Bank Debt versus Mutual Fund Equity in Liquidity Provision*

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Abstract

We propose a unified framework to compare and quantify liquidity provision by traditional debt-issuing commercial banks and equity-issuing non-banks. We first show that both types of financial intermediaries provide liquidity by insuring against idiosyncratic liquidity risks as in Diamond and Dybvig (1983) but with distinct frictions. The fixed value of debt induces *panic runs* whereas the flexible payoff of equity renders investor redemptions more sensitive to news on fundamentals, i.e., a *flow-to-fundamentals* relationship. Both frictions constrain liquidity provision by generating premature liquidation of long-term investments. Informed by the theory, we develop the Liquidity Provision Index (LPI) as the first empirical measure of liquidity provision that can be generally applied across demandable debt and demandable equity issuing financial institutions. At the end of 2017, bond mutual fund shares provide a significant amount of liquidity amounting to one quarter of that by uninsured bank deposits. We confirm that the majority of the gap arises from the difference in contract forms instead of regulatory features such as deposit insurance. We further use the switch by institutional prime Money Market Funds (MMF) from a fixed to a floating value during the MMF Reform to corroborate the capacity of demanable equity in liquidity provision. Over time, the gap between bank and fund liquidity provision has continuously narrowed. Quantitative Easing and post-crisis liquidity regulation have contributed to the migration of liquidity provision away from the traditional deposit-taking banking sector to equity-funded non banks.

1 Introduction

A predominant function of financial intermediaries is liquidity provision. In a world where investors face idiosyncratic liquidity risks, banks provide liquidity by issuing debt claims redeemable at short notice, that is, liquid demand deposits, from investing in a portfolio of illiquid loans. Liquidity provision is achieved by pooling and thus insuring against idiosyncratic liquidity shocks across investors so that collectively, more illiquid assets can be funded using liquid short-term claims (Diamond and Dybvig, 1983).

While the literature has primarily attributed liquidity provision to banks, financial intermediation has increasingly moved beyond the traditional banking sector. In particular, intermediaries issuing demandable equity have played an increasingly important role. For example, US open-end mutual funds saw their total assets increase from 5% of GDP in the 1980s to 100% in 2019 with a record $21.3 trillion assets under management. These trends beg the question of whether liquidity provision is still only confined to debt-issuing intermediaries. After all, open-end mutual funds also invest in significant amounts of illiquid assets like corporate bonds and loans and issue fund shares redeemable at short notice (Chernenko and Sunderam, 2017, Goldstein, Jiang and Ng, 2017). Further, if equity-issuing intermediaries indeed provide liquidity to investors, what are the underlying frictions and financial stability implications? How do they compare to those by debt-issuing intermediaries like banks?[1]

This paper presents the first unified framework for liquidity provision by debt- and equity-issuing financial intermediaries. We theoretically show that intermediaries issuing demandable equity are able to create liquidity just like debt-issuing intermedaries do because as long as investors' collectively hold their funds in the intermediary and intermediaries optimally use liquid assets to meet redemptions first, investors' idiosyncratic liquidity risks can be shared and more illiquid assets can be held until maturity.

However, the choice between debt and equity is a deciding factor for the efficiency of liquidity provision and financial stability. We find that equity's flexible NAV value helps mitigate panic runs, which have long been viewed as a major drawback of debt contracts, but renders investor redemptions more sensitive to news about economic fluctuations. These volatile *flows*

[1] To be exact, we consider open-end equity contracts redeemable at short notice so although banks also issue equity, bank equity does not directly fall into our consideration of liquidity provision.

to fundamentals in turn expose liquidity provision by funds more to aggregate shocks in the economy.[2]

Informed by the theory, we design the first empirical measure of liquidity provision that is applicable to both demandable debt and demandable equity issuing financial institutions. The Liquidity Provision Index (LPI) measures how much more investors expect to obtain by redeeming shares (demandable debt) compared to directly selling the underlying portfolio of assets. We find that a dollar invested in bond fund shares provides significant amount of liquidity amounting to a quarter of that by bank deposits. We further apply the LPI to Money Market Funds (MMF), and use the switch by institutional prime funds from a fixed to a flexible value during the MMF Reform to corroborate the capacity of demandable equity in liquidity provision. Moreover, we find that the gap between bank and fund LPI has increasingly narrowed over time, and that Quantitative Easing and post-crisis liquidity regulation at banks have contributed to the migration of liquidity provision away from the traditional deposit-taking banking sector.

A key novelty of our paper is to jointly analyze liquidity provision by bank debt and fund equity under a common theoretical framework. Similar to Diamond and Dybvig (1983), investors are subject to idiosyncratic liquidity shocks in an incomplete market. They invest through an intermediary, who chooses a portfolio of liquid cash and an illiquid long-term investment project at the beginning ($t = 0$), considering investors' liquidity shocks in the short-run ($t = 1$) and the uncertain but potentially higher project return in the long run ($t = 2$). The illiquid project incurs a liquidation cost when sold prematurely in the spirit of Shleifer and Vishny (1992, 1997). There is also aggregate uncertainty over the long-run return of the project as in Allen and Gale (1998). Investors receive a noisy private signal about these future economic fundamentals at $t = 1$ and decide whether to withdraw their funds from the intermediary or not.[3]

In this setting, we formulate liquidity provision as the difference between an intermediary's expected contract payment to investors withdrawing at short notice ($t = 1$) and the direct liqui-

[2]Our baseline model highlights that the use of debt exposes banks to panic runs whereas the use of redeemable equity with perfectly adjusting values induces volatile flows-to-fundamentals relationship in funds. In reality, institutional differences between banks and mutual funds could induce fundamentals-driven, non-panic-based runs in banks (e.g., Allen and Gale, 1998) and panic-based runs in mutual funds (Chen, Goldstein and Jiang, 2010, Goldstein, Jiang and Ng, 2017, Zeng, 2017). While our theory presents a benchmark case focusing on the most salient difference between debt and equity contracts, our empirical analysis will consider these frictions and their implications for our findings.

[3]To be exact, only those without a liquidity shock consider to withdraw or not. Investors hit with a liquidity shock always consume at $t = 1$.

dation value of its underlying asset portfolio. Bank debt provides liquidity because idiosyncratic liquidity risks are pooled across investors so that more of the illiquid long-term project can be held to maturity. In the same context, an intermediary issuing redeemable equity contracts also pools investors' resources at the intermediary level to invest in more long term projects so that early redemptions can be met at net asset values (NAVs) higher than the direct liquidation value of the underlying assets. The use of debt funding is therefore not a necessary condition for liquidity provision.

Nevertheless, the choice of debt versus equity funding determines how losses from premature liquidations are distributed among investors and thus the financial stability implications of liquidity provision. On one hand, debt-issuing intermediaries are susceptible to panic runs because the fixed face value of debt promised to withdrawing depositors imposes all losses from premature asset liquidations on depositors remaining in the bank and thereby induces a first-mover advantage to withdraw.[4] In contrast, the use of demandable equity with flexible NAVs allows losses from premature liquidations to be proportionally born by redeeming and non-redeeming shareholders and thereby removes the first-mover advantage that leads to runs.[5] The comparative advantage of equity over debt in sufficiently bad states of the world is illustrated in Figures 1 and 2.

On the other hand, demandable equity bears the disadvantage of rendering liquidity provision more sensitive to fluctuations in fundamentals. When equity investors receive private news about project returns, they decide whether to stay in the fund to earn the long run return or to redeem their shares and store cash on their own. Flexible NAVs cause redemption decisions to be continuously revised with changes in fundamentals, resulting in volatile *flows-to-fundamentals* and premature asset liquidations, which ultimately constrain liquidity provision. In contrast, the fixed payment promised by bank debt renders deposit withdrawals less sensitive to changes in fundamentals as long as they do not fall below the threshold for panic-runs as in Figures 1 and 2.

[4] Specifically, banks meet withdrawal requests by prematurely selling the long-term projects and paying out the face value of debt on a first-come-first-serve basis. Knowing that funds will eventually run out, investors withdraw early in the hopes of getting paid first once they believe that others will. Applying the global games technique as in Goldstein and Pauzner (2005), we show that panic runs occur only when fundamentals fall below a threshold.

[5] One interpretation of this benchmark case of demandable equity is open-end mutual funds using swing pricing to incorporate all possible liquidation-induced costs into the end-of-trading-day NAV, as the U.S. SEC has recommended since 2016.

One advantage of our theoretical notion of liquidity provision is that it can be adapted into a sufficient statistic for liquidity provision in practice—the Liquidity Provision Index (LPI). The LPI captures the difference between an intermediary's expected contract payment to investors withdrawing at short notice and the direct liquidation value of the underlying asset portfolio per dollar invested. It is the first measure of liquiity provision that is generally applicable to financial institutions regardless of the contractual form of their liabilities and only requires public data as inputs .[6]

Calculating the LPI involves three steps. We first determine the model-implied contract payment of demandable shares (demandable debt) for any given proportion of early redemptions (withdrawals). For example, when outflows are very small, funds can first use cash to meet redemptions so the contract payment is 100% of the initial NAV. As outflows increase, funds resort to selling increasingly illiquid assets with higher haircuts, continuously decreasing the NAV. For banks, incurred haircuts also increase with withdrawals but uninsured depositors receive a fixed value unless the bank defaults. Given these differences, expected contract payments depend on the distribution of fundamentals, for which we use the observed distribution of flows at the intermediary level as an empirical proxy. Using observed outflows for calculating the LPI has two key advantages. The distribution of economic fundamentals in the theory cannot be easily quantified but investor flows that arise from fundamentals are observable and measurable. Moreover, observed flows incorporate the presence of regulations and frictions outside of the model, which allows the LPI to be a realistic measure of financial institutions' liquidity provision in practice. Finally, to set a baseline for liquidity provision, we deduct the direct liquidation value of the underlying portfolio.

Our baseline results apply the LPI to a laboratory of commercial banks and bond mutual funds. We find that fund shares provide a significant amount of liquidity relative to uninsured deposits. At the end of 2017, for a dollar invested in uninsured bank deposits, investors expect 19.0 cents more upon early withdrawal than under direct liquidation of the underlying portfolio; whereas for the average dollar invested in redeemable bond shares, they expect 4.3 cents more. In other words, liquidity provided by bond mutual fund shares amounts to a quarter of the liquidity provided by bank deposits. Further, the gap in liquidity production capacity between banks and

[6]Debt-issuing intermediaries include commercial banks, prime money market funds (MMF) pre MMF Reform, asset-backed commercial paper (ABCP) conduits, and leveraged hedge funds; whereas equity-issuing intermediaries comprise of open-end mutual funds, prime money market funds post MMF reform, unleveraged hedge funds, pension funds, and insurance companies.

funds has increasingly narrowed from 2011 to 2017. We find evidence that the fall in LPI is driven by an expansion in central bank reserves following Quantitative Easing and the incidence of the Liquidity Coverage Ratio. Both policies increase the proportion of liquid assets on bank balance sheets, which raises the direct liquidation value of the portfolio and shrinks the capacity for liquidity provision by banks.

The LPI is designed to serve as a consistent measure of liquidity provision in practice, where flows and portfolio choices of financial institutions may be influenced by the regulatory environment. To isolate the impact of contractual forms on liquidity provision, we further use the LPI to perform several additional checks. First, we limit deposit flows to the uninsured portion of bank deposits, which are are consistent with demandable debt in our model.[7] Nevertheless, explicit and implicit guarantees may still indirectly affect the estimated LPI of uninsured deposits through equilibrium flows and portfolio choice. To this end, we use cross-sectional variation in bank liability structure to project that a hypothetical bank without any deposit insurance or non-deposit liabilities bank provides two-thirds of the estimated average bank LPI, which implies that the bulk of the difference in bank and fund LPI arises from the use of demandable debt versus equity.

Finally, we identify the effect of debt versus equity funding on liquidity provision by applying the LPI to a sample of MMFs around the 2016 MMF Reform. The reform required institutional prime MMFs to switch from fixed to floating NAVs, which represents a transition from debt-funding to equity-funding.[8] At the same time, retail prime MMFs were exempt from changing their NAV reporting and provide a natural control group for other changes in the economy during the reform period. Our difference-in-differences estimates confirm an economically and statistically significant 19% drop in liquidity provision due to the reform, corroborating that liquidity provision by demandable equity is significant but relatively lower than that of debt.

The banking literature on liquidity provision has mostly centered around deposit-issuing banks as in Diamond and Dybvig (1983), Diamond and Rajan (2001), Kashyap, Rajan and Stein

[7] Uninsured deposits are of high empirical relevance. Egan, Hortacsu and Matvos (2017) estimate that uninsured comprise half of all consumer deposits in large US commercial banks and show that they are subject to runs. One notable example of a modern commercial bank run is that on Washington Mutual (WaMu).

[8] Institutional prime MMFs were indeed subject to panic runs before the reform, notable examples including the Reserve Primary Fund, the first MMF that "broke the buck," and the Putnam Fund, the first MMF that suddenly closed, both due to severe runs in 2008.

(2002), and Goldstein and Pauzner (2005), for example.[9] Hanson, Shleifer, Stein and Vishny (2015) consider money-like debt claims issued by banks, which have access to deposit insurance, versus shadow banks, which require asset sales at short notice. [10] Dang et al. (2017) compare banks to financial markets focusing on banks' ability to keep information on fundamentals secret. Our paper focuses on the contractual form of intermediary liabilities, which is a fundamental difference between banks and non-banks. We jointly analyze liquidity provision by demandable equity and debt in a common theoretical framework based on Diamond and Dybvig (1983) to show that equity also insures against liquidity risk but with different frictions. Our focus on demandable claims is fundamentally different from Jacklin (1987) who focuses on how tradable but non-demandable claims may provide liquidity.[11]. Furthermore, Jacklin (1987) does not consider aggregate risks and thus cannot accommodate the notion of debt versus equity contracts.

The comparative advantage of equity with flexible NAVs in reducing the incidence of panic runs speaks to the literature on mutual fund flows' financial stability implications. Closely related is Chernenko and Sunderam (2017) who empirically examine the relationship between fund cash management and maturity mismatch. Chen, Goldstein and Jiang (2010) and Goldstein, Jiang and Ng (2017) find that panic runs may occur in mutual funds holding illiquid assets when adjustments in fund NAVs are imperfect. Using a dynamic model, Zeng (2017) illustrates that fund managers' cash management and an inherent non-commitment problem may also lead to panic runs. Our theory analyzes a benchmark fund with fully flexible NAVs to provide a direct comparison with demandable debt issuing intermediaries in liquidity provision. Nevertheless, we do not deny the presence of potential frictions in NAV adjustment in practice and our empirical LPI uses the actual observed fund flows to provide a realistic quantification of liquidity provision.

Finally, our paper contributes to the measurement of liquidity provision by financial intermediaries. The prior literature has focused on the banking sector (Berger and Bouwman, 2009, Brunnermeier, Gorton and Krishnamurthy, 2012, Bai, Krishnamurthy and Weymuller, 2018), as the theoretical foundation of liquidity provision by other types of intermediaries has not yet been

[9]Other papers on the role of banks include Diamond (1984) on bank monitoring and Rajan (1992) on relationship banking. Recently, Drechsler, Savov and Schnabl (2017) and Li, Ma and Zhao (2020) analyze how market power renders banks special in deposit-taking. They do not focus on liquidity provision as in Diamond and Dybvig (1983).

[10]Other papers on shadow bank liquidity provision include Gorton and Metrick (2010), Stein (2012), Sunderam (2015), Nagel (2016) and Xiao (2019).

[11]In reality, the tradable but non-demandable claims considered by Jacklin (1987) most closely resemble shares of close-end mutual funds rather than open-end mutual funds

laid out. We apply our model to develop the LPI as the first empirical measure of liquidity provision that can be applied beyond the traditional banking sector to include both demandable debt and demandable equity issuing financial institutions. The LPI we propose is simple to estimate with publicly available data and may serve as a useful tool for researchers and policy-makers to assess the efficiency and implications of liquidity provision by non-bank financial institutions.

The remainder of the paper is organized as follows. Section 2 lays out the theoretical framework for liquidity provision by debt and equity issuing intermediaries. Section 3 explains the construction of the LPI and Section 4 presents the estimation results for liquidity provision by bond mutual funds, commercial banks, and MMFs. Section 5 concludes.

2 Theoretical Framework

The economy has three dates, $t = 0, 1, 2$, with no time discount. There is a $[0, 1]$ continuum of ex-ante identical households, each of which has one unit of consumption good as the initial endowment at $t = 0$, called "cash", which serves as the numeraire. Each household is uncertain about her preferences over consumption at $t = 1$ and $t = 2$. At the beginning of $t = 1$ a household learns her preferences privately: with probability π she is an early-type and gets utility $u(c_1)$ from date-1 consumption only, while with probability $1 - \pi$ she is a late-type and gets utility $u(c_2)$ from date-2 consumption only. Let the primitive flow utility function, $u(c)$, be increasing, concave, and satisfy the Inada conditions.[12]

The market is incomplete in the sense that no Arrow-Debreu securities are available. The consumption good, cash, can be directly consumed at any given date or transferred to the next date via one of two technologies: 1) a long-term, illiquid investment project denoted as "project", and 2) a short-term liquid asset denoted as "storage".[13,14]

The project is risky, illiquid, and available for investment at $t = 0$ only. One unit of cash invested in the project at $t = 0$ yields R units of cash at $t = 2$, where R is a random variable

[12] Notably, Inada conditions require $u(0) = 0$.

[13] Note that we do not attach any exogenous utility or convenience value to holding cash per se. Instead, liquidity creation arises endogenously through liquidity insurance in an incomplete market setting as in Diamond and Dybvig (1983).

[14] Although the original Diamond and Dybvig (1983) model does not have storage, the portfolio choice between storage and an illiquid project has been introduced to the Diamond and Dybvig (1983) framework since Cooper and Ross (1998) and Ennis and Keister (2006).

that follows a distribution of $G(\cdot)$ with support $[0, +\infty)$. Denote R as the fundamentals of the economy; since R is uncertain, the economy entails aggregate risks.[15] Only the distribution of R, $G(\cdot)$, is common knowledge to households at $t = 0$, and we assume $E[R] > 1$ so that the project generates a higher expected return than cash. At $t = 1$, the project has not yet come to fruition and retains a value of one. If it is prematurely liquidated at this point, a liquidation discount will be incurred in the spirit of Shleifer and Vishny (1992, 1997).[16] Denote the cash value obtainable when l of the project is liquidated as:

$$C(l) = l - \frac{\phi}{2} l^2 \,, \tag{2.1}$$

where $0 < \phi \leq 1$ captures the extent of project illiquidity. This parametric form (2.1) can be micro-founded by a downward-sloping demand for the illiquid project: the more of the project is prematurely liquidated, the lower the marginal liquidation value.[17]

Storing cash is riskless. An intermediary always yields one unit of cash on the next date when using the storage technology. Households are less efficient at storing cash and only obtain γ units when investing one unit from $t = 1$ to $t = 2$. Specifically, we assume

$$\gamma = 1 - \kappa n \,, \tag{2.2}$$

where $0 \leq \kappa < 1$ captures the decreasing returns to scale when a late household operates the storage and n is the population of late households who use this storage.[18] This can be thought of as households finding it more costly to physically store cash or being less efficient at investing

[15] The original Diamond and Dybvig (1983) model does not have aggregate risks, but they have been introduced in the follow-up literature, notably, Allen and Gale (1998) and Goldstein and Pauzner (2005).

[16] The original Diamond and Dybvig (1983) model does not have this liquidation cost. But it has been introduced by Cooper and Ross (1998) to consider inefficient liquidation and its interplay with bank liquidity holdings, and has been so far considered as a standard element of the Diamond and Dybvig (1983) paradigm.

[17] Formally, let the marginal liquidation value of the project be $1 - \phi z$, where z is the total unit of projects being liquidated. As a result, if a total of l illiquid project were prematurely liquidated at $t = 1$, the amount of cash raised would be

$$\int_0^l (1 - \phi z) dz \,.$$

Calculating this integral yields (2.1).

[18] An alternative specification is to explicitly assume γ to be a decreasing function of the units of cash stored by late households. This specification would not affect the economic insight because it is mathematically equivalent to assume γ to be decreasing in n in a non-linear way. To see the intuition, notice that any household using this storage always stores a positive amount of cash, and hence specification (2.2) already implies that γ is decreasing in cash stored by late households.

cash in capital market securities. It is important to note that intermediaries' storage efficiency does not determine the presence of liquidity provision. In other words, all predictions still carry through if the relative inefficiency parameter κ is equal to 0. The purpose of introducing κ is to capture the extent of the flow-to-fundamentals relationship, which will eventually influence the magnitude of liquidity provision.

At the beginning of $t = 1$, every household i receives a private signal of R:

$$s_i = \theta(R) + \varepsilon_i \, .$$

where $\theta(R) \in [0, 1)$ is strictly increasing in R, and ε_i is i.i.d. and arbitrarily small.[19]

To uncover the similarities and differences between debt and equity in liquidity provision, we compare two scenarios with different intermediary arrangements, keeping the same underlying economy. In both scenarios, a representative financial intermediary is present who makes portfolio choices $(x_{k,0}, y_{k,0})$ at $t = 0$ on households' behalf, where $x_{k,0}$ is the amount of cash and $y_{k,0}$ the amount of projects. The subscript $k \in \{b, f\}$ denotes the intermediary type: a bank or a fund. Since the intermediary is representative, it maximizes households' utility and breaks even in equilibrium. Another interpretation is that the intermediary is mutually owned by all the households without any agency frictions.

In the first scenario, a representative bank offers a standard demandable debt contract $(c_{b,1}, c_{b,2})$ to households, where the cash payment at $t = 1$, $c_{b,1}$, is subject to a sequential service constraint as in Diamond and Dybvig (1983).[20]

In the second scenario, a representative open-end mutual fund offers an NAV-based, pro-rata equity contract $(c_{f,1}(\lambda(R)), c_{f,2}(\lambda(R)))$ in which the cash payments are the end-of-date net asset values (NAVs). These payments are contingent on the number of households redeeming at $t = 1$, λ, which is in turn determined by economic fundamental R in equilibrium. Thus, this payment structure represents a redeemable equity contract.

It is important to note that we derive optimal debt and equity contracts based on the type of debt and equity most closely aligned with those in the real world, i.e., demandable deposits and

[19] This information structure is similar to that in Allen and Gale (1998) and more formally that in Goldstein and Pauzner (2005).

[20] Our model insights will not change if we instead assume a pro rata rule following, for example, Allen and Gale (1998). We do not consider other versions of the sequential service constraint such as that in Green and Lin (2003).

open end mutual fund shares with flexible NAV. In this sense, we solve for constrained optimal contracts just as in Diamond and Dybvig (1983). We do not focus on an unconstrained optimal contract or mechanism design problem over a general contract space.

2.1 Liquidation Value and Liquidity Provision

Given our focus on liquidity provision by intermediaries, we first define a benchmark case of how much short-term consumption a household can enjoy by liquidating a given portfolio at short notice without an intermediary. Formally, we denote this as the liquidation value. Note that the liquidation value is general in the sense that it can be defined on any portfolio and thus is a function of (x_0, y_0); it does not necessarily rely on any equilibrium concept.

Definition 1. *Given any t-0 portfolio (x_0, y_0), its liquidation value at $t = 1$ is given by*

$$\begin{aligned} c_1(x_0, y_0) &\doteq x_0 + y_0 - \frac{\phi}{2} y_0^2 \\ &= 1 - \frac{\phi}{2} y_0^2 \,. \end{aligned} \tag{2.3}$$

As (2.3) indicates, the liquidation value is lower than the fair value of the portfolio at $t = 1$ due to the liquidation discount as specified in (2.1). The less liquid a portfolio, the lower the liquidation value.

According to Diamond and Dybvig (1983), creating deposits that are more liquid than the underlying portfolio held by banks can be viewed as a liquidity insurance arrangement in which depositors share the risk of liquidating an illiquid portfolio at short notice and at a loss. Definition 1 allows us to naturally extend this insight to non-bank, equity-issuing financial intermediaries to define a unified notion of liquidity provision as the difference between the expected intermediary contract payment and the liquidation value of the underlying portfolio. Formally, the equilibrium amount of liquidity provision for any intermediary is defined as follows.

Definition 2. *For a dollar invested in claims issued by an intermediary $k \in \{b, f\}$, the amount of liquidity provision is defined as*

$$E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0}) \,, \tag{2.4}$$

where the contract payment $c^*_{k,1}(R)$ and intermediary portfolio holdings $(x^*_{k,0}, y^*_{k,0})$ are equilibrium outcomes given the intermediary arrangement (bank or fund) and the underlying economy, the expectation $E[\cdot]$ is taken over fundamental R, and $c_1(\cdot,\cdot)$ is the liquidation value function as defined in Definition 1.

Intuitively, definition 2 indicates that the amount of liquidity provision to a household subject to realized liquidity shocks is affected by both sides of the intermediary's balance sheet. Specifically, liquidity provision is the difference between the expected contract payment (liability side of the intermediary) and the liquidation value of the underlying portfolio (asset side of the intermediary). It is positive if the claims issued by the intermediary are more liquid, i.e., have a higher value upon liquidation at short notice, than the underlying assets it holds. Looking ahead, we will show that the two sides of the intermediary balance sheet are jointly determined in equilibrium, and that the equilibrium amount of liquidity provision is characterized by the endogenous frictions stemming from the different contractual forms.

One nice feature about Definition 2 is that, since the liquidation value $c_1(x^*_{k,0}, y^*_{k,0})$ is independent to fundamental R, (2.4) can be re-expressed as

$$E[c^*_{k,1}(R) - c_1(x^*_{k,0}, y^*_{k,0})]\,,$$

which (before taking the expectation) allows our theory to capture ex-post liquidity provision for any given realization of R. It thus allows us to compare the relative frictions of bank debt versus fund equity in liquidity provision for any given state of the underlying economy.

Our definition of liquidation value and liquidity provision has two advantages. Theoretically, it is in line with the original insight of liquidity provision by Diamond and Dybvig (1983) and allows for a unified comparison across debt- and equity-issuing financial intermediaries.[21] Empirically, all essential inputs for calculating the liquidation value and the magnitude of liquidity provision are observable, which allows for a direct application of the model to measure liquidity provision by real-world intermediaries such as banks, open-end mutual funds, and money market funds (MMFs).

[21]In Appendix A, we further show that the notion of liquidation value is tightly linked to the equilibrium outcome in autarky, which serves as a direct welfare benchmark but is not empirically observable.

2.2 Bank Debt

At $t = 0$, the representative bank offers a demandable debt contract $(c_{b,1}, c_{b,2})$, which is subject to a sequential service constraint at $t = 1$, to households. Notably, $c_{b,1}$ represents the face value of debt, which is agreed on at $t = 0$. Because the bank breaks even, $c_{b,2}$ is automatically determined once $c_{b,1}$ is chosen so that the face value of debt, $c_{b,1}$, is sufficient to capture the contract offered to depositors.[22] At $t = 0$, the bank also chooses the optimal portfolio $(x_{b,0}, y_{b,0})$ to maximize ex-ante expected household utility.

Although the debt face value $c_{b,1}$ is fixed and independent of the number of withdrawals as long as the bank is solvent, the sequential service constraint implies that the actual payments of $(c_{b,1}, c_{b,2})$ implicitly depend on λ_b given any initial bank portfolio choice $(x_{b,0}, y_{b,0})$ as shown in Table 1:

Table 1: Ex-post bank debt payments

t-1 withdrawal	$\lambda_b \leq \frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$	$\lambda_b > \frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$
t-1 payment	$c_{b,1}$	$c_{b,1}$ with probability $q(\lambda_b)$
t-2 payment	$\frac{x_{b,1} + y_{b,1}R}{1-\lambda_b}$	0

where

$$q(\lambda_b) = \frac{1-\frac{\phi}{2}y_{b,0}^2}{\lambda_b c_{b,1}} \tag{2.5}$$

is the probability of withdrawing households to be served when the bank fails and the sequential service constraint is binding, and $(x_{b,1}, y_{b,1})$ is the bank's remaining portfolio after meeting date-1 withdrawals:

$$x_{b,1} = \max\{x_{b,0} - \lambda_b c_{b,1}, 0\},$$

and

$$y_{b,1} = \begin{cases} y_{b,0} & \text{if} \quad x_{b,0} \geqslant \lambda_b c_{b,1}, \\ y_{b,0} - l_b & \text{if} \quad x_{b,0} < \lambda_b c_{b,1}, \end{cases} \tag{2.6}$$

[22]Following Diamond and Dybvig (1983), debt means that $c_{b,1}$ does not depend on λ_b (unless a run occurs), regardless of whether $c_{b,2}$ depends on λ_b or R. This focus on t-1 consumption is consistent with our definition of fund equity, which hinges on date-1 NAV $c_{f,1}(\lambda_f)$ directly depending on λ_f and and in turn on R in equilibrium because λ_f depends on R.

where l_b, the unit of projects that the bank has to liquidate at $t = 1$, is determined by

$$\lambda_b c_{b,1} = x_{b,0} + l_b - \frac{\phi}{2} l_b^2 \,, \tag{2.7}$$

if the bank is solvent at $t = 1$, and clearly $l_b = y_{b,0}$ if the bank fails.

According to Definition 2, bank debt provides liquidity when $E[c^*_{b,1}] > c_1(x^*_{b,0}, y^*_{b,0})$. Looking forward, we provide sufficient conditions to ensure that this is true in equilibrium at the end of this subsection. Nevertheless, the actual debt payment schedule shown in Table 1 suggests that the debt-issuing bank is indeed vulnerable to potential panic runs due to strategic uncertainty among late households.

The intuition of the strategic uncertainty and the underlying first-mover advantage is the following. suppose the bank has enough cash in the sense that $x_{b,0}$ is large enough to meet withdrawal by the π population of early households.[23] If a late household expects all other households to wait until $t = 2$, it is her best response to wait because that would allow her to get a strictly positive t-2 consumption level. Nevertheless, if she expects all other households to withdraw at $t = 1$, nothing would remain at $t = 2$ so that her best response would be to run as well. Thus, the design of $(c_{b,1}, c_{b,2})$ must take the probability of panic runs at $t = 1$ into account.

We work backwards by analyzing late households' run decision, the bank's optimal choice of debt face value $c_{b,1}$ and the portfolio choice problem. In doing so, we apply the global games technique following Goldstein and Pauzner (2005) to pin down the probability of panic runs. The advantage of using global games is to link bank run probability to fundamentals while preserving the panic-based nature of runs. This is important in our framework because it enables a parallel comparison between bank debt and fund equity in liquidity provision given the same fundamentals.

To formulate the analysis, we introduce a mild distributional assumption regarding households' private signal s_i. Specifically, we assume θ to be uniformly distributed, and that ε_i, the i.i.d. noise term, is uniformly distributed over $[-\varepsilon, \varepsilon]$, where ε is arbitrarily small. As shown in Goldstein and Pauzner (2005), under these mild distributional assumptions, there exists a unique t-1 equilibrium in which a panic run occurs when R is below a threshold R^*, and any equilibrium

[23] The coordination failure among late households is essentially the same when $x_{b,0}$ is small but the verbal discussion is more involved. And in a t-0 equilibrium, the bank always holds a sufficiently high $x_{b,0}$ to meet early household withdrawal.

must be such a threshold equilibrium.[24] Thus, we follow the same logic to show that such a threshold equilibrium exists in our framework, and then analyze the property of the threshold R^*.

Proposition 1. *Given the date-0 fund position $(x_{b,0}, y_{b,0})$, the promised debt value $c_{b,1}$, and the signal s_i received by households at $t = 1$, there exists a unique run threshold $R^* > \underline{R}$, where $\underline{R} > 0$ is given in the appendix.*

Proposition 1 shows that bank debt subjects the bank to panic runs, which is coordinated by fundamentals. An important observation from Proposition 1 is that the run threshold R^* can be either larger or smaller than 1. We depict the idea of Proposition 1 in Figure 1, where the red step curve illustrates the relationship between late households' withdrawal decision and economic fundamentals.

After illustrating that bank liquidity provision entails panic runs, we analyze how bank contract design and portfolio choice affect the magnitude of runs given economic fundamentals:

Proposition 2. *Given the date-0 fund position $(x_{b,0}, y_{b,0})$, the promised debt value $c_{b,1}$, and the signal s_i received by households at $t = 1$, the unique run threshold $R^*(c_{b,1}, x_{b,0})$ is increasing in $c_{b,1}$ and decreasing in $x_{b,0}$.*

Proposition 2 shows that a higher promised debt value $c_{b,1}$ renders panic runs more likely. On one hand, a higher $c_{b,1}$ directly indicates more liquidity provision at $t = 1$. On the other hand, the induced higher run threshold suggests more premature liquidation, hurting liquidity provision to early investors. It follows that panic runs are a disadvantage of using bank debt in providing liquidity – a key conceptual point of this paper. Proposition 2 also implies that stored cash $x_{b,0}$ helps mitigate panic runs. This observation stems from the role of stored cash in helping liquidity provision at $t = 1$. A higher amount of stored cash helps reduce the first-mover advantage and thus decreases the run threshold.

Proposition 2 is economically important because it illustrates that panic runs indeed comprise the main friction for bank debt to provide liquidity. To see this, consider the bank's optimization

[24] More formally, following Goldstein and Pauzner (2005), we need to assume an "upper dominance region," which means that late households never run when $R \to +\infty$, to ensure the existence of a threshold run equilibrium. This is a technical assumption to ensure that the global games technique works. Since this is a well understood technical point in the literature and is not crucial to our economic mechanism, we omit the details and refer interested readers to Goldstein and Pauzner (2005) for the economic motivation of that upper dominance region.

problem at $t = 0$. The representative bank solves the optimal debt face value $c_{b,1}$ and portfolio allocation $(x_{b,0}, y_{b,0})$ to maximize the expected utility of households:

$$\max_{c_{b,1},x_{b,0},y_{b,0}} \int_0^{R^*} u(c_{b,1})q(1)dG(R) + \int_{R^*}^{+\infty} (\pi u(c_{b,1}) + (1-\pi)u(c_{b,2}))\, dG(R) \tag{2.8}$$

where the function q is given by (2.5) and R^* is determined according to Proposition 1. As (2.8) indicates, a higher run possibility (i.e., a higher run threshold R^*) unambiguously leads to a lower ex-ante expected household utility. Thus, according to Proposition 2, at the margin, the bank has to either provide a lower deposit value $c^*_{b,1}$ or increase its cash position $x^*_{b,0}$ to reduce panic runs, both leading to lower equilibrium liquidity provision according to Definition 2.

Finally, we ensure that the bank in our model indeed provides liquidity ex-ante according to Definition 2. It suffices to give a set of sufficient conditions that ensures $E[c^*_{b,1}(R)] > c_1(x^*_{b,0}, y^*_{b,0})$ without fully solving (2.8).[25] When the distribution of $G(R)$ reflects sufficiently good fundamentals (e.g., $G(R)$ follows an exponential distribution with a sufficiently small rate), we have $c^*_{b,1} \to \frac{1}{\pi} > 1$ whereas $c_1(x^*_{b,0}, y^*_{b,0}) < 1$. Thus, by standard continuity argument, there exists a distribution $G(R)$ that ensures the bank to provide liquidity in our model. For any distribution $G'(R)$ that first-order stochastic dominates $G(R)$, the bank provides liquidity ex-ante.[26]

2.3 Fund Equity

The open-end mutual fund offers an NAV-based equity contract $(c_{f,1}(\lambda_f(R)), c_{f,2}(\lambda_f(R)))$, which is demandable at the end of each date and whose payments are the end-of-day NAVs.[27] To capture the essence of equity, we consider fully adjustable NAV in the sense that the contract payment is explicitly written on the number of shareholders who actually redeem at $t = 1$, denoted by λ_f. In practice, open-end mutual funds achieve this by "striking the NAV" – a standard industry practice during the trading day to form the estimated amount of redemption requests, perform the necessary asset transactions, and pre-calculate the end-of-day NAVs. We essentially take this process to be frictionless in the model. In equilibrium, withdrawals λ_f will in

[25] This approach is standard in the literature, for example, Theorem 3 in Goldstein and Pauzner (2005).

[26] Theoretically, we can also easily show that, under the same set of sufficient conditions, the bank provides a higher expected t-1 consumption than the autarky. See Appendix A for a formal analysis.

[27] Relatedly, Green and Lin (2003) considers an alternative, hypothetical equity contract implied by a direct mechanism design problem in the Diamond and Dybvig (1983) setting. As acknowledged by Green and Lin (2003), however, that hypothetical equity contract is not observed in reality.

turn be determined by fundamentals R, which is why contract payments are indirectly affected by R and take the form of $(c_{f,1}(\lambda_f(R)), c_{f,2}(\lambda_f(R)))$.[28]

We work backwards on the fund's optimal contract design and portfolio choice, taking household decisions into account. First, at $t = 1$, the fund pays out the end-of-day NAV_1 to redeeming households. The representative fund, which maximizes expected shareholder utility, will first deploy stored cash to meet redemptions at $t = 1$ because this avoids premature liquidation of the project that has a higher expected return and also a liquidation cost. If the cash stored no longer suffices to pay all redeeming households, the fund will resort to liquidating the project prematurely, raise cash, adjust the end-of-day NAV downwards, and pay all redeeming households the resulting NAV.[29] Hence, at $t = 1$, the fund NAV is determined by

$$\begin{aligned} c_{f,1}(\lambda_f) &= NAV_1(\lambda_f) \\ &= x_{f,0} + y_{f,0} - \frac{\phi}{2} l_f^2(\lambda_f) \\ &= 1 - \frac{\phi}{2} l_f^2(\lambda_f) \\ &= 1 - \frac{\phi}{2} (y_{f,0} - y_{f,1}(\lambda_f))^2 \,, \end{aligned} \tag{2.9}$$

where $l_f = y_{f,0} - y_{f,1}$ is the unit of prematurely liquidated projects at $t = 1$. $y_{f,1}$ is the fund's remaining position in the illiquid project after meeting date-1 redemptions, which is a function of λ_f in equilibrium. l_f also depends on λ_f and is determined by

$$\lambda_f NAV_1(\lambda_f) = \min\left\{\lambda_f, x_{f,0} + l_f - \frac{\phi}{2} l_f^2\right\} \,, \tag{2.10}$$

[28]Notice that this is an equilibrium outcome and not because contract payments are directly written on households' private signal s_i.

[29]In reality, the Investment Company Act of 1940 prohibits open-end mutual funds from borrowing or lending (i.e., using or extending credit lines), even within the same fund family. Within-fund-family overnight borrowing and lending is legal only upon application to and approval from the US SEC under emergency situations. The same legal restriction applies to using redemption gates to suspend daily redemption to shareholders. Mutual funds may also opt to redeem in-kind rather than redeem in-cash; however, to protect their reputation to facilitate future share distribution, mutual funds are extremely reluctant to use redemption in-kind even in bad times. Consistent with this argument, there has been no evidence of open-end mutual funds using redemption in-kind.

where the LHS is the total amount of cash distributed to redeeming investors and the RHS is the amount of available cash, both evaluated at the end of $t = 1$. And clearly, we have

$$y_{f,1} = \begin{cases} y_{f,0} & \text{if} \quad x_{f,0} \geqslant \lambda_f \, , \\ y_{f,0} - l_f & \text{if} \quad x_{f,0} < \lambda_f \, , \end{cases} \tag{2.11}$$

Two important remarks are in order. First, although households' private signal s_i at $t = 1$ is not contractable and cannot be written into the NAV, fundamentals R influence equilibrium payments through affecting the number of households who actually redeem at $t = 1$. Thus, the equilibrium equity contract payment is written in the form of $(c_{f,1}(\lambda_f(R)), c_{f,2}(\lambda_f(R)))$. Second, the value of projects that are not liquidated at $t = 1$ remains at 1 because they have not yet come to fruition. This valuation is reflected in the NAVs as a result of its flexibly adjusting contract value.

Without fully solving for the equilibrium, we first show that fund equity indeed provides liquidity in the sense of Diamond and Dybvig (1983) in any equilibrium:

Proposition 3. *Given any equilibrium fund portfolio $(x^*_{f,0}, y^*_{f,0})$, the fund provides liquidity ex-ante as defined in Definition 2, that is,*

$$E[c^*_{f,1}(\lambda_f(R))] > c_1(x^*_{f,0}, y^*_{f,0}) \, ,$$

where the t-1 NAV function $c^*_{f,1}(\cdot)$ is given by (2.9).

The proof of Proposition 3 is straightforward and thus we give it here to help build intuition. Directly comparing (2.9) and the liquidation value as defined in Definition 1 shows that $c_{f,1}(\lambda_f(R)) \geq c_1(x_{f,0}, y_{f,0})$, and this inequality takes a strict form if $y_{f,1} > 0$. Intuitively, unless the fund is fully liquidated, the end-of-day NAV promised to households who redeem at $t = 1$ is strictly higher than the liquidation value of the underlying fund portfolio. Notice that this statement is true regardless of how many households actually redeem, and thus is also true regardless of the fundamental. Taking expectations with respect to the fundamental thus yields the result.

The intuition behind Proposition 3 is that, by pooling resources among early and late households at the intermediary level, issuing demandable equity, and paying out early households at the NAV, the fund is able to provide liquidity insurance among all households exactly in the spirit of Diamond and Dybvig (1983).[30] Notably, the fund equity under its payment structure (2.9) does not have any debt-like feature, suggesting that a debt contract is not a necessary condition for liquidity provision. Again, the key is to create a pool of resources to share idiosyncratic liquidity risks, be it a bank or a fund.

Nevertheless, the characteristics of liquidity provision by bank debt and fund equity differ because the former pays a fixed face value unless default whereas the latter promises a flexible and information sensitive payment. The following lemma offers a first insight into the difference:

Lemma 1. *Given any fund portfolio $(x_{f,0}, y_{f,0})$, the consumption promised to early households is decreasing in the number of redeeming households, that is, $\partial c_{f,1}(\lambda_f)/\partial \lambda_f \leq 0$, and this inequality takes a strict form if $l_f > 0$.*

Lemma 1 illustrates that, by issuing equity, funds are able to flexibly adjust their actual, ex-post liquidity provision at $t = 1$ once liquidation losses realize. In contrast, banks would have to honor the fixed debt value $c_{b,1}$ to redeeming households unless the bank fails. As we will show later, the flexible contract value of equity allows funds to eliminate panic runs but comes at the cost of flows to fundamentals, which represent the comparative advantage and disadvantage of liquidity provision by equity-issuing intermediaries respectively.

Having analyzed the NAV at $t = 1$, the NAV at $t = 2$ is determined by

$$c_{f,2}(\lambda_f) = NAV_2(\lambda_f) = \frac{1}{1-\lambda_f}\left(x_{f,1} + y_{f,1}R\right), \tag{2.12}$$

where

$$x_{f,1} = \max\{x_{f,0} - \lambda_f c_{f,1}(\lambda_f), 0\},$$

and $y_{f,1}$ is given by (2.11).

Taken together, the NAV rules (2.9), (2.10), and (2.12) and Lemma 1 lead to an important result:

[30] Similarly to the analysis of bank above, the ability of fund equity liquidity provision is general and holds qualitatively if we instead compare the optimally determined $E[c^*_{f,1}(\lambda_f)]$ to the autarky outcome $c^*_{a,1}$ in equilibrium under certain sufficient conditions. See Appendix A for a formal analysis.

Proposition 4.

$$\begin{cases} NAV_1(\lambda_f) > NAV_2(\lambda_f) & \textit{if } R < 1\,, \\ NAV_1(\lambda_f) = NAV_2(\lambda_f) & \textit{if } R = 1\,, \\ NAV_1(\lambda_f) < NAV_2(\lambda_f) & \textit{if } R > 1\,. \end{cases}$$

Proposition 4 has two important implications. The first one is that fund equity with fully flexible NAV is not subject to panic runs. This is because any late households' redemption decision is uniquely determined by the comparison between NAV_2 and NAV_1, which in turn as shown by Proposition 4 solely depends on economic fundamentals R but not any strategic motives by households. In contrast, in the original Diamond and Dybvig (1983) framework, bank liquidity provision goes hand in hand with panic-based bank runs.

The second implication of Proposition 4 is that the comparison between NAV_2 and NAV_1 is directly linked to fundamentals R, which households infer from the signal they receive. This direct link between fund NAVs and fundamentals immediately leads to the friction underlying fund liquidity provision, as we illustrate below. In contrast, for a bank, the deposit value is indirectly linked to fundamentals, which serve as a coordination device for run decisions.

The above two implications play a key role in understanding late households' optimal redemption decision at $t = 1$ because they compare NAV_2 and NAV_1 to decide whether to redeem or not. Suppose that after observing s_i, w_f late households choose to redeem at $t = 1$, the total portion of redeeming households will be

$$\lambda_f = \pi + w_f$$

since early households always redeem. Given (2.9) and (2.12), the amount of late householders w_f who choose to redeem at $t = 1$ will be

$$\begin{cases} w_f = 0 & \text{if} \quad u(c_{f,1}(\lambda_f)) < E[u(c_{f,2}(\lambda_f))|s_i]\,, \\ w_f \in (0, 1-\pi) & \text{if} \quad u(c_{f,1}(\lambda_f)(1-\kappa w_f)) = E[u(c_{f,2}(\lambda_f))|s_i]\,, \\ w_f = 1-\pi & \text{if} \quad u(c_{f,1}(\lambda_f)(1-\kappa w_f))) > E[u(c_{f,2}(\lambda_f))|s_i]\,. \end{cases} \tag{2.13}$$

Solving (2.13) with Proposition 4 in mind yields the late households' optimal redemption decision at $t = 1$:

Proposition 5. *Given date-0 fund position* $(x_{f,0}, y_{f,0})$ *and signal* s_i *received by households at* $t = 1$*, late households withdrawing from the fund amount to*

$$w_f^* = \begin{cases} 0 & \text{if} \quad R \geq 1 \\ \frac{1}{2}\left(1 - \pi - \sqrt{\frac{2}{4(1-x_{f,0})(R-1) + \kappa(1-\pi)^2}}\right) & \text{if} \quad R < 1 \text{ and } x_{f,0} \geq \pi \\ \frac{1-R}{\kappa} & \text{if} \quad R < 1 \text{ and } x_{f,0} < \pi \end{cases} \tag{2.14}$$

subject to $w_f^* \leq 1 - \pi$.

We denote the negative relationship between redemptions w_f and fundamentals R as the flows-to-fundamentals relationship.[31] Notice that in contrast to withdrawals induced by bank runs, there is no strategic element driving the flows to fundamentals. Figure 1 illustrates the result of Proposition 5 for funds and Proposition 1 for banks. The blue and red lines indicate the relationship between equilibrium outflows and economic fundamentals for debt-issuing banks and equity-issuing funds respectively. The abrupt increase in liquidations at R^* for the red line indicates the presence of bank runs whereas the negative slope of the blue line corresponds to the flows-to-fundamentals relationship for funds.

Given the flows-to-fundamentals relationship, the fund liquidates the project in order to meet redemption requests. The following relationship between fund flows and fundamentals results:

Proposition 6. *Given the date-0 fund position* $(x_{f,0}, y_{f,0})$ *and the signal* s_i *received by households at* $t = 1$*, the fund liquidates*

$$l_f^* = \begin{cases} 0 & \text{if} \quad x_{f,0} \geq \pi + w_f^* \\ \frac{1 - \sqrt{1 + 2\phi\left(\pi^2 + x_{f,0} - \pi(1 + x_{f,0})\right)}}{(1-\pi)\phi} & \text{if} \quad R \geq 1 \text{ and } x_{f,0} < \pi + w_f^* \\ \frac{1 - \sqrt{1 + 2\phi\left((\pi + w_f^*)^2 + x_{f,0} - (\pi + w_f)(1 + x_{f,0})\right)}}{(1 - \pi - w_f^*)\phi} & \text{if} \quad R < 1 \text{ and } x_{f,0} < \pi + w_f^* \end{cases} \tag{2.15}$$

units of project at $t = 1$ *subject to* $l_f^* \leq y_{f,0}$*, where* w_f^* *is given by Proposition 5.*

[31] Further, w_f^* is also increasing in $1 - \pi$ and ϕ while decreasing in $x_{f,0}$ and κ.

Proposition 6 is graphically illustrated in Figure 2 along with the corresponding result for debt-issuing banks. It shows both bank debt and fund equity are subject to premature liquidations albeit with different characteristics arising from panic runs at banks and the flows-to-fundamentals relationship at funds.[32]

Taken together, Propositions 5 and 6 show that liquidity provision by equity-issuing funds is exposed to premature liquidations arising from the sensitivity of equity valuation to fluctuations in economic fundamentals. This flow-to-fundamentals friction hurts NAV_1 and thereby limits equilibrium liquidity provision according to Definition 2.

Proposition 6 also suggests that more stored cash between $t = 0$ and 1 can alleviate the sensitivity of liquidations to fundamentals through three channels. First, when $R \geq 1$ but $x_{f,0} < \pi$, the fund can directly use stored cash to meet early household redemptions, incurring a lower liquidation l_f. Second, a higher $x_{f,0}$ translates to lower $y_{f,0}$, which leads to a lower premature project liquidation l_f. Finally, because the marginal project liquidation value is decreasing in the unit of projects liquidated, a higher $x_{f,0}$ also implies a weaker flow-to-fundamentals relationship. The fund will take these channels into account when maximizing ex-ante household utility to choose a storage-project mix at $t = 0$ that optimally trades off liquidity provision to early households and return generation to late households.

Taking the above analysis into account, the representative fund solves the optimal portfolio allocation $(x_{f,0}, y_{f,0})$ at $t = 0$ to maximize the expected utility of all households:

$$\max_{x_{f,0}, y_{f,0}} E\left[\lambda_f u(c_{f,1}(\lambda)) + (1 - \lambda_f) u(c_{f,2}(\lambda))\right] \tag{2.16}$$

subject to (2.9), (2.11), (2.12), and (2.15). As we have already shown in Proposition 3, fund equity indeed provides liquidity ex-ante in any equilibrium.

3 Liquidity Provision Index

This section develops an empirical measure for the notion of liquidity provision conceptualized in our model. In line with Definition 2, the LPI captures how much the expected contract

[32] Also note that l_f^* is increasing in ϕ, decreasing in $x_{f,0}$, and concave in R due to the flexible adjustment of NAV_1.

payment by an intermediary exceeds the direct liquidation value of the underlying assets per dollar invested. In other words, it is the empirical proxy for the expected improvement in short-term consumption $E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0})$ as in Definition 2.[33]

The LPI is designed to quantify iquidity provision by both demandable debt and demandable equity issuing financial institutions in practice. In other words, the measurement of liquidity provision remains consistent in the presence of regulations and frictions that may influence investors' redemption decisions and intermediary's portfolio choice in practice. The key lies in directly using the observed distribution of outflows to calculate expected contract payments. This method is consistent with the theory because fundamentals R affect liquidity provision through determining investors' outflows.[34] At the same time, observed flows already reflect the equilibrium outcome so that we can remain agnostic about the measurement of fundamentals R and its empirical mapping to investor decisions, in which factors outside of our model (e.g. regulation) may also play a role. We also use the actual portfolio holdings of banks and funds, which may arise from the contractual form of liabilities as well as other institutional features. In this sense, the LPI on its own is not meant as an identification strategy for the pure effect of debt versus equity in our model. To this end, we will apply the LPI in additional tests in Section 4.

In this section, we first illustrate the construction of the LPI for debt and equity funded intermediaries with a simple example in Subsection 3.1. Then we provide a formal step-by-step explanation of the construction and discuss the connection to the theory in Subsection 3.2. The LPI is generally applicable, but for the ease of exposition, we will henceforth use banks and funds interchangeably with demandable debt and demandable equity issuing intermediaries.

3.1 An Example

Consider a hypothetical mutual fund that holds 10% cash and 90% of corporate bonds, where the corporate bonds are illiquid and can only be converted to 70% of their fair value upon early redemption. When all investors demand to redeem their shares early, the per unit contract payment would only be $1 \times 0.1 + 0.7 \times 0.9 = 0.73$ because all illiquid assets would be pre-maturely

[33] The LPI remains agnostic about the long run consumption (i..e, $E[c^*_{k,2}]$ in the model), which is an important but separate question.

[34] See Propositions 1 or bank debt and 5 for fund equity.

liquidated and incur a discount. Notice that an investor directly holding 0.1 of cash and 0.9 of the illiquid assets receives the same 0.73 if she has to liquidate early because she would also have to sell her entire portfolio as well. In this sense, holding a share at the fund does not improve the consumption value when all other fund investors also withdraw early because all liquidity risk is systematic and liquidity insurance is ineffective.

However, as long as not all investors redeem early, idiosyncratic liquidity risk is pooled at the fund level and holding fund shares reduces the average discount suffered and improves the amount of cash obtainable upon short notice. For example, if total outflows amount to less than 10% of the funds assets, they can be met by just using the fund's cash so that no liquidation discounts are incurred. If redemptions exceed 10%, more illiquid assets would have to be liquidated and the contract payment would decrease until it reaches 0.73 at a 100% outflows. This trend is depicted graphically by the blue line in the upper panel of Figure 3. The dotted line on the same graph indicates the liquidation value of the underlying portfolio, which is 0.73.

The difference between the solid blue line and the dotted blue line represents the fund's contribution to liquidity for a given level of outflows. Hence, we can integrate over the distribution of outflows to calculate expected liquidity provision. Note that Figure 3 does not imply that all outflow volumes will actually occur with equal probability. Rather, we let the data speak by taking the empirical distribution of fund flows as the equilibrium outcome of early redemptions. This allows us to calculate the LPI for each fund as the difference between the expected contract payment and the direct liquidation value per dollar investment.

The LPI for banks can be constructed in a very similar way. Consider a hypothetical bank with 10% cash and 90% corporate loans, where the corporate loans can only be converted to 60% of their fair value upon early redemption. The direct liquidation value of the portfolio is $1 \times 0.1 + 0.6 \times 0.9 = 0.64$. The bank can provide a higher contract payment than the direct liquidation value as long as not all depositors prematurely withdraw. However, the liquidation value does not continuously decrease with the proportion of outflows because the face value of deposit contracts is fixed until the default threshold is crossed. The contract payment and direct liquidation value for our hypothetical bank are indicated by the solid red line and the dotted red line in the lower panel of Figure 3. Finally, we can use the bank's distribution of deposit flows to calculate by how much the expected contract payment exceeds the direct liquidation value of assets, which is the bank LPI.

3.2 Construction

More formally, we can generalize the LPI construction into three steps. Recall that our goal is to measure how much the expected contract payment by an intermediary exceeds the direct liquidation value of the underlying assets per dollar invested, i.e., the empirical proxy of $E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0})$.

Step 1: The first step is to calculate the contract payment by outflows as in Figure 3. This requires knowing how much of each asset is liquidated to meet a given proportion of redemptions/withdrawals and at what cost.

Let $\hat{\lambda}_i$ be the amount of outflows at bank (fund) i as a percentage of total assets. $\hat{\lambda}_i$ is the empirical counterpart to the number of investors that withdraw in the model, λ_k. We also extend the two-asset world of the model to allow for a complete portfolio distribution. Let vector Y_{it} be bank (fund) i's asset portfolio distribution at time t with Y_{ijt} denoting the portfolio weight on asset $j \in J$. The asset index, j, is ranked in increasing order with the haircut of the asset h_{jt}, i.e., $h_{jt} \leq h_{j't}$ for any $j \leq j'$. In other words, more liquid assets have smaller indexes. The ranking of liquidation haircuts and the distribution of assets determine which assets are to be liquidated for a given outflow as banks (funds) meet redemption requests by selling more liquid assets first.[35] To illustrate, recall the above example where a bank (fund) holding 10% of cash can meet outflows of up to 10% without losses. As outflows increase, banks (funds) use increasingly more illiquid asset categories to meet their redemption requests until even the most illiquid assets are liquidated when outflows (hypothetically) reach 100%.

While the contract payment of fund shares follows that of fund assets due to flexible NAVs, bank deposits do not lose value until there are not enough funds left to pay the face value. To focus on the difference in contract forms, we consider liquidity provision by uninsured deposits throughout our calculations, for which the contract payment will fall below the face value if proceeds from asset liquidations are insufficient. Nevertheless, the LPI framework is able to accommodate the presence of insured deposits and their effect on uninsured deposits (see Appendix B for details). Denoting $1 - H_t(\hat{\lambda}_i, Y_{it})$ as the contract payment for investors withdrawing (redeeming) a dollar of deposits (fund shares) when outflows are $\hat{\lambda}_i$ under bank (fund) asset

[35] Jiang, Li and Wang (2020), among others, identify this order of liquidation in the data.

distribution Y_{it}, we have for bank i:

$$H_t(\hat{\lambda}_i, Y_{it}) = \begin{cases} 0 & \text{if the bank survives} \\ \sum_{j=1}^{J} Y_{ijt} h_{jt} & \text{if the bank fails}\,, \end{cases} \tag{3.1}$$

and for fund i:

$$H_t(\hat{\lambda}_i, Y_{it}) = \sum_{j=1}^{J_{\hat{\lambda}}-1} Y_{ijt} h_{jt} + \widehat{Y_{iJ_{\hat{\lambda}}t}} h_{J_{\hat{\lambda}}t}\,, \tag{3.2}$$

where $\widehat{Y_{iJ_{\hat{\lambda}}t}}$ is the position of the least liquid asset being liquidated to meet $\hat{\lambda}_i$ while assets more liquid than $J_{\hat{\lambda}}$ are fully liquidated.

Step 2: Since contract payments vary with outflows, the expected contract payment depends on the distribution of outflows. Our model predicts how early redemptions/withdrawals vary with economic fundamentals for a given asset portfolio of a bank (fund). Since economic fundamentals are difficult to quantify, our empirical approach directly measures the realized flows, which is the equilibrium outcome of the decision-making process. In reality, institutional features like implicit guarantees at banks and sticky NAVs at mutual funds may also influence investor flows. These effects are incorporated in the observed outflows we use, which renders the LPI an accurate measure of liquidity provision in practice. Nevertheless, we will introduce further tests based on the LPI in order to isolate the pure effect of debt versus equity in Section 4.

We estimate the distribution of observed outflows for each fund and bank ,$F(\hat{\lambda}_i)$, and obtain their expected contract payment as

$$\int (1 - H_t(\hat{\lambda}_i, Y_{it})) dF(\hat{\lambda}_i)\,. \tag{3.3}$$

Viewed through the lens of the model, this expected contract payment of bank deposits (fund shares) proxies for the expected consumption available to bank depositors (fund share holders) withdrawing prematurely at the end of the first period, $E[c^*_{k,1}(\lambda)]$. Conceptually, the difference between $t = 0$ and 1 in Diamond and Dybvig (1983)-type models captures an infinitely short time period. In the same vein, our empirical construction of the contract payment as in (3.3) also reflects a very short time period in which the net return on assets are negligible.[36,37]

Step 3: Finally, we calculate the direct liquidation value of the bank's (fund's) assets and substract it from the expected contract payment by the bank (fund). Notice that the direct liquidation value of the bank (funds) is equivalent to the effective contract payment of the bank (fund) where all investors withdraw (redeem), i.e., $\hat{\lambda}_i = 1$, and all assets are sold off at short notice, i.e., $1 - H_t(1, Y_{it})$.

Taken together, the LPI of bank (fund) i at time t is its expected contract payment relative to the direct liquidation value of its assets:

$$LPI_{it} = \int (1 - H_t(\hat{\lambda}_i, Y_{it}))dF_i(\hat{\lambda}_i) - (1 - H_t(1, Y_{it})), \tag{3.4}$$

where $F_i(\hat{\lambda}_i)$ is the estimated distribution of bank deposit (fund) outflow $\hat{\lambda}_i$ and $H_t(\hat{\lambda}_i, Y_{it})$ is given by (3.1) and (3.2).

To summarize, we have the following close mapping between the theory and empirics as shown in Table 2:

Table 2: Mapping between equilibrium outcomes and empirical counterparts

	Outflows	Asset holdings	Contract Payment	Liquidation Value
Theory	λ_k	$(x^*_{k,0}, y^*_{k,0})$	$E[c^*_{k,1}(\lambda_k)]$	$c_1(x^*_{k,0}, y^*_{k,0})$
Empirics	$\hat{\lambda}_i$	Y_{it}	$\int(1 - H_t(\hat{\lambda}_i, Y_{it}))dF(\hat{\lambda}_i)$	$1 - H_t(1, Y_{it})$

[36]This is consistent with our model that between $t = 0$ and 1, the investment project has not yet come to fruition and thus retains a value of one, and thus the fund's NAV is also one at the beginning of $t = 1$. For a bank, although the level of the optimal deposit value at $t = 1$ can be theoretically different from one in a static setting following Diamond and Dybvig (1983), the effective ultra-short-term net asset return is still negligible.

[37]While the focus of the LPI is short-term liquidity provision, or $E[c^*_{k,1}]$, the long run consumption, $E[c^*_{k,2}]$, is also an important element of our model and relates to the cost of liquidity provision.

4 Liquidity Provision by Bank Debt and Fund Equity

In this section, we apply the LPI to compare and contrast liquidity provision by debt and equity issuing financial institutions in practice. Subsection 4.1 introduces the data, while Subsection 4.2 presents the baseline estimates of liquidity provision by commercial banks and bond mutual funds.

To isolate the effect of debt versus equity conceptualized in our model, we conduct a number of tests to purge out the effect of deposit insurance and non-deposit bank liabilities in Subsection 4.3. In Subsection 4.4, we further apply the LPI to MMFs. The difference-in-difference analysis we perform around the 2016 MMF Reform identifies the effect of debt versus equity funding on liquidity provision.

We proceed to analyze the determinants of bank and fund LPI variations in Subsection 4.5. Finally, Subsection 4.6 examines liquidity provision in the time series and provides evidence for post-crisis liquidity regulation and Quantitative Easing to have narrowed the gap in liquidity provision between banks and bond funds from 2011 to 2017.

4.1 Data

We use bank call reports and mutual fund holdings from the CRSP database to obtain the distribution of assets A_{it} for banks and funds respectively. Please refer to Table **??** for a detailed mapping between balance sheet variables and our asset categories.

For haircuts, we use collateral haircuts in repo markets and discounts on loan sales in secondary markets. Similar to Bai, Krishnamurthy and Weymuller (2018), we collect securities haircuts from the New York Fed's repo data series, commercial loan haircuts from the Loan Syndications and Trading Association and real estate loan haircuts from the Federal Home Loan Banks. Also following Bai, Krishnamurthy and Weymuller (2018), we smooth the haircut series using principal components to remove outliers.[38] Figure 4 plots the haircut series for different asset categories over our sample period. As expected, safe assets such as Treasuries have smaller and less volatile haircuts whereas relatively illiquid assets such as loans have higher and more volatile haircuts.

[38]Specifically, we first extract the first principal component PC_t based on the panel of haircuts. Then, we regress each raw haircut time-series on PC_t and use the predicted value of the regression as the smoothed haircut.

To account for the potential price impact of bank and fund asset liquidations on the effective haircuts, we repeat our LPI calculation with price-impact adjusted haircuts in Appendix **??**.

Regarding flows, we calculate uninsured deposit flows as percentage change in total uninsured deposits using bank call reports from 2010 to 2017. We augment the resulting flow distribution with data on bank failures. For each mutual fund, flows are calculated using its monthly (portfolio-level) total net asset changes using the CRSP database. For positive flows, we assume that no assets are prematurely liquidated so that the effective contract payment is equal to one.

4.2 Baseline LPI Estimates for Banks and Bond Mutual Funds

We follow the procedure described in Section 3.2 to construct the quarterly LPI for each bank and fund in our sample.

We first construct the bank- and fund-level contract payments as a function investor outflows. We report the aggregate result in Figure 6. This aggregate result can be thought of as a representative bank holding the average portfolio of the banking sector and a representative fund holding the average portfolio as the bond mutual fund sector. Similar to the example in Section 3.1, the contract payment for funds continuously declines with early redemptions because the value of fund shares adjusts to reflect the incurred haircuts when increasingly illiquid assets are liquidated. Banks also sell more illiquid assets as deposit withdrawals increase but the uninsured debt contract guarantees a constant contract payment until the bank defaults, i.e., deposit withdrawals cannot be met at their promised value.

When outflows reach 100%, an intermediary has to fully liquidate the underlying portfolio, and the contract payment of uninsured bank deposits and mutual fund shares are 78 and 95 cents per dollar respectively as shown in Figure 6. These are also the direct liquidation values of the representative fund and bank's underlying portfolio, showing that bank assets are relatively more illiquid than fund assets. One rationale provided by our theory is that the use of debt versus equity contracts induce different incentives for holding liquid assets. Deposit-funded banks hold liquid assets to reduce the incidence of panic runs whereas equity funded funds hold liquid assets to alleviate flows to fundamentals. The aggregate result is consistent with the latter concern being more pronounced in the data.

We then estimate the flow distribution for each bank and fund in our sample. Figure 5 plots the distribution of bank and fund flows from 2011 to 2017 for a representative bank and a representative fund. As explained before, the empirical flow distributions capture the effect of economic fundamentals on investor flows in equilibrium. In practice, imperfections in the adjustment of fund NAVs could also have influenced the observed outflows. These imperfections could arise from NAV adjustments being lagged as in Chen, Goldstein and Jiang (2010) and Goldstein, Jiang and Ng (2017) or not being perfectly forward looking as in Zeng (2017). Since the observed outflows would be more pronounced than under perfectly flexible NAVs, the fund LPI we obtain can be seen as a lower bound to fund LPI with frictionless NAVs.[39]

Finally, we use the estimated flow distribution and contract payment curve for each bank and fund to calculate the LPI for each intermediary-quarter. Figure 7 plots the distribution of average bank and fund level LPIs. It shows that from 2011 to 2017, the average dollar invested in funds and banks provide 4 cents and 22 cents of liquidity respectively. In other words, the LPI shows that liquidity provision by mutual funds is economically significant and comprises about a fifth of that by commercial banks per unit investment.

These LPI estimates are realistic in the sense that they can accommodate the influence of real-life frictions and the regulatory environment on liquidity provision by banks and funds. As explained in Appendix B, deposit insurance along with other similar regulatory policies do not affect the validity of the LPI construction. This feature of the LPI is very valuable for quantifying liquidity provision in practice.

4.3 Effect of Deposit Insurance and Non-deposit Liabilities

To further determine the pure effect of using debt versus equity funding on liquidity provision, we conduct a number of additional tests. First, our calculation only considers the uninsured portion of deposits of bank LPI. However, regulatory features such as deposit insurances and non-deposit liabilities may indirectly affect the magnitude of LPI through bank portfolio choice as well. Since the ideal experiment of the same bank operating with and without deposit insurance does not

[39]Going forward, the introduction of swing pricing could potentially alleviate frictions in the adjustment of NAVs and improve the liquidity provision capacity by funds. Evidence for outflows becoming less negative following the introduction of swing pricing has been established in the UK market by Jin et al. (2020).

exist, we perform two tests to show that regulation can only explain a limited portion of the difference in liquidity provision by debt versus equity funded intermediaries.

We first relate bank-level LPIs to the ratio of insured deposits in the data and project the LPI that would have applied to a bank without insured deposits in Table 3. The constant term in column (1), which is statistically significant, indicates that without deposit insurance, the LPI for uninsured deposits at an average bank in our sample would be 0.16, compared to the average LPI of 0.22 for banks. Similarly, column (2) indicates that a linear-projected bank without insured deposits or non-deposit liabilities would have had an LPI of 0.15 for its uninsured deposits. In other words, the bulk of bank LPI is not driven by deposit insurance, and the ratio of bank versus fund LPI increases from one-fifth to one-quarter when the effect of deposit insurance and non-deposit liabilities at banks is removed.

4.4 Application: MMF LPI around the MMF Reform

There may be remaining concerns of regulatory differences other than deposit insurance such as implicit guarantees. To this end, we apply the LPI to a laboratory of MMFs before and after the MMF Reform to isolate the effect of debt versus equity in liquidity provision. In October 2016, the Securities and Exchange Commission implemented the MMF Reform. Among other changes, institutional prime MMFs were required to switch from reporting a $1 fixed share price to floating net asset values, which effectively corresponds to a switch from demandable debt to equity contracts in our framework. Importantly, among other dimensions of the reform, only the floating NAV rule did not apply to retail prime funds who continued reporting a $1 fixed share price. This setting naturally lends itself to a difference-in-differences analysis with institutional and retail prime funds as treatment and control groups to study the difference in liquidity provision by debt and equity funded intermediaries.

Figure 10 plots the LPI of institutional and retail prime MMFs from three years before to three years after the reform.[40] The lower panel repeats the analysis with the subset of funds that appeared in both pre and post reform periods. For both samples, institutional and retail prime fund LPI largely followed the same pattern before October 2016, agreeing with the parallel

[40] To account for changes in the sensitivity of fund flows, the flow distribution for the LPI calculation are constructed separately for the pre and post reform periods respectively.

trends assumption. Since the onset of the reform, institutional MMFs experienced a significantly larger drop in LPI than retail funds.

We corroborate our findings with a formal difference-in-differences test in Table 4. The coefficients on the interaction term reflects the change in LPI due to a switch from debt to equity contracts. Using the full sample of funds, the baseline result in Column (1) shows that the LPI drops by 40/(225-13)=19% when institutional MMFs switch from fixed to floating NAVs. This result confirms that demandable equity can provide significant amounts of liquidity, albeit 19% less than that of demandable debt. To ensure that flows during the implementation period do not drive the results, Columns (3) and (4) repeat the analysis excluding flows from August to October 2016. Columns (5) and (6) shifts the treatment date a year back to account for potential anticipation effects. Even-numbered columns focus on the subsample of funds appearing in both pre and post period respectively. The coefficients remain statistically significant and similar in magnitude.

4.5 Determinants of Liquidity Provision in the Cross-section

We proceed with the determinants of variations in LPI in the cross section of funds and banks. For banks, as seen in Table 3, deposit insurance cannot fully explain but has a positive effect on liquidity provision, which is consistent with lower bank run probability allowing for more liquidity provision in the theory. The same trend is also depicted in the upper panel of Figure 9. Other factors, such as leverage ratio and the proportion of non-deposit funding may also affect the probability of runs while banks of different asset sizes may be subject to different regulatory constraints. The lower panel of Figure 9 shows that the relationship between LPI and deposit insurance remains robust to the inclusion of these variables.

Regarding mutual fund fund LPIs, our model shows that liquidity provision by equity is constrained by flows to fundamentals. In line with the theory, we find that the more sensitive fund flows are to changes in fund returns, the lower its liquidity provision (see Figure 8). While fund returns are used as an empirical proxy for economic fundamentals, other features of the fund, such as size, age and expense ratio, may vary simultaneously. The lower panel of Figure 8 shows that the negative relationship between liquidity provision and fund flow sensitivity is preserved after adding fund-level characteristics as controls. The corresponding regression results in Table 5 corroborate the results.

4.6 Liquidity Provision in the Time-series

Finally, we examine how liquidity provision has evolved over time. Plotting the quarterly weighted average of bank and fund level LPIs, Figure 11 shows that the difference has been narrowing over time with bank LPI sharply decreasing from 0.285 in 2011 to 0.190 in 2017 and fund LPI increasing from 0.039 in 2011 to 0.043 in 2017. In other words, within six years, liquidity provided by an average dollar invested in funds has approximately increased from a seventh to a quarter of an dollar invested in commercial banks.

While a full characterization of the trend's determinants is beyond the scope of this paper, we highlight that changes in the regulatory landscape have had a significant impact on the capacity of liquidity provision by commercial banks.

First is the increase in central bank reserves following Quantitative Easing. Excess reserves held with the Federal Reserve are liquid assets on bank balance sheets. The overall effect of more reserves on liquidity provision can go in two different ways. It could have a positive effect because, as Proposition 2 suggests, more liquid bank balance sheets are less prone to runs. This is illustrated by the rightward shift of the default threshold for outflows from $\hat{\lambda}$ to $\hat{\lambda}'$ in Figure 12. On the other hand, a portfolio with more reserves also has a higher liquidation value under direct holding by investors, which decreases the potential capacity for liquidity provision by the intermediary. This effect is reflected by the increase in liquidation value of the bank portfolio from C to $C^{'}$ in Figure 12. Correspondingly, the potential increase and decrease in liquidity provision are indicated by the areas shaded in green and orange respectively.

Empirically, we find evidence consistent with the latter effect being dominant, i.e., QE decreases the capacity of liquidity provision by banks. As shown in Figure 13, the expansion in excess reserves from \$1 trillion in 2011Q3 to more then \$2.5 trillion in 2014Q3 is mirrored by a corresponding sharp fall in bank LPI during the same period. Sorting banks into quartiles of reserve uptake as a proportion of balance sheet size, we observe that the LPI drops consistently more for banks in the upper quartile, which lends further support for the aggregate effect. Therefore, QE lowers bank LPI through raising the liquidation value under direct holding, which limits how much banks can contribute to liquidity provision in non-default states. In the limit, narrow banks, which only hold liquid assets such as reserves provid negligible amounts of liquidity relative to traditional commercial banks.

In this context, the Liquidity Coverage Ratio (LCR) has similar effects on bank liquidity provision as Quantitative Easing. In the U.S., the LCR stipulates that banks with \$250 billion or more in total assets or \$10 billion or more in on-balance sheet foreign exposures are required to hold sufficient amounts of High Quality Liquid Assets (HQLA), which include cash, central bank reserves, and some agency MBS, to cover expected net cash outflows for a 30-day stress period. Banks with \$50 billion or more in consolidated assets are also subject to a less stringent LCR requirement. Similar to the QE case, the LCR requires large banks to hold a higher fraction of liquid assets on their balance sheets, which raises the default threshold in terms of outflows but also increases the liquidation value of the benchmark asset portfolio (see Figure 12).

We find evidence for an overall negative impact of the LCR on bank liquidity provision within our sample period. Figure 14 shows that banks most impacted by the LCR, i.e., above \$250 billion in total assets, also experience the most pronounced decline in LPI relative to those without and with a less stringent LCR requirement. This is again consistent with the interpretation that the LCR moves commercial banks more towards a narrow-banking business model, for which the gap between the liquidation value of bank assets and the contract payment of bank liabilities (i.e., deposits) is limited and liquidity provision is diminished.

5 Conclusion

This paper demonstrates that open-end equity issued by non-bank intermediaries is able to provide liquidity in the sense of Diamond and Dybvig (1983) just like demandable debt issued by the traditional banking sector. Liquidity creation stems from the pooling of idiosyncratic liquidity shocks at the intermediary level, which occurs independently of the claims issued by the intermediary as long as they are redeemable at short notice. The characteristics of liquidity provision, however, are different. Equity is not prone to panic runs as in the case of debt because flexible NAVs removes the first mover advantage in redemptions. However, the continuous adjustment of equity's contract value also renders investor flows and liquidity provision more sensitive to fluctuations in the economy.

Based on the theory, we develop the Liquidity Provision Index (LPI) as a parsimonious measure of liquidity creation across different types of intermediaries. It captures the extra proceeds an investor expects to obtain by withdrawing a debt or equity claim from an intermediary relative to directly holding and selling the underlying portfolio of assets herself. Applied to deposit-issuing commercial banks and equity-issuing bond mutual funds, we find that the average LPI of funds is economically significant and about a quarter of the LPI of banks at the end of 2017. The LPI gap between banks and funds has also been continuously narrowing over time, coinciding with an increase in liquid assets on bank balance sheets following Quantitative Easing and the Liquidity Coverage Ratio. These results highlight a new side effect of unconventional monetary policy and post crisis liquidity regulation.

The migration of liquidity provision away from deposit-issuing banks to equity-issuing financial institutions like bond mutual funds bears far-reaching implications. Not only will liquidity provision become more exposed to volatile flows to fundamentals, as we have shown, assets held by intermediaries will also become more prone to premature liquidations as the underlying economy deteriorates. This was already evident during the Covid-19 crisis, when mutual funds suffered heightened outflows, collectively liquidated their portfolios, and induced significant strains in Treasury and corporate bond markets Ma et al. (2019). Therefore, the consequences of increased reliance on liquidity transformation by equity-issuing non banks provide important considerations for the regulation of banks and financial institutions going forward.

Figure 1: Investor Withdrawals and Economic Fundamentals

This graph depicts withdrawals (ω) by late households against variation in economic fundamentals (R). The blue line represents the equilibrium withdrawal decisions of fund investors, where the negative slope indicates a flows-to-fundamentals relationship for funds. The red line corresponds to withdrawal decisions of bank investors, where the abrupt rise in withdrawals at R^* indicates the presence of panic runs, with R^* being the run threshold.

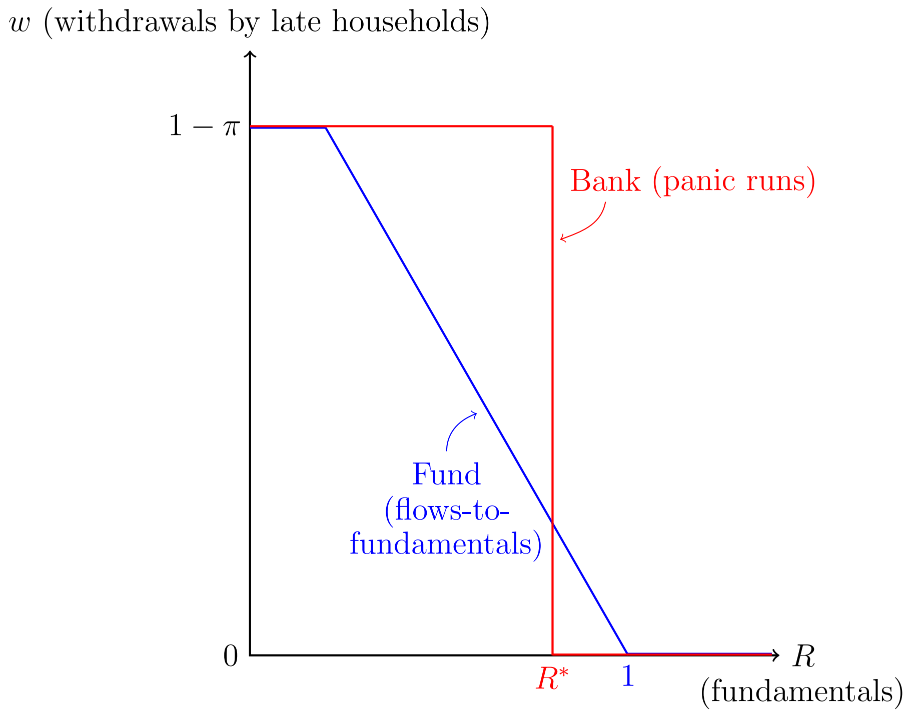


Figure 2: Premature Liquidations and Economic Fundamentals

This graph depicts the level of premature liquidations (l) against economic fundamentals (R). The blue line represents the premature liquidations at the fund, where the negative slope arises from a flows-to-fundamentals relationship. The red line corresponds to premature liquidations at the bank, where the abrupt rise in liquidations at R^* indicates the presence of panic runs, with R^* being the run threshold. Note that the fund curve is currently depicted under $x_{f,0} \geq \pi$. The flat portion for funds (when $R > 1$) is strictly higher than 0 if $x_{f,0} < \pi$.

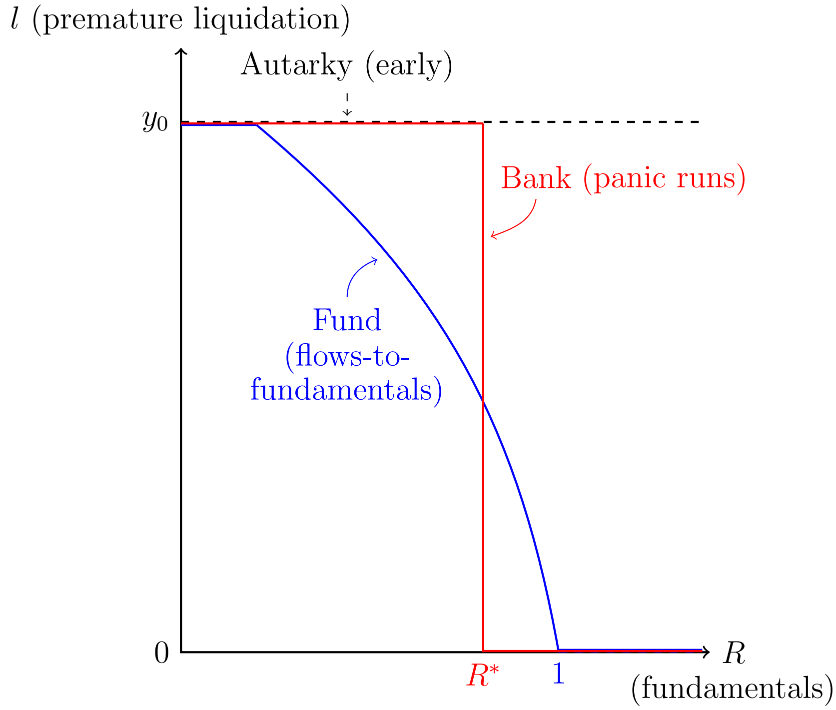


Figure 3: Contract Payment, Liquidation Value and Liquidity Provision (Example)

The upper panel plots the contract payment of fund shares for a given proportion of outflows. The hypothetical fund considered holds 10% of cash and 90% of corporate bonds, where the latter incurs a haircut of 30% upon early redemption. The dotted line is the liquidation value of the portfolio of assets when directly held and sold by an investor at short notice. The lower panel plots the contract payment of bank deposits for a given proportion of outflows.The hypothetical bank considered holds 10% of cash and 90% of corporate loans, where the latter incurs a haircut of 40% upon early redemption. The dotted line is the liquidation value of the portfolio of assets when directly held and sold by an investor at short notice.

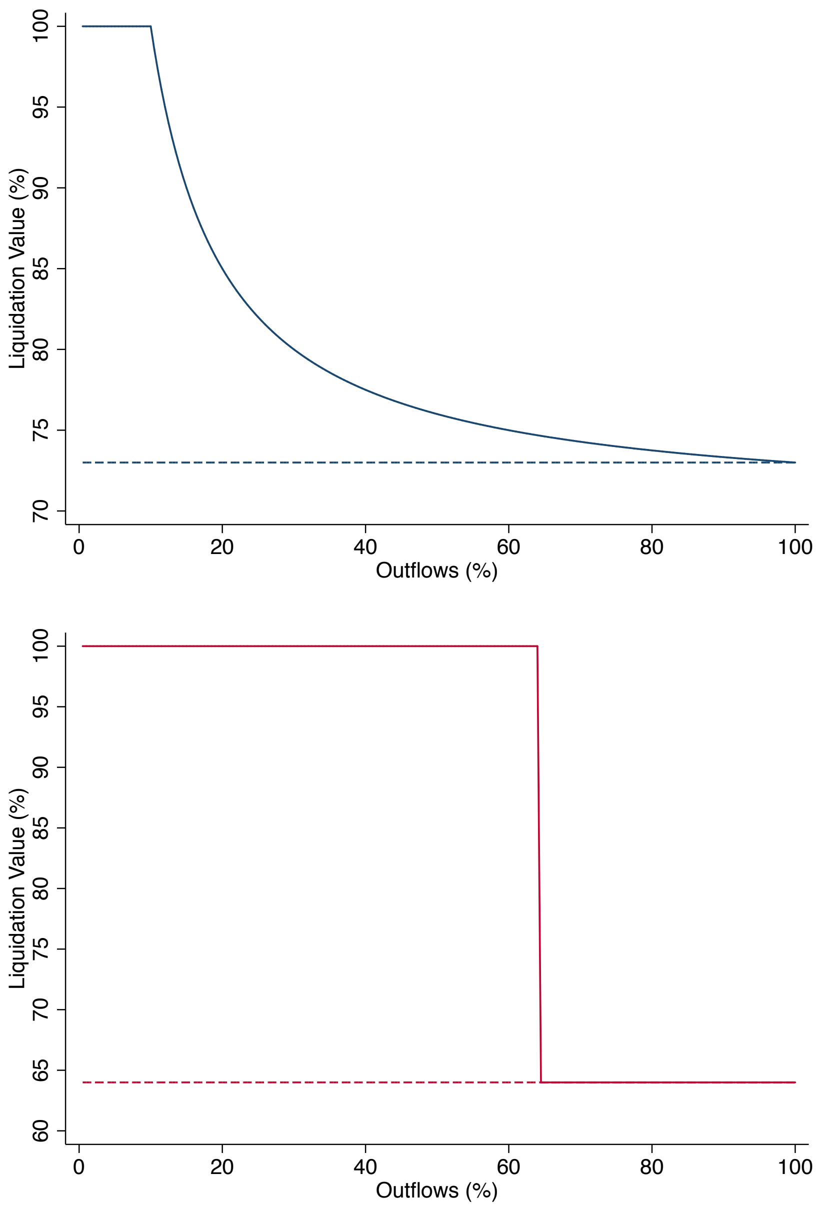


Figure 4: Haircuts

This graph plot the average market haircuts for different asset categories over time. Securites haircut data (upper panel) is obtained from the Federeal Reserve Bank of New York's pulished repo series. Haircuts for commercial loans, real estate loans and personal loans (lower panel) are from the Loan Syndications and Trading Association (LSTA), the Federal Home Loan Banks website and the Federal Reserve respectively. To remove outliers in the original data, we calculate the first principal component of the underlying series and plot the predicted value from the loadings regression $h_k = a_k + b_k PC_t + \epsilon_{kt}$ for each asset category k.

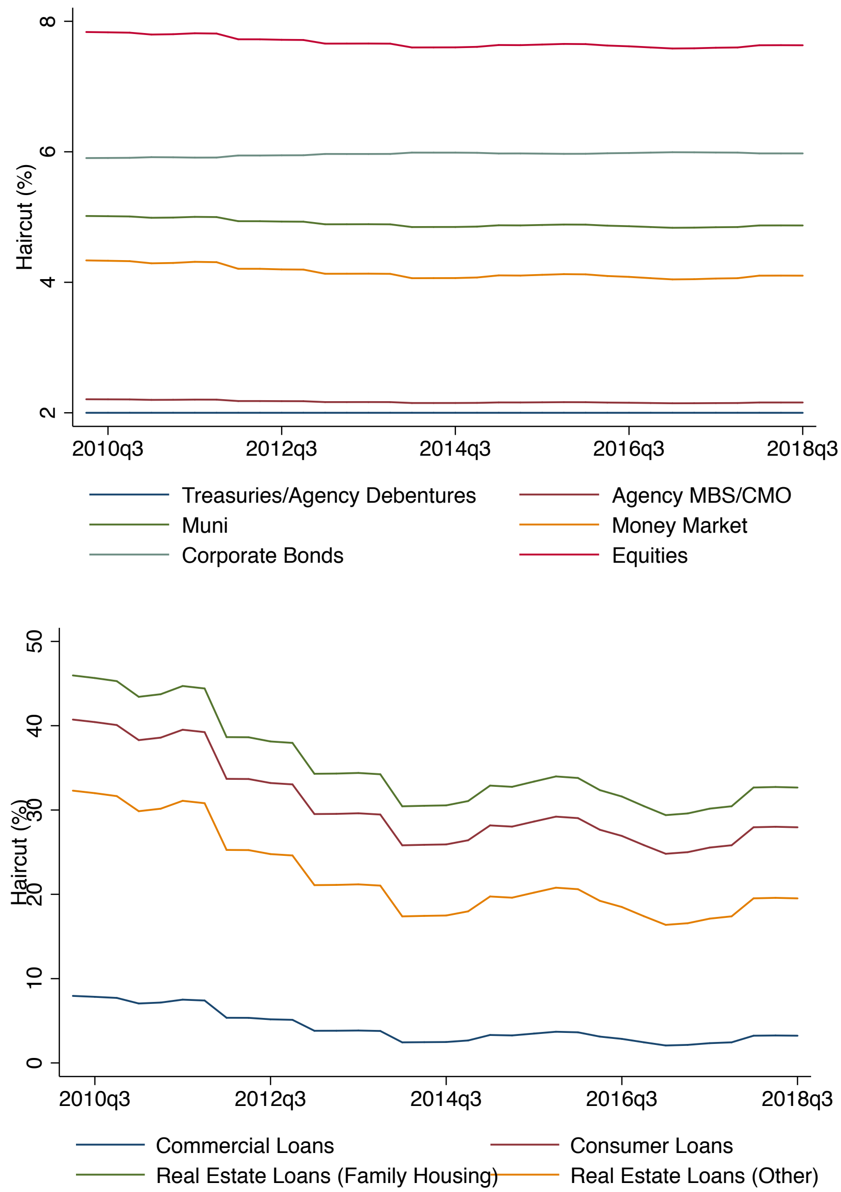
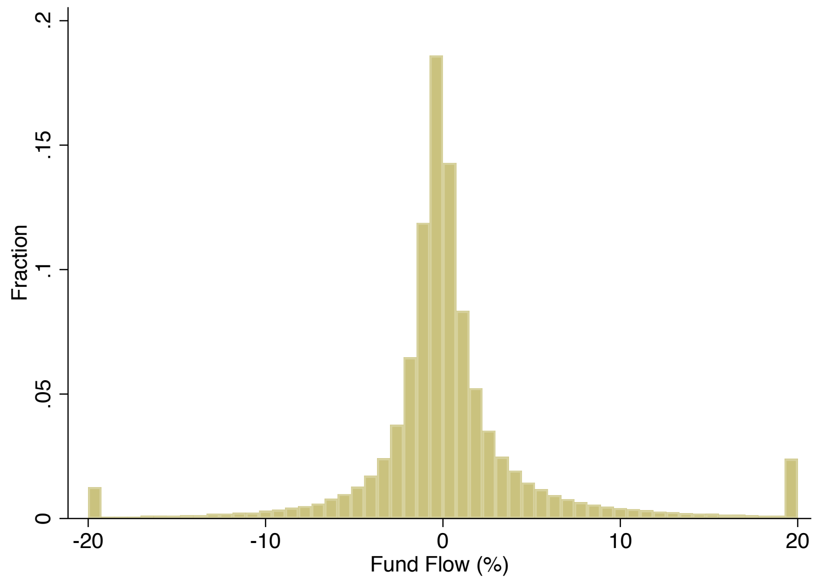


Figure 5: Distribution of Bank and Fund Flows

This graph plots frequency distribution of monthly flows of funds (upper panel) and bank liabilities (lower panel). The sample covers all commercial banks and mutual funds in our sample from 2011 to 2017. For presentation purposes, the distribution is truncated at -20% and 20% with observations in the tail assigned to the last bin.



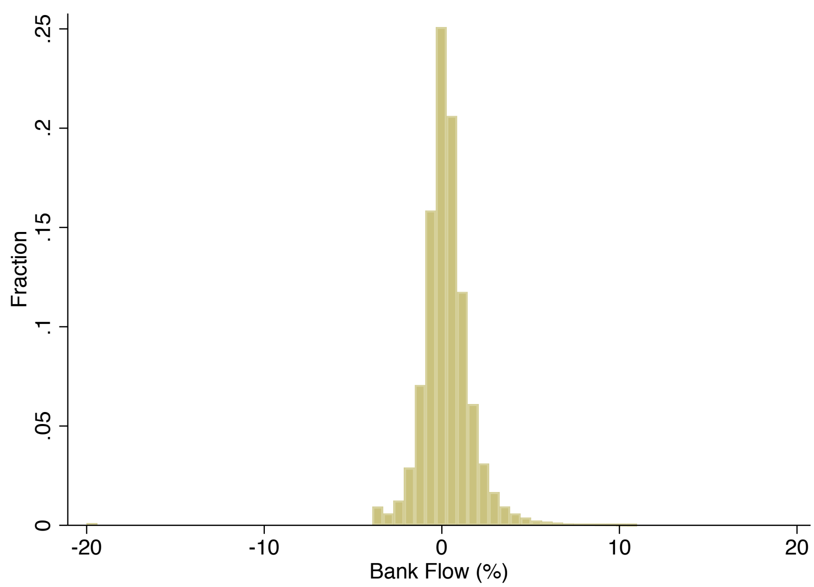
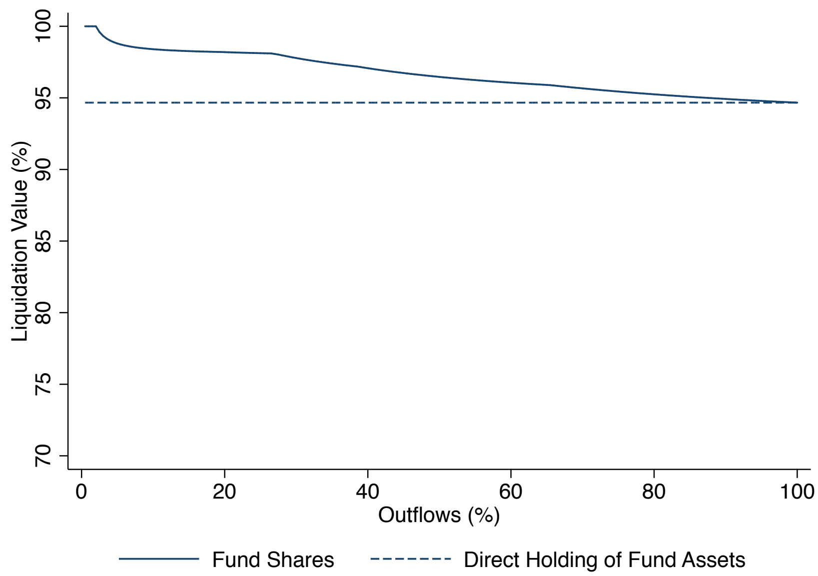


Figure 6: Contract Payment, Liquidation Value and Liquidity Provision

The upper panel plots the contract payment of fund shares for a given proportion of outflows. The fund asset porfolio reflects the volume weighted average of all corporate bond mutual funds' portfolios during our sample period from 2011 to 2017. The dotted lines represent the liquidation value of the portfolio of fund assets when directly held and sold by an investor at short notice. The lower panel plots the contract payment of bank deposits for a given proportion of outflows. The bank asset porfolio reflects the volume weighted average of all commercial banks' portfolios during our sample period from 2011 to 2017. The dotted lines represent the liquidation value of the portfolio of bank assets when directly held and sold by an investor at short notice.



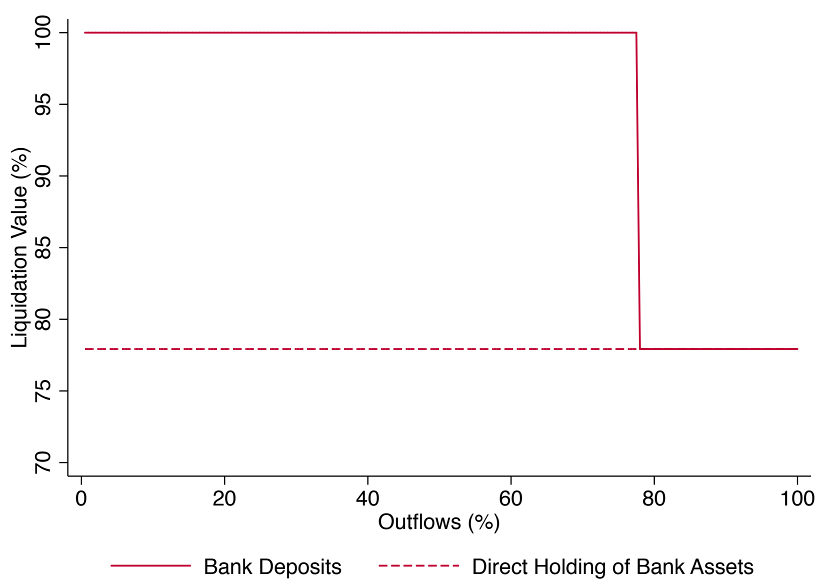


Figure 7: Cross-section of Bank and Fund Liquidity Provision

This graph plots the distribution of average commercial bank and bond mutual fund LPIs in the cross-section. The LPI for each bank and fund is calculated as the average LPI over the sample period from 2011 to 2017.

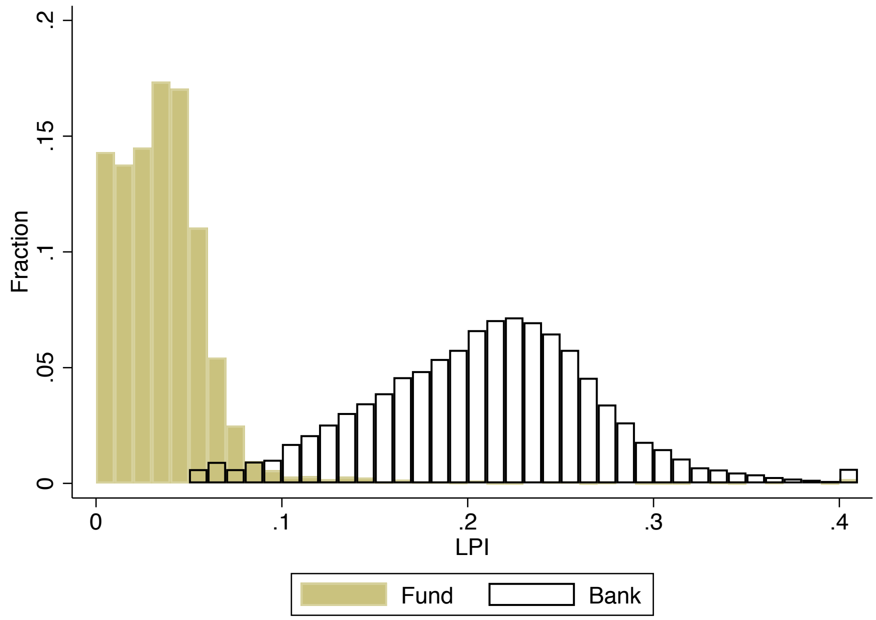


Figure 8: Sensitivity of Fund Liquidity Provision

This graph plots average LPI against fund flow sensitivity at the fund level for bond mutual funds. Fund level LPIs are averaged over the sample period from 2011 to 2017. Fund flow sensitivity is obtained by regressing fund flows against lagged fund returns and lagged fund flows from 2011 to 2017. The upper panel is a univariate binned plot while the lower panel also controls for log(fund age), log(assets) and expense ratio by residualizing the variables on the controls before binning and plotting.

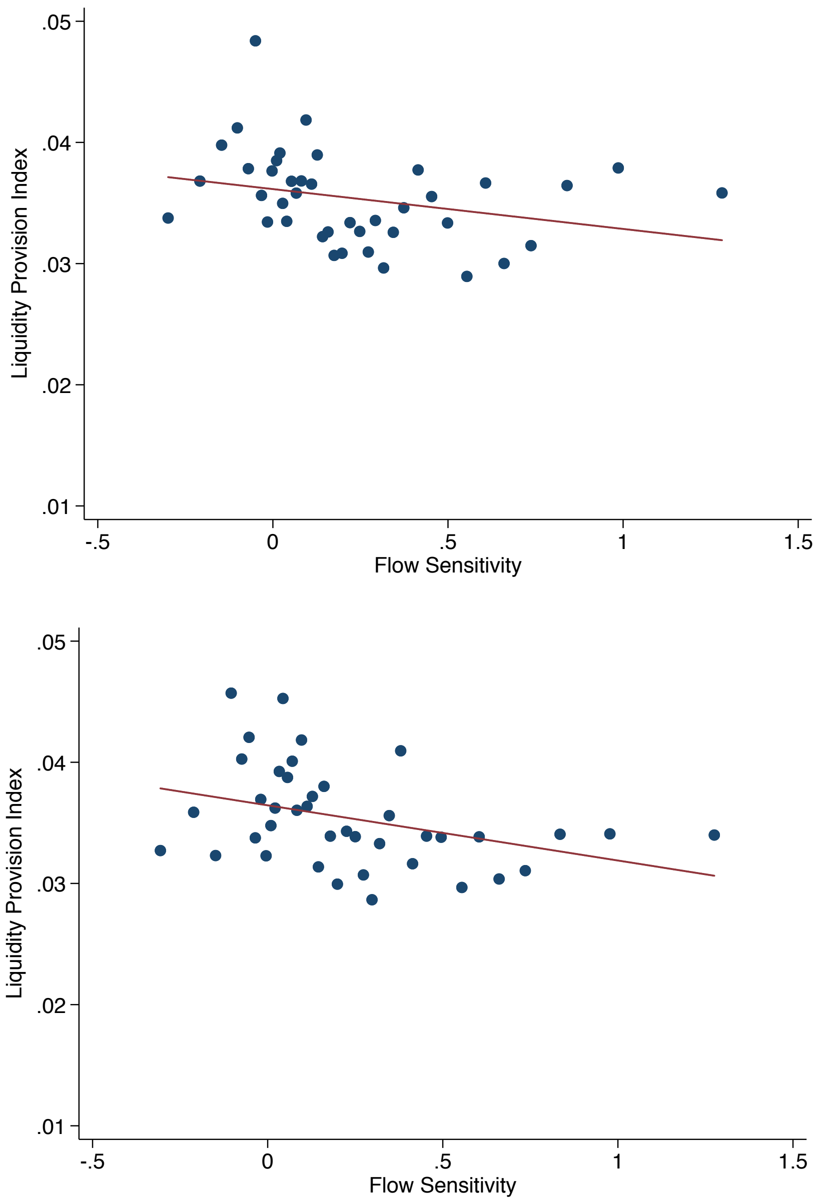
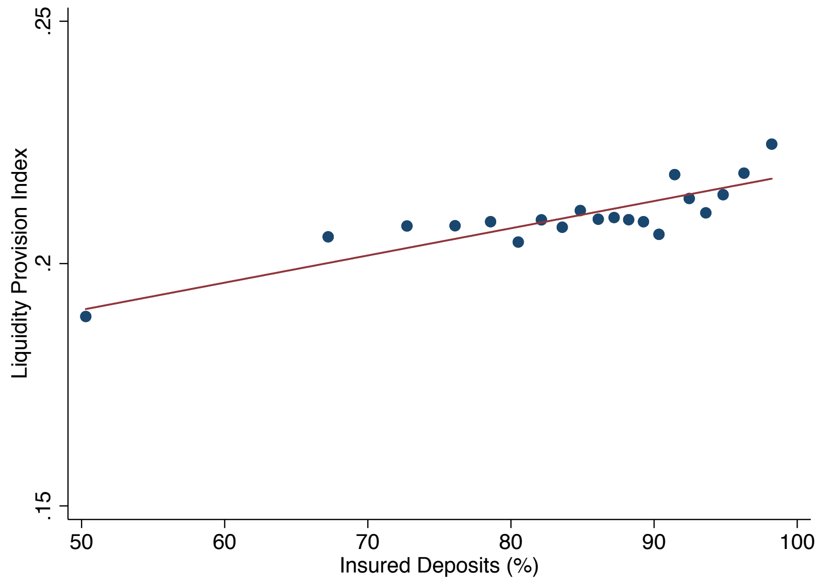


Figure 9: Sensitivity of Bank Liquidity Provision

This graph plots average LPI against the ratio of insured deposits over total deposits at the bank level for commercial banks. Bank level LPIs and insured deposit ratios are averaged over the sample period from 2011 to 2017. The upper panel is a univariate binned plot while the lower panel also controls for log(assets), equity ratio and the ratio of non-deposit liabilities over total assets by residualizing the variables on the controls before binning and plotting.



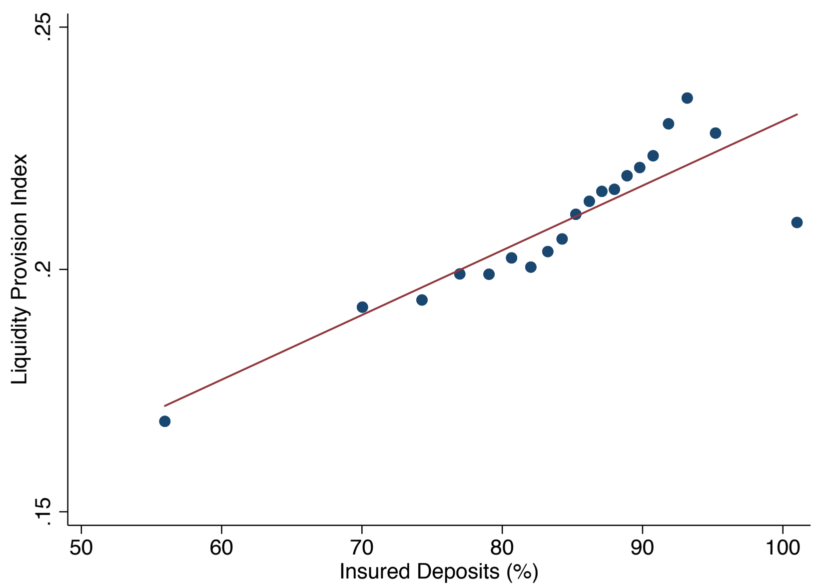


Figure 10: MMF Liquidity Provision and Floating NAVs

This graph plots average LPIs of institutional and retail prime MMFs from September 2013 to September 2019. The vertical line marks the implementation of the MMF Reform, which requires institutional prime funds to report floating NAVs. The upper panel covers the full sample whereas the lower panel restricts fund share-classes that appear at least once in both the pre and post reform period.

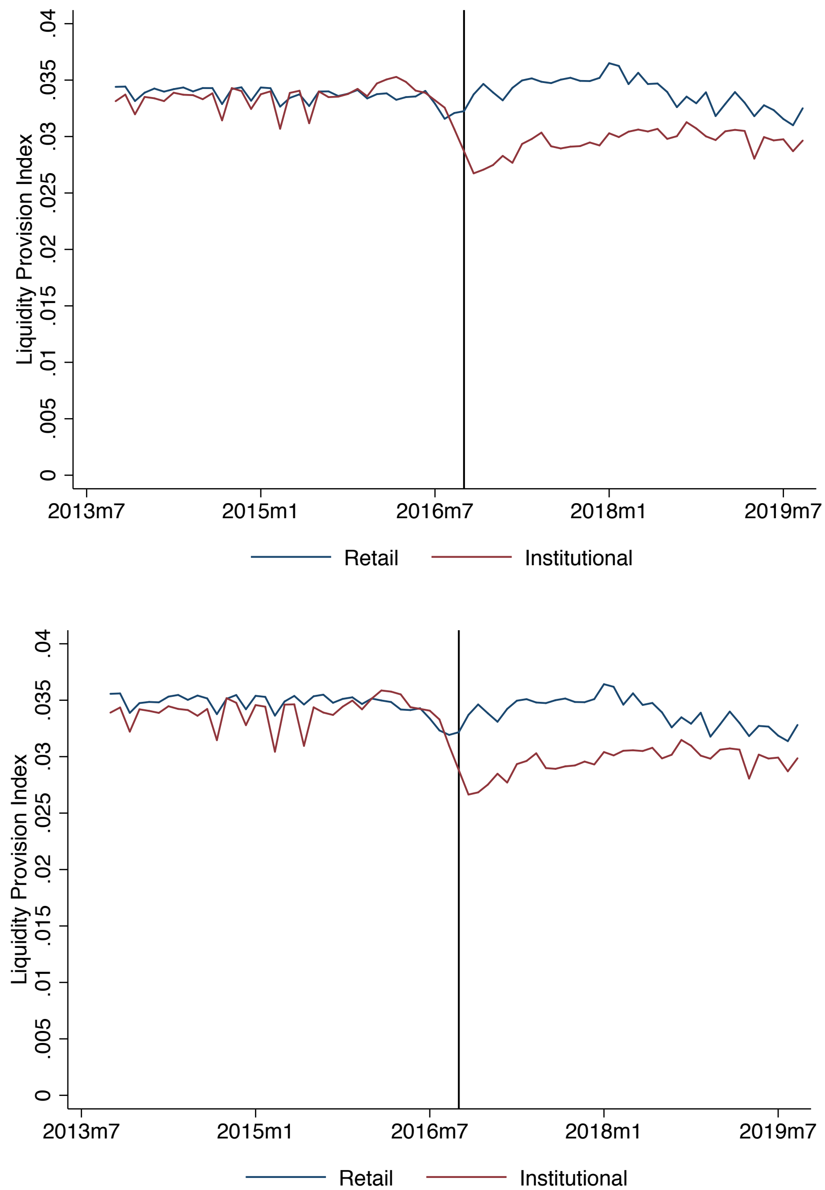


Figure 11: Liquidity Provision Index from 2011 to 2017

This graph plots the average LPI for commercial banks and bond mutual funds from 2011 to 2017. We first calculate the LPI for each bank and fund in each quarter and then plot the asset-size weighted LPI from 2011 to 2017.

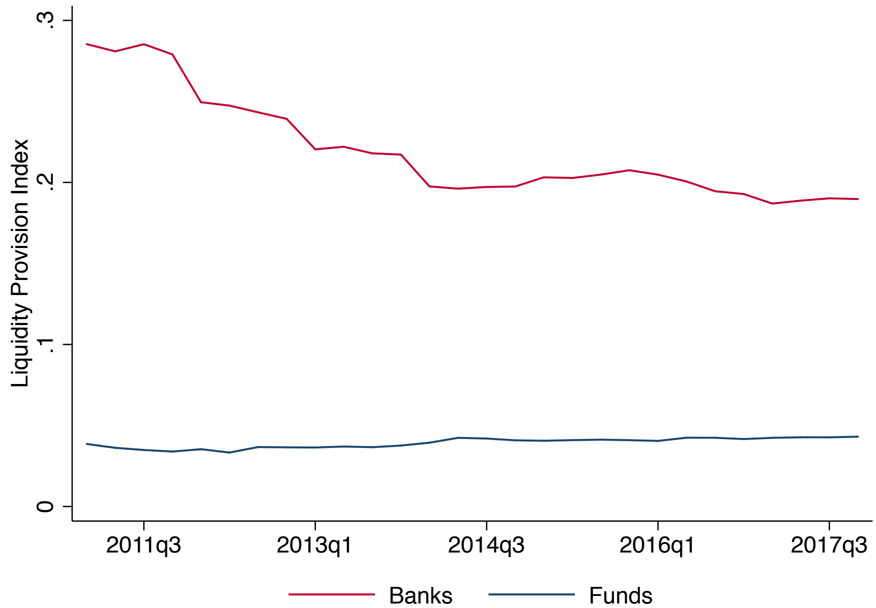


Figure 12: Liquidity Provision Pre- and Post-QE and LCR

This graph plots illustrates the contract payment of bank deposits when a given percentage of bank assets have been withdrawn before and after the implementation of QE (LCR). The liquidation value follows the orange line pre QE (LCR) and shifts to the green line post QE (LCR). This shifts the default threshold for outflows from $\hat{\lambda}$ to $\hat{\lambda}'$ and the liquidation value of the asset portfolio under direct holding from C to C'.

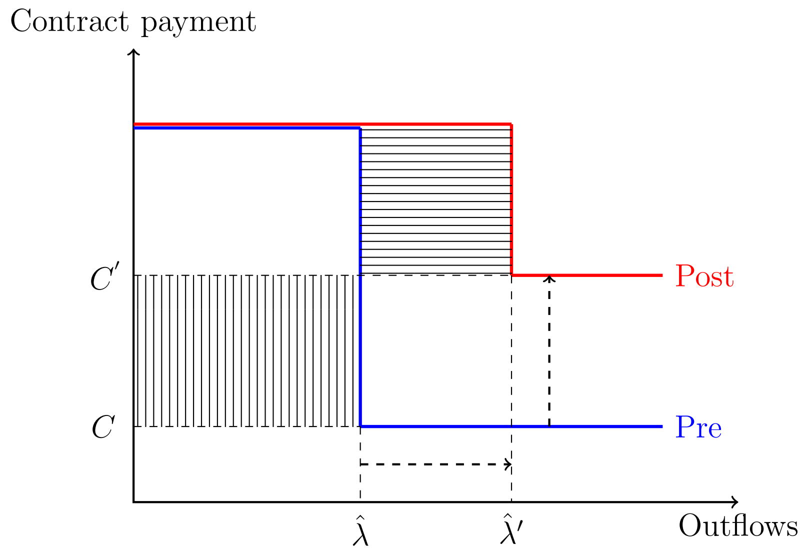


Figure 13: Bank Liquidity Provision and Excess Reserves

This graph plots the LPI for commercial banks by reserve uptake quartile (left axis) and the aggregate volume of excess reserves (right axis) from 2011 to 2017. Reserve uptake is measured by the percentage change in reserves as a fraction of total assets from 2011Q1, the beginning of the sample, to 2014Q3, when reserve levels peak. The median LPI in each quartile is selected, normalized by its initial value in 2011Q1 and plotted.

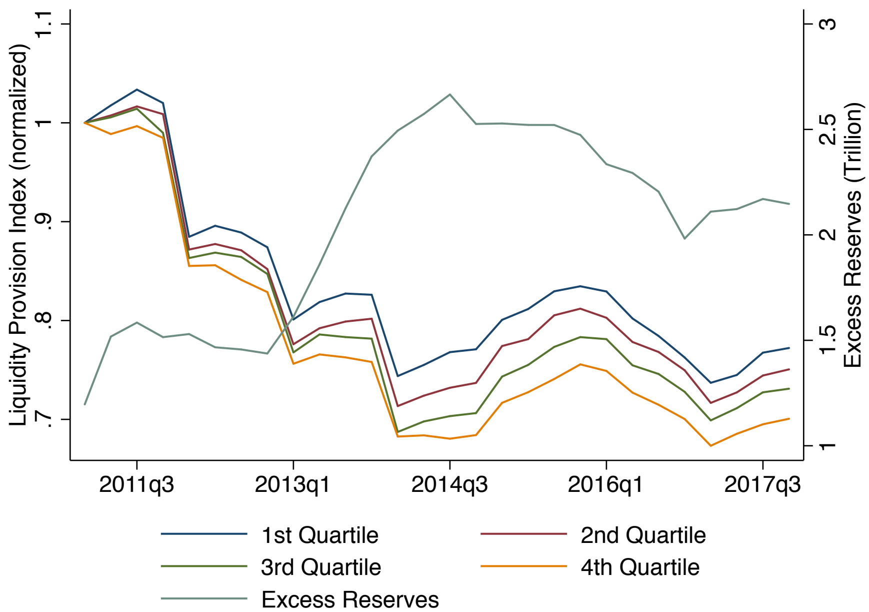


Figure 14: Bank Liquidity Provision and the Liquidity Coverage Ratio

This graph plots the LPI for commercial banks by asset size groups for the Liquidity Coverage Ratio. Banks are sorted by asset size into those above 250 billion, between 50 and 250 billion and below 50 billion. The median LPI in each asset group is selected, normalized by its initial value in 2011Q1 and plotted.

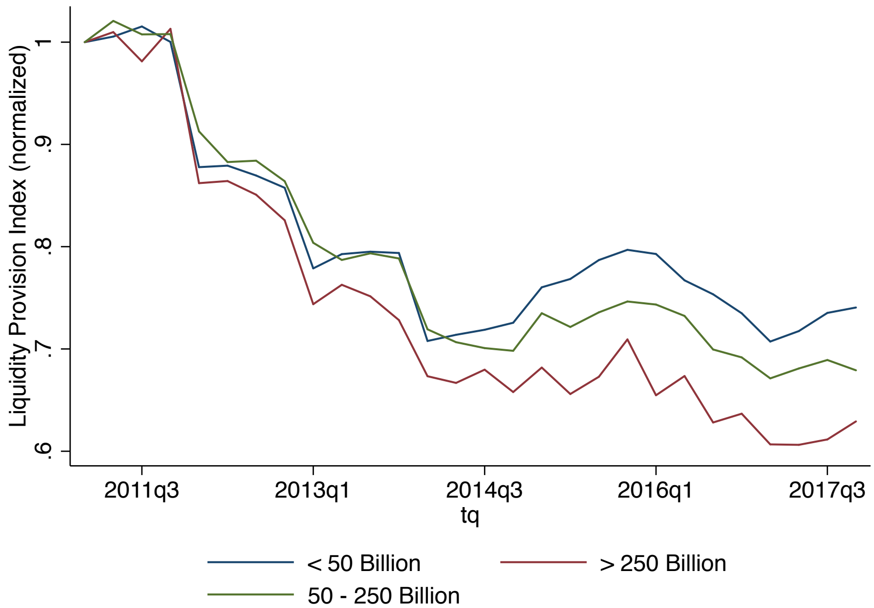


Table 3: Bank LPI and Insured Deposits Ratios

This table shows the relationship between bank LPI and the ratio of insured deposits. Control variables include non deposit ratio over all liabilities, and bank size measured by log(assets). Bank LPI and all other variables are averaged over the sample period from 2011 to 2017. Note that the constant term in the second column represents the expected LPI of banks without insured deposits or non-deposit liabilities.

	(1) LPI	(2) LPI	(3) LPI
Insured Deposits Ratio	0.055***	0.064***	0.144***
	[0.006]	[0.006]	[0.006]
Non-deposits Ratio		0.052***	0.034***
		[0.008]	[0.008]
Log(assets)			0.017***
			[0.001]
Constant	0.164***	0.147***	-0.127***
	[0.005]	[0.006]	[0.010]
Observations	7535	7535	7535

Table 4: The Effect of Floating NAV on MMF Liquidity Provision

This table shows the effect of the 2016 MMF Reform on liquidity provision by institutional prime funds versus retail prime funds. *Institutional Fund* is a dummy variable for the treatment group as the floating NAV requirement of the MMF reform only applies to institutional but not retail share-classes . *Post Reform* is an indicator variable for the treatment period. For columns (1) to (2), the treatment period begins with the official implementation date of the reform in October 2016. Columns (3) and (4) repeat the analysis in columns (1) and (2) but exclude flows around the implementation period from August to October 2016. Columns (5) and (6) account for potential anticipation effects by setting the treatment period to begin one year earlier in October 2015. Columns (2), (4) and (6) restrict the sample to the set of fund share classes that appear in both the pre and post period, that is, those funds which survived the reform. The dependent variable is share-class level LPI averaged for both the pre and post period. The coefficient on the interaction variable, *Post Reform* $*$ *Institutional Fund*, corresponds to the difference-in-differences result. Standard errors are clustered at the share-class level.

	(1) LPI	(2) LPI	(3) LPI	(4) LPI	(5) LPI	(6) LPI
Post Reform	-0.0003	-0.0011***	-0.0007	-0.0015***	-0.0009*	-0.0014***
	[0.0004]	[0.0004]	[0.0007]	[0.0005]	[0.0006]	[0.0003]
Inst Fund	-0.0013***	-0.0020***	-0.0002	-0.0006	-0.0006	-0.0011**
	[0.0005]	[0.0005]	[0.0006]	[0.0005]	[0.0006]	[0.0004]
Post Reform * Inst Fund	-0.0040***	-0.0032***	-0.0035***	-0.0033***	-0.0042***	-0.0044***
	[0.0007]	[0.0007]	[0.0009]	[0.0007]	[0.0007]	[0.0005]
Constant	0.0225***	0.0230***	0.0334***	0.0344***	0.0337***	0.0346***
	[0.0003]	[0.0003]	[0.0005]	[0.0004]	[0.0005]	[0.0003]
Observations	649	439	666	470	744	622
Adj. R-squared	0.17	0.24	0.07	0.13	0.15	0.27

Table 5: Fund LPI and Fund Flow Sensitivity

This table shows the relationship between fund LPI and the sensitivity of fund flows to lagged fund returns. Other control variables include the expense ratio, log(total net assets) and log(fund age). Fund LPI and all other variables are averaged over the sample period from 2011 to 2017.

	(1) LPI	(2) LPI	(3) LPI	(4) LPI
Flow Sensitivity	-0.329**	-0.328**	-0.330**	-0.456***
	[0.158]	[0.158]	[0.158]	[0.154]
Expense Ratio		0.014	0.014	0.013
		[0.020]	[0.020]	[0.019]
Log(Total Net Assets)			0.000	0.001***
			[0.000]	[0.000]
Log(Fund Age)				-0.009***
				[0.001]
Constant	0.036***	0.036***	0.035***	0.049***
	[0.001]	[0.001]	[0.002]	[0.002]
Observations	2626	2626	2626	2626

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Appendix

A Autarky, Liquidation Value, and Liquidity Provision

In this appendix, we first formally characterize an autarky outcome and show that it is tightly linked to the definition of liquidation value as in Definition 1. Then we show that our definition of liquidity provision, that is, Definition 2, is tightly related but preferable to an alternative definition based on the autarky.

Consider households live in autarky without access to financial intermediaries. They choose their portfolio $(x_{a,0}, y_{a,0})$ at $t = 0$, where $x_{a,0}$ is the amount of cash stored in the storage and $y_{a,0}$ the amount of projects invested before knowing their types.

At the beginning of $t = 1$, the household learns her type and receives the signal s_i. An early household always liquidates all her projects regardless of s_i, and consume the cash:

$$\begin{aligned} c_{a,1} &= x_{a,0} + y_{a,0} - \frac{\phi}{2} y_{a,0}^2 \\ &= 1 - \frac{\phi}{2} y_{a,0}^2 \,, \end{aligned} \tag{A.1}$$

where $(x_{a,0}, y_{a,0})$ represents her portfolio choice at $t = 0$. An important observation from (A.1) is that what an early household can get in the autarky is indeed the liquidation value of her portfolio, as defined in Definition 1.

Late households, however, will only liquidate the project and store the proceeds as cash if she expects a sufficiently poor performance after observing s_i. Formally, conditional on s_i, the late household's optimal portfolio choice problem at $t = 1$ is given by

$$\max_{l_a \geqslant 0} E\left[u(c_{a,2}) | s_i\right] \,,$$

where

$$c_{a,2} = x_{a,1}(1 - \kappa) + y_{a,1} R \,, \tag{A.2}$$

where in turn

$$\begin{cases} x_{a,1} = x_{a,0} + l_a - \dfrac{\phi}{2} l_a^2 \,, \\ y_{a,1} = y_{a,0} - l_a \end{cases}$$

are the late household's position at the end of $t = 1$.[41] It is easy to show that, given the date-0 position $(x_{a,0}, y_{a,0})$ and the signal R received at $t = 1$, an early household always liquidates all the illiquid asset holding $y_{a,0}$ regardless of R, while a late household optimally liquidates

$$l_a = \frac{1}{\phi}\left(1 - \frac{R}{1-\kappa}\right) \tag{A.3}$$

units of project at $t = 1$, subject to $l_a \geq 0$ and $l_a \leq y_{a,0}$.

Taking the decisions at $t = 1$ into account, households optimally choose their portfolios at $t = 0$ before knowing their type:

$$\max_{x_{a,0}} E[\pi u(c_{a,1}) + (1-\pi)u(c_{a,2})]\,,$$

subject to conditions (A.1), (A.2), (A.3). Without fully solving for the equilibrium, it is straightforward to see that the optimal $c^*_{a,1}$ is strictly lower than 1 when the household invests in a positive amount of projects at $t = 0$, that is, $y^*_{a,0} > 0$, and hence $x^*_{a,0} < 1$.

The analysis above about the autarky allows us to link our definition of provision, as in Definition 2, to an alternative definition in which one compares the t-1 consumption level promised by an intermediary and the autarky outcome. Formally, by the Principle of Optimization, we have:

Proposition 7. *If an intermediary $k \in \{b, f\}$ provides liquidity in the sense that $E[c^*_{k,1}] - c^*_{a,1} > 0$, it must provide liquidity in the sense that $E[c^*_{k,1}] - c_1(x^*_{k,0}, y^*_{k,0}) > 0$.*

Proposition 7 suggests that liquidity provision based on Definition 2 is a necessary condition of liquidity provision under the alternative definition based on autarky. Thus, given the advantages of Definition 2 that 1) it is conceptually tightly linked to the insight of Diamond and Dybvig (1983) and 2) is all relies on empirically observable outcomes and thus can be more directly mapped to empirics, we use it as our preferred definition of liquidity provision.

B Deposit Insurance

In this appendix, we provide a detailed example showing that our framework, both theoretically and empirically, is robust to the consideration of deposit insurance and other similar institutional and regulatory features. Our empirical LPI construction is indeed general enough to handle deposit insurance. It also illustrates that deposit insurance contributes to bank liquidity provision but only partially.

[41] One unit of cash invested in the storage yields $1 - \kappa$ units of cash since one unit of late households is using the storage.

Consider a hypothetical bank that holds 10% cash and 90% of illiquid loans, where the loans, if liquidated at short notice, can be only recovered at 60% of their fair value (i.e., the haircut is 40%). Different from our baseline model, suppose this bank has 50% insured deposits and 50% uninsured deposits. Also suppose that the empirical distribution of outflows is uniform on [0,1]. (To be precise, we only take the 50-50 mix of insured and uninsured deposits as exogenously given in this example; the distribution of outflows and the bank portfolio should be viewed as endogenously determined by household and bank optimization.) Under this example, the liquidation value of the bank portfolio is $1 \times 0.1 + 0.6 \times 0.9 = 0.64$. Thus, for an insured depositor, a dollar invested in the insured deposit generates an LPI of $1 - 0.64 = 0.36$. While for an uninsured depositor, a dollar invested in the uninsured deposit generates an LPI of $(1 - 0.64) \times 0.64 = 0.2304$.

Now suppose for this same bank, all deposits become uninsured. According to the theoretical model of Allen et al. (2018),[42] few insured deposits lead to a higher run threshold (i.e., a higher run probability), and consequently, a more liquid bank portfolio. Consistent with this argument, suppose the distribution of outflows becomes a triangular distribution with a densify function of $f(\lambda) = 2\lambda$, and the bank portfolio becomes 20% cash and 80% loans. Under this case, the liquidation value of the bank portfolio becomes $1 \times 0.2 + 0.6 \times 0.8 = 0.68$, which is higher than before. For an uninsured depositor, a dollar invested in the uninsured deposit generates an LPI of $\int_0^{0.68} (1 - 0.68) 2\lambda d\lambda = 0.1479$. Compared to the case above, the LPI decreases by $0.2304 - 0.1479 = 0.0825$, which suggests that for this actual bank with a 50-50 mix of insured and uninsured deposits, deposit insurance contributes 0.0825 towards its total LPI of 0.2304.

The example above has two important implications. First, it clearly illustrates that, what our LPI captures is indeed the amount of liquidity provision by uninsured bank deposits. In other words, by design, the LPI is independent to the consideration of deposit insurances. Second, because the LPI construction uses information about the two equilibrium outcomes of 1) the empirical distribution of flows and 2) the bank portfolio, and deposit insurance affects bank liquidity through affecting these two equilibrium outcomes, our LPI as an empirical measure of bank liquidity provision is indeed robust to the consideration of deposit insurances.

C Proofs

Proof of Proposition 1. Denote the run threshold as $R' = R(\theta')$, that is, if household i observes a private signal $s_i < \theta'$ she runs; otherwise she stays. Then the population of households who runs, λ_b,

[42]Similar to our baseline model, Allen et al. (2018) build a global-games based model with a different focus on the interplay between general government guarantees (including deposit insurance) and bank runs.

can be written as

$$\lambda_b\left(\theta,\theta'\right)=\begin{cases}1 & \text{if } \theta\leq\theta'-\varepsilon\\ \pi+(1-\pi)\left(\frac{\theta'-\theta+\varepsilon}{2\varepsilon}\right) & \text{if } \theta'-\varepsilon<\theta\leq\theta'+\varepsilon\\ \pi & \text{if } \theta>\theta'+\varepsilon\end{cases}.$$

Let $v\left(R\left(\theta\right),\lambda_b\right)$ be the difference of utilities between staying and running, then

$$v\left(R\left(\theta\right),\lambda_b\right)=\begin{cases}u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta)}{1-\lambda_b}\right)-u\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right) & \text{if } \pi\leq\lambda_b<\frac{x_{b,0}}{c_{b,1}}\\ u\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R(\theta)}{1-\lambda_b}\right)-u\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right) & \text{if } \frac{x_{b,0}}{c_{b,1}}\leq\lambda_b<\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}\\ -q\left(\lambda_b\right)u\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right) & \text{if } \frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}<\lambda_b\leq 1\end{cases},$$

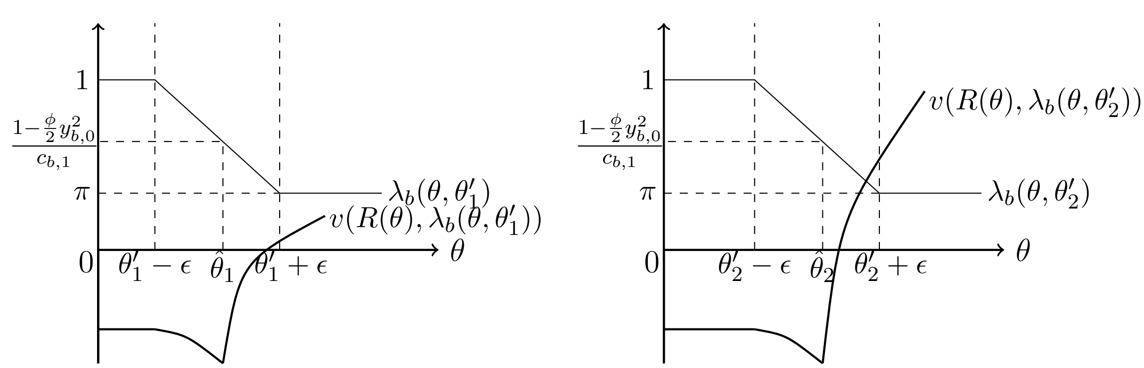
where $l_b\left(\lambda_b,c_{b,1}\right)$ satisfies $\lambda_b c_{b,1}=x_{b,0}+l_b-\frac{\phi}{2}l_b^2$. If household i observes signal s_i, given that other households use the threshold strategy, she will run if $\int_{s_i-\varepsilon}^{s_i+\varepsilon}v\left(R\left(\theta\right),\lambda_b\left(\theta,\theta'\right)\right)d\theta>0$; or stay otherwise.

To prove that there exists a unique run threshold R^*, we need to prove that there is a unique θ^* such that if $\theta'=\theta^*$, the household who observes signal $s_i=\theta'=\theta^*$ is indifferent between run and stay. That is,

$$V\left(\theta^*\right)\equiv\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon}v\left(R\left(\theta\right),\lambda_b\left(\theta,\theta^*\right)\right)d\theta=0.$$

The graph of $v\left(R\left(\theta\right),\lambda_b\left(\theta,\theta'\right)\right)$ is depicted in Figure A1, where $\hat{\theta}$ satisfies $\lambda_b\left(\hat{\theta},\theta'\right)=\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$. It is easy to check that v is constant on $(0,\theta'-\varepsilon)$, decreasing on $\left(\theta'-\varepsilon,\hat{\theta}\right)$ and increasing on $(\hat{\theta},1)$. Figure A1 also illustrates how v changes when θ' increases. As θ' increases, the integral of v on $(0,\theta'-\varepsilon)$ remains the same since v does not directly depends on θ in this interval and the length of the interval is constant given ε; on $(\hat{\theta},\theta'+\varepsilon)$, for any given λ_b, if θ' goes up, v increases because all θ in this interval goes up. Thus the integral on $(\hat{\theta},\theta'+\varepsilon)$ increases. In summary, $V\left(\theta'\right)$ is increasing in θ'.

Figure A1: The graph of $v(R(\theta),\lambda_b(\theta,\theta'))$



Since $G(\cdot)$ is supported on $[0,+\infty)$, $R \to +\infty$ when $\theta \to 1$. That is, $\lim_{\theta' \to 1} V(\theta') = +\infty$. Furthermore, $V(0) < 0$. Then by the intermediate value theorem, there exists θ^* such that $V(\theta^*) = 0$. The uniqueness of θ^* follows by the monotonicity of $V(\cdot)$ and so does R^*.

Intuitively, the threshold R^* must larger than the lower dominance region $\underline{R}$ since the households face more risks in the sense that bankrupt may happen. $\underline{R}$ is pinned down by letting households be indifferent between run and stay if there is no withdrawal, that is, $u\left(\frac{x_{b,1}+y_{b,1}\underline{R}}{1-\lambda_b}\right) = u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)$, or, $\underline{R} = \frac{c_{b,1}(1-\kappa)(1-\pi)+\pi c_{b,1}-x_{b,0}}{1-x_{b,0}}$. ∎

Proof of Proposition 2. In the proof of Proposition 1, we know that $V(\cdot)$ is increasing and θ^* is pinned down by $V(\theta^*) = 0$. Thus, to prove that $\frac{d\theta*}{dc_{b,1}} > 0$, it is sufficient to prove that $V(\theta')$ is decreasing in $c_{b,1}$. Note that when $\theta \in (\theta'-\varepsilon, \theta'+\varepsilon)$, $\pi + (1-\pi)\left(\frac{\theta'-\theta+\varepsilon}{2\varepsilon}\right) = \lambda_b$, so we have $d\theta = -\frac{2\varepsilon}{1-\pi}d\lambda_b$. Then $V(\theta')$ can be rewritten as

$$
\begin{aligned}
V(\theta') &= \int_{\pi}^{1} v\left(R\left(\theta' + \varepsilon\left(1 - 2\frac{\lambda-\pi}{1-\pi}\right)\right), \lambda_b\right)\frac{2\varepsilon}{1-\pi}d\lambda_b \\
&= \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \frac{2\varepsilon}{1-\pi}\left(u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'+\varepsilon\left(1-2\frac{\lambda-\pi}{1-\pi}\right)\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \\
&+ \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \frac{2\varepsilon}{1-\pi}\left(u\left(\frac{(y_{b,0}-l_b(\lambda_b,c_{b,1}))R\left(\theta'+\varepsilon\left(1-2\frac{\lambda-\pi}{1-\pi}\right)\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \\
&- \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} \frac{2\varepsilon}{1-\pi}q(\lambda_b)\,u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)d\lambda_b.
\end{aligned}
$$

Let $V = 0$, which pins down θ^* and thus R^*. And since V is continuous in ε, we can take the limit of the equation above at $\varepsilon \to 0$:

$$
\begin{aligned}
\int_{\pi}^{1} v\left(R(\theta^*), \lambda_b\right)d\lambda_b = &\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta')}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \\
&+ \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\left(u\left(\frac{(y_{b,0}-l_b(\lambda_b,c_{b,1}))R(\theta')}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \quad \text{(C.1)} \\
&- \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} q(\lambda_b)\,u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)d\lambda_b.
\end{aligned}
$$

Multiplying both sides of (C.1) with $c_{b,1}$ and taking derivative with respect to $c_{b,1}$, we have the derivative expressed by

$$\begin{aligned}
& c_{b,1}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(-\frac{\lambda_b}{1-\lambda_b}u'\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta^*\right)}{1-\lambda_b}\right)-(1-\kappa\left(\lambda_b-\pi\right))u'\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right)\right)d\lambda_b \\
+ \quad & c_{b,1}\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\left(-\frac{\frac{\partial l_b}{\partial c_{b,1}}R\left(\theta^*\right)}{1-\lambda_b}u'\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta^*\right)}{1-\lambda_b}\right)-(1-\kappa\left(\lambda_b-\pi\right))u'\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right)\right)d\lambda_b \\
- \quad & \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1}\frac{1-\frac{\phi}{2}y_{b,0}^2}{\lambda_b}(1-\kappa\left(\lambda_b-\pi\right))u'\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right)d\lambda_b \\
+ \quad & \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta^*\right)}{1-\lambda_b}\right)-u\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right)\right)d\lambda_b \\
+ \quad & \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\left(u\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta^*\right)}{1-\lambda_b}\right)-u\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right)\right)d\lambda_b \\
- \quad & \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1}q\left(\lambda_b\right)u\left(c_{b,1}(1-\kappa\left(\lambda_b-\pi\right))\right)d\lambda_b\,,
\end{aligned}$$

where $\frac{\partial l_b}{\partial c_{b,1}}=\frac{\lambda_b}{1-\phi l_b}$ and $\frac{\partial q}{\partial c_{b,1}}=-\frac{1-\phi y_{b,1}^2}{\lambda_b c^2}<0$. Note that it suffices to prove that

$$\begin{aligned}
& c_{b,1}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}-\frac{\lambda_b}{1-\lambda_b}u'\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b \\
+ \quad & c_{b,1}\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}-\frac{\frac{\partial l_b}{\partial c_{b,1}}R\left(\theta'\right)}{1-\lambda_b}u'\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta^*\right)}{1-\lambda_b}\right)d\lambda_b \\
+ \quad & \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b \\
+ \quad & \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}u\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b<0\,.
\end{aligned} \tag{C.2}$$

Integrating by parts, we have the LHS of (C.2) re-expressed as:

$$\begin{aligned}
& -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right) \\
+ \quad & \frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\pi\left(1-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right) \\
+ \quad & \frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(x_{b,0}+y_{b,0}R\left(\theta'\right)-2\lambda_b c_{b,1}\right)u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b
\end{aligned}$$

$$+ \quad c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)$$

$$+ \quad \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^{2}}{c_{b,1}}}\left(c_{b,1}g_{\lambda_b}\left(c_{b,1},\lambda_b\right)+1\right)u\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b, \tag{C.3}$$

where $g\left(c_{b,1},\lambda_b\right)=\frac{(1-\lambda_b)\lambda_b}{1-c_{b,1}-\phi y_{b,0}l_b\left(\lambda_b,c_{b,1}\right)}$.

We first consider the first three terms of (C.3). Since $u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta^*)}{1-\lambda_b}\right)$ is decreasing in λ_b and $x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)<0$ for $c_{b,1}>1$, the sum of these three terms is less than:

$$\begin{aligned}
& -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right)\\
& +\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\pi\left(1-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
& +\frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(x_{b,0}+y_{b,0}R\left(\theta'\right)-2c_{b,1}\frac{\frac{x_{b,0}}{c_{b,1}}+\pi}{2}\right)u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b\\
< \quad & -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right)\\
& +\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\pi\left(1-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
& +\frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\left(y_{b,0}R\left(\theta'\right)-c_{b,1}\pi\right)\left(\frac{x_{b,0}}{c_{b,1}}-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
< \quad & -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right)\\
& +\frac{R\left(\theta'\right)y_{b,0}x_{b,0}}{c_{b,1}\left(x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)\right)}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\\
& -\pi u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
< \quad & -\frac{x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)-\pi u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)<0\,.
\end{aligned}$$

We then consider the last two terms of (C.3), which decrease in ϕ. When $\phi=0$, we have $l_b=\pi c_{b,1}-x_{b,0}$, and $g_{\lambda_b}\left(c_{b,1},\lambda_b\right)=\frac{1-2\lambda_b}{1-c_{b,1}}$. Then the sum of these two terms becomes

$$c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}\left(1+\frac{c_{b,1}\left(1-2\lambda_b\right)}{1-c_{b,1}}\right)u\left(\frac{\left(1-\lambda_b c_{b,1}\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b$$

$$
\begin{aligned}
&< c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}\left(1+\frac{c_{b,1}\left(1-2\frac{1+x_{b,0}}{2c_{b,1}}\right)}{1-c_{b,1}}\right)u\left(\frac{\left(1-\lambda_b c_{b,1}\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b \\
&= c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}-\frac{x_{b,0}}{1-c_{b,1}}u\left(\frac{\left(1-\lambda_b c_{b,1}\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b \\
&< c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)-\frac{x_{b,0}}{1-c_{b,1}}\frac{1-x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right) \\
&= -\frac{x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)<0\,.
\end{aligned}
$$

In summary, V is decreasing in $c_{b,1}$ and thus $\frac{\partial\theta^*}{\partial c_{b,1}}>0$. That is, R^* is increasing in $c_{b,1}$.

To see why θ^* is decreasing in $x_{b,0}$, take the derivative of (C.1) with respect to $x_{b,0}$ and note that $y_{b,0}=1-x_{b,0}$, we have

$$
\begin{aligned}
&\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}-\frac{R\left(\theta'\right)-1}{1-\lambda_b}u'\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b \\
&+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\frac{\left(\frac{1}{1-\phi l_b}-1\right)R\left(\theta'\right)}{1-\lambda_b}u'\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b \\
= &\frac{R\left(\theta'\right)-1}{R\left(\theta'\right)-c_{b,1}-\left(R\left(\theta'\right)-1\right)x_{b,0}}\left(\left(1-\pi\right)u\left(\frac{x_{b,0}-c_{b,1}\pi+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)-\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,1}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right. \\
&\left.+\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b\right) \\
&+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\frac{\left(\frac{1}{1-\phi l_b}-1\right)R\left(\theta'\right)}{1-\lambda_b}u'\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b.
\end{aligned}
$$

Similar to above, we can prove that the first term is positive. Furthermore, note that $\phi l_b=1-\sqrt{1-2\phi\left(\lambda_b c_{b,1}-x\right)}<1$, the second them is also positive. Following the same logic, V is increasing in $x_{b,0}$, and thus R^* is decreasing in $x_{b,0}$. ∎

Proof of Proposition 4. Let $C(l_f)$ be the amount of cash raised by prematurely liquidating l_f project at $t=1$. Assume that $C(0)=0$ and for any $l_f>0$, $0<C(l_f)<l_f$. Note that the parametric form of $C(l_f)$ in our baseline model satisfies these conditions. We consider two cases below.

First, consider $l_f > 0$. Note that equations (2.9) and (2.10) give two ways to calculate NAV_1:

$$NAV_1(\lambda_f) = 1 - l_f + C(l_f) \tag{C.4}$$
$$= \frac{x_{f,0} + C(l_f)}{\lambda_f}. \tag{C.5}$$

Solving (C.5) as an equation of λ_f yields:

$$\lambda_f = \frac{x_{f,0} + C(l_f)}{1 - l_f + C(l_f)}. \tag{C.6}$$

Plugging (C.6) into the expression of NAV_2 (2.12):

$$\begin{aligned} NAV_2(\lambda_f) &= \frac{y_{f,0} - l_f}{1 - \lambda_f} R \\ &= \frac{1 - l_f + C(l_f)}{1 - x_{f,0} - l_f}(y_{f,0} - l_f)R, \end{aligned}$$

which by (C.4) and the fact that $x_{f,0} + y_{f,0} = 1$ immediately leads to

$$NAV_2(\lambda_f) = NAV_1(\lambda_f)R.$$

Hence, $NAV_2(\lambda_f) > NAV_1(\lambda_f)$ if and only if $R > 1$ when $l_f > 0$.

Then, consider $l_f = 0$. In this case, $NAV_1(\lambda_f) = 1$, and

$$\begin{aligned} NAV_2(\lambda_f) &= \frac{x_{f,0} - \lambda_f + y_{f,0}R}{1 - \lambda_f} \\ &= 1 + \frac{y_{f,0}}{1 - \lambda_f}(R - 1) \qquad \text{(C.7)} \\ &= NAV_1(\lambda_f) + \frac{y_{f,0}}{1 - \lambda_f}(R - 1), \end{aligned}$$

where (C.7) uses $x_{f,0} + y_{f,0} = 1$. This implies that $NAV_2(\lambda_f) > NAV_1(\lambda_f)$ if and only if $R > 1$ when $l_f = 0$. ∎

Proof of Proposition 5. Because ε_i is arbitrarily small, there is no fundamental uncertainty between $t = 1$ and 2. Thus, late households' problem reduces to:

$$\begin{cases} w_f = 0 & \text{if} \quad c_{f,1}(\lambda_f) < E[c_{f,2}(\lambda_f)|R], \\ w_f \in (0, 1 - \pi) & \text{if} \quad c_{f,1}(\lambda_f)(1 - \kappa w_f) = E[c_{f,2}(\lambda_f)|R], \\ w_f = 1 - \pi & \text{if} \quad c_{f,1}(\lambda_f)(1 - \kappa w_f)) > E[c_{f,2}(\lambda_f)|R]. \end{cases} \tag{C.8}$$

Note that (C.8) is reduced as a quadratic equation of w_f. Directly solving for w_f under the constraints yields the desired results. ∎

Proof of Proposition 6. This proof is directly built on the results of Proposition 5. There are three cases.

Case 1. When $x_{f,0} \geq \pi + w_f^*$, stored cash is sufficient to meet the redemption needs regardless of the flow-to-fundamental relationship. As a result, $l_f^* = 0$.

Case 2. When $R \geq 1$ and $x_{f,0} < \pi$ (note that $w_f^* = 0$ when $R \geq 1$), only early households redeem at $t = 1$ but stored cash is not sufficient to meet their redemption needs. Thus, solving (2.10) at $\lambda_f = \pi$, which is a quadratic function of l_f, yields the desired result.

Case 3. When $R < 1$ and $x_{f,0} < \pi + w_f^*$, all early households and some late households redeem at $t = 1$ and stored cash is not sufficient to meet their redemption needs. Thus, solving (2.10) at $\lambda_f = \pi + w_f^*$, which is a quadratic function of l_f, yields the desired result. ∎

Table OA5: Asset Category Sources

This table shows the sources for banks and funds for each asset class used in the LPI calculation. Bank asset holdings are obtained using bank balance sheet data from call reports. The bank holdings variables all come from RCFD (for example, the corresponding cash variable is RCFD0010) except for real estate loans which also take variables from RCON. Mutual fund holdings data is obtained from the CRSP database, and fund cash holdings are taken from CRSP mutual funds summary data. For fund holdings, all asset classes except cash holdings are categorized directly from securities-level holdings using the mapping shown.

Category	Bank Source (RCFD)	Fund Holdings Mapping
1. Treasuries &	3531, 0213, 1287	US Government & Agency Bills, Bonds,
Agency Debentures	3532, 1290, 1293, 1295, 1298	Notes, Strips, Trust Certificates
2. Agency MBS & CMO	G301, G303, G305, G307, G313, G315, G317, G319	Agency MBS, TBA MBS, CMO,
	K143, K145, K151, K153, G379, G380, K197	Pass Through CTF, REMIC, ARM
3. Commercial Loans	F610, F614, F615, F616, K199, K210, F618	Syndicated Loans, Term Deposits,
	$(2122 - 3123) - 1975 - 1410$ if ≥ 0	Term Loans
4. Money Market		Money Market, CDs, Corporate Paper
5. Municipal	8499, 8497, 3533	Municipality Debt
6. Corporate Bonds	G386, 1738, 1741, 1743, 1746	Bonds, MTN, Foreign Gov'ts & Agencies
7. Equities	A511	Equities, Funds, Convertible bonds
8. Private ABS & CMO	G309, G311, G321, G323, K147, K149, K155, K157	ABS, CMO, CDO, CLO, Covered Bonds
9. Consumer Loans	1975	
10. Real Estate Loans (Family)	$1410 * (RCON3465/RCON3385)$	
11. Real Estate Loans (Other)	$1410 * (RCON3466/RCON3385)$	
12. Cash	0010	CRSP Mutual Funds summary Cash %
13. Fixed Assets	3541, 3543, total assets - sum of above variables	