Pandemic Lockdown:
The Role of Government Commitment*

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Abstract

This paper studies optimal lockdown policies in a dynamic economy without government commitment. A lockdown imposes a cap on labor supply, which lowers economic output but improves health prospects. A government would like to commit to limit the extent of future lockdowns in order to increase investment by supporting a more optimistic economic outlook. However, such a commitment may not be credible since investment decisions are sunk at the time when the government decides on lockdowns. Rules that limit a government's future policy discretion can improve the efficiency of lockdowns, even in the presence of noncontractible information.

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1 Introduction

The ongoing COVID-19 pandemic has highlighted a great rise in not only empidemiological but also policy uncertainty.[1] In response to the pandemic, governments across the world implemented lockdown policies to limit the spread of infections. In numerous cases, these policies were first scheduled to end in the near future and then extended. For instance, New York Governor Andrew Cuomo imposed a statewide stay-at-home order on March 22, 2020, with an initial end date of April 19. This lockdown was later extended, first until April 29 and then until May 15, though targeted restrictions remain in place until the present day. Similar lockdown extensions were implemented at the discretion of many other regional and national governments. Such policy discretion can cause additional—i.e., over and above the pandemic—uncertainty among businesses that need to make forward-looking decisions subject to sunk costs. Examples of sunk costs include airlines investing in fleet maintenance, hotels deciding how many employees to retain on payroll, and restaurants placing ingredient orders ahead of reopening. Through these forward-looking decisions, current economic activity depends on the degree to which businesses expect governments to revise their plans for reopening the economy in the future.

In this paper, we study the role of government commitment and time consistency in designing lockdown policy. We consider a dynamic economy that embeds sequential government policy decision-making into a general SIR model of pandemics (Kermack and McKendrick, 1927; Ferguson et al., 2020). Each period, firms invest, the government chooses a lockdown policy, and workers supply labor. A lockdown imposes an upper bound on labor supply, limiting both economic activity and disease spread. Our framework is general and subsumes key mechanics of many other macroeconomic SIR models with lockdown or disease-mitigation policies in the literature.[2] A key feature of our model is that investment is determined before a lockdown policy is chosen. We think of this as capturing the kinds of investments in maintenance, employee retention, and inventory that businesses make while anticipating the ensuing trajectory of lockdown

[1] Following the onset of the COVID-19 pandemic, Baker et al. (2020) document a fourfold increase in their Economic Policy Uncertainty index, reaching its highest value on record. Hassan et al. (2020) track firm-level risks and sentiments due to government-related and other factors using text analysis of earnings conference calls. A recent report by McKinsey & Company, Smit et al. (2020) conclude that "lockdowns also cause uncertainty to remain high, as the extent of the structural damage to the economy becomes less predictable the longer lockdowns stay in place" and that "this uncertainty is paralyzing."

[2] See Acemoglu et al. (2020), Alfaro et al. (2020), Alvarez et al. (2020), Atkeson (2020a,b), Baqaee and Farhi (2020a,b), Berger et al. (2020), Birinci et al. (2020), Chari et al. (2020), Craig and Hines (2020), Eichenbaum et al. (2020a,b), Fang et al. (2020), Farboodi et al. (2020), Glover et al. (2020), Gregory et al. (2020), Jones et al. (2020), Kaplan et al. (2020), Krueger et al. (2020), and Piguillem and Shi (2020), among others.

policies during a pandemic. Through the forward-looking nature of investment, current economic activity depends on firms' expectations of lockdown policy.

Lockdowns induce both output costs and health benefits (Gourinchas, 2020; Hall et al., 2020). In our model, output decreases with the intensity of the lockdown through two channels. First, it decreases statically through lower labor supply, which is directly curbed by the lockdown. Second, it decreases dynamically through lower investment in expectation of lower future marginal returns to investment, which is indirectly reduced by the lockdown. Under government commitment, the optimal lockdown policy equates its marginal output costs and health benefits.

Our main result concerns the effect of a government's lack of commitment on optimal lockdown policy. A government would like to commit to the extent of future lockdowns in order to support more optimistic firm expectations in the present. However, such a commitment is not generally credible since investment decisions are sunk when the government decides on future lockdowns. Faced with a sunk investment, a government without commitment wants to impose a more stringent lockdown relative to the optimal policy under commitment because it does not fully internalize the associated reduction in returns to investment. Firms rationally anticipate the government's lack of commitment, causing them to invest less than they would in anticipation of the policy under commitment. Through this mechanism, lack of commitment distorts the efficient levels of investment and output associated with lockdown policy.

We then study how a government can improve the efficiency of its lockdown policy by using contingent plans that depend on new information that arrives during a lockdown. Examples of such information include estimates of disease mortality, the state of the economy, the medical system's capacity, or progress on vaccine development. Some of this information may be relevant for the payoffs and costs of lockdown policy. We show that a rule that imposes state-contingent limits on future policy discretion can attain the efficient allocation.

Finally, we extend our model to the case of noncontractible information that affects the efficiency of lockdown policy. In this setting, a rigid rule that constrains the government's ability to react to such information may be inefficient because future policy flexibility is valuable. Nevertheless, we show that an appropriately designed rule continues to be socially beneficial even when policy flexibility is valuable in such an environment. The reason is that a rule can be designed to preclude ex-ante inefficiently severe lockdowns that may be chosen by a future government without commitment. As such, a marginally binding rule increases social welfare by raising investment and output while continuing to allow for efficient lockdowns.

Our theoretical characterization of welfare-improving policy rules is in line with recent examples of real-world lockdown policies. For example, in his daily briefing on May 4, 2020, New York Governor Andrew Cuomo committed to a list of four quantifiable core factors that would need to be met in order for regional economies to reopen for business.[3] This qualitatively resembles our prescription that optimal lockdown rules should be conditioned on future information arrival. Elsewhere, Florida governor Ron DeSantis announced on September 25, 2020, a lower bound on restaurant capacity regardless of local restrictions. The stated goal of this lower bound was to reduce future lockdown policy discretion.[4] Again, this qualitatively resembles our finding that an unconditional limit on future lockdowns can improve the efficiency of lockdown policies.

Importantly, our analysis does not imply that lockdowns are harmful or necessarily too strict. In fact, reducing or lifting the lockdown in our model is detrimental if the resulting health costs exceed the economic gains. The government in our model chooses a current lockdown policy that is ex-post optimal. Our analysis points to the value of rules that define limits on the extent of future lockdowns. Such rules are net beneficial if their expected economic gains from stimulating investment toward its efficient level exceed their health costs. Naturally, there are other reasons why a government may choose inefficiently lax lockdowns. Our model abstracts from policy biases involving insufficient degrees of lockdowns by assuming that policies are chosen by a rational and benevolent government that maximizes long-run social welfare.[5] The mechanism we highlight here would act against political economy considerations that lead to departures from the benevolent-government assumption.

Related literature. This paper relates to the nascent literature on optimal policy in a pandemic, with recent contributions by Acemoglu et al. (2020), Alfaro et al. (2020), Alvarez et al. (2020),

[3]New York Governor Andrew Cuomo announced at his daily New York coronavirus press conference on May 4, 2020: "As long as your rate of transmission is manageable and low, then reopen your businesses and reopen the businesses in phases, so you're increasing that activity level while you're watching the rate of transmission. Rate of transmission goes up, stop the reopening, close the valve, close the valve right away. So reopen businesses, do it in phases and watch that rate of transmission. If it [the transmission rate R_t] gets over 1.1 stop everything immediately."

[4]Florida governor Ron DeSantis announced at a news conference on September 25, 2020: "We're also cognizant about the need for business certainty. There have been some local closures and other types of restrictions and so the order that I'm signing today will guarantee restaurants operate—will not allow closures. They can operate at a minimum of 50 percent regardless of local rule."

[5]This assumption may be violated if political economy considerations lead the government to overweigh immediate economic gains relative to future health costs of relaxing a lockdown, akin to the mechanism in Aguiar and Amador (2011).

Atkeson (2020a,b), Baqaee and Farhi (2020a,b), Berger et al. (2020), Birinci et al. (2020), Chari et al. (2020), Craig and Hines (2020), Eichenbaum et al. (2020a,b), Fang et al. (2020), Farboodi et al. (2020), Glover et al. (2020), Gregory et al. (2020), Jones et al. (2020), Kaplan et al. (2020), Krueger et al. (2020), and Piguillem and Shi (2020), among others. The analysis contained in this literature focuses on the optimal design of government policy, including the timing and intensity of lockdowns, under the assumption that the optimal policy can be enforced at all dates and under all contingencies. Our work contributes to this literature by highlighting that this analysis omits an important aspect of lockdown design, namely that the optimal policy may be hard to enforce due to issues of time-inconsistency. What distinguishes our work is the focus on the value of government commitment and the design of rules to improve efficiency absent such commitment in the context of lockdown policy.

That prior work on policy responses to a pandemic has ignored issues of time inconsistency is perhaps surprising given the parallel insights from an older literature that studies government commitment in the context of capital taxation, including the important contributions by Kydland and Prescott (1980), Chari and Kehoe (1990), Klein et al. (2008), Aguiar et al. (2009), and Chari et al. (2019). Like in the previous work on capital taxation, lack of commitment in our model reduces economic activity by distorting investment. However, our work incorporates two new insights that are central to the context of pandemics. First, a lockdown distorts investment not directly via taxation but indirectly by lowering the marginal returns to investment through a cap on labor supply. Second, investment distortions from lockdown do not directly affect the government budget but instead affect the future health state of the economy.

Our study of government commitment in the context of a pandemic also represents a methodological advance over previous work in the literature on capital taxation. Since the health state of our economy follows a general SIR model of pandemic spread, the value of a given health state cannot be represented by a univariate, concave function as in a typical model of optimal fiscal policy. This means that the usual methods for comparative statics cannot be applied here. Instead, we characterize the time-inconsistency of optimal lockdown policy under weak assumptions on the economic environment and the SIR model of disease dynamics.

Our analysis of rules for lockdown policy in the presence of noncontractible information relates to a growing literature on commitment versus flexibility in macroeconomics (Athey et al., 2005; Amador et al., 2006; Halac and Yared, 2014, 2018; Moser and Olea de Souza e Silva, 2019). Prior work in this area has focused on rules for either savings or for monetary and fiscal policy. Our work adds to this literature and a nascent strand of papers on the economics of pandemics with a theoretical analysis of the optimality of rules for lockdown policy. Our result that rules can strictly increase social welfare, even if flexibility is valuable, is reminiscent of similar insights in the context of savings or fiscal and monetary policy. However, our results do not directly follow from the methods developed in prior work, which rely on stronger assumptions on the utility function and the information structure than we require in our setting. By extending these insights and applying them to optimal lockdown design, we highlight a hitherto overlooked aspect of the policy debate around responses to pandemics.

2 Model

We consider a general infinite-horizon model of an economy during a pandemic. Each period has four stages. First, firms make an irreversible investment that enhances future productivity, such as rent, utilities, overhead expenses, software licenses, marketing, recruiting, and personnel costs. Second, after the investment is undertaken, the government chooses a lockdown policy. A lockdown imposes a cap on labor supply, which reduces economic output but inhibits disease spread. Third, production then takes place and all proceeds are paid to firms and workers. Fourth and finally, the pandemic evolves according to an SIR model of disease spread, which depends on the lockdown policy. A key feature of our model is that investment is determined before lockdown policy is chosen. We think of this as capturing the fact that business purchases of irreversible inputs must be made in advance of production and in anticipation of future policies.

2.1 Economic Environment

Periods are indexed by $t = 0, 1, \ldots$. At every date t, competitive firms make an irreversible investment x_t. The government then chooses a lockdown policy $\ell_t \in [0, 1]$ representing a cap on labor supply. If $\ell_t = 1$, then there is no lockdown while if $\ell_t = 0$, there is maximal lockdown. The economy is initialized with a continuum of mass 1 of workers that supply up to one unit of labor inelastically subject to the binding upper bound ℓ_t.

Workers consume their wage income

$$c_t = w_t \ell_t, \tag{1}$$

where c_t is consumption and w_t is the market wage. The irreversible investment x_t combined with labor ℓ_t generates output y_t according to the following production function:

$$y_t = f\left(x_t, \ell_t, \Omega_t\right),$$

where Ω_t is the health state at date t that is described in detail in the next subsection. The dependence of the production function on the health state captures the possibility that the pandemic—in addition to making people sick and killing people—decreases output by debilitating the workforce, by changing the share of the labor force working from t the office and from home, and by inducing protective but productivity-reducing social distancing efforts even absent any lockdown (Dingel and Neiman, 2020; Mongey et al., 2020). We assume that the function $f(\cdot)$ is continuously differentiable, increasing, and globally concave in x_t and ℓ_t, with $\lim_{x_t \to 0} \partial f(\cdot)/\partial x = \lim_{\ell_t \to 0} \partial f(\cdot)/\partial \ell = \infty$ and $\lim_{x_t \to \infty} \partial f(\cdot)/\partial x = \lim_{\ell_t \to \infty} \partial f(\cdot)/\partial \ell = 0$. From here on, we make the following key assumption:

Assumption 1. *The production function $f\left(x_t, \ell_t, \Omega_t\right)$ satisfies*

$$\frac{\partial^2 f(\cdot)}{\partial x_t \partial \ell_t} > 0.$$

Assumption 1 states that investment x_t and labor ℓ_t are q-complements in production. It implies that there are higher marginal returns to investment x_t when labor ℓ_t is greater and vice versa, which is intuitive under our interpretation of x_t being investment that enhances future productivity.

Firm owners maximize profits

$$\pi_t = y_t - x_t - w_t \ell_t,$$

where we have set the price of the irreversible investment x_t equal to that of output, which is normalized to 1. In a competitive equilibrium, the marginal product of investment satisfies the following firm optimality condition:

$$1 = \frac{\partial f\left(x_t, \ell_t, \Omega_t\right)}{\partial x}. \tag{2}$$

Equation (2) implies that in a competitive equilibrium where investment adjusts to the anticipated level of labor supply, investment can be written as

$$x_t = x^*\left(\ell_t, \Omega_t\right), \tag{3}$$

where the function $x^*(\cdot)$ satisfies $\partial x^*\left(\ell_t, \Omega_t\right)/\partial \ell > 0$ by Assumption 1. In other words, firms invest less in anticipation of a more stringent lockdown as a result of the q-complementarity between investment and labor in production.

Labor is competitively supplied so wages equal the marginal product of labor given by

$$w_t = \frac{\partial f\left(x_t, \ell_t, \Omega_t\right)}{\partial \ell}. \tag{4}$$

From equation (4), consumption given by (1) satisfies

$$c_t = c^*\left(x_t, \ell_t, \Omega_t\right), \tag{5}$$

where the function $c^*(\cdot)$ is continuously differentiable in x_t and ℓ_t and strictly increasing in x_t by Assumption 1.[6]

2.2 Disease Spread, Lockdown Policy, and Welfare

We model disease spread as following an SIR model (Kermack and McKendrick, 1927; Ferguson et al., 2020), which we allow to depend on a lockdown policy, as in Atkeson (2020a), Eichenbaum et al. (2020a), and Alvarez et al. (2020). Specifically, we define the health state of the economy in period t as $\Omega_t = \{S_t, I_t, R_t, D_t\}$, where $S_t \in [0,1]$ is the mass of susceptible individuals, $I_t \in [0,1]$ is the mass of infected and contagious individuals, $R_t \in [0,1]$ is the mass of recovered individuals,

[6] We do not require that $c^*(\cdot)$ be globally increasing in ℓ_t, though this is the case for commonly used production functions, such as those in the Cobb-Douglas family.

and $D_t \in [0,1]$ is the mass of deceased individuals. It follows that

$$S_t + I_t + R_t + D_t = 1.$$

An SIR model defines a mapping $\Gamma(\cdot)$ that implies a law of motion of the health state,

$$\Omega_{t+1} = \Gamma(\ell_t, \Omega_t),$$

which depends on the degree of lockdown at date t.[7] The initial health state Ω_0 is taken as given.

Social welfare equals the discounted sum of a utility stream,

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \Omega_t), \tag{6}$$

where $\beta \in (0,1)$ is the discount factor, and $u(\cdot)$ is a strictly increasing and strictly concave utility function of consumption c_t and also depends on the health state Ω_t.[8] Utility depends directly on the health state, which captures the costs of illness and mortality associated with disease spread. Moreover, utility also indirectly depends on disease spread through the level of consumption c_t, since the health state Ω_t directly enters the production function $f(\cdot)$.

We do not impose restrictions on how the health state and lockdown impact output, utility, and disease dynamics in the economy other than the following assumption, which we maintain from here on:

Assumption 2. *The functions $f(x_t, \ell_t, \Omega_t)$, $u(c_t, \Omega_t)$, and $\Gamma(\ell_t, \Omega_t)$ are continuously differentiable in all elements of Ω_t.*

This technical assumption guarantees that the government's problem is well-behaved and that we can rely on first-order conditions (FOCs) in the proofs of our results. Note that these assumptions are satisfied in many recent macroeconomic models with SIR modules in which disease dynamics respond smoothly to lockdown policies, such as Eichenbaum et al. (2020a) and Alvarez et al. (2020).

[7] All of our results extend to a setting where the health state is also a function of time, a feature which would capture factors such as the evolving constraints on the medical system and the changing likelihood of vaccine discovery.

[8] Our assumption that the government only weighs worker consumption is not necessary to derive our main result in Proposition 1, namely that the optimal lockdown policy is time-inconsistent. We make this assumption to simplify the exposition here.

2.3 Timeline

At the beginning of each period, the health state Ω_t is taken as given. The intra-period order of events at each date t is as follows:

1. Firms choose irreversible investment x_t;
2. The government chooses lockdown policy ℓ_t;
3. Production occurs subject to lockdown policy, output y_t is produced, and workers and firm-owners are paid and consume;
4. The health state evolves according to the transition function Γ.

It is worth highlighting that investments are made before the lockdown policy is chosen each period. This sequencing captures the long-term investments that businesses make in anticipation of the future trajectory of lockdown policies. Our modeling approach is motivated by recent survey evidence documenting that businesses that expect a more prolonged crisis—due to the pandemic and associated lockdowns—are also more likely to expect to shut down operations (Bartik et al., 2020). We will explore the implications of this sequencing of investment and lockdown decision for the optimal policy under commitment compared to that under lack of commitment.

3 Optimal Policy under Commitment

Suppose that the government commits to an optimal lockdown policy sequence $\{\ell_t^c\}_{t=0}^{\infty}$ at time 0. This means that the government internalizes the fact that investment optimally adjusts to anticipated labor supply as determined by future lockdown policy. Given firm optimality in (3), this policy sequence induces an optimal investment sequence $\{x_t^c\}_{t=0}^{\infty}$ under government commitment.

After substituting the investment function $x_t = x^*(\ell_t, \Omega_t)$ from (3) and the consumption function $c_t = c^*(x_t, \ell_t, \Omega_t)$ from (5) into the social welfare function (6), the government with commitment solves the following sequence problem:

$$\max_{\{\ell_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c^*\left(x^*\left(\ell_t, \Omega_t\right), \ell_t, \Omega_t\right), \Omega_t\right) \right\} \tag{7}$$

$$\begin{aligned} \text{s.t.} \quad & \ell_t \in [0,1], \quad \forall t \geq 0, \\ & \Omega_{t+1} = \Gamma\left(\ell_t, \Omega_t\right), \quad \forall t \geq 0, \\ & \Omega_0 \text{ given.} \end{aligned}$$

Importantly, substituting the optimal firm investment response $x^*\left(\ell_t, \Omega_t\right)$ into the welfare function before deriving the optimal lockdown sequence $\{\ell_t^c\}_{t=0}^{\infty}$ that solves the program in (7) means that the government with commitment takes into account the reaction of investment to its policies in all periods. The problem of the government with commitment can be written recursively as

$$V^c\left(\Omega\right) = \max_{\ell \in [0,1]} \left\{ u\left(c^*\left(x^*\left(\ell, \Omega\right), \ell, \Omega\right), \Omega\right) + \beta V^c\left(\Gamma\left(\ell, \Omega\right)\right) \right\}, \tag{8}$$

where $V^c\left(\Omega\right)$ denotes the value of health state Ω to the government with commitment. The solution to program (8) induces an optimal lockdown policy under commitment as a function of the prevailing health state Ω, denoted $\ell^c(\Omega)$. This lockdown policy in turn yields an optimal investment level under commitment that depends only on the health state Ω, denoted $x^c(\Omega) = x^*(\ell^c(\Omega), \Omega)$.

Standard arguments together with Assumption 2 imply that $V^c\left(\Omega\right)$ is continuously differentiable in all of its arguments. This means that the necessary FOC for interior optimal levels of lockdown under commitment $\ell^c \in (0,1)$ is

$$\frac{\partial u\left(\cdot\right)}{\partial c} \left[\frac{\partial c^*\left(\cdot\right)}{\partial x} \frac{\partial x^*\left(\cdot\right)}{\partial \ell} + \frac{\partial c^*\left(\cdot\right)}{\partial \ell} \right] = -\beta \frac{dV^c\left(\cdot\right)}{d\ell}. \tag{9}$$

In choosing the degree of lockdown, the government weighs two opposing forces, as in Gourinchas (2020) and Hall et al. (2020). On the one hand, it considers the *economic costs* captured by the left-hand side of (9). The economic costs are twofold. First, conditional on the level of investment, a lockdown has a direct impact on output and consumption by limiting labor supply. Second, a lockdown has an indirect impact on output by reducing the marginal product of investment which reduces investment. The government's ability to commit gives it the ability to take into account both of these factors and anticipate firms' reaction to the policy.

On the other hand, the government considers the discounted future *health benefits* in terms of reduced mortality from inhibiting the disease spread, as captured by right-hand side of (9). Using the envelope condition, the marginal health cost of lockdown can be represented recursively as

$$\frac{dV^c\left(\Omega'\right)}{d\ell} = \frac{du\left(c^*\left(x^c\left(\ell',\Omega'\right),\ell',\Omega'\right),\Omega'\right)}{d\ell} + \beta\frac{dV^c\left(\Gamma\left(\ell',\Omega'\right)\right)}{d\ell}, \tag{10}$$

where $\Omega' = \Gamma(\ell, \Omega)$ denotes next period's health state and ℓ' denotes the level of next period's optimal lockdown. By use of the envelope condition, the optimal lockdown policy function $\ell^c(\Gamma(\ell, \Omega))$ was replaced with the level of next period's optimal lockdown ℓ' on the right-hand side of equation (10). This equation illustrates that present lockdown dynamically impacts all future health states, which in turn impact welfare both through their direct health costs and through their indirect effect on consumption.

4 Optimal Policy under Lack of Commitment

Under lack of commitment, investment is treated as sunk at the time when lockdown policy is chosen. The government at date t chooses an optimal degree of lockdown that depends on investment x_t and the health state Ω_t, which we denote $\ell^*(x_t, \Omega_t)$. Firms in turn anticipate the government's policy and decide on the optimal investment level $x^*(\ell_t, \Omega_t)$ that depends on the expected lockdown ℓ_t and the health state Ω_t. We consider a Markov perfect equilibrium (MPE), in which policies can be expressed as functions of only the health state Ω_t, namely $x^n(\Omega_t)$ and $\ell^n(\Omega_t)$. In any MPE, $x^n(\Omega_t) = x^*(\ell^n(\Omega_t), \Omega_t)$ and $\ell^n(\Omega_t) = \ell^*(x^n(\Omega_t), \Omega_t)$, as the government and firms take each other's policy functions as given when choosing their actions under the prevailing health state.

The problem of the government without commitment in an MPE can be written recursively as

$$\begin{aligned} W^n\left(x,\Omega\right) &= \max_{\ell\in[0,1]} \left\{u\left(c^*\left(x,\ell,\Omega\right),\Omega\right) + \beta V^n\left(\Gamma\left(\ell,\Omega\right)\right)\right\}, \\ V^n\left(\Omega'\right) &= u\left(c^*\left(x^n\left(\Omega'\right),\ell^n\left(\Omega'\right),\Omega'\right),\Omega'\right) + \beta V^n\left(\Gamma\left(\ell^n\left(\Omega'\right),\Omega'\right)\right), \end{aligned} \tag{11}$$

where $W^n(x, \Omega)$ denotes the value of investment x and health state Ω, while $V^n(\Omega')$ denotes the continuation value of next period's health state $\Omega' = \Gamma(\ell, \Omega)$ to the government without commitment. Note that $W^n(x, \Omega)$ depends on the current period's investment and health state, while $V^n(\Omega')$ depends only on next period's health state. This reflects the fact that next period's MPE investment policy $x^n(\Omega')$ is already consistent with the future MPE lockdown policy $\ell^n(\Omega')$ by the government without commitment, and vice versa. Importantly, by not substituting the current period's optimal investment response when solving its problem, the government without commitment treats current investment as sunk when deciding on lockdown policy.

Consider the government's FOC in a differentiable MPE for interior lockdown $\ell^n \in (0, 1)$ under lack of commitment:

$$\frac{\partial u(\cdot)}{\partial c} \frac{\partial c^*(\cdot)}{\partial \ell} = -\beta \frac{dV^n(\cdot)}{d\ell}. \tag{12}$$

Assumption 1 implies that, holding all else—including investment and the health state—fixed, the left-hand side of the optimality condition under lack of commitment in equation (12) is strictly less than that under commitment in equation (9). This captures the fact that a government without commitment undervalues the economic cost of a lockdown relative to a government with commitment. Specifically, a government without commitment does not take into account that a more stringent lockdown changes ex-ante firm expectations in a way that reduces the level of investment, which in turn reduces future output and consumption.

Now consider the right-hand side of (12). The envelope condition can be written as

$$\begin{aligned}\frac{dV^n(\Omega')}{d\ell} = {} & \frac{du(c^*(x^n(\ell', \Omega'), \ell', \Omega'), \Omega')}{d\ell} + \beta \frac{dV^n(\Gamma(\ell', \Omega'))}{d\ell} \\ & + \left[\frac{du(c^*(x^n(\ell', \Omega'), \ell', \Omega'), \Omega')}{d\ell'} + \beta \frac{dV^n(\Gamma(\ell', \Omega'))}{d\ell'} \right] \frac{d\ell^n(\Omega')}{d\ell},\end{aligned} \tag{13}$$

where $\Omega' = \Gamma(\ell, \Omega)$ denotes next period's health state as a function of the current lockdown level and health state, ℓ' denotes the level of optimal lockdown under lack of commitment next period, and $\ell^n(\Omega')$ is next period's MPE lockdown policy under lack of commitment as a function of next period's health state.

The first line on the right-hand side of the envelope condition under no commitment in (13) is analogous to that under commitment in (10). It represents the payoff from changing the future health state by changing the lockdown today, holding fixed the optimal future lockdown policy.

The second line on the right-hand side of (13) is unique to the case of lack of commitment. It corresponds to the strategic effect of a lockdown on future policy, since changing the future health state also changes future lockdown incentives. Under commitment, the term analogous to that in brackets in the second line of (13) is zero because the government with commitment takes into account firms' reaction to its choice of policy, as captured by the FOC (9). Under lack of commitment, however, equation (12) and Assumption 1 together imply that the term in brackets is positive.[9]

Note that our general model is rather complex in that the value of a given health state cannot be represented by a univariate, concave function of the state, as in a typical model of optimal fiscal policy. Nevertheless, under the weak conditions spelled out above, we obtain the following result:

Proposition 1 (Time Inconsistency)**.** *Suppose that the optimal policy under commitment* $\{\ell_t^c\}_{t=0}^{\infty}$ *admits an interior solution in some period* t*. Then the optimal policy under commitment is not time-consistent.*

Proof. See Appendix A.1. □

Proposition 1 states that lack of government commitment may result in an inefficient lockdown policy. The idea behind the proof is as follows: If the optimal lockdown policy under lack of commitment mirrored that under commitment, then the no-commitment government's option to deviate would have zero value because a deviation would be associated with weakly negative payoff. But at an interior solution where $\ell_t \in (0, 1)$ for some t, the optimality condition (12) under no commitment calls for a strictly lower value of ℓ_t than condition (9) under commitment. Therefore, the optimal policy is time-inconsistent whenever it is interior.

The intuition for this result is that, absent commitment, the government takes firm investment as given and thus undervalues the economic cost of a lockdown, leading to an inefficient choice of lockdown. By anticipating this behavior, firms invest less than they would under commitment. For this reason, the optimal policy under lack of commitment differs from that under commitment.[10]

Note that Proposition 1 is silent on whether the optimal lockdown policy under commitment is more or less stringent than that under lack of commitment. This is due to two key differences

[9]While we can sign the term in brackets, we cannot sign the overall strategic effect since the sign of $d\ell^n\left(\Gamma\left(\ell, \Omega\right)\right)/d\ell$ is ambiguous due to the nonlinear dynamics of the SIR model. If, for example, a marginal increase in ℓ causes a large (small) share of the population to become recovered and immune, then the optimal future $\ell^n\left(\Gamma\left(\ell, \Omega\right)\right)$ may increase (decrease).

[10]In case Assumption 1 is reversed—i.e., if x and ℓ are q-substitutes in production—then our main result continues to hold but the intuition is also reversed.

between the optimal policies with and without commitment. The first difference is a static one: starting from an ℓ_t that is interior under commitment and given a health state Ω_t, investment x_t, and continuation value $V(\Omega_{t+1})$, a government without commitment would choose a strictly lower ℓ_t than a government with commitment. This is due to the fact that the government without commitment treats investment x_t as sunk when it decides on lockdown policy at time t. The second difference is a dynamic one: given the inefficient policy choice of a government without commitment, investment and the health state will evolve differently in a dynamic model under lack of commitment versus under commitment. Since the sign of the second difference is not clear without further model restrictions, the net effect of lack of commitment on lockdown policies is theoretically ambiguous. This also means that a full characterization of the MPE lockdown policy $\ell^n(\cdot)$ is difficult and beyond the scope of the current paper.

5 Value of Rules

We have established that the optimal lockdown policy is not time-consistent. Time-inconsistency in policy arises due to dynamic government optimization in the space of Markov strategies. Deviations from the policy under commitment occur because a government without commitment chooses a lockdown that is ex-post optimal but leads to ex-ante inefficient investment in expectation of the no-commitment outcome. This raises the possibility that constraints on government policy can prevent ex-ante inefficient policy outcomes.

5.1 Optimality of Limiting Future Policy Discretion

In our environment, a credible lockdown policy plan can be socially optimal. Suppose that rather than choosing a lockdown policy $\ell_t \in [0,1]$ with discretion, the government is constrained to choosing a policy $\ell_t \in \mathcal{L}_t(\Omega_t)$ from a subset of the policy space $\mathcal{L}_t(\Omega_t) \subseteq [0,1]$ that depends on the prevailing health state Ω_t. As an example of a particularly heavy-handed policy constraint, consider $\mathcal{L}_t(\Omega_t) = \{\ell_t^c(\Omega_t)\}$. Then the policy decision is constrained to the optimum under commitment, $\ell_t(\Omega_t) = \ell_t^c(\Omega_t)$. Clearly, this policy constraint implements the efficient outcome as it mimics the time-consistent policy choice.

Going beyond this extreme example, we can study rules that constrain the extent of a lockdown. Consider a state-contingent rule $\mathcal{L}_t(\Omega_t) = \{\ell_t | \ell_t \geq \underline{\ell}_t(\Omega_t)$ so that a government at date

t can choose any policy ℓ_t that exceeds $\underline{\ell}_t(\Omega_t)$ with discretion. In other words, the government commits to limiting the stringency of the lockdown.[11] We then have the following result:

Proposition 2 (Value of Rules)**.** *Consider a rule $\{\underline{\ell}_t(\Omega_t)\}_{t=0}^{\infty}$ such that $\underline{\ell}_t(\Omega_t) = \ell_t^c(\Omega_t)$ for all periods t and all health states Ω_t. Then there exists an MPE subject to this rule in which the government without commitment chooses the optimal policy under commitment.*

Proof. See Appendix A.2. □

Proposition 2 shows that the introduction of rules that impose a limit on the severity of lockdown can implement the optimal policy and therefore improve the efficiency and welfare in an economy without government commitment. The idea behind the proof is as follows: A rule that takes the form of a lower bound only allows for upward deviations in labor supply from ℓ_t to some $\tilde{\ell}_t > \ell_t$, i.e., to a less strict lockdown. But if a surprise relaxation of lockdown were optimal to a government without commitment given sunk investment $x^*(\ell_t, \Omega_t)$ as a function of the anticipated lockdown ℓ_t and health state Ω_t, then a government with commitment could have implemented the same lockdown relaxation with firms anticipating it, leading to investment $x^*(\tilde{\ell}_t, \Omega_t)$. Since an anticipated lockdown relaxation yields higher investment and thus consumption, due to q-complementarity between investment and labor in production, such a deviation contradicts the optimality of the original lockdown policy.

The intuition for this result is that a lower bound on labor supply stops the government without commitment from making short-sighted policy decisions when investment is treated as sunk. An upper bound on labor supply is not necessary because lack of commitment does not tempt a government to impose too lax a lockdown. This is because q-complementarity between investment and labor in production (Assumption 1) implies that one-shot deviations from an equilibirum under commitment by a government without commitment are profitable only in the direction of stricter, not less strict, lockdown policy. For this reason, an upper bound on labor supply cannot improve the efficiency of lockdown policy.

Note that the rule described in Proposition 2 is less strict than one dictating the exact level of lockdown every period. At the same time, an upper bound on lockdowns may be overly strict if good reasons for imposing future lockdowns materialize themselves. While our analysis so far

[11] This lower-bound rule matches Florida governor Ron DeSantis' announcement in a news conference on September 25, 2020, of a state-wide 50-percent minimum capacity for restaurants.

has abstracted from such reasons by assuming that the ex-post efficiency of future lockdowns can be guaranteed ex-ante, we now turn to the case when future policy flexibility is valuable.

5.2 Uncertainty and Noncontractible Information

We have shown thus far that in an economy in which full information on the health state is available, a government without commitment would like to choose a more stringent lockdown than is optimal, but a rule that limits future lockdown can increase welfare by mitigating this commitment problem. In practice, of course, government policy depends not only on the health state but also on new information that arrives during a lockdown. Such information may include estimates of disease mortality, the state of the economy, the likelihood of vaccine discovery, or the medical system's capacity. However, information on future realizations of these variables may be hard to verify or incorporate into a written contract.

This motivates our study of the design of rules under uncertainty and noncontractible information. We show that a modification of our previous result (Proposition 2) extends to an environment that incorporates such considerations. Specifically, we show that rules that constrain future government policy either as a function of future information revelation—as seen in the US state of New York—or unconditionally—as in the US state of Florida—can improve welfare.

To capture this idea, suppose that a state variable θ_t is realized in addition to the prevailing health state Ω_t before investment $x_t = x^*(\ell_t, \Omega_t, \theta_t)$ is made in anticipation of lockdown $\ell_t = \ell^*(x_t, \Omega_t, \theta_t)$ in period t. Let θ_t be independently and identically distributed with associated probability density function (pdf) $g(\theta_t)$ over support $\theta_t \in [\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} < \overline{\theta}$ for all t. After substituting the modified consumption function $c_t = c^*(x_t, \ell_t, \Omega_t, \theta_t)$ based on (5), social welfare at time $t = 0$ given a sequence of investment and lockdown policies $\{x_t, \ell_t\}_{t=0}^{\infty}$ is

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0[u\left(c^*\left(x_t, \ell_t, \Omega_t, \theta_t\right), \Omega_t, \theta_t\right)] \qquad (14) \\ \text{s.t.} \quad &\Omega_{t+1} = \Gamma\left(\ell_t, \Omega_t, \theta_t\right), \quad \forall t \geq 0, \\ &\theta_t \sim g\left(\theta_t\right), \quad \forall t \geq 0, \\ &\Omega_0 \text{ given,} \end{aligned}$$

where the expectation $\mathbb{E}_0[\cdot]$ is taken over the time-0 and future realizations of θ_t.

Note that the stochastic state θ_t enters the problem in multiple places. It indirectly enters the consumption function $c^*(\cdot)$ through its effect on production. At the same time, it directly enters the utility function $u(\cdot)$ and the SIR model $\Omega(\cdot)$. Of course, while they are taken as given in (14), the optimal policy functions for investment $x^*(\ell_t, \Omega_t, \theta_t)$ and lockdown $\ell^*(x_t, \Omega_t, \theta_t)$ will also depend on θ_t.

The optimal lockdown policy under commitment depends on the health state Ω_t and the realization of θ_t, denoted $\ell^c(\Omega_t, \theta_t)$. This policy function implicitly takes into account the optimal investment under commitment, $x^c(\Omega_t, \theta_t) = x^*(\ell_t^c(\Omega_t, \theta_t), \Omega_t, \theta_t)$. Analogously, the optimal lockdown policy in an MPE under lack of commitment depends only on the health state Ω_t and the realization of θ_t, denoted $\ell^n(\Omega_t, \theta_t)$. This policy function implicitly takes into account the MPE choice of investment under lack of commitment, $x^n(\Omega_t, \theta_t) = x^*(\ell_t^n(\Omega_t, \theta_t), \Omega_t, \theta_t)$.

Suppose that θ_t represents contractible information. From this and an argument analogous to that in Proposition 2, it follows that a rule that imposes a sequence of lower bounds $\{\underline{\ell}_t(\Omega_t, \theta_t)\}_{t=0}^{\infty}$ on labor supply, so that $\ell_t \geq \underline{\ell}_t(\Omega_t, \theta_t)$ for $\underline{\ell}_t(\Omega_t, \theta_t) = \ell^c(\Omega_t, \theta_t)$ in all periods t, can increase social welfare by inducing the government without commitment to choose the policy under commitment.

In practice, some of the information in θ_t may not be contractible. In this case, a rigid plan may be too constraining since policy flexibility in responding to realizations of θ_t is valuable. We show that bounded discretion in the form of a rule $\underline{\ell}_t(\Omega_t) > 0$ that constrains the government to policies $\ell_t \in [\underline{\ell}_t(\Omega_t), 1]$ independent of θ_t can still improve welfare in this case. To this end, consider the recursive formulation of the problem faced by a government without commitment:

$$W^n(x, \Omega, \theta) = \max_{\ell \in [0,1]} \left\{ u\left(c^*\left(x, \ell, \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell, \Omega, \theta\right), \theta'\right)] \right\}, \tag{15}$$

$$V^n\left(\Omega', \theta'\right) = u\left(c^*\left(x^n\left(\Omega', \theta'\right), \ell^n\left(\Omega', \theta'\right), \Omega', \theta'\right), \Omega', \theta'\right) + \beta \mathbb{E}_{\theta''}[V^n\left(\Gamma\left(\ell^n\left(\Omega', \theta'\right), \Omega', \theta'\right), \theta''\right)],$$

where $\Omega' = \Gamma(\ell, \Omega, \theta)$ and $\mathbb{E}_{\theta'}[\cdot]$ denotes the expectation over next period's realization of θ'. From here on, we operate under the following simplyfing assumption:

Assumption 3. *The optimal lockdown policy under lack of commitment $\ell^n(\Omega_t, \theta_t)$ is strictly decreasing in θ_t over interior $\ell^n(\Omega_t, \theta_t) \in (0,1)$ and continuous in a neighborhood below $\overline{\theta}$ for all Ω_t. Moreover, the pdf $g(\cdot)$ is strictly positive and continuous in a neighborhood below $\overline{\theta}$.*

According to Assumption 3, higher values of the noncontractible state are associated with stricter optimal lockdown policies under lack of commitment. Then we obtain the following result:

Proposition 3 (Value of Rules under Uncertainty)**.** *Suppose the optimal lockdown policy under lack of commitment is interior at time 0, so $\ell^n(\Omega_0, \theta_0) < 1$ for some realization of θ_0 with positive probability. Then there exists a rule that imposes a lower bound $\underline{\ell}_t(\Omega_t)$ on labor supply in some period t that strictly increases social welfare under lack of commitment.*

Proof. See Appendix A.3. □

Proposition 3 shows that the introduction of rules increases social welfare even if there is a future policy discretion is valuable. The idea behind the proof is as follows: A government lacking commitment chooses a more severe lockdown in the future than is socially desirable. As such, a marginally binding rule increases social welfare by raising investment and output at no efficiency cost to future lockdown policy design. To demonstrate this, a key part of the argument is that the most extreme possible lockdown policy would never be optimal for a government with commitment under any realization of new information. This is natural in our setting in which the production technology satisfies an Inada condition—completely shutting down the economy yields unbounded marginal gains from opening the economy slightly. [12]

The intuition for this result is that a marginally binding rule does not prevent efficient lockdowns while successfully limiting the damages of excessive lockdowns in the future. By preventing only the most extreme variants of future lockdown policies, such a rule can improve firm expectations and improve the efficiency of lockdown policy.

6 Concluding Remarks

We have analyzed the value of government commitment in choosing a lockdown policy. A government would like to commit to limit the extent of future lockdowns in order to support more optimistic firm expectations in the present. However, such a commitment is not credible since investment decisions are sunk when the government makes the lockdown decision. This gives value to rules limiting future lockdown policy discretion.

[12] For an example of a case in which extreme choices are sometimes optimal even under commitment, see Halac and Yared (forthcoming) for a discussion of threshold contracts with escape clauses.

Our analysis leaves several interesting avenues for future research. First, the generality of our approach suggests that time-consistency considerations could be relevant in the realms of many lockdown decision problems. For instance, it would be interesting to characterize the optimal policy response to widespread employee furloughs. Payroll subsidies and cheap access to credit for businesses have been widely advocated during the global COVID-19 pandemic. However, their efficiency under lack of government commitment could be drastically different from that under commitment, which previous work has exclusively focused on. Time-inconsistency is also relevant in other domains, such as school and college decisions to reopen in anticipation of future shutdowns. Insights similar to our characterization of government policy under lack of commitment may apply in such contexts.

Second, we have evaluated the effect of rules that limit lockdowns assuming that governments adhere to such rules. In practice, rules may be broken and the private sector may be uncertain about the government's commitment to respecting them. In the context of capital taxation, Phelan (2006) and Dovis and Kirpalani (2019) show that this consideration leads the private sector to dynamically update its beliefs about a government's ability to commit. We conjecture that in our framework, this uncertainty could cause firms to react to lockdown extensions by becoming increasingly pessimistic about the government's ability to commit to lifting a future lockdown. This could lead to further declines in investment and economic activity as well as political economy repercussions in response to lockdown extensions.

Finally, our analysis ignores the availability of monetary and fiscal policy tools, as in Guerrieri et al. (2020). In our framework, these tools could not only mitigate the immediate economic costs of a pandemic, but also boost investment, thus counteracting future economic costs from underinvestment due to the government's lack of commitment. We leave the exploration of how optimal lockdown policy interacts with monetary and fiscal policy under lack of government commitment as an interesting subject of further research.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Proof. To prove that the optimal lockdown policy is not time-consistent, we want to show that $\ell_t^c \neq \ell_t^n$ for some t. Let t be a period in which $\ell_t^c \in (0,1)$, which exists by assumption.

Suppose, by way of contradiction, that there exists an MPE under no commitment that coincides with the optimal policy under commitment in all possible states and all periods. For a government choosing lockdown ℓ_t given health state Ω_t, this would mean that the (continuation) value function would be the same with and without commitment. Therefore,

$$\frac{dV^c(\cdot)}{d\ell_t} = \frac{dV^n(\cdot)}{d\ell_t}, \tag{16}$$

so the derivative of the continuation value with respect to current lockdown is the same with and without commitment. However, if (16) holds, then the two optimality conditions (9) and (12) cannot simultaneously hold because their left-hand sides are not equal by Assumption 1:

$$-\beta\frac{dV^c(\cdot)}{d\ell_t} = \frac{\partial u(\cdot)}{\partial c_t}\left[\frac{\partial c^*(\cdot)}{\partial x_t}\frac{\partial x^c(\cdot)}{\partial \ell_t} + \frac{\partial c^*(\cdot)}{\partial \ell_t}\right] > \frac{\partial u(\cdot)}{\partial c_t}\frac{\partial c^*(\cdot)}{\partial \ell_t} = -\beta\frac{dV^n(\cdot)}{d\ell_t}$$

This poses a contradiction with the equality in (16), proving the claim that the policy under lack of commitment does not coincide with that under commitment. Therefore, the optimal lockdown policy is not time-consistent.

□

A.2 Proof of Proposition 2

Proof. To prove that a rule consisting of a lower bound $\underline{\ell}_t(\Omega_t) = \ell_t^c(\Omega_t)$ on ℓ_t supports an MPE that attains the efficient allocation, we want to show that there exists no profitable deviation from this allocation by a government without commitment adhering to this rule.

Consider a government today choosing the optimal policy under the state-contingent rule and expecting all future governments to choose lockdown equal to the state-contingent rule. Therefore, the government's state-contingent policy is given by $\{\ell_t(\Omega_t)\}_{t=0}^{\infty}$ such that $\ell_t(\Omega_t) = \underline{\ell}_t(\Omega_t) =$

$\ell_t^c(\Omega_t)$ in all states and all periods, which induces a sequence of investments $\{x_t^c\}_{t=0}^{\infty}$ such that $x_t = x_t^c$ in all states and all periods. Now consider in any period t the problem of the government without commitment, which anticipates all future governments to follow the optimal policy under commitment and also for investment to match that under the commitment policy. Under this construction, comparing the FOC of the government under lack of commitment (12) with that under commitment (9), the unconstrained government without commitment would like to choose a value of ℓ_t that is strictly lower than $\underline{\ell}_t(\Omega_t)$. Clearly, this is not possible given the rule, which constrains the government to choose $\ell_t \geq \underline{\ell}_t(\Omega_t)$. Thus, there are two possibilities: either $\ell_t = \underline{\ell}_t(\Omega_t) = \ell_t^c(\Omega_t) < 1$ and there exists a profitable upward deviation from ℓ_t to some $\tilde{\ell}_t \in (\ell_t, 1]$ in period t, or else the current allocation constitutes an MPE.

Suppose by way of contradiction there exists such a profitable upward deviation from $\ell_t < 1$ to $\tilde{\ell}_t > \ell_t$ in period t given sunk investment x_t and health state Ω_t. For this to be the case, we must have

$$u(c^*(x^*(\ell_t, \Omega_t), \tilde{\ell}_t, \Omega_t)) + \beta V^c(\Gamma(\tilde{\ell}_t, \Omega_t)) > u(c^*(x^*(\ell_t, \Omega_t), \ell_t, \Omega_t)) + \beta V^c(\Gamma(\ell_t, \Omega_t)). \tag{17}$$

Note that because this deviation is unanticipated, investment $x^*(\ell_t, \Omega_t)$ remains at its level in ex-ante expectation of lockdown ℓ_t under any ex-post investment $\tilde{\ell}_t$. We now show that if the inequality in (17) were to hold, then the government under commitment could profitably deviate from its investment strategy, thus contradicting the optimality of the original MPE.

Consider the same deviation from $\ell_t < 1$ to $\tilde{\ell}_t > \ell_t$ by a government with commitment. Since firms anticipate the lockdown policy in period t under commitment, Assumption 1 implies that due to q-complementarity between x_t and ℓ_t in production, the optimal investment would also adjust upward from $x_t = x^*(\ell_t, \Omega_t)$ to $\tilde{x}_t = x^*(\tilde{\ell}_t, \Omega_t) > x_t$. Since consumption in (5) is strictly increasing in x_t, this deviation yields a strictly greater benefit to the government with commitment compared to that under commitment.

We conclude that equation (17) characterizing the deviation by the government without commitment can hold only if there exists a profitable deviation by the government with commitment. This contradicts the optimality of the original MPE, thus invalidating the existence of a profitable upward deviation by the government without commitment. Therefore, the allocation under commitment together with a rule consisting of a lower bound $\underline{\ell}_t(\Omega_t) = \ell_t^c(\Omega_t)$ on ℓ_t in all states and all periods also constitutes an MPE under lack of commitment. $\square$

A.3 Proof of Proposition 3

Proof. First, note that lockdown under full commitment and under lack of commitment is never maximal due to the Inada conition on the production function $f(\cdot)$ with respect to labor input ℓ.

Since the statement of the proposition concerns the existence of a rule in some period t, we will consider period $t = 0$. Now contemplate a rule that imposes a lower bound $\underline{\ell}_0(\Omega_0) = \underline{\ell}(\Omega_0; \varepsilon) = \ell^n\left(\Omega_0, \overline{\theta} - \varepsilon\right)$, for some $\varepsilon > 0$, on labor supply ℓ_0 at time 0. We will establish that such a rule strictly increases social welfare for small enough $\varepsilon > 0$. For the remainder of the proof, we consider a perturbation only at time $t = 0$, which we treat as the current period, and will drop all time subscripts.

For a given state (Ω, θ), let $x^n \equiv x^n(\Omega, \theta)$ and $\ell^n(\Omega, \theta)$ denote the MPE investment policy and lockdown policy under no commitment in the absence of a rule, and let $x^r \equiv x^r(\Omega, \theta; \varepsilon)$ and $\ell^r(\Omega, \theta; \varepsilon)$ denote the MPE investment policy under no commitment subject to the rule $\underline{\ell}(\Omega; \varepsilon)$, all from a period-0 perspective. Now let us look at the welfare in an economy subject to such a rule relative to that in an economy without rules. By Assumption 3, $\ell^n(\Omega, \theta)$ is weakly decreasing in θ, so the difference in social welfare conditional on $\theta < \overline{\theta} - \varepsilon$ is zero since the policy under no commitment is unchanged. The difference in social welfare from realizations $\theta \in \left[\overline{\theta} - \varepsilon, \overline{\theta}\right]$ is nonzero and equals

$$\int_{\theta=\overline{\theta}-\varepsilon}^{\overline{\theta}} \left\{ \begin{array}{l} \left[u\left(c^*\left(x^r, \ell^r(\Omega, \theta; \varepsilon), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^r(\Omega, \theta; \varepsilon), \Omega, \theta\right), \theta'\right)]\right] \\ -\left[u\left(c^*\left(x^n, \ell^n(\Omega, \theta), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^n(\Omega, \theta), \Omega, \theta\right), \theta'\right)]\right] \end{array} \right\} g(\theta) d\theta, \quad (18)$$

where $\mathbb{E}_{\theta'}[\cdot]$ denotes the current period's expectation over next period's realization of θ'. We first establish that (18) is bounded from below by

$$\int_{\theta=\overline{\theta}-\varepsilon}^{\overline{\theta}} \left\{ \begin{array}{c} \left[u\left(c^*\left(x^n, \ell^n(\Omega, \overline{\theta}-\varepsilon), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^n(\Omega, \overline{\theta}-\varepsilon), \Omega, \theta\right), \theta'\right)]\right] \\ -\left[u\left(c^*\left(x^n, \ell^n(\Omega, \theta), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^n(\Omega, \theta), \Omega, \theta\right), \theta'\right)]\right] \end{array} \right\} g(\theta) d\theta, \quad (19)$$

where we replaced the θ-dependent terms $\ell^r(\Omega, \theta; \varepsilon)$ in the first line of (18) with $\ell^n(\Omega, \overline{\theta} - \varepsilon)$ for all θ in (19). Take an arbitrary $\theta \in \left[\overline{\theta} - \varepsilon, \overline{\theta}\right]$. Note that $\ell^r(\Omega, \theta; \varepsilon) \geq \ell^n(\Omega, \overline{\theta} - \varepsilon)$ by design of the rule. Then there are two cases to consider.

Case 1: If $\ell^r(\Omega, \theta; \varepsilon) = \ell^n(\Omega, \overline{\theta} - \varepsilon)$, then the pointwise variant of the lower bound in (19) is trivially satisfied with equality at at any point that falls under Case 1.

Case 2: If $\ell^r(\Omega, \theta; \varepsilon) > \ell^n\left(\Omega, \overline{\theta} - \varepsilon\right)$, then for this to be an MPE, the government without commitment must weakly prefer choosing $\ell^r(\Omega, \theta; \varepsilon)$ over $\ell^n\left(\Omega, \overline{\theta} - \varepsilon\right) < \ell^r(\Omega, \theta; \varepsilon)$:

$$\begin{aligned} & u\left(c^*\left(x^r, \ell^r(\Omega, \theta; \varepsilon), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^r(\Omega, \theta; \varepsilon), \Omega, \theta\right), \theta'\right)] \\ \geq & u\left(c^*\left(x^r, \ell^n(\Omega, \overline{\theta} - \varepsilon), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^n(\Omega, \overline{\theta} - \varepsilon), \Omega, \theta\right), \theta'\right)] \end{aligned} \tag{20}$$

Furthermore, since x and ℓ are q-complements in production by Assumption 1, $\ell^n\left(\Omega, \overline{\theta} - \varepsilon\right) < \ell^r(\Omega, \theta; \varepsilon)$ implies that $x^n < x^r$ and thus

$$\begin{aligned} & u\left(c^*\left(x^r, \ell^n\left(\Omega, \overline{\theta} - \varepsilon\right), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^n\left(\Omega, \overline{\theta} - \varepsilon\right), \Omega, \theta\right), \theta'\right)] \\ > & u\left(c^*\left(x^n, \ell^n\left(\Omega, \overline{\theta} - \varepsilon\right), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^n\left(\Omega, \overline{\theta} - \varepsilon\right), \Omega, \theta\right), \theta'\right)]. \end{aligned} \tag{21}$$

Combining equations (20) and (21), we see that

$$\begin{aligned} & u\left(c^*\left(x^r, \ell^r(\Omega, \theta; \varepsilon), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^r(\Omega, \theta; \varepsilon), \Omega, \theta\right), \theta'\right)] \\ > & u\left(c^*\left(x^n, \ell^n\left(\Omega, \overline{\theta} - \varepsilon\right), \Omega, \theta\right), \Omega, \theta\right) + \beta \mathbb{E}_{\theta'}[V^n\left(\Gamma\left(\ell^n\left(\Omega, \overline{\theta} - \varepsilon\right), \Omega, \theta\right), \theta'\right)]. \end{aligned} \tag{22}$$

From the inequality in (22) it follows that the pointwise variant of the lower bound in (19) is satisfied with strict inequality at any point that falls under Case 2.

Combining Cases 1 and 2, we conclude that (19) indeed represents a lower bound on (18). All that remains to be shown is that the value of (19) is strictly positive for small enough $\varepsilon > 0$. To see that this is the case under the stated assumption of interior lockdown $\ell^n(\Omega, \theta) < 1$, recall that the optimal lockdown is strictly more severe under lack of commitment than under commitment for interior levels of lockdown. This implies that, for small enough $\varepsilon > 0$, for all $\theta \in \left[\overline{\theta} - \varepsilon, \overline{\theta}\right]$ we have that welfare strictly increases when we replace $\ell^n(\Omega, \theta)$ by $\ell^n(\Omega, \overline{\theta} - \varepsilon) > \ell^n(\Omega, \theta)$, where the strict inequality follows from Assumption 3, which states that $\ell^n(\cdot)$ is strictly decreasing. Since the density $g(\cdot)$ is strictly positive and continuous in a neighborhood below $\overline{\theta}$ by Assumption 3, the interval $\left[\overline{\theta} - \varepsilon, \overline{\theta}\right]$ defines a strictly positive probability mass. Combining the last two insights, the expression in (19) is strictly positive for small enough $\varepsilon > 0$.

This concludes the proof that the imposition of such a rule strictly increases welfare. $\square$