Repo Priority Right and the Bankruptcy Code

Jun Kyung Auh∗ and Suresh Sundaresan†

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ABSTRACT

This paper shows that when the bankruptcy code protects the creditors’ rights with no impairments to secured creditors, issuance of debt such as repo with exemption from automatic stay adds no value. When the bankruptcy process admits violations of absolute priority rules or results in collateral impairments to secured creditors, the liability structure includes short-term debt, with safe harbor protection when the pledged collateral satisfies a minimum liquidity threshold. Safe harbor rights lead firms to issue more short-term debt, less long-term debt and increase the long-term spreads.

Keywords: Bankruptcy Code, Super Seniority, Maturity Structure

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∗Georgetown University (email: junkyung.auh@georgetown.edu)
†Columbia Business School (email: ms122@columbia.edu)
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I Introduction

“Safe harbor” provisions to creditors refer to specific contractual rights in a loan agreement. When a borrower, who has pledged some collateral, with safe harbor protection to borrow cash, files for bankruptcy, the cash lender does not have to join the other creditors in the bankruptcy process and can walk free with the pledged collateral. Thus, the safe harbor provisions provide super-senior rights to lenders in seizing the collateral which is exempt from the automatic stay provisions of the bankruptcy code. In the absence of any safe harbor provisions, claims of creditors will be subject to automatic stay. Furthermore, any pre-bankruptcy transfers of a subset of creditors can be clawed back to facilitate orderly liquidation or reorganization.

In this paper, we examine, from the perspective of a value-maximizing borrower, the following questions pertaining to the creditors’ rights. First, in the presence of safe harbor protection, what is the optimal level of leverage and maturity composition debt? How do the properties of the bankruptcy code influence this decision? How does the “run risk” of short-term debt affect the optimal default decision on long-term, unsecured debt? Does the safe harbor protection lead to “excessive” use of short-term debt (relative to the alternative of secured short-term debt subject to automatic stay)? Does the presence of safe harbored short-term debt lead to an increase in the expected costs associated with bankruptcy and an increase in long-term credit spreads? We believe that answers to these questions are of first order importance in better understanding the borrowers’ optimal response to enhanced rights provided to short-term creditors.

Safe harbor treatment to repurchase agreements (repo) lending came about in the wake of two bankruptcies in 1982 in the United States (Drysdale and Lombard Wall). The bankruptcy court overseeing the latter insolvency announced that the firm’s repos would be treated as secured loans, rather than as outright transactions, and issued a temporary restraining order prohibiting the sale of the repo securities. This led to considerable confusion about the ownership of repo collateral under bankruptcy.

Garbade (2006) provides a concise account of these developments. Paul Volcker [who was the
Federal Reserve Chairman at that time), wrote to the Congress in 1982 arguing that repo agreements should be accorded the safe harbor status. One key argument made by Volcker in this context was the possibility of a systemic crisis if such rights are not provided to cash lenders. To quote Volcker: “... Exemption from automatic stay for repo collateral may prevent a manageable and isolated problem of one dealer from becoming a more generalized one.”

He, however, argued that such an exemption should be narrowly applied with only Treasury and other safe assets qualifying for exemption from automatic stay. In 1984, the bankruptcy code was amended and safe harbor privileges were accorded to repo agreements with Treasury securities and other securities with the full guarantee of the United States. The foregoing discussions make it clear that the original purpose for giving safe harbor protection was to prevent systemic risk that may arise when a major repo borrower fails, and thus enmeshing a number of repo lenders and other significant counterparties in lengthy and costly bankruptcy proceedings that can precipitate a domino effect. Duffie and Skeel (2012), Acharya and Oncu (2012) provide a backdrop to this and other arguments that have been presented about safe harbor provisions, and the effect such provisions can have on systemic risk.

The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA) modified the bankruptcy code further to give similar protection to repo loans based on mortgage collateral. As a result, in the United States, short-term loans/debt provided by lenders as in repo, are protected by safe harbor provisions, for a wide range of underlying collateral. Once the bankruptcy code is amended this way, the borrowers will respond to optimally rearrange their liability structure and leverage. Such an optimal response by the borrower has not been studied in the literature and is the subject matter of this paper. In their response, borrowers and lenders will only internalize their private costs and benefits. They are unlikely to take into account the potential social costs which may be imposed by the “run” decisions of cash lenders holding collateral that is exempt from automatic stay. Such social costs are the focus of a number of papers, which have debated and recommended a “rollback” of safe harbor rights to

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1 Volcker (1982) also noted that repo agreements are important in the conduct of open market operations and in the financing of Government debt.
2 See, Volcker, Paul, (1982), Letter addressed to Bob Dole, Chairman, Subcommittee on Courts Committee on the Judiciary United States Senate, September 29th.
3 Asset-backed securities such as asset-backed commercial paper are also protected by bankruptcy-remote structures, allowing the lenders to have easier access to the collateral in the SPV.
illiquid collateral and institutional remedies for dealing with repo runs.\textsuperscript{4} Our paper provides an initial step in understanding how debtors respond to safe harbor rights when they act only in their own self interest. This understanding is a necessary input into the process whereby economists, legal scholars and regulators attempt to understand how such actions may lead to systemic crises.

In this paper, we develop a structural model of default in which the optimal capital structure and the optimal mix of short-term debt and long-term debt, known as the liability structure are determined endogenously. Specifically, the paper links the optimal liability structure and the emergence of secured short-term debt protected by “safe harbor” structures to the underlying bankruptcy code, and its lack of effectiveness in protecting the rights of creditors, or entailing dead-weight losses.\textsuperscript{5} In our setting, the borrower is able to optimally trade off between the “run risk” of safe harbored short-term debt and the “run-free” long-term debt depending on the inefficiencies in bankruptcy code, the availability of eligible collateral, and the secondary market liquidity of the collateral to increase the overall value of the firm. We also present a simple modification of the model to admit secured short-term debt subject to automatic stay. We show that the pledged collateral must satisfy a minimum liquidity threshold for the firm to issue debt with exemption from automatic stay. This threshold depends on the properties of the bankruptcy code and the properties of the asset.

Our paper falls between two strands of literature. The first strand is the literature that focuses on how short-term debt such as repo financing can exacerbate aggregate risk by precipitating the fire-sale of assets, when there is an aggregate macro shock. The second strand is the literature that attempts to explore how a firm selects its optimal capital/liability structure.

The road map of the paper is as follows. In Section II, we place the results of this paper in the context of the pertinent literature. Section III presents motivating empirical patterns. Section IV outlines the payoff functions to creditors and borrowers when the borrowers optimally

\textsuperscript{4}See, Acharya and Oncu (2012), Duffie and Skeel (2012), Morrison et al. (2014), and Adams and Roe (2014)

\textsuperscript{5}There can be other reasons for creating bankruptcy-remote SPV structures, relating to capital relief, and accounting disclosures. These are not treated in our analysis
decide to restructure under the shadow of a bankruptcy code. Section V contains the results of the paper on restructuring, optimal leverage, liability structure, and how they relate to the underlying bankruptcy code. Section VI delivers results about the relationship between the secondary market liquidity of the assets pledged and the incentives of the borrower to use safe harbor debt. Section VII concludes.6

II Related Literature

A number of authors have weighed in on the super-senior rights of repo lenders and the holders of collateral in derivatives transactions. Several authors, notably Gorton and Metrick (2012) have identified the “repo run” during the recent financial crisis in 2008 due to the declining value of collateral and increased “hair cuts” as an important triggering event in pushing the financial sector to the brink of insolvency. Krishnamurthy et al. (2012) argue that the “run” was more severe in repos backed by private sector securities, and that the crisis looked less like a traditional bank run of depositors and more like a credit crunch among dealer banks.

There is a series of papers that discuss the special treatment of bankruptcy code on derivatives.7 Roe (2011) concludes that “the Bankruptcy Code’s safe-harbor, super-priorities for derivatives and repurchase agreements are ill conceived.” He observes that “not only do the provisions facilitate runs on financial institutions during financial crises, they also seriously weaken counter-parties’ ex-ante incentives for financial stability.” Roe (2011) suggests the following counter-factual: in the absence of safe harbor provisions, we should ordinarily expect players to substitute such protected type of liability for other financing channels. Bolton and Oehmke (2014) also conclude that safe harbor provisions in derivative markets may lead to undesirable outcomes. They suggest that such provisions for derivatives can lead to inefficiencies by shifting credit risk to the firm’s creditors. They further note that as a result of

6In an online appendix, we provide the following additional results: Section A endogenizes the optimal sharing rules through a bargaining game, and finds an interior level of pledging of assets. Section B provides a formal rationale for why short-term debt is the preferred instrument for safe harbor provisions. Section C shows that the qualitative results go through when we hold the total recovery a constant.

7This discussion can be traced back to Edwards and Morrison (2005).
safe harbor provisions, firms may take on derivative positions that are too large from a social perspective. Acharya et al. (2012) argue that the automatic stay provisions may be ex-post optimal for loans made under repo agreements, due to the fire-sale of collateral of firms that were highly levered. They are not concerned with the issue of a firm’s optimal liability/capital structure, ex-ante. Antinolfi et al. (2012) argue that the exemption from bankruptcy may result in increased repo lending, and thus leading to damaging fire sales in the secondary markets. Duffie and Skeel (2012) and Skeel and Jackson (2012) have argued that repo loans should not be insulated from bankruptcy provisions such as automatic stay, although they have concluded that repo loans based on cash-like securities need not be subjected to automatic stay. They suggest that repo loans based on illiquid collateral should be covered by automatic stay.

In the context of this strand of literature, we make the following contributions: first, we examine what should be the optimal response of the borrower when faced with the choice of issuing short-term, secured debt (which subjects the borrower to run risk) and long-term unsecured debt. We show that the borrower will internalize the run risk and choose in general a finite amount of secured, short-term debt. In our framework, when the illiquidity of the pledged collateral reaches a certain threshold low value, borrowers will optimally choose not to issue short-term debt protected by safe harbor provisions.

The optimal capital literature in a structural setting has tended to work with a single, homogeneous debt setting, following the insights of Merton (1974), Black and Scholes (1973), Black and Cox (1976) and Leland (1994). More recently, some papers have attempted to determine the optimal liability structure. Hackbarth and Mauer (2011) emphasize the importance of priority structure of debt in the context of growth options, but they do not address the problem run risk posed by short-term creditors nor do they model super senior rights. Hackbarth et al. (2007) consider bank debt and public debt, and they model a situation where the bank debt can be negotiated outside the bankruptcy code. However, they treat both debt as perpetual: in their setting, there is no short-term debt, which can expose the borrower to run risk.

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8The policy implications of systemic risk arising from the fire sale of repo collateral is examined by Acharya and Oncu (2012) and Oehmke (2014). We abstract from this question.
9Black and Cox (1976) consider both senior and subordinated debt of finite maturity.
He and Xiong (2012) provide a dynamic model of debt runs. They derive an equilibrium and interpret it in a model in which each creditor, in deciding whether or not to roll-over his debt, must reflect on other creditors’ roll-over actions. Chen et al. (2012) also build a dynamic model of debt structure and make a distinction between idiosyncratic risk and systematic risk of firms, and they note that this distinction plays an important role in the choice of maturity structure and rollover decisions. Brunnermeier and Oehmke (2013) develop a model that endogenously determines maturity structure of financial institutions in the presence of multiple creditors. The model has a prediction that, during the economic turmoil, each creditor has an incentive to shorten their loan to the bank, which would result excessive short-term financing and unnecessary roll-over risk.

These papers, by and large, are not concerned with the question of the link between the bankruptcy code and the emergence of safe harbor protected debt as an optimal outcome. Nor are they concerned explicitly about the optimal liability structure of the firms when there is a possibility of a run by short-term lenders protected by super-senior provisions. These issues form the basis of our paper. Our paper thus focuses on the question of the level of short-term debt that a firm should optimally select, and abstracts from the question of the implications of such debt in the aggregate when there is a macro-economic shock. We believe that this is the first necessary step in tackling challenging policy question regarding creditors’ right and its implications.

To address this question, first we derive the optimal liability structure of the firm, and show the conditions under which debt with or without safe harbor protection emerges as a component of the optimal liability structure. Second, we show that the borrowers recognize that the short-term lenders (such as money market lenders) have an incentive to run and make their loans risk-free. This opportunistic behavior of short-term lenders, who run with their safe harbored collateral, is fully internalized by the borrowers when they choose their optimal capital and liability structure. Third, we explore, for given dead-weight losses, how the extent to which absolute priority rights are respected influences the optimal liability structure and the use of short-term debt with super-senior provisions. By virtue of the fact that we have a structural framework, we can compute the credit spreads of long-term debt, which internal-
izes the run risk of safe harbored short-term debt, and effectively reduces the long-term debt capacity of the firm issuing short-term debt protected by safe harbor. Our paper shows that safe harbored short-term debt may increase the value of the firm by lowering the expected dead-weight costs of bankruptcy and escaping potential APR violations.

III Empirical Motivation from the 1984 Bankruptcy Reform

In 1984, the Bankruptcy Amendments and Federal Judgeship Act was enacted by the Congress, which granted super-seniority to repo lender’s claims on the collateral pledged against their loan. This act was motivated in part by the bankruptcies of two repo dealers and the desire of the Federal Reserve to preserve the smooth functioning of repo markets. Drysdale, one of dealers in the repo market, filed bankruptcy on 17 May 1982. This was followed by the insolvency of Lombard Wall, another dealer, on August 12, 1982. Then, on August 17, 1982, the bankruptcy court overseeing the insolvency issued a temporary restraining order prohibiting sale of the repo securities. This raised doubts about the rights of creditors who had lent cash against collateral. The Federal Reserve, which conducts its open market operations through repo markets, wanted to preserve the integrity of repo markets and successfully persuaded the Congress to enact the law strengthening the rights of repo cash lenders.

After a period of great uncertainty from August 1982 to July 1984, Congress passed the act, which exempted Treasury securities, federal agency bonds, bank CDs, and bankers’ acceptances from automatic stay in repo contracts, granting “safe harbor provision” to repo debt. The 1984 bankruptcy reform was thus an autonomous event enhancing the rights of repo cash lenders. We document below that this act led to (i) an increase in the use of short-term (repo) debt, (ii) a decrease in the use of long-term debt, and (iii) an increase in the spreads of long-term debt. These stylized facts are broadly consistent with the main predictions of the model developed in the paper.

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10 The primary problem was the neglect of accrued interest in repo contracts, as noted in Garbade (2006). The cash loaned did not reflect the accrued interest of the collateral, introducing additional counter-party exposure.

11 This act was signed into law on July 10, 1984. See Garbade (2006).
III.a. Data

In order to test the effects of the Bankruptcy Amendments and Federal Judgeship Act of 1984, on the use of safe harbored debt, liability structure and long-term credit spreads, we specify our sample period to be from 1980 to 1990. We have used two databases: (1) Flow of funds data from the Federal Reserve Board and (2) debt issuances data from the SDC platinum database.

The first database contains information on the amount of repo financing of broker-dealers during our sample period. To the best of our knowledge, this is only publicly available data that shows the amount of repo financing of broker-dealers in this period. The second database contains a comprehensive set of information on bond issuances during the sample period. With the bond-specific information in this database, we calculate the yield to maturity and duration of each bond. We also calculate the credit spreads by taking the difference between the bond yield and yields of Treasury securities with a matching duration at the quarter of issuance. In the database, there are multiple offerings from an issuer during a quarter. Using this issuance data, we construct issuer-quarter panel data by aggregating the issue amount and computing the weighted average of the spreads per issuer and quarter (weighted by the issue amount). The resulting database consists of a total of 3,820 issuer-quarter observations (1,602 unique bond issuers), with information of their issuance spread, total amount of offering, and their most frequently observed credit rating per each quarter.\footnote{SDC database includes bond ratings from two agencies (S&P and Moody’s). We map Moody’s rating into S&P in order to unify the rating.} We further augment the data with Compustat for leverage information when issuers are public firms. Table 1 provides summary statistics of of the SDC database at the individual bond level and the issuer-quarter level.

[ Insert Table 1 here. ]
III.b. **Repo Lenders’ Rights and the Use of Repo Debt**

Controlling for the new issuance of Treasury securities, Figure 1 illustrates several dynamics on usage of repo financing before and after the law change. In the figure, the circle (triangle) markers indicate the aggregated quantity of repo debt by broker-dealers with respect to the total new issuance of Treasury securities per quarter before (after) the 1984 act was passed in the Congress. Specifically, the filled circles correspond to the period of uncertain regulatory treatment on the rights of repo cash lender upon bankruptcy (from August 17, 1982 to July 10, 1984). Within each shape of markers, the darker color indicates later time in the pre and post period of the reform.

Before the 1984 act, the figure shows that the repo amount is almost perfectly lined up with amount of available collateral (circle). However, the filled markers indicate that there was a shrinkage of repo debt during the uncertain period when the bankruptcy court overseeing the insolvency issued a temporary restraining order prohibiting the sale of the repo securities. After the 1984 act, there was a remarkable jump in the usage of repo debt. Light-colored triangles connect darker-colored circles and darker-colored triangles: this patterns shows that the transition happened right after the safe-harbor provision was granted to the repo. To see its statistical significance, we create a ratio of broker-dealer's repo position over amount of Treasury security supply. In a two sample \(t\)-test, the mean difference in the ratio between pre and post-1984 act periods is very significant.\(^\text{13}\)

[ Insert Figure 1 here. ]

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\(^{13}\)\(t\)-statistic is about 6.4.

III.c. **Repo Lenders’ Rights and Long-term Credit Spreads**

Controlling for credit quality, we next show that the the long-term spread increased after safe harbor status was given to repo debt. With the improved access to repo collateral, this regulation change makes the long-term creditor worse off: in the event of bankruptcy, the pledged assets will not be available to the long-term creditors, adversely affecting the recovery
rates to long-term creditors.

In the time series, the spread can generally change for two additional reasons. One reason could be the dynamics in the macro economy that affect all the borrowers in the cross-section. Another reason could be the idiosyncratic changes in the credit risk of the borrower, including issuers’ leverage ratio. We therefore need to control for these two dimensions.

The 1984 Act affected only financial firms because the Act only changes bankruptcy treatment of repo, a dominant debt instrument used by financial firms. This fact allows us to analyze the long-term credit spreads in a difference-in-differences set up, where we regard financial firms as a treated group and non-financial firms as a control group.

Any potential macro-economic changes on spreads can be, therefore, mitigated by taking the differences between financial firms and non-financial firms. We further controlled for idiosyncratic changes of the credit risk by credit ratings, issuers’ total leverage, and issuer characteristics. Specifically, we include the rating and issuer fixed effects in the regression specification. The fixed effect enables us to compare the spread changes within each rating class or each issuing firm. Using the issuer-quarter panel data from the SDC database, for borrower \( i \) at each quarter \( t \), we estimate the following equation:

\[
\text{Spread}_{i,t} = \alpha_t + \alpha^i + \beta_1 \cdot \text{Fin}_i + \beta_2 \cdot \text{Post}_t + \beta_3 \cdot \text{Fin}_i \cdot \text{Post}_t + \lambda \cdot \text{Leverage}_i + \varepsilon_{i,t},
\]

where \( \text{Post} \) is a dummy variable which is 1 if \( t \) is after the safe-harbor provision passed the Congress (1983 Q3) otherwise 0, and \( \text{Fin} \) is a dummy variable that gives us 1 if the borrower \( i \) is financial firms, according to SIC (SIC 6000 firms). \( \alpha^t \) is the time fixed effect, and \( \alpha^i \) controls for the issuer-specific heterogeneity by its credit rating fixed effect or issuer fixed effect.\(^{14}\) \( \text{Leverage} \) is the total leverage ratio of issuer \( i \) at the corresponding quarter \( t \) in order to further control inter-rating-variation when we use rating fixed effects.

Table 2 shows the regression results. Across all specifications with different combination of control variables, we consistently document the following observations: (a) financial firms,
on average, had lower spread comparing to non-financial firms by 17 to 72 basis points ($\beta_1$), (b) all firms had lower spread by 28 to 87 basis points during the post-1984 act period ($\beta_2$), (c) however, the spread of financial firms increased by 18 to 66 basis points after the 1984 act ($\beta_3$). Recall that in each specification, we control for issuer’s credit quality by its credit rating, leverage, and firm fixed effect. Hence, the spread difference between financial and non-financial firms is estimated within a rating category or an issuer itself, controlling for both macro and issuer-specific credit risk. In the following sections, we develop a structural framework to explain the above stylized facts.

[ Insert Table 2 here. ]

IV Bankruptcy Code & Safe Harbor Debt

The bankruptcy code and its implications for the design of debt contracts has been stressed in the work of Hart (2000), in which he identifies the following desirable goals for an “optimal” bankruptcy code: first, the bankruptcy code must deliver an efficient ex-post outcome in terms of maximizing the value of assets available to all claimants. Second, the bankruptcy code must deliver ex-ante efficiency inducing borrowers to commit themselves to service their contractual debt obligations. Such a commitment should be enforced by penalizing borrowers in bankruptcy states. Finally, Hart (2000) suggests that a good bankruptcy procedure must respect absolute priority rules (APR), with the exception that some portion of value should possibly be reserved for shareholders.15

We explore, in the context of a structural model of default, how these desired properties of the bankruptcy code influence, ex-ante, the choice of firm’s optimal liability structure and optimal capital structure. Specifically, we assume that the unlevered asset value of the borrowing firm

15The proviso that “some portion of value should possibly be reserved for shareholders” arises because the borrowers can take excessive amount risk near bankruptcy in the absence of such provisions. In our model, this possibility is ruled out as the investment policy is assumed to be fixed. See Milbradt and Oehmke (2014) and Cheng and Milbradt (2011) who study the link between investment decisions and optimal maturity structure.
follows a Geometric Brownian Motion (GBM) process:

\[
d\frac{V}{V} = (r - \delta)dt + \sigma dW^Q
\]  

(2)

where, \( r \) is risk-free rate, \( \delta \) is the dividend yield, \( \sigma \) is the volatility of asset return and \( W^Q \) is standard Wiener process under risk neutral measure \( Q \).

The specification above implies that the investment policy is fixed. This assumption, while restrictive, allows us to focus on the optimal liability structure and optimal leverage decisions of the borrower.\(^{16} \) In our stylized setting, the borrower can issue two types of debt: one type is instantaneously maturing debt, which serves as a metaphor for short-term secured debt. The debt can be simply secured within the bankruptcy code, and be subject to automatic stay, or lenders of this type may require the super-seniority provisions of safe harbor. We assume that they will be able to monitor the firm closely and run at the right moment to make their debt risk-free. The other type of debt is the long-term, unsecured debt, which in our setting is simply a perpetual debt with a specified coupon. The setting is very similar to the classic structural models of default such as Merton (1974), and Leland (1994). What distinguishes our setting from these papers is the fact that we consider two types of liabilities, and that the short-term lenders can run to make their debt risk-free.\(^{17} \) The key to our model is the nature of the bankruptcy code, and the implied restructuring possibilities that the bankruptcy code presents to lenders and borrowers.

**IV.a. Bankruptcy, Run Risk and Restructuring**

The presence of a bankruptcy code can lead to endogenous debt restructuring as noted in papers such as Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), François

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\(^{16}\) It is useful to think of \( V \) as the after-tax value of the assets of a firm, which has 100% equity. The claims to the cash flows of such a firm will be split between the firm's equity holders and the government, which levies taxes. It is easy to begin with a process for the earnings before interest and taxes (EBIT) and derive the process in Equation (2) as done in Goldstein et al. (2001). For simplicity of exposition, we work with the specification in Equation (2).

\(^{17}\) The firm decides on the levels of short-term debt and long-term debt at \( t = 0 \) and commits until default occurs endogenously. Developing a fully dynamic liability structure model with safe harbor debt is beyond the scope of this paper.
and Morellec (2004), and in Broadie et al. (2007). In these papers, a well defined bankruptcy code, with an automatic stay provision and an associated exclusivity period allows the borrowers to file for bankruptcy and suspend their contractual debt payments as they decide whether to restructure or liquidate. These papers show, under the shadow of such a bankruptcy code, lenders and borrowers will restructure their claims at the endogenous restructuring boundary. They provide micro-foundations for the restructuring triggers and relate them to the provisions of the bankruptcy code. In this context, the default event can be generally interpreted as a restructuring event between debt holder and equity holder where liquidation of the firm is the special case of the restructuring event. Fan and Sundaresan (2000) present a model of endogenous restructuring under the shadow of Chapter 7 liquidation. Consistent with the approaches taken by these papers, we focus our attention on the endogenously determined restructuring boundary $V_B$, which is the threshold level of the value of the firm at which the borrower decides to restructure. The restructuring boundary will be optimally chosen by the borrower. When the borrower has raised capital by issuing both short-term and long-term debt, the question of restructuring becomes more complicated.

The firm’s short-term debt is assumed to mature at each instant. This nature of the maturity gives the short-term creditor a priority over the long-term creditor. The advantage of short-term debt in our context is that it is essentially risk-free: with the GBM process in Equation (2) for the underlying value of the firm’s assets and continuous monitoring, the short-term creditors (who have full information) can promptly act and be repaid the amount lent at the restructuring boundary, provided the collateral is free from automatic stay. Thus, the short-term lender seizes the assets through super-senior rights (with exemptions from automatic stay) optimally to make their claims risk-free. Appendix B exploits briefly the trade offs about providing safe harbor rights to short-term versus long-term debt. We refer to the point at which the run occurs as the “run boundary” and denote it by $V_R$. The firm ceases to exist the first time when $V$ reaches either $V_B$ or $V_R$. These two channels may have different costs, and in turn they influence the liability structure decisions by the borrower as shown later in the paper.

The motive for issuing debt can either be due to the tax code or due to agency theoretic con-
siderations. Following the structural models of default, we will assume that the interest expenses associated with servicing debt contracts are tax-deductible. Thus, value is created by short-term debt (and long-term debt) in this model. We assume that the tax rate is \( \tau > 0 \).\(^{18}\)

**IV.b. Equity, Short-term Secured Debt & Long-term Unsecured Debt**

We denote the value of short-term debt as \( S \). A fraction \( \theta \in [0, \bar{\theta}] \) of the assets of the firm is pledged to the short-term creditors. Here, we assume that \( \bar{\theta} \) is the maximum fraction of the assets that is eligible as collateral. If these pledged assets are held outside the bankruptcy code (i.e., exempt from automatic stay, as in safe harbor), the short-term lenders can liquidate them at a proportionate liquidation cost of \( \beta > 0 \), which reflects the secondary market liquidity of the assets unencumbered by automatic stay.\(^{19}\) The short-term creditors get the assets that have been pledged outside the bankruptcy code. The parameter \( \theta \) reflects the fraction of assets of the borrower that are eligible to be pledged to secure short-term financing. Not all borrowers will have the same level of eligible collateral. We denote by \( \bar{\theta} < 1 \) as the upper limit on this fraction. In our model, \( \bar{\theta} \) implicitly captures the costs associated with pledging assets so that they are exempt from automatic stay. There may be both explicit and implicit costs: securing assets outside may require the borrowing firm to set up a collateral management program, monitor covenants that long-term creditors may impose by way of negative pledges, restrictions on “sale” of assets, etc.\(^{20}\) Such costs can vary from one borrower to another, depending upon the characteristics of the assets that the borrower has: volatility of assets and secondary market liquidity would be key characteristics. Moreover, long-term creditors might

\(^{18}\)One may question whether repo debt, an important short-term financing channel for financial firms, would yield any tax benefits, since the transaction is technically selling the asset to the lender with an agreement to buy it back. However, according to GAAP, the difference between original sales proceeds and repurchase amount is considered as interest expenses (see FASB ASC paragraph 860-10-40-24(c)). Also court decisions have ruled that, for tax purposes, repo transactions constitute money borrowings from secured loan. (see Nebraska v. Lowenstein, 513 U.S. 123 (1994); American National Bank of Austin v. United States, 421 F. 2nd 442 (5th Cir. 1970); First American National Bank of Nashville v. United States, 467 F.2nd 1098 (6th Cir. 1972); Rev. Rul. 77-59, 1977-1 C.B. 196.)

\(^{19}\)He and Milbradt (2012) model how corporate default decisions interact with the endogenous secondary market liquidity through the rollover channel, and identify the potential for a feedback loop between default and secondary market liquidity.

\(^{20}\)We can model these costs, and derive the main implications of the paper, with qualitatively similar results. This will, however, generally makes the model much more complicated and necessitate numerical analysis. The current specification delivers the basic economic intuition and allows us to obtain analytic characterization of main results.
stipulate some restrictions on the amount that may be pledged.

The vector \( \{V_0, \bar{\theta}, \beta, \sigma, \delta\} \) reflects the asset composition of the borrowing firm in this model. A firm with a low \( \sigma \) and a low \( \beta \) is endowed with a higher level of attractive and liquid collateral available for pledging. The choice of \( \theta \) is endogenous and depends on the asset composition and the parameters of the bankruptcy code as we show later. We take the vector of asset composition as given, exogenously, and focus on the firm’s leverage and liability decisions, given these parameters and certain properties of the bankruptcy code. The choice of the asset mix along the dimensions of liquidity and volatility is a very interesting problem, but one that we do not investigate in this paper.

Formally, at the run boundary, \( V_R \), the payoffs to the short-term lender will be given by the expression below.

\[
S(V_R) = \theta(1 - \beta)V_R = S
\]  

(3)

From the short-term creditor’s perspective, they will stop lending just at the point that they can recover their principal amount. This is the instant when \( V_R = \frac{S}{\theta(1-\beta)} \). It is worth noting that \( S \), which is the par value of short-term debt, will be determined endogenously, and hence the run boundary will be endogenous as well.\(^{21}\) While our theory will focus on safe harbored short-term debt, the model is general enough to treat secured debt inside the bankruptcy code as we will explain below. We now turn to the payoff functions of long-term debt and equity at either the default point or the restructuring point. Consider a firm that operates in a bankruptcy code in which, upon filing for Chapter 11, the firm will either emerge with restructured debt and equity claims or liquidate under Chapter 7. We assume that the underlying bankruptcy code will lead to an endogenous restructuring boundary \( V_B \) at which the borrowers (equity holders) get a payout as shown in Equation (4) .\(^{22}\)

\[
E(V_B) = (1 - \theta)\psi_EV_B
\]  

(4)

\(^{21}\)If the run occurs first, the firm ceases to exist and the long-term creditors and equity holders are paid off as follows: \( D(V_R) = \psi_D(1 - \theta)V_R \), and \( E(V_R) = \psi_E(1 - \theta)V_R \). As we will show later, the run boundary in equilibrium coincides with the optimal restructuring boundary \( V_B \).

\(^{22}\)Default happens when either a run happens or when equity holders default on long-term debt.
If we take the interpretation that $V_B$ is the bankruptcy boundary, then clearly, the parameter $\psi_E$ captures the extent to which the borrowers (equity holders) are able to extract some surplus upon bankruptcy. On the other hand, if we view $V_B$ as the restructuring point, then $\psi_E$ captures the degree to which the underlying bankruptcy code allows borrowers to extract some value through restructuring under the shadow of the bankruptcy code. If we set $\psi_E = 0$ then the equity holders get nothing, and the absolute priority rule (APR) is fully respected in either bankruptcy or in the restructuring that may occur in the shadow of the bankruptcy code. The APR violations (i.e., $\psi_E > 0$) are economic rents that the borrowers are extracting from the long-term lenders due to some deficiency in the bankruptcy code: such deficiencies may include excessive stay period, greater bargaining rights to debtors, and leakages of value during the stay period to the detriment of creditors. This presents another possible reason as to why safe harbor can add value to the firm: safe harbor diminishes this avenue of rent extraction by equity holders through pledging collateral that is subject to exemption from automatic stay.

Long-term debt holders will receive at $V_B$ the following payout.

$$D(V_B) = (1 - \theta)\psi_D V_B$$

If $V_B$ represents the bankruptcy boundary, then $\psi_D$ indicates the fraction of the firm’s assets that the long-term creditor gets at default. Therefore $1 - (\psi_D + \psi_E) \equiv \alpha \geq 0$ is the total loss in the bankruptcy process. Alternatively, if $V_B$ is the restructuring boundary, under the shadow of bankruptcy, $\psi_D$ will measure the recovery to the long-term creditors in the restructuring boundary. We assume that $\psi_D + \psi_E \leq 1$ with the understanding that when $\psi_D + \psi_E = 1$ (or $\alpha = 0$), the restructuring process (under the shadow of the bankruptcy code) is fully efficient in the sense that there are no dead-weight losses, and the entire residual value is split between equity holders and long-term debt holders. Thus, we distinguish two things: the dead-weight losses associated with restructuring, and the violation of creditors’ rights to the full residual value of the firm: the magnitude of $1 - (\psi_D + \psi_E) \equiv \alpha \geq 0$ addresses the question of “ex-post efficient outcomes” that is delivered by the bankruptcy code, and $\psi_E$ measures the APR.
violations. In Section A, we provide the micro-foundation that establishes the link between the provisions of the bankruptcy code and the sharing rules $\psi_D$ and $\psi_E$.

We formally distinguish between secured debt within the rubric of bankruptcy code (i.e., secured debt that is subject to automatic stay), and secured safe harbored debt as follows. Secured debt holders, when they are subject to automatic stay cannot immediately seize their collateral, as they must wait for the stay period to be completed. During this period, it is conceivable that their collateral may lose some value, thus imposing losses on them. One consequence of this might be that conservative lenders of short-term credit may assume that their costs of liquidating collateral after the expiration of automatic stay will be higher than $\beta$.

The presence of collateral will make APR violations less important to short-term creditors. To reflect these considerations, more generally, we assume that the liquidation costs, $\hat{\beta}$, faced by secured short-term debt holders who are subject to automatic stay is bounded as follows: $\beta < \hat{\beta} \leq \alpha$. By virtue of these arguments, the safe harbor provision provides an advantage to the lender in seizing the collateral at a lower cost $\beta$. We will formally show that the relationship between $\beta$, $\hat{\beta}$ and $\alpha$ will determine whether or not it is optimal to issue short-term secured debt, and conditional on issuing such short-term secured debt, whether it should be safe harbored or not.\textsuperscript{23}

V Optimal Restructuring and Liability Structure

In this section, we begin by asking how the borrowing firm determines its leverage, maturity structure, and bankruptcy point under a specific bankruptcy code. Let $C$ be the dollar coupon rate promised by the borrower to long-term creditors. Let $S$ be the par value of short-term debt issued by the borrower, which matures at each instant. We begin by asking the following question: given a liability structure, $\{C, S\}$, how would the borrower select the optimal

\textsuperscript{23}Some caveats about our specification should be noted: first, we assume that the pledging of assets with exemptions from automatic stay is not costly. Second, it is possible that the carve outs at a low cost $\beta$ may cause some inefficiencies in the restructuring following the run. We address these issues briefly, later in the paper.
restructuring boundary? As in any structural model of default, we assume that the borrower commits to maintain this level and composition of debt until default. Typically, negative pledges are placed to prohibit the further issuance of debt and the sale of assets. Such provisions can serve to limit significant changes in the capital and liability structure. Based on data gathered from Capital IQ, out of 6,315 issuances of financial debt from Jan 1967- April 2012, 88.9% have negative pledge covenants or restrictions of asset sales. In addition, 69.7% have negative pledge covenants or cross acceleration provisions.24

Let us first suppose that the borrower ignores the run risk presented by the short-term lenders in selecting the equity-value maximizing restructuring boundary, $V_B$. Alternatively, let us suppose that the short-term creditors do not run until the firm chooses to default. We will later explicitly incorporate the run risk imposed by the short-term lenders in the determination of the optimal liability structure and the restructuring boundary by the borrower. This problem is no different from the one studied in the literature before except that we replace the debt cash flows $C$ in the single debt case to $C + Sr$ in the context of our model. This leads to the restructuring boundary, which is stated below in Proposition 1.25

**Proposition 1.** Optimal restructuring boundary for equity holders is

$$V_B = \lambda \left( \frac{C + Sr}{r} \right),$$

where $\lambda = (1 - \tau)(\frac{1}{1 + x})\left(1 - (1 - \theta)\psi_E\right) > 0$ and $x$ is the positive root of the characteristic equation

$$0 = \frac{1}{2}x^2\sigma^2 - x(r - \delta - \frac{\sigma^2}{2}) - r.$$

Note that as the APR violations admitted by the bankruptcy code increases, i.e., as $\psi_E$ increases, the borrower would like to restructure early to get his share of the residual value of the firm earlier. Finally, note that when $\theta = 1$, APR violations do not influence the choice of the restructuring boundary. This is due to the fact that the debt is secured by all assets that the borrower is able to pledge, and the collateral posted is exempt from automatic stay.

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24Examining the issues studied in this paper in the context of a dynamic capital structure model is a worthwhile future research endeavor.

25Proofs of all propositions, theorems and corollaries are placed in the appendix.
Now we relax the assumption that there is no run risk. In the presence of a run risk, the borrower will rationally anticipate that the short-term lenders will run precisely when the market value of the collateral held by them reaches the value $S$. In other words, when $V \downarrow V_R \equiv \frac{S}{\theta(1-\beta)}$, the lenders will run and refuse to lend any further. We prove in the appendix that the borrowers will reflect the actions of the lenders and choose $S^*$ such that $S^* = V_B \theta(1 - \beta)$. This is stated in Proposition 2.

**Proposition 2.** Borrowers pick $S^*$ such that (a) it is equity value-maximizing, and (b) the short-term debt is risk-free. In other words, $S^*$ is chosen so that $V_B$, the optimal default boundary, is set to be equal to the “run” boundary of short-term creditors, $V_R$, i.e., $V_R = V_B$, and $S^* = V_B \theta(1 - \beta)$.

Figure 2 provides the economic intuition behind Propositions 1 and 2. We provide the proof in the appendix. Let us set $S = 0$ and $\psi_E = 0$. In this case, there is only long-term debt, and the default boundary coincides with the one found by Leland (1994) as can be seen from Proposition 1. This is the Y-intercept of the line $V_B$ in Figure 2. As we increase $S$, it is intuitive that $V_B$ should increase, but so does $V_R$. In fact, $V_R$ increases much more rapidly, as the short-term lender will exit sooner to make her loans risk-free. The point where $V_R$ and $V_B$ intersect pins down the level of $S^*$ that the borrower will choose to pick the restructuring (bankruptcy) boundary, which maximizes her equity value.

We now turn to the value-maximizing choice of optimal liability structure, $\{C^*, S^*\}$. The optimal liability structure which maximizes the total value of the firm can be determined as shown in the appendix. We present below the optimal liability structure, and the ratio of the optimal level of short-term debt to the optimal level of long-term debt in Lemma 1.$^{26}$

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$^{26}$With this approach, we treat that $S, C$ are a firm’s choice variable at the inception. In reality, however, one can consider a dynamic adjustment of liability structure. A dynamic capital structure problem, with endogenous levels of short-term and long-term debt is clearly the preferred goal. Given the complexity of treating both long-term and short-term debt, with the latter exempt from automatic stay and exposing the issuer to “run” risk, we have abstracted from this goal. Existing dynamic models do not treat the complexities that are addressed in our paper.
Lemma 1. The optimal levels of short-term secured debt and unsecured long-term debt are:

\[
C^* = rV \left( \frac{1 - \lambda \theta (1 - \beta)}{\lambda} \right) \left( \frac{\tau}{(1 + x) (\tau + \lambda (\alpha - (\alpha - \beta) \theta))} \right)^{1/x}
\]

\[
S^* = V \theta (1 - \beta) \left( \frac{\tau}{(1 + x) (\tau + \lambda (\alpha - (\alpha - \beta) \theta))} \right)^{1/x}.
\]

Lemma 1 immediately gives us the optimal liability ratio between short-term and long-term debt for \( \theta > 0 \):

\[
\frac{C^*}{S^*} = \frac{1 - \lambda \theta (1 - \beta)}{\lambda \theta (1 - \beta)}.
\] (6)

In addition, Lemma 1 shows how the firm chooses the optimal mix of short-term and long-term debt levels, fully internalizing the run risk posed by the short-term lenders. It also links the optimal liability structure to the deep parameters of the model such as (a) the fraction of assets that are eligible for pledging, (b) deviations from APR admitted by the bankruptcy code, (c) the extent to which the financial distress and the resulting restructuring is costly, (d) the volatility of the underlying assets of the borrower, and (e) the secondary market liquidity of the collateral.

We discuss below in detail these implications of Lemma 1. In the top panel of Figure 3, we plot along the X-axis, \( \psi_E \), which measures the extent to which the bankruptcy code admits deviations from APR, and along the Y-axis, the value-maximizing ratio of short-term debt to long-term debt implied by our model. These are plotted for three levels of \( \bar{\theta} \): the lowest line refers to the lowest \( \bar{\theta} \) and, the topmost line refers to the highest \( \bar{\theta} \). We note that when the bankruptcy code admits no violations of APR, generally the firm is able to support a much higher level of long-term debt in all the three cases, with the additional observation that the firm with a greater fraction of eligible collateral for pledging tends to use more short-term debt. As the bankruptcy code begins to admit greater deviations from APR, all the firms (irrespective of their \( \bar{\theta} \)) tend to increase their use of short-term debt, although the incentives are much greater for the firms with lowest \( \bar{\theta} \) as they have the lowest level of short-term debt to begin with.
The bottom panel of Figure 3 plots along the X-axis, $\bar{\theta}$, which measures the fraction of eligible collateral that may be used for pledging, and along the Y-axis, the value-maximizing ratio of short-term debt to long-term debt implied by our model. These are plotted for three levels of $\psi_D$: the lowest line refers to the lowest $\psi_D$, and the topmost line refers to the highest $\psi_D$. We note that when the bankruptcy code results in high financial distress costs associated with restructuring, generally the firm is able to support a much lower level of long-term debt. The firm with the greater fraction of eligible collateral for pledging tends to use more short-term debt. In this case, we assume that $\psi_E = 0$ so that there are no deviations from APR. This implies that even in the absence of APR deviations, firms in our model have an incentive to use short-term debt. This incentive arises due to the fact that once the assets are pledged outside of the bankruptcy code, the costs associated with recovery, as summarized by $\beta$ may be much less than the costs, $\alpha$, when the assets are inside the jurisdiction of the bankruptcy code and subject to automatic stay. An important example of the increased costs is the possible loss of collateral value and interest during the stay period from the perspective of the short-term creditors.\(^{27}\)

The bottom panel of Figure 3 shows yet another role played by safe harbor: when the restructuring process is perceived to be very costly, moving assets outside the bankruptcy code can add value to the borrowing firm as a whole. To the extent that we believe that the financial distress resolution process faced by financial institutions is much costlier than the ones faced by non-financial firms, then it is reasonable to expect financial firms to use more of these arrangements than non-financial firms. Their ability to use safe harbored debt and SPVs may also be further enhanced by the fact that they may be holding more eligible collateral.

[ Insert Figure 3 here. ]

The top panel of Figure 4 examines the relationship between optimal leverage and the volatility of the underlying assets of the borrower. In this figure, we also report the level of optimal

\(^{27}\)In this context, Morrison et al. (2014) note the following: “Interest rate shifts may change the value of the underlying collateral and interest is not necessarily available even to secured lenders. A long stay might be costly to non-safe-harbored repo debt, as it can be for many secured creditors. This difficulty might warrant amendments to the Bankruptcy Code that ensure better adequate protection of the counterparty’s interest in the collateral, as valued on the filing date.” These observations underscore the role played by the parameters $\beta$, $\hat{\beta}$ and $\alpha$. 
leverage in an otherwise identical Leland (1994) economy with a single issue of unprotected debt.

The optimal leverage in our model is generally higher than Leland (1994). This is due to the fact that the firm is able to trade off between short-term and long-term debt to optimally take advantage of any inefficiencies in the bankruptcy code. With a costly bankruptcy code, the firm will issue more short-term debt protected by safe harbor. Likewise when the firm has more eligible collateral, it can take advantage of that to increase its optimal leverage.

Note that at very low levels of asset volatility, the borrower tends to use higher levels of leverage, irrespective of the fraction of the assets that are eligible to be posted as collateral. This is due to the fact that at such low levels of $\sigma$ the borrower's ability to issue long-term debt at a lower spread is also high, and this compensates for the inability to issue short-term debt when $\bar{\theta}$ is low. This effect can also be appreciated by reviewing the optimal liability structure in the bottom panel of Figure 4, for various levels of $\bar{\theta}$. It is clear that the firm's value in our economy is higher due to the increased level of optimal leverage.

Note the liability structure of less risky firms tend to have a greater proportion of short-term debt. In addition, if the firm has a greater fraction of eligible collateral, it is able to sustain a greater proportion of short-term debt in its liability structure. We should also note that the asset volatility of financial firms tend to be much lower than those of non-financial firms. This will also, cetaris-paribus, promote the use of short-term debt, which one observes in real life.

[ Insert Figure 4 here. ]

V.a. Creditor's Rights and the Relevance of Safe Harbor Debt

In this section we examine how the liability structure and creditor's rights influence the total value of the firm. In our model, the total value of the firm, denoted by $v$ is simply the following:

$$v = S + D + E,$$
where $E$ is the value of equity, $D$ is the value of long-term debt and $S$ is the value of short-term debt. The expressions for these claims on the assets of the firm are provided in the appendix. The price $p$ of a claim that pays $\$1$ when $V$ reaches $V_B$ and zero elsewhere can be derived as:

$$p = \left( \frac{V}{V_B} \right)^x.$$  

28 We formalize the economic intuition that with a bankruptcy code that fully protects the rights of creditors, there is no role for safe harbor. Hence, we begin with the assumption that APR is strictly enforced. When APR is respected, we have $\psi_E = 0$. Using this condition in Lemma 1, and substituting in the values of $E$, $S$ and $D$, we can now write the total value of the borrower’s firm as follows:

$$v(V) = V + \frac{V_B}{\lambda} \tau (1 - p) + p V_B \left( \theta ((1 - \psi_D) - \beta) - (1 - \psi_D) \right)$$ 

(7)

Recall that when $\psi_E = 0$, $(1 - \psi_D)$ becomes the total loss from the bankruptcy, i.e., $\alpha = (1 - \psi_D)$ and $\beta$ is the cost of liquidating collateral outside of the bankruptcy code. Then the last term of the Equation (7) becomes $p V_B (\theta (\alpha - \beta) - \alpha)$. Therefore, when $\alpha = \beta$, the firm’s value is independent of $\theta$. This is in view of the fact that $p$ is independent of $\theta$ when $\alpha = \beta$, in the absence of APR violations.29 This leads to our Theorem 1, which is stated next.

**Theorem 1.** When the bankruptcy code fully protects creditor’s rights (i.e., $\psi_E = 0$) and the recovery rates to the creditor under the bankruptcy code are identical to the recovery rates when the debt is protected by safe harbor ($\beta = \alpha$), then the firm’s value is unaffected by the presence of safe harbor rights. The value of the firm is independent of the amount of the pledged collateral.

An important implication of the theorem is the following: under the hypothesized conditions of Theorem 1, the optimal level of pledged assets, $\theta^* \in [0, \bar{\theta}]$ does not affect the overall value of the firm. Since the firm is indifferent between pledging collateral with exemption from

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28 Note that in equilibrium, $V_R = V_B$, and hence the state price may also be specified in terms of the run boundary.

29 It can be shown that

$$p = \left[ \frac{\tau}{(1 + x)(\tau - \lambda \theta (1 - \beta) + \lambda (1 - (1 - \theta)(\psi_D + \psi_E)))} \right].$$

This is independent of $\theta$ under the conditions specified in Theorem 1.
automatic stay or issuing secured debt with automatic stay, the optimal liability structure cannot be uniquely determined. What we can say is that when \( \alpha = \beta \), the level of short-term debt can vary from zero to \( \bar{\theta} \). For this special case, safe harbor adds no value to the borrowing firm, relative to the case with only long-term debt. The value of the firm is the same as the one obtained by Leland (1994) for the unsecured long-term debt issuance case, irrespective of \( \theta \). This suggests that the basic framework of Leland (1994) admits a richer set of liability structures than a quick reading of the paper might imply. This point is more nuanced, when one considers the long-term credit spreads implied by structural models of default. While the total value of the firm is the same for \( \theta^* \in [0, \bar{\theta}] \), the long-term spreads implied by the model for the range of admissible values of \( \theta \) might be very different from the ones implied by the original version of Leland (1994) for the case in which only unsecured long-term debt was considered. We provide in Figure 5, the long-term spreads implied by the model for admissible values of \( \theta \), noting that the value of the firm is identical to the one found by Leland (1994) when \( \beta = 1 - \psi_D \). As \( \beta \) decreases, the use of short-term debt with exemption from automatic stay increases. This causes the long-term credit spreads to go up.

[ Insert Figure 5 here. ]

Now we examine the value of the borrowing firm when \( \psi_E > 0 \). Here, there are two cases to consider. The first case wherein \( \psi_D + \psi_E = 1 \) (or, \( \alpha = 0 \)) implies that the bankruptcy code in the background results in a restructuring process that avoids dead-weight losses. The second case is one in which \( \psi_D + \psi_E < 1 \), so that there are some dead-weight losses associated with restructuring in the shadow of the bankruptcy code. The value of the firm in this more general case is

\[
v(V) = V + \frac{V_B}{\lambda} \tau (1 - p) + pV_B \left( \theta(1 - \beta) - (1 - (1 - \theta)(\psi_D + \psi_E)) \right)
\] (8)

In Theorem 1, we pinned down the secondary market liquidity of the pledged collateral, \( \beta = 1 - \psi_D \), and assumed that \( \psi_E = 0 \). In the more general case, the secondary market liquidity is not restricted, and we admit \( \beta \in (0, 1) \). The total value of the firm in Equation (8) can be either increasing in \( \theta \) or decreasing in \( \theta \) depending on how liquid or illiquid the pledged collateral is. Hence, we must first characterize the effect of secondary market liquidity on the incentives to
VI Asset Liquidity

The liquidity of the collateral in the secondary market and its appropriateness for backing safe harbor debt, which is exempt from automatic stay, has received some discussion in the literature. Duffie and Skeel (2012) have argued the following: “.. repos [and certain closely related (Qualified Financial Contracts) – QFCs] that are backed by liquid securities should be exempt from automatic stays, or receive an effectively similar treatment. Repos backed by illiquid assets, on the other hand, should not be given this safe harbor.” In our model, we can address this question formally, from the perspective of the debtor, as opposed to the perspective of a regulator: will the borrower optimally choose to issue safe-harbored debt only when the secondary market liquidity of its eligible collateral is better than a certain liquidity threshold? Intuitively, if $\beta$ (which represents the secondary market friction parameter of the safe harbored asset in our model) is better than a certain liquidity threshold level, it might be optimal for the borrower to issue safe-harbored debt. Theorem 1 says that if there are no APR violations and the restructuring process is no more costly than liquidating collateral in the safe harbor, then the firm value is independent of the amount pledged. In fact, if the restructuring cost $1 - \psi_D = 0$ then, Theorem 1 implies that $\theta^* = 0$ as well to prevent a welfare loss. This implies that there is no place for safe harbor under such conditions.

If the bankruptcy code implies a costly restructuring ($\psi_D$ is low), then the constraint on secondary market liquidity becomes less binding. In a similar vein, if the bankruptcy code results in more APR violations, the constraint on market liquidity becomes more relaxed. This is to say that if the bankruptcy code is efficient, only very liquid assets with very low values of $\beta$ can be used as the collateral. We formalize this intuition next.

Recall that $\lambda$ is as defined in Proposition 1. Differentiating Equation (8) with respect to $\theta$, and
simplifying, we get the following result:

\[
\frac{dv}{d\theta} = V_B \left[ \frac{\tau(1-p)\psi_E}{1+x(1-\tau)} + (1-\beta)p - p(\psi_D + \psi_E) \right] 
\]

In Equation (9), the first term is positive. Equation (9) implies that when the secondary market liquidity is such that \( (1-\beta) - (\psi_D + \psi_E) \equiv \alpha - \beta > 0 \), the value of the firm increases in \( \theta \) and \( \theta^* = \bar{\theta} \). Equation (9) also shows that when \( \psi_E = 0 \), \( \frac{dv}{d\theta} = 0 \) when \( \beta \) takes a threshold value \( \underline{\beta} = 1 - \psi_D \). If \( \beta > \underline{\beta} \), then \( \theta^* = 0 \), and if \( \beta < \underline{\beta} \), then \( \theta^* = \bar{\theta} \). Let us consider now the case where \( \psi_E > 0 \). Note from Equation (9) that when \( (1-\beta) = \psi_D + \psi_E \), i.e., \( \alpha = \beta \), the value of the firm is increasing in \( \theta \), and \( \theta^* = \bar{\theta} \). This result shows the pernicious effects of the violations of APR on the incentives to use debt that is exempt from automatic stay. In fact, Equation (9) suggests that the value of the firm may be increasing for a range of \( \beta \) values when liquidation outside the bankruptcy code is costlier than liquidating collateral inside the bankruptcy code, when \( \psi_E > 0 \).

We therefore consider the case when \( \psi_E > 0 \) and \( 1 - \beta < \psi_D + \psi_E \) (i.e., \( \beta > \alpha \)). It is useful to note the following: \( \frac{dv}{d\theta}_{|\beta=0} > 0 \), and hence \( \theta^* = \bar{\theta} \). When \( \beta = 1 \), it is clear that \( V_B \to \infty \), and it is never optimal to issue short-term debt, and from Proposition 3 we have that \( S^* = 0 \). This implies that \( \theta^* = 0 \). We formalize the main implications of the secondary market liquidity on the optimal level of pledging in the next theorem.

**Theorem 2.** When APR violation exists (\( \psi_E > 0 \)), even if liquidation cost is greater than the bankruptcy loss (i.e., \( \beta > \alpha \)), it may be optimal to issue short-term debt protected by safe harbor as much as possible (\( \theta^* = \bar{\theta} \)), as long as the liquidation loss of the pledged assets does not exceed a threshold level of illiquidity \( \underline{\beta} \) (i.e., \( \beta < \underline{\beta} \)), where

\[
\underline{\beta} = \alpha + \frac{\psi_E(\alpha + \psi_D\tau)(1+x)}{(1-\psi_E)(1-\tau)}
\]

This is a stark contrast to the first case when APR is fully respected. In that case, as we showed in Theorem 1, when \( \alpha = \beta \), the firm value is independent of \( \theta \). An implication of
Theorem 2 is that when $\beta < \beta$, the firm will secure all its eligible assets for pledging in safe harbor to issue riskless short-term debt.

Equation (10) shows that when $\psi_E > 0$, the second term is positive, and the liquidity threshold can actually increase in $\psi_E$. The top panel in Figure 6 shows how the liquidity threshold interacts with the bankruptcy code. Holding $\alpha$ fixed, as we increase $\psi_E$, APR violations increase. This permits even less liquid collateral to be posted as collateral to issue short-term debt. Holding $\psi_E$ fixed, as we increase $\alpha$, it follows that $\psi_D$ must decrease. This makes pledging even relatively illiquid assets to issue short-term debt valuable.

In the bottom panel of Figure 6, we plot the relationship between APR violations and the optimal liquidity threshold for different levels of asset volatility. Here we keep $\psi_D$ fixed. Note that higher APR violations leads to a less binding liquidity constraint at first. However, when $\psi_E$ is very high, $\alpha$ approaches zero and the bankruptcy code produces very few dead-weight losses. This tightens the liquidity threshold.

Our results imply that public policy recommendations on the desirability of (or, lack thereof) safe harbor, should also take into account the nature of the bankruptcy code that is already in place. In this context, Morrison et al. (2014) have proposed a similar rollback of safe harbor rights for illiquid collateral, but they have simultaneously proposed an amendment to the bankruptcy code that will facilitate better access to collateral for short-term creditors who are subject to automatic stay. We have already expressed in Equation (8) the value of the firm as a function of $\theta$ in this general case when $\psi_E > 0$.

Thus, our model allows the exploration of the following counterfactual: suppose that legislation is passed to eliminate safe harbor rights. The borrower now has the flexibility only to issue secured debt subject to automatic stay. Under this counterfactual, when there are APR violations, i.e., $\psi_E > 0$, and the short-term secured creditors can access their collateral after the stay period at a cost $\hat{\beta}$, the borrower will have an incentive to issue secured debt inside the bankruptcy code. This is due to the fact that long-term debt is exposed to APR violations, but secured short-term debt avoids APR violations and enables short-term creditors to access their collateral at a cost $\hat{\beta}$. 

27
In Figure 7, we examine the optimality of issuing secured short-term debt within the bankruptcy code (i.e., \( \hat{\beta} \leq \alpha \)). To illustrate the point, we assume the most adverse situation where \( \hat{\beta} = \alpha \). The top panel plots the firm value as a function of \( \theta \) for different levels of APR violations. When there are no APR violations, there is no advantage to issuing short-term, secured debt within the bankruptcy code in our model (note that the value of the firm is unaffected by \( \theta \) when \( \psi_E = 0 \)). On the other hand, for \( \psi_E > 0 \), there is an advantage to issuing short-term secured debt, as the value of the firm is increasing in \( \theta \): this reflects the fact that long-term debt is adversely affected by deviations from APR, but short-term debt, by virtue of its security, is able to avoid those potential adverse effects.

The bottom panel plots the ratio of short-term debt to long-term debt as a function of \( \bar{\theta} \) for different levels of \( \psi_D \), under the assumption that there are APR violations. The greater the value of \( \psi_D \) is, the higher are the benefits of issuing short-term, secured debt.

It is also intuitive to expect that as \( \beta \) increases, there will be a point at which it is never optimal to use the safe harbor so that \( \theta^* = 0 \). This is of less economic interest, as \( \beta \) increases, the run boundary \( V_R \) becomes very large, causing the firm to issue very little short-term debt.

We further show that firms would choose the maximum level of secured short-term debt even when the inside bankruptcy cost and outside liquidation loss are identical (i.e., \( \alpha = \beta \)), so long as the code does not guarantee creditors' right (\( \psi_E > 0 \)). This result implies that, even if such losses are same, firms would circumvent APR violation by setting assets aside through the issuance of secured short-term debt. Figure 8 illustrates this result.

The figure also indicates that firm value always stays low when APR violation exists (with an exception when the entire asset can be pledged, i.e., \( \hat{\theta} = 1 \)), and that asset securing is a way to mitigate the problem of value-loss. The firm value is lower with APR violation, not because of short-term debt usage, but due to the creditor-unfriendly bankruptcy code itself.
It should also be noted that, the total recovery value of a firm is endogenous in our model: the aggregate recovery through Equations (3)-(5) and $V_B$ are endogenously chosen by $\theta$. The firm can choose the relevant composition of debt to maximize its value. However, to further distinguish the driver of our main implication, we show, in Appendix C, that the same result can be obtained when we force the total recovery value to be constant across firms’ choice of asset securing ($\theta$). This indicates that the choice of recovery value does not alone lead our results.

**VII Conclusion**

We have developed a structural model of optimal liability structure and optimal leverage in which the borrower internalizes the risk of a run by short-term lenders in choosing his liability structure and leverage. When there are violations of APR and dead-weight losses associated with the underlying restructuring process, we show that the borrower has an incentive to use safe-harbored debt such as secured repo debt. The model has predictions about how firms will change their liability structure and leverage when the rights of creditors are improved so that their collateral is exempt from automatic stay.

An important implication of the model is that the properties of the bankruptcy code play an important role in whether there will be incentives for borrowers to use the exemptions from automatic stay for issuing short-term debt. The model predicts that the introduction of safe harbor rights will lead to (a) greater use of short-term debt, (b) a lowering of the use of long-term debt, and (c) an increase in the spreads of long-term debt. The model also shows that the borrower will only pledge assets with a minimal level of secondary market liquidity to issue debt with exemption from automatic stay: this minimum level will be informed by the features of the bankruptcy code and the properties of the asset, such as its volatility.

Some caveats are in order: our model does not consider dynamic adjustments to leverage
and liability structure. This is clearly an important question. He and Xiong (2012) have considered a framework to include this possibility. We also do not allow for variations in the investment opportunity set to the borrower. Clearly, these issues warrant further research.
References


Figure 1: **Quantity adjustment of repo with respect to creditor right changes:** This figure plots broker-dealers’ repo debt position as a function of new Treasury security issuance (collateral supply). Circled markers show repo quantity before the 1984 act that granted safe-harbor provision to repo transaction has been effective. Red-filled circle markers are corresponding repo quantity between time period when there was uncertainty on bankruptcy treatment (between 1982Q3 and 1983Q2). Triangled markers represent repo quantity after the act is effective. Within each marker type, the darker color indicates the later time in each period.

**Interpretation:** Legal interpretation of creditors’ rights upon bankruptcy affects the usage of repo contracts, after adjusting for the supply of collateral: when creditors’ rights are enhanced, repo usage increases.
Figure 2: **Illustration of choosing $V_B$ and $S$ for a given $C$**: This figure illustrates the equity holder's optimal choice of restructuring boundary and the level of short-term debt. For a given level of $C$, the equity holder chooses $V_B$ and $S$. The solid line with steeper slope is $V_R$ imposed by the short-term lender. The solid line with flatter slope is $V_B$, the optimal restructuring boundary associated with each value of $S$. The dotted line is 45 degrees line. With a given $C$, the optimal choice is to pick $S$ and $V_B$ at the point where two solid lines cross. In this figure, $C = 0.045$, $\theta = 0.8$, $\psi_D = 0.7$ and $\psi_E = 0$ is assumed. Also the following deep parameter values have been assumed: $V_0 = 1$, $r = 0.04$, $\delta = 0.02$, $\tau = 0.35$, $\sigma = 0.25$, $\beta = 0.05$.

**Interpretation**: The level of optimal repo cash borrowing is chosen by the borrower such that the optimal point at which the repo cash lenders decide to run coincides with the point at which the borrower optimally decides to default.
Figure 3: **Effect of APR violations and eligibility to pledge on optimal liability structure:** These figures plot short-term debt to long-term debt ratio \((S/D)\) with respect to the APR violation, \(\psi_E\) (Panel (a)) and eligibility to pledge, \(\bar{\theta}\) (Panel (b)). The top figure is with three different value of \(\bar{\theta} = 0.525\) (solid), 0.5 (dashed) and 0.475 (dotted), keeping \(\psi_D = 0.5\). The bottom figure uses three different value of \(\psi_D = 0.9\) (solid), 0.5 (dashed) and 0.1 (dotted), keeping \(\psi_E = 0.0\). For both figures, the following deep parameter values have been assumed: \(V_0 = 1, r = 0.04, \delta = 0.02, \tau = 0.35, \sigma = 0.5\) and \(\beta = 0.05\).

**Interpretation:** More short-term debt with enhanced creditors’ rights is used when the bankruptcy code does not protect creditors’ rights and when the borrowing firm has more assets to pledge as collateral.
Figure 4: **Effect of asset volatility on optimal liability structure and leverage:** These figures plot short-term debt to long-term debt ratio (top) and total leverage of a firm (bottom) with respect to the asset volatility ($\sigma$). The solid line uses $\theta = 0.8$ and the dashed and dotted line use $\theta = 0.6$, $\theta = 0.4$ respectively. In the bottom pane, thicker line represents the leverage in Leland (1994) with only unprotected long-term debt, keeping other parameters same. For all three lines, the following deep parameter values have been assumed: $V_0 = 1$, $r = 0.04$, $\delta = 0.02$, $\tau = 0.35$, $\beta = 0.05$, $\psi_D = 0.5$ and $\psi_E = 0.1$.

**Interpretation:** Allowing short-term borrowing with enhanced creditors’ rights results in higher leverage, compared to Leland (1994) benchmark.
Figure 5: **Long-term spread by asset pledging:** This plot displays the spread of long-term debt with respect to the pledging action ($\theta$). Here, three levels of liquidity cost ($\beta$) are considered: $\beta = 0.3$ (dotted), $\beta = 0.4$ (dashed), and $\beta = 0.5$ (solid). We assume the following bankruptcy code parameters: $\psi_D = 0.5$, $\psi_E = 0.1$. Also, the following deep parameter values have been assumed: $V_0 = 1$, $r = 0.04$, $\delta = 0.02$, $\sigma = 0.5$ and $\tau = 0.35$.

**Interpretation:** Rational unsecured long-term lenders internalize the level of credit-enhanced short-term debt issued by the firm, ex-ante, and demand a higher credit risk premium.
Figure 6: **Constraint on liquidity of pledged asset to justify safe harboring activity:** These figures visualize the minimum level of liquidity that collateralized asset should satisfy for safe harboring to be beneficial. The top figure represents the minimum liquidity constraints ($\beta$) with respect to APR violation ($\psi_E$). With asset volatility $\sigma = 0.5$, three different levels of $\psi_E$ are considered: $\psi_E = 0.0$ (dotted), $\psi_E = 0.1$ (dashed), and $\psi_E = 0.2$ (solid). The bottom figure represents the illiquidity threshold in the APR violation ($\psi_E$) and total cost of bankruptcy ($\alpha$). Here, $\psi_D$ is fixed at 0.5. Therefore, moving $\psi_E$ from 0 to 0.5 implies higher APR violation and more efficient bankruptcy code. Three different levels of asset volatility ($\sigma$) are considered: $\sigma = 0.2$ (dotted), $\sigma = 0.2$ (dashed), and $\sigma = 0.2$ (solid). For both, the following deep parameter values have been assumed: $V_0 = 1$, $r = 0.04$, $\delta = 0.02$ and $\tau = 0.35$.

**Interpretation:** When the bankruptcy code permits APR violations, ex-post, the borrowing firm may have an incentive to pledge even illiquid collateral under safe harbor.
Figure 7: **Case of secured but not safe harbored debt:** These plots describe the case when the short-term debt is secured debt but not safe harbored. Since the short-term debt is under the bankruptcy code, they share the bankruptcy cost but there is no APR violation because it is secured. Panel (a) shows that firm value is increasing in $\theta$ when $\psi_E > 0$. Solid and dashed line use $\psi_E = 0.2, 0.1$, respectively and 0 for dotted line. Panel (b) is the corresponding debt structure with respect to $\theta$ when $\psi_E = 0.2$ with three different values of $\psi_D = 0.3$ (dotted), 0.4 (dashed) and 0.5 (solid). For both figures, other parameters are used as follows: $V_0 = 1$, $r = 0.04$, $\delta = 0.02$, $\tau = 0.35$ and $\sigma = 0.25$.

**Interpretation:** Asset pledging and issuance of short-term debt achieve a higher firm value when APR violation is allowed.

(a) Firm value with respect to asset pledge ($\theta$)

(b) Debt structure with respect to pledgibility ($\bar{\theta}$)
Figure 8: **Firm value under APR violation with identical inside and outside loss:** This plot shows the firm value with respect to a firm’s choice $\theta$, keeping the inside and outside bankruptcy cost identical (i.e., $\alpha = \beta$). The firm value is presented with three different levels of APR violation ($\psi_E = \{0.0\ (dotted), 0.1\ (dashed), 0.2\ (solid)\}$), keeping $\alpha \equiv 1 - (\psi_E + \psi_D) = \beta = 0.5$ to highlight the effect of creditor-friendliness of the bankruptcy code. Other parameters are used as follows: $V_0 = 1$, $r = 0.04$, $\delta = 0.02$, $\tau = 0.35$ and $\sigma = 0.25$.

**Interpretation:** The borrowing firm may use safe harbored short-term debt, even when liquidation losses within the bankruptcy code and under safe harbor are the same so long as there are APR violations.
Table 1: **Summary statistics:** This table presents summary statistics of SDC corporate bond database in our sample period (1980-1990). The top panel corresponds to individual bond level information. Yield is yield to maturity and we augment missing yield from the database when we have enough information (coupon, price, etc.) to calculate the yield. Spread is yield minus reference yield of the Treasury securities. When spread is missing, and we can calculate the yield, we calculate the spread from yield of a duration-matched Treasury security at the quarter of issuance. Time to maturity is in year. Duration is Macaulay duration. Coupon is in percentage point. Rating is unified coarse rating which is number coded (AAA=1, AA=2, A=3, BBB=4, BB=5, B=6, CCC=7, CC=8). The mean or median rating indicates average rating in the sample period is between A and BBB. Callability is the fraction of callable security. Financial Bond is the fraction of bond issue from financial borrowers (SIC 6000 firms).

(I) Bond level Data

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Table 2: **Difference-in-differences analysis on spread within rating class:** This table presents the regression analysis specified in Equation (1):

\[
Spread_{i,t} = \alpha_t + \alpha_i + \beta_1 \cdot Fin_i + \beta_2 \cdot Post_t + \beta_3 \cdot Fin_i \cdot Post_t + \lambda \cdot Leverage_i + \varepsilon_{i,t},
\]

where \( Fin \) is a dummy variable which is 1 if the issuer is financial borrower (SIC6000), otherwise zero. \( Post \) is 1 if the timing is after the safe harbor is granted (after the regulation passed the Congress at 1983Q2). Quantity is the total issue quantity per quarter per issuer. As described in Table 1, Rating is unified and number-coded coarse rating. Specification (1) has only Rating fixed effects. Specification (2) has Rating fixed effect and the issue quantity is controlled for. Specification (3) has both Time and Rating fixed effect and the issue quantity is controlled for. Asterisks denote statistical significance at the \( 0.01(***), 0.05(**) \) and \( 0.1(*) \) levels.

**Interpretation:** Rational unsecured long-term lenders internalize the level of credit-enhanced short-term debt issued by the firm due to the regulation change, ex-ante, and demand a higher credit risk premium.

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\( t \) statistics in parentheses
* \( p<0.10 \), ** \( p<0.05 \), *** \( p<0.01 \)
Appendix: Proofs

Proof of Proposition 1

Starting from Equation (2), we first find valuations for equity claim. The value of equity satisfies the following ODE:

$$\frac{1}{2}\sigma^2 E_{VV} V^2 + (r - \delta) E_E V - r E + \delta V - (C + Sr)(1 - \tau) = 0. \quad (a.1)$$

The boundary conditions are:

$$E(V \uparrow \infty) = V - (C/r + S)(1 - \tau)$$
$$E(V_B) = (1 - \theta) \psi E_V$$
$$E_V(V_B) = (1 - \theta) \psi_E$$

The general solution of the Equation (a.1) is known as

$$a_0 + a_1 V + a_2 V^{-x}$$

where $x > 0$ is the root of the characteristic equation below:

$$\frac{1}{2} x^2 \sigma^2 - x \left( r - \delta - \frac{\sigma^2}{2} \right) - r = 0.$$ 

Using the boundary conditions, we determine $a_0, a_1, a_2$ and $V_B$. Defining $p \equiv (V/V_B)^{-x}$, we obtain the valuation function for the equity, $E(V)$:

$$E(V) = V - (C/r + S)(1 - \tau)(1 - p) - (1 - (1 - \theta)\psi_E)V_Bp \quad (a.2)$$

$V_B$ can be determined from maximizing Equation (a.2) such that $\frac{\partial E}{\partial V}|_{V=V_B} = 0$, yielding the expression in the lemma:

$$V_B = \lambda \left( \frac{C + Sr}{r} \right).$$
where \( \lambda = (1 - \tau)(\frac{\tau}{1+\tau})\frac{1}{1-(1-\theta)\psi_E} > 0 \). Note that when \( \psi_E = 0 \), \( V_B \) and \( p \) are independent of \( \theta \).

□

**Proof of Proposition 2**

The optimal level of two types of debt is determined to maximize the firm value. Therefore, we need to find valuation for the levered firm. The proof for Proposition 1 gives us equity valuation. We need to find valuations for the rest of claims: long-term and short-term debt. First, we solve the ODE satisfied by the value of long-term debt:

\[
\frac{1}{2}\sigma^2D_{VV}V^2 + (r - \delta)V_D - rD + C = 0,
\]

with boundary conditions

\[
D(V \uparrow \infty) = C/r \\
D(V_B) = (1-\theta)\psi_D V_B.
\]

The second boundary condition specifies that short term debt is secured by a fraction \( \theta \) of assets and short-term lenders have seniority to long-term debt holders. The debt value is:

\[
D(V) = C/r(1-p) + p(1-\theta)\psi_D V_B.
\] (a.3)

Next, the valuation function for the short-term debt, \( B \), is always \( S \) due to the fact that the short-term lender can always recover the full amount, i.e.,

\[
B(V) = S.
\] (a.4)

Then, we add up Equation (a.2), (a.3), and (a.4) to compute the firm value, \( v \):

\[
v(V) = E(V) + D(V) + B(V) \\
= V + \tau(C/r + S)(1-p) + SpV_B + p(1-\theta)(\psi_D + \psi_E)V_B.
\]
We can rewrite the expression for the value of the firm as
\[
v(V) = V + \tau \left( \frac{C}{r} + S \right)(1 - p) + Sp - pV_B[1 - (1 - \theta)(\psi_D + \psi_E)]. \tag{a.5}
\]

Now, we investigate the relationship between the default boundary \(V_B\) and run boundary \(V_R\). We can have one of three cases: \(V_B > V_R\), \(V_B < V_R\), and \(V_B = V_R\), which we exam each of them in the following.

**Case 1: \(V_B > V_R\)**

First, we investigate the case, when \(V_B > V_R\). Taking the derivative of the firm’s value in Equation (a.5) with respect to \(C\) and setting it to zero, we get:
\[
\frac{\partial v}{\partial C} = \frac{\tau (1 - p)}{r} - \frac{\tau xp}{r} + \frac{Sxp\lambda}{rV_B} - \frac{[1 - (1 - \theta)(\psi_D + \psi_E)](1 + x)p\lambda}{r} = 0, \tag{a.6}
\]
where \(V_B\) and \(\lambda\) are defined in Proposition 1. Also, with respect to \(S\), we get:
\[
\frac{\partial v}{\partial S} = \tau(1 - p) - xp\tau + p + \frac{Sxp\lambda}{V_B} - \frac{[1 - (1 - \theta)(\psi_D + \psi_E)](1 + x)p\lambda}{1}. \tag{a.7}
\]

Using Equation (a.6) and (a.7), we get the following implication:
\[
\frac{\partial v}{\partial S} = p > 0
\]

This implies that the firm will have an incentive to increase \(S\) and thereby implying that \(V_B > V_R\) is not an equilibrium.

**Case 2: \(V_B < V_R\)**

Next, we investigate the case, when \(V_B < V_R\). For a given short-term debt \(S\), it is easy to find the expression for run boundary and its state-contingency price: \(V_R \equiv \frac{s}{(1-\beta)p}\) and \(p = \left(\frac{V}{V_R}\right)^{-x}\).
Since \( V_R \) will be breached earlier than \( V_B \), the value of the firm in this case is:

\[
v(V) = V + \tau \left( \frac{C}{r} + S \right) (1 - p) + Sp - pV_R[1 - (1 - \theta)(\psi_D + \psi_E)]
\]

Taking the derivative of the above firm’s value with respect to \( S \), and setting it equal to zero, we get:

\[
\frac{\partial v}{\partial S} = \tau(1 - p) - \tau \left( \frac{C}{r} + S \right) \frac{xp}{S} + p(1 + x) - [1 - (1 - \theta)(\psi_D + \psi_E)] \frac{(1 + x)p}{(1 - \beta)\theta} = 0 \quad (a.8)
\]

Taking the derivative of the firm’s value with respect to \( C \), we get:

\[
\frac{\partial v}{\partial C} = \frac{\tau(1 - p)}{r} > 0. \quad (a.9)
\]

Equation (a.8) and (a.9) mean that the firm will have an incentive to increase \( C \) and thereby implying that \( V_B < V_R \) is also not an equilibrium. From the two cases above, we conclude that \( V_B = V_R \) is the unique equilibrium. □

**Proof of Lemma 1**

Result of Proposition 2 immediately yields:

\[
V_B = \lambda \left[ \frac{C}{r} + S \right] = V_R = \frac{S}{\theta(1 - \beta)}. \quad (a.10)
\]

This obtains:

\[
\frac{C}{r} = S \frac{1 - \lambda \theta(1 - \beta)}{\lambda \theta(1 - \beta)}. \quad (a.11)
\]

Using Equation(a.10) and (a.11) in Equation (a.5) we can express the value of the firm as follows:

\[
v(V) = V + \frac{S\tau}{\lambda \theta(1 - \beta)} - pS \left[ \frac{\tau}{\lambda \theta(1 - \beta)} - 1 + \frac{1 - (1 - \theta)(\psi_D + \psi_E)}{\theta(1 - \beta)} \right].
\]
Differentiating the value of the firm with respect to $S$ and setting it to zero, we obtain:

$$\tau = p(1+x) [\tau - \lambda \theta (1-\beta) + \lambda \{1 - (1-\theta)(\psi_D + \psi_E)\}].$$

Rearranging the above expression yields:

$$S^* = V\theta (1-\beta) \left[ \frac{\tau}{(1+x)(\tau - \lambda \theta (1-\beta) + \lambda \{1 - (1-\theta)(\psi_D + \psi_E)\})} \right]^{\frac{1}{x}}. \quad (a.12)$$

Using this expression for $S^*$ in Equation (a.12) obtains the proposition:

$$C^* = rV \frac{1 - \lambda \theta (1-\beta)}{\lambda} \left[ \frac{\tau}{(1+x)(\tau - \lambda \theta (1-\beta) + \lambda \{1 - (1-\theta)(\psi_D + \psi_E)\})} \right]^{\frac{1}{x}}. \quad \square$$

**Proof of Theorem 2**

We start from our expression of $C^*$ and $S^*$ in Lemma 1. Using our definition of $\alpha$, we can rewrite them as follows:

$$C^* = rV \left( \frac{1 - \lambda \theta (1-\beta)}{\lambda} \left[ \frac{\tau}{(1+x)(\tau - \lambda \theta (1-\beta) + \lambda \{1 - (1-\theta)(\psi_D + \psi_E)\})} \right]^{\frac{1}{x}} \right) \quad (a.13)$$

$$S^* = V\theta (1-\beta) \left[ \frac{\tau}{(1+x)(\tau + \lambda (\alpha - (\alpha - \beta)\theta))} \right]^{\frac{1}{x}}. \quad (a.14)$$

Using the expression for $C^*$, $S^*$ in Equations (a.13), (a.14) and the solution for $V_B$ in Equation (a.10), we can find $p^*$ as follows:

$$p^* = \frac{\tau}{(x+1) \left( \frac{1 - \tau x (\alpha (1-\theta) + \beta \theta)}{x+1} \right) p^* + (\alpha - \beta) p^*}. \quad \text{(48)}$$

When we rewrite the Equation (9) in the paper, we have

$$\frac{\partial v}{\partial \theta} = V_B \left[ \frac{\tau \psi_E}{x+1(1-\tau)} + \left( \frac{\alpha - \beta}{x+1(1-\tau)} \right) p^* \right].$$

$$= V_B \left[ \left( \frac{\tau \psi_E}{x+1(1-\tau)} \right) (1-p^*) + (\alpha - \beta) p^* \right].$$
By observing the equation, it is easy to check $\frac{\partial v}{\partial \theta}$ is always positive if $\alpha > \beta$. Hence, if $\alpha > \beta$, it will be always better off to pledge as much as a firm can: $\theta^* = \bar{\theta}$. However, $\frac{\partial v}{\partial \theta}$ can be negative with some high level of $\beta > \alpha$.

To find level of $\beta$ that switches the sign of the above derivative, we first find the first order condition $\partial v / \partial \theta = 0$. Since $V_B > 0$, it is zero if and only if

$$p^* = \left( \frac{V}{V_B} \right)^{-x} = \frac{\tau \psi_E}{\tau \psi_E + \frac{x}{1+x}(\alpha - \beta)(1 - \tau)}.$$ 

Equating the above two equations for $p^*$ and and solve for $\theta$, we have the expression of $\hat{\theta}^*$ that makes $\partial v / \partial \theta = 0$:

$$\hat{\theta}^* = \frac{\alpha(1 - \tau)(1 + \psi_E x) + (1 - \psi_E)(\psi_E \tau(x + 1) - \beta(1 - \tau))}{\psi_E(x)((\alpha - \beta)(1 - \tau) - \psi_E \tau) - \psi_E \tau).$$

Next step is to find the second derivatives of $v$ with respect to $\theta$: $\frac{\partial^2 v}{\partial \theta^2}$:

$$\frac{\partial^2 v}{\partial \theta^2} = \frac{V(1 - \tau)(x + 1)(\beta(1 - \psi_E) - \alpha)\tau^2}{h(\theta)^2} \left( \frac{1}{h(1 - \psi_E(1 - \theta))} \right)^{\frac{x + 1}{\tau}} h(\theta)^\frac{1}{\tau},$$

where $h(\theta) = \tau[(1 - \psi_E) + \psi_E \theta + x(\psi_D(1 - \theta) + \theta(1 - \beta))] + x\alpha(1 - \theta) + x\beta\theta$.

We can see that $h(\theta)$ is positive, hence $\frac{\partial^2 v}{\partial \theta^2}$ is also positive in any cases. This means that there is an unique minimum point of $v$ with respect to $\theta$. Therefore, the $\hat{\theta}^*$ expression we found in the first order constraint above pick up the level of $\theta$ that minimizes the firm value, $v$. Note that this $\hat{\theta}^*$ is not constrained by any range of $\theta$. However, by construction, the $\theta$ can take a value from $[0, \bar{\theta}]$. Note is that the $\hat{\theta}^*$ is monotonically increasing in $\beta$. We can check this from the following:

$$\frac{\partial \hat{\theta}^*}{\partial \beta} = \frac{(1 + x)(1 - \tau)(x(\alpha + \psi_D \tau) + \tau(1 - \psi_E))}{(\psi_E \tau + x(\beta - \alpha + \tau - (\psi_D + \beta) \tau)^2} > 0.$$ 

Hence, if $\beta$ is at some level of $\beta$, say $\underline{\beta}$, then the $\hat{\theta}^*$ that minimizes the firm value will be equal to zero. Hence, for any $\beta < \underline{\beta}$, $\hat{\theta}^*$ will lie left side of 0 which is the minimum possible value of $\theta$ and $\frac{\partial v}{\partial \theta} > 0$ for the range of any $\theta \in [0, \bar{\theta}]$. This implies $\theta^* = \bar{\theta}$ for $\beta < \underline{\beta}$. 

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To find the expression for $\beta$, we first find an expression of $\beta$ that makes $\frac{\partial v}{\partial \theta}$ equal to 0:

$$\beta = \frac{\tau \psi_E (\psi_E + (1 - \psi_D) x) \bar{\theta} + \psi_E (1 - \psi_E + \psi_D x) + \alpha \psi_E (1 - \bar{\theta}) x}{(1 - \psi_E (1 + \theta x))(1 - \tau)}.$$

We now pin down the threshold level of illiquidity ($\bar{\beta}$) by replacing $\bar{\theta}$ of the above equation with 0, which is the minimum possible value of $\bar{\theta}$ and obtains the theorem:

$$\bar{\beta} = \alpha + \frac{\psi_E (1 + x) (\alpha + \tau \psi_D)}{(a - \psi_E)(1 - \tau)}.$$

This result implies that, whenever $\beta < \bar{\beta}$, $\frac{\partial v}{\partial \bar{\theta}} > 0$ for any $\theta \in [0, \bar{\theta}]$, hence $\theta^* = \bar{\theta}$. It can be further understood in two angles. First, when $\psi_E$ is zero, then $\bar{\beta}$ collapses to $\alpha$, recovering our result in Theorem 1. Second, for a given $\alpha$, as we allow APR violation ($\psi_E > 0$), the $\beta$ becomes larger than $\alpha$. This is intuitive because pledging assets becomes a better solution to avoid the APR violation, so the liquidity constraint is less binding. □
Online Appendix
A Model Extension

A.a. Optimal Interior $\theta^*$

In our formulation above, we have assumed that pledging a fraction $\theta$ of the assets to borrow short term from lenders who are protected by safe harbor provisions, does not impose any additional costs on long-term lenders and equity holders, other than the unavailability of the collateral that was pledged. This is clear from the specifications of Equation (3), (4) and (5).

It may be more realistic to specify that the payoffs to long-term lenders and equity holders may be adversely impacted by the roll risk of repo cash lenders, leading to a bankruptcy outcome. To allow for this possibility, we can modify the payoff functions $(1 - \theta)$ in Equation (4) and (5) to an arbitrary $f(\theta)$ as follows.

When a fraction $\theta$ is pledged, the costs are reflected in what the remaining creditors and equity holders will get under restructuring.

\[
D(V_B) = f(\theta) \cdot \psi_D V_B \\
E(V_B) = f(\theta) \cdot \psi_E V_B
\]

where, we place the following restrictions on the $f(\cdot)$ function: $f' < 0$, $f(\bar{\theta}) = 0$, and $f(0) = 1$. Intuitively, we are saying that as higher amounts are pledged to issue short-term debt with safe harbor privileges, there may be additional costs imposed on the remaining creditors of the issuing firm. For example, if a run by short-term lenders were to occur, the ensuing panic may cause the recovery for the remaining claimants in the resulting bankruptcy to be far lower.

It turns out, that our analysis can be derived with this more general specification. With a
more general specification sketched in this section, we can prove the following results:

\[ S^* = V\theta(1-\beta)\left[\frac{\tau}{(1+x)(\tau-\hat{\lambda}\theta(1-\beta)+\hat{\lambda}\{1-f(\theta)(\psi_D+\psi_E)\})}\right]^{\frac{1}{\hat{\lambda}}} \] (A.15)

\[ C^* = rV\frac{1-\hat{\lambda}\theta(1-\beta)}{\lambda}\left[\frac{\tau}{(1+x)(\tau-\hat{\lambda}\theta(1-\beta)+\hat{\lambda}\{1-f(\theta)(\psi_D+\psi_E)\})}\right]^{\frac{1}{\hat{\lambda}}} \] (A.16)

\[ v(V) = V + \frac{S\tau}{\lambda\theta(1-\beta)} - pS\left[\frac{\tau}{\lambda\theta(1-\beta)} - 1 + \frac{1-f(\theta)(\psi_D+\psi_E)}{\theta(1-\beta)}\right] \] (A.17)

where,

\[ \hat{\lambda} = (1-\tau)\left(\frac{x}{1+x}\right)\frac{1}{1-f(\theta)\psi_E} > 0 \]

With the results above, it is straightforward to characterize the value-maximizing choice of \( \theta^* \). We had earlier chosen the simple specification \( f(\theta) = 1-\theta \) to get tractable solutions, but the main qualitative implications carry through for the more general functional form \( f(\theta) \).

However, in this subsection, we showcase that the interior \( \theta \) can be obtained with a specific functional form of \( f(\theta) \). Let us consider the following specification:

\[ f(\theta) = \frac{\log(1+\bar{\theta}-\theta)}{\log(1+\bar{\theta})} \] (A.18)

The function satisfies the conditions above: \( f' = \frac{-1}{(1+\bar{\theta}-\theta)\log(1+\bar{\theta})} \) and \( f(\bar{\theta}) = 0 \). Note that \( f'' = \frac{f'}{(1+\bar{\theta}-\theta)} < 0 \). This indicates that the cost of pledging imposed on the recovery is convex in \( \theta \). Using Equation (A.15), (A.16) and this function \( f(\theta) \), we express the firm value \( v \) in Equation (A.17) in \( \theta \). The equity holder optimizes the \( \theta \in (0,\bar{\theta}) \) by maximizing the firm value \( v \). With a standard set of parameters, the following figure illustrates that \( v \) is hump-shaped in \( \theta \). \( \theta^* \) is found at the maximum level of \( v \).

[ Insert Figure A.1 here. ]

We summarize the most relevant results here:
1. With higher degree of APR violations, keeping total bankruptcy loss fixed, firms pledge more assets and avoid the costs of APR violations.

2. If the firm has more volatile assets (i.e., $\sigma$ is high), the firm pledges less collateral. This result can be interpreted as follows. As $\sigma$ increases, the default boundary would go down. This is because the equity holder would not want to give up the firm sooner when the asset value of the firm increases. This gives the equity holder an incentive to delay the run by the short-term lender. As a result, the firm takes less short-term debt by pledging fewer asset.

3. With an interior solution of $\theta^*$, the minimum required liquidity $\beta$ shown in Equation (10) can be analyzed in a similar context but with slightly different definition: $\beta$ is such that $\theta^* = 0$. In other words, when the collateral asset does not satisfy the minimum liquidity constraints, that is $\beta > \beta$, the firm optimally pledges nothing. As in the simpler case, with more severe APR violation, the admissible set of collateral is larger in terms of the required liquidity (higher $\beta$).

In this subsection, we have used a non-linear function, $f(\theta)$, to obtain an interior solution for $\theta^*$, which is the optimal level of assets that the borrower will pledge. The use of a non-linear function $f(\theta)$, however, reduces the tractability of the model, which is the reason why we have used a linear function $f(\theta)$ for the main results.

A.b. Micro-founding Restructuring and Sharing Rules

In the formulation above, we operated in a reduced-form setting and did not establish a direct link between the sharing rules proposed in the restructuring rules in Equation (4) and (5) and the provisions of the underlying code as discussed by Mella-Barral (1999), François and Morellec (2004), and in Broadie et al. (2007).

We establish the micro foundations in two ways. First, we demonstrate the link between the sharing rules and the parameters of the bankruptcy code, by using the approach of François
and Morellec (2004). This approach illustrates how the provisions of the bankruptcy code influence the choice of the restructuring boundary and the payoffs to borrowers and lenders at the boundary. Second, we formulate a simple bargaining game to show that the sharing rules can be derived endogenously.

A.b.1. Sharing Rules and the Bankruptcy Code

Two key parameters of the bankruptcy code that are modeled by François and Morellec (2004), and Broadie et al. (2007) are the following: (a) the length of the automatic stay, denoted as $d$, and (b) the flow rate of costs, $\phi$, associated with the firm being in the Chapter 11 process, attempting to restructure its loans. The parameter $\phi$ captures the rate at which the Chapter 11 process is dissipating the resources of the firm per unit time. Using the approach of François and Morellec (2004), we can also link the parameter $\psi_E$ to these parameters and the bargaining power $\eta$ of the borrowers as implied by the provisions of the bankruptcy code as in Fan and Sundaresan (2000).

In Figure A.2, we plot the implied $\psi_E$ for different values of $d$ in years, and for different bargaining powers $\eta$ (top panel) and for different flow rate of costs (dissipative dead-weight losses of being in the Chapter 11 process) $\phi$ (bottom panel). Note that if the bankruptcy code allows the borrower to remain in the Chapter 11 process for a long period with automatic stay in effect, then the implied deviations from APR can be rather high. As seen in the top panel, for greater bargaining power of the borrower, the implied violations of APR are higher. The bottom panel shows that, as the costs associated with the Chapter 11 process decrease, there is more room for a shareholder's strategic behavior, hence the APR violations increase. These are fairly intuitive conclusions.

[ Insert Figure A.2 here. ]

Thus, Figure A.2 illustrates the manner in which the length of the automatic stay ($d$) and frictional costs in the Chapter 11 process ($\phi$) manifest themselves in our model through $\psi_E$. 
A.b.2. A Simple Bargaining Game to Endogenize Sharing Rules

Beyond mapping our results to the existing literature, we provide a micro-foundation for our sharing rule in this subsection under the feature of the Chapter 7 provision. For simplicity, we do not consider a restructuring game in which the sharing rules are tied to the value of the firm under a new reorganization plan (we believe that such a formulation will complicate the analysis, without necessarily altering the major results).

We start by noting that short-term lenders under safe harbor provision are not subject to any APR violations because they have the collateral. Moreover they can stop the lending before the equity holder tries to renegotiate. Therefore, as in our main model in Equation (4) and (5), the sharing rule is only relevant to the equity holder and long-term lender, after the short-term lenders first take $\theta$ fraction of the asset. As a credible threat, if long-term creditors do not negotiate, the equity holder can take the firm to the court, and there will be a recovery cost associated with the Chapter 7 provision for long-term lenders. Specifically, we assume that a fraction $\gamma \in (0, 1)$ is lost. Hence, the recovery for long-term debt holder is only $(1 - \gamma)(1 - \theta)V_B$. In the presence of this threat, we show how the sharing between the equity holder, and long-term creditors arises from the following game.

First, upon a successful renegotiation, a fraction $\mu$ goes to the equity holder and a fraction $1 - \mu$ goes to the long-term creditor out of the available firm’s asset $(1 - \theta)V_B$. The renegotiation is however costly: there is a fixed cost of renegotiation, $K$. Therefore, the continuation value is $\mu((1 - \theta)V_B - K)$ for the equity holder and $(1 - \mu)((1 - \theta)V_B - K)$ for the long-term creditor. Therefore incremental value upon a renegotiation is $\mu((1 - \theta)V_B - K)$ for the equity holder and $(1 - \mu)((1 - \theta)V_B - K) - (1 - \gamma)(1 - \theta)V_B$ for the long-term creditors.

The equity holder and long-term creditor now enter into a Nash bargaining game, determining the shares to the equity holder, $\mu$. For a given bargaining power of the equity holder over the long-term creditor $\eta \in (0, 1)$, the game is specified as below:

$$
\mu^* = \arg \max_\mu \left[ \mu((1 - \theta)V_B - K) \right]^\eta \cdot \left[ (1 - \mu)((1 - \theta)V_B - K) - (1 - \gamma)(1 - \theta)V_B \right]^{1-\eta}
$$
The solution for the above problem is:

\[ \mu^* = \eta \left[ 1 - (1 - \gamma) \frac{(1 - \theta) V_B}{(1 - \theta) V_B - K} \right] \]

The degree of APR violation is measured by the value of \( \mu^* \). If the equity holder can extract larger economic rent from the long-term creditor (higher \( \mu^* \)), it precisely implies a more severe APR violation. In order to provide a mapping from this micro-founded result to our simplified specification in Equation (4) and (5), we express \( \psi_D \) and \( \psi_E \) with the deep parameters in this game.

\[ \psi_D = \frac{(1 - \theta) V_B - K}{(1 - \theta) V_B} - \eta \left[ \frac{(1 - \theta) V_B - K}{(1 - \theta) V_B} - (1 - \gamma) \right] \]  
(A.19)

\[ \psi_E = \eta \left[ \frac{(1 - \theta) V_B - K}{(1 - \theta) V_B} - (1 - \gamma) \right] \]  
(A.20)

Since we operate in a general context where renegotiation is considered as a credit event, we can also provide an expression for the bankruptcy loss parameter \( \alpha = 1 - (\psi_D + \psi_E) \) in our main model.

\[ \alpha = \frac{K}{(1 - \theta) V_B} \]

Note that the necessary assumption to justify the feasibility of this game is the loss from renegotiation is smaller than the loss incurred by the Chapter 7 provision: \( \gamma > \alpha \). This implies that \( \psi_E \) in Equation (A.20) and the second term of \( \psi_D \) in Equation (A.19) are positive.

Observation of \( \psi_E \) in Equation (A.20) provides several implications: the degree of APR violation increases as the threat of Chapter 7 (higher \( \gamma \)) is larger, as the equity holder’s negotiation power is higher (larger \( \eta \)), and it decreases as renegotiation is more costly (higher \( K \)). These implications are compatible with ones in the previous subsection where we use features of the Chapter 11.
Figure A.1: **Firm value with respect to \( \theta \) with \( f(\theta) \) in Equation (A.18):** This plot showcases that an interior solution for \( \theta \) can be found with a specific functional form of \( f(\theta) \) as in Equation (A.18). The following parameters are used: \( V_0 = 1, r = 0.04, \delta = 0.02, \sigma = 0.25, \tau = 0.35, \beta = 0.01, \psi_E = 0.2, \psi_D = 0.5 \) and \( \bar{\theta} = 1.0 \).
Figure A.2: **Illustrations of APR violations arising from the provisions of the bankruptcy code:** These plots depict how our reduced form of APR violation parameter, $\psi_E$, is linked to the characteristics of the bankruptcy code at a more fundamental level, using the framework of Francois and Morellec (2004). Both figures map the length of automatic stay of the code to the parameter $\psi_E$, which captures APR violations. Panel (a) varies the bargaining power of share holder by using $\eta = 0.7$ (solid), 0.6 (dashed) and 0.5 (dotted), keeping $\phi = 0.03$. Panel (b) varies the cost of being in the Chapter 11 by using $\phi = 0.03$ (solid), 0.02 (dashed) and 0.01 (dotted), keeping $\eta = 0.5$. For both figures, total bankruptcy loss is fixed at 0.2 and other parameters are used as follows: $V_0 = 1$, $r = 0.04$, $\delta = 0.02$, $\tau = 0.35$, $\sigma = 0.25$, $\beta = 0.05$. 

(a) APR, stay period ($d$) and bargaining power ($\eta$) 

(b) APR, stay period ($d$) and Ch.11 cost ($\phi$)
Why are Short-term Debt Safe Harbored?

The decision to make short-term debt (specifically, repo) was made by the Congress, as noted earlier. In this section, we motivate why typically only short-term debt with a few days to maturity are issued with the safe harbor protection in real life, from the perspective of the borrower. The reasoning is formalized below.

The borrower can issue short-term debt and pledge collateral outside the automatic stay. The short-term creditor can refuse to roll over at each instant unless two conditions are satisfied. First, the borrower must make the contractual interest payments. Second, the borrower must post sufficient collateral so that the loan is always risk-free. If either of these conditions fails, the lender can refuse to roll over the debt. This is a powerful/credible threat that the short-term creditor has which will then incentivize the borrower to honor the contractual commitments.

We can now imagine the borrower issuing long-term debt with safe harbor provision. In this case the borrower must pay at each instant the contractual coupon and ensure that the collateral is always adequate to make the long-term debt risk-free. In view of the fact that the long-term debt holder has already committed to provide a loan for a long maturity, he lacks the ability of the short-term debt holder who can refuse to roll over the loan. Of course, the long-term debt holder can write covenants which can stipulate that they have the right to “walk free” with the collateral at any time to render their debt risk-free, and without joining the other creditors in the bankruptcy process. Such covenants would require the borrower to “top up” the collateral at each instant to make the long-term contract essentially risk-free. If such contracts can be written and enforced without any costs, then long-term debt may also be issued with safe harbor provisions. We formalize this economic intuition in the next proposition.

**Proposition 3.** When financial covenants can be enforced instantaneously without any costs to ensure that the value of the collateral is always equal to par, than the borrower is indifferent between issuing short-term debt or long-term debt with safe harbor protection.
Two points are worth making in this context: with these additional covenants, the long-term debt tends to look more like short-term debt in states of the world where the borrower will have to top up the collateral. In a sense, these states are the ones that really matter as the likelihood of a loss is present only in these states. Since the threat of refusing to roll over the loan is much more powerful than enforcing financial covenants, short-term debt becomes a more natural candidate for being issued with safe harbor provisions. With costly enforcement of covenants, short-term debt with safe harbor will be generally preferred over long-term debt with safe harbor.

**Proof for Proposition 3**

In order for the short-term or long-term debt to be protected by safe harbors, the borrower must move some assets outside the automatic stay provisions of the Code. We will formulate the problem for long-term debt, although the arguments are identical for short-term debt. Let us denote $F$ to be the par value of long-term debt, which we assume to have infinite maturity. Let the coupon rate per unit time be $c$. At time $t = 0$ the borrower can set aside a fraction $\theta$ of assets into the safe harbor. The value of the sheltered assets at $t = 0$ is $\theta V_0 (1 - \beta) \geq F$. Since the option to default rests with the borrower, as long as the coupon payments are met, the long-term creditor will not be able to seize the assets held in safe harbor. In addition, as the asset value declines, the creditor is exposed to default risk. In order to protect the creditor from default risk, it is necessary that the assets are always held at a level $F$. This would imply that in states of the world when $V_t < F \theta (1 - \beta)$. The long-term bond covenants must require the borrower to move additional assets to “top up” levels of the safe harbored collateral to $F$. We will model below this requirement, which would imply that the borrower will have to issue additional equity in states where $V_t < F \theta (1 - \beta)$ to top up the collateral so that the value of always equal to $F$. In contrast, with short-term debt, (instantaneous maturity), the lender can always enforce the contract by refusing to lend, when $V_t \downarrow F$, and refuse to roll over the loan. This is the key difference between short-term and long-term debt in our model. It follows then that as long as covenants are enforced promptly and without costs, the long-term
debt is also viable for safe harbor provisions. We formalize these ideas below and derive the optimal default boundary.

The equity holders’ (the borrower) problem can be divided into two regions. In the first region we have: \( V_t \geq \frac{F}{\theta(1-\beta)} \). Here, the collateral has sufficient assets to protect the long-term debt investors. In this region (region 1), equity value satisfies the following ODE:

\[
\delta V - F c(1 - \tau) - r G + G V (r - \delta) V + \frac{1}{2} G_{VV} \sigma^2 V^2 = 0
\]

In the second region we have: \( V_t < \frac{F}{\theta(1-\beta)} \). Here, the collateral is insufficient to protect the long-term debt investors. In this region (region 2), equity value satisfies the following ODE, reflecting the additional equity issuance to top up the collateral:

\[
\delta V - F c(1 - \tau) - [F - V \theta(1 - \beta)] - r E + E V (r - \delta) V + \frac{1}{2} E_{VV} \sigma^2 V^2 = 0
\]

In moving from region 1 to region 2 we note that distinction between short-term debt and long-term debt. In the case of short-term debt, the onus is upon the borrower to top up the collateral: in the absence of such an action, the short-term creditor will simply refuse to roll over the loan. On the other hand, in the case of long-term debt, the creditors will have to rely on enforcing the loan covenants to ensure that the collateral is topped up. In its absence, the covenants should allow the long-term creditors to accelerate the payment of principal. If such enforcements are costly, then there will be dead-weight losses in moving from region 1 to region 2, which will be reflected in the ODE, and the option of issuing long-term debt will become suboptimal.

We need enforce the following boundary conditions to solve for the value of the borrower in issuing safe harbored long-term or short-term debt. \( E(V_B) = 0, E_V(V_B) = 0, G(\frac{F}{\theta(1-\beta)}) = E(\frac{F}{\theta(1-\beta)}), G_V(\frac{F}{\theta(1-\beta)}) = E_V(\frac{F}{\theta(1-\beta)}), \) and \( G(V \uparrow \infty) = V - \frac{F c(1 - \tau)}{r} \).

It is the enforcement mechanism that is different. Let \( \lambda_1 > 0 \) and \( \lambda_2 < 0 \) be the two roots of
the following characteristic equation:

\[-r - (r - \delta)\lambda + \frac{1}{2}\lambda(1 + \lambda)\sigma^2 = 0\]

We can show that the solution to the above problem takes the following form.

\[
G(V; V_B) = V - \frac{Fc(1 - \tau)}{r} + a_1 V^{-\lambda_1}
\]

\[
E(V; V_B) = \frac{F + Fc(1 - \tau)}{r} + \frac{\delta + \theta(1 - \beta)}{\delta} V + b_1 V^{-\lambda_1} + b_2 V^{-\lambda_2}
\]

where \(a_1, b_1\) and \(b_2\) are constants that are as shown below.

\[
a_1 = \frac{F}{r\delta} \left( \frac{\delta}{\lambda_1} + (r - \delta) \right) \left[ \frac{F}{\theta(1 - \beta)} \right] + \frac{1}{\lambda_1} V_B^{\lambda_1} \left( 1 + \frac{\theta(1 - \beta)}{\delta} \right)
\]

\[
b_1 = \frac{1}{\lambda_1} V_B^{\lambda_1} \left( 1 + \frac{\theta(1 - \beta)}{\delta} \right) - \frac{\theta(1 - \beta)}{\delta} \left( \frac{1 + \lambda_1}{\lambda_1 - \lambda_2} \right) \left[ \frac{F}{\theta(1 - \beta)} \right]^{1 + \lambda_1}
\]

\[
b_2 = \frac{\theta(1 - \beta)}{r} \left( \frac{1}{\lambda_1 - \lambda_2} \right) \left[ \frac{F}{\theta(1 - \beta)} \right]^{1 + \lambda_1}
\]

The optimal default boundary \(V_B\) is found by maximizing the equity value and is presented below:

\[
\frac{F}{r} \frac{1 + \lambda_2}{\lambda_1 - \lambda_2} \left( \frac{F}{V_B \theta(1 - \beta)} \right)^{\lambda_2} - \frac{F \lambda_2}{r} \frac{1 + \lambda_1}{\lambda_1 - \lambda_2} \left( \frac{F}{V_B \theta(1 - \beta)} \right)^{\lambda_1} = \frac{F + Fc(1 - \tau)}{r} - \frac{1 + \lambda_1}{\lambda_1} \left( 1 + \frac{\theta(1 - \beta)}{\delta} \right)
\]

Since the debt is risk-free, \(c = r\), in equilibrium. It follows that the topping up of the collateral will occur until the equity value goes to zero, at which point the creditors will walk free with the collateral, rendering the debt risk-free. This proves that with prompt and costless enforcement of covenants, long-term debt can also be issued with safe harbor provisions.

Note that if the enforcement of covenants is costly, equity holders will prefer to issue short term debt with safe harbor rights and save costly negotiations.
C Setting Total Recovery Constant

We now explore a possibility that differential total recovery value with respect to firms’ choice
of pledging may drive our main result. First, we obtain the total recovery value $R$, we add up
all the recovery value given by Equation (3)-(5). At the equilibrium, we have that

$$R(\theta) = [(1 - \theta) \cdot (\psi_D + \psi_E) + \theta \cdot (1 - \beta)] \cdot V_B$$

For a given range of $\theta$ between [0,1] and with a standard parameter set, we plot the total
recovery value with different levels of APR violation $(\psi_E)$ while keeping $\alpha = \beta$.

Panel (a) of Figure C.3 shows that the total recovery in our model is, in fact, endogenous.
As a firm endogenously chooses $\theta$, the total recovery given default will be determined. This
illustration suggests that $R'(\theta) \geq 0$ and it is also easy to show analytically using results in
Lemma 1 and Proposition 2: as long as $\beta \leq \alpha$, the equilibrium total recovery rate increases in
$\theta$. As a special case, when $\psi_E = 0$ the total recovery value is irrelevant to securing exercises:
for every value of $\theta$, $R$ is a constant. However, as $\psi_E > 0$, total recovery is increasing in $\theta$,
indicating that a firm can enjoy higher recovery value as it pledges more asset.

In order to verify that differential recovery value is not driving our main results, we construct
the recovery-neutral benchmark. In particular, we make the total recovery rate fixed across $\theta$
by finding $\beta(\theta)$ such that $\frac{\partial}{\partial \theta}[(1 - \theta) \cdot (\psi_D + \psi_E) + \theta \cdot (1 - \beta(\theta))] = 0$. $^{30}$ This condition implies that

$$\beta'(\theta) = \frac{\alpha - \beta(\theta)}{\theta}.$$ 

With a boundary condition $\beta(1) = 0$, the above differential equation yields:

$$\beta(\theta) = -\frac{(1 - \theta)}{\theta} \alpha.$$ 

We substitute our parametric liquidity cost $\beta$ with the above function $\beta(\theta)$, and produce our

$^{30}$A more general version $R(\theta) = [(1 - \theta) \cdot (\psi_D(\theta) + \psi_E(\theta)) + \theta \cdot (1 - \beta(\theta))]$ is reduced to this specification based
on the assumption that $\psi_D$ and $\psi_E$ are given by the underlying bankruptcy code, which are independent of firms’
choice for $\theta$. We thank the referee for the general specification.
main result again: the firm value with respect to $\theta$ corresponding to Figure 8.

Panel (b) of Figure C.3 indicates that such a “recovery-neutral” benchmark delivers qualitatively identical results of the version with parametric $\beta$, ensuring our results are not driven by mechanical recovery rate change. The main implication of this exercise is that even when we force the recovery rate to be constant across firms’ choice, when the bankruptcy code is not creditor-friendly and therefore allows APR violation, secured short-term debt provides a way to circumvent this problem and hence enhances the firm valuation. This result allows us to keep our parametric $\beta$ in our main model which is simpler and better tractable.
Figure C.3: **Total recovery and firm value with constant total recovery:** The upper panel (a) shows the total recovery value with respect to a firm’s choice \( \theta \), where the total recovery value \( R(\theta) \) is defined by \( [(1 - \theta) \cdot (\psi_D + \psi_E) + \theta \cdot (1 - \beta)] \cdot V_B \). In this figure, the inside bankruptcy cost and outside liquidation loss are identical (i.e., \( \alpha = \beta \)). The total recovery value is presented with three different levels of APR violation (\( \psi_E = \{0.0 \text{ (dotted)}, 0.1 \text{ (dashed)}, 0.2 \text{ (solid)}\} \)), keeping \( \alpha \equiv 1 - (\psi_E + \psi_D) = \beta = 0.5 \). The lower panel (b) shows the firm value with respect to a firm’s choice \( \theta \), keeping the total recovery value constant across firms’ choice variable \( \theta \) (i.e., \( \frac{\partial}{\partial \theta} \[(1 - \theta) \cdot (\psi_D + \psi_E) + \theta \cdot (1 - \beta(\theta))] = 0 \)). The firm value is presented with three different levels of APR violation (\( \psi_E = \{0.0 \text{ (dotted)}, 0.1 \text{ (dashed)}, 0.2 \text{ (solid)}\} \)), keeping \( \alpha \equiv 1 - (\psi_E + \psi_D) = \beta = 0.5 \). For both figures, other parameters are used as follows: \( V_0 = 1, r = 0.04, \delta = 0.02, \tau = 0.35, \sigma = 0.25 \).