Multi-Period Contracting and Salesperson Effort Profiles: The Optimality of “Hockey Stick,” “Giving Up” and “Resting on Laurels”

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ABSTRACT

We study multi-period salesforce incentive contracting where salespeople can engage in effort gaming, a phenomenon that has extensive empirical support. Focusing on a repeated moral hazard scenario with two independent periods and a risk-neutral agent with limited liability, we conduct a theoretical investigation to understand which effort profiles the firm can expect under the optimal contract. We show that various effort profiles that may give the appearance of being sub-optimal, such as postponing effort exertion (“hockey stick”) and not exerting effort after a bad or a good initial demand outcome (“giving up” and “resting on laurels,” respectively) may indeed be induced optimally by the firm. The reason is that, under certain conditions that depend on how severe the contracting frictions are and how effective effort exertion is in increasing demand, the firm wants to concentrate rewards on extreme demand outcomes; this induces gaming and reduces expected demand, but also makes motivating effort cheaper thus saving on incentive payments. On introducing dependence between time periods, such as when the agent can transfer demands between periods, this insight continues to hold and, furthermore, “hockey stick,” “giving up” and “resting on laurels” can be optimal for the firm even under repeated short time horizon contracting. Our results imply that one must carefully consider the setting and environmental factors when making inferences about contract effectiveness from dynamic effort profiles of agents.

Keywords: Salesforce compensation; dynamic incentives; effort gaming; “hockey stick;” sales push out and pull in.

INTRODUCTION

Salesforce expenditures account for 10%–40% of the revenues of US firms (Albers and Mantrala 2008), which is of the order of hundreds of billions of dollars annually (Zoltners et al. 2008). Compensation contracts used by firms to reward salespeople are usually comprised of a fixed part (e.g., base salary) and a variable part (e.g., commissions on sales or discrete bonuses awarded on achieving a quota of sales in a specified time period). According to Joseph and Kalwani (1998), who conducted a survey of Fortune 500 firms, over 90% use quota-based rewards in their compensation plans. The advantage of quota-based reward plans is that they provide stronger incentives to salespeople to reach a high level of sales (Hedges 2015), as variable compensation is rewarded only in that case.

Firms employ salespeople for extended periods of time, and when quota-based incentives are used in such a multi-period, long time horizon setting, the issue of dynamic gaming of effort arises. This is
because in a multi-period scenario the agents may strategically adjust their effort exertion over time, based on how uncertain outcomes are realized and how the contract will determine reward in current and future periods. One might intuit that in a multi-period scenario the principal would always want the agent to consistently exert high effort (the leftmost plot in Figure 1). However, empirical research has carefully documented the effort exertion profiles of agents induced by different types of contracts in multi-period scenarios, and consistent effort exertion is often not the case.

As a canonical example, consider a scenario in which outcomes are measured every quarter and the salesperson is paid a bonus if a particular sales quota is reached in six months (i.e., two quarters). To reach his six-month quota with minimum effort, the agent may strategically shirk work in the first quarter hoping for a high demand outcome without much effort, and exert greater effort only in the second quarter based on the outcome of the first quarter. Such effort postponement is often observed in reality and is sometimes known as the “hockey stick” pattern because effort exertion is flat in early periods and increases sharply in later periods, thus taking the shape of a hockey stick, as the middle plot in Figure 1 illustrates (Chen 2000). Oyer (1998) analyzes aggregate data (from the Survey of Income and Program Participation (SIPP)) spanning multiple industries where quota-based plans are used and detects that firms’ sales increase at the end of a fiscal year, suggesting that salespeople postpone effort exertion until the end of a compensation window to meet their quotas and get bonuses. Steenburgh (2008) analyzes individual salesperson-level data from a Fortune 500 company that sells durable office products and uses quota-based plans, and finds similar patterns to those reported in Oyer (1998). Misra and Nair (2011) analyze data from a Fortune 500 contact lens manufacturer and find evidence of shirking of effort by agents in the early part of the compensation cycle. As in Oyer (1998) and Steenburgh (2008), they find higher sales at the ends of quarters compared to early in the quarters, which again suggests that agents tend to increase effort as they reach closer to the end of a compensation window. Chung and Narayandas (2017) conduct a field experiment and also report delaying effort as an issue of concern in long time horizon contracts.

For similar strategic reasons related to effort gaming, we can observe a “giving up” effort profile under a quota-based contract (Steenburgh 2008, Jain 2012, Chung et al. 2014, Chung and Narayandas 2017). Suppose that in the first quarter the salesperson exerts effort but demand realization is low; then he is far away from achieving the six-month quota so does not exert effort in the second quarter, i.e., the salesperson “gives up.” Chung et al. (2014) analyze data from a Fortune 500 office durable goods
manufacturer and find that weak performers may give up if they realize that sales quotas under the long time horizon contract become unachievable. Chung and Narayandas (2017) also find empirical evidence that under a monthly quota plan, salespeople who had a series of bad draws early in the month may decide to give up late in the month because there is no chance that they can meet or exceed the quota set by the firm.

Alternatively, sales agents who have already achieved sales close to the quota in early sales cycles may not have the incentive to put in much effort later and “rest on laurels” instead (Chung et al. 2014, Misra and Nair 2011, Chung and Narayandas 2017). For example, suppose that in the first quarter the salesperson exerts effort and demand turns out to be high, bringing the salesperson reasonably close to the six-month quota. The salesperson then does not exert much effort in the second quarter, being almost assured of the bonus at the end of the two quarters. Chung et al. (2014) present evidence that the best performers will reduce productivity after getting close to or attaining quotas. On a similar note, Misra and Nair (2011) show that agents may shirk after they bring in enough sales to bring them close to their quotas. In both the “giving up” and the “resting on laurels” scenarios, an agent’s effort level is expected to decline over time, as the rightmost plot in Figure 1 illustrates.

An agent’s effort gaming — postponement of effort exertion, and shirking after either bad or good early outcomes — is usually considered an undesirable outcome from the firm’s point of view because it involves not exerting effort before the sales quota is reached. In this paper, we conduct a theoretical investigation on whether these effort profiles are necessarily suboptimal for the firm. A different way of asking this question is: Is it always optimal for the firm to consistently induce high effort from the agent? If not, what effort profile(s) will be induced under the optimal incentive contract? An associated question here is: What is the optimal contract structure? Specifically, should the firm reward only extreme sales outcomes to motivate effort or also reward intermediate sales outcomes? If the firm only rewards extreme sales outcomes, it would be more exposed to gaming, but would it be too costly to prevent gaming by rewarding intermediate sales outcomes as well?

To answer these questions, we build a stylized principal-agent model with moral hazard in which a firm interacts with a salesperson for two time periods. The firm uses a contract that is determined at the start of the first period and pays once at the end of the second period based on the outcomes of the two periods. We assume the demand outcome in each period to be stochastically dependent on the effort exerted in that period, and assume the demand outcomes in the two periods to be independent of each
other. The reward to the agent is based on the history of demand outcomes in the two periods. Under the two-period contract, the agent can dynamically adjust his effort level in the later period based on the early period’s demand outcome which, in turn, also influences his first-period effort exertion decision. We assume that the firm and the salesperson are risk neutral, and that the agent has limited liability. Limited liability can be thought of as protection from downside risk for the salesperson, i.e., he will be guaranteed a minimum payment even in the case of an unfavorable market outcome (which is a robust feature of real-world compensation plans). This assumption aligns well with industry practice, yet limited liability introduces contracting frictions.

We find that, under different conditions, the optimal two-period contract can be a “gradual contract” that rewards the salesperson for all demand realizations (with higher reward for higher demand realizations), or an “extreme contract” that rewards the salesperson only for the maximum possible demand realization across the two periods. Interestingly, we find that different effort exertion profiles are possible under the different optimal contracts. When the optimal contract is a gradual contract, the agent exerts effort in both periods. However, when the optimal contract is an extreme contract, then we observe two types of effort patterns that, as discussed earlier, are characterized by effort gaming — no effort exertion in the first period followed by effort exertion in the second period only in case of high demand realization in the first period (the “hockey stick” effort profile), and effort exertion in the first period followed by effort exertion in the second period only in the case of high demand realization in the first period (the “giving up” effort profile). Note that both of these effort profiles are characterized by probabilistic effort exertion in the second period, based on the outcome of the first period. Our results show that observing less than high effort exertion either early or late may not necessarily imply that the contract is “not effective” in inducing demand — this may, in fact, be optimal for the firm in the sense that this is exactly the effort profile it expects when choosing the optimal contract. Our results imply that one must carefully understand and consider the setting and environmental factors when making inferences about contract effectiveness from dynamic effort profiles of agents. In particular, in the presence of contracting frictions, on observing less than high effort exertion by salespeople in some time periods, one should not rush to the conclusion that this must mean that the system is in a sub-optimal state and that the firm should want to change something to induce consistently high effort exertion.

Which effort profile we would expect depends on the optimal contract which, in turn, is based on the tension between two countervailing forces — the “demand effect” (which refers to how much demand is
expected from that contract; the firm wants this to be high) and the “incentive effect” (which refers to how much the firm will have to pay per unit of expected demand under that contract; the firm wants this to be low). Extreme contracts, that induce effort gaming, reduce the expected demand but also reduce the expected incentive cost per unit of expected demand. Intuitively speaking, this is because, under moral hazard, when lesser effort is expected to be exerted the detection of whether or not it has been exerted becomes easier and so the incentive that must be given to have the effort exerted is smaller. In other words, an extreme contract that rewards only for high demand outcomes achieves effort exertion more efficiently compared to a gradual contract that rewards for all demand outcomes. The insight that bonuses for high demand outcomes motivate more effort is recognized in the industry (Hedges 2015), and also aligns with the findings in Chung et al. (2014) that bonuses, especially end of year bonuses, perform well through higher effort motivation in spite of gaming effects being present.

The hockey stick pattern is especially interesting as it implies that under the optimal contract the firm induces delaying of effort. The hockey stick pattern is generated when the optimal contract is an extreme contract which makes implementing effort cheaper in the second period when the first period outcome is already revealed. Furthermore, inducing low effort in the first period also reduces the expected incentive payment because there is a lower probability of the extreme sales quota being met. While this effort postponement is typically interpreted negatively (Chen 2000), and as something to avoid, our analysis shows that it indeed can be generated under an optimal contract and even with independent periods, because even though the firm reduces expected demand generated, it also reduces the expected incentive payment to generate this demand.

A number of papers, including Oyer (1998), Steenburgh (2008), Misra and Nair (2011), Jain (2012), Chung et al. (2014) document another kind of gaming (in addition to effort gaming) in a dynamic incentives setting—they show that in a multi-period setting with non-linear contracts, sales agents transfer demand between periods. Specifically, they may pull in orders from future periods if they would otherwise fall short of a sales quota in one period, whereas they may push out orders to the future if quotas are either unattainable or have already been achieved. We extend our basic model to study strategic sales pull in and push out behavior which makes the two periods interdependent. We find that if sales pull in and push out is possible, the “hockey stick” and “giving up” effort profiles can be also generated under short term horizon contracting (in our model, when the firm uses a sequence of period-by-period contracts). In other words, the agent’s incentive to transfer sales under a sequence of
short-term contracts plays a similar role as the agent’s dynamic gaming under a non-linear two-period contract, in terms of being used by the principal to save on incentive cost.

Finally, we extend our basic model in another way to make the two time periods interdependent. Specifically, we introduce the idea of an exogenous and limited amount of product inventory that has to be sold across the two periods; this makes the contract design decisions for the principal in the two time periods dependent. We find that with limited inventory the principal’s incentive to induce effort in the first period is lesser, i.e., to save on incentive cost, the principal may optimally desire effort postponement by the agent hoping for a high demand outcome in first period, and if the first-period demand outcome is low then the agent can exert effort in the second period. Naturally, this again leads to a hockey stick effort profile.

Our research is related to the literature seeking to explain an agent’s dynamic effort gaming under a long time horizon contract. Consider the hockey stick pattern characterized by an increasing effort profile over time, which has attracted much attention. Existing explanations for this phenomenon are, broadly speaking, of three types: (i) those that invoke behavioral explanations or argue that this is due to suboptimal behavior from either the principal or the agent, (ii) those that take a behavioral goal attainment perspective, and (iii) those that rely on demand shifting across time periods to explain this effort profile. Chen (2000) shows that if quotas are not in line with an agent’s level of productivity, the salesperson may find it optimal to wait before exerting effort to resolve uncertainty over the realization of early demand shocks, i.e., the agent will postpone effort. Chung et al. (2014) focus on suboptimal gaming behaviors committed by the agent— they discover from a counterfactual analysis that effort concentration in later periods can arise from agents’ myopic behaviors, while a forward-looking agent would smooth out efforts over time to take into account the uncertainty in future demand shocks. Also taking a behavioral perspective, Jain (2012) studies a scenario where agents lack in self-control. Then, the firm can take advantage of an agent’s lack of self-control to maximize its profits by paying a single bonus at the end, which essentially encourages effort postponement. The goal attainment literature also adopts a behavioral perspective to explain an agent’s procrastination. Kivetz et al. (2006) show the goal-gradient phenomenon wherein an agent works harder towards a goal as he gets closer to achieving it. Heath et al. (1999) use goal-serving as a reference point to explain effort postponement in agents, the idea being that goals have diminishing returns, and thus combining multiple short-term goals into a long-term goal will result in less effort exertion earlier on. Distinct from these explanations, our
work provides the novel insight that agents’ gaming behaviors can be optimally induced by the firm to improve its profits. The hockey stick effort pattern can also be observed if salespeople pull in orders from future periods when they are short of sales to meet the current quota. This has been documented by a number of papers on salesforce compensation (Oyer 1998, Steenburgh 2008, Kishore et al. 2013), accounting (Healy 1985) and navy recruitment (Asch 1990). However, we show that demand borrowing is not necessarily needed to obtain the hocky stick profile (recall that our basic model assumes this away but still produces the hockey stick profile).

Our research is related to the body of work on dynamic incentives with repeated moral hazard. One stream of this work assumes the firm to be risk neutral but agents to be risk averse, which leads to contracting frictions. Under this paradigm, a seminal paper, Hölmstrom and Milgrom (1987) shows that a linear contract is optimal for the principal when a number of other assumptions hold. We note that the gradual two-period contract that we derive as optimal for the firm under certain conditions can be interpreted as a linear contract as well, but it is only optimal for a certain region of the parameter space (recall that we assume the agent to be risk neutral). A number of papers in the risk aversion paradigm revisit the assumptions of Hölmstrom and Milgrom (1987) and show the optimality of non-linear contracts (Rogerson 1985, Spear and Srivastava 1987, Schättler and Sung 1993, Sung 1995, Hellwig and Schmidt 2002, Sannikov 2008, Rubel and Prasad 2015).

A second stream of the work on dynamic incentives assumes agents to be risk neutral with limited liability, which is a different source of contracting friction (our paper falls under this paradigm). Bierbaum (2002) studies how to induce high effort from the agent in each of two periods (which may not be profit maximizing for the principal), while we allow different effort profiles to be induced by the optimal contracts under different conditions. Kräkel and Schöttner (2016) study the firm’s choice between commissions and bonuses and determines conditions under which one or the other (or a combination) is optimal in a dynamic setting, in a special case when an agent’s outside option is equal to his limited liability. Schöttner (2016) studies optimal contracting when the agent’s effort costs change over time. These papers, however, assume equal values of the reservation utility and the limited liability, which is not without loss of generality. In fact, our most interesting results on effort patterns are obtained for the case when the value of the outside option is greater than the value of the limited liability, which is a reasonable assumption. We also note that Carroll (2015) assumes limited liability with risk neutrality

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1Ridlon and Shin (2013) show that an increasing effort profile over time is obtained in a multi-person, multi-period sales contest with handicapping.
and shows that a linear contract is optimal, but that this result is driven by the assumption that the objective of the principal is to write a robust contract to optimize the worst-case outcome than to optimize the expected profit (which we assume and is the more standard assumption).

More broadly, our research adds to the extensive literature on salesforce incentives in marketing which, in addition to the papers already cited, includes Raju and Srinivasan (1996), Godes (2004), Simester and Zhang (2010) and Zhang (2016), among many others. Our extension with limited inventory is related to the work on salesforce compensation when operational considerations are important (Chen 2000, Plambeck and Zenios 2003, Dai and Jerath 2013, Saghaian and Chao 2014, Dai and Jerath 2016, Dai and Jerath 2019).

The rest of the paper is organized as follows. In the following section, we describe the model. Next, we conduct the analysis and obtain our key insights regarding the different forces at play, and show under what conditions an agent’s dynamic gaming can be optimally induced by the principal. Following this, we allow the agent to push out and pull in sales between periods, and we allow for periods to be dependent by assuming that the principal has limited inventory to be sold in the two periods. In the final section, we conclude with a discussion. The proofs are provided in an Appendix and an Online Appendix (indicated appropriately).

**MODEL**

We develop a stylized agency theory model in which a firm (the principal) hires a salesperson (the agent) to exert demand-enhancing effort. There are two time periods denoted by \( t \in \{1, 2\} \). Demand in both periods is uncertain and the outcomes in each period are assumed to be independent of the other period, conditional on the effort exerted in that period. Let \( D_t \) be the demand realization in period \( t \), which can be either \( H \) or \( L \) with \( H > L > 0 \). The agent’s effort increases the probability of realizing high demand levels. The effort level in period \( t \), denoted by \( e_t \), can be either 1 or 0, i.e., the agent either “works” or “shirks” in each period; however, the principal does not observe the effort level. For instance, we can think of effort level 0 as a salesperson making a client visit (which is observable and verifiable) and effort level 1 as the salesperson’s additional effort spent in talking to and convincing the client to make the purchase (which the firm cannot observe or verify). Without effort exertion \( (e_t = 0) \) demand is realized as \( H \) with a probability of \( q \), and with effort exertion \( (e_t = 1) \) this probability increases to

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2Our insights hold qualitatively with three periods. A generalization to \( n \) periods is not analytically tractable, but we expect our results to hold qualitatively in that scenario as well.
\( p \) \((0 < q < p < 1)\). A larger \( p \) implies greater effectiveness of the salesperson’s effort, while \( q \) can be interpreted as the natural market outcome. We assume that all the demand created can be met and each unit sold gives a revenue of 1 and has a marginal cost of zero. The cost of effort is given by \( \phi > 0 \) for \( e_t = 1 \) and is normalized to zero for \( e_t = 0 \).

We assume that both the firm and the salesperson are risk neutral. Unlike the firm, however, the salesperson has limited liability, implying that he must be protected from downside risk. Specifically, we assume that the salesperson has a limited liability of \( K \) in each period, i.e., to employ the agent for one period, the principal must guarantee a compensation of at least \( K \) under any demand outcome. Limited liability is a widely observed feature of salesforce contracts in the industry, and this assumption is a standard one in the literature to prevent the firm from being sold to a risk-neutral agent (Sappington 1983, Park 1995, Kim 1997, Oyer 2000, Laffont and Martimort 2009, Simester and Zhang 2010, Dai and Jerath 2013, Carroll 2015). The limited liability assumption also implies the existence of a wage floor for the salesperson, which is aligned with industry practice. We assume that the salesperson’s reservation utility is \( U \) for each period, which can be thought of as coming from the agent’s outside option. The agent’s limited liability can be different from his reservation utility (Kim 1997, Oyer 2000). For instance, if the salesperson’s alternative employment opportunities are attractive, then the agent’s reservation utility will be higher than his limited liability. In the analysis, we assume this to be the case and, in accordance with this, we assume \( U \geq K \).

The agent is reimbursed for effort using an incentive contract. Effort is unobservable to the firm and demand is random but can be influenced by effort, so the firm and the agent sign an outcome-based contract. The firm can propose a long time horizon contract, which in this case is a two-period contract that is determined at the beginning of the first period and pays once at the end of the second period based on the outcomes of the two periods.

The timeline of the game is as follows. At the beginning of period 1, i.e., \( T = 1 \), the principal proposes the contract and the agent decides whether or not to accept the offer. If accepted, the agent

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3If the salesperson’s skills are most valuable in a sales context and he cannot expect comparable compensation in other professions, then the reservation utility, which can be thought of as coming from the outside option, may be lower than the limited liability, i.e., \( U < K \). We note that the results for \( U < K \) are qualitatively the same as for \( U = K \).

4The discrete demand distribution that we have assumed ensures that effort will not change the support of the demand distribution; otherwise, the principal may be able to infer the agent’s effort from the demand outcome and would induce the agent to work by imposing a large penalty for demand outcomes that cannot be obtained under equilibrium effort but can be obtained under off-equilibrium effort, as argued in Mirrlees (1976).

5Note that we do not allow renegotiation under long-term contracting, and this is critical to our results. As Fudenberg et al. (1990) argue, long-term contracting is valuable only if optimal contracting requires commitment to a plan today that would not otherwise be adopted tomorrow.
then decides on his effort in the first period, \( e_1 \). At the end of \( T = 1 \), the agent and the principal observe the demand outcome for the first period, \( D_1 \). The agent then chooses his second period effort \( e_2 \). At the end of \( T = 2 \), the agent and the principal observe the second period demand outcome \( D_2 \). The firm realizes the profit and the agent gets paid according to the contract. We make the assumption that when the agent is indifferent between exerting effort or not, he will choose to exert effort. We now proceed to the analysis.

**ANALYSIS**

**Benchmark 1: First-Best Scenario**

We start by presenting the first-best solution (for instance, if the agent’s effort is observable). In this case, the two periods are independent and equivalent and it is sufficient to study just one period. We obtain the following first-best solution (details of the formulation and the analysis are provided in Section A1 in the Appendix).

**Result 1** *(Optimal First-Best Solution)* The first-best contract instructs the agent to exert effort \( e^*_{FB} = 1 \) and pays a fixed salary equal to the agent’s cost of effort \( s^*_{FB} = \phi \), if and only if \( H - L \geq \frac{\phi}{p-q} \). Otherwise, the principal chooses not to direct effort exertion, i.e., \( e^*_{FB} = 0 \), and \( s^*_{FB} = 0 \).

From Result 1, we can infer that the principal would like the agent to exert effort when the upside market potential is large, or when the effectiveness of the agent’s effort is high. To rule out the trivial case in which the firm is not interested in motivating effort even in the first-best scenario, we only consider the parameter space with \( H - L \geq \frac{\phi}{p-q} \) in the rest of the paper.

**Benchmark 2: One-Period Scenario**

Suppose that the firm could only contract with the agent one period at a time; however, there is moral hazard and limited liability. This is a standard textbook problem, and we obtain the following result (details of the formulation and the analysis are provided in Section A2 in the Appendix).

**Result 2** *(Optimal One-Period Solution)* The optimal one-period contract gives the salesperson a fixed salary of\( \max\{K, U - \frac{q}{p-q} \phi\} \), no bonus if the realized demand is \( L \), and a bonus of \( \frac{\phi}{p-q} \) if the realized demand is \( H \). Under this contract, the salesperson exerts effort.

\(^6\)The firm’s profit can be ensured to be non-negative by assuming a large-enough value of the low demand level, \( L \). This would not affect the equilibrium optimal effort, which depends on the difference between the high demand level and the low demand level, \( H - L \).
In this case, the firm gives the salesperson a fixed salary to cover his limited liability while ensuring his participation. The notable feature of this contract is that a bonus, equal to $\frac{\alpha}{p-q}$, is given only if the realized demand is $H$; this bonus decreases as the effectiveness of the agent’s effort, $p$, increases because a larger $p$ implies that effort exertion is easier to detect.

**Two-Period (Long Time Horizon) Contract**

In this scenario, the firm proposes a long time horizon two-period contract at the beginning of the first period and pays once at the end of the second period based on the outcomes of the two periods. A key feature of this scenario introduced due to unobservability of effort and the contract paying at the end of two periods is that the agent can “game” the system—the agent can choose effort in period 2 based on the outcome of period 1 (and, realizing this, can also choose the effort in period 1 strategically). We denote the two-period effort profile by $\langle e_1, e_H^2, e_L^2 \rangle$, where the second period’s effort $e_D^2$ is contingent on the first period’s demand realization, $D_1 \in \{H, L\}$.

In full generality, this contract involves a guaranteed salary for employing the agent for two periods, plus a bonus issued at the end of the two periods that is contingent on the whole history of outputs. We denote the fixed salary by $S$, and denote the bonus paid at the end of $T = 2$ by $b_2(D_1, D_2)$. Such a contract thus stipulates four possible bonuses, $b_2(L, L), b_2(L, H), b_2(H, L)$ and $b_2(H, H)$.\(^\text{7}\) To prevent the agent from restricting sales to $L$ when demand is $H$, we impose a constraint on the two-period contract given by $b(H, H) \geq \max\{b(H, L), b(L, H)\}$, i.e., the bonus paid when demand in both periods is realized as $H$ should be no lower than the bonus paid when demand in only one of the periods is realized as $H$. (Note that our key insights hold even without this assumption as shown in Section A3.4 in the Appendix, but this is a natural assumption and simplifies the analysis.) Under this constraint, we obtain the following lemma (the detailed proof is in Section A3.2 in the Appendix).

**Lemma 1** In the weakly dominant two-period contract, $b_2(H, L) = b_2(L, H)$.

Lemma 1 directly implies the following lemma.

**Lemma 2** It is optimal for the principal to pay the agent at the end of two periods a bonus according to cumulative sales (which can be $2L, H + L$ or $2H$).

\(^\text{7}\)Due to the discrete nature of our demand model, it suffices to focus on a fixed salary with lump sum bonus, as any combination of commissions and bonuses can be rewritten as a fixed salary and a bonus payment corresponding to each possible sales history.
Lemma 2 significantly simplifies the analysis. We denote the fixed salary by $S$, normalize the bonus payment when the total sales across two periods are $2L$ as 0, denote the bonus payments when the total sales are $H+L$ and $2H$ by $B_1$ and $B_2$, respectively, and define $D = D_1 + D_2$. We formulate the principal’s problem as follows.

$$\max_{B_1, B_2} E[D|e_1, e_2^H, e_2^L] - E[S + B_1 + B_2|e_1, e_2^H, e_2^L]$$

s.t. 

1. $U_A(e_2^H) > U_A(\tilde{e}_2^H)$  \hspace{1cm} (IC$^H_2$)
2. $U_A(\tilde{e}_2^L) > U_A(e_2^L)$  \hspace{1cm} (IC$L_2$)
3. $U_A(e_1|e_2^H, e_2^L) > U_A(\tilde{e}_1|e_2^H, e_2^L)$  \hspace{1cm} (IC$^H_1$)
4. $U_A(e_1, e_2^H, e_2^L) \geq 2U$  \hspace{1cm} (PC)
5. $S, S + B_1, S + B_2 \geq 2K$  \hspace{1cm} (LL)

In the above, $U_A(\cdot)$ denotes the agent’s expected net utility. (IC$^H_2$) stands for the agent’s incentive compatibility constraint in the second period following $D_1 = H$, where $U_A(e_2^H)$ represents the agent’s net payoff in Period 2 upon exerting effort $e_2^H$. (Then, if the agent exerts effort, he will get net utility $S + B_2 - \phi$ with probability $p$ and $S + B_1 - \phi$ otherwise; without exerting effort, he will get net utility $S + B_2$ with probability $q$ and $S + B_1$ otherwise.) To induce $e_2^H$, the principal needs to ensure that the agent gets a higher payoff upon exerting effort $e_2^H$, compared with a different effort level $\tilde{e}_2^H$. Similarly, (IC$L_2$) stands for the incentive compatibility constraint for inducing effort level $e_2^L$ in the second period following $D_1 = L$. (IC$^H_1$) represents the incentive compatibility constraint in the first period. $U_A(e_1|e_2^H, e_2^L)$ denotes the agent’s expected net utility across two periods upon exerting $e_1$ in the first period, given that the agent is induced to exert effort $e_2^H$ or $e_2^L$ in the second period based on the outcome of the first period. If $e_1 = 1$, his total net utility will be $U_A(e_2^H) - \phi$ with probability $p$ and $U_A(e_2^L) - \phi$ with probability $1 - p$; if $e_1 = 0$, his expected net utility will be $U_A(e_2^H)$ with probability $q$ and $U_A(e_2^L)$ with probability $1 - q$.

To induce $e_1$, the principal needs to ensure that the agent gets a higher total net utility on exerting $e_1$, compared with a different effort level $\tilde{e}_1$. (PC) is the agent’s participation constraint, which ensures that the agent’s expected net utility across the two periods is no less than twice his per-period reservation utility, $2U$. (LL) is the agent’s limited liability constraint, which ensures that the agent’s two-period

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8Lemma 1 holds without discounting and with risk-neutral agents. As shown by Spear and Srivastava (1987) and Sannikov (2008), if agents discount their future utility, or if the agent is risk averse, a path-dependent contract can be optimal.
compensation is no less than his limited liability over two periods, i.e., $2K$.

To arrive at an optimal contract for the principal, it is crucial to understand how the agent’s effort profile in the two periods changes with the bonuses $B_1$ (which is provided if demand is $H + L$) and $B_2$ (which is provided if demand is $2H$) in the two-period contract. The following proposition describes the expected effort profile $(e_1, E[e_2])$, where $e_1$ is an agent’s effort level in the first period, and $E[e_2]$ is an agent’s expected effort level in the second period (since $e_2$ depends on $D_1$, which is random). For instance, if the agent exerts effort in period 1 and will exert effort in period 2 only if the outcome in period 1 is $H$, then $e_2 = 1$ with probability $p$ and $e_2 = 0$ with probability $1 - p$, so we write this expected effort profile as $(1, p)$.

**Proposition 1 (Agent’s Response to Two-period Contract)** Given $B_1$ and $B_2$, the agent’s expected effort profile $(e_1, E[e_2])$ is as follows (the regions referred to are as per Figure 2 and they are analytically defined in Table A2 in Section A3.1 in the Appendix):

- A constant work expected effort profile, $e = (1, 1)$, is induced when both $B_1$ and $B_2 - B_1$ are large (Region IV), a constant shirk expected effort profile, $e = (0, 0)$, is induced when both $B_1$ and $B_2 - B_1$ are small (Region I).

- A “hockey stick” expected effort profile, $e = (0, q)$ or $e = (0, 1 - q)$, where the agent exerts more effort in expectation in the second period than in the first period is induced when $B_1$ is small and $B_2 - B_1$ is moderate (Region II) or when $B_1$ is moderate and $B_2 - B_1$ is small (Region VI), respectively.

- A “giving up” expected effort profile, $e = (1, p)$, where the agent exerts effort in the first period and in the second period upon a high first-period demand outcome, is induced when $B_1$ is small and $B_2 - B_1$ is large (Region III).

- A “resting on laurels” expected effort profile, $e = (1, 1 - p)$, where the agent exerts effort in the first period and in the second period upon a low first-period demand outcome, is induced when $B_1$ is large and $B_2 - B_1$ is small (Region V).

Figure 2 illustrates Proposition 1 graphically. The $x$-axis, $B_1$, is the incremental reward when total sales increase from $2L$ to $H + L$; the $y$-axis, $B_2 - B_1$, is the incremental reward when total sales increase.

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The convention we follow is that when we refer to the 3-tuple $\langle e_1, e^H_2, e^L_2 \rangle$, we use angular brackets, and when we refer to the 2-tuple $(e_1, E[e_2])$ we use parentheses.
from $H + L$ to $2H$. Based on Proposition 1, if both rewards are small, there is no effort exertion in either period, denoted by $e = (0, 0)$, which is Region I. If both rewards are large, the agent will put in effort in both periods, i.e., $e = (1, 1)$, which is Region IV. For other regions, the effort exertion decisions are more nuanced. If the demand outcome in period 1 is $L$ and the agent does not secure the bonus $B_1$, he will not exert additional effort in period 2 if $B_1 \leq \frac{\phi}{p-q}$. If the demand outcome in period 1 is $H$ and the agent secures the bonus $B_1$, he will not exert additional effort in period 2 if $B_2 - B_1 \leq \frac{\phi}{p-q}$. In other words, $B_1$ and $B_2 - B_1$ motivate the agent to exert effort in the second period if demand in the first period turns out to be $L$ and $H$, respectively.

Furthermore, the agent’s effort exertion at $T = 1$ depends on the values of both $B_1$ and $B_2 - B_1$. In Regions II and VI, the agent does not work in period 1 and chooses to “ride his luck” in period 1 — this results in a “hockey stick” effort profile where the expected effort level is higher in the second period than in the first period. The difference between Regions II and VI is that in Region II, the agent works in period 2 if the demand outcome in period 1 is favorable, i.e., $H$, and in Region VI, he works in period 2 if the demand outcome in period 1 is unfavorable, i.e., $L$. In Regions III and V, the agent works in period 1. However, in Region III, he works in period 2 only if the demand outcome in period 1 is favorable, i.e., $H$ — this results in a “giving up” effort profile where the agent does not exert effort in period 2 if the early demand outcome is unfavorable and the bonus becomes unattainable. In Region V, he works in period 2 if the demand outcome in period 1 is unfavorable, i.e., $L$ — this results in a “resting on laurels” effort profile where the agent does not exert effort in period 2 if the early demand outcome is favorable, i.e., $H$.

We now determine the optimal compensation plan for the salesperson which balances the expected revenue $E[D]$ and the expected compensation cost $E[S+B_1+B_2]$. The following proposition characterizes the optimal two-period contract for the principal.

**Proposition 2 (Optimal Two-period Contract)** The optimal two-period contract is as follows (the regions referred to are as per Figure 3 and they are analytically defined in Table A4 in Section A3.3 in the Appendix):

- **In Region I**, the optimal contract is a “zero-bonus contract” which only offers a fixed salary $S^*_I = 2U$.

- **In Region II**, the optimal contract is an “extreme, low-powered contract” which rewards bonuses
only at $2H$, with $B_1 = 0, B_2^* = \frac{\phi}{p-q}$ and a fixed salary of $S_{II}^* = \max\{2U - \frac{q}{p-q}\phi, 2K\}$. 

- In Region III, the optimal contract is an “extreme, high-powered contract” with $B_1^* = 0, B_2^* = \frac{1+p-q}{p(p-q)}\phi$ and a fixed salary of $S_{III}^* = \max\{2U - \frac{q}{p-q}\phi, 2K\}$. 

- In Region IV, the optimal contract is a “gradual contract” which rewards bonuses at both $H + L$ and $2H$, with $B_1^* = \frac{\phi}{p-q}, B_2^* = 2\frac{\phi}{p-q}$ and a fixed salary of $S_{IV}^* = \max\{2U - \frac{2q}{p-q}\phi, 2K\}$. 

The regions of the parameter space under which different contracts are optimal are illustrated in Figure 3, and the contract forms themselves and the outcomes are illustrated graphically in detail in Figure 4. Corollary 1 presents an interesting implication of Proposition 2. 

**Corollary 1** Under a two-period contract, both a “hockey stick” expected effort profile given by $(0,q)$, and a “giving up” expected effort profile given by $e = (1,p)$, may be optimally induced by the firm. 

Proposition 2 and Corollary 1 provide interesting and important implications for observed effort profiles of agents. First, under certain conditions, a pattern of delaying effort, i.e., the “hockey stick” effort profile, is optimally induced from the agent under the optimal contract chosen by the firm. Here, the effort profile is $\langle 0, 1, 0 \rangle$ and the expected effort profile is $(0,q)$, i.e., effort is not exerted in the first period but then exerted in the second period with probability greater than zero, specifically, only if the first period demand is realized as high. This happens when the firm finds it optimal to use an extreme yet low-powered contract (see the leftmost column of Figure 4). Specifically, this is a contract in which $B_1 = 0$ and $B_2 = \frac{\phi}{p-q}$, i.e., it gives a bonus only if realized demand is $2H$, but this bonus is small (note from the benchmark one-period analysis that $\frac{\phi}{p-q}$ is the bonus given under the optimal contract on obtaining demand $H$ in one period, while here this bonus is given on obtaining demand $2H$ in two periods).

Under certain conditions, a pattern of “giving up” in the second period is also optimally induced by the firm. Here, the effort profile is $\langle 1, 1, 0 \rangle$ and the expected effort profile is $(1,p)$, i.e., effort is exerted in the first period but then exerted in the second period with probability less than 1, specifically, only if the first period demand is realized as high. The optimal contract to induce such an effort profile is to use the extreme and high-powered contract (see the middle column of Figure 4) that provides a bonus $B_1 = 0$ and a large bonus $B_2 = \frac{1+p-q}{p(p-q)}\phi$ (which is larger than $\frac{\phi}{p-q}$).
Finally, in certain parameter spaces, the principal finds it optimal to motivate the effort profile \( \langle 1, 1, 1 \rangle \) and the expected effort profile \( (1, 1) \), i.e., effort is induced under all conditions in both periods. The contract adopted for this is a “gradual contract” (see the rightmost column of Figure 4) that rewards bonuses at both \( H + L \) and \( 2H \); specifically, \( B_1 = \frac{\phi - q}{p-q} \) and \( B_2 = 2 \frac{\phi - q}{p-q} \).

To understand the intuition behind these results, we discuss two effects that are operative, namely the “demand effect” and the “incentive effect.” Note that we can rewrite the principal’s profit, given by \( E[D] - E[S + B_1 + B_2] \), as a product of \( E[D] \), which is the expected demand generated, and \( 1 - \frac{E[S + B_1 + B_2]}{E[D]} \), which is the net expected profit for inducing each unit of expected demand. The former term is relevant to the demand effect and the latter term is relevant to the incentive effect.

First, we discuss the demand effect. This is straightforward— in terms of the expected demand generated under each contract, implementing the “hockey stick” expected effort profile, \( e = (0, q) \), generates the least expected demand \( E[D] \), followed by the “giving up” expected effort profile, \( e = (1, p) \), followed by constant expected effort profile, \( e = (1, 1) \). Next, we discuss the incentive effect. The term \( \frac{E[S + B_1 + B_2]}{E[D]} \) can be interpreted as the expected incentive payment per unit of expected demand, and the firm prefers this to be small. It turns out that inducing the “hockey stick” expected effort profile \( e = (0, q) \) has the smallest \( \frac{E[S + B_1 + B_2]}{E[D]} \), followed by inducing the “giving up” expected effort profile \( e = (1, p) \), followed by inducing the constant expected effort profile, \( e = (1, 1) \). To see this note that, essentially, the principal needs to implement three effort choices— \( e_1, e_H^2 \) and \( e_L^2 \). Among these, it is most efficient to implement \( e_H^2 = 1 \), followed by \( e_1 = 1 \), followed by \( e_L^2 = 1 \). Intuitively, it is “cheaper” to implement \( e_H^2 = 1 \) and \( e_1 = 1 \) (than \( e_L^2 = 1 \)), because in both cases the principal can concentrate reward at the highest sales level of \( 2H \) by adopting an extreme contract. Furthermore, comparing between \( e_H^2 = 1 \) and \( e_1 = 1 \), it is cheaper to implement \( e_H^2 = 1 \) because the first-period outcome is already revealed to be \( H \); for \( e_1 = 1 \) the first-period outcome will be \( H \) only with probability \( p \) if the salesperson works. It is “costliest” to implement \( e_L^2 = 1 \), because the first period outcome is already revealed to be \( L \) and the principal therefore must offer a bonus \( B_1 > 0 \) at the less extreme sales level \( H + L \); this gives the agent an incentive to ride his luck in the first period, hoping for a high demand outcome to secure bonus payments, and makes it harder for the principal to motivate effort. To obtain the expected effort profiles \( (0, q) \), \( (1, p) \) and \( (1, 1) \), the principal must implement \( \langle 0, 1, 0 \rangle \), \( \langle 1, 1, 0 \rangle \) and \( \langle 1, 1, 1 \rangle \), respectively. Given the above discussion, one can see that \( (0, q) \) ranks as most efficient based on the incentive effect, followed by \( (1, p) \), followed by \( (1, 1) \).
Therefore, the hockey stick, giving up and constant expected effort profiles (induced by the extreme low-powered, extreme high-powered and gradual contracts, respectively) are ordered from worst to best by the demand effect, and in exactly the opposite direction by the incentive effect;\(^{10}\) i.e., the demand effect and the incentive effect are countervailing forces acting in opposite directions, thus creating a tension in the model. Combining the demand effect and the incentive effect, we obtain the overall outcomes of which contract is optimal under different parametric conditions, as shown in Figures 3(a) and 3(b). Consider, for both figures, how the contracts and outcomes change as we vary \(U - K\) (following the horizontal dotted lines). When the limited liability, \(K\), is sufficiently high, i.e., we are in Region I where \(U - K\) is small, the friction from moral hazard is large enough such that the principal does not want to induce effort exertion. As limited liability decreases and we are in Region II, the friction due to moral hazard becomes smaller, and the principal starts to motivate effort using the extreme two-period contract, which provides effective incentives. However, since the limited liability is still relatively high in this scenario, the principal only induces \(e^H_2 = 1\) (which, as we discussed earlier, is the cheapest way to have effort exerted) through a small ultimate bonus and the expected effort profile is the hockey stick profile \(e = (0, q)\).

As the limited liability continues to decrease further and we are in Region III, the principal implements \(e_1 = 1\) through a high ultimate bonus to leverage the demand effect, and the expected effort profile is the giving up effort profile \(e = (1, p)\). When limited liability becomes small enough, i.e., we are in Region IV, the demand effect dominates the incentive effect. The principal implements constant expected effort profile \(e = (1, 1)\) using the gradual two-period contract.

Next, using Figure 3(a), we discuss the optimal contract against \(p - q\), which increases with the effectiveness of the agent’s effort. We vary \(p - q\) in Figure 3(a) along the vertical dotted line, keeping \(H - L\) and \(U - K\) fixed. Generally speaking, the firm needs to give a more effective agent a lower incentive to work because the outcome is a better signal of effort exerted and therefore more easily detectable; in other words, as \(p\) increases, the frictions due to moral hazard decrease. Consequently, on the lines of the intuition discussed earlier, the principal starts to implement \(e = (0, q)\), followed by \(e = (1, p)\), followed by \(e = (1, 1)\). However, if \(p\) becomes very large, the principal will induce \(e = (1, p)\) again. This is because the demand losses due to no effort exertion in the second period upon low demand realization are insignificant when \(p\) is very large: when \(p\) increases, the demand loss, given by \((p - q)(H - L)\) if

\(^{10}\)We note here that, for a fixed set of parameters, the fixed salaries weakly decrease as we go from the extreme low-powered contract, to the extreme high-powered contract, to the gradual contract.
it happens, gets larger, but the probability of effort not being exerted in the second period, given by \((1 - p)\), gets smaller.

Finally, using Figure 3(b), we discuss the optimal contract against \(H - L\), which is another metric of the impact of the agent’s effort. We vary \(H - L\) in Figure 3(b) along the vertical dotted line, keeping \(p - q\) and \(U - K\) fixed. As \(H - L\) increases, the demand effect becomes stronger; therefore, the principal starts to implement \(e = (0, q)\), followed by \(e = (1, p)\), followed by \(e = (1, 1)\).

To summarize our insights, different from the lay view that the “hockey stick” phenomenon is somehow suboptimal for the firm, our analysis suggests that it can be optimally induced by the firm even though it could choose a contract in which consistently high effort could be motivated. The intuition is that delaying effort can be optimal for agent when a low ultimate bonus payment is associated with a high quota level, and inducing delayed effort can simultaneously be optimal for the firm to save on incentive cost when the moral hazard friction is high (as in the case of high limited liability). On similar lines, we can also explain the optimality of the “giving up” effort profile. We highlight that we obtain these results under the assumption that periods have independent demand outcomes (conditional on effort).

**Discussion on Contract Structures**

It is worth noting that the gradual long time horizon contract is essentially a commission-based contract. This is because every additional sales outcome \(H\) brings in the same additional reward \(\phi_{p - q}\). With linear sales commissions there is a constant incentive to work and no incentive to delay effort or give up. However, if we assume ex ante that only a commission contract is allowed, then the optimal long-term contract will either be a commission-based contract (i.e., the gradual contract) that induces \(e = (1, 1)\), or a contract that pays only a base salary and induces \(e = (0, 0)\). Referring to Figure 3, in Regions I and IV the outcomes would be the same, while in Regions II and III the principal would make less profit with only a linear contract allowed than it does with non-linear contracts allowed, but there would be no effort gaming.

Furthermore, the gradual contract is a replicate of a sequence of optimal short time horizon contracts, i.e., a sequence of *period-by-period contracts* where the principal specifies a one-period contract at the beginning of the first period, and then specifies another one-period contract at the beginning of the second period (see Result 2). Therefore, whenever the principal prefers to induce constant effort exertion in the two-period contract, the principal is indifferent between a sequence of period-by-period contracts and a
two-period contract. Note that there is no effort gaming in this situation. We state this as a corollary.

**Corollary 2** With independent periods, whenever the principal finds it optimal to induce constant effort exertion in the two-period contract, the principal is indifferent between a sequence of period-by-period contracts and a two-period contract. In this situation, there is no effort gaming.

A point of discussion in the salesforce compensation literature is about the time horizon of contracts, i.e., whether a firm should use long term or short term contracts. In a recent review article, Coughlan and Joseph (2012) list this as a very important yet under-researched issue in salesforce management. While it is straightforward that a fully flexible long time horizon contract will be weakly dominant over a sequence of short time horizon contracts, our result above shows that under certain conditions long time horizon contracting will be strictly better than short time horizon contracting, while under other conditions both will achieve the same result.

**EXTENSIONS**

Until now, we have assumed that the two time periods are independent of each other. In this section, we study scenarios in which the two periods are interdependent. There may be many ways due to which the periods can be interdependent. We consider two such ways: (i) demand transfer in the presence of sales push out and pull in which introduces dependence due to the agent’s incentive to transfer sales, and (ii) a limited amount of product to sell across the two periods which introduces dependence due to an external constraint. We show that with inter-dependent periods, new forces emerge that make it optimal for the principal to choose a contract that induces a “hockey stick” effort profile under long time horizon contracting and even in short time horizon contracting.

**Sales Push Out and Pull In Between Periods**

Salespeople working under quota-based plans may resort to modifying demand in particular periods to meet quotas in those periods. Oyer (1998) empirically demonstrates the existence of demand pull in and push out between fiscal cycles when salespeople face non-linear contracts. In particular, Oyer (1998) reveals that sales agents will pull in orders from future periods if they would otherwise fall short of a sales quota in one cycle, whereas they push out orders to the future if quotas are either unattainable or have already been achieved. In previous sections, we assume away such sales push out and pull in phenomena.
by assuming that the agent cannot shift sales between two periods (yet still obtain the hockey stick and
giving up effort profiles).

In this section, we relax this assumption and allow the agent to push extra sales to, or borrow sales
from, the later period. With this ability of the agent, the two-period optimal contract (which pays at
the end of the two periods) is not affected, but the period-by-period contract (which pays in the interim)
has to be reanalyzed. We provide a sketch of the analysis below, with details provided in Section A4 in
the Appendix. A main takeaway from this analysis is that in the presence of sales push out and pull in
effects, we can observe effort dynamics, e.g., delayed effort, under optimal short time horizon contracting
as well.

We assume that in the period-by-period contract, at the end of the first period, the agent observes
the actual sales $D_1$ ahead of the principal. He can then strategically push out sales to, or pull in sales
from, the second period. The principal only observes the sales level after the agent’s manipulation, which
we denote by $D'_1$, and pays the agent according to $D'_1$. For instance, the principal will observe $D'_1 = H$
if $D_1 = H$, or if $D_1 = L$ and the agent pulls in $H - L$ from the second period.\footnote{We assume that the agent can pull in at most $L$ from the second period to the first, and we focus on the case when $H < 2L$. This ensures that even if $D_1 = L$, the agent can manage to report $D'_1 = H$ by pulling in $H - L < L$ from $T = 2$.} Likewise, observing $D'_1 = L$ may imply that $D_1 = L$, or that $D_1 = H$ and the agent pushed out the demand $H - L$ to the
second period.

To show the incentives and dynamics at play with sales push out and pull in, we provide the following
illustration. Consider first the principal’s problem at $T = 2$. At the beginning of the second period, a
new contract is initiated. To induce a specific effort level in the second period, the principal will set
the second period’s quota level and bonus value based on the observed earlier outcome $D'_1$, which we
denote by $\chi_2(D'_1)$ and $b_2(D'_1)$, respectively. While we relegate the details to Appendix A4, the result is
that to induce $\langle e^H_2, e^L_2 \rangle = (1, 0)$ the principal sets $\chi_2(D'_1) = 2H - D'_1$, and to induce $\langle e^H_2, e^L_2 \rangle = (0, 1)$,
the principal sets $\chi_2(D'_1) = H + L - D'_1$. In both cases, $b_2(D'_1) = \frac{\phi}{p - q}$. This implies that the principal
readjusts the quota level but not the bonus amount to achieve a desired effort profile in the second period.
Anticipating this, if the first period’s quota level is not high enough, the agent, rather than exerting
effort, prefers to pull in sales from the second period to achieve the first-period quota and obtain the
first-period bonus. To induce $e_1 = 1$, in turn, the principal must set $\chi_1$ high enough (for instance
$H + L$), such that it would become impossible for the agent to simply secure early bonus by pulling in
sales if $D_1 = L$, but it is still achievable in case $D_1 = H$. Once we recognize the above incentives, the
intuition for the rest of the analysis is quite similar to that highlighted in the main model. Table A5 in Section A4 in the Appendix presents the optimal short-term contract to induce different effort profiles. We summarize the results in the following proposition and illustrate them in Figure 5.

**Proposition 3** In the presence of sales push out and pull in, under short time horizon contracting, the “hockey stick” expected effort profile, $e = (0, q)$, the “giving up” expected effort profile, $e = (1, p)$, and the “resting on laurels” expected effort profile, $e = (1, 1 - p)$, are optimal for the principal to induce in the respective regions in Figure 5.

Proposition 3 suggests that with sales push out and pull in, agents’ dynamic gaming can be expected in certain parameter spaces, even if the principal designs her contract optimally by accounting for the gaming behaviors induced from agents. If we compare the effort outcomes in Figure 5 and Figure 3, we find that short-term contracting in the presence of sales push out and pull in achieves the same outcome as long-term contracting in the parameter space where the “hockey stick” expected effort profile, $e = (0, q)$, or the “giving up” expected effort profile, $e = (1, p)$, is optimally induced. This suggests that even if each contract signed only spans one period, the principle can take advantage of the agent’s push out and pull in behaviors to induce dependence between two periods and provide as efficient incentives as a two-period contract does. Essentially, the principal wants to make the agent’s later effort choice dependent on earlier outcomes to save on incentive cost. In the long time horizon contract with independent periods, it is achieved through an agent’s dynamic gaming in response to a non-linear two-period contract. In this section, it is achieved by an agent’s incentive to push out and pull in sales in response to a sequence of short time horizon contracts. Therefore, in this part of the parameter space, the ability of the agent to push out and pull in sales *benefits* the principal even when it can only sign period-by-period contracts by enabling more efficient effort exertion than would be possible if this ability were absent.

Furthermore, a “resting on laurels” expected effort profile $e = (1, 1 - p)$ is optimal for the principal under short time horizon contracting when agents can transfer sales, in the parameter space where, roughly speaking, constant effort exertion $e = (1, 1)$ is optimal under long time horizon contracting. To see this, note that in the presence of sales push out and pull in, the short time horizon contract fails to induce $e = (1, 1)$. This is because when the principal anticipates that agents may push out and pull in sales, and agents also realize that the principal will respond optimally to it, the principal is unable to induce consistently high efforts. It is also worth noting that the period-by-period contract in the
presence of sales push out and pull in still performs weakly worse for the principal than the two-period contract. Namely, it performs the same as the two-period contract for inducing $e = (1, p), e = (0, q)$ and $e = (0, 0)$, but it fails to induce $e = (1, 1)$. This is because, given any period-by-period contract, the agent can always rely on pushing out or pulling in sales (rather than exerting effort in both periods) to obtain the same expected bonus as he would get by exerting effort in both periods.

**Inter-dependent Periods with Limited Inventory**

In this section, we assume that the principal has a limited amount of product to sell across the two periods, such that the demand outcome in the first period can change incentive provision for inducing demand in the second period. We extend the model by assuming that the principal has limited inventory, denoted by $\Omega > 0$, to be sold across two periods. The inventory cannot be replenished before period 2 starts and any demand more than $\Omega$ is lost, i.e., actual sales $\bar{D} = \min\{D_1 + D_2, \Omega\}$. Therefore, the two periods become dependent through $\Omega$. We assume zero inventory costs for simplicity. We keep everything else in the model the same as before. To focus on the interesting cases, we only consider the case when $H - L \geq \frac{p}{(p-q)^2} \phi$ so that the market upside potential is large enough to justify effort induction in both periods given unlimited inventory. In this case, we solve the firm’s contracting problem by replacing the total demand $D$ by its truncated value $\bar{D} = \min\{D_1 + D_2, \Omega\}$. Recall that we assume that compensation cannot be decreasing in total sales. We obtain the following proposition.

**Proposition 4** With limited inventory, the “hockey stick” expected effort profile, $e = (0, 1 - q)$, and the “resting on laurels” expected effort profile, $e = (1, 1 - p)$, are optimally induced expected effort profiles by the firm under the long time horizon contract in the respective regions in Figure 6(a).

The proof of the above proposition is in Section OA2 in the Online Appendix. We find that under a two-period contract with limited inventory, both a “hockey stick” effort profile $e = (0, 1 - q)$ and the “resting on laurels” effort profile $e = (1, 1 - p)$ can be optimally induced expected effort profiles by the firm. The expected effort profile $e = (0, 1 - q)$ indicates that the agent does not exert effort in the first period and exerts effort in the second period only if the demand outcome of the first period is low (note the difference from the hockey stick expected effort profile $e = (0, q)$ obtained in the main analysis, where the agent does not exert effort in the first period and exerts effort in the second period only if the demand outcome of the first period is high). The expected effort profile $e = (1, 1 - p)$ indicates that the agent exerts effort in the first period and exerts effort in the second period only if the demand
outcome of the first period is low; this can be interpreted as “resting on laurels” in the second period as no effort is exerted if the first period is a success (note that this is also different from the “giving up” effort profile).

A key insight is that when the working environment is easy enough for the agent, i.e., the total amount of product to be sold, $\Omega$, is small, or $H$ or $p$ is large, the principal may have the incentive to induce agents to postpone effort, i.e., work hard only in the late period and this too only if the first-period demand outcome is unfavorable. Furthermore, as a result of limited inventory, the conclusion from the basic model that firms weakly prefer two-period contracting over period-by-period contracting does not always hold true. In the current scenario, the period-by-period contract outperforms the two-period contract when limited liability is very small or very large, since it gives the principal more flexibility in adjusting the contracts (note that we maintain the assumption that compensation is non-decreasing in sales). Specifically, the principal prefers the period-by-period contract to the two-period contract in Regions III and IV in Figure 6(b) (the detailed solution is in Section OA3 in the Online Appendix).

In Region III, where the agent’s limited liability is large, the period-by-period contract implements $e = (0, 1 - q)$ and performs the best for the principal. This is because, under the constraint that compensation cannot be decreasing in sales the two-period contract rewards the agent more than the period-by-period contract when demand outcomes at both periods are $H$. In Region IV, where the agent’s limited liability is small, the period-by-period contract implements $e = (1, 1 - p)$ and performs the best for the principal. The two-period contract cannot replicate the period-by-period contract for inducing $e = (1, 1 - p)$, because it suffers from the agent’s dynamic gaming. To ensure early effort exertion, the principal pays higher bonus under the two-period contract when demand outcomes in both periods are $H$, compared with under the period-by-period contract. In those scenarios, a period-by-period contract that gives the principal flexibility to adjust quota levels performs better than a two-period contract.

**CONCLUSIONS AND DISCUSSION**

Firms employ and reward salespeople over multiple time periods. We address a question that arises in this context: Is an agent’s dynamic gaming under a long time horizon contract, for example, “delaying effort”, “giving up” and “resting on laurels,” necessarily suboptimal for the firm? We employ a two-period repeated moral hazard framework with stochastic demand and unobservable effort, and assume the agent to be risk neutral with limited liability.
We show that the two-period expected effort profile under the optimal contract may not always be high effort exertion in both periods; under different conditions, the principal may optimally induce, in the expected sense, lower than the highest level of effort exertion in one of the periods. Figure 1 shows three canonical expected effort profiles that the firm may optimally induce — the leftmost plot indicates high effort in both periods, the middle plot indicates no effort in the first period and low expected effort exertion in the second period (the “hockey stick” profile), and the rightmost plot indicates high effort in the first period but low expected effort in the second period (the “giving up” or “resting on laurels” profile). The latter two expected effort profiles appear to be sub-optimal behaviors by the agent, even though the firm may want exactly these effort profiles under optimality. In essence, we show that a number of different effort profiles are possible under the optimal contract, and high effort exertion in every period is actually not always desired by the principal. Therefore, one has to be careful in making inferences about contract effectiveness from realized multi-period effort exertion profiles of agents.

Effort profiles characterized by gaming are induced when the firm uses a contract over a long time horizon that enables the firm to reward the salesperson only when a relatively high quota level is reached. However, the firm always has the option to implement a contract that prevents gaming by rewarding the agent for intermediate demand outcomes. The reason that the firm may choose an extreme contract and not prevent gaming is because the extreme contract enables the firm to efficiently implement effort (the “incentive effect”) even though it reduces the expected demand (the “demand effect”). We show that our results get strengthened if we make the different time periods interdependent by allowing the salesperson to shift demand between periods or by assuming that a specific amount of inventory must be sold across the two periods.

We conclude with a brief discussion of some of our assumptions and limitations. We have assumed binomial demand and binary effort levels. However, since our main insights are driven by the tradeoff between providing incentives at the cost of gaming losses, we expect them to hold in a continuous setting as well. By the same token, if we allow periods to be dependent in other ways (e.g., a high demand outcome in the early period makes a high demand outcome in the second period more or less likely), our key insights will hold. Finally, we have not considered phenomena such as “racheting” in which future quota targets for a salesperson are determined based on past performance; we leave these considerations, that fall under an asymmetric information paradigm, for future research.

12The low value can vary under different conditions, we keep it at one level in the figure for simplicity.
References


Figure 1: Canonical Effort Profiles in a Multi-Period Sales Scenario

Figure 2: Agent’s Optimal Effort Responses Based on Bonuses $B_1$ (given when final sales are $L + H$) and $B_2$ (given when final sales are $2H$). The regions are defined in Table A2 in Section A3.1 in the Appendix.
Figure 3: Optimal Two-period Contracts and Effort Outcomes. The optimal contract in Region II is an extreme, low-powered contract, in Region III is an extreme, high-powered contract and in Region IV is a gradual contract (in Region I the agent is not offered any incentive payment). The regions are defined in Table A4 in Section A3.3 in the Appendix.
Figure 4: Each column of this figure corresponds to one of the optimal contracts — the left column to the extreme low-powered contract (Region II of Figure 3), the middle column to the extreme high-powered contract (Region III of Figure 3), and the right column to the gradual contract (Region IV of Figure 3). The figures in the top row illustrate the final bonuses, \( B_0, B_1, \) and \( B_2 \) (at the leaves of the trees), and the induced effort levels, \( e_1, e_L^1, \) and \( e_H^1 \) (in the circular nodes), under each contract. The figures in the second row depict \( e_1 \) and \( E[e_2] \) under each contract. The figures in the third row illustrate the form of the contract. The fixed salaries from the left column to the right are \( S_{II} = \max\{2U - \frac{q}{p-q} \phi, 2K\}, S_{III} = \max\{2U - \frac{q}{p-q} \phi, 2K\}, S_{IV} = \max\{2U - \frac{2q}{p-q} \phi, 2K\}, \) respectively.
Figure 5: Effort Outcomes Under the Optimal Contracts with Sales Push Out and Pull In. The optimal contracts and regions are defined in Table A5 in Section A4 in the Appendix.

Figure 6: Effort Outcomes and Optimal Contracts Under Limited Inventory

(a) Effort Outcomes Under the Optimal Two-Period Contract. The regions are defined in Table A6 in Section A5 in the Appendix.

(b) Contract Comparison. The regions are defined in Table A7 in Section A5 in the Appendix.
Appendix

A1 First-Best Solution

The firm can implement any effort level $e_t$ in either period, by reimbursing the agent a fixed salary $s_t$ which must be at least $K$ while ensuring the agent’s participation. The principal’s problem in each period is the following.

$$\max_{s_t} \quad E[D_t|e_t] - E[s_t|e_t]$$

s.t. $U_A(e_t) \geq U$ \hspace{1cm} (PC$_t$)

$\quad s_t \geq K$ \hspace{1cm} (LL$_t$)

Here, (PC$_t$) is the agent’s participation constraint, where $U_A(e_t)$ stands for the salesperson’s expected net utility on exerting effort $e_t$, which is equal to $s_t - \phi$ if the agent exerts effort and is equal to $s_t$ if the agent does not exert effort. It states that to employ the sales agent, the principal needs to provide a fixed salary that makes the agent’s expected net utility from exerting effort $e_t$ no less than his outside option, which simplifies as $s_t \geq U + \phi$ if effort is exerted, and as $s_t \geq U$ if effort is not exerted. (LL$_t$) stands for the agent’s limited liability constraint, which ensures that the agent receives a fixed salary $s_t$ no less than his limited liability $K$. Notice that given $U \geq K$, (LL$_t$) holds whenever (PC$_t$) is satisfied under the first-best solution.

If the contract specifies effort exertion in period $t \in \{1, 2\}$, i.e., $e_t = 1$, the principal’s expected profit is equal to the expected market demand subject to the agent’s effort exertion, $pH + (1 - p)L$, minus the minimal salary to ensure effort exertion, $U + \phi$, i.e., $pH + (1 - p)L - (U + \phi)$. If $e_t = 0$, the principal gets the natural market outcome and pays the minimal salary to employ the salesperson, i.e. $qH + (1 - q)L - U$. This leads to the following first-best solution in Result 1.

Intuitively, the firm would like to direct the salesperson to work hard if and only if the increase in the expected demand subject to the agent’s effort exertion (given by $(p - q)(H - L)$) outweighs the marginal cost for soliciting effort $\phi$. In other words, the principal only needs to compensate the agent for his outside option plus cost of effort. Therefore the principal solicits effort exertion if and only if $H - L \geq \frac{\phi}{p-q}$. The first-best solution is not affected by the agent’s limited liability $K$, because under the assumption $U \geq K$ the limited liability constraint is automatically satisfied when the participation constraint is met.
A2 Optimal One-Period Contract

Consider the problem for period $t \in \{1, 2\}$. Since demand follows a binomial distribution, the principal offers quota-bonus contracts with quota levels $\chi_t \in \{H, L\}$ and bonuses $b_{\chi_t,t} \geq 0$, where the bonus $b_{\chi_t,t}$ is paid to the salesperson if and only if the sales reach the quota $\chi_t$, together with a fixed salary of $s_t$. Indeed, it suffices for the principal to consider only two of the decision variables. Without loss of generality, we normalize $b_{L,t}$ to 0 and simplify the notation of $b_{H,t}$ as $b_t$, i.e., the principal does not issue bonus when the demand outcome is $L$ and issues bonus $b_t$ when the demand outcome is $H$.

The principal’s problem in each period is the following.

$$\max_{s_t, b_t} \quad E[D_t|e_t] - E[s_t + b_t|e_t]$$

s.t.  
\begin{align*}
U_A(e_t) &> U_A(\tilde{e}_t) \quad \text{(IC)} \\
U_A(e_t) &\geq U \quad \text{(PC)} \\
s_t, s_t + b_t &\geq K \quad \text{(LL)}
\end{align*}

The participation constraint (PC) and the limited liability constraint (LL) can be interpreted in a similar way as in the first-best scenario. In addition, the contract needs to satisfy an incentive compatibility constraint (IC), which states that to induce effort $e_t$, the principal needs to ensure that the agent gains a higher net utility by exerting effort $e_t$ compared with a different effort level $\tilde{e}_t$.

Before solving the optimal contract for the principal, we first derive the best contract for the principal to induce any given effort level. To implement $e_t = 1$, from the incentive compatibility constraint (IC), the principal needs to set $b_{H,t}$ satisfying $s_t + pb_t - \phi \geq s_t + qb_t$, which simplifies into $b_t \geq \frac{\phi}{p-q}$. The participation constraint (PC) requires that the agent’s expected utility from exerting effort is no lower than his reservation utility, that is, $s_t + p\frac{\phi}{p-q} - \phi \geq U$. To meet the limited liability constraint (LL) we need the guaranteed salary no less than the agent’s limited liability, i.e., $s_t \geq K$. The solution is that to implement $e_t = 1$, the principal offers a fixed salary $s_t = \max\{K, U - q\frac{\phi}{p-q}\}$, and a bonus $b_t = \frac{\phi}{p-q}$ if the demand outcome is high. To implement $e_t = 0$, it is enough for the principal to only offer the agent a fixed salary $s_t = \max\{K, U\}$.

Comparing the two cases, the increase in expected payment to the agent when inducing effort
(compared with not inducing effort) can be simplified as

\[
E[s_t + b_t|e_t = 1] - E[s_t + b_t|e_t = 0] = \begin{cases} 
\phi, & \text{if } U - K > \frac{q}{p-q} \phi, \\
\frac{p}{p-q} \phi - (U - K), & \text{if } 0 \leq U - K < \frac{q}{p-q} \phi.
\end{cases}
\]

Furthermore, inducing effort from the agent, compared with not inducing effort, increases the expected demand by \(E[D_t|e_t = 1] - E[D_t|e_t = 0] = (p - q)(H - L)\). The overall solution to the optimal period-by-period contract is specified in the following.

<table>
<thead>
<tr>
<th>Region</th>
<th>(U - K)</th>
<th>(H - L)</th>
<th>(e_t^*)</th>
<th>(s_t^*)</th>
<th>(b_t^*)</th>
<th>Principal’s Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0 \leq U - K &lt; \frac{q}{p-q} \phi</td>
<td>\frac{p}{p-q} \phi &lt; H - L &lt; \frac{p}{(p-q)^2} \phi - \frac{U - K}{p-q}</td>
<td>0</td>
<td>U</td>
<td>0</td>
<td>2qH + (2 - 2q)L - 2U</td>
</tr>
<tr>
<td>II</td>
<td>U - K \geq \frac{q}{p-q} \phi</td>
<td>H - L \geq \frac{p}{(p-q)^2} \phi - \frac{U - K}{p-q}</td>
<td>1</td>
<td>U - \frac{q}{p-q} \phi</td>
<td>\frac{p}{p-q} \phi</td>
<td>2pH + (2 - 2p)L - 2U - 2\phi</td>
</tr>
<tr>
<td></td>
<td>0 \leq U - K &lt; \frac{q}{p-q} \phi</td>
<td>H - L \geq \frac{p}{(p-q)^2} \phi - \frac{U - K}{p-q}</td>
<td>1</td>
<td>K</td>
<td>\frac{p}{p-q} \phi</td>
<td>2pH + (2 - 2p)L - 2K - 2p\frac{\phi}{p-q}</td>
</tr>
</tbody>
</table>

Table A1: Optimal Period-by-Period Contract. Regions as in Figure A1.

Figure A1: Optimal Period-by-Period Contract and Effort Outcomes. The regions are defined in Table A1. In Region II no effort is induced in the period-by-period contracting case, and in Region II high effort is induced in both periods.

Figure A1 depicts the optimal period-by-period contract with respect to the range of the demand distribution \((H - L)\), the agent’s effectiveness parameter \((p - q)\), and the agent’s reservation utility
relative to his limited liability \((U - K)\). In Region I, the principal wants to induce effort in the first-best scenario but not in the period-by-period contracting scenario. In Region II, the principal wants to induce effort in the period-by-period scenario. The optimal one-period contract induces the agent to exert effort in each period by issuing bonus \(\frac{\phi}{p - q}\) upon high demand realization.

A3 Two-Period (Long Time Horizon) Contract

A3.1 Agent’s Effort Responses

<table>
<thead>
<tr>
<th>Region</th>
<th>((B_1, B_2))</th>
<th>((e_1, E[e_2]))</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(0 \leq B_1 &lt; \frac{\phi}{p - q}), (0 \leq B_2 - B_1 &lt; \frac{\phi}{p - q})</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>II</td>
<td>(B_2 - B_1 \geq \frac{\phi}{p - q}), (pB_2 + (1 - p - q)B_1 &lt; \frac{\phi}{p - q} + \phi)</td>
<td>((0, q))</td>
</tr>
<tr>
<td>III</td>
<td>(0 \leq B_1 &lt; \frac{\phi}{p - q}), (pB_2 + (1 - p - q)B_1 \geq \frac{\phi}{p - q} + \phi)</td>
<td>((1, p))</td>
</tr>
<tr>
<td>IV</td>
<td>(B_1 \geq \frac{\phi}{p - q}), (B_2 - B_1 \geq \frac{\phi}{p - q})</td>
<td>((1, 1))</td>
</tr>
<tr>
<td>V</td>
<td>(B_2 - B_1 &lt; \frac{\phi}{p - q}), (qB_2 + (1 - p - q)B_1 \geq \frac{\phi}{p - q} - \phi)</td>
<td>((1, 1 - p))</td>
</tr>
<tr>
<td>VI</td>
<td>(B_1 \geq \frac{\phi}{p - q}), (qB_2 + (1 - p - q)B_1 &lt; \frac{\phi}{p - q} - \phi)</td>
<td>((0, 1 - q))</td>
</tr>
</tbody>
</table>

Table A2: Agent’s Effort Responses to Two-Period Contracts. Regions are depicted in Figure 2.

A3.2 General Two-Period Contract

By enumerating the optimal contract to incentivize any possible effort profile, we show that under the weakly-dominant long-term contract, \(b_2(L, H) = b_2(H, L)\). Therefore, it is sufficient for the principal to focus on the long-term contract that pays at the end according to cumulative sales.

In the following, we use the labels \((IC^H_{2\text{-ge}})\), \((IC^H_{2\text{-l}})\), \((IC^L_{2\text{-ge}})\), \((IC^L_{2\text{-l}})\), \((IC_{1\text{-ge}})\), \((IC_{1\text{-l}})\) and \((LL)\) to denote the following constraints:

\[(IC^H_{2\text{-ge}})\] denotes \(b_2(H, H) - b_2(H, L) \geq \frac{\phi}{p - q}\) and \((IC^H_{2\text{-l}})\) denotes \(b_2(H, H) - b_2(H, L) < \frac{\phi}{p - q}\);

\[(IC^L_{2\text{-ge}})\] denotes \(b_2(L, H) \geq \frac{\phi}{p - q}\) and \((IC^L_{2\text{-l}})\) denotes \(b_2(L, H) < \frac{\phi}{p - q}\);

\[(IC_{1\text{-ge}})\] denotes \(U_H - U_L \geq \frac{\phi}{p - q}\) and \((IC_{1\text{-l}})\) denotes \(U_H - U_L < \frac{\phi}{p - q}\);

\[(LL)\] denotes \(S + b_2(H, H), S + b_2(H, L), S + b_2(L, H) \geq 2K\).
To induce $e = (1, 1)$, the principal’s problem is:

$$
\begin{align*}
\min_{S, b_2(H, H), b_2(H, L), b_2(L, H)} & \quad S + p^2b_2(H, H) + p(1 - p)(b_2(H, L) + b_2(L, H)) \\
\text{s.t.} & \quad S + p^2b_2(H, H) + p(1 - p)(b_2(H, L) + b_2(L, H)) - 2\phi \geq 2U \quad (PC) \\
\text{and} & \quad (IC_2^H\text{-ge}), (IC_2^L\text{-ge}), (IC_1\text{-ge}), (LL)
\end{align*}
$$

In $(IC_1\text{-ge})$, $U_H = S + pb_2(H, H) + (1 - p)b_2(H, L) - \phi$ is the agent’s expected utility in the second period given $D_1 = H$, and $U_L = S + pb_2(L, H) - \phi$ is the agent’s expected utility in the second period given $D_1 = L$.

The optimal contract to induce this effort profile is given by: $S = 2\max\{K, U - \frac{q}{p-q}\phi\}$, $b_2(L, H) = \frac{\phi}{p-q}$, $b_2(H, L) \leq \frac{\phi}{p-q}$, $pb_2(H, H) + (1 - p)b_2(H, L) = (1 + p)\frac{\phi}{p-q}$. The following history-independent contract lies within the optimal contract set: $S = 2\max\{K, U - \frac{q}{p-q}\phi\}$, $b_2(L, H) = \frac{\phi}{p-q}$, $b_2(H, L) = 2\frac{\phi}{p-q}$. The expected payment to the agent is $\max\{2\frac{p}{p-q}\phi + 2K, 2\phi + 2U\}$.

To induce $e = (1, p)$, the principal’s problem is:

$$
\begin{align*}
\min_{S, b_2(H, H), b_2(H, L), b_2(L, H)} & \quad S + p^2b_2(H, H) + p(1 - p)b_2(H, L) + p(1 - q)b_2(L, H) \\
\text{s.t.} & \quad S + p^2b_2(H, H) + p(1 - p)b_2(H, L) + p(1 - q)b_2(L, H) - (1 + p)\phi \geq 2U \quad (PC) \\
\text{and} & \quad (IC_2^H\text{-ge}), (IC_2^L\text{-li}), (IC_1\text{-ge}), (LL)
\end{align*}
$$

In $(IC_1\text{-ge})$, $U_H = pb_2(H, H) + (1 - p)b_2(H, L) - \phi$, $U_L = qb_2(L, H)$. The optimal contract to induce this effort profile is given by: $S = 2\max\{K, U - \frac{q}{2p-q}\phi\}$, $b_2(L, H) = 0$, $0 \leq b_2(H, L) \leq \frac{1 + p-q}{1-p}\frac{\phi}{p-q}$, $pb_2(H, H) + (1 - p)b_2(H, L) = (1 + p - q)\frac{\phi}{p-q}$. The following history-independent contract lies within the optimal contract set: $S = 2\max\{K, U - \frac{q}{2p-q}\phi\}$, $b_2(L, H) = 0$, $b_2(H, H) = 0$, $b_2(H, H) = (1 + \frac{1-q}{p})\frac{\phi}{p-q}$.

To induce $e = (1, 1 - p)$, the principal’s problem is:

$$
\begin{align*}
\min_{S, b_2(H, H), b_2(H, L), b_2(L, H)} & \quad S + pqb_2(H, H) + p(1 - q)b_2(H, L) + (1 - p)pb_2(L, H) \\
\text{s.t.} & \quad S + pqb_2(H, H) + p(1 - q)b_2(H, L) + (1 - p)pb_2(L, H) - (2 - p)\phi \geq 2U \quad (PC) \\
\text{and} & \quad (IC_2^H\text{-li}), (IC_2^L\text{-ge}), (IC_1\text{-ge}), (LL)
\end{align*}
$$

In $(IC_1\text{-ge})$, $U_H = qb_2(H, H) + (1 - q)b_2(H, L)$, $U_L = pb_2(L, H) - \phi$. 

Electronic copy available at: https://ssrn.com/abstract=3517558
To induce $e = (0, q)$, the principal’s problem is:

$$
\min_{S, b_2(H,H), b_2(H,L), b_2(L,H)} \quad S + q p b_2(H,H) + q(1 - p)b_2(H,L) + (1 - q)q b_2(L,H)
$$

$$
s.t. \quad S + q p b_2(H,H) + q(1 - p)b_2(H,L) + (1 - q)q b_2(L,H) - q \phi \geq 2U \quad (PC)
$$

and

$$(IC_2^H - ge), (IC_2^L - l), (IC_1 - l), (LL)$$

In $(IC_1 - l)$, $U_H = p b_2(H,H) + (1 - p)b_2(H,L) - \phi$, $U_L = q b_2(L,H)$. The unique optimal contract to induce this effort profile is given by: $S = 2 \max\{ K, U - \frac{q}{p - q} \phi \}$, $b_2(L, H) = \frac{\phi}{p - q}$, $b_2(H, L) = 0$, $b_2(H, H) = \frac{\phi}{p - q}$. The above contract is clearly history-independent.

To induce $e = (0, 1 - q)$, the principal’s problem is:

$$
\min_{S, b_2(H,H), b_2(H,L), b_2(L,H)} \quad S + q^2 b_2(H,H) + q(1 - q)b_2(H,L) + (1 - q)q b_2(L,H)
$$

$$
s.t. \quad S + q^2 b_2(H,H) + q(1 - q)b_2(H,L) + (1 - q)q b_2(L,H) - (1 - q)\phi \geq 2U \quad (PC)
$$

and

$$(IC_2^H - l), (IC_2^L - ge), (IC_1 - l), (LL)$$

In $(IC_1 - l)$, $U_H = q b_2(H,H) + (1 - q)b_2(H,L)$, $U_L = p b_2(L,H) - \phi$. The unique optimal contract to induce this effort profile is given by: $S = 2 \max\{ K, U - \frac{q}{2(p - q)} \phi \}$, $b_2(L, H) = \frac{\phi}{p - q}$, $b_2(H, L) = 0$, $b_2(H, H) = \frac{\phi}{p - q}$. Here, the principal offers $b_2(H, H) = \frac{\phi}{p - q}$ due to the non-decreasing constraint. $b_2(L, H)$ needs to be at least $\frac{\phi}{p - q}$ to induce $e_2^L = 1$. Because $b_2(H, H)$ cannot be lower than $b_2(L, H)$ according to the non-decreasing constraint, we will also have $b_2(H, H) = \frac{\phi}{p - q}$. Indeed, the non-decreasing constraint matters only when inducing $e = (0, 1 - q)$. We will reach at the same contract for inducing any other profile regardless of imposing the non-decreasing constraint or not.

Note that the optimal contract for inducing $e = (0, 1 - q)$ turns out to be history-dependent. However, we can prove that inducing $e = (0, 1 - q)$ is sub-optimal for the principal when the two periods are independent (see Appendix A3.3 for details) and the non-decreasing constraint is
imposed, thus is out of consideration.

- The cases when the principal would like to induce effort \( e = (1, 0) \) or \( e = (0, 1) \) are trivially dominated by the case when he would like to induce \( e = (1, 1) \) so there is no need for consideration.

We observe that, although the contract to induce \( e = (0, 1 - q) \) is history dependent, it is suboptimal for the principal; in all other cases, the optimal contract is characterized by \( b_2(L, H) = b_2(H, L) \). Therefore, when the two periods are independent, it suffices for the principal to pay the agent the same at the end of the second period based on the cumulative sales across two periods.

We summarize in Table A3 the expected sales and payments to the agent for different effort profiles that the principal induces. The principal’s profit can be obtained from \( E[D] - (S + E[B]) \).

<table>
<thead>
<tr>
<th>Region</th>
<th>((e_1, E[e_2]))</th>
<th>(E[D])</th>
<th>(S + E[B])</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>((0, 0))</td>
<td>(2qH + (2 - 2q)L)</td>
<td>(\max{2K, 2U})</td>
</tr>
<tr>
<td>II</td>
<td>((0, q))</td>
<td>((pq + 2q - q^2)H + (2 - pq - 2q + q^2)L)</td>
<td>(\max{2K + \frac{pq}{p-q}\phi, 2U + q\phi})</td>
</tr>
<tr>
<td>III</td>
<td>((1, p))</td>
<td>((p + p^2 + q - pq)H + (2 - p^2 - q + pq)L)</td>
<td>(\max{2K + \frac{p^2 + p - pq}{p-q}\phi, 2U + (1 + p)\phi})</td>
</tr>
<tr>
<td>IV</td>
<td>((1, 1))</td>
<td>(2pH + (2 - 2p)L)</td>
<td>(\max{2K + \frac{2p}{p-q}\phi, 2U + 2\phi})</td>
</tr>
<tr>
<td>V</td>
<td>((1, 1 - p))</td>
<td>((2p - p^2 + pq)H + (2 - 2p + p^2 - pq)L)</td>
<td>(\max{2K + \frac{2p - p^2 + pq}{p-q}\phi, 2U + (2 - p)\phi})</td>
</tr>
<tr>
<td>VI</td>
<td>((0, 1 - q))</td>
<td>((p + q^2 + q - pq)H + (1 - q)(2 - p + q)L)</td>
<td>(\max{2K + \frac{p + q^2 - pq}{p-q}\phi, 2U + (1 - q)\phi})</td>
</tr>
</tbody>
</table>

Table A3: The Expected Sales and Payments to the Agent for Different Effort Profiles that the Principal Induces in a Two-period Contract

### A3.3 Optimal Two-Period Contract

The regions in Figure 3 and the principal’s profit in each region are as follows. Note that the non-negativity of the principal’s profit can be ensured by choosing a sufficiently high value of the level of the low demand outcome, \( L \).
We first rule out the optimality of inducing $e = (0, 1 - q)$ and $e = (1, 1 - p)$ for the principal. Inducing $e = (0, 1 - q)$ is sub-optimal for the principal due to the following reasons. The saving in expected payment in inducing $e = (0, 1 - q)$ compared with inducing $e = (1, p)$ is given by $(p + q)\phi$, regardless of the value of $U - K$. In addition, the loss in expected demand in inducing $e = (0, 1 - q)$ compared with inducing $e = (1, p)$ is given by $(p^2 - q^2)(H - L)$. Therefore, inducing $e = (0, 1 - q)$ is dominated by inducing $e = (1, p)$ if $H - L \geq \frac{\phi}{p - q}$, the is the parameter space we consider.

We can rule out the optimality of inducing $e = (1, 1 - p)$ using a similar rationale. In particular, when $U - K \geq 0$, inducing $e = (1, 1 - p)$ is dominated by inducing $e = (1, 1)$ given $H - L \geq \frac{\phi}{p - q}$, the parameter space we focus on. When $U - K < 0$, inducing $e = (1, 1 - p)$ is dominated by inducing either $e = (1, 1)$ or $e = (0, 0)$ for the principal and is also sub-optimal. In particular, when $H - L < \frac{2p^2 - q^2}{(2 - p)(p - q)}$, we can prove that the incremental expected demand when inducing $e = (1, 1 - p)$ compared with inducing $e = (0, 0)$, given by $(2 - p)(p - q)(H - L)$, is no more than the incremental expected payment, given by $\frac{2p^2 - q^2 + pq}{p - q} \phi$. Therefore inducing $e = (0, 0)$ dominates inducing $e = (1, 1 - p)$ for the principal when $H - L < \frac{2p^2 - q^2}{(2 - p)(p - q)} \phi$. Since $\frac{2p^2 - q^2 + pq}{(2 - p)(p - q)} \phi > \frac{\phi}{p - q}$, inducing $e = (1, 1 - p)$ will be dominated either by inducing $e = (1, 1)$ or inducing $e = (0, 0)$ for the principal and thus is sub-optimal.

Next, we compare the principal’s profits for inducing the remaining effort profiles, i.e., $e = (0, 0)$, $e = (0, q)$, $e = (1, p)$, and $e = (1, 1)$. We now solve for the optimal two-period contract.

**Case 1:** $0 \leq U - K \leq \frac{q^2}{2(p - q)} \phi$.
In this case, the expected payments to the agent can be simplified as,

\[
E[S + B] = \begin{cases} 
2U, & \text{if } e = (0, 0) \\
2K + \frac{pq}{p-q} \phi, & \text{if } e = (0, q) \\
2K + \frac{p^2 + p - pq}{p-q} \phi, & \text{if } e = (1, p) \\
2K + \frac{2p}{p-q} \phi, & \text{if } e = (1, 1) 
\end{cases}
\]

Comparing the expected demands and payments for inducing \(e = (0, 0), e = (0, q), e = (1, p)\) and \(e = (1, 1)\), we get the optimal contract for the principal as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{p(1-p+q)}{(1-p)(p-q)^2} \phi, \\
(1, p), & \text{if } 0 \leq U - K < \frac{pq^2}{2(1+p-q)(p-q)} \phi, H - L > \frac{p^2 + p - pq}{(1+p)(p-q)^2} \phi - \frac{2(U-K)}{(1+p)(p-q)}, \text{ or,} \\
(0, q), & \text{if } \frac{pq^2}{2(1+p-q)(p-q)} \phi \leq U - K < \frac{q^2}{2(p-q)} \phi, \frac{p(1+p-2q)}{(1+p)(p-q)^2} \phi \leq H - L < \frac{p(1+p-2q)}{(1+p)(p-q)^2} \phi, \\
(0, 0), & \text{if } 0 \leq U - K < \frac{pq^2}{2(1+p-q)(p-q)} \phi, H - L \leq \frac{p^2 + p - pq}{(1+p)(p-q)^2} \phi - \frac{2(U-K)}{(1+p)(p-q)}, \text{ or,} \\
& \frac{pq^2}{2(1+p-q)(p-q)} \phi \leq U - K < \frac{q^2}{2(p-q)} \phi, H - L \leq \frac{p}{(p-q)^2} \phi - \frac{U-K}{q(p-q)},
\end{cases}
\]

- **Case 2:** \(\frac{q^2}{2(p-q)} \phi < U - K \leq \frac{q}{2(p-q)} \phi\).

The expected payment for inducing \(e = (0, q)\) becomes \(q\phi + 2U\) in this case. The other payments are the same as in Case 1. The solution for this case is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{p(1-p+q)}{(1-p)(p-q)^2} \phi, \\
(1, p), & \text{if } \frac{p + (p-q)^2}{(1+p-q)(p-q)^2} \phi - \frac{2(U-K)}{(1+p)(p-q)} \phi \leq H - L < \frac{p(1+p-2q)}{(1+p)(p-q)^2} \phi, \\
(0, q), & \text{if } \frac{\phi}{p-q} \leq H - L \leq \frac{p + (p-q)^2}{(1+p-q)(p-q)^2} \phi - \frac{2(U-K)}{(1+p)(p-q)} \phi. \\
(0, 0), & H - L < \frac{\phi}{p-q}.
\end{cases}
\]

- **Case 3:** \(\frac{q}{2(p-q)} \phi < U - K \leq \frac{q}{p-q} \phi\).

The expected payment for inducing \(e = (1, p)\) becomes \((1 + p)\phi + 2U\) in this case. The other
payments are the same as in Case 2. The solution for this case is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)}, \\
(1, p), & \text{if } \frac{\phi}{p-q} \leq H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)}, \\
(0, 0), & \text{if } H - L \leq \frac{\phi}{p-q},
\end{cases}
\]

- **Case 4:** \( U - K > \frac{q}{p-q} \phi \).

The expected payment for inducing \( e = (1, 1) \) becomes \( 2\phi + 2U \) in this case. The other payments are the same as in Case 3. The solution for this case is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{1}{p-q}, \\
(0, 0), & H - L < \frac{1}{p-q}.
\end{cases}
\]

### A3.4 Optimal Two-Period Contract without Non-decreasing Constraint

In the main analysis, we impose the non-decreasing constraint on the long time horizon contract to avoid the agent destroying sales. Indeed, if we relax this constraint, even with independent periods, the second type of “hockey stick” effort profile \( e = (0, 1-q) \) can be optimally induced by the principal in Region VI in Figure A2. In other words, if the non-decreasing constraint is removed, a “hockey stick” effort profile can be generated in a larger parameter space in equilibrium.

![Figure A2: Optimal Two-Period Contract and Effort Profile without Non-decreasing Constraints](https://ssrn.com/abstract=3517558)
A4  Period-by-Period Contract with Sales Push Out and Pull In

We derive the optimal period-by-period contract using backward induction. We start from the principal’s problem of inducing a specified effort profile in the second period.

- **Case 1:** Consider the case when the principal induces $e^H_2 = 1$, i.e., effort is exerted by the agent when the first period’s sales realization is $D_1 = H$.

  Given the reported sales level $D'_1$, even though the principal cannot observe the real realization of $D_2$ but can only observe the reported sales level $D'_2$, he can still infer $D_2$ from $D'_2$ by readjusting the quota level at the second period and setting bonus value high enough to ensure that if $D_2 = H$ the agent will not restrict sales. In particular, given $D_1 = H$, no matter what sales level the agent reports for the early period $D'_1$, if $D_2$ also realizes as $H$, the principal expects to observe the second period’s sales level as $D'_2 = 2H - D'_1$ (conditional on the bonus being high enough).

  Consequently, to induce $e^H_2 = 1$, it suffices to set the quota level $\chi^H_2$ equal to $2H - D'_1$ and to provide bonus $b_2$ equal to $\phi - q$. Additionally, since $\chi_2 = 2H - D'_1 \geq H + L - D'_1$, in case $D_1 = L$, no matter how much the agent reports, the later quota level will never be met. This implies that inducing $e^H_2 = 1$ will lead to $e^L_2 = 0$.

  Combined together, by setting $\chi_2 = 2H - D'_1 \geq H$ and $b_2 = \phi - q$, the principal induces $\langle e^H_2, e^L_2 \rangle = \langle 1, 0 \rangle$. We will discuss the level of the fixed salary later, as the agent decides whether to accept the principal’s contract at the beginning by weighing his utilities across two periods, anticipating that the principal may readjust quota levels later.

- **Case 2:** Inducing $e^L_2 = 1$, i.e., motivating effort exertion when the first period’s sales realization is $D_1 = L$.

  In a similar way as the case above, we have that the principal needs to set the quota level at $\chi^L_2 = H + L - D'_1$ and the bonus level at $b_2 = \phi - q$ to induce $e^L_2 = 1$. Also, since $\chi_2 = H + L - D'_1$, the quota level in the second period will always be met in case $D_1 = H$ so that there will be no effort exerted. This implies $e^L_2 = 1$ will lead to $e^H_2 = 0$. Combined together, by setting $\chi_2 = H + L - D'_1 \geq H$ and $b_2 = \phi - q$, the principal induces $\langle e^H_2, e^L_2 \rangle = \langle 0, 1 \rangle$.

  To summarize, to induce a specific effort level, the principal can adjust the quota level in the second period $\chi_2$ based on reported sales $D'_1$ in the first period. To induce $\langle e^H_2, e^L_2 \rangle = \langle 1, 0 \rangle$, the principal sets
\(\chi_2(D'_1) = 2H - D'_1\). To induce \(e^H_2, e^L_2\) = (0, 1), the principal sets \(\chi_2(D'_1) = H + L - D'_1\). In both cases, the principal offers a bonus of \(b_2 = \frac{\phi}{p-q}\) once the sales meet the quota level. Furthermore, inducing \(e^H_2 = 1\) and \(e^L_2 = 1\) is not incentive compatible when a salesperson can push or pull in sales since the range of quota levels required to motivate \(e^H_2 = 1\) has no overlap with that to induce \(e^L_2 = 1\).

Now we move to the principal’s problem in the first period. To induce \(e_1 = 1\), the principal needs to set the corresponding quota level \(\chi_1\) at such a level that, after accounting for sales push out and pull in, when \(D_1 = L\) the agent cannot meet the quota, and when \(D_1 = H\) he can meet the quota; this is derived as \(2L < \chi_1 \leq H + L\). To see this, given \(\chi_1 > 2L\), when \(D_1 = L\), the agents cannot make \(\chi_1\) even by pulling in all available sales \(L\) from the second period. Given \(\chi_1 \leq H + L\), when \(D_1 = H\), the agent can meet the quota by pulling in \(\chi - H < L\). Without loss of generality, it is enough to consider \(\chi_1 = H + L\).

The early bonus level \(b_1\) as well as the two fixed wage levels \(s_1\) and \(s_2\) are chosen by accounting for the second period’s effort profile. Now we discuss all the possible scenarios.

1. To induce \(e = (1, p)\), i.e., \(e = (1, 1, 0)\), the principal needs to provide sufficient \(b_1\) for agents to exert effort, which is given by \((1 - q)\frac{\phi}{p-q}\). This is because, if \(D_1 = H\), the agent earns \(s_1 + b_1 + s_2 + p\frac{\phi}{p-q} - \phi\); if \(D_1 = L\), the agent earns \(s_1 + s_2\). To induce \(e_1 = 1\), the principal needs to make sure \(b_1 + p\frac{\phi}{p-q} - \phi \geq \frac{\phi}{p-q}\), which simplifies into \(b_1 \geq (1-q)\frac{\phi}{p-q}\). Following this, the principal pays the agent \(s_1 + s_2 + p(b_1 + p\frac{\phi}{p-q}) = s_1 + s_2 + p(1 + p - q)\frac{\phi}{p-q}\) in expectation. Fixed wages are chosen such that the fixed wage in each period is no lower than the limited liability, and the two fixed wages combined can ensure the agent’s participation, namely, \(s_1 \geq K, s_2 \geq K, s_1 + s_2 \geq 2U + p(1 + p - q)\frac{\phi}{p-q} - (1 - p)\phi = 2U + \frac{\phi}{p-q}\phi\).

To summarize, to induce \(e = (1, p)\), at \(T = 1\), the principal sets \(\chi_1 = H + L\) and \(b_1 = (1 - q)\frac{\phi}{p-q}\). Under this contract, the agent exerts effort at \(T = 1\). If \(D_1 = H\), the agent pulls in \(L\) from \(T = 2\) to meet the early quota. Then at \(T = 2\), the principal readjusts the quota level to \(\chi_2 = H - L\) and sets \(b_2 = \frac{\phi}{p-q}\) to encourage effort exertion. If \(D_1 = L\), the agent reports \(D_1 = L\) as it is. The principal then readjusts the quota level at the second period to \(\chi_2 = 2H - L\) which is higher than \(H\) so the quota level is not achievable and the agent gives up.

2. Similarly, to induce \(e = (1, 1 - p)\), i.e., \(e = (1, 0, 1)\), the principal sets \(\chi_1 = H + L\) and offers \(b_1 = q\frac{\phi}{p-q}\). To see this, if \(D_1 = H\), the agent gets paid \(s_1 + b_1 + s_2 + \frac{\phi}{p-q}\), otherwise, the agent gets paid \(s_1 + s_2 + p\frac{\phi}{p-q} - \phi\). It requires \(b_1 + \frac{\phi}{p-q} - (p\frac{\phi}{p-q} - \phi) \geq \frac{\phi}{p-q}\) to motivate \(e_1 = 1\), which is equivalent to \(b_1 \geq q\frac{\phi}{p-q}\). The principal pays the agent \(s_1 + s_2 + (2p - p^2 + pq)\frac{\phi}{p-q}\) on expectation. Finally, the fixed wages are set such that each period’s wage is no lower than the limited liability and the two wages in
combination will ensure the agent’s participation, i.e., \( s_1 \geq K, s_2 \geq K, s_1 + s_2 \geq 2U + \frac{2q}{p-q} \phi \).

In this scenario, if \( D_1 = H \), agents again pull in \( L \) from the second period to earn early bonus; the principal later sets \( \chi_2 = 0 \) inducing no effort. If \( D_1 = L \) instead, agents cannot make early bonus and simply report \( D_1 = L \); the principal will set \( \chi_2 = H \) to induce effort.

(3) In other circumstances when the principal does not want to induce early effort, the contract is straightforward to derive and is shown in Table A5.

<table>
<thead>
<tr>
<th>Region</th>
<th>( r_1, r_2 )</th>
<th>( r_1 + r_2 )</th>
<th>( \chi_1 )</th>
<th>( b_1 )</th>
<th>( h(D_1) )</th>
<th>( s_1(D_1) )</th>
<th>Principal’s Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(0,0)</td>
<td>0</td>
<td>0</td>
<td>( 2L )</td>
<td>0</td>
<td>2(2-H) + (2-2L)E - max(2K, 2L)</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>(0,q)</td>
<td>max(2K + ( \frac{pq}{p-q} (2L) ))</td>
<td>0</td>
<td>0</td>
<td>( 2H - D_1 )</td>
<td>(pq - 2q)(H + (2 - 2L)E - max(2K, 2L) + ( \frac{pq}{p-q} (2L) ))</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>(1,q)</td>
<td>max(2K + ( \frac{pq}{p-q} (2L + (1 + pq)q) ))</td>
<td>( H + L )</td>
<td>( 1-q )</td>
<td>( 2H - D_1 )</td>
<td>(pq - 2q + ( 2p-q ))H + (2 - 2L)E - max(2K, 2L) + ( \frac{pq}{p-q} (2L + (1 + pq)q) ))</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>(1-q)</td>
<td>max(2K + ( \frac{pq}{p-q} (2L + (1-q)(pq)) ))</td>
<td>0</td>
<td>0</td>
<td>( 2H - D_1 )</td>
<td>(pq - 2q + ( 2p-q ))H + (2 - 2L)E - max(2K, 2L) + ( \frac{pq}{p-q} (2L + (1-q)(pq)) ))</td>
<td></td>
</tr>
</tbody>
</table>

Table A5: Period-by-Period Contract with Sales Push Out and Pull In. Regions are depicted in Figure 5.

Based on Table A5, we can prove that in the presence of sales push out and pull in, inducing \( e = (0,1-q) \) is dominated by inducing \( e = (1,p) \) for the principal. As a result, it is optimal for the principal to induce \( e = (0,q), e = (1,p), e = (1,1-p) \), or \( e = (0,0) \), depending on the parameter space as Figure 5 presents.

### A5 Inter-dependent Periods with Limited Inventory

With limited inventory, the optimal two-period contract and the outcomes are as per the following table.\(^{13}\)

\[ \omega_5 = H + L - \frac{1-q}{pq} (H - L) + \frac{p^2 + p-q}{(p^2 - q)(p-q)} \phi, \omega_6 = H + L - \frac{1-q}{q} (H - L) + \frac{p+q-(p-q)^2}{q(p-q)^2} \phi, \omega_7 = 2L + \frac{p+q-(p-q)^2}{(1-p)(p-q)} \phi, \mu_2 \equiv \frac{1}{2} \left( \frac{(p-q+q^2)}{p-q} \right) \phi - (1-q)(p-q)(H - L) \], \( \mu_3 = \frac{1}{2} \left[ \frac{(p+q-p^2+pq)}{p-q} \phi - (1-p)(p-q)(H - L) \right] \)

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\(^{13}\) Electronic copy available at: https://ssrn.com/abstract=3517558
Table A6: Optimal Two-Period Contract and Effort Profile with Limited Inventory. Regions are depicted in Figure 6(a).

Figures 6(a) illustrates the parametric regions with the different effort profiles under the optimal contract. In our previous analysis, we showed that without limited inventory, the optimal contract is either a gradual contract inducing \( e = (1,1) \) or an extreme contract inducing \( e = (1,p) \) or \( e = (0,q) \). In this scenario, if \( \Omega \) is relatively high, we are in Region IV in which a gradual contract induces \( e = (1,1) \) or in Region III in which an extreme contract induces \( e = (1,p) \). For a small \( \Omega \), we are in Region V in which a gradual contract induces effort \( e = (1,1-p) \). In this case, the agent still exerts early effort, but will exert effort in the second period only when the first period’s outcome is \( L \). For a yet smaller \( \Omega \), we are in Region VI in which a history-dependent contract inducing effort \( e = (0,1-q) \) is optimal for the principal. Under this contract, the principal offers \( b_2(H,L) = 0 \) and \( b_2(L,H) = b_2(H,H) = \frac{\phi}{p-q} \). In this case, the bonus payment is not affected by the first period’s demand outcome, and will be issued if the second period realizes as \( H \).  

In the presence of limited inventory, the period-by-period contract and the two-period contract compare as per the following table.  

<table>
<thead>
<tr>
<th>Region</th>
<th>( \Omega )</th>
<th>( U - K )</th>
<th>( (e_1, E[e_2]) )</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>I ( { \omega_2, \frac{\omega_2}{p-q} - \phi } ) ( (0, \omega_2) )</td>
<td>( 2L, \omega_2 - \frac{2U-K}{(1-q)(p-q)} ) ( 2L, \omega_2 - \frac{2U-K}{(1-q)(p-q)} )</td>
<td>( (0,0) )</td>
<td>( S = 2U, B_1 = 0, B_2 = 0 ) ( S = 2U, B_1 = 0, B_2 = 0 )</td>
<td></td>
</tr>
<tr>
<td>III ( { \frac{\phi}{p-q} \omega_3 } ) ( { \omega_2, \frac{\phi}{p-q} \omega_3 } ) ( { \omega_2, \frac{\phi}{p-q} \omega_3 } ) ( { \omega_2, \frac{\phi}{p-q} \omega_3 } )</td>
<td>( H + L + \frac{\phi}{q-q} ) ( 2H ) ( (1,p) )</td>
<td>( (1,p) )</td>
<td>( S = 2U, B_1 = 0, B_2 = \frac{\phi}{p-q} \omega_3 ) ( S = 2K, B_1 = 0, B_2 = \frac{\phi}{p-q} \omega_3 )</td>
<td></td>
</tr>
<tr>
<td>IV ( { \omega_3, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } )</td>
<td>( H + L + \frac{\phi}{q-q} ) ( 2H ) ( (1,1) )</td>
<td>( (1,1) )</td>
<td>( S = 2K, B_1 = \frac{\omega_3}{p-q}, B_2 = 2 \frac{\omega_3}{p-q} )</td>
<td></td>
</tr>
<tr>
<td>V ( { \omega_3, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } )</td>
<td>( \max { \omega_6 - \frac{2U-K}{(1-q)(p-q)}, \omega_6 - \frac{2U-K}{(1-q)(p-q)} } ) ( 2L + \frac{\phi}{p-q}, H + L + \frac{\phi}{p-q} ) ( 2L + \frac{\phi}{p-q}, H + L + \frac{\phi}{p-q} ) ( (0,1) )</td>
<td>( (0,1) )</td>
<td>( S = 2U, b_2(L,H) = \frac{\phi}{p-q}, b(H,L) = 0, b(H,H) = \frac{\phi}{p-q} ) ( S = 2U, b_2(L,H) = \frac{\phi}{p-q}, b(H,L) = 0, b(H,H) = \frac{\phi}{p-q} )</td>
<td></td>
</tr>
<tr>
<td>VI ( { \omega_3, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } ) ( { \omega_2, \frac{\omega_3}{p-q} } )</td>
<td>( 2L + \frac{\phi}{p-q}, \max { \omega_6 - \frac{2U-K}{(1-q)(p-q)}, \omega_6 - \frac{2U-K}{(1-q)(p-q)} } ) ( 2L + \frac{\phi}{p-q}, H + L + \frac{\phi}{p-q} ) ( 2L + \frac{\phi}{p-q}, H + L + \frac{\phi}{p-q} ) ( (0,1) )</td>
<td>( (0,1) )</td>
<td>( S = 2U, b_2(L,H) = \frac{\phi}{p-q}, b(H,L) = 0, b(H,H) = \frac{\phi}{p-q} ) ( S = 2K, b_2(L,H) = \frac{\phi}{p-q}, b(H,L) = 0, b(H,H) = \frac{\phi}{p-q} )</td>
<td></td>
</tr>
</tbody>
</table>

Note that this is the only case where the non-decreasing constraint (that compensation should not be decreasing in sales) binds in the optimal contract. In particular, to induce \( c_2^* = 1 \), we need \( b_2(L,H) \) is at least \( \frac{\phi}{p-q} \). Given \( b_2(L,H) = \frac{\phi}{p-q} \), \( b_2(H,H) \) cannot be less than \( \frac{\phi}{p-q} \) due to the non-decreasing constraint \( b_2(H,H) \geq b_2(L,H) \).

Note that with limited inventory the effort profile \( (0,q) \) is not induced under the optimal contract, while without limited the effort profile \( (0,1-q) \) is not induced under the optimal contract.
<table>
<thead>
<tr>
<th>Region</th>
<th>$U - K$</th>
<th>$\Omega$</th>
<th>$(e_1, K[e_2])$</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$2L &lt; \Omega &lt; \omega_1 - \frac{4L}{q}$</td>
<td>$2L &lt; \omega_1 - \frac{4L}{q}$</td>
<td>$(0, U)$</td>
<td>Period-by-period / Two-period</td>
</tr>
<tr>
<td>II</td>
<td>$\omega_1 - \frac{4L}{q} &lt; \Omega &lt; \omega_2 - \frac{2H(1-K)}{q}$</td>
<td>$\omega_2 - \frac{2H(1-K)}{q} &lt; \Omega &lt; \omega_3 - \frac{2H(1-L)}{p}$</td>
<td>$(0, \mu_2)$</td>
<td>Period-by-period/Tw</td>
</tr>
</tbody>
</table>
Online Appendix for: Multi-Period Contracting and Salesperson Effort Profiles: The Optimality of “Hockey Stick,” “Giving Up” and “Resting on Laurels”

OA1 Optimal Period-by-period Contract with Limited Inventory

Recall that in the previous analysis with a period-by-period contract, independence across the two periods implied that the optimal contract stays the same for \( t = 1 \) and \( t = 2 \). However, in the presence of limited inventory, the principal’s decision at \( T = 2 \), after observing \( D_1 \), is affected by the remaining inventory level \( \Omega - D_1 \). In other words, with limited inventory, the principal’s decision variables become \((s_1, s_2^{D_1})\) and \((b_1, b_2^{D_1})\), where it will dynamically adjust contract terms at period 2 depending on the realization of \( D_1 \) as \( H \) or \( L \), and the effort levels induced correspondingly are \((e_1, e_2^{D_1})\). We obtain that with limited inventory, the optimal period-by-period contract and the outcomes are as per the following table. 17

<table>
<thead>
<tr>
<th>Region</th>
<th>( \Omega )</th>
<th>((e_1, E[e_2]))</th>
<th>((s_1, s_2^H, s_2^L))</th>
<th>((b_1, b_2^H, b_2^L))</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(2L, \omega_1 = \frac{U-K}{p-q})</td>
<td>(0, 0)</td>
<td>(K, K, K)</td>
<td>(0, 0, 0)</td>
</tr>
<tr>
<td>II</td>
<td>(\omega_1 = \frac{U-K}{p-q}, \max(\omega_2 - \frac{1+q-p}{q(p-q)}(U-K), \omega_2' - \frac{1+q-p}{q(p-q)}(U-K)))</td>
<td>((0, 1-q))</td>
<td>(U, U, K)</td>
<td>(0, 0, \frac{p}{p-q})</td>
</tr>
<tr>
<td>III</td>
<td>(\max(\omega_2 - \frac{1+q-p}{q(p-q)}(U-K), \omega_2' - \frac{1+q-p}{q(p-q)}(U-K)), \omega_1 = \frac{U-K}{p-q})</td>
<td>((1, 1-p))</td>
<td>(K, U, K)</td>
<td>(\frac{p}{p-q}, 0, \frac{p}{p-q})</td>
</tr>
<tr>
<td>IV</td>
<td>(\omega_1 = \frac{U-K}{p-q}, 2H)</td>
<td>((1, 1))</td>
<td>(K, K, K)</td>
<td>(\frac{p}{p-q}, \frac{p}{p-q}, \frac{p}{p-q})</td>
</tr>
</tbody>
</table>

Below, we prove the above results. When the inventory level is high enough (Region IV) and does not lead to a bottleneck, the principal induces \( e = (1, 1) \), consistent with the case without inventory concerns. For a smaller \( \Omega \) (Region III), although \( e_1 = 1 \) remains, \( e_2 \) becomes contingent on \( D_1 \); a successful first period will cause an inventory shortage later and no extra effort is needed. The expected effort in the second period thus is the probability of realizing \( D_1 \) as \( L \), which is \( 1 - p \), and the resulting effort profile is \( e = (1, 1-p) \). If \( \Omega \) is further below a threshold (Region II), the principal abandons early effort induction. This leads to an equilibrium effort profile \( e = (0, 1-q) \). For a yet smaller \( \Omega \) (Region I), inventory levels are too low to justify any effort induction, i.e. \( e = (0, 0) \). It is noteworthy that the set of optimal effort profiles excludes \( e = (0, 1) \) and \( e = (1, 0) \).

With limited inventory, we first update the expected demands generated under different effort profiles. Here, we write the effort level in the second period in terms of its expectation. For instance, effort profile \( e = (0, 1-q) \) implies that the principal does not induce effort in the first period, and has a probability of \( 1-q \) to induce effort in the second period (i.e., when the first period demand realizes as \( L \)). The result is given by Table OA1.

\[\begin{align*}
\omega_1 &\equiv H+L+\frac{p}{(p-q)}\phi, \\
\omega_2 &\equiv H+L-\frac{1-p}{q}(H-L)+\frac{p^2+pq}{q(p-q)}\phi, \\
\omega_2' &\equiv 2L+\frac{p^2+pq}{(1-p)(p-q)}\phi, \\
\omega_3 &\equiv 2L+\frac{p}{(p-q)}\phi, \\
\omega_4 &\equiv \frac{1+q-p}{(1-p)(p-q)}\phi
\end{align*}\]

17
\[
\begin{align*}
(e_1, E[e_2]) & & 2L < \Omega \leq H + L & & H + L < \Omega \leq 2H \\
(0, 0) & & (2q - q^2)\Omega + (1 - q)^22L & & q^2\Omega + (2q - q^2)(H + L) + (1 - q)^22L \\
(0, 1 - q) & & (p + q - pq)\Omega + (1 - p)(1 - q)2L - (q^2 + p - pq)\frac{1}{p-q} & & q^2\Omega + (p - q^2) + q - pq)(H + L) + (1 - p)(1 - q)2L - (q^2 + p - pq)\frac{1}{p-q} \\
(0, q) & & (2q - q^2)\Omega + (1 - q)^22L - pq\frac{1}{p-q} & & pq\Omega + (1 + q^2 - q - pq)(H + L) + (1 - q)^22L - pq\frac{1}{p-q} \\
(1, 1 - p) & & (2p - p^2)\Omega + (1 - p)^22L - (2p - p^2 + pq)\frac{1}{p-q} & & pq\Omega + (2p - p^2 - pq)(H + L) + (1 - p)^22L - (2p - p^2 + pq)\frac{1}{p-q} \\
(1, p) & & (p + q - pq)\Omega + (1 - p)(1 - q)2L - (p^2 + p - pq)\frac{1}{p-q} & & p^2\Omega + (p - p^2 + q - pq)(H + L) + (1 - p)(1 - q)2L - (p^2 + p - pq)\frac{1}{p-q} \\
(1, 1) & & (2p - p^2)\Omega + (1 - p)^22L - 2p\frac{1}{p-q} & & p^2\Omega + (2p - 2p^2)(H + L) + (1 - p)^22L - 2p\frac{1}{p-q}
\end{align*}
\]

Table OA1: Expected Demand under Period-by-period Contract with Limited Inventory

![Graph](image)

Figure OA1: Optimal Period-by-period Contract with Limited Inventory

We then derive the expected payments to the agent when different effort profiles are induced. Based on the solution for the period-by-period contract, if the induced effort level is \(e_t = 1\), then the principal pays \(\max\{K + \frac{p}{p-q}\phi, U + \phi\}\) to the agent in expectation. If the induced effort level is \(e_t = 0\), then the principal pays \(\max\{K, U\}\) to the agent. We thus get the expected payments to the agent when the two periods are dependent, by adding up the expected payments to the agent in each period. Throughout this online appendix, we use \(E[S + B]\) as the total expected payment to the agent during the two periods, including both fixed salaries and bonuses. The result is given by the following table.

\[
\begin{align*}
(e_1, E[e_2]) & & E[S + B] \\
(0, 0) & & 2\max\{K, U\} \\
(0, 1 - q) & & (1 - q)\max\{K + \frac{p}{p-q}\phi, U + \phi\} + (1 + q)\max\{W, U\} \\
(0, q) & & q\max\{K + \frac{p}{p-q}\phi, U + \phi\} + (2 - q)\max\{K, U\} \\
(1, 1 - p) & & (2 - p)\max\{K + \frac{p}{p-q}\phi, U + \phi\} + p\max\{K, U\} \\
(1, p) & & (1 + p)\max\{K + \frac{p}{p-q}\phi, U + \phi\} + (1 - p)\max\{K, U\} \\
(1, 1) & & 2\max\{K + \frac{p}{p-q}\phi, U + \phi\}
\end{align*}
\]

Table OA2: Expected Payment under Period-by-period Contract with Limited Inventory
We solve the optimal period-by-period contract for the principal when the two periods are dependent, by considering \( U - K \) in different ranges. Before solving the problem, we first rule out the optimality of inducing \( e = (1, p) \) and inducing \( e = (0, q) \) for the principal.

- Below, we first prove that it is sub-optimal for the principal to induce \( e = (1, p) \).
  - When \( \Omega < H + L \), we can observe that inducing \( e = (1, p) \) is dominated by inducing \( e = (0, 1 - q) \) for the principal. This is because, given \( \Omega < H + L \), inducing \( e = (1, p) \) generates the same demand in expectation as inducing \( e = (0, 1 - q) \), while leading to a higher payment in expectation.
  - When \( \Omega > H + L \) and \( H - L \geq \frac{p}{(p - q)^2} \varphi \), inducing \( e = (1, p) \) is dominated by inducing \( e = (1, 1) \) for the principal. Indeed, given \( \Omega > H + L \), the difference in expected demands between inducing \( e = (1, 1) \) and inducing \( e = (1, p) \) is given by,

\[
E_{\Omega > H + L}[D|e = (1, 1)] - E_{\Omega > H + L}[D|e = (1, p)]
\]

\[
= \left( 2p(1 - p) - (1 - p)(p + q) \right) (H + L) + \left( (1 - p)^2 - (1 - p)(p - q) \right) (H + L) 2L
\]

\[
\]

Meanwhile, the difference in their expected payments is,

\[
E[S+B|e = (1, 1)] - E[S+B|e = (1, p)] = \begin{cases} 
\frac{p(1-p)}{p-q} \varphi, & \text{if } U - K \leq 0, \\
\frac{p(1-p)}{p-q} \varphi + (1 - p)(U - K), & \text{if } 0 < U - K \leq \frac{q}{p-q} \varphi, \\
(1 - p) \varphi, & \text{if } U - K > \frac{q}{p-q} \varphi.
\end{cases}
\]

The above expression is no larger than \( \frac{p(1-p)}{p-q} \varphi \), where the maximal value takes place when \( U - K \leq 0 \). We can see the incremental demand in inducing \( e = (1, 1) \) relative to inducing \( e = (1, p) \) is no less than the maximal incremental payment in this scenario. That is, given \( H - L \geq \frac{p}{(p - q)^2} \varphi \),

\[
(1 - p)(p - q)(H - L) \geq \frac{p(1-p)}{p-q} \varphi.
\]

This implies that \( e = (1, p) \) is dominated by inducing \( e = (1, 1) \) when \( \Omega > H + L \) and \( H - L \geq \frac{p}{(p - q)^2} \varphi \).

- When \( \Omega > H + L \) and \( H - L \leq \frac{p}{(p - q)^2} \varphi \), we will show that inducing \( e = (1, p) \) is dominated by either inducing \( e = (1, 1) \) or inducing \( e = (0, 0) \). Consider the two dimensional parameter space \((U - K, \Omega)\).

* Below, we first show that at \( \Omega = 2H \) and \( U - K = (p - q) \left( \frac{p}{(p - q)^2} \varphi - (H - L) \right) \) (referred as “focal point”), the principal is indifferent between inducing \( e = (1, 1) \), \( e = (1, p) \) and \( e = (0, 0) \). Since \( \frac{p}{p-q} \varphi - (p-q)(H - L) \in \left( 0, \frac{q}{p-q} \varphi \right) \), we can write down the expected payments at the focal point without ambiguity. Let’s compare the cases for inducing
$e = (1, p)$ and $e = (1, 1)$. The difference in their expected demands at $\Omega = 2H$ is $(1 - p)(p - q)(H - L)$. At $U - K = - \frac{p}{p-q} \phi - (p - q)(H - L)$, the difference in their expected payments is, $\frac{p(1-p)}{p-q} \phi - (1 - p)(U - K) = (1 - p)(p - q)(H - L)$, which is equal to the difference in their expected demands. We then compare the cases for inducing $e = (1, p)$ and inducing $e = (0, 0)$ at $\Omega = 2H, U - K = - \frac{p}{p-q} \phi - (p - q)(H - L)$. Given $\Omega = 2H$, the difference in their expected demands is $(1 - p)(p - q)(H - L)$. The different in their expected payments is $\frac{p(1+p)}{p-q} \phi - (1 + p)(U - K)$, which is equal to the incremental payment (i.e., $(1 - p)(p - q)(H - L)$) when $U - K = - \frac{p}{p-q} \phi - (p - q)(H - L)$.

* We have shown that without inventory concern, the principal is indifferent between inducing $e = (1, 1), e = (1, p), e = (0, 0)$ at $\Omega = 2H$ and $U - K = (p-q)\left(\frac{1}{(p-q)^2} \phi - (H-L)\right)$. Now consider $\forall \Omega \in (H + L, 2H)$.

* $\forall \Omega \in (H + L, 2H)$, when $U - K > - \frac{p}{p-q} \phi - (p - q)(H - L)$, compared to the focal point, the incremental demand in inducing $e = (1, 1)$ relative to inducing $e = (1, p)$ remains the same, but the incremental payment becomes smaller. This implies that $\forall \Omega \in (H + L, 2H)$, inducing $e = (1, p)$ performs worse for the principal relative to inducing $e = (1, 1)$ when $U - K > - \frac{p}{p-q} \phi - (p - q)(H - L)$.

* $\forall \Omega \in (H + L, 2H)$ when $U - K < - \frac{p}{p-q} \phi - (p - q)(H - L)$, compared to the focal point, the incremental demand in inducing $e = (1, p)$ relative to inducing $e = (0, 0)$ gets smaller, but the incremental payment becomes larger. This implies that inducing $e = (1, p)$ performs worse for the principal relative to inducing $e = (0, 0)$ when $U - K < - \frac{p}{p-q} \phi - (p - q)(H - L)$.

* Combined together, $\forall \Omega \in (H + L, 2H)$, we have that inducing $e = (1, p)$ is sub-optimal for the principal given $H - L < \frac{p}{(p-q)^2} \phi$.

- Put the three cases above together, we can reach the conclusion that inducing $e = (0, q)$ is sub-optimal for the principal.

* In a similar way, we can prove that inducing $e = (0, q)$ is sub-optimal for the principal.

- When $\Omega < H + L$, we can observe that inducing $e = (0, q)$ is dominated by inducing $e = (0, 0)$ for the principal.

**Proof.** This is because, given $\Omega < H + L$, inducing $e = (0, q)$ generates the same demand in expectation as inducing $e = (0, 0)$, while leading to a higher payment in expectation.

- When $\Omega > H + L$ and $H - L \geq \frac{p}{(p-q)^2} \phi$, inducing $e = (0, q)$ is dominated by inducing $e = (1, 1 - p)$ for the principal. Indeed, given $\Omega > H + L$,

$$E_{\Omega> H+L}[D|e = (1, 1 - p)] - E_{\Omega> H+L}[D|e = (0, q)]$$

$$= \left( \frac{p(2 - p - q) - q(2 - p - q)}{H+L} \right) (H+L) + \left( (1-p)^2 - (1-q)^2 \right) 2L$$

$$= (2 - p - q)(p - q)(H - L).$$
When $\Omega < U_e$ relative to inducing $e = (1, 1 - p)$ scenario. That is, $H[\ast \forall \Omega \in (H + L, 2H)]$ below, we first show that without inventory concern at $\Omega = 2H, U - K = \frac{p}{p - q}\phi - (p - q)(H - L)$, the principal is indifferent between inducing $e = (0, q)$, $e = (1, 1 - p)$ and $e = (0, 0)$. Notice that $\frac{p}{p - q}\phi - (p - q)(H - L)$ lies between 0 and $\frac{q}{p - q}\phi$, therefore we can write down the expected payments at the focal point without ambiguity. Let’s compare the case for inducing $e = (0, q)$ and $e = (1, 1 - p)$ at the focal point of $\Omega = 2H, U - K = \frac{p}{p - q}\phi - (p - q)(H - L)$. The difference in their expected demands at $\Omega = 2H$ is $(2 - p - q)(p - q)(H - L)$. In addition, at $U - K = \frac{p}{p - q}\phi - (p - q)(H - L)$, we get that the difference in their expected payments, $\frac{q(2 - p - q)(p - q)(H - L)}{p - q} = (2 - p - q)(p - q)(H - L)$, which is equal to the difference in their expected demands. Now compare the cases for inducing $e = (0, q)$ and inducing $e = (0, 0)$. Given $\Omega = 2H$, the difference in their expected demands is $q(p - q)(H - L)$. The difference in their expected payments is $\frac{p}{p - q}\phi - q(U - K)$, which is equal to the incremental payment (i.e., $q(p - q)(H - L)$) when $U - K = \frac{p}{p - q}\phi - (p - q)(H - L)$. Therefore, we have shown that without inventory concern, the principal is indifferent between inducing $e = (1, 1 - p), e = (0, q), e = (0, 0)$ at the focal point of $\Omega = 2H, U - K = \frac{p}{p - q}\phi - (p - q)(H - L)$. Now let’s consider the case with $\forall \Omega \in (H + L, 2H)$. 

* $\forall \Omega \in (H + L, 2H)$, when $U - K > \frac{p}{p - q}\phi - (p - q)(H - L)$, compared to the focal point, the incremental demand in inducing $e = (1, 1 - p)$ relative to inducing $e = (0, q)$ remains the same, but the incremental payment becomes smaller. This implies that inducing $e = (1, 1 - p)$ performs worse for the principal relative to inducing $e = (0, q)$ in this case.

* $\forall \Omega \in (H + L, 2H)$ when $U - K < \frac{p}{p - q}\phi - (p - q)(H - L)$, compared to the focal point, the incremental demand in inducing $e = (0, q)$ relative to inducing $e = (0, 0)$ gets smaller, but the incremental payment becomes larger. This implies that inducing $e = (0, q)$ is dominated by inducing $e = (0, 0)$ when in this case.

* To summarize, it is sub-optimal for the principal to induce $e = (0, q)$ when $H + L < \Omega < \frac{p}{p - q}\phi - \frac{q}{p - q}\phi$, otherwise the incremental demand in inducing $e = (1, 1 - p)$ relative to inducing $e = (0, q)$, is no less than the maximal incremental payment in this scenario. That is,

$$(2 - p - q)(p - q)(H - L) \geq \frac{p(2 - p - q)}{p - q}\phi, \text{ given } H - L \geq \frac{p}{(p - q)^2}\phi$$

This implies that $e = (0, q)$ is dominated by inducing $e = (1, 1 - p)$ when $\Omega > H + L$ and $H - L \geq \frac{p}{(p - q)^2}\phi$.

- When $\Omega > H + L$ and $H - L < \frac{p}{(p - q)^2}\phi$, we will show that inducing $e = (0, q)$ is dominated by either inducing $e = (1, 1 - p)$ or inducing $e = (0, 0)$.
2H and \( H - L < \frac{p}{(p-q)^2} \phi \).

- Put the three cases above together, we reach the conclusion that inducing \( e = (0, q) \) is suboptimal for the principal under the period-by-period contract with limited inventory.

Now we have ruled out the optimality of inducing \( e = (1, p) \) and \( e = (0, q) \) for the principal. Below, we focus on the remaining possible effort profiles, i.e., \( e = (1,1-p), (0,1-q), (0,0) \) and compare the principal’s profits under these effort profiles. To solve the problem, we start our analysis by considering \( U - K \) in different ranges. In the following analysis, we focus on the parameter space where the upside demand potential is large enough, i.e., \( H - L \geq \frac{p}{(p-q)^2} \phi \). The other case when \( H - L < \frac{p}{(p-q)^2} \phi \) is less interesting, since in that case, when \( U - K < 0 \), the principal will not induce effort in equilibrium regardless of the inventory level.

**Case 1:** \( U - K \geq \frac{q}{p-q} \phi \):

- First, we compare the principal’s profits when inducing \( e = (1,1) \) and when inducing \( e = (1,1-p) \), by considering \( \Omega \in [2L, H + L) \) and \( \Omega \in [H + L, 2H) \) separately.

  * For \( \Omega \in [2L, H+L) \), inducing \( e = (1,1) \) generates the same profit as inducing \( e = (1,1-p) \), while paying more to the agent in expectation, thus is dominated by inducing \( e = (1,1-p) \).

    For \( \Omega \in [H + L, 2H) \),

    \[
    E_{\Omega>H+L}[D|e = (1,1)] - E_{\Omega>H+L}[D|e = (1,1-p)]
    = (p^2 - pq) \Omega + \left(2p(1-p) - p(2-p-q)\right)(H + L)
    = p(p-q)\left(\Omega - (H + L)\right)
    \]

- Therefore, the principal prefers inducing \( e = (1,1) \) to inducing \( e = (1,1-p) \), when the incremental demand is larger then the incremental payment, which requires,

  \[
  p(p-q)\left(\Omega - (H + L)\right) > p\phi,
  \]

  \[
  \Omega > H + L + \frac{1}{p-q} \phi
  \]

  As a result, for \( \Omega \in [H + L + \frac{1}{p-q} \phi, 2H) \), the principal prefers \( e = (1,1) \) to \( e = (1,1-p) \).

  For \( \Omega \in [H + L, H + L + \frac{1}{p-q} \phi) \), the principal prefers \( e = (1,1-p) \) to \( e = (1,1) \).

  * Combined together, we have that when \( \Omega > H + L + \frac{1}{p-q} \phi \), inducing \( e = (1,1) \) generates a higher profit for the principal compared with inducing \( e = (1,1-p) \).
– Next, we compare the principal’s profits when inducing $e = (1, 1 - p)$ and when inducing $e = (0, 1 - q)$.

* For $\Omega < H + L$,

$$E_{2L<\Omega<H+L}[D|e = (1, 1 - p)] - E_{2L+\Omega<H+L}[D|e = (0, 1 - q)] = (1 - p)(p - q)(\Omega - 2L),$$

$$E[S + B|e = (1, 1 - p)] - E[S + B|e = (0, 1 - q)] = (1 + q - p)\phi.$$

Therefore the principal prefers inducing $e = (1, 1 - p)$ to inducing $e = (0, 1 - q)$, when the incremental demand is larger than the incremental payment, which requires,

$$(1 - p)(p - q)(\Omega - 2L) > (1 + q - p)\phi,$$

$$\Omega < 2L + \frac{1 + q - p}{(1 - p)(p - q)} \phi \equiv \omega_4.$$

Notice that the above value is smaller than $H + L$ (since $H - L \geq \frac{p}{(p - q)^2}\phi$ implies that $H - L \geq \frac{1 + q - p}{(1 - p)(p - q)}\phi$). Therefore, the principal prefers inducing $e = (1, 1 - p)$ to inducing $e = (0, q)$ when $\omega_4 < \Omega < H + L$.

* For $\Omega > H + L$, inducing $e = (1, 1 - p)$ dominates inducing $e = (0, q)$. The reason is as follows. The difference in the expected demands between inducing the above two effort profiles is given by,

$$E_{\Omega>H+L}[D|e = (1, 1 - p)] - E_{\Omega>H+L}[D|e = (0, 1 - q)] = \left(pq - q^2\right)\Omega + \left(p(2 - p - q) - (1 - q)(p + q)\right)(H + L)$$

$$= (p - q)\left(q(\Omega - (H + L)) + (1 - p)(H - L)\right).$$

The difference in the expected payments to the agent is the same as what we just derived. Thus the principal prefers inducing $e = (1, 1 - p)$ to inducing $e = (0, 1 - q)$ when,

$$\left(p - q\right)\left(q\left(\Omega - (H + L)\right) + (1 - p)(H - L)\right) > (1 + q - p)\phi,$$

i.e., $\Omega > H + L - \frac{1 - p}{q}(H - L) + \frac{1 + q - p}{q(p - q)}\phi$.

However, $H + L - \frac{1 - p}{q}(H - L) + \frac{1 + q - p}{q(p - q)}\phi$ is smaller than $H + L$, since $H - L \geq \frac{p}{(p - q)^2}\phi \geq \frac{1 + q - p}{(1 - p)(p - q)}\phi$.

* Combined, if $H + L \geq \frac{p}{(p - q)^2}\phi$, the principal prefers inducing $e = (1, 1 - p)$ to inducing $e = (0, 1 - q)$ when $\Omega > \omega_4$ (which is smaller than $H + L$).

– Finally, we compare the principal’s profits when inducing $e = (0, 1 - q)$ and when inducing $e = (0, 0)$. 

Electronic copy available at: https://ssrn.com/abstract=3517558
For $\Omega < H + L$,

$$E_{\Omega < H + L}[D|e = (0, 1 - q)] - E_{\Omega < H + L}[D|e = (0, 0)]$$

$$= (1 - q)(p + q) - 2q(1 - q)\Omega + \left((1 - p)(1 - q) - (1 - q)^2\right)2L$$

$$= (1 - q)(p - q)(\Omega - 2L),$$

$$E[S + B|e = (0, 1 - q)] - E[S + B|e = (0, 0)] = (1 - q)\phi.$$

Therefore the principal prefers inducing $e = (0, 1 - q)$ to inducing $e = (0, 0)$, when the incremental demand is larger then the incremental payment, which requires,

$$\left(1 - q\right)(p - q)(\Omega - 2L) > (1 - q)\phi,$$

$$\Omega > 2L + \frac{1}{p - q}\phi.$$

We can see that $2L + \frac{1}{p - q}\phi$ is indeed smaller than $H + L$ since $H - L > \frac{1}{p - q}\phi$.

For $\Omega > H + L$, we can easily see that inducing $e = (0, 0)$ is sub-optimal compared with inducing $e = (0, q)$. The reason is that when $\Omega > H + L$, inducing $e = (0, 0)$ will generate the same profit for the principal as inducing $e = (0, 0)$ while paying a higher expected payment.

The conclusion is that under this scenario, if $\Omega > 2L + \frac{1}{p - q}\phi$, it generates a higher profit for the principal by inducing $e = (0, 1 - q)$ compared with inducing $e = (0, 0)$.

To summarize, given $U - K > \frac{q}{p - q}\phi$, the optimal contract for the principal with limited inventory is,

$$(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } \Omega \in [H + L + \frac{1}{p - q}\phi, 2H), \\
(1, 1 - p), & \text{if } \Omega \in [\omega_4, H + L + \frac{1}{p - q}\phi), \\
(0, 1 - q), & \text{if } \Omega \in [2L + \frac{1}{p - q}\phi, \omega_4), \\
(0, 0), & \text{if } \Omega \in [2L, 2L + \frac{1}{p - q}\phi). 
\end{cases}$$

Case 2: $0 < U - K \leq \frac{q}{p - q}\phi$:

We first compare the principal’s profits when inducing $e = (1, 1)$ and when inducing $e = (1, 1 - p)$.

Since compared with Case 1, inducing $e = (1, 1)$ will only become less attractive relative to inducing $e = (1, 1 - p)$, the cutoff value of $\Omega$ for the principal to be indifferent between the two contracts should be larger than $H + L$. The difference in the expected demands is the same as in Case 1. But the difference in their expected payments to the agent
Next, we compare the principal’s profits when inducing \( e \) profiles the same as in Case 1, given by \( e = (0, 1) \). For \( \Omega^* \), overall, we have that compared with inducing \( e = (1, 1) \), the difference in their expected payments only becomes larger compared with Case 1.

Then the principal prefers inducing \( e = (1, 1) \) to inducing \( e = (1, 1 - p) \), when

\[
p(p - q) \left( \Omega - (H + L) \right) > \frac{p^2}{p - q} \phi - p(U - K),
\]

where \( \omega_1 = H + L + \frac{p}{(p-q)^2} \phi \).

- For \( \Omega < H + L \), again, inducing \( e = (1, 1) \) is dominated by inducing \( e = (1, 1 - p) \) since the difference in their expected payments only becomes larger compared with Case 1.

- Overall, we have that compared with inducing \( e = (1, 1 - p) \), inducing \( e = (1, 1) \) performs better for the principal when \( \Omega > \omega_1 - \frac{U - K}{p - q} \).

Next, we compare the principal’s profits when inducing \( e = (1, 1 - p) \) and when inducing \( e = (0, 1 - q) \). The difference in the expected demands between inducing the above two effort profiles the same as in Case 1, given by \( (p - q) \left( q(\Omega - (H + L)) + (1 - p)(H - L) \right) \).

- For \( \Omega \in [H + L, 2H) \),

\[
E[S + B|e = (1, 1 - p)] - E[S + B|e = (0, 1 - q)]
= (2 - p)K + pU + \frac{2p - p^2}{p - q} \phi - ((1 - q)K + (1 + q)U + \frac{p - pq}{p - q} \phi) - (1 + q - p) (U - K)
= \frac{p - p^2 + pq}{p - q} \phi - (1 + q - p) (U - K).
\]

Then the principal prefers inducing \( e = (1, 1 - p) \) to inducing \( e = (0, 1 - q) \), when the incremental demand is larger then the incremental payment, which requires,

\[
(p - q) \left( q(\Omega - (H + L)) + (1 - p)(H - L) \right) > \frac{p^2}{p - q} \phi - (1 + q - p) (U - K),
\]

where \( \omega_2 = H + L + \frac{1-p}{q(p-q)^2} \phi \). Here, if \( H - L < \frac{q(1-p+q)}{(1-p)(p-q)^2} \phi \), the above cut off value, \( \omega_2 = \frac{1-p}{q(p-q)^2} (U - K) \) is smaller than \( H + L \) for \( \forall U - K \in (0, \frac{q}{p-q} \phi) \).

For \( H - L > \frac{q(1-p+q)}{(1-p)(p-q)^2} \phi \), it exceeds \( H + L \) only when \( U - K < \mu_1 \equiv \frac{p}{p-q} \phi - \frac{(1-p)(p-q)}{1+q-p} (H - L) \), where \( \mu_1 \in (0, \frac{q}{p-q} \phi) \). Thus the condition for \( e = (1, 1 - p) \) to dominate \( e = (0, 1 - q) \) can be further simplified as: if \( H - L < \frac{q(1-p+q)}{(1-p)(p-q)^2} \phi \), we need \( \Omega > H + L \), and if \( H - L > \frac{q(1-p+q)}{(1-p)(p-q)^2} \phi \), we need \( \Omega > \omega_2 - \frac{1-p}{q(p-q)^2} (U - K) \) and \( 0 < U - K < \mu_1 \).
Finally, we compare the principal’s profits when inducing \( e = (0, 1 - q) \) and when inducing \( e = (0, 0) \).

* For \( \Omega \in [2L, H + L] \), the difference in expected demands under the cutoff value is,

\[
E_{2L < \Omega \leq H + L}[D | e = (1, 1 - p)] - E_{2L < \Omega < H+L}[D | e = (0, 1 - q)]
\]
\[
= (1 - p)(p - q)(\Omega - 2L).
\]

Then the principal prefers inducing \( e = (1, 1) \) to inducing \( e = (1, 1 - p) \) when,

\[
(1 - p)(p - q)(\Omega - 2L) > \frac{p - p^2 + pq}{p - q} \phi - \left(1 + q - p\right)(U - K),
\]
\[
\Omega > \omega_2' - \frac{1 + q - p}{(1 - p)(p - q)}(U - K) \quad \text{and} \quad \Omega < H + L,
\]

where \( \omega_2' \equiv 2L + \frac{p - p^2 + pq}{(1 - p)(p - q)} \phi \). Indeed, \( \omega_2' - \frac{1 + q - p}{(1 - p)(p - q)}(U - K) \) and \( \omega_2 - \frac{1 + q - p}{q(p - q)}(U - K) \) intersect at \( \Omega = H + L \).

* Put together, we have the principal prefers inducing \( e = (1, 1) \) to inducing \( e = (1, 1 - p) \) when the following conditions are met:

1. if \( H - L < \frac{q (1 - p + q)}{(1 - p)(p - q)^2} \phi \),

\[
\Omega > \omega_2 - \frac{1 + q - p}{q(p - q)}(U - K) \quad \text{and} \quad 0 < \mu < \mu_1, \quad \text{or}
\]
\[
\Omega > \omega_2' - \frac{1 + q - p}{(1 - p)(p - q)}(U - K) \quad \text{and} \quad \mu_1 < \mu < \frac{q}{p - q} \phi;
\]

2. if \( H - L > \frac{q (1 - p + q)}{(1 - p)(p - q)^2} \phi \),

\[
\Omega > \omega_2' - \frac{1 + q - p}{(1 - p)(p - q)}(U - K) \quad \text{and} \quad 0 < \mu < \frac{q}{p - q} \phi.
\]

These can be written together as

\[
\Omega > \max \left\{ \omega_2 - \frac{1 + q - p}{q(p - q)}(U - K), \omega_2' - \frac{1 + q - p}{(1 - p)(p - q)}(U - K) \right\} \quad \text{and} \quad 0 < \mu < \frac{q}{p - q} \phi.
\]

Finally, we compare the principal’s profits when inducing \( e = (0, 1 - q) \) and when inducing \( e = (0, 0) \).

* For \( \Omega \) smaller than \( H + L \), the difference in the expected demands between inducing the above two effort profiles is given by the same value as in Case 1, \( (1 - q)(p - q)(\Omega - 2L) \).

The difference in the expected payments to the agent between inducing the above two effort profiles is

\[
E_{\Omega}[S + B | e = (0, 1 - q)] - E_{\Omega}[S + B | e = (0, 0)]
\]
\[
= \left( (1 - q)K + (1 + q)U + \frac{p - pq}{p - q} \phi \right) - 2U
\]
\[
= \frac{p - pq}{p - q} \phi - (1 - q)(U - K).
\]

Therefore the principal prefers inducing \( e = (0, 1 - q) \) to inducing \( e = (0, 0) \), when the
incremental demand is larger then the incremental payment, which requires,

\[(1 - q)(p - q)(\Omega - 2L) > \frac{p - q}{p - q} \phi - (1 - q)(U - K),\]

\[\omega_3 - \frac{U - K}{p - q} < \Omega < H + L,\]

where \(\omega_3 = 2L + \frac{p}{(p - q)^2} \phi\). Indeed, we can prove that given \(H - L > \frac{1}{p - q} \phi\), we have \(\omega_3 - \frac{U - K}{p - q} < H + L\).

* For \(\Omega > H + L\), inducing \(e = (0, 0)\) generates the same profit as inducing \(e = (0, 1 - q)\) but paying more thus is dominated by inducing \(e = (0, 0)\).

- Combined together, given \(0 < U - K \leq \frac{q}{p - q} \phi\), the optimal contract for the principal with limited inventory is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } \Omega \in \left[\omega_1 - \frac{U - K}{p - q}, 2H\right], \\
(1, 1 - p), & \text{if } \Omega \in \left[\max\{\omega_2 - \frac{1 + q - p}{q(p - q)}(U - K), \omega_2 - \frac{1 + q - p}{(1 - p)(p - q)}(U - K)\}, \omega_1 - \frac{U - K}{p - q}\right], \\
(0, 1 - q), & \text{if } \Omega \in \left[\omega_3 - \frac{U - K}{p - q}, \max\{\omega_2 - \frac{1 + q - p}{q(p - q)}(U - K), \omega_2 - \frac{1 + q - p}{(1 - p)(p - q)}(U - K)\}\right], \\
(0, 0), & \text{if } \Omega \in \left[2L, \omega_3 - \frac{U - K}{p - q}\right]. 
\end{cases}
\]

- **Case 3: \(U - K \leq 0\):**

  - First, we compare the principal’s profits when inducing \(e = (1, 1)\) and when inducing \(e = (1, 1 - p)\).

    * For \(\Omega > H + L\),

\[
E_{\Omega > H + L}[D|e = (1, 1)] - E_{\Omega > H + L}[D|e = (1, 1 - p)] = (p^2 - pq) \Omega + (2p(1 - p) - p(2 - p - q))(H + L) = p(p - q)\left(\Omega - (H + L)\right),
\]

\[
E[S + B|e = (1, 1)] - E[S + B|e = (1, p)] = \left(2K + \frac{2p}{p - q}\phi\right) - \left(2K + \frac{2p - p^2}{p - q}\phi\right) = \frac{p^2}{p - q}\phi.
\]

Therefore the principal prefers inducing \(e = (1, 1)\) to inducing \(e = (1, 1 - p)\), when the incremental demand is larger then the incremental payment, which requires,

\[
p\left(p - q\right)\left(\Omega - (H + L)\right) > \frac{p^2}{p - q} \phi,
\]

\[\Omega > H + L + \frac{p}{(p - q)^2} \phi = \omega_1.\]
For $\Omega < H + L$, it is easy to see that it generates a higher profit for the principal by inducing $e = (1, 1)$ compared with inducing $e = (1, 1 - p)$. Thus, inducing $e = (1, 1)$ dominates inducing $e = (1, 1 - p)$ when $\Omega > \omega_1$.

Next, we compare the principal’s profits when inducing $e = (1, 1 - p)$ and when inducing $e = (0, 1 - q)$.

For $\Omega$ greater than $H + L$,

$$E_{\Omega > H + L}[D|e = (1, 1 - p)] - E_{\Omega > H + L}[D|e = (0, 1 - q)] = \left( \frac{pq - q^2}{p - q} \right) \Omega + \left( \frac{p(2 - p - q) - (1 - q)(p + q)}{p - q} \right) (H + L)$$

$$= \left( p - q \right) \left( q(\Omega - (H + L)) + (1 - p)(H - L) \right).$$

Next, we compare the principal’s profits when inducing $e = (1, 1 - p)$ and when inducing $e = (0, 1 - q)$.

For $\Omega < H + L$, it generates a higher profit for the principal by inducing $e = (1, 1 - p)$ compared with inducing $e = (0, 1 - q)$ if

$$(p - q)(1 - p)(\Omega - 2L) > \frac{p - p^2 + pq}{p - q} \phi,$$

$$\Omega > 2L + \frac{p - p^2 + pq}{(1 - p)(p - q)^2} \phi = \omega_2'$$

Indeed $\omega_2$ and $\omega_2'$ intersect at $H - L = \frac{q(1 - p + q)}{(1 - p)(p - q)^2} \phi$. Thus inducing $e = (1, 1 - p)$ dominates inducing $e = (0, 1 - q)$ if $\Omega > \max\{\omega_2, \omega_2'\}$.

Finally, we compare the principal’s profits when inducing $e = (0, 1 - q)$ and when inducing $e = (0, 0)$. 

Electronic copy available at: https://ssrn.com/abstract=3517558
* For $\Omega$ smaller than $H + L$,

\[
E_{\Omega < H+L}[D|e = (0, 1 - q)] - E_{\Omega < H+L}[D|e = (0, 0)]
= (1 - q)(p + q - 2q(1 - q))\Omega + \left((1 - p)(1 - q) - (1 - q)^2\right)2L
= (1 - q)(p - q)\left(\Omega - 2L\right),
\]

\[
E[S + B|e = (0, 1 - q)] - E[S + B|e = (0, 0)]
= \left(2K + \frac{p - pq}{p - q}\phi\right) - 2K
= \frac{p - pq}{p - q}\phi.
\]

Therefore the principal prefers inducing $e = (0, 1 - q)$ to inducing $e = (0, 0)$, when the incremental demand is larger than the incremental payment, which requires,

\[
(1 - q)(p - q)\left(\Omega - 2L\right) > \frac{p - pq}{p - q}\phi,
\]

\[
\Omega > 2L + \frac{p}{(p - q)^2}\phi = \omega_3, \text{ and } \Omega < H + L.
\]

We can see that $\omega_3 < H + L$ when $H - L > \frac{p}{(p - q)^2}\phi$.

* For $\Omega > H + L$, it generates a higher profit for the principal by inducing $e = (0, 1 - q)$ compared with inducing $e = (0, 0)$, if

\[
(1 - q)(p - q)(H - L) > \frac{p - pq}{p - q}\phi,
\]

\[
H - L > \frac{p}{(p - q)^2}\phi \text{ and } \Omega > H + L.
\]

* Overall, it generates a higher profit for the principal by inducing $e = (0, 1 - q)$ compared with inducing $e = (0, 0)$ when $\Omega > \omega_3$, with $\omega_3 < H + L$.

– As a summary, given $U - K \leq 0$, the optimal contract for the principal with limited inventory is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } \Omega \in [\omega_1, 2H), \\
(1, 1 - p), & \text{if } \Omega \in [\max\{\omega_2, \omega'_2\}, \omega_1), \\
(0, 1 - q), & \text{if } \Omega \in [\omega_3, \max\{\omega_2, \omega'_2\}], \\
(0, 0), & \text{if } \Omega \in [2L, \omega_3).
\end{cases}
\]

Combined, we have the optimal period-by-period contract for the principal with limited inventory as follows:
OA2  Optimal Two-period Contract with Limited Inventory

- Below, we first rule out the optimality for the principal to induce \( e = (1, p) \) given \( H - L > \frac{p}{p-q} \phi \).

  - When \( \Omega < H + L \), we can observe that inducing \( e = (0, q) \) is dominated by inducing \( e = (0, 0) \) for the principal. This is because, given \( \Omega < H + L \), inducing \( e = (0, q) \) generates the same demand in expectation as inducing \( e = (0, 0) \), while leading to a higher payment in expectation.

  - When \( H + L < \Omega < H + L + \frac{\phi}{p-q} \), inducing \( e = (0, q) \) is still dominated by inducing \( e = (0, 0) \) for the principal. In this scenario, the difference between expected demands between inducing \( e = (0, q) \) and inducing \( e = (0, 0) \) is given by,

    \[
    E_{H+L<\Omega \leq H+L+\frac{\phi}{p-q}}[D|e = (0, q)] - E_{H+L<\Omega \leq H+L+\frac{\phi}{p-q}}[D|e = (0, 0)] = q(p-q)(\Omega - (H + L)),
    \]

    \[
    \leq q(p-q)\left(\frac{\phi}{p-q}\right) = q\phi.
    \]

Meanwhile the difference in their expected payments is,

\[
E[S+B|e = (0, q)] - E[S+B|e = (0, 0)] = \begin{cases} \frac{pq}{p-q} \phi, & \text{if } U - K \leq 0, \\ \frac{pq}{p-q} \phi - 2(U - K), & \text{if } 0 < U - K \leq \frac{q^2}{2(p-q)\phi}, \\ q\phi, & \text{if } U - K > \frac{q^2}{2(p-q)\phi}. \end{cases}
\]

which is at least \( q\phi \) (the minimal value is obtained when \( U - K > \frac{q^2}{2(p-q)\phi} \)).

As we can see, compared with inducing \( e = (0, 0) \), when the principal induces \( e = (0, q) \), the minimal incremental payment is exactly equal to the maximal incremental demand. This
implies that the increase in expected payment cannot be justified by the increase in expected demand, therefore \( e = (0, q) \) is dominated by inducing \( e = (0, 0) \), given \( H + L < \Omega < H + L + \frac{\phi}{p-q} \).

- Finally, when \( \Omega > H + L + \frac{\phi}{p-q} \), inducing \( e = (0, q) \) is still dominated by inducing \( e = (1, p) \) for the principal. With \( \Omega > H + L + \frac{\phi}{p-q} \), the difference in expected demands between inducing \( e = (1, p) \) and inducing \( e = (0, q) \) is given by,

\[
E_{\Omega > H + L + \frac{\phi}{p-q}}[D|e = (1, p)] - E_{\Omega > H + L + \frac{\phi}{p-q}}[D|e = (0, q)]
= (p - q)\left(p(\Omega - (H + L)) + (1 - q)(H - L)\right),
> \left(p - q\right)\left(p\frac{\phi}{p-q} + (1 - q)(H - L)\right)
\]

Meanwhile the difference in their expected payments is no larger than \( p^2 + p - 2pq \frac{\phi}{p-q} \). Below, we show that compared with inducing \( e = (0, q) \), when the principal induces \( e = (1, p) \), even the maximal incremental payment is smaller than the minimal incremental demand in this scenario, thus the increase in demand will more than compensate the increase in payment. The condition for this to hold is,

\[
\left(p - q\right)\left(p\frac{\phi}{p-q} + (1 - q)(H - L)\right) > p^2 + p - 2pq \frac{\phi}{p-q},
\]

i.e., \( H - L > \frac{p}{(p-q)^2} \phi \),

which is the parameter space we focus on. This implies that \( e = (0, q) \) is dominated by inducing \( e = (0, 0) \) when \( \Omega > H + L + \frac{\phi}{p-q} \).

- Put the three cases above together, we reach the conclusion that inducing \( e = (0, q) \) is suboptimal for the principal (in the interesting parameter space where \( H - L > \frac{p}{(p-q)^2} \phi \)).

We now solve the optimal two-period contract with limited inventory by considering \( \Omega \) lying in the following intervals: \( H + L + \frac{1}{p-q} \phi \leq \Omega < 2H \), \( 2L + \frac{1}{p-q} \phi \leq \Omega < H + L + \frac{1}{p-q} \phi \), and when \( \Omega < 2L + \frac{1}{p-q} \phi \). The expected payment for inducing each effort profile remains the same as that in the two-period contracting case with independent periods (see Table A3 for details), and the expected demand for inducing each effort profile remains the same as that in the period-by-period contracting case with dependent periods (see Table OA1 for details).

- **Case 1**: \( H + L + \frac{1}{p-q} \phi \leq \Omega < 2H \).

  - In this case, we first rule out the optimality of inducing \( e = (1, 1 - p) \) for the principal. Indeed, inducing \( e = (1, 1 - p) \) is dominated by inducing \( e = (1, 1) \) in this region. The difference in the expected payment between inducing \( e = (1, 1 - p) \) and inducing \( e = (0, 1 - q) \) is \( p\phi \) regardless
of the value of \( U - K \), i.e., \( \forall U - K \),

\[
E[S + B|e = (1, 1)] - E[S + B|e = (1, 1 - p)],
\]

\[
= \max\left\{ \frac{p^2 + p - pq}{p - q} \phi + 2K, (1 + p)\phi + 2U \right\} - \max\left\{ \frac{p + q^2 - pq}{p - q} \phi + 2K, (1 - q)\phi + 2U \right\}
= p\phi.
\]

The difference in their expected demands is given by,

\[
E_{\Omega \geq H + L + \frac{\phi}{p - q}}[D|e = (1, 1)] - E_{\Omega \geq H + L + \frac{\phi}{p - q}}[D|e = (1, 1 - p)]
= p(p - q)(\Omega - (H + L)),
\]

\[
\geq p(p - q)\frac{\phi}{p - q} = p\phi.
\]

That means, the incremental demand in inducing \( e = (1, 1) \) relative to inducing \( e = (1, 1 - p) \) is no smaller than the incremental payment, therefore \( e = (1, 1 - p) \) is sub-optimal given \( \Omega \geq H + L + \frac{1}{p - q}\phi \).

- Next, we rule out the optimality of inducing \( e = (0, 1 - q) \) for the principal under this scenario, by showing that inducing \( e = (0, 1 - q) \) is dominated by inducing \( e = (1, p) \) in this region.

The difference in the expected demands they generate, given by \((p^2 - q^2)(\Omega - (H + L))\), is no smaller than than the difference in their expected payments to the agent, given by \((p + q)\phi\), when \( \Omega \geq H + L + \frac{1}{p - q}\phi \).

From the analysis above, the possible optimal contracts for the principal are among inducing \( e = (1, 1), e = (1, p), e = (0, 0) \).

- We compare the principal’s profits under \( e = (1, 1) \) and \( e = (1, p) \). We summarize the difference in their expected payments as,

\[
E[S + B|e = (1, 1)] - E[S + B|e = (1, p)]
= \begin{cases} 
\frac{p - p^2 + pq}{p - q} \phi, & \text{if } U - K \leq \frac{q}{2(p - q)} \phi, \\
\frac{p + q - p^2 + pq}{p - q} \phi - 2(U - K), & \text{if } \frac{q}{2(p - q)} \phi < U - K \leq \frac{q}{p - q} \phi, \\
(1 - p)\phi, & \text{if } U - K > \frac{q}{p - q} \phi.
\end{cases}
\]

The incremental demand is \((1 - p)(p - q)(H - L)\) in this scenario. Given \( \Omega \geq H + L + \frac{\phi}{p - q} \), we can show that the principal is indifferent between inducing \( e = (1, 1) \) and inducing \( e = (1, p) \) when \( U - K \) is at \( \mu_3 \) define below. That is,

\[
(1 - p)(p - q)(H - L) = \frac{p + q - p^2 + pq}{p - q} \phi - 2(U - K),
\]

i.e., \( U - K = \frac{1}{2} \left[ \frac{p + q - p^2 + pq}{p - q} \phi - (1 - p)(p - q)(H - L) \right] \equiv \mu_3 \in \left( \frac{q}{2(p - q)} \phi, \frac{q}{p - q} \phi \right) \).

For \( U - K \geq \mu_3 \), the incremental payment becomes smaller thus inducing \( e = (1, 1) \) dominates
inducing \( e = (1, p) \). For \( U - K < \mu_3 \), the incremental payment becomes larger thus inducing \( e = (1, p) \) dominates inducing \( e = (1, 1) \).

We now compare the principal’s profits under \( e = (1, p) \) and \( e = (0, 0) \). The difference in the expected demands between inducing the above two effort profiles is given by,

\[
E_{\Omega \geq H + L + \frac{\phi}{p-q}} [D| e = (1, p)] - E_{\Omega \geq H + L + \frac{\phi}{p-q}} [D| e = (0, 0)]
\]

\[
= (p^2 - q^2) \Omega + \left( (1-p)(p+q) - 2q(1-q) \right)(H + L) - \left( (1-p)(1-q) - (1-q^2) \right)2L
\]

\[
= (p^2 - q^2) \left( \Omega - (H + L) \right) + (p - q) \left( 1 - q \right) \left( H - L \right).
\]

The difference in the expected payments to the agent between inducing the above two effort profiles is,

\[
E[S + B| e = (1, p)] - E[S + B| e = (0, 0)]
\]

\[
= \begin{cases} 
\frac{p^2 + p - pq}{p-q} \phi, & \text{if } U - K \leq 0, \\
\frac{p^2 + p - pq}{p-q} \phi - 2(U - K), & \text{if } 0 < U - K \leq \frac{q}{2(p-q)} \phi, \\
(1 + p) \phi, & \text{if } U - K > \frac{q}{2(p-q)} \phi.
\end{cases}
\]

Therefore the principal prefers inducing \( e = (0, 1 - q) \) to inducing \( e = (0, 0) \), when the incremental demand is larger then the incremental payment, which requires,

1. If \( U - K > 0 \),

\[
(p^2 - q^2) \left( \Omega - (H + L) \right) + (p - q) \left( 1 - q \right) \left( H - L \right) > \frac{p^2 + p - pq}{p-q} \phi,
\]

i.e., \( \Omega > H + L - \frac{1-q}{p+q} (H - L) + \frac{p^2 + p - pq}{(p^2 - q^2)(p-q)} \phi \equiv \omega_5 \);

2. If \( 0 < U - K < \frac{q}{2(p-q)} \phi \):

\[
(p^2 - q^2) \left( \Omega - (H + L) \right) + (p - q) \left( 1 - q \right) \left( H - L \right) > \frac{p^2 + p - pq}{p-q} \phi - 2(U - K),
\]

\[
\Omega > \omega_5 - \frac{2(U-K)}{p^2-q^2}.
\]

We can show that \( \omega_5 - \frac{2(U-K)}{p^2-q^2} > H + L + \frac{\phi}{p-q} \) only when \( U - K < \mu_2 \), with \( \mu_2 \in \left( 0, \frac{q}{2(p-q)} \phi \right) \).

If \( U - K > \frac{q}{2(p-q)} \phi \), inducing \( e = (1, p) \) also dominates inducing \( e = (0, 0) \) since the difference in their expected payments will become smaller as \( U - K \) increases beyond \( \frac{q}{2(p-q)} \phi \).

To summarize, inducing \( e = (1, p) \) dominates inducing \( e = (0, 0) \) when \( \mu_2 < U - K < \mu_3 \), or when \( 0 < U - K < \mu_2 \) and \( \Omega > \omega_5 - \frac{2(U-K)}{p^2-q^2} \), or when \( U - K < 0 \) and \( \Omega > \omega_5 \).

Therefore, given \( H + L + \frac{1}{p-q} \phi \leq \Omega < 2H \), the solution for the optimal contract for the principal is,
Case 2: $2L + \frac{1}{p-q} \phi \leq \Omega < H + L + \frac{1}{p-q} \phi$.

- First, we rule out the optimality of inducing $e = (1, 1)$ under this scenario. Compared with inducing $e = (1, 1 - p)$, the incremental payment for inducing $e = (1, 1)$ is still $\phi$. However, the difference in their expected demands is given by,

$$E_{2L + \frac{1}{p-q} \phi \leq \Omega < H + L + \frac{1}{p-q} \phi}[D|e = (1, 1)] - E_{2L + \frac{1}{p-q} \phi \leq \Omega < H + L + \frac{1}{p-q} \phi}[D|e = (1, 1 - p)]$$

$$\leq p(p-q)(\Omega - (H + L)),$$

$$\leq p(p-q)\frac{\phi}{p-q} = p\phi.$$

That means, the incremental demand in inducing $e = (1, 1)$ relative to inducing $e = (1, 1 - p)$ is no larger than the incremental payment, therefore $e = (1, 1)$ is sub-optimal.

- Next, we rule out the optimality of inducing $e = (1, p)$ for the principal by showing that inducing $e = (1, p)$ is dominated by inducing $e = (0, 1 - q)$ in this region. The maximal difference in the expected demands they generate, given by $(p^2-q^2)\frac{1}{p-q} \phi = (p+.q)\phi$, is the same as the difference in their expected payments to the agent, given by $(p+q)\phi$. Therefore, inducing $e = (1, p)$ is dominated by inducing $e = (0, 1 - q)$ for the principal.

Now we compare the principal’s profits under inducing $e = (1, 1 - p)$, $e = (0, 1 - q)$, $e = (0, 0)$.

- First consider the pair of $e = (1, 1 - p)$ and $e = (0, 1 - q)$.

$$E[S + B|e = (1, 1 - p)] - E[S + B|e = (0, 1 - q)]$$

$$= \begin{cases} 
\frac{p-(p-q)^2}{p-q} \phi, & \text{if } U - K \leq \frac{q}{2(p-q)} \phi, \\
\frac{p+q-(p-q)^2}{p-q} \phi - 2(U - K), & \text{if } \frac{q}{2(p-q)} \phi < U - K \leq \frac{q}{p-q} \phi, \\
(1 - p + q)\phi, & \text{if } U - K > \frac{q}{p-q} \phi.
\end{cases}$$

$$E[D|e = (1, 1 - p)] - E[D|e = (0, 1 - q)]$$

$$= \begin{cases} 
(p-q)\left(q(\Omega - (H + L)) + (1-p)(H - L)\right), & \text{if } \Omega > H + L, \\
(1-p)\left(p-q\right)(\Omega - 2L), & \text{if } \Omega < H + L.
\end{cases}$$

We find the boundary conditions where the principal is indifferent between inducing $e = (0, 1 - q)$ and inducing $e = (1, 1 - p)$. The result is that,
* Consider the case when \( U - K > \frac{q}{p-q} \phi \). At \( \Omega = 2L + \frac{1-p+q}{(1-p)(p-q)} \phi = \omega_4 \), inducing \( e = (0, 1-q) \) and inducing \( e = (1, 1-p) \) generate the same profits for the principal. Here, \( \omega_4 \) is the same as we defined in Appendix OA1, and it is smaller than \( H + L \). As such, the difference in their expected demands \( (1-p)(p-q)(\Omega - 2L) = (1-p+q) \phi \) is equal to the difference in their expected payments which is also given by \( (1-p+q) \phi \). Following this, given \( U - K > \frac{q}{p-q} \phi \), for \( \Omega > \omega_4 \), inducing \( e = (1, 1-p) \) dominates inducing \( e = (0, 1-q) \) and for \( \Omega < \omega_4 \), it is the opposite.

* Then consider the case when \( U - K \leq \mu_3 \). Indeed, the above two cutoff values on \( \Omega \) intersect at \( \Omega = \mu_3 \), inducing \( e = (1, 1-p) \) and \( e = (0, 1-q) \) generate the same profits for the principal if the following condition is met,

\[
(p-q) \left( q(\Omega - (H + L)) + (1-p)(H - L) \right) = \frac{p+q-(p-q)^2}{p-q} \phi - 2(U - K),
\]

\[i.e., \Omega = H + L - \frac{1-p}{q} (H - L) + \frac{p+q-(p-q)^2}{q(p-q)^2} \phi - \frac{2(U-K)}{q(p-q)}.\]

Denote \( \omega_6 = H + L + \frac{1-p}{q} (H - L) + \frac{p+q-(p-q)^2}{q(p-q)^2} \phi \), then the boundary condition for this case is \( \Omega = \omega_6 - \frac{2(U-K)}{q(p-q)} \). Notice that \( \omega_6 - \frac{2(U-K)}{q(p-q)} \) intersects with \( \Omega = H + L + \frac{\phi}{p-q} \) at \( U - K = \mu_3 \), which aligns with our observation.

* For the case when \( \mu_3 < U - K < \frac{q}{p-q} \phi \) and \( \Omega > H + L \), inducing \( e = (0, 1-q) \) and inducing \( e = (1, 1-p) \) generate the same profits for the principal when,

\[
(1-p) \left( p - q \right) \left( \Omega - 2L \right) = \frac{p+q-(p-q)^2}{p-q} \phi - 2(U - K),
\]

\[i.e., \Omega = 2L + \frac{p+q-(p-q)^2}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)}.\]

Denote \( \omega_6' = 2L + \frac{p+q-(p-q)^2}{(1-p)(p-q)} \phi \), then the boundary condition for this case is \( \Omega = \omega_6' - \frac{2(U-K)}{(1-p)(p-q)}. \)

* Indeed, the above two cutoff values on \( \Omega \) intersect at \( \Omega = H + L \), therefore, when \( \mu_2 < U - K < \frac{q}{p-q} \phi \), the principal is indifferent between inducing the two effort profiles when \( \Omega = \max \left\{ \omega_6 - \frac{2(U-K)}{(1-p)(p-q)}, \omega_6' - \frac{2(U-K)}{q(p-q)} \right\} \).

Combined together, under this scenario, we have inducing \( e = (1, 1-p) \) dominates inducing \( e = (0, 1-q) \) when \( \omega_4 \leq \Omega < H + L + \frac{1}{p-q} \phi, U - K > \frac{q}{p-q} \phi \), or when \( \max \left\{ \omega_6 - \frac{2(U-K)}{(1-p)(p-q)}, \omega_6' - \frac{2(U-K)}{q(p-q)} \right\} \leq \Omega < H + L + \frac{1}{p-q} \phi, \mu_3 < U - K < \frac{q}{p-q} \phi \).

- Now consider the pair of \( e = (0, 1-q) \) and \( e = (0, 0) \). The difference in the expected payments
to the agent between inducing the above two effort profiles is

\[ E[S + B|e = (0, 1 - q)] - E[S + B|e = (0, 0)] = \begin{cases} 
\frac{p+q^2-pq}{p-q} \phi, & \text{if } U - K \leq 0, \\
\frac{p+q^2-pq}{p-q} \phi - 2(U - K), & \text{if } 0 < U - K \leq \frac{q}{2(p-q)} \phi, \\
(1 - q)\phi, & \text{if } U - K > \frac{q}{2(p-q)} \phi.
\end{cases} \]

The difference in the expected demands between inducing the above two effort profiles is given by,

\[ E[D|e = (0, 1 - q)] - E[D|e = (0, 0)] = \begin{cases} 
(1 - q)(p - q)(H - L), & \text{if } \Omega > H + L, \\
(1 - q)(p - q)(\Omega - 2L), & \text{if } \Omega < H + L.
\end{cases} \]

We find boundary condition where the principal is indifferent between inducing \( e = (0, 1 - q) \) and inducing \( e = (0, 0) \). The result is as follows.

* Consider the case when \( U - K > \frac{q}{2(p-q)} \phi \). In this case, at \( \Omega = 2L + \frac{1}{p-q} \phi \) (which is smaller than \( H + L \)), inducing \( e = (0, 1 - q) \) and inducing \( e = (0, 0) \) generate the same profits for the principal. As such, the difference in their expected demands \( (1 - q)(p - q)(\Omega - 2L) = (1 - q)\phi \) is equal to the difference in their expected payments which is also given by \( (1 - q)\phi \). Following this, given \( U - K > \frac{q}{2(p-q)} \phi \), inducing \( e = (0, 1 - q) \) dominates inducing \( e = (0, 0) \) for \( \Omega > 2L + \frac{\phi}{p-q} \), and inducing \( e = (0, 1 - q) \) is dominated by inducing \( e = (0, 0) \) for \( \Omega < 2L + \frac{\phi}{p-q} \).

* When \( U - K < \mu_2 \), inducing \( e = (0, 0) \) dominates inducing \( e = (0, 1 - q) \). This is because at \( \Omega = H + L + \frac{\phi}{p-q} \) and \( U - K = \mu_2 \), the principal is indifferent between inducing \( e = (1, p) \) and inducing \( e = (0, 1 - q) \). According to Case 1, at this point, he is indifferent between inducing \( e = (1, p) \) and inducing \( e = (0, 0) \), therefore, he is also indifferent between inducing \( e = (0, 1 - q) \) and inducing \( e = (0, 0) \). To the southwestern corner of this point, as the inventory level becomes smaller the difference in their expected demands becomes smaller. In addition, as \( U - K \) becomes smaller, the difference in their expected demands becomes larger. Accordingly, inducing \( e = (0, 1 - q) \) will perform worse relative to inducing \( e = (0, 0) \). As a result, we have inducing \( e = (0, 1 - q) \) is dominated by inducing \( e = (0, 0) \) when \( U - K < \mu_2 \) and when \( \Omega < H + L + \frac{\phi}{p-q} \).

* When \( \mu_2 < U - K < \frac{q}{2(p-q)} \phi \), for the case when \( \Omega < H + L \), inducing \( e = (0, 1 - q) \) and inducing \( e = (0, 0) \) generate the same profits for the principal when,

\[ (1 - q)(p - q)(\Omega - 2L) = \frac{p+q^2-pq}{p-q} \phi - 2(U - K), \]

i.e., \( \Omega = 2L + \frac{p+q^2-pq}{(1-q)(p-q)^2} \phi - \frac{2(U-K)}{(1-q)(p-q)} \).

Denote \( \omega_7 = 2L + \frac{p+q^2-pq}{(1-q)(p-q)^2} \phi \), then the boundary condition for this case is \( \Omega = \omega_7 - \).
Indeed, we can show that the above cutoff value on $\Omega$ is equal to $H + L$ at $U - K = \mu_2$, which makes sense since at $\Omega = H + L$ and $\mu_2$, the principal is indifferent between inducing the two effort profiles. Therefore, when $\mu_2 < U - K < \frac{q}{p-q}\phi$, the principal is indifferent between inducing the two effort profiles when $\Omega = \max\{\omega_5 - \frac{2(U-K)}{(1-q)(p-q)}, \omega_5' - \frac{2(U-K)}{q(p-q)}\}$.

* To summarize, inducing $(0, 1 - q)$ dominates inducing $e = (0, 0)$ for the principal when

$$2L + \frac{1}{p-q}\phi \leq \Omega < \max\{\omega_6 - \frac{2(U-K)}{(1-p)(p-q)}, \omega_6' - \frac{2(U-K)}{q(p-q)}, H + L + \frac{1}{p-q}\phi\}, U - K > \frac{q}{2(p-q)}\phi$$

or when $\omega_7 - \frac{2(U-K)}{(1-q)(p-q)} \leq \Omega < H + L + \frac{1}{p-q}\phi, \mu_2 < U - K < \frac{q}{2(p-q)}\phi$.

**Case 3:** $\Omega < 2L + \frac{1}{p-q}\phi$. Under this scenario, it is easy to see that the principal’s optimal strategy is not to induce effort, i.e., $e = (0, 0)$.

In summary, the overall solution to the optimal two-period contract under limited inventory for the principal is as follows:

$$(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H + L + \frac{1}{p-q}\phi \leq \Omega < 2H, U - K > \mu_3, \\
(1, p), & \text{if } H + L + \frac{1}{p-q}\phi \leq \Omega < 2H, \mu_2 \leq U - K < \mu_3, \text{ or}
\text{ if } \omega_5 - \frac{2(U-K)}{p^2-q^2} \leq \Omega < 2H, 0 \leq U - K < \mu_2, \text{ or}
\text{ if } \omega_5 - \frac{2(U-K)}{p^2-q^2} \leq \Omega < 2H, \leq U - K < 0,
(1, 1 - p), & \text{if } \omega_4 \leq \Omega < H + L + \frac{1}{p-q}\phi, U - K > \frac{q}{p-q}\phi, \text{ or}
\text{ if } \max\{\omega_6 - \frac{2(U-K)}{q(p-q)}, \omega_6' - \frac{2(U-K)}{(1-p)(p-q)}, H + L + \frac{1}{p-q}\phi\}, U - K > \frac{q}{2(p-q)}\phi,
(0, 1 - q), & \text{if } 2L + \frac{1}{p-q}\phi \leq \Omega < \max\{\omega_6 - \frac{2(U-K)}{q(p-q)}, \omega_6' - \frac{2(U-K)}{(1-p)(p-q)}, H + L + \frac{1}{p-q}\phi\}, U - K > \frac{q}{2(p-q)}\phi,
(0, 0), & \text{if } 2L + \frac{1}{p-q}\phi \leq \Omega < \max\{\omega_6 - \frac{2(U-K)}{q(p-q)}, \omega_6' - \frac{2(U-K)}{(1-p)(p-q)}, H + L + \frac{1}{p-q}\phi\}, U - K > \frac{q}{2(p-q)}\phi, \text{ or}
\text{ if } 2L \leq \omega_5 - \omega_7 \frac{2(U-K)}{p^2-q^2}, 0 \leq U - K < \mu_2, \text{ or}
\text{ if } 2L \leq \omega_5 - \omega_7, U - K < 0.
\end{cases}$$

**OA3 Comparison between Two-Period Contract and Period-by-Period Contract with Limited Inventory**

We prove the results with the aid of Figure 6(b), which provides a detailed comparison between the period-by-period and the two-period contract with dependent periods. We will show that (1) the period-by-period contract performs the same as the two-period contract in Region I and Region II, (2) the period-by-period contract performs better than the two-period contract in Region III and Region IV and, and (3) the period-by-period contract performs worse than the two-period contract in Region V and Region VI.

**First,** we prove that the period-by-period contract performs the same as the two-period contract in Region I and Region II.
– In Region I, the principal induces \( e = (0, 0) \) under both the period-by-period contract and the two-period contract. The principal gets the profits regardless which contract he adopts.

– In Region II, the principal induces \( e = (1, 1) \) under both the period-by-period contract and the two-period contract. The principal gets the same profits regardless which contract he adopts.

• We then prove that the period-by-period contract performs better than the two-period contract in Region III. We start our proof by establishing the boundary condition where the principal is indifferent between the two-period contract and the period-by-period contract in equilibrium.

\[ l_1 : U - K \leq 0, \Omega = \omega_8 \equiv H + L + \frac{p^2}{(p+q)(p-q)^2}\phi. \]

* Under this case, the principal induces \( e = (0, 1 - q) \) under the two-period contract, but he induces \( e = (1, 1) \) under the period-by-period contract. With \( \Omega = H + L + \frac{p^2}{(p+q)(p-q)^2}\phi > H + L \), the difference in their expected demands are

\[
E_{\text{period-by-period}}[D|e = (0, 1 - q)] - E_{\text{two-period}}[D|e = (1, p)] \\
= -\left(p^2 - q^2\right)\left(\Omega - (H + L)\right), \\
= -\frac{p^2}{p - q}\phi.
\]

The difference in their expected payments is given by,

\[
E_{\text{period-by-period}}[S+B|e = (0, 1 - q)] - E_{\text{two-period}}[S+B|e = (1, p)] \\
= \left(\frac{2K + \frac{p - pq}{p - q}\phi}{p - q}\right) - \left(\frac{2K + \frac{p^2 + p - pq}{p - q}\phi}{p - q}\right), \\
= -\frac{p^2}{p - q}\phi,
\]

which is equal to the difference in expected payments above.

* As \( \Omega \) decreases from \( \omega_8 \), the two-period contract inducing \((1, p)\) still generates higher demand relative to the period-by-period contract inducing \((0, q)\), however, the difference between the two demand levels decreases as \( \Omega \) decreases from \( \omega_8 \). As a result, in equilibrium, the period-by-period contract performs better than the two-period contract below \( l_1 \).

\[ l_2 : 0 < U - K < \frac{q^2}{(1+q)(p-q)}\phi \text{ and } \Omega = \omega_8 - \frac{1+q}{p^2-q^2}(U - K). \]

* Under this case, the principal induces \( e = (0, 1 - q) \) under the two-period contract, but induces \( e = (1, 1) \) under the period-by-period contract. With \( \Omega = H + L + \frac{p^2}{(p+q)(p-q)^2}\phi > \)

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\( H + L \), the difference in their expected demands is
\[
E_{\text{period-by-period}}[D|e = (0, 1 - q)] - E_{\text{two-period}}[D|e = (1, p)]
= -(p^2 - q^2)\left(\Omega - (H + L)\right),
= (1 + q)(U - K) - \frac{p^2}{p - q}\phi.
\]

The difference in their expected payments is given by,
\[
E_{\text{period-by-period}}[S + B|e = (0, 1 - q)] - E_{\text{two-period}}[S + B|e = (1, p)]
= \left( (1 - q)K + (1 + q)U + \frac{p - pq}{p - q}\phi \right) - \left( 2K + \frac{p^2 + p - pq}{p - q}\phi \right),
= (1 + q)(U - K) - \frac{q^2}{p - q}\phi,
\]
which is equal to the difference in expected payments above.

- To the lower-left of \( l_2 \), as \( \Omega \) decreases, the two-period contract inducing \((1, p)\) still generates higher demand relative to the period-by-period contract inducing \((0, 1 - q)\), however, the difference between the two demand levels decreases. In addition, as \( U - K \) decreases, the two-period contract inducing \((1, p)\) pays the agent more relative to the period-by-period contract inducing \((0, 1 - q)\), and the difference between the two payments increases. As a result, in equilibrium, the period-by-period contract performs better than the two-period contract to the lower-left of \( l_2 \).

- \( l_3 : U - K = \frac{q^2}{(1+q)(p-q)}\phi, \forall \Omega \in \left[ 2L + \frac{1}{p-q}, (p - \frac{q^2}{1+q})\phi, H + L + \phi \right] \).
- In this case, both contracts, if optimally chosen, will induce \( e = (0, 1 - q) \). Under the period-by-period contract, the expected payment to the agent is given by,
\[
E_{\text{period-by-period}}[S + B|e = (0, 1 - q)] - E_{\text{two-period}}[S + B|e = (0, 1 - q)]
= \left( (1 - q)K + (1 + q)U + \frac{p(1 - q)}{p - q}\phi \right) - \left( 2K + \frac{p^2 + p - pq}{p - q}\phi \right),
= (1 + q)(U - K) - \frac{q^2}{p - q}\phi.
\]
Thus the expected payments to the agent are the same under the period-by-period contract and the two-period contract when \( U - K = \frac{q^2}{(1+q)(p-q)} < \frac{q}{2} \). In addition, both contracts induce \( e = (0, 1 - q) \) at optimal, thus generate the same expected demands. Together, we have that the principal’s equilibrium profits stay the same under the two types of contracts.

- To the left of \( l_3 \), as \( U - K \) decreases, the two-period contract inducing \((0, 1 - q)\) pays the agent more relative to the period-by-period contract inducing \((0, 1 - q)\), and the difference between the two salaries increases. As a result, in equilibrium, the period-by-period contract performs better than the two-period contract to the left of \( l_3 \).
- $U - K \leq 0, \Omega \in [2L, \omega_3]$ ; $0 < U - K \leq \frac{q^2}{(1 + q)(p - q)}\phi, \Omega \in [2L, \omega_3 - \frac{U - K}{1 - U - K}],$

* This is the boundary condition where the principal is indifferent between inducing $e = (0, 0)$ and inducing $e = (0, 1 - q)$ under the period-by-period contract. Also, at this area, the principal induces $e = (0, 0)$ under the two-period contract. Therefore, the two types of contract perform the same for the principal.

* Above of this line, as $\Omega$ increases, the principal gets a higher profit by inducing $e = (0, 1 - q)$ under the period-by-period contract compared with inducing $e = (0, 0)$, but he gets the same profit under the two-period contract by inducing $e = (0, 0)$. As a result, the period-by-period contract performs better than the two-period contract in equilibrium.

• Similarly, we characterize the boundary condition of Region IV where the principal also prefers the period-by-period contract to the two-period contract.

- $l_4 : U - K = \frac{q^2}{p - q}\phi, \Omega \in [\omega_4, H + L + \frac{\phi}{p - q}],$ the principal is indifferent from the optimally chosen period-by-period contract and the optimally chosen two-period contract.

* Under both types of contracts, the principal induces $e = (1, 1 - p)$ and pays $2U + (2 - p)\phi$.

* To the left of $l_2$, as $U - K$ decreases, the two-period contract still induces $e = (1, 1 - p)$, however the period-by-period contract pays the agent less relative to the two-period contract, therefore, is preferred by the principal.

- $l_5 : U - K \in [\omega_4, H + L]$ and $\Omega = \omega_9 = \frac{2 - p}{(1 - p)(p - q)}(U - K)$, where $\omega_9 \equiv 2L + \frac{p + q + pq - p^2 - q^2}{(1 - p)(p - q)^2}$.

* Under this case, the principal again induces $e = (0, 1 - q)$ under the two-period contract, but he induces $e = (1, 1 - p)$ under the period-by-period contract. With $\Omega = 2L + \frac{p + q + pq - p^2 - q^2}{(1 - p)(p - q)^2} - \frac{2 - p}{(1 - p)(p - q)}(U - K) < H + L$, the difference in their expected demands is

$$E_{\text{period-by-period}}[D|e = (1, 1 - p)] - E_{\text{two-period}}[D|e = (0, 1 - q)]$$

$$= (1 - p)(p - q)(\Omega - 2L),$$

$$= - (2 - p)(U - K) + \frac{p + q + pq - p^2 - q^2}{p - q}\phi.$$ 

The difference in their expected payments is again given by,

$$E_{\text{period-by-period}}[S + B|e = (1, 1 - p)] - E_{\text{two-period}}[S + B|e = (0, 1 - q)]$$

$$= (2 - p)K + pU + \frac{2p - p^2}{p - q}\phi - (2U + (1 - q)\phi),$$

$$= - (2 - p)(U - K) + \frac{p + q + pq - p^2 - q^2}{p - q}\phi,$$

which is equal to the difference in expected payments above.

* To the upper-right of $l_5$, as $\Omega$ increases, the two-period contract inducing $(0, 1 - q)$ still generates less demand relative to the period-by-period contract inducing $(1, 1 - p)$, and the difference between the two demand levels increases. In addition, as $U - K$ increases, the
two-period contract inducing \((0, 1 - q)\) pays the agent less relative to the period-by-period contract inducing \((1, 1 - p)\), but the difference between the two payments decreases. As a result, in equilibrium, the period-by-period contract performs better than the two-period contract to the upper-right of \(l_5\).

\[
-l_6 : \Omega \in \left[ H + L, H + L + \frac{\phi}{p - q} \right] \text{ and } \Omega = \omega' - \frac{(2-p)(U-K)}{q(p-q)} , \text{ where } \omega' = H + L - \frac{1-p}{q}(H-L) + \frac{p+q+pq-p^2-q^2}{q(p-q)} \phi. 
\]

* Under this case, the principal induces \(e = (0, 1 - q)\) under the two-period contract, but induces \(e = (1, 1 - p)\) under the period-by-period contract. With \(\Omega = H + L - \frac{1-p}{q}(H-L) + \frac{p+q+pq-p^2-q^2}{q(p-q)} \phi - \frac{(2-p)(U-K)}{q(p-q)}\), the difference in their expected demands is

\[
E_{\text{period-by-period}}[D|e = (1, 1 - p)] - E_{\text{two-period}}[D|e = (0, 1 - q)] = q(p-q) \left( \Omega - (H + L) \right) + \left( 1 - p \right)(p-q) \left( H - L \right),
\]

\[
= - (2-p)(U-K) + \frac{p+q+pq-p^2-q^2}{p-q} \phi.
\]

The difference in their expected payments is given by,

\[
E_{\text{period-by-period}}[S + B|e = (1, 1 - p)] - E_{\text{two-period}}[S + B|e = (0, 1 - q)] = \left( 2 - p \right)K + pU + \frac{2p-p^2}{p-q} \phi - \left( 2U + (1-q) \phi \right),
\]

\[
= - (2-p)(U-K) + \frac{p+q+pq-p^2-q^2}{p-q} \phi,
\]

which is equal to the difference in expected payments above.

* To the upper-right of \(l_6\), the period-by-period contract performs better than the two-period contract due to the same reason as the case above.

\[
-l_7 : \Omega > H + L + \frac{\phi}{p - q}, \mu < \mu_3, \Omega = \omega_{10} + \frac{2-p}{\mu(p-q)}(U-K), \text{ where } \omega_{10} = H + L + \frac{1-p}{p}(H-L) - \frac{p+q+pq-2p^2}{p(p-q)} \phi.
\]

* Under this case, the principal again induces \(e = (1, p)\) under the two-period contract, and induces \(e = (1, 1 - p)\) under the period-by-period contract. With \(\Omega = H + L + \frac{1-p}{p}(H-L) - \frac{p+q+pq-2p^2}{p(p-q)} \phi + \frac{2-p}{p(p-q)}(U-K)\), the difference in their expected demands is

\[
E_{\text{period-by-period}}[D|e = (1, 1 - p)] - E_{\text{two-period}}[D|e = (1,p)] = p \left( q - p \right) \left( \Omega - (H + L) \right) - \left( 1 - p \right)(q-p) \left( H - L \right),
\]

\[
= - (2-p)(U-K) + \frac{p+q+pq-2p^2}{p-q} \phi.
\]
The difference in their expected payments is again given by,

\[ E_{\text{period-by-period}}[S + B|e = (1, 1 - p)] - E_{\text{two-period}}[S + B|e = (1, p)] \]

\[ = \left( (2 - p)K + pU + \frac{2p - p^2}{p - q} \phi \right) - \left( 2U + (1 + p)\phi \right), \]

\[ = - (2 - p)(U - K) + \frac{p + q + pq - 2p^2}{p - q} \phi, \]

which is equal to the difference in expected payments above.

* To the lower-right of \( l_7 \), relative to the two-period contract inducing \( e = (1, p) \), the period-by-period contract inducing \( (1, 1 - p) \) induces more demand as \( \Omega \) decreases. In addition, it pays lower as \( U - K \) increases. As a result, in equilibrium, the period-by-period contract performs better than two-period contract to the lower-right of \( l_7 \).

\[ l_8 : \Omega > H + L + \frac{p}{p - q}, \mu_3 < U - K < \frac{q}{p - q} \phi, \Omega = \omega_1 - \frac{1}{p - q}(U - K). \]

* Under this case, the principal again induces \( e = (1, 1) \) under the two-period contract, and induces \( e = (1, 1 - p) \) under the period-by-period contract. With \( \Omega = H + L + \frac{p}{(p - q)^2} \phi - \frac{1}{p - q}(U - K) \), the difference in their expected demands is

\[ E_{\text{period-by-period}}[D|e = (1, 1 - p)] - E_{\text{two-period}}[D|e = (1, 1)] \]

\[ = - p \left( p - q \right) \left( \Omega - (H + L) \right), \]

\[ = p(U - K) - \frac{p^2}{p - q} \phi. \]

The difference in their expected payments is again given by,

\[ E_{\text{period-by-period}}[S + B|e = (1, 1 - p)] - E_{\text{two-period}}[S + B|e = (1, 1)] \]

\[ = \left( (2 - p)K + pU + \frac{2p - p^2}{p - q} \phi \right) - \left( 2K + 2 \frac{p}{p - q} \phi \right), \]

\[ = p(U - K) - \frac{p^2}{p - q} \phi, \]

which is equal to the difference in expected payments above.

* To the lower-left of \( l_8 \), as \( \Omega \) decreases, the two-period contract inducing \( (1, 1) \) still generates higher demand relative to the period-by-period contract inducing \( (1, 1 - p) \), but the difference between the two demand levels decreases. In addition, as \( U - K \) decreases, the two-period contract pays the agent less relative to the period-by-period contract, and the difference between the two payments decreases. As a result, in equilibrium, the period-by-period contract performs better than two-period contract to the lower-left of \( l_8 \).

• Now we prove that the period-by-period contract performs worse than the two-period contract in Region V and Region VI.
In Region B, the principal induces $e = (0, 1 - q)$ under the two-period contract, and induces either $e = (0, 1 - q)$ or $e = (0, 0)$. In the parameter space $e = (0, 1 - q)$ is induced under the period-by-period contract, the two-period contract performs better the period-by-period contract. This is because Region V moves away from $l_3$ in the opposite direction as Region III does, thus we also get the opposite conclusion that the two-period contract performs better than the period-by-period contract to the right of $l_3$. Also, Region V and Region IV move away from $l_5$ and $l_6$ in the opposite directions, thus we get that the two-period contract performs better than the period-by-period contract to the lower-left of $l_5$ and $l_6$.

In the parameter space where $e = (0, 0)$ is induced under the period-by-period contract, the two-period contract performs better since the principal’s profit will be higher when inducing $e = (0, 1 - q)$ under the two-period contract.

In region VI, the principal induces $e = (1, p)$ under the two-period contract, and induces either $e = (0, 1 - q)$ or $e = (1, 1)$ under the period-by-period contract. Region VI and Region III move away from $l_1$ and $l_2$ in the opposite directions, therefore, we will get the result that the two-period contract performs better than the period-by-period contract above $l_1$ and to the upper-right of $l_6$. Additionally, Region VI and Region IV move away from $l_7$ in the opposite directions, therefore, we get that the two-period contract performs better than the period-by-period contract to the upper-left $l_7$. 

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