen inflation, so monetary policy takes effect more slowly than exchange rate shocks. In modeling the real exchange rate, we face a tension arising from the well-known Backus-Smith puzzle. While under complete international financial markets the real exchange rate of a country is predicted to depreciate when consumption is high, empirically real exchange rates are close to uncorrelated with real economic fundamentals or even depreciate when consumption is low (Backus and Smith (1993)). We choose the reduced-form specification (9) to be consistent with the empirical evidence, so as to generate empirically relevant bond risk predictions. We keep the specification of exchange rates quite general, but one possible driver of exchange rates could be shocks to the risk-bearing capacity of financial intermediaries, as in Maggiori and Gabaix (2015). One important implication of (9) is that domestic inflation tends to devalue LC bonds for international agents and drives LC bond risks, consistent with the empirical evidence in panels A and B of Figure 2.\(^{10}\)

We can now price three different bonds: (a) an FC bond, which pays one unit of real international consumption; (b) a nominal LC bond, which delivers \( \exp(-\pi_2) \) units of real domestic consumption; and (c) a real LC bond, which delivers one unit of real domestic consumption. Bond prices are given by the following equations (see Appendix B for details):

\[ q_1^{FC} = \beta, \tag{10} \]
\[ q_1^{LC, real} = \beta. \tag{11} \]
\[ q_1^{LC} = E_1 \left[ \exp \left( \log \beta - \phi x_2 - \frac{1}{2} \phi^2 \sigma_x^2 \right) \exp(-\pi_2) \right], \quad \phi = \theta \lambda \frac{\sigma^*}{\sigma_x^2}. \tag{12} \]

LC bonds are priced as if the international investor had effective risk aversion \( \phi \) over

\(^{10}\)Given that there is some evidence that real exchange rates tend to depreciate in recessions, particularly in EMs, Appendix B.5 solves an extended model with this feature and finds that model implications are unchanged.
local output $x_2$. Expression (12) shows that the international investor is effectively more risk-averse over local output if risk aversion $\theta$ is high or if the local output loading onto world output $\lambda$ is high. The ratio of the variances enters, because if local output is more volatile than world output, world output moves less than one-for-one with local output, so international investors appear less risk-averse over local output variation. The real exchange rate does not enter into the pricing of real and nominal LC bonds, because in expectation one unit of real domestic consumption buys one unit of real international consumption and exchange rate shocks are uncorrelated with all other shocks. Finally, we denote one-period log bond yields by

$$y_{1}^{LC} = -\log q_{1}^{LC}, \quad y_{1}^{FC} = -\log q_{1}^{FC}. \quad (13)$$

We assume that domestic equity is a claim on domestic output and is priced by the same international investor, giving the equity risk premium faced by the international investor as

$$E_{1}(r_{2}^{e}) + \frac{1}{2} \text{Var}_{1}(r_{2}^{e}) - y_{1}^{FC} = \theta \text{Cov}_{1}(x_{2}^{*}, x_{2}) = \phi \sigma_{x}^{2}. \quad (14)$$

Equity is in zero supply to financial investors, thereby not entering into domestic consumption. The expression for the equity premium will be useful in Section 4 to calibrate the magnitude of risk premia.

We abstract from the risk of outright sovereign default. Under the assumption of simultaneous default, which Du and Schreger (2016a) and Jeanneret and Souissi (2016) show is empirically plausible, LC debt and FC debt by the same issuer bear the same default risk premium. Even then, issuing FC debt may be costly if it precludes the option to use inflation to avoid outright default. In the current framework, exchange rate volatility is the main driver making FC debt issuance costly, so adding such an additional cost of FC debt would act similarly to increasing the exchange rate volatility. For an analysis of the choice of the currency denomination of sovereign debt with strategic default, see Engel and Park (2016).
3.3 Budget Constraint

To focus on the portfolio choice component of the government’s decision, we assume that the government must raise a fixed amount \( V \). The government chooses face values \( D^{FC} \) and \( D^{LC} \) to satisfy the period 1 budget constraint:\(^{11}\)

\[
D^{FC}_1 q^{FC}_1 + D^{LC}_1 q^{LC}_1 = V. \tag{15}
\]

Let \( s_1 \) denote the share of nominal LC bonds in the government’s portfolio:

\[
s_1 = \frac{q^{LC}_1 D^{LC}_1}{V}. \tag{16}
\]

We define the debt portfolio log return from period 1 to period 2 in excess of the domestic consumption risk-free bond:\(^{12}\)

\[
xr^d_2 = \log \left( \frac{D^{FC}_1 \exp \left( -\varepsilon_2 + \frac{1}{2} \sigma^2 \right) + D^{LC}_1 \exp (-\pi_2)}{\beta^{-1} V} \right). \tag{17}
\]

3.4 Log-Quadratic Expansion for Loss Function

This section derives a log-quadratic expansion of the government loss function, which provides intuition and is used for the log-linear analytic solution. In contrast, the numerical solutions do not rely on the log-quadratic expansion, instead using the exact expressions in Sections 3.1 through 3.3. We obtain the following second-order expressions for LC bond

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\(^{11}\)Here, we do not explicitly allow the government to issue inflation-indexed LC debt. In contrast to the hypothetical real LC bond considered in the previous section, in practice inflation-indexed bond issuance appears to be costly. Inflation-indexed bond issuance can be costly for reasons analogous to those for FC debt, if indexation is imperfect, either because the inflation index does not correspond perfectly to the domestic borrower’s consumption basket or because indexation occurs with lags. In addition, empirical evidence from the U.S. suggests that inflation-indexed debt requires a substantial liquidity premium (Pflueger and Viceira (2016)). For this reason, in our empirical analysis we combine inflation-indexed and FC debt to capture inflation-insulated debt issuance.

\(^{12}\)Taking the expectation over \( \varepsilon_2 \), the average cost in terms of domestic consumption of repaying a unit face value FC bond is greater than 1. While the mean exchange rate is 1, the mean inverse exchange rate is not equal to 1 due to Jensen’s inequality. To purchase one unit of international consumption, the domestic borrower expects to give up more than one unit of real domestic consumption, because he has to average over states with different exchange rates. This divergence between the expected return on risk-free real FC and LC bonds is also known as Siegel’s paradox (Siegel (1972), Karolyi and Stulz (2003)).
prices and risk premia:

\[ q_1^{LC} \approx \beta \exp \left( -E_1 \pi_2 + \frac{1}{2} \text{Var}_1 \pi_2 + \phi \text{Cov}_1 (x_2, \pi_2) \right), \]  

\[ y_1^{LC} - E_1 \pi_2 + \frac{1}{2} \text{Var}_1 \pi_2 - y_1^{FC} = -\phi \text{Cov}_1 (x_2, \pi_2). \]  

The output-inflation covariance \( \text{Cov}_1 (x_2, \pi_2) \) enters as a risk premium term. Intuitively, a positive output-inflation covariance means that the issuer does not inflate during bad times, making LC bonds safe from investors’ point of view and increasing the value to investors. Provided that \( \pi_2 \) is a function of local output \( x_2 \) and the commitment state, as is the case in equilibrium, (19) also equals the possibly more familiar expression in terms of global output \(-\theta \text{Cov}_1 (x^*_2, \pi_2)\). We expand bond portfolio excess returns log-quadratically following Campbell and Viceira (2002):

\[ xr_2^d + \frac{1}{2} (xr_2^d)^2 \approx (1 - s_1) \left( \varepsilon_2 + \frac{1}{2} (\varepsilon^2_2 + \sigma^2_\varepsilon) \right) + s_1 \left( -(\pi_2 - E_1 \pi_2) + \frac{1}{2} ((\pi_2 - E_1 \pi_2)^2 - \text{Var}_1 \pi_2) - \phi \text{Cov}_1 (x_2, \pi_2) \right) \]  

Substituting back into a log-quadratic expansion of the loss function, taking expectations, and ignoring policy-independent terms gives the expected loss:

\[ E_1 L_2 = \underbrace{aE_1 \pi_2^2}_{\text{Inflation Cost}} + \underbrace{s_1 \bar{D}(\gamma - \phi) \text{Cov}_1 (x_2, \pi_2)}_{\text{Hedging - Nominal Risk Premium}} + \underbrace{\frac{\gamma^2 s_2^2 \bar{D}^2 \text{Var}_1 \pi_2}{2}}_{\text{Volatility LC Debt}} + \underbrace{\frac{\gamma^2 \bar{D}^2 (1 - s_1)^2 \sigma^2_\varepsilon + \bar{D} (1 - s_1) \sigma^2_\varepsilon}{2}}_{\text{Volatility + Convexity FC Debt}}. \]  

Here, we define \( \bar{D} = \beta^{-1} V \). We divide the expected loss into four terms. The first term, Inflation Cost, is simply the expected welfare cost of inflation. The second term, Hedging – Nominal Risk Premium, is new and is the focus of our analysis. This term captures the welfare benefits and costs of the state contingency of LC debt. Since this term depends on the comovement of expected inflation and output, it corresponds most closely to the expected inflation-output beta and the bond-stock beta, which are our key empirical proxies, rather
than the noisier realized inflation-output beta. There are two opposing forces: the welfare benefit of domestic consumption smoothing from a positive inflation-output covariance is counteracted by the risk premium that can be earned by selling insurance to risk-averse investors. If $\gamma > \phi$, the model formalizes the intuition from the introduction, where a government inflates in bad times in order smooth consumption, and the benefits of doing so outweigh the risk premium that needs to be paid for this insurance. In contrast, if $\phi > \gamma$, the benefit to the government from selling insurance to foreign investors outweighs the desire to smooth domestic consumption. In this case, the loss function decreases with the inflation-output covariance, because a government that inflates during good times and deflates during bad times earns a risk premium from risk-averse investors, thereby raising average domestic consumption. To preview our results, one of the most important considerations in solving this problem is to understand when the government can credibly promise a less negative or even positive inflation-output covariance. As long as the investor has non-zero risk aversion ($\phi > 0$), the government wants to limit the tendency to inflate during bad states of the world ex ante, but may deviate ex post.

The final two terms capture losses from consumption volatility induced by the volatility in debt repayments. The volatility of debt repayments enters into expected domestic consumption utility, because domestic consumers have a non-diversified, non-zero debt position, and consumption utility is concave. The third term, Volatility LC Debt, captures the utility losses from consumption volatility caused by the fact that inflation volatility induces movements in the real amount repaid on LC debt. If the country has no LC debt ($s_1 = 0$), this effect disappears. The final term, Volatility + Convexity FC Debt, captures losses from borrowing in foreign currency induced by fluctuations in the exchange rate and disappears if the country has no FC debt ($s_1 = 1$). Exchange rate volatility lowers expected consumption through a convexity effect and induces variation in domestic real consumption, which is costly due to utility curvature. In the same way that inflation volatility induces fluctuations in consumption by inducing volatility in LC debt repayments, so do exchange rate fluctuations through their effect on real debt repayments on FC debt. In addition, FC debt is costly because the expected inverse exchange rate is greater than 1 over the expected exchange rate.
3.5 Analytic Solution

This section presents the analytic model solution. For solution details, see Appendix B.

3.5.1 Inflation Policy Functions

We first solve for the inflation policy function as a function of local output and the LC debt share. There will be two inflation policy functions: one for the commitment state and one for the no-commitment state. We first characterize inflation policy functions conditional on the LC debt share $s_1$ and then solve for the optimal $s_1$. If the no-commitment state is realized, which occurs with probability $1 - p$, the government re-optimizes the ex post loss function (4), taking as given any quantities that were determined in period 1. Combining a log-linear expansion of the relation between real debt repayments and inflation with a log-quadratic expansion for the government loss function (4) gives the ex post optimal inflation policy that the government will implement when it loses commitment:\textsuperscript{13}

$$\bar{\pi}_{2}^{nc} = \frac{s_1 \bar{D}}{2\alpha} - \gamma \frac{s_1 \bar{D}}{2\alpha} x_2. \quad (22)$$

The first term in (22) captures the standard inflation bias of a myopic government. The bias increases with the amount of LC debt $s_1 \bar{D}$ and decreases in the real marginal cost of inflation $\alpha$. The second term captures inflation cyclicality, showing that the incentive to inflate is greatest during recessions, when output is low and the marginal utility of consumption is high for domestic consumers. The degree of countercyclicality depends on $\gamma \frac{s_1 \bar{D}}{2\alpha}$. This term is intuitive, because $\gamma$ is the curvature of the domestic agents’ consumption utility and determines how much the marginal utility of consumption increases in low-consumption states. The amount of LC debt $s_1 \bar{D}$ and the cost of generating inflation $\alpha$ enter similarly as for the inflation level. Risk premia do not enter the no-commitment inflation policy function, because from the perspective of a reoptimizing government they are predetermined.

In period 1, the government chooses the commitment inflation rule to minimize (21),

\textsuperscript{13}The log-linear expansion simplifies the effect that inflation has on the real value of debt. It effectively approximates that a one percentage point of extra inflation provides the same real debt relief at any level of inflation. This approximation is reasonable for low to moderate inflation regimes, as in our empirical sample. It greatly enhances tractability of the analytic solution, because it allows us to solve separately for the optimal inflation policies across the commitment and no-commitment states.
while internalizing that with probability $1 - p$ its future self will deviate and choose inflation according to (22). This gives the commitment policy function:

$$\pi^c_2 = (\phi - \gamma) \frac{s_1 \bar{D}}{2\alpha} x_2.$$  \hfill (23)

The commitment inflation rule (23) exhibits no inflationary bias on average. Inflation-cyclicality, as captured by the inflation-output slope coefficient also changes and has a new non-negative term $\phi$. The slope coefficient in (23) is positive, and the government wants to commit to procyclical inflation if and only if investors have higher effective risk aversion than the government, because government debt has hedging value to investors and sells at a premium.

### 3.5.2 Inflation Moments and LC Debt Share

Analogously to our empirical analysis, we define the inflation-output beta as the slope from regressing period 2 log inflation $\pi_2$ onto period 2 log output $x_2$. The mean, variance, and inflation-output beta for period 2 inflation then equal:

$$E_1(\pi_2) = (1 - p) s_1 \bar{D} \frac{1}{2\alpha},$$  \hfill (24)

$$Var_1(\pi_2) = \left( \frac{s_1 \bar{D}}{2\alpha} \right)^2 \left( p(1 - p) + \left( \gamma^2 - p\phi (2\gamma - \phi) \right) \sigma^2_x \right),$$  \hfill (25)

$$Beta(\pi_2, x_2) = \frac{(p\phi - \gamma) s_1 \bar{D}}{2\alpha}.$$  \hfill (26)

We can gain intuition by considering two special cases with zero credibility ($p = 0$) and full credibility ($p = 1$). With $p = 0$, the government has no ability to commit and the inflation-output beta reduces to $Beta(\pi_2, x_2) = -\frac{\gamma s_1 \bar{D}}{2\alpha}$. A government without commitment is always tempted to inflate during recessions, leading to countercyclical inflation and a negative inflation-output beta.

With full credibility ($p = 1$), the inflation-output beta becomes $Beta(\pi_2, x_2) = \frac{(\phi - \gamma) s_1 \bar{D}}{2\alpha}$, which is greater than the inflation-output beta with zero commitment as long as effective investor risk aversion $\phi$ is positive. In particular, when $\phi = \gamma$ the full credibility government’s inflation-output beta is zero and inflation is constant. More generally, provided that $\phi > 0$, ...
increases with credibility \( p \). While it is well understood that a lack of credibility can lead to an inflationary bias, our contribution is to show that a lack of credibility also affects sovereign debt portfolio choice through an inflation cyclicality channel.

In the model, LC bond payoff surprises are perfectly negatively correlated with inflation and equity payoff surprises are perfectly positively correlated with output, so the bond-stock beta is exactly the negative inflation-output beta. This links back to our empirical evidence, where we proxy for inflation cyclicality both directly and with the negative beta of LC bond returns with respect to the local stock market, which can be measured more precisely using higher-frequency financial data.

Substituting (24) through (26) into the expected loss function (21) and taking the first-order condition with respect to the LC debt share \( s_1 \) gives an intuitive expression for this debt share (assuming an interior solution):

\[
s_1 = \frac{2\alpha \left[ \gamma + 1/D \right] \sigma_x^2}{(1 - p) (1 + \phi^2 \sigma_x^2) - (\phi - \gamma)^2 \sigma_x^2 + 2\alpha \gamma \sigma_x^2},
\]

(27)

The model provides guidance regarding how we should think about measuring the LC debt share in the data. The LC debt share \( s_1 \) plays two different roles in the model, each of which is more accurately represented by different measures in the data. The optimal inflation policy functions (22) and (23) depend on how many resources one percentage point of inflation transfers from foreigners to domestic agents. On the other hand, the equilibrium LC debt share (27) reflects the government’s choice of a debt portfolio. Since a central government arguably controls how much LC debt it issues, but not necessarily how much of that debt is held by foreigners, we use as the main variable in our empirical analysis the LC debt share in all central government debt. All our empirical results are robust to using the LC debt share of externally-held debt and the LC debt share of long-term debt – two types of debt that provide a domestic borrower with particularly strong real debt relief for each percentage point of inflation.
3.5.3 Comparative Statics

From (27), we derive the comparative static for the LC debt share with respect to credibility:

\[
\frac{ds_1}{dp} = s_1^2 \frac{1 + \phi^2 \sigma_x^2}{2\alpha \left[\gamma + 1/D\right] \sigma_x^2} > 0. \tag{28}
\]

Provided that the LC debt share \(s_1\) is at an interior solution, it increases with credibility. As credibility increases, the government faces smaller risk premia for issuing LC debt. Moreover, the probability of inefficiently high inflation for a government with LC debt declines. Both of these factors reinforce each other to increase the LC debt share for high-credibility governments.

Next we explore the model implications for the relation between inflation cyclicality and LC debt shares. Combining (26) and (28) gives the total derivative:

\[
\frac{dBeta(\pi_2, x_2)}{ds_1} = \frac{\partial Beta(\pi_2, x_2)}{\partial s_1} + \frac{\partial Beta(\pi_2, x_2)}{\partial p} \frac{1}{\frac{ds_1}{dp}} = \left(\frac{p\phi - \gamma}{2\alpha}\right) D + \frac{\phi \bar{D} \left[\gamma + 1/D\right] \sigma_x^2}{s_1 \left(1 + \phi^2 \sigma_x^2\right)}. \tag{29}
\]

Our main stylized empirical fact, which finds that LC debt shares are positively related to inflation-output betas, predicts \(\frac{dBeta(\pi_2, x_2)}{ds_1} > 0\). The model inflation-output beta varies with the LC debt share \(s_1\) through two channels. First, the direct effect of a higher LC debt share is to increase both the consumption-smoothing benefits of countercyclical inflation and the amount of real consumption that can be gained from making LC debt safe for investors. Through this channel, the effect of increasing the LC debt share \(s_1\) is proportional to the inflation-output beta (26). The first term in (29) is negative if the government is more risk-averse with respect to domestic output than investors are (\(\gamma > \phi\)), or if credibility \(p\) is low. In this case, to generate a positive relation between inflation-output betas and LC debt shares as in the data, the second term would have to be sufficiently positive to outweigh the direct effect.

Second, the equilibrium relation between inflation-output betas and LC debt shares reflects the effect of credibility on both variables. Expression (26) shows that the inflation-
output beta increases with credibility (strictly, if $\phi > 0$), because with higher credibility we need to put a higher weight on the stable inflation policy. Since the LC debt share also increases with credibility, variation in credibility induces a non-negative relation between LC debt shares and inflation-output betas. This second channel is larger if effective investor risk aversion $\phi$ is high. The reason is that a high-credibility government has a stronger incentive to limit inflation state contingency when risk premia are large.

The case $\phi = 0$ illustrates that limited commitment alone cannot plausibly generate the upward-sloping relation between inflation-output betas and LC debt shares in the data. Risk-averse investors are therefore essential to matching the downward-sloping empirical relation between inflation-output betas and LC debt shares. In the absence of risk premia, a high-credibility government optimally follows a countercyclical inflation policy that generates inflation only in bad states of the world to smooth consumption, generating a negative inflation-output beta (26) in contrast with the empirical evidence that countries with high LC bond shares have zero or even positive inflation-output betas and zero or negative bond-stock betas. Moreover, (29) is negative, so the model predicts a downward-sloping relation between LC debt shares and inflation-output betas. Intuitively, because domestic consumption is far from perfectly hedged, as appears plausible empirically, the marginal benefit from further consumption hedging to the government is high. Consequently, a higher LC debt share increases the benefit of each additional percentage point of inflation, increasing the incentive to vary inflation countercyclically over the business cycle.\(^{14}\)

### 4 Calibrating the Model

In this section, we calibrate the model to examine whether the forces discussed in Section 3 can quantitatively replicate the empirical patterns and to assess the numerical accuracy of the analytic solution. The analytic solution helps us select parameter values without an expensive grid search. We use global solution methods to solve for the full nonlinear

\(^{14}\)Only if domestic consumption is close to perfectly hedged, which appears less empirically plausible, can the model generate an upward-sloping relation between LC debt shares and inflation-output betas, because then the marginal benefit of inflation variation decreases with the amount of LC debt outstanding. However, with close to perfect domestic consumption hedging, the model counterfactually predicts negative inflation-output betas and positive bond-stock betas for LC debt issuers. Formally, we capture limited consumption smoothing in the analytic solution by taking an expansion with $D^3$ small. For a solution that keeps third-order terms in $D$, see Appendix B.
solution (i.e., not the analytic solution). Table 3 reports calibration parameters, and Table 4 compares empirical and model moments.

We solve the model for two calibrations that differ only in terms of credibility $p$. The high-credibility calibration uses $p_H = 1$, corresponding to full credibility, while the low-credibility calibration has $p_L < 1$. We choose the low-credibility calibration to target the difference in empirical moments between emerging markets and developed markets, reported in the leftmost column of Table 4.

We set the government’s borrowing need to 13% of GDP, corresponding to the average share of external sovereign debt in emerging markets. We set exchange rate volatility to $\sigma_\varepsilon = 14\%$ to match the median annual volatility of emerging market exchange rate returns since 1990. A substantial cost of borrowing in FC implies that the share of LC debt falls relatively slowly with respect to $p$ in equilibrium, ensuring that even low-credibility countries have some LC debt.

With (22) and (24), we have that $E_1\pi_{2,L} = (1 - p_L)E_1\pi_{2,L}^{nc}$. Identifying $E_1\pi_{2,L}$ with average emerging market survey inflation in excess of developed market survey inflation and $E_1\pi_{2,L}^{nc}$ with maximum emerging market survey inflation in excess of average developed market survey inflation pins down $p_L = 1 - \frac{2.00\%}{6.07\%} = 0.67$. We calibrate the inflation cost to match average emerging market survey inflation in excess of developed market survey inflation of 2.0%. With (24) we obtain:

$$\alpha = \frac{(1 - p_L)s_{1,L}\bar{D}}{2E_1\pi_{2,L}} = \frac{0.33 \times 0.5 \times 0.13}{2 \times 0.02} = 0.5. \quad (30)$$

We explore model implications for a wide range of values for $\phi$. We set $\phi = \gamma$ for our benchmark calibration. The benchmark case of equal government and effective investor risk aversion has appealing implications. It implies that a full-credibility issuer chooses an all LC debt portfolio and perfect inflation targeting, with no inflation variability, similarly to developed countries in our sample.\(^{16}\) We choose government and effective investor risk

\(^{15}\)We minimize the Euler equation error for the inflation policy function in the no-commitment state over the no-commitment policy function. We then minimize the loss function over the commitment policy function and the LC debt share. Both commitment and no-commitment policy functions for log inflation are quadratic in log output. For details and a sensitivity analysis of model moments to individual parameters, see Appendix B.

\(^{16}\)In our sample, the mean beta of local equity returns on U.S. equity returns is 0.97 and the mean beta of local GDP growth on U.S. GDP growth 0.86. Therefore, assuming equal risk aversion ($\gamma = \theta$) between the
aversion ($\gamma$ and $\phi$) to match the empirical difference in inflation-output betas of $-0.21$. We substitute into (26):

$$Beta_L(\pi_2, x_2) - Beta_H(\pi_2, x_2) = -\gamma \bar{D}_{1,L} \frac{1-p_L}{2\alpha} (1-p_L) = -\gamma \times \left[ \frac{0.13 \times 0.5}{2 \times 0.5} \times 0.33 \right],$$

indicating that we need risk aversion on the order of $\gamma = 10$ to match the empirical difference in inflation-output betas across emerging and developed markets. While a risk-aversion parameter of 10 is high, it is at the upper end of values considered plausible by Mehra and Prescott (1985).

Finally, high output volatility $\sigma_y = 8\%$ is needed to generate a plausible level for the equity premium. While this volatility is higher than emerging market output volatility in our sample, a higher volatility may be priced into asset markets if emerging markets are subject to crashes and crises. We do not attempt to explain the equity volatility puzzle (Shiller, 1981; LeRoy and Porter, 1981), which can be resolved if consumption and dividend growth contain a time-varying long-run component (e.g., Bansal and Yaron (2004)) or if preferences induce persistent fluctuations in risk premia (e.g., Campbell and Cochrane (1999)).

Table 4 shows that the calibration matches the empirical moments quite well. We obtain average low-commitment inflation of around 3% and maximum no-commitment inflation of 8%. The inflation-output beta for the low-credibility calibration is -0.27 compared to a high-credibility beta of 0, matching the difference in betas in the data. The small difference between the global and analytic solutions reassures us that our approximations capture the main forces at play.

### 4.1 Policy Functions

Figure 3 contrasts government policy functions for inflation and real debt repayments as functions of log output. The top two panels show log inflation (left) and the conditional expected real debt portfolio excess excess return (right), averaged across commitment and no-commitment states. Blue solid lines correspond to low credibility and red dashed lines correspond to high credibility. All policy functions in Figure 3 use numerical solution methods.

government and investors, the benchmark of $\gamma \approx \phi$ is natural.
The left panels of Figure 3 illustrate the inflation policy function features discussed in Section 3.5. The top left panel shows that the low-credibility government implements a state-contingent inflation policy function that is higher on average than for the high-credibility government, and especially so during low-output states. The middle and lower panels of Figure 3 decompose the differences between high- and low-credibility governments across commitment and no-commitment states. In the commitment state, the low-credibility government sets inflation to zero, similarly to the high-credibility government. In the no-commitment state, the low-credibility government inflates away its LC debt and chooses especially high inflation in low-output states. The low-credibility government reaches the no-commitment state with positive probability $1 - p_L > 0$, while the high-credibility government reaches it with probability 0, so the average inflation profile for the low-credibility government is higher and more countercyclical.

The right panels of Figure 3 show real debt portfolio excess returns, which are related to inflation by taking the expectation of (17) with respect to $\varepsilon_2$. The top right panel shows that countercyclical inflation translates into procyclical real debt repayments for the low-credibility country. Moreover, the low credibility country faces real debt repayments that are higher on average because of LC bond risk premia.

Even in the commitment state, credibility affects real excess returns of the sovereign bond portfolio, even though inflation in this state is close to zero. Credibility enters because ex ante LC bond prices reflect non-zero inflation expectations and inflation risk premia, which can raise the cost of repaying LC debt ex post. The low-credibility government’s real debt repayments are highest in the commitment state, because this is a state of surprisingly low inflation relative to ex ante investor expectations. With high average inflation expectations, the low-credibility government has to issue a large face value of LC debt to raise a given amount of real resources, so in a state of low realized inflation, real debt repayments are high. In the no-commitment state, real debt portfolio excess returns are close to zero on average, reflecting higher average inflation, and lowest in recessions, when inflation is high.

4.2 Comparative Statics

In this section, we analyze how LC debt issuance, inflation, inflation-output betas, and LC risk premia vary with credibility and investor risk aversion.
4.2.1 Credibility

Figure 4 shows that changes in credibility, or the probability of honoring the previously announced inflation plan, can explain substantial differences along key dimensions. An increase in credibility makes it less likely that the government will be tempted to inflate away the debt, leading to lower inflation expectations. A low-credibility government is especially tempted to inflate away the debt during recessions, generating an upward-sloping relation between inflation-output betas and credibility. Risk-averse international investors require a return premium for holding LC bonds that lose value precisely when marginal utility is high, driving up LC risk premia for low-credibility governments. Finally, low-credibility governments issue a smaller share of LC debt, to constrain themselves from inflating in low-output states, thereby reducing the real costs of inflation and risk premia.

4.2.2 Investor Risk Aversion

Figure 5 shows that model predictions vary substantially with investor risk aversion. In the case with risk-neutral investors ($\phi = 0$), investors charge no risk premium for inflation-output covariances. In this case, the low-credibility government has a high LC debt share, generates high inflation, and generates a strongly negative inflation-output beta. In fact, both low- and high-credibility governments generate almost identical inflation-output betas, indicating clearly that this case cannot explain the cross-country variation in inflation cyclicality in the data.

While the benchmark calibration in Tables 3 and 4 replicates the empirical fact that inflation-output betas are greater in developed markets than in emerging markets and generates zero inflation-output betas for high-credibility issuers, the model can easily generate even positive inflation-output betas if investors are effectively more risk-averse than the government ($\phi = 12$). With highly risk-averse investors, it is the government that sells insurance to the global investor by issuing LC debt, similarly to the setting considered in Farhi and Maggiori (2016), rather than the risk-neutral investor insuring the government by buying it. Higher investor risk aversion than government risk aversion could be due to political economy reasons that induce the government not to fully adjust for risk. For instance, the risk of losing elections may lead to a divergence between private and government incentives, especially during low-output states, much as in Aguiar and Amador (2011), where a lower
discount factor driven by political economy forces can engender a bias toward more debt.

5 Testing Additional Empirical Implications

The model presented in the previous two sections highlights the importance of monetary policy credibility for the level and cyclicality of LC risk and sovereign debt portfolios. This section tests additional model predictions and provides direct evidence for our proposed mechanism. We provide evidence for the following three predictions: First, we predict that countries with positive bond-stock betas have higher LC bond risk premia. Second, we predict that low-credibility countries have higher LC bond risk premia. Third, we predict an inverse relation between LC debt shares and LC bond risk premia.

5.1 Empirical Drivers of Risk Premia

In the model, bond risk premia act as an important channel linking monetary policy credibility, bond return cyclicality, and sovereign debt portfolios. We measure ex ante risk premia for our cross-section of countries to correspond to the left-hand side of (19):

\[ \overline{RP} = \bar{y}^{LC} - \bar{\pi} + \frac{1}{2} Var_{\pi} - \left( \bar{y}^{US} - \bar{\pi}^{US} + \frac{1}{2} Var_{\pi}^{US} \right). \]  

(32)

A bar indicates the mean from 2005 to 2014. Intuitively, (32) removes average local inflation from LC bond yields to isolate the risk premium component. Unlike in the model, we correct for the fact that U.S. inflation is non-zero. In Appendix A, we show that results are quantitatively and qualitatively robust to adjusting LC bond yields for default risk using synthetic default-free LC bonds as in Du and Schreger (2016a).\(^{17}\)

In the model, bond risk premia are driven by return comovements with the international SDF (7). In our empirical analysis, we proxy for this with the beta of LC bond log excess returns with respect to log excess returns on the U.S. S&P 500 (bond-S&P betas). For instance, for Brazil the bond-S&P beta would represent the slope coefficient of Brazilian LC bond log excess returns with respect to U.S. stock log excess returns. Here, U.S. stock

\(^{17}\)Due to our short sample, ex post bond risk premia, measured as realized excess returns, are extremely noisy. We therefore prefer ex ante measures, corresponding to those that governments see when making issuance decisions.
returns proxy for world stock returns if the U.S. equity market is well integrated with the rest of the world. While bond-S&P betas are unlikely to explain all cross-sectional variation in LC bond risk premia, showing a qualitatively and quantitatively significant relation will provide important evidence for our proposed channel. We decompose each country’s risk premium into two components by estimating the following regression:

\[ \bar{RP}_i = \mu + \kappa b(bond, S&P)_i + \varepsilon_i. \]  

(33)

Column (2) of Table 5 estimates regression (33) and finds a statistically significant and quantitatively meaningful estimate for \( \kappa \). A one-unit increase in the bond-S&P beta is associated with an increase in the risk premium of 10 percentage points in annualized units, which is the same order of magnitude as the U.S. equity premium. The bond-S&P beta not only carries an economically and statistically significant price of risk but also explains a substantial portion of cross-sectional variation in LC bond risk premia, with an R-squared of 30%. The estimated slope coefficient is similar in column (1), where we use the beta with respect to the local stock market instead of the S&P, supporting the notion that LC bonds that are the best hedges for the issuer tend to require the highest risk premia. In Appendix A.6, we show that the key risk premium relation in Table 5, column (2), remains highly statistically significant when using generalized method of moments to account for the fact that bond-S&P betas are estimated.

Next we interact the bond-stock beta with the local-S&P beta, as a proxy for the co-movement between local and global SDFs. Column (3) shows that results are unchanged, indicating that the co-movement between local and global SDFs is sufficiently large and consistent across countries that local inflation cyclicality indeed drives the cross-section of LC bond risk premia, as in the model. Column (4) further addresses concerns that cross-country differences in the local-global loadings might directly drive differences in LC bond risk premia. We regress risk premia onto the local-global beta \( b(stock, S&P) \) directly, which does not enter significantly and has no explanatory power for LC bond risk premia.

Table 5 also provides evidence on the link between bond risk premia and monetary policy credibility using two de facto measures that we construct. We prefer de facto measures of central bank credibility to de jure ones because recent measures of de jure central bank independence have been found to be uncorrelated with average inflation (Crowe and Meade, 2007). Using Financial Times articles over the period 1995—2015, we construct the corre-
lation between the keywords “debt” and “inflation” for each country as a proxy for inverse inflation credibility. The intuition is that if inflation is solely determined by the central bank and debt is determined by the fiscal authority, these topics should be discussed separately, and the correlation should be low. On the other hand, if inflation and debt are determined by the same central government, we would expect newspaper articles to discuss both jointly, and the correlation should be high. We count the number of articles containing both keywords and the country name and divide them by the geometric average of the articles that contain one of the keywords combined with the country name. Consistent with the model, column (5) of Table 5 shows that this de facto monetary policy credibility measure is strongly correlated with risk premia, with an $R^2$ of 47.2%.

Column (6) uses the gap between announced inflation targets and survey expectations to measure inverse inflation credibility. If credibility is low, we expect survey inflation to exceed announced inflation targets. We define the “credibility gap” as the greater of the average difference between the central bank inflation target and survey inflation expectations and zero. Over the past decade, on average, the emerging markets in the sample have a mean credibility gap of 0.6 percent, whereas the developed markets in the sample have a mean credibility gap of 0.1 percent. Column (6) suggests that a 0.5 percentage point increase in the credibility gap, corresponding to the average difference between emerging and developed countries, is associated with a 2 percentage point increase in LC bond risk premia, which is economically large and in line with model predictions.

### 5.2 Evidence on Bond Risk Premia and Debt Portfolio Choice

Next we turn to the model prediction that LC debt shares are negatively related to LC bond risk premia, and in particular to the component of LC bond risk premia that derives from bond return comovements with the international investor’s SDF. Consistent with this prediction, Table 6 shows a negative and statistically significant relation between LC debt shares on the left-hand side and LC bond risk premia on the right-hand side. LC bond risk premia explain a substantial 45% of variation in LC debt shares. A 2.4 percentage point increase in LC bond risk premia, roughly the average difference between emerging and developed countries, is associated with a $2.4 \times 6.6 = 16$ percentage point decrease in the LC debt share. Next we decompose the risk premium into a world CAPM component—the
component explained by the bond-S&P beta—and the alpha with respect to the U.S. S&P:

\[ RP_{CAPM,i} = \hat{\kappa}b(bond, S&P)_i, \ a_{CAPM,i} = \overline{RP}_i - RP_{CAPM,i}. \]  

(34)

where \( \hat{\kappa} \) is the slope coefficient estimated in Table 5, column (2). The estimated alpha \( a_{CAPM} \) may reflect measurement error of the CAPM risk premium, for instance if the S&P is an imperfect proxy for the world portfolio, or pricing errors on the part of investors, so we would expect LC debt shares to decrease with both \( RP_{CAPM} \) and \( a_{CAPM} \). Table 6, column (2), supports the notion that sovereign issuers reduce LC issuance in response to higher LC bond risk premia, and that the riskiness of LC bonds for U.S. investors, as proxied by the bond-S&P beta, accounts for a substantial portion the downward-sloping relation between LC debt issuance and LC risk premia. Columns (2) and (3) show that while both components of the risk premium contribute significantly to the explanatory power of risk premia for LC debt shares, our proxy for the CAPM component enters with a larger coefficient and explains more than half the R-squared in column (1). Columns (4) through (6) show that the relation between risk premia and LC debt shares is robust to controlling for log GDP, foreign exchange rate regime, commodity share of exports, and average inflation.

5.3 Time Series Changes in LC Debt Issuance

One of the most striking developments in international bond markets over the past two decades is how many countries have gone from having very little LC debt during the 1990s to substantial LC debt shares in the 2000s. We now show evidence that the bond risks channel of monetary policy credibility can help us understand changes in LC debt issuance. Our analysis is constrained by the fact that our main proxy for the hedging properties of LC bonds—the bond-stock beta—can be constructed only if LC debt is actually available, which for many countries in our sample was not the case during the 1990s. We therefore rely on decade-by-decade estimates of inflation forecast betas to measure the risks of hypothetical LC bond risks over time. Figure 6 shows that the strongest increases in inflation-output forecast betas were accompanied by the most marked increases in LC debt shares, supporting the notion that the bond risks channel of monetary policy credibility explains not only level differences in LC debt shares across countries but also changes since the 1990s. Looking only at emerging countries, shown in green, the upward-sloping relation in Figure 6 looks
even stronger. On the other hand, we should not be surprised to see that changes in LC debt shares for developed markets are zero. Developed markets in the data correspond to model governments that hit the 100% LC debt share constraint, so we should not expect them to change their LC debt shares for small changes in credibility. This evidence also shows that our main stylized empirical fact holds in changes, thereby controlling for omitted variables that are constant at the country level, such as natural resource endowments.\textsuperscript{18}

6 Conclusion

This paper argues that differences in monetary policy credibility, combined with investors that require a risk premium for holding positive-beta bonds, explain the relation between sovereign debt portfolios and government bond risks across countries. We document that sovereigns whose LC bonds tend to lose value during recessions and hence provide the borrower with consumption-smoothing benefits, issue little LC debt. We explain this stylized fact with a model in which risk-averse investors charge a premium for holding LC bonds that lose value during recessions, thereby making LC debt expensive for low-credibility governments and driving them toward FC debt issuance. Importantly, both limited commitment on the issuer’s part and investor risk aversion are necessary to match the empirical evidence. The key contribution of the paper is to demonstrate how the interaction of lender risk aversion and monetary credibility can explain why countries with positive bond-stock betas, which would seemingly achieve most consumption-smoothing from issuing LC debt, have the lowest LC debt share. Our simple framework gives rise to a number of testable predictions on inflation, inflation cyclicality, sovereign debt portfolios, and proxies of effective monetary policy credibility, which we verify in the data.

\textsuperscript{18}We compute inflation-output forecast betas and LC debt shares separately for the decades 1995—2004 and 2004—2015 for 20 countries. The rest of 10 sample countries are excluded due to missing forecasting data for the 1995—2004 period.
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Figure 1: Local Currency Debt Shares and Bond Betas

Note: This figure shows the share of LC debt as a fraction of central government debt (in %) over the period 2005--2014. Bond-stock betas are estimated as the slope coefficient of quarterly LC bond log excess returns onto local stock market log excess returns over the same time period

$$x_{t+\Delta t}^{LC} = b_0 + b(bond, stock) \times x_{t+\Delta t}^m + \epsilon_t.$$ 

Three-letter codes indicate currencies. Emerging markets are shown in red and developed markets in green. The highest and lowest observations are winsorized.
Figure 2: Local Currency Debt Shares, Inflation Betas, and Bond-S&P Betas

Note: Panels A, B, and D plot the share of LC debt in the sovereign debt portfolio on the y-axis against expected inflation-output betas, realized inflation-output betas, and the beta of LC bond returns with S&P returns, respectively. Panel C shows bond betas against local stock returns on the y-axis against bond-S&P betas on the x-axis. Developed markets are denoted by green dots, and emerging markets are denoted by red dots. The three-letter currency code is used to label countries. The highest and lowest observations are winsorized. More details on variable definitions can be found in Section 2.
Figure 3: Policy Functions

Note: The solid blue lines indicate the low-credibility calibration, while the dashed red lines indicate the high-credibility calibration. Left panels show log inflation. Right panels show real debt portfolio excess returns in percent, following equation (17). The y-axis shows log output in percent deviations from the steady-state. “Average” refers to the weighted average across commitment and no-commitment states, where the weights are given by credibility $p$. 
Note: This figure shows average inflation, the inflation-output beta, LC bond risk premia, and the LC debt share while varying credibility $p$. All other parameters are held constant at values shown in Table 3.
Figure 5: Varying Effective Investor Risk Aversion

Note: This figure shows average log inflation, the inflation-output beta, LC bond risk premia, and the LC debt share against effective investor risk aversion $\phi$ for low-credibility (blue solid) and high-credibility (red dashed) calibrations. All other parameters are held constant at values shown in Table 3.
Figure 6: Changes 1995—2004 versus 2005—2014

Note: This figure shows decade-over-decade changes in the inflation forecast beta on the x-axis and changes in LC debt shares on the y-axis, where changes are from 1995—2004 versus 2005—2014. The highest and lowest observations are winsorized.
Table 1: Summary Statistics for Developed and Emerging Markets (2005—2014)

<table>
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<tr>
<td></td>
<td>π</td>
<td>Survey π</td>
<td>y^{LC}</td>
<td>b(\tilde{\pi}, \tilde{gdp})</td>
<td>b(\pi, IP)</td>
<td>b(bond, stock)</td>
<td>b(stock, S&amp;P)</td>
<td>s</td>
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<td>(A) Developed Markets (N = 11)</td>
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<tr>
<td>Mean</td>
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<td>1.83</td>
<td>2.62</td>
<td>0.42</td>
<td>0.05</td>
<td>-0.10</td>
<td>0.95</td>
<td>89.27</td>
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<td>S.d.</td>
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<td>1.24</td>
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<td>0.06</td>
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<td>11.23</td>
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<td>Max</td>
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<td>-0.18</td>
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<td>1.00</td>
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<td>-0.07</td>
<td>0.58</td>
<td>11.97</td>
</tr>
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<td>(C) Full Sample (N = 30)</td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
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<td>3.10</td>
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<td>1.55</td>
<td>100.00</td>
</tr>
<tr>
<td>Min</td>
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<td>0.32</td>
<td>0.61</td>
<td>-0.25</td>
<td>-0.50</td>
<td>-0.18</td>
<td>0.58</td>
<td>11.97</td>
</tr>
<tr>
<td>(D) Mean Difference between Emerging and Developed Markets</td>
<td>-2.391***</td>
<td>-2.004***</td>
<td>-3.388***</td>
<td>0.215**</td>
<td>0.0736*</td>
<td>-0.160***</td>
<td>-0.04</td>
<td>26.16***</td>
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</table>

Note: This table reports summary statistics for the cross-sectional mean of seven variables for developed and emerging market groups. The variables include (1) \( \pi \), realized inflation (%), (2) Survey \( \pi \), survey inflation (%), (3) \( y^{LC} \), five-year LC bond yield, (4) \( b(\tilde{\pi}, \tilde{gdp}) \), inflation-output forecast beta, (5) \( b(\pi, IP) \), realized inflation-output beta, (6), (7) \( b(stock, S&P) \) beta of local log equity excess returns with respect to log U.S. S&P 500 excess returns, and (8) \( s \), percentage share of LC debt in total sovereign debt portfolios. Panel (A) reports results for developed markets. Panel (B) reports results for emerging markets. Panel (C) reports results for the pooled sample. Panel (D) tests the mean difference between developed and emerging markets. Robust standard errors are reported in parentheses. Significance levels are denoted by *** p<0.01, ** p<0.05, * p<0.1.
Table 2: Cross-Sectional Regression of Local Currency Debt Shares on Nominal Risk Betas

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<th>(3)</th>
<th>(4)</th>
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<tr>
<td>( b(bond, stock) )</td>
<td>(-116.3^{***} )</td>
<td>(-106.3^{**} )</td>
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</tr>
<tr>
<td></td>
<td>(21.40)</td>
<td>(31.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b(\tilde{\pi}, \tilde{gdp}) )</td>
<td></td>
<td></td>
<td>(57.63^{***} )</td>
<td>(8.986)</td>
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<tr>
<td>( b(\pi, IP) )</td>
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<td></td>
<td>(126.6^{***} )</td>
<td>(31.84)</td>
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<tr>
<td>log(GDP)</td>
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<td></td>
<td></td>
<td>(1.092 )</td>
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<td></td>
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<td>(3.815)</td>
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<tr>
<td>FX Regime</td>
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<td>Commodity Share</td>
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<td>Constant</td>
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<td>(57.24^{***} )</td>
<td>(72.00^{***} )</td>
<td>(71.80^{*} )</td>
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<tr>
<td></td>
<td>(3.460)</td>
<td>(4.857)</td>
<td>(3.993)</td>
<td>(3.948)</td>
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Note: This table shows the cross-country regression results of the LC debt share, \( s \) (between 0 and 1), on measures of inflation cyclicality. The independent variables in the first three columns are the bond-stock beta \( b(bond, stock) \), the inflation forecast beta \( b(\tilde{\pi}, \tilde{gdp}) \) and the realized inflation-output beta \( b(\pi, IP) \), respectively. In column (4), we control for the mean log per capita GDP level between 2005 and 2014, log(GDP), the average exchange rate classification used in Reinhart and Rogoff (2004), FX regime, and the commodity share of exports. The “Commodity Share” is defined as the sum of “Ores and Metals” and “Fuel” exports as a percentage of total merchandise exports from World Bank World Development Indicators. More details on variable definitions can be found in section 2. The top and bottom observations are winsorized. Robust standard errors are used in all regressions with the significance level indicated by *** \( p<0.01 \), ** \( p<0.05 \), * \( p<0.1 \).
Table 3: Calibration Parameters

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<th>High Credibility</th>
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<tr>
<td>Inflation Cost</td>
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<td>Output Vol.</td>
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<td>Government Risk Aversion</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Effective Investor Risk Aversion</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Exchange Rate Vol.</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>

Note: All parameters are in annualized natural units.

Table 4: Empirical and Model Moments

<table>
<thead>
<tr>
<th>Data</th>
<th>Emerging-Developed</th>
<th>Low Credibility</th>
<th>High Credibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Inflation</td>
<td>2.00</td>
<td>2.99</td>
<td>0.00</td>
</tr>
<tr>
<td>No-Commitment Inflation</td>
<td>6.07</td>
<td>8.48</td>
<td>12.00</td>
</tr>
<tr>
<td>Inflation Beta</td>
<td>-0.21</td>
<td>-0.27</td>
<td>-0.01</td>
</tr>
<tr>
<td>LC Debt Share</td>
<td>0.63</td>
<td>0.54</td>
<td>1.00</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.35</td>
<td>6.25</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Note: All moments are in annualized natural units. The empirical moment for average inflation is the difference between average survey inflation for emerging and developed markets in Table 1. The empirical inflation-output beta is computed as the difference between average expected inflation-output betas in emerging and developed markets. The empirical no-commitment inflation is computed as the difference between maximum emerging market survey inflation and average developed market survey inflation in Table 1. The equity risk premium is the average local equity excess return in our sample. All model moments are computed using global solution methods.
Table 5: Empirical Drivers of Bond Risk Premia

<table>
<thead>
<tr>
<th>LC Bond Risk Premium</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b(bond, stock)$</td>
<td>15.30***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.117)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b(bond, S&amp;P)$</td>
<td>11.36***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.194)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b(bond, stock) \times b(stock, S&amp;P)$</td>
<td>14.60***</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(2.805)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b(stock, S&amp;P)$</td>
<td></td>
<td></td>
<td>1.041</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.553)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>News Correlation</td>
<td></td>
<td></td>
<td></td>
<td>36.08***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(9.524)</td>
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<td></td>
</tr>
<tr>
<td>Credibility Gap</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.637***</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.989)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.773***</td>
<td>2.151***</td>
<td>1.661***</td>
<td>0.856</td>
<td>-5.559***</td>
<td>0.508</td>
</tr>
<tr>
<td></td>
<td>(0.264)</td>
<td>(0.449)</td>
<td>(0.248)</td>
<td>(1.383)</td>
<td>(1.735)</td>
<td>(0.392)</td>
</tr>
<tr>
<td>Observations</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>22</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.610</td>
<td>0.302</td>
<td>0.616</td>
<td>0.010</td>
<td>0.472</td>
<td>0.323</td>
</tr>
</tbody>
</table>

Note: This table regresses the empirical risk premium proxy (32) on bond-stock betas and measures of monetary policy credibility. $b(bond, stock)$ is the beta of LC bond excess returns with respect to the local stock market. $b(bond, S&P)$ is the beta of LC bond returns with respect to U.S. S&P returns. $b(bond, stock) \times b(stock, S&P)$ is the interaction of bond-local stock return betas and the beta of local on U.S. equity returns. $b(stock, S&P)$ is the beta of local on U.S. equity returns. “News Count” is the correlation of the keywords “debt” and “inflation” in Financial Times articles 1996—2015 from ProQuest Historical Newspapers. We compute the correlation as the number of articles mentioning both “debt” and “inflation” divided by the geometric average of articles that mention either “debt” or “inflation.” We require articles to also mention the country name. The inflation credibility gap is measured as the mean difference between the survey inflation expectations from Consensus Economics and the announced inflation target since 2005. The top and bottom observations are winsorized. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.
Table 6: Local Currency Debt Share and Bond Risk Premia

<table>
<thead>
<tr>
<th>LC Debt Share</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Premium</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>-6.581***</td>
<td>(0.871)</td>
<td>-5.841***</td>
<td>(0.915)</td>
<td>-5.481***</td>
<td>(0.939)</td>
<td>-5.725***</td>
</tr>
<tr>
<td>$RP_{CAPM}$</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>-8.937***</td>
<td>(2.600)</td>
<td>-8.937***</td>
<td>(1.886)</td>
<td></td>
<td>(0.985)</td>
<td></td>
</tr>
<tr>
<td>$a_{CAPM}$</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>-5.562***</td>
<td>(0.985)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (GDP)</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
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<tr>
<td>2.985</td>
<td>(2.939)</td>
<td>2.777</td>
<td>(3.392)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>FX Regime</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>1.975</td>
<td>(3.103)</td>
<td>1.984</td>
<td>(3.151)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Commodity Share</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>-0.144</td>
<td>(0.171)</td>
<td>-0.137</td>
<td>(0.197)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Inflation</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>-1.943*</td>
<td>(1.092)</td>
<td>-0.303</td>
<td>(1.791)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Constant</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>85.70***</td>
<td>(3.796)</td>
<td>70.88***</td>
<td>(3.739)</td>
<td>82.85***</td>
<td>(3.574)</td>
<td>53.80*</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>R-squared</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
<td>s s s s s</td>
</tr>
<tr>
<td>0.451</td>
<td>0.251</td>
<td>0.476</td>
<td>0.498</td>
<td>0.468</td>
<td>0.498</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table regresses the average LC debt share onto our empirical risk premium proxy, defined in equation (32). $RP_{CAPM}$ is the risk premium component explained by the bond-S&P beta and $a_{CAPM}$ is the corresponding alpha, as defined in (34). The FX Regime is from Reinhart and Rogoff (2004). The “Commodity Share” is defined as the sum of “Ores and Metals” and “Fuel” exports as a percentage of total merchandise exports from World Bank World Development Indicators. The top and bottom observations are winsorized. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.
Appendix - For Online Publication Only

This online appendix consists of Section A “Empirical Robustness” and Section B “Model Appendix.”

A Empirical Robustness

A.1 Long-Term Debt

The cross-sectional relationship between LC risk betas and LC debt shares is robust to measuring the LC debt share only in long-term debt, as shown in Figure A.1. We obtain face values and issuance dates for all historical individual sovereign bond issuances from Bloomberg for 14 emerging markets and estimate the long-term LC debt share as the outstanding amount of LC debt with five or more years remaining to maturity relative to all outstanding debt with five or more years remaining to maturity.

Figure A.1: LC Debt Share in Long-Term Debt versus Bond-Stock Beta

Note: This figure plots the bond-stock beta on the x-axis and the share of LC debt in all outstanding long-term debt on the y-axis. Long-term debt is defined as having a remaining time to maturity of five or more years. The share of LC debt in long-term debt is estimated from individual bond issuance data from Bloomberg.

A.2 TIC Data

In this section, we demonstrate that our results are robust to examining external debt separately, rather than all central government debt as in the main paper. The primary reason for not doing so is that it would be reduce country coverage. We approximate foreign-owned debt by looking only at the debt owned by U.S. domiciled investors. U.S. investors report their security level holdings
Figure A.2: Nominal Share in External Debt
as part of the Treasury International Capital (TIC) data. Here, we calculate the LC debt share in the U.S. portfolio by dividing the total value of government debt owned by U.S. investors in the borrowing country’s currency by the total amount of that country’s sovereign debt owned by U.S. investors. Figure A.2 shows that the negative relation between bond-stock betas and LC debt shares is robust to using this alternative measure of LC debt shares.

### A.3 LC versus FC Bond Betas

So far, we have assumed that real exchange rates are uncorrelated with all other shocks. This is clearly a simplification. However, what matters for the domestic borrower’s choice between LC and FC debt is the relative hedging properties and the relative risk premia of these two types of debt. If real exchange rate cyclicality is similar across countries, inflation cyclicality and LC bond return cyclicality are the relevant margins for understanding cross-sectional differences in LC debt shares. Figure A.3 shows the relation between LC debt shares on the x-axis against LC bond betas in excess of FC bond betas. We see that countries with lower LC debt shares have higher LC bond betas in excess of FC betas, confirming the empirical evidence in Figure 1.

**Figure A.3: LC minus FC Bond-Stock Betas**

Note: This figure shows the difference between unhedged LC bond betas and FC betas for dollar investors on the x-axis against LC debt shares on the y-axis.
Figure A.4: Local Currency Bond Betas Excluding 2008—2009

Note: This figure shows LC bond-stock betas excluding the period 2008–2009 on the x-axis and LC bond-stock betas for the full sample (including 2008—2009) on the y-axis. The highest and lowest observations are winsorized.

A.4 Robustness to Excluding the Financial Crisis

One important period in the middle of our sample is the financial crisis of 2008—2009. While this period marked an important recession for the U.S. and many other countries, we show in this section that our main empirical results are not driven by the financial crisis.

Figure A.4 shows our baseline LC bond-stock beta on the y-axis against a LC bond-stock beta excluding the financial crisis period on the x-axis. We see that bond-stock betas are extremely similar when excluding the financial crisis, indicating that our key bond cyclicality measure is not driven by a small number of observations. Figure A.5 shows that our main stylized fact in Figure 1 remains unchanged if we exclude the crisis period in our construction of LC bond betas.

A.5 Default-Adjusted Bond Risk Premia

To adjust for default risk, we construct a synthetic default-free nominal bond yield. We follow Du and Schreger (2016a) by combining a U.S. Treasury with a fixed-for-fixed cross-currency swap
Figure A.5: Local Currency Debt Shares and Bond Betas Excluding 2008—2009

Note: This figure differs from Figure 1 only in that it excludes 2008—2009 from the computation of LC bond betas on the x-axis. The highest and lowest observations are winsorized.
Figure A.6: Local Currency Debt Shares and Default-Adjusted Bond Betas

Note: This figure differs from Figure 1 only in that it excludes 2008—2009 from the computation of LC bond betas on the x-axis. The highest and lowest observations are winsorized.
to create a synthetic default-free local bond. In the absence of financial market frictions and sovereign default risk, we would expect $y^{LC} = y^{LC*}$. Figure A.6 plots the LC debt share against default-adjusted bond-stock betas, which are computed by replacing LC bond yields by synthetic default-free LC bond yields in the computation of LC bond returns. The strong similarity to Figure 1 shows that our main empirical finding is robust to adjusting for the default component of LC bond returns.

We now show that results in Section 5 are robust to using default-adjusted bond risk premia. We define an alternative measure of the risk premium that removes sovereign default risk from the nominal bond yield:

$$\bar{RP}_{alt} = y^{LC*} - \bar{\pi} + \frac{1}{2} Var \bar{\pi} - \left( y^{US} - \bar{\pi}^{US} + \frac{1}{2} Var \bar{\pi}^{US} \right)$$

$$\equiv (y^{US} + ccs) - \bar{\pi} + \frac{1}{2} Var \bar{\pi} - \left( y^{US} - \bar{\pi}^{US} + \frac{1}{2} Var \bar{\pi}^{US} \right), \tag{A.1}$$

where $ccs$ denotes the fixed-for-free cross-currency swap rate. Tables A.1 and A.2 replicate the empirical results in Section 5 using default-adjusted risk premia and find the results qualitatively unchanged.
\begin{table}
\centering
\caption{Empirical Drivers of Default-Adjusted Bond Risk Premia}
\begin{tabular}{lrrrrrr}
\hline
 & (1) & (2) & (3) & (4) & (5) & (6) \\
\hline
\text{LC Bond Risk Premium} & RP & RP & RP & RP & RP & RP \\
\text{\textit{b(bond, stock)}} & 9.659\text{***} & & & & & \\
 & (2.036) & & & & & \\
\text{\textit{b(bond, S&P)}} & 8.600\text{**} & & & & & \\
 & (3.235) & & & & & \\
\text{\textit{b(bond, stock)}\times\textit{b(stock, S&P)}} & 9.437\text{***} & & & & & \\
 & (1.819) & & & & & \\
\text{\textit{b(stock, S&P)}} & \text{\textit{-0.0482}} & & & & & \\
 & (1.395) & & & & & \\
\text{News Correlation} & 21.79\text{***} & & & & & \\
 & (6.540) & & & & & \\
\text{Credibility Gap} & & & 2.657\text{***} & & & \\
 & & & (0.895) & & & \\
\text{Constant} & 1.020\text{***} & 1.345\text{***} & 0.957\text{***} & 1.111 & -3.410\text{**} & 0.120 \\
 & (0.250) & (0.356) & (0.243) & (1.284) & (1.257) & (0.357) \\
\hline
\text{Observations} & 28 & 28 & 28 & 28 & 28 & 21 \\
\text{R-squared} & 0.457 & 0.250 & 0.478 & 0.000 & 0.331 & 0.314 \\
\hline
\end{tabular}
\end{table}

Note: This table shows that Table 5 is robust to using the default-adjusted LC bond risk premium (A.1). \textit{b(bond, stock)} is the beta of LC bond excess returns with respect to the local stock market. \textit{b(bond, S&P)} is the beta of LC bond returns with respect to U.S. S&P returns. \textit{b(bond, stock)}\times\textit{b(stock, S&P)} is the interaction of bond-local stock return betas and the beta of local on U.S. equity returns. \textit{b(stock, S&P)} is the beta of local on U.S. equity returns. “News Count” is the correlation of the keywords “debt” and “inflation” in Financial Times articles for 1996—2015 from ProQuest Historical Newspapers. We compute the correlation as the number of articles mentioning both “debt” and “inflation” divided by the geometric average of articles that mention either “debt” or “inflation.” We require articles to also mention the country name. The inflation credibility gap is measured as the mean difference between the survey inflation expectations from Consensus Economics and the announced inflation target since 2005. Robust standard errors are used in all regressions, with the significance level indicated by \textit{***} \textit{p}<0.01, \textit{**} \textit{p}<0.05, \textit{*} \textit{p}<0.1.
Table A.2: Local Currency Debt Share and Default-Adjusted Bond Risk Premia

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
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<tbody>
<tr>
<td>LC Debt Share</td>
<td>s</td>
<td>s</td>
<td>s</td>
<td>s</td>
<td>s</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>(1.215)</td>
<td>(1.897)</td>
<td>(2.574)</td>
<td>(2.732)</td>
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<tr>
<td>$RP_{CAPM}$</td>
<td>-13.42***</td>
<td>-10.94***</td>
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<tr>
<td></td>
<td>(4.339)</td>
<td>(3.309)</td>
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<tr>
<td>$a_{CAPM}$</td>
<td>-5.654***</td>
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</tr>
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<td>(1.248)</td>
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<td></td>
<td>(3.510)</td>
<td>(4.027)</td>
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<tr>
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<td>2.645</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>(3.646)</td>
<td>(3.786)</td>
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<td>Commodity Share</td>
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<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.212)</td>
<td>(0.222)</td>
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<td>Average Inflation</td>
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<td>-3.020</td>
<td>-0.166</td>
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</tr>
<tr>
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<td></td>
<td>(2.568)</td>
<td>(2.854)</td>
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<tr>
<td>Constant</td>
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<td>69.18***</td>
<td>82.98***</td>
<td>15.64</td>
<td>88.52***</td>
<td>17.07</td>
</tr>
<tr>
<td></td>
<td>(3.841)</td>
<td>(4.005)</td>
<td>(4.534)</td>
<td>(36.24)</td>
<td>(7.297)</td>
<td>(44.58)</td>
</tr>
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<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.404</td>
<td>0.257</td>
<td>0.461</td>
<td>0.479</td>
<td>0.426</td>
<td>0.479</td>
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</tbody>
</table>

Note: This table shows that Table 6 is robust to using a default-adjusted risk premium on the right-hand side. This table regresses the average LC debt share onto the alternative empirical risk premium proxy, defined in equation (A.1). $RP_{CAPM}$ is the risk premium component explained by the bond-S&P beta, and $a_{CAPM}$ is the corresponding alpha, corresponding to the fitted value and residual of Table A.1, column (2). The FX Regime is from Reinhart and Rogoff (2004). The “Commodity Share” is defined as the sum of “Ores and Metals” and “Fuel” exports as a percentage of total merchandise exports from World Bank World Development Indicators. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.
A.6 GMM Estimation

In this subsection, we re-estimate the key risk premium regression in Table 5, column (2), using the generalized method of moments (GMM) to account for the fact that betas are not known but are generated. We use \( \alpha_i \) and \( \beta_i \) to denote the intercept and slope coefficient in a regression of country \( i \)'s LC bond log excess returns onto U.S. stock market log excess returns. In the main paper, we use a two-step estimation procedure. In the first step, we estimate \( \alpha_i \) and \( \beta_i \) for each \( i \). In the second step, we estimate \( \mu \) and \( \kappa \) in a regression (33) that treats \( \beta_i \) as observed variables.

To estimate \( \alpha_i, \beta_i, \mu, \) and \( \kappa \) in a single step, we define the following GMM moments, which we expect to have a population mean of zero:

\[
g_{i,t} = \begin{cases} 
R_P - \mu - \kappa \beta_i & \text{for } 1 \leq i \leq N \\
(R_P - \mu - \kappa \beta_i) \beta_i & \text{for } N + 1 \leq i \leq 2N \\
x_{t}^{LC,i} - \alpha_i - \beta_i x_{t}^{m,USD} & \text{for } 2N + 1 \leq i \leq 3N \\
(x_{t}^{LC,i} - \alpha_i - \beta_i x_{t}^{m,USD}) x_{t}^{m,USD} & \text{for } 3N + 1 \leq i \leq 4N 
\end{cases}
\]  

(A.2)

Here, \( N \) denotes the number of countries in the sample and the parameter vector to be estimated is

\[
\begin{align*}
\mathbf{b} & = [\mu, \kappa, \mathbf{\alpha}, \mathbf{\beta}]', \\
\mathbf{\alpha} & = [\alpha_1, \alpha_2, \ldots, \alpha_N], \\
\mathbf{\beta} & = [\beta_1, \beta_2, \ldots, \beta_N].
\end{align*}
\]

The first \( 2N \) moment conditions in (A.2) are for the cross-sectional regression in the second stage. Moment conditions \( 2N + 1 \) through \( 4N \) are for the first-stage regressions. In sample, the \( 4N \) moments (A.2) cannot all simultaneously be set to zero, because we only have \( 2N + 2 \) parameters.

The GMM estimator \( \hat{b} \) then is defined by setting

\[
A \times \frac{1}{T} \sum_{t=1}^{T} g_{i,t}(\hat{b}) = 0,
\]

(A.3)

where \( A \) is a weighting matrix of size \( (2N + 2) \times 4N \) that has full rank. Letting \( \hat{b}_0 \) denote the true population value for \( b \), the estimated parameter vector \( \hat{b} \) then has the standard GMM asymptotic distribution

\[
\begin{align*}
\hat{b} & \sim \mathcal{N}(b_0, V) \\
V & = T^{-1} (AD)^{-1} ASA' (AD)^{-1}
\end{align*}
\]

(A.4)

where \( D = E \left[ \frac{\partial g}{\partial b} \right] \) is the sample average of the derivative of \( g \) with respect to the parameter vector \( b \) and \( S \) is the spectral density matrix of \( g_{i,t} \) at frequency zero.

For our GMM estimation, we choose a weighting matrix \( A = [2N + 2 \times 4N] \) that guarantees that the GMM point estimates for \( \mu \) and \( \kappa \) are identical to those obtained from the two-step
procedure, but of course we expect the GMM estimation to generate different standard errors. We can achieve this by setting
\[ A = \begin{bmatrix} 1_{1 \times N} & 0_{1 \times N} & 0_{1 \times 2N} \\ 0_{1 \times N} & 1_{1 \times N} & 0_{1 \times 2N} \\ 0_{2N \times N} & 0_{2N \times N} & I_{2N} \end{bmatrix}. \] (A.6)

Here 0_{M \times P} and 1_{M \times P} define block matrices of all zeros and ones with size [M \times P], respectively. We use I_{2N} to denote the identity matrix of size 2N. For our application, we obtain the consistent estimator for D:
\[ \hat{D} = \begin{bmatrix} -1_{N \times 1} & -\beta & 0_{N \times N} & -\kappa I_N \\ -\beta & -\beta^2 & 0_{N \times N} & -2\kappa \times \text{diag}(\beta) \\ 0_{N \times 1} & 0_{N \times 1} & -I_N & -I_N \sum_{t=1}^T x_{t \text{m,USD}}^{m,USD} T^{-1} \\ 0_{N \times 1} & 0_{N \times 1} & -I_N \sum_{t=1}^T x_{t \text{m,USD}}^{m,USD} T^{-1} & -I_N \sum_{t=1}^T (x_{t \text{m,USD}}^{m,USD})^2 T^{-1} \end{bmatrix}, \] (A.7)

where diag(\beta) denotes the matrix with the elements of \beta along the diagonal. We estimate the upper left [2N \times 2N] submatrix of S from the cross-section of countries by assuming that the residuals in the cross-country regression of risk premia onto true bond-S&P betas are identically and independently distributed and uncorrelated with elements 2N + 1 through N of g_t. Without these additional assumption we cannot estimate S, since elements 1 through 2N of g_t are constant over time, because RP_i is the country-specific risk premium averaged over the sample period. The spectral density for moments 2N + 1 through 4N is estimated from the time series with a Newey-West kernel with m = 120 lags to account for serial correlation and overlapping return observations:
\[ \hat{S} = \begin{bmatrix} I_N \hat{s}_1 & I_N \hat{s}_{12} & 0_{N \times 2N} \\ I_N \hat{s}_{12} & I_N \hat{s}_2 & 0_{N \times 2N} \\ 0_{N \times N} & 0_{N \times N} & T^{-1} \sum_{t=1}^T \left( \tilde{g}_t \tilde{g}_t' + \sum_{i=1}^m \left( 1 - \frac{i}{m+1} \right) \left[ \tilde{g}_t \tilde{g}_{t+i} + \tilde{g}_{t+i} \tilde{g}_t' \right] \right) \end{bmatrix}. \] (A.8)

Here
\[ \hat{s}_1 = \frac{T}{N - 2} \sum_{t=1}^T \sum_{i=1}^N g_{i,t}^2, \] (A.9)
\[ \hat{s}_2 = \frac{T}{N - 2} \sum_{t=1}^T \sum_{i=N+1}^{2N} g_{i,t}^2, \] (A.10)
\[ \hat{s}_{12} = \frac{T}{N - 2} \sum_{t=1}^T \sum_{i=1}^N g_{i,t} g_{i+N,t}, \] (A.11)

and \( \tilde{g}_t \) refers to the vector containing elements \( g_{2N+1,t} \) through \( g_{4N,t} \). The GMM standard errors of the second-stage coefficients are then given by the first two elements of
\[ SE(\hat{\mu}) = \sqrt{V(1,1)}, \quad (A.12) \]
\[ SE(\hat{\kappa}) = \sqrt{V(2,2)}. \quad (A.13) \]

The point estimates and standard errors for \( \hat{\mu} \) and \( \hat{\kappa} \) are reported in Table A.3. Column (1) starts by replicating Table 5. It uses the same two-stage estimation procedure as in Table 5, but in contrast to Table 5 it does not winsorize. Comparing A.3 column (1) with Table 5 column (2) shows that winsorizing has no qualitative or quantitative effect on the results. Column (2) in Table A.3 reports results from the GMM procedure, that estimates \( b \) in one single step. The point estimates are identical to column (1) and the standard errors are only slightly larger without affecting statistical significance, as one would expect if bond betas \( \beta \) are precisely estimated. Taken together, Table 5 suggests that bond betas are sufficiently precisely estimated over our sample period that generated regressors are not a significant concern for our benchmark results.

<table>
<thead>
<tr>
<th>LC Bond Risk Premium</th>
<th>(1) OLS</th>
<th>(2) GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b(bond, S&amp;P) )</td>
<td>10.835**</td>
<td>10.835***</td>
</tr>
<tr>
<td></td>
<td>(3.948)</td>
<td>(4.067)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.114***</td>
<td>2.114***</td>
</tr>
<tr>
<td></td>
<td>(0.458)</td>
<td>(0.472)</td>
</tr>
</tbody>
</table>

Observations 30

Note: This table estimates the slope of bond risk premia (32) on bond-S&P betas. The specification in column (1) is analogous to Table 5, column (2), but uses non-winsorized data. Column (2) is estimated using the GMM procedure outlined above to account for generated regressors. Significance level is indicated by *** \( p<0.01 \), ** \( p<0.05 \), * \( p<0.1 \).
B Model Appendix

The Model Appendix is structured as follows:

- Subsection B.1 provides details for the analytic model solution.
- Subsection B.2 provides details for the numerical model solution.
- Subsection B.3 analyzes the accuracy of the analytic solution.
- Subsection B.4 analyzes the sensitivity of calibration moments to moving one parameter at a time.
- Subsection B.5 describes and solves analytically an extended model, in which the real exchange rate is allowed to depreciate during recessions.
- Subsection B.6 describes, solves analytically, and provides a simple calibration for an extended model, in which debt is held domestically instead of foreign investors.

B.1 Analytic Model Solution

In this section, we solve the model analytically. Throughout the appendix we use the notation

\[ L = E_1 L_2. \]  

We will also drop time 1 subscripts for the LC debt share and simply write it as \( s \) for conciseness.

B.1.1 FC Bonds

The log pricing kernel for pricing assets in international consumption units is

\[ m^*_{22} = \log \beta - \theta x^*_{22} - \frac{1}{2} \theta^2 (\sigma^*)^2. \]  

The price of an FC bond hence is

\[ q_1^{FC} = \beta E_1 \left[ exp \left( -\theta x^*_{22} - \frac{1}{2} \theta^2 (\sigma^*)^2 \right) \right], \]  

\[ = \beta. \]  

B.1.2 LC Bonds

Now let’s consider a bond that pays off \( f(x_2, I_c) \) domestic consumption units in period 2. Here, \( I_c \) is defined as an indicator function, such that \( I_c = 1 \) if the commitment state holds in period 2 and \( I_c = 0 \) otherwise. If \( f(x_2, I_c) = 1 \), we are considering a real LC bond. Nominal LC bond payoffs can also be written in this form, because we assumed that period 2 inflation conditional on the
commitment state depends only on local output. We can hence write inflation as \( \pi_2 = \pi_2(x_2,I_c) \).

If \( f(x_2,I_c) = \exp(-\pi_2(x_2,I_c)) \), we are considering a nominal LC bond.

To obtain expressions for LC bond prices in terms of local output and inflation, we decompose world output into a component that is perfectly correlated with local output plus a shock that is uncorrelated with local output \( \eta_2^* \perp x_2 \):

\[
x_2^* = \lambda^* x_2 + \eta_2^*.
\]  

(B.5)

Here, \( \lambda^* \) is the regression coefficient of world output onto local output:

\[
\lambda^* = \frac{\text{Cov}(x_2^*, x_2)}{\text{Var}(x_2^*)},
\]  

(B.6)

\[
= \lambda^* \frac{(\sigma^*)^2}{(\sigma_x)^2}.
\]  

(B.7)

One way of seeing that this is the correct expression for \( \lambda^* \) is by noting that (8) and (B.7) imply

\[
\text{Cov}(x_2^* - \lambda^* x_2, x_2) = \text{Cov}(x_2^*, \lambda^* x_2 + \eta_2) + \text{Cov}(-\lambda^* x_2, x_2),
\]  

(B.8)

\[
= \lambda^* (\sigma^*)^2 - \lambda^* \sigma_x^2,
\]  

(B.9)

\[
= 0.
\]  

(B.10)

The log pricing kernel for pricing assets with payoff in local consumption combines the log SDF (7) with the log real exchange rate from (9):

\[
m_2 = \log \beta - \theta x_2 - \frac{1}{2} \theta^2 (\sigma^*)^2 + \varepsilon_2 - \frac{1}{2} \sigma_{\varepsilon}^2.
\]  

(B.11)

The price of the LC bond is

\[
q_1^{\text{LC}:f} = \beta E_1 \left[ \exp \left( -\theta x_2^* - \frac{1}{2} \theta^2 (\sigma^*)^2 + \varepsilon_2 - \frac{1}{2} \sigma_{\varepsilon}^2 \right) f(x_2, I_c) \right],
\]  

(B.12)

\[
= \beta E_1 \left[ \exp \left( -\theta \lambda^* x_2 - \theta \eta_2 - \frac{1}{2} \theta^2 (\sigma^*)^2 + \varepsilon_2 - \frac{1}{2} \sigma_{\varepsilon}^2 \right) f(x_2, I_c) \right].
\]  

(B.13)

But with \( \eta_2^* \perp x_2 \) and \( I_c \perp \{x_2, x_2^*, \varepsilon_2\} \), the expression for \( q_1^{\text{LC}:f} \) simplifies to

\[
q_1^{\text{LC}:f} = \beta E_1 \left[ \exp \left( -\theta \lambda^* x_2 - \frac{1}{2} \theta^2 (\lambda^*)^2 (\sigma_x^2) \right) f(x_2, I_c) \right].
\]  

(B.14)

In the last step, we have used that \( (\sigma^*)^2 = (\lambda^* \sigma_x)^2 + (\sigma_\eta^*)^2 \).

Defining effective risk aversion as

\[
\phi = \theta \lambda^* = \theta \lambda \frac{(\sigma^*)^2}{\sigma_x^2},
\]  

(B.15)
the price of a nominal LC bond then can be written as

$$q_1^{LC} = \beta E \left[ \exp \left( -\phi x_2 - \frac{1}{2} \phi^2 \sigma^2_x \right) \exp(-\pi_2 (x_2, I_c)) \right]. \quad (B.16)$$

The price of a real LC bond then equals

$$q_1^{LC,real} = \beta E \left[ \exp \left( -\phi x_2 - \frac{1}{2} \phi^2 \sigma^2_x \right) \right] = \beta. \quad (B.17)$$

### B.1.3 Log-Quadratic Loss Function

Using a log-quadratic expansion of the form

$$\exp(z) - 1 = z + \frac{1}{2} z^2$$

the loss function (4) becomes (ignoring constants)

$$L_2 \approx \alpha \pi_2^2 - \left( c_2 + \frac{1}{2} c_2^2 \right) + \frac{\gamma}{2} c_2^2. \quad (B.18)$$

Defining $\bar{D} = \beta^{-1}V$, we expand consumption in terms of output and the excess return on the debt portfolio:

$$c_2 + \frac{1}{2} c_2^2 \approx \bar{X} \left( \exp \left( x_2/\bar{X} \right) - 1 \right) - \bar{D} \left( \exp \left( x r_d^2 \right) - 1 \right), \quad (B.19)$$

$$= x_2 + \frac{1}{2} \frac{x_2^2}{\bar{X}} - \bar{D} \left( x r_d^2 + \frac{1}{2} (x r_d^2)^2 \right). \quad (B.20)$$

We expand bond portfolio excess returns log-quadratically similarly to Campbell and Viceira (2002):

$$x r_d^2 + \frac{1}{2} (x r_d)^2 \approx \exp(x r_d^2) - 1, \quad (B.21)$$

$$\approx \left( 1 - s_1 \right) \left( \exp \left( \varepsilon_2 + \frac{1}{2} \sigma^2_\varepsilon \right) - 1 \right) + s_1 \left( \frac{\beta}{q_1^{LC}} \exp(-\pi_2) - 1 \right), \quad (B.22)$$

$$= \left( 1 - s_1 \right) \left( \varepsilon_2 + \frac{1}{2} (\varepsilon_2^2 + \sigma^2_\varepsilon) \right) \quad (B.23)$$

$$+ s_1 \left( - (\pi_2 - E_1 \pi_2) + \frac{1}{2} (\pi_2 - E_1 \pi_2)^2 - Var_1 \pi_2 \right) \phi Cov_1 (x_2, \pi_2).$$

Substituting back into the loss function (B.18), ignoring policy independent terms, and taking expectations over $x_2$, $\pi_2$, and $\varepsilon_2$ gives the expected loss (21).
B.1.4 Full Commitment

We first consider the special case with \( p = 1 \) and derive the optimal inflation policy taking the LC debt share \( s \) as given. The inflation policy function minimizes the approximate loss function (21), which is the expectation of a quadratic equation in log local inflation and log local output. Log output is exogenously given, while log inflation is a choice variable. As is standard in quadratic programming, optimal log inflation is hence linear in log output. Therefore, we need to solve for constants \( \pi \) and \( \delta \) such that:

\[
\pi_2 = \pi + \delta x_2. \tag{B.25}
\]

We can then substitute this policy function into the quadratic loss function (21) and simplify:

\[
L^{p=1} = \alpha \left( \pi^2 + \delta^2 \sigma_x^2 \right) + s\tilde{D} (\gamma - \phi) \delta \sigma_x^2 + \frac{\gamma}{2} \tilde{D}^2 s^2 \delta^2 \sigma_x^2 + \frac{\gamma \tilde{D}^2 \sigma_x^2}{2} (1 - s)^2 + \tilde{D} \sigma_x^2 (1 - s). \tag{B.26}
\]

Expression (B.26) shows that it is optimal to set expected inflation to zero \( \bar{\pi} = 0 \). The first-order condition for \( \delta \) becomes

\[
\delta = \frac{(\phi - \gamma) s \tilde{D}}{2\alpha + \gamma s^2 \tilde{D}^2}. \tag{B.27}
\]

The mean, variance, and output beta for period 2 inflation hence become

\[
E_1 \pi_2 = 0, \tag{B.28}
\]
\[
Var_1 \pi_2 = \frac{(\phi - \gamma)^2 s^2 \tilde{D}^2 \sigma_x^2}{(2\alpha + \gamma s^2 \tilde{D}^2)^2}, \tag{B.29}
\]
\[
Beta(\pi_2, y_2) = \frac{(\phi - \gamma) s \tilde{D}}{2\alpha + \gamma s^2 \tilde{D}^2}. \tag{B.30}
\]

Next, we keep only first- and second-order terms in \( \tilde{D} \) to clarify the intuition of our results for the case where external debt-to-GDP ratios are not large, as appears to be the case for emerging markets. Intuitively, this corresponds to looking at the limit where \( \tilde{D} \) goes to zero sufficiently slowly that \( \tilde{D} \) and \( \tilde{D}^2 \) have a substantial effects on equilibrium quantities, but \( \tilde{D}^3 \) does not. In our calibration \( \tilde{D} = 0.13 \), so \( \tilde{D}^2 = 0.02 \) and \( \tilde{D}^3 = 0.0022 \), and we are plausibly in a situation where \( \tilde{D}^3 \) is small relative to average inflation and bond risk premia. Formally, we express \( \frac{\tilde{D}}{2\alpha + \gamma s^2 \tilde{D}^2} \) as a Taylor expansion in \( \tilde{D} \) and keep only terms that are second-order or lower in \( \tilde{D} \). The Taylor expansion \( \frac{1}{1+x} = 1 - x + x^2 + O(x^3) \) gives

\[
\frac{\tilde{D}}{2\alpha + \gamma s^2 \tilde{D}^2} = \frac{\tilde{D}}{2\alpha} \left( 1 - \frac{\gamma s^2 \tilde{D}^2}{2\alpha} \right) + O(\tilde{D}^3), \tag{B.31}
\]
\[
= \frac{\tilde{D}}{2\alpha} + O(\tilde{D}^3). \tag{B.32}
\]
Substituting (B.32) into (B.27) gives the optimal commitment inflation policy function as

\[ \pi_c^2 = \frac{(\phi - \gamma) s\bar{D}}{2\alpha} x_2. \] 

(B.33)

Combining (B.28) through (B.30) with (B.32) gives expressions (24) through (26) for the special case \( p = 1 \).

**B.1.5 No Commitment**

Next we consider the special case with \( p = 0 \), corresponding to the case where the period 2 government re-optimizes with certainty. Similarly to the use of a log-linear intertemporal budget constraint in Campbell and Viceira (2002), we approximate period 2 debt portfolio returns with the following first-order expansion for tractability:

\[ xr_2^d \approx (1 - s)\varepsilon_2 - s(\pi_2 - E_1\pi_2). \] 

(B.34)

Real interest rate shocks are not known at the time when the no-commitment government chooses inflation. Substituting (B.34) into (21), taking expectations over \( \varepsilon_2 \), and omitting terms that are independent of \( \pi_2 \), the no-commitment government in period 2 hence chooses \( \pi_{nc}^2 \) to minimize

\[ L_{nc}^2 = \alpha \pi_2^2 - s\bar{D}(\pi_2 - E_1\pi_2) + \frac{\gamma}{2} \left( x_2 + s\bar{D}(\pi_2 - E_1\pi_2) \right)^2. \] 

(B.35)

Since the no-commitment loss function (B.35) is a quadratic function in log local output and log inflation, it follows that no-commitment log inflation is linear in log output. The first-order condition for inflation becomes

\[ \pi_2 = \frac{s\bar{D}(1 - \gamma x_2)}{2\alpha + \gamma s^2 \bar{D}^2} + \frac{\gamma s^2 \bar{D}^2}{2\alpha + \gamma s^2 \bar{D}^2} E_1\pi_2. \] 

(B.36)

We can now use (B.36) to solve for the expectation, variance, and output beta of period 2 inflation

\[ E_1\pi_2 = \frac{s\bar{D}}{2\alpha}, \] 

(B.37)

\[ Var_1\pi_2 = \frac{\gamma^2 s_x^2}{(2\alpha + \gamma s^2 \bar{D}^2)^2} \bar{D}^2 s^2, \] 

(B.38)

\[ Beta(\pi_2, x_2) = \frac{-\gamma}{2\alpha + \gamma s^2 \bar{D}^2} s\bar{D}. \] 

(B.39)

Substituting (B.37) and (B.32) into (B.36) and keeping terms that are \( O(\bar{D}^2) \) gives the no-commitment inflation policy function

\[ \pi_{nc}^2 = \frac{s\bar{D}(1 - \gamma x_2)}{2\alpha}. \] 

(B.40)
Substituting (B.32) into (B.37) through (B.39) gives expressions (24) through (26) for the special case $p = 0$.

Throughout the paper, we ignore third- and higher-order terms in $\bar{D}$, effectively focusing on the empirically relevant case where $\bar{D}$ is small. Expression (B.39) shows what happens if we do not make this approximation and instead $\bar{D}$ is large. Expression (B.39) shows that $Beta(\pi_2, x_2)$ initially decreases in the amount of LC debt $s\bar{D}$, but for large values of $s\bar{D}$ it increases in $s\bar{D}$. The intuition for the large $s\bar{D}$ case is that when the amount of LC debt is large, domestic consumption is close to perfectly hedged and close to constant, because the government inflates to exactly offset all fluctuations in output. When $\bar{D}$ is large, an increase in the LC debt share may therefore reduce the amount of inflation variation that is required to obtain perfect consumption hedging and make inflation-output betas less negative. Since empirical emerging market external Debt/GDP is small and emerging market consumption appears far from fully hedged by variations in the real value of the government debt portfolio, we report approximations for the empirically relevant case with $\bar{D}$ small.

By using the first-order expansion (B.34), we ignore convexity in the relation between debt repayments and log inflation, or the fact that the first percentage point of inflation reduces real debt more than the last percentage point of inflation. However, the fact that the inflation policy function (B.40) is independent of risk premia is not a result of the first-order approximation. Risk premia are given as of period 2 and would not enter into the optimal no-commitment inflation choice even if we were using a second-order expansion. With a second-order expansion for debt portfolio excess returns we would continue to obtain that a higher LC debt share generates an incentive to inflate in recessions, which is the key feature of no-commitment inflation that drives our results. The benefit of using a first-order expansion is that the solution for the optimal inflation policy becomes separable, with the optimal commitment and no-commitment inflation policy functions given by (B.33) and (B.40) even if $p$ differs from the two special cases so far.

### B.1.6 Partial Commitment

Now we consider the case for general $0 \leq p \leq 1$. We use $\pi_2^{nc} = \pi_2(x_2, I_c = 0)$ to denote the inflation policy function if commitment fails in period 2 and $\pi_2^c = \pi_2(x_2, I_c = 1)$ to denote the inflation policy function if commitment holds in period 2. In the no-commitment state, the government chooses inflation to minimize (B.35) with first-order condition

$$
\pi_2^{nc} = \frac{s\bar{D} (1 - \gamma x_2)}{2\alpha + \gamma s^2 \bar{D}^2} + \frac{\gamma s^2 \bar{D}^2}{2\alpha + \gamma s^2 \bar{D}^2} \left( p\bar{\pi} + (1 - p)E_1 \pi_2^{nc} \right),
$$

(B.41)

showing that at the optimal value for $\bar{D}$, $\pi_2^{nc}$ is linear in log output. The commitment inflation policy minimizes (21), which is the expectation of a quadratic function in log inflation and log output. The optimal commitment inflation policy therefore is the solution to a quadratic program and takes the log-linear form:

$$
\pi_2^c = \bar{\pi} + \delta x_2.
$$

(B.42)
We next derive the optimal expressions for $\bar{\pi}$ and $\delta$. Expected no-commitment inflation equals

$$E_{1}\pi^{nc}_{2} = \frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} + \frac{\gamma s^2\bar{D}^2}{2\alpha + p\gamma s^2\bar{D}^2}p\bar{\pi}. \tag{B.43}$$

Unconditional expected inflation equals

$$E_{1}\pi_{2} = p\bar{\pi} + (1 - p)E_{1}\pi^{nc}_{2}, \tag{B.44}$$

$$= (1 - p)\frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} + \frac{2\alpha + \gamma s^2\bar{D}^2}{2\alpha + p\gamma s^2\bar{D}^2}p\bar{\pi}. \tag{B.45}$$

The variance and covariance of period 2 inflation then become

$$Var_{1}\pi_{2} = p(1 - p)\left(\frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} - \frac{2\alpha}{2\alpha + p\gamma s^2\bar{D}^2}\right)^2$$

$$+ p\delta^2\sigma_x^2 + (1 - p)\frac{\gamma^2\sigma_x^2}{(2\alpha + \gamma s^2\bar{D}^2)^2}s^2\bar{D}^2, \tag{B.46}$$

$$Cov(x_2, \pi_2) = p\delta\sigma_x^2 - (1 - p)\frac{\gamma\sigma_x^2}{2\alpha + \gamma s^2\bar{D}^2} s\bar{D}. \tag{B.47}$$

The inflation sensitivity in the commitment state $\delta$ enters into the variance and covariance exactly as before, but scaled by the commitment probability $p$. It hence follows that the optimal commitment sensitivity $\delta$ takes the form

$$\delta = -\frac{(\gamma - \phi)}{2\alpha + \gamma s^2\bar{D}^2}s\bar{D}. \tag{B.48}$$

Average commitment inflation $\bar{\pi}$ is chosen to minimize

$$\alpha \left( (1 - p)\frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} + \frac{2\alpha + \gamma s^2\bar{D}^2}{2\alpha + p\gamma s^2\bar{D}^2}p\bar{\pi} \right)^2$$

$$+ \left( \alpha + \frac{\gamma}{2}\bar{D}^2s^2 \right)p(1 - p)\left( \frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} - \frac{2\alpha}{2\alpha + p\gamma s^2\bar{D}^2}\right)^2. \tag{B.49}$$

Dividing by common factors, optimal $\bar{\pi}$ minimizes

$$\alpha \left( (1 - p)s\bar{D} + (2\alpha + \gamma s^2\bar{D}^2)p\bar{\pi} \right)^2 + \left( \alpha + \frac{\gamma}{2}\bar{D}^2s^2 \right)p(1 - p)\left( s\bar{D} - 2\alpha\bar{\pi} \right)^2. \tag{B.50}$$

Taking the first-order condition of (B.50) with respect to $\bar{\pi}$ then shows that inflation in the commitment state equals

$$\bar{\pi} = 0. \tag{B.51}$$
With the two inflation policy functions, we can then solve for closed-form expressions for the expectation, variance, and output beta for period 2 inflation, taking the LC debt share $s$ as given:

$$E_1(\pi_2) = (1-p)\frac{s\bar{D}}{2\alpha + p\gamma s^2 D^2}, \quad \text{(B.52)}$$

$$Var_1(\pi_2) = p(1-p)\left(\frac{s\bar{D}}{2\alpha + p\gamma s^2 D^2}\right)^2 + \gamma^2 - p\phi (2\gamma - \phi)\bar{D}^2 s^2 \sigma_x^2, \quad \text{(B.53)}$$

$$Beta(\pi_2, y_2) = \frac{(p\phi - \gamma) \bar{D}s}{2\alpha + \gamma s^2 D^2}, \quad \text{(B.54)}$$

Substituting in the inflation policy functions, the expected loss function simplifies to

$$L = \frac{1-p}{2} - \frac{s^2 \bar{D}^2}{2\alpha + p\gamma s^2 D^2} + \frac{1}{2}\gamma^2 - p\phi (2\gamma - \phi)\bar{D}^2 s^2 \sigma_x^2 + \frac{\gamma^2 - p\phi (2\gamma - \phi)}{(2\alpha + \gamma s^2 D^2)^2} (1-s)^2$$

$$+ \bar{D}\sigma_x^2 (1-s). \quad \text{(B.55)}$$

Combining (B.55) with (B.32) gives the expected loss function

$$L = (1-p)\frac{s^2 \bar{D}^2}{4\alpha} - \frac{(\phi - \gamma)^2}{4\alpha} s^2 \bar{D}^2 \sigma_x^2 + \frac{(1-p)\phi^2}{4\alpha} s^2 \bar{D}^2 \sigma_x^2$$

$$+ \frac{\gamma \bar{D}^2 \sigma_x^2}{2} (1-s)^2 + \bar{D}\sigma_x^2 (1-s). \quad \text{(B.56)}$$

Combining (B.52) through (B.54) with (B.32) gives expressions (24) through (26).

**B.1.7 First–Order Condition for LC Debt Share**

Taking the first-order condition of (B.56) with respect to $s$ gives

$$s = \frac{2\alpha \left[ \gamma + \frac{1}{\bar{D}} \right] \sigma_x^2}{(1-p)(1+\phi^2 \sigma_x^2) - (\phi - \gamma)^2 \sigma_x^2 + 2\alpha \gamma \sigma_x^2}. \quad \text{(B.57)}$$

Provided that $s$ is at an interior solution, the optimal $s$ is given by (B.57). From (B.57), we derive the comparative static for the LC debt share with respect to credibility:

$$\frac{ds}{dp} = \frac{s^2}{2\alpha \left[ \gamma + \frac{1}{\bar{D}} \right] \sigma_x^2}, \quad \text{B.58}$$

$$> 0.$$

**B.1.8 Solution Summary**

Substituting (B.32) and (B.52) into (B.41) gives the no-commitment inflation policy function
Combining (B.32) with (B.48) and (B.51) gives the commitment inflation policy function

\[ \pi^c_2 = (\phi - \gamma) \frac{s_1 \bar{D}}{2\alpha} x_2. \]  

(B.60)

Substituting (B.32) into (B.52) through (B.54) gives (24) through (26) in the main text.

B.2 Numerical Solution

We solve the model numerically using global projection methods. To reduce the dimensionality of the optimization problem, we use the following numerical steps.

1. Starting from the analytic solution, we choose the no-commitment policy function to minimize the expected Euler equation error while holding constant the LC debt share and the commitment policy function.

2. We choose the commitment policy function to minimize the expected loss function, while holding constant the LC debt share and the no-commitment policy function.

3. We alternate steps 1 and 2 until the maximum absolute change in both policy functions is less than \(10^{-12}\). This gives the loss function at a given LC debt share.

4. We optimize over the LC debt share to minimize the expected loss function, where for every single value of \(s\) we repeat steps 1 through 3 to evaluate the loss function.

B.2.1 Functional Form

We solve for commitment- and no-commitment inflation policies of the form

\[ \pi^{nc}_2 = b_1(s) + b_2(s) x_2 + b_3(s) x_2^2, \]  

(B.61)

\[ \pi^c_2 = c_1(s) + c_2(s) x_2 + c_3(s) x_2^2, \]  

(B.62)

where all coefficients may depend on the LC debt share \(s\). We start our optimization routine at the analytic solution, that is

\[ b_1 = \frac{s \bar{D}}{2\alpha}, \]  

(B.63)

\[ b_2 = \frac{-s \bar{D} \gamma}{2\alpha}, \]  

(B.64)

\[ b_3 = 0, \]  

(B.65)

\[ c_1 = c_2 = c_3 = 0. \]  

(B.66)
The starting value for the LC debt share is as given in Table 3 in the main paper.

For given no-commitment and commitment policy functions and a given LC debt share, we compute LC bond prices numerically as

$$q_{LC}^1 = \frac{1 - p}{\beta} \left[ \exp \left( -\phi x_2 - \frac{1}{2} \phi^2 \sigma_x^2 \right) \exp(-\pi_c^n) \right]$$

We evaluate the expectation in (B.67) using Gauss-Legendre quadrature with 30 node points, truncating the interval at -6 and +6 standard deviations.

**B.2.2 No Commitment Policy Function**

We choose the vector \((b_1, b_2, b_3)\) to minimize the expected squared Euler equation error

$$\text{Error} = 2\alpha \pi_c^n - E_{\varepsilon_2} \left( C_{c,1}^{nc,\gamma} \right) \tilde{D} \frac{\beta}{q_1^{LC}} \exp(-\pi_c^n),$$

where we evaluate no-commitment consumption numerically according to

$$C_{c}^{nc} = \tilde{X} \exp(x_2/\tilde{X}) - \tilde{D} \left( (1 - s) \exp \left( \varepsilon_2 + \frac{1}{2} \sigma_\varepsilon^2 \right) + s \frac{\beta}{q_1^{LC}} \exp(-\pi_c^n) \right).$$

We evaluate the expectation \(E[\text{Error}^2]\) again using Gauss-Legendre quadrature with 30 nodes and truncating the interval at -6 and +6 standard deviations.

**B.2.3 Commitment Policy Function**

For a given LC debt share and a given no-commitment policy function, the commitment inflation policy function minimizes

$$E_1 \left[ \alpha \pi_c^n \right] - pE_1 \left[ \frac{C_{c}^{nc,1-\gamma}}{1 - \gamma} \right] - (1 - p)E_1 \left[ \frac{C_{c}^{nc,1-\gamma}}{1 - \gamma} \right]$$

where we evaluate commitment consumption numerically:

$$C_{c}^{c} = \tilde{X} \exp(x_2/\tilde{X}) - \tilde{D} \left( (1 - s) \exp \left( \varepsilon_2 + \frac{1}{2} \sigma_\varepsilon^2 \right) + s \frac{\beta}{q_1^{LC}} \exp(-\pi_c^n) \right),$$

and no-commitment consumption and the ratio of bond prices are given by (B.69) and (B.67). All expectations are again evaluated numerically using Gauss-Legendre quadrature using the same grid points as before.
B.2.4 Numerical Model Moments

We use Gauss-Legendre quadrature to evaluate inflation moments for the numerical solution according to

\[
E_1 \pi_2 = p E_1 \pi_c^2 + (1 - p) E_1 \pi_{nc}^2, \quad \text{(B.72)}
\]

\[
\text{Var}_1 \pi_2 = p E_1 (\pi_c^2) + (1 - p) E_1 (\pi_{nc}^2) - (E_1 \pi_2)^2, \quad \text{(B.73)}
\]

\[
\text{Beta}(\pi_2, x_2) = \frac{(p E_1 ((\pi_c - E_1 \pi_2) x_2) + (1 - p) E_1 ((\pi_{nc} - E_1 \pi_2) x_2)) / \sigma_x^2. \quad \text{(B.74)}
\]

We obtain the LC bond risk premium as

\[
RP^{LC} = \log(q_1^{FC}) - \log(q_1^{LC}) - E_1 \pi_2 + \frac{1}{2} \text{Var}_1 \pi_2. \quad \text{(B.75)}
\]

B.3 Accuracy of Analytic Solution

Table A.2 shows model moments for the analytic model solution. The analytic solution is remarkably accurate, with all model moments broadly similar to Table 4.

Figure A.1 compares analytic (solid blue) and numerical (dashed red) policy functions while holding the LC debt share constant at its numerical solution value of 0.54. Given the simplicity of the analytic solution, it is very accurate. The main difference is that the numerical policy function is more convex, bounding no-commitment inflation away from zero in high-output states.

Finally, Figures B.2 and B.3 reproduce the comparative statics with respect to credibility \( p \) and risk aversion \( \phi \) using the analytic solution. The strong similarity with Figures 4 and 5 indicates that the analytic model solution is indeed valuable for understanding the drivers of LC bond issuance, both qualitatively and even quantitatively.

Table B.1: Analytic Solution for Model Moments

<table>
<thead>
<tr>
<th>Data / Model</th>
<th>Emerging-Developed</th>
<th>Low Credibility</th>
<th>High Credibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Inflation</td>
<td>2.00</td>
<td>2.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Average No-Commitment Inflation</td>
<td>6.07</td>
<td>6.15</td>
<td>13.00</td>
</tr>
<tr>
<td>Inflation Beta</td>
<td>-0.21</td>
<td>-0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>LC Debt Share</td>
<td>0.63</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.35</td>
<td>6.25</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Note: All moments are in natural annual units. The difference between this table and Table 4 in the main paper is that this table uses the analytic models solution instead of global solution methods.
Figure B.1: Numerical Policy Functions

Note: The top panel shows log inflation (in percent) against log output in percent deviations from the steady state in the commitment state. The bottom panel shows log inflation in the no-commitment state. The solid blue lines show the analytic solutions, while the dashed red lines show numerical solutions using projection methods.
Figure B.2: Analytic Solution: Varying Credibility

Note: This figure differs from Figure 4 only in that it shows the analytic solution instead of the numerical model solution.
Figure B.3: Varying Effective Investor Risk Aversion

Note: This figure varies from Figure 5 only in that it uses the analytic solution instead of the numerical model solution. This figure shows average log inflation, the inflation-output beta, LC bond risk premia, and the LC debt share against effective investor risk aversion $\phi$ for low-credibility (solid blue) and high-credibility (dashed red) calibrations. All other parameters are held constant at values shown in Table 3.
B.4 Calibration Sensitivity

Table B.2 shows how the model properties change as we change one calibration parameter at a time. Column (1) reproduces the benchmark calibration for reference. Columns (2) and (3) show the effect of setting effective investor risk aversion to 0 and 12, respectively. For $\phi = 0$, both high- and low-credibility governments hit the constraint of a 100% LC debt share and therefore the inflation-output betas are almost equal. For $\phi = 12$, the model generates reasonable LC debt shares, a positive inflation-output beta for the high-credibility government, and a negative inflation-output beta for the low-credibility government, similarly to the data. Increasing the inflation cost $\alpha$ compresses inflation-output betas toward zero but does not change the model properties otherwise. Choosing a lower output volatility leaves inflation-output betas and LC debt shares largely unchanged but reduces the equity risk premium. Column (6) shows that reducing both investor and government risk aversion to 5 compresses the variation in inflation-output betas and risk premia across low- and high-credibility governments, but preserves the signs. Column (7) shows that, on the other hand, increasing the debt-to-GDP ratio increases the gap between high-credibility and low-credibility inflation-output betas. Finally, column (8) shows that reducing the exchange rate volatility to $\sigma_\varepsilon = 0.11$ reduces the LC debt share for the low-credibility government but still generates an economically meaningful negative inflation-output beta.
Table B.2: Calibration Sensitivity

<table>
<thead>
<tr>
<th>Low Credibility</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
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<td></td>
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<tr>
<td>Average Inflation</td>
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<td>2.90</td>
<td>2.58</td>
<td>4.61</td>
<td>1.91</td>
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<td>8.25</td>
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<td>5.62</td>
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<td>-0.29</td>
<td>-0.13</td>
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<td>-0.18</td>
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<td>0.51</td>
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<td>0.66</td>
<td>0.61</td>
<td>0.40</td>
<td>0.34</td>
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<tr>
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<td>7.51</td>
<td>6.26</td>
<td>3.19</td>
<td>3.13</td>
<td>6.26</td>
<td>6.26</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Inflation</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>No-Commitment Inflation</td>
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<td>11.96</td>
<td>11.53</td>
<td>6.98</td>
<td>11.21</td>
<td>11.34</td>
<td>20.49</td>
<td>12.07</td>
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<tr>
<td>Inflation Beta</td>
<td>-0.01</td>
<td>-1.34</td>
<td>0.25</td>
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<tr>
<td>LC Debt Share</td>
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<tr>
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<td>6.26</td>
<td>0.00</td>
<td>7.51</td>
<td>6.26</td>
<td>3.19</td>
<td>3.13</td>
<td>6.26</td>
<td>6.26</td>
</tr>
</tbody>
</table>

Note: This table varies one parameter at a time, as indicated in each column. All other parameters are as in Table 3.
B.5 Model Extension: Real Exchange Rate Correlations

There is some empirical evidence that a country’s currency loses value in real terms during recessions. We can extend the model to account for this by modeling the real exchange rate as correlated with local output:

\[ \varepsilon_2 = \rho x_2 + \nu_2, \quad (B.76) \]

where \( \nu_2 \) is independent of \( x_2 \), \( x^*_2 \), and \( I_c \). When \( \rho = 0 \), (B.76) nests the uncorrelated exchange rate model in the main text (9). In this section, we consider the case where \( \rho > 0 \), so the value of LC depreciates in real terms during recessions. We take \( \rho \) as given and outside the government’s control.

We show that the analytic solution for inflation policy functions and the LC debt share \( s \) in the extended model are similar to before. In particular, we show that the comparative static of inflation-output betas with respect to the LC debt share \( s \) continues to be negative when investors are risk neutral, in contrast to the data. We hence continue to need investors who are effectively risk-averse (\( \phi > 0 \)) to explain our main empirical stylized fact. The analytic solution for the extended model with \( \rho > 0 \) proceeds analogously to that in section B.1.

B.5.1 Bond Prices

Defining the log SDF \( m_2 \) and \( m^*_2 \) as in section B.1, the price of an FC bond is exactly as before and is given by:

\[ q^{FC}_1 = \beta. \quad (B.77) \]

The new parameter \( \rho \) enters in two ways into LC bond prices:

\[ q^{LC}_1 = E_1 [\exp(m_2 - \pi_2)], \quad (B.79) \]

\[ = \beta \left[ \exp(-\phi x_2) \right] E_1 \left[ \exp(-\phi^2 x^2 - \frac{1}{2} \rho^2 x^2 - \frac{1}{2} \phi \rho x^2) \right]. \quad (B.80) \]

\[ = \beta \exp(-\rho \phi \sigma^2_x) \left[ \exp(-\phi x_2) \right] \quad (B.81) \]

Effect 1: First, if the real exchange rate depreciates in bad times (\( \rho > 0 \)), this makes investing in LC bonds risky, driving down the LC bond price. This first effect disappears if international investors are risk-neutral (\( \phi = 0 \)) and does not depend on inflation cyclical.

Effect 2: Second, \( \rho \) affects how inflation state contingency enters into nominal LC bond prices. With \( \rho > 0 \), the price of LC bonds that pay out during expansions increases relative to the price of LC bonds that pay out during recessions. The intuition is that because the real exchange rate
is higher during expansions, an LC bond whose payoffs are concentrated in expansions provides investors with higher expected payoffs measured in international consumption units for any given expected payoff in local consumption units, driving up the LC bond price. This second effect is present even in the case when investors are risk-neutral ($\phi = 0$). Even if investors are risk-neutral over international consumption goods, the marginal utility they derive from one unit of local goods varies across states of the world depending on the real exchange rate.

Similarly, the price of a real (inflation-indexed) LC bond is

$$q_{1,\text{real}}^L = E_1 [\exp(m_2)],$$

$$= \beta \exp \left( -\rho \phi \sigma_{x}^2 \right).$$

We now define the debt portfolio log return in excess of a real LC risk-free bond:

$$x_{r_2}^d = \log \left( \frac{D_1^{FC} \exp \left( -\varepsilon_2 + \frac{1}{2} \sigma_{\varepsilon}^2 \right) + D_1^{LC} \exp(-\pi_2)}{\beta^{-1} \exp \left( \rho \phi \sigma_{x}^2 \right) V} \right).$$

Note that (B.84) nests the previous definition (17) if $\rho = 0$. Extending the definition for $\bar{D}$ to

$$\bar{D} = \beta^{-1} \exp(\rho \phi \sigma_{x}^2) V,$$

we can write period 2 consumption as

$$C_2 = X_2 - \bar{D} \exp(x_{r_2}^d).$$

### B.5.2 Log-Quadratic Loss Function

We obtain the following second-order expansion for nominal LC bond prices:

$$q_1^{LC} \approx \beta \exp \left( -\rho \phi \sigma_{x}^2 - E_1 \pi_2 + \frac{1}{2} Var_1 \pi_2 + (\phi - \rho) \text{Cov}_1(x_2, \pi_2) \right).$$

The approximate risk premium on LC bonds becomes

$$y_1^{LC} - E_1 \pi_2 + \frac{1}{2} Var_1 \pi_2 - y_1^{FC} = -(\phi - \rho) \text{Cov}_1(x_2, \pi_2) - \rho \phi \sigma_{x}^2.$$

A log-quadratic expansion of debt portfolio excess returns gives:

$$x_{r_2}^d + \frac{1}{2} \left( x_{r_2}^d \right)^2 \approx \exp \left( x_{r_2}^d \right) - 1,$$

$$\approx (1 - s) \left( -\varepsilon_2 + \frac{1}{2} (\varepsilon_{\varepsilon}^2 + \sigma_{\varepsilon}^2) - \rho \phi \sigma_{x}^2 \right)$$

$$+ s \left( -(\pi_2 - E_1 \pi_2) + \frac{1}{2} ((\pi_2 - E_1 \pi_2)^2 - Var_1 \pi_2) - (\phi - \rho) \text{Cov}_1(x_2, \pi_2) \right).$$
Substituting into the log-quadratic expansion for the loss function, taking expectations, and dropping policy-independent terms shows that in the extended model, the expected loss function needs to be modified to:

\[ E_1 L_2 = \alpha E_1 \pi_2^2 + \bar{D} E_1 \left[ x r_2^d + \frac{1}{2} (x r_2^d)^2 \right] + \frac{\gamma}{2} E_1 \left[ -2 \bar{D} x_2 x r_2^d + \bar{D}^2 (x r_2^d)^2 \right], \]

(B.91)

\[ = \alpha E_1 \pi_2^2 + \bar{D} \left[ (1 - s) \left( \sigma_x^2 - \rho \phi \sigma_x^2 \right) - s (\phi - \rho) \text{Cov}\left( x_2, \pi_2 \right) \right] + \frac{\gamma}{2} \left[ 2 \bar{D} \left[ (1 - s) \rho \sigma_x^2 + s_1 \text{Cov}\left( \pi_2, x_2 \right) \right] + \bar{D}^2 \left[ (1 - s)^2 \sigma_x^2 + s^2 \text{Var}\left( \pi_2 \right) \right] \right], \]

(B.92)

\[ = \alpha E_1 \pi_2^2 + s \bar{D} \left( \gamma - (\phi - \rho) \right) \text{Cov}\left( x_2, \pi_2 \right) + \frac{\gamma}{2} \bar{D}^2 s^2 \text{Var}\left( \pi_2 \right), \]

(B.93)

\[ = \alpha E_1 \pi_2^2 + s \bar{D} \left( \gamma - (\phi - \rho) \right) \text{Cov}\left( x_2, \pi_2 \right) + \frac{\gamma}{2} \bar{D}^2 s^2 \text{Var}\left( \pi_2 \right), \]

(B.94)

The expected loss function (B.94) is analogous to (21) in the benchmark model. The new parameter \( \rho \) enters in two instances. First, it generates an additional cost of FC debt issuance, thereby strengthening further the incentives to borrow in LC, acting effectively like an increase in exchange rate volatility in the benchmark model. The intuition is that if the real exchange rate depreciates during bad times, it is more likely that the real cost of servicing FC debt spikes during recessions, when marginal utility of consumption is already high. Second, a positive value for \( \rho \) alters risk premia, thereby affecting how the inflation-output covariance enters into the expected loss function. However, the government continues to face a trade-off between smoothing domestic consumption and selling insurance to international investors, provided that \( \phi - \rho > 0 \). To preview, we will find in this extended model, that the empirical observation that inflation-output betas increase with the LC debt share implies not just \( \phi > 0 \) but the even stronger relation \( \phi > \rho \). So, if anything, this modification strengthens the conclusion that we need investor risk aversion to explain the stylized empirical fact.

### B.5.3 Inflation Policy Functions

We again first solve for the inflation policy functions while holding fixed the LC debt share \( s \). Substituting the log-linear relation between real debt portfolio excess returns and inflation surprises (B.34) into the loss function, we find that the no-commitment inflation policy minimizes the same ex-post loss function (B.35) as in the benchmark model. In addition, the period 1 expected loss function (B.94) takes the same form as (21), with \( \phi \) replaced by \( \phi - \rho \) plus an additional term that is independent of inflation and hence does not enter into the optimal inflation policy choice while holding fixed \( s \). Keeping again only \( \mathcal{O}(D^2) \) terms, it is hence clear that the policy function in the
The no-commitment state is identical to before and equals
\[
\pi^nc_2 = \frac{sD}{2\alpha} - \gamma \frac{sD}{2\alpha} x_2. \tag{B.95}
\]

The inflation policy function in the commitment state is analogous with effective investor risk aversion \(\phi\) replaced by \(\phi - \rho\):
\[
\pi^c_2 = (\phi - \rho - \gamma) \frac{sD}{2\alpha} x_2. \tag{B.96}
\]

The inflation-output beta is the weighted average of inflation-output betas in the commitment and no-commitment states:
\[
\text{Beta}(\pi_2, x_2) = (p(\phi - \rho) - \gamma) \frac{sD}{2\alpha}. \tag{B.97}
\]

### B.5.4 First-Order Condition for LC Debt Share

Using again (B.32) to keep only \(O(D^2)\) terms, the expected loss function becomes:
\[
E_1L_2 = (1 - p) \frac{s^2D^2}{4\alpha} - \frac{(\phi - \rho - \gamma)^2}{4\alpha} s^2D^2\sigma_x^2 + \frac{(1 - p)(\phi - \rho)^2}{4\alpha} s^2D^2\sigma_x^2
+ \frac{\gamma D^2}{2} \frac{(\sigma_e^2 + (\gamma - \phi)\rho\sigma_x^2)}{(1 - s)^2} + \bar{D} \left( \sigma_e^2 + (\gamma - \phi)\rho\sigma_x^2 \right) (1 - s). \tag{B.98}
\]

Taking the first-order condition of (B.98) with respect to \(s\) gives
\[
s = \frac{2\alpha \left[ \gamma + 1/\bar{D} \right] (\sigma_e^2 + (\gamma - \phi)\rho\sigma_x^2)}{(1 - p) (1 + (\phi - \rho)^2\sigma_x^2) - (\phi - \rho - \gamma)^2 \sigma_x^2 + 2\alpha\gamma (\sigma_e^2 + (\gamma - \phi)\rho\sigma_x^2)}. \tag{B.99}
\]

Equation (B.99) shows that, provided that the optimal LC debt share \(s\) is at an interior solution, \(s\) increases with credibility and the functional form is very similar to before.

### B.5.5 Comparative Statics

The comparative static of the LC debt share at an interior solution with respect to credibility becomes
\[
\frac{ds}{dp} = \frac{s^2}{2\alpha \left[ \gamma + 1/\bar{D} \right]} \frac{1 + (\phi - \rho)^2\sigma_x^2}{(\sigma_e^2 + (\gamma - \phi)\rho\sigma_x^2)}, \tag{B.100}
\]
\[
> 0.
\]
The comparative static of the inflation-output beta (equation (29) in the main text) becomes:

$$\frac{dBeta(\pi_2, x_2)}{ds} = \frac{\partial Beta(\pi_2, x_2)}{\partial s} + \frac{\partial Beta(\pi_2, x_2)}{\partial p} \frac{1}{\frac{ds}{dp}}, \quad \text{(B.101)}$$

$$= \left( p(\phi - \rho) - \gamma \right) \frac{\bar{D}}{2\alpha} + \left( \frac{\phi - \rho \bar{D}}{s} \left( \gamma + \frac{1}{\bar{D}} \left( \sigma_x^2 + (\gamma - \phi)\rho \sigma_x^2 \right) \right) \right) \frac{1}{1 + (\phi - \rho)^2 \sigma_x^2}. \quad \text{(B.102)}$$

Equation (B.102) is analogous to (29) in the benchmark model and continues to have the same main terms, except the expressions are now somewhat more complicated. We can again consider the case with risk-neutral investors ($\phi = 0$), in which case the comparative static (B.102) becomes

$$\frac{dBeta(\pi_2, x_2)}{ds} = \left( -\rho p + \gamma \right) \frac{\bar{D}}{2\alpha} - \rho \frac{\bar{D}}{s} \left( \gamma + \frac{1}{\bar{D}} \left( \sigma_x^2 + (\gamma - \phi)\rho \sigma_x^2 \right) \right) \frac{1}{1 + \rho^2 \sigma_x^2} < 0. \quad \text{(B.103)}$$

Expression (B.103) shows that when investors are risk-neutral, the model continues to have the counterfactual implication that in equilibrium the inflation-output beta should decrease with LC debt shares. The conclusion that we need risk aversion to explain our main empirical stylized fact therefore continues to hold in the extended model. The intuition is that risk-neutral investors value LC bonds that pay out during good times—after all, this is when one local good buys a large number of international goods. With $\rho > 0$, LC debt hence generates an even stronger incentive to inflate in bad states of the world, thereby leading to a downward-sloping relation between the LC debt share and inflation-output betas. The equilibrium effect with risk-neutral international investors ($\phi = 0$) is also unambiguously negative and hence further contributes to the downward-sloping model-implied relation between inflation-output betas and LC debt shares. In order to generate an upward-sloping relation between LC debt shares $s$ and inflation-output betas as in the data, we hence need $\phi > 0$ similarly to the model with $\rho = 0$.

### B.5.6 Quantitative Implications

We find that for a plausible value of $\rho$ the quantitative model implications are almost unchanged relative to the benchmark model. Since we have found the analytic model solution to be quite accurate in the benchmark case (see Table B.1), we use the analytic model solution for an easy quantitative comparison in this subsection. We set $\rho = 0.4$, corresponding to the empirical value reported in Favilukis et al. (2015). All other parameter values are as in Table 3. Table B.3 compares the analytic solution for the benchmark model ($\rho = 0$, columns (1) and (2)) and the model with correlated exchange rates ($\rho = 0.4$, columns (3) and (4)). We can see that an empirically plausible real exchange rate correlation leaves the model implications qualitatively and even quantitatively unchanged.
Table B.3: Model Extension – Correlated Real Exchange Rate: Empirical and Model Moments

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Model</th>
<th>Correlated Exchange Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low $p$</td>
<td>High $p$</td>
</tr>
<tr>
<td>Average Inflation</td>
<td>2.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Average No-Commitment Inflation</td>
<td>6.15</td>
<td>13.00</td>
</tr>
<tr>
<td>Inflation Beta</td>
<td>-0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>LC Debt Share</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.25</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Note: All parameters are as in Table 3, except that we allow real exchange rates to be correlated with output ($\rho = 0.4$) for columns (3) and (4). All moments are in annualized natural units. All model moments are computed using the analytic solution.

B.6 Model Extension: Domestic Debt

This section solves a model extension, where debt is held by domestic agents and the government’s incentive to inflate the debt arises because it does not have access to lump-sum taxation. We show that the functional form of the analytic solution and quantitative model implications are very similar to the benchmark model. The model in this section has three key features that distinguish it from the benchmark model and make it a model of domestic debt: First, debt repayments no longer decrease domestic consumption one-for-one. Instead, the government needs to raise taxes to repay debt, leading to quadratic deadweight losses, as in the classic framework of Barro (1979). Second, investors and the government both care about the same consumption good. Third, investors and the government have the same coefficient of risk aversion.

Similarly to before, the government faces a trade-off between paying a lower risk premium on its debt by providing investors with a safe security and lowering average tax distortions by smoothing taxes across states of the world. We solve the model analytically and find that the expected loss function, inflation policy functions, and inflation betas take similar forms as before. In particular, the model can generate a positive comparative static of inflation-output betas with respect to the LC debt share, as in the data, through an equilibrium effect. The equilibrium effect operates through the effect of credibility, whereby credible governments choose higher LC debt shares and less countercyclical inflation, thereby paying lower risk premia on LC debt.

B.6.1 Domestic Debt Framework

We now assume that the government does not have access to lump-sum taxes. Instead, we assume that taxes are distortionary, so a real debt repayment of $D_2$ incurs a deadweight cost of $\tau D_2^2$, as in Barro (1979). For instance, this functional form for deadweight costs may arise if the government has access only to labor taxes, which distorts labor supply. If raising taxes leads to costly distortions, the government has an incentive to use the “inflation tax” to reduce the real debt burden and hence...
real taxes. The government resource constraint becomes

\[ C_2 = X_2 - \tau D_2^2, \quad (B.104) \]

implying that zero taxes have no deadweight costs, but increasing tax burdens have increasing marginal cost. The government’s loss function is given similarly to before:

\[ L_2 = \alpha \pi_2^2 - \frac{C_2^{1-\gamma}}{1-\gamma}. \quad (B.105) \]

For simplicity, we continue to assume that bonds are priced by an investor whose log SDF is perfectly correlated with the exogenous component of local output with risk aversion \( \gamma \):

\[ m_2 = \log(\beta) - \gamma x_2 - \frac{\gamma^2}{2} \sigma_x^2. \quad (B.106) \]

The SDF (B.106) captures that domestic investors require risk premia to hold assets whose payoffs co-move with local economic conditions. To keep the solution tractable, (B.106) abstracts from terms in consumption that depend on debt repayments. It would be straightforward to extend (B.106) to allow for investor risk aversion that differs from the government’s risk aversion, as might be the case if the government weights the worst states of the world less than consumers for political economy reasons.

The model timing is identical to the model in the main text. We require again that the government in period 1 chooses the optimal commitment inflation policy as a function of only local output \( x_2 \).

### B.6.2 Bond Prices

LC bonds have real payoffs per face value \( \exp(-\pi_2) \) and FC bonds have real payoffs \( \exp(-\varepsilon_2 + \frac{1}{2} \sigma^2) \). LC and FC bond prices are priced by a domestic agent with SDF (B.106) over local consumption goods, so bond prices are given by:

\[ q_{1}^{FC} = E_t \left[ \exp \left( m_2 - \varepsilon_2 + \frac{1}{2} \sigma^2 \right) \right] = \beta \exp(\sigma^2), \quad (B.107) \]

\[ q_{1}^{LC} = E_t \left[ \exp \left( \log(\beta) - \gamma x_2 - \frac{1}{2} \gamma^2 \sigma_x^2 \right) \exp(-\pi_2) \right]. \quad (B.108) \]

Note that now since investors are domestic, we obtain a new \( \exp(\sigma^2) \) term in the FC bond price (B.107). This obtains because investors and the government now care about the same consumption bundle, whereas in the benchmark model the real exchange rate drives a wedge between the price of the investor’s consumption bundle and the domestic consumption bundle.
B.6.3 Log-Quadratic Loss Function

We again derive a log-quadratic approximation to the expected loss function. Expanding consumption log-quadratically in terms of output and the excess return on the debt portfolio,

\[ c_2 + \frac{1}{2} c_2^2 \approx C_2 - 1, \]  
\[ X \exp \left( \frac{x_2}{X} \right) - \tau D^2 \left( \exp(x_r^d) \right)^2 - 1, \]  
\[ X \left( \exp \left( \frac{x_2}{X} \right) - 1 \right) - \tau D^2 \left[ \left( \exp(x_r^d) - 1 \right)^2 + 2 \exp(x_r^d) - 2 \right] + X - \tau D^2 - 1, \]
\[ \approx x_2 + \frac{1}{2} \frac{x_2^2}{X} - 2\tau D^2 \left[ x_r^d + \left( x_r^d \right)^2 \right] + X - \tau D^2 - 1. \]  

We again make the normalization that steady-state consumption in period 2 equals one on average, so \( \bar{X} = 1 + \tau D^2 \). Substituting this into (B.111), log-quadratic expansion for consumption becomes

\[ c_2 + \frac{1}{2} c_2^2 \approx x_2 + \frac{1}{2} \frac{x_2^2}{X} - 2\tau D^2 \left[ x_r^d + \left( x_r^d \right)^2 \right]. \]  

The second-order approximation to the log bond portfolio excess return is

\[ x_r^d + \frac{1}{2} \left( x_r^d \right)^2 = (1 - s) \left( \varepsilon_2 + \frac{1}{2} (\varepsilon_2^2 - \sigma_2^2) \right) \]
\[ + s \left( - (\pi_2 - E\pi_2) + \frac{1}{2} \left( (\pi_2 - E\pi_2)^2 - Var\pi_2 \right) - \gamma Cov(x_2, \pi_2) \right). \]  

Substituting back into the loss function for for \( x_r^d + \frac{1}{2} \left( x_r^d \right)^2 \), and ignoring policy independent
terms, gives
\[ L_2 \approx \alpha \pi_2^2 - \left( c_2 + \frac{1}{2} \sigma_2^2 \right) + \frac{\gamma}{2} \sigma_2^2, \] (B.114)
\[ \approx \alpha \pi_2^2 - \left( x_2 + \frac{1}{2} \sigma_2^2 \right) - 2\tau D^2 \left[ x r_2^d + \left( x r_2^d \right)^2 \right] + \frac{\gamma}{2} \left( x_2 - 2\tau D^2 x r_2^d \right)^2, \] (B.115)
\[ \approx \alpha \pi_2^2 + 2\tau D^2 \left[ x r_2^d + \frac{1}{2} \left( x r_2^d \right)^2 \right] + \tau D^2 \left( x r_2^d \right)^2 + \frac{\gamma}{2} \left( -4\tau D^2 x r_2^d x_2 + 4\tau^2 D^4 \left( x r_2^d \right)^2 \right), \]
\[ \approx \alpha \pi_2^2 + 2\tau D^2 \left[ (1 - s) \left( \varepsilon_2 + \frac{1}{2} \left( \varepsilon_2 - \sigma_2^2 \right) \right) + s \left( -\left( \pi_2 - E_1 \pi_2 \right) + \frac{1}{2} \left( \left( \pi_2 - E_1 \pi_2 \right)^2 - Var_1 \pi_2 \right) - \gamma Cov_1 \left( x_2, \pi_2 \right) \right) \right]
+ \left( \tau D^2 + 2\gamma \tau^2 D^4 \right) \left( 1 - s \right)^2 \varepsilon_2 - 2 \left( 1 - s \right) \sigma_2 \varepsilon_2 \left( \pi_2 - E_1 \pi_2 \right) + s^2 \left( \pi_2 - E_1 \pi_2 \right)^2
- 2\gamma \tau D^2 x_2 \left( (1 - s) \varepsilon_2 - s \left( \pi_2 - E_1 \pi_2 \right) \right). \] (B.116)

Debt repayments now are costly only to the extent that they induce deadweight losses, so they appear pre-multiplied by \( 2\tau D^2 \). Taking expectations gives the expected loss function
\[ E_1 L_2 = \alpha E_1 \pi_2^2 + 2\tau D^2 \times s \left( \gamma - \gamma \right) Cov_1 \left( \pi_2, x_2 \right) \]
\[ + \left( \tau D^2 + 2\gamma \tau^2 D^4 \right) s^2 Var_1 \pi_2 + \left( \tau D^2 + 2\gamma \tau^2 D^4 \right) \left( 1 - s \right)^2 \sigma_2 \] (B.117)

The expected loss function (B.117) takes a form very similar to equation (21) in the main text. Since the government has the same risk aversion over consumption as local bond investors, the consumption hedging and nominal risk premium terms are exactly equally strong ex ante and offset each other, giving the government an incentive to choose a commitment inflation policy that does not inflate in bad times, similarly to the model in the main text. Both inflation volatility and exchange rate volatility are now costly because the government wants to smooth tax distortions across states of the world, in addition to the desire to smooth consumption fluctuations over time because of utility curvature as in (21).

**B.6.4 Full Commitment Inflation Policy**

We start by solving for the optimal inflation policy functions at a given LC debt share \( s \). First, we consider the special case \( p = 1 \). Under full commitment, the loss function is strictly increasing in \( E_1 \pi_2 \) and \( Var_1 \pi_2 \) for any given LC debt share \( s \). It is therefore clear that the optimal inflation policy sets inflation constant at zero.

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B.6.5 No Commitment Inflation Policy

Next, we solve for the optimal inflation policy in the special case \( p = 1 \). Using the log-linear expansion for real debt portfolio returns (B.34), the government in the no-commitment state minimizes the no-commitment loss function

\[
L^{nc} = \alpha \pi_2^2 - 2\tau \bar{D}s (\pi_2 - E_1 \pi_2) + \left( \tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4 \right) s^2 (\pi_2 - E_1 \pi_2)^2 + 2\gamma \tau \bar{D}^2 x_2 s (\pi_2 - E_1 \pi_2). \tag{B.118}
\]

The first-order condition for no-commitment inflation becomes

\[
\pi_{nc}^2 = \frac{2\tau \bar{D}s}{2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2} (1 - \gamma x_2) + \frac{2 \left( \tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4 \right) s^2}{2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2} E_1 \pi_2. \tag{B.119}
\]

Unconditional expected inflation equals

\[
E_1 \pi_2 = \frac{\tau \bar{D}}{\alpha} s. \tag{B.120}
\]

Inflation variance and the inflation beta equal

\[
Var_1 \pi_2 = \gamma^2 \left( \frac{2\tau \bar{D}s}{2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2} \right)^2 \sigma_x^2, \tag{B.121}
\]

\[
Beta_1 (\pi_2, x_2) = -\gamma \frac{2\tau \bar{D}s}{2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2}. \tag{B.122}
\]

B.6.6 Partial Commitment Inflation Policy

We now solve for the inflation policy function at a given LC debt share \( s \) for the general case \( 0 \leq p \leq 1 \). We use \( \pi_2^{nc} = \pi_2(x_2, I_c = 0) \) to denote the inflation policy function if commitment fails in period 2 and \( \pi_2^c = \pi_2(x_2, I_c = 1) \) to denote the inflation policy function if commitment holds in period 2. The no-commitment inflation policy minimizes the period 2 loss function (B.118) and hence is given by the first-order condition:

\[
\pi_{2}^{nc} = \frac{2\tau \bar{D}s(1 - \gamma x_2)}{2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2} + \frac{2 \left( \tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4 \right) s^2}{2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2} (p \bar{\pi} + (1 - p) E_1 \pi_2^{nc}). \tag{B.123}
\]

Combining (B.117) and (B.123) shows that commitment inflation minimizes the expectation of a quadratic function in log inflation and log output. Log inflation hence solves a quadratic program and is linear in log output. We hence need to solve for constants \( \bar{\pi} \) and \( \delta \) such that:

\[
\pi_2^c = \bar{\pi} + \delta x_2. \tag{B.124}
\]
We next derive the expressions for optimal $\bar{\pi}$ and $\delta$. Expected no-commitment inflation then equals:

$$E_1\pi_2^{nc} = \frac{2\tau \bar{D}^2 s}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} + \frac{2ps^2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4)}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} \bar{\pi}. \quad (B.125)$$

Combining (B.124) and (B.125), unconditional expected inflation equals

$$E_1\pi_2 = p\bar{\pi} + (1 - p)E_1\pi_2^{nc}. \quad (B.126)$$

The inflation variance and the inflation-output covariance can then be expressed as:

$$Var_1\pi_2 = p(1 - p)\left( \frac{s \times 2\tau \bar{D}^2}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} - \frac{2\alpha}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} \bar{\pi} \right)^2 + p\delta^2 \sigma_x^2 + (1 - p)\frac{\gamma^2 \sigma_x^2}{(2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2) \bar{x}^2} (2\tau \bar{D})^2, \quad (B.127)$$

$$Cov(x_2, \pi_2) = p\delta^2 \sigma_x^2 - (1 - p)\frac{\gamma^2 \sigma_x^2}{(2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2) \bar{x}^2} \times 2\tau \bar{D}^2 s. \quad (B.128)$$

Inflation sensitivity in the commitment state $\delta$ enters into inflation variances and covariances exactly as with full commitment, scaled by the commitment probability $p$. It hence follows that the optimal commitment inflation sensitivity is

$$\delta = 0. \quad (B.129)$$

Ignoring terms in the loss function that are independent of $\bar{\pi}$, average commitment inflation $\bar{\pi}$ is chosen to minimize

$$\alpha E_1\pi_2^2 + (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2 Var_1\pi_2 \quad (B.130)$$

$$\alpha (E_1\pi_2^2 + (\alpha + (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2) Var_1\pi_2 \quad (B.131)$$

Substituting in for $E_1\pi_2$ and $Var_1\pi_2$ and dropping terms that are independent of $\bar{\pi}$, it follows that $\bar{\pi}$ equivalently minimizes:

$$\alpha \left( \frac{(1 - p) \times 2\tau \bar{D}^2 s}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} + \frac{2\alpha + 2s^2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4))}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} \right)^2 p\bar{\pi}^2 \quad (B.132)$$

$$+ (\alpha + (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2) \times p(1 - p) \left( \frac{s \times 2\tau \bar{D}^2}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} - \frac{2\alpha}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} \bar{\pi} \right)^2. \quad (B.133)$$
Dividing by common factors, optimal $\bar{\pi}$ minimizes
\[
\alpha ( (1 - p)s \times 2\tau \bar{D}^2 + (2\alpha + 2s^2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4)) p\bar{\pi})^2 \\
+ (\alpha + (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2) p(1 - p) (s \times 2\tau \bar{D}^2 - 2\alpha \bar{\pi})^2,
\]
(B.132)
showing that the optimal $\bar{\pi}$ is given by:
\[
\bar{\pi} = 0.
\]
(B.133)

With (B.125), (B.129), and (B.133), we can express expected inflation, the inflation variance, and the inflation-output beta, at a given LC debt share $s$ as:

\[
E_1\pi_2 = \frac{(1 - p) \times 2\tau \bar{D}^2 s}{2\alpha + 2ps^2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4)},
\]
(B.134)

\[
Var_1\pi_2 = p(1 - p) \left( \frac{s \times 2\tau \bar{D}^2}{2\alpha + 2ps^2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4)} \right)^2 \\
+ (1 - p) \frac{\gamma^2 \sigma_x^2}{(2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2)^2} s^2 (2\tau \bar{D}^2)^2, 
\]
(B.135)

\[
Beta_1 (\pi_2, x_2) = -(1 - p) \frac{\gamma}{2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2} \times 2\tau \bar{D}^2 s.
\]
(B.136)
B.6.7 First-Order Condition for LC Debt Share

Substituting in the expressions for inflation moments (B.134) through (B.136), the expected loss function simplifies to

\[ E_{1}L_{2} = \alpha \left( \frac{(1-p) \times 2\tau \bar{D}^2 s}{2\alpha + 2ps^2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4)} \right)^2 \]

\[ + (\alpha + (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2) \left( p(1-p) \left( \frac{s \times 2\tau \bar{D}^2}{2\alpha + 2ps^2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4)} \right)^2 \right) \]

\[ + (1-p) \frac{\gamma^2 \sigma_x^2}{(2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2)} s^2 (2\tau \bar{D}^2)^2 \]

\[ + (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) (1-s)^2 \sigma_x^2, \quad (B.137) \]

\[ = (1-p) \frac{(2\tau \bar{D}^2 s)^2}{2\alpha + 2ps^2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4)} \]

\[ + \frac{1-p}{2} \frac{\gamma^2 \sigma_x^2}{(2\alpha + 2 (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) s^2)} (2\tau \bar{D}^2 s)^2 \]

\[ + (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) (1-s)^2 \sigma_x^2. \quad (B.138) \]

Next, we keep only terms that are \( O(\tau^2) \), an approximation that is justified if the deadweight costs from taxation are not too large. In contrast to externally held debt, \( \bar{D} \) is plausibly large for domestically held debt, and for this reason we do not approximate around \( \bar{D} \) small. We use the following Taylor expansion for the function \( \frac{1}{1+t} \)

\[ \frac{1}{1+t} = 1 - t + O(t^2). \quad (B.139) \]

to obtain

\[ E_{1}L_{2} = (1-p) \frac{2(\tau \bar{D}^2 s)^2}{\alpha} \]

\[ + (1-p) \frac{\gamma^2 \sigma_x^2}{\alpha} (\tau \bar{D}^2 s)^2 \]

\[ + (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) (1-s)^2 \sigma_x^2 + O(\tau^3). \quad (B.140) \]
Taking the first-order condition of (B.140) with respect to $s$ gives
\[
s = \frac{2\alpha \left(1 + 2\gamma \tau \bar{D}^2\right) \sigma^2}{\left(1 - p\right) \left(4\tau \bar{D}^2 + 2\gamma^2 \tau \bar{D}^2 \sigma^2\right) + 2\alpha \left(1 + 2\gamma \tau \bar{D}^2\right) \sigma^2},
\]
(B.141)
\[
= \frac{2\alpha \left(1/\left(2\tau \bar{D}^2\right) + \gamma\right) \sigma^2}{\left(1 - p\right) \left(2 + \gamma^2 \sigma^2\right) + 2\alpha \left(1/\left(2\tau \bar{D}^2\right) + \gamma\right) \sigma^2},
\]
(B.142)

Similarly to equation (28) in the main text, it follows that $s$ increases with credibility provided that it is at an interior solution. The comparative static of $s$ with respect to $p$ is now given by
\[
\frac{ds}{dp} = s^2 \frac{2 + \gamma^2 \sigma^2}{2\alpha \left(1/\left(2\tau \bar{D}^2\right) + \gamma\right) \sigma^2} \geq 0.
\]
(B.143)

Intuitively, a low credibility government anticipates that it will inflate away debt in the worst states of the world, thereby imposing deadweight costs on tax payers and paying higher LC risk premia. A low credibility government hence chooses to borrow in FC to deliberately limit the incentive to create countercyclical inflation in period 2.

**B.6.8 Solution Summary**

From the functional form of (B.117) it follows that terms in the optimal inflation policy that are quadratic in the tax loss parameter $\tau$ lead to terms in the loss function that are third-order in $\tau$. Consistent with the approximation for the loss function (B.140), we keep only inflation policy terms that are $O(\tau)$. Combining the Taylor expansion (B.139) with (B.125), (B.129), and (B.133) gives the inflation policy functions as
\[
\pi_{2c}^{nc} = \frac{\tau \bar{D}^2 s (1 - \gamma x_2)}{\alpha},
\]
(B.144)
\[
\pi_2^c = 0.
\]
(B.145)

Note the similarity between (B.144)-(B.145) and (23)-(22) in the main text when investors and the government are equally risk averse ( $\phi = \gamma$). Keeping only $O(\tau)$ terms, the inflation-output beta becomes
\[
\text{Beta}_1 (\pi_2, x_2) = -(1 - p) \frac{\gamma \tau \bar{D}^2 s}{\alpha}.
\]

We hence see that the inflation beta is negative and increases with credibility $p$. The intuition is that a credible government is more likely to implement the stable inflation policy, thereby avoiding countercyclical inflation and high LC bond risk premia.
B.6.9 Comparative Statics

The total derivative of the inflation-output beta with respect to the LC debt share is given by

\[
\frac{dBeta(\pi_2, x_2)}{ds} = \frac{\partial Beta(\pi_2, x_2)}{\partial s} + \frac{\partial Beta(\pi_2, x_2)}{\partial p} \frac{1}{ds/dp},
\]

\[
= -(1-p) \gamma \frac{\tau \bar{D}}{\alpha} + \gamma \frac{\tau \bar{D}^2 s}{\alpha} \frac{2\alpha (1/(2\tau \bar{D}^2) + \gamma) \sigma^2_x}{s^2 + \gamma^2 \sigma_x^2}
\]

Expression (B.147) is analogous to equation (29) in the main paper. As in the benchmark model with \( \phi \leq \gamma \), the direct effect generates a downward-sloping relation between LC debt shares and inflation-output betas. The intuition is that the higher the LC debt share \( s \), the stronger the incentive to inflate in bad states of the world. This is counteracted by the equilibrium effect, which generates an upward-sloping relation between LC debt shares and inflation-output betas. Investors in the domestic model are always risk averse over domestic output, so there is no direct comparison to the \( \phi = 0 \) case in the international model. However, we note that we continue to have the same main terms in (B.147) as in (29). In particular, we see that only the equilibrium effect can explain the empirical stylized fact of an upward-sloping relation between inflation betas and LC debt shares. The intuition is that a high-credibility government commits to non-cyclical inflation so as to lower the risk premium paid on LC debt. At the same high-credibility issuers also choose larger LC debt shares \( s \), contributing to an upward-sloping relation between \( s \) and inflation-output betas.

B.6.10 Quantitative Implications

We compare a numerical version the domestic debt model with the benchmark model. For the domestic debt model, we choose a debt-to-GDP ratio of \( \bar{D} = 1 \), corresponding to the high level of overall debt-to-GDP ratios for many countries in our sample, especially once pensions and social security are included, and a small tax distortion parameter \( \tau = 0.065 \). A tax distortion parameter of \( \tau = 0.065 \) implies that the log-linear approximate marginal benefit of reducing the real face value of debt by 1 percentage point is the same as in the benchmark model and equal to \( 2 \times \tau \times \bar{D} = 0.13 \) percentage points. All other parameters are set as in the benchmark model. Since we have found that the analytic solution is accurate for the benchmark model (see Table B.1), we use the analytic model solution to compare the two models. Overall we find that analytic model moments in Table B.4 are similar to Table 4, indicating that the bond risks channel of monetary policy credibility remains quantitatively important in the domestic debt setting.
<table>
<thead>
<tr>
<th></th>
<th>Benchmark Model</th>
<th>Domestic Debt Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low $p$</td>
<td>High $p$</td>
</tr>
<tr>
<td>Average Inflation</td>
<td>2.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Average No-Commitment Inflation</td>
<td>6.15</td>
<td>13.00</td>
</tr>
<tr>
<td>Inflation Beta</td>
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<td>0.00</td>
</tr>
<tr>
<td>LC Debt Share</td>
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<td>1.00</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.25</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Note: This table compares the benchmark model (columns (1) and (2)) to the domestic debt model (columns (3) and (4)). The domestic debt model assumes a debt-to-GDP ratio of $\bar{D} = 1$ and tax distortion parameter $\tau = 0.065$. All other parameters are as in Table 3. All moments are in annualized natural units and are computed using analytic model solutions.