

Token-based Platform Finance*

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Abstract

We develop a dynamic model of platform economy where tokens serve as a means of payments among platform users and are issued to finance investment in platform productivity. Tokens are optimally issued to reward platform owners when the productivity-normalized token supply is low and burnt to boost the franchise value when the productivity-normalized supply is high. Although token price is determined in a liquid market, the platform's financial constraint generates an endogenous token issuance cost, causing underinvestment through the conflict of interest between insiders (platform owners) and outsiders (users). Blockchain technology mitigates underinvestment by addressing the platform's time-inconsistency problem.

Keywords: Blockchain, Cryptocurrency, Dynamic Corporate Financing, Durable Goods, Gig Economy, Optimal Token Supply, Time Inconsistency, Token/Coin Offering.

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1 Introduction

Digital platforms are reshaping the organization of economic activities. Traditional platforms rely heavily on payment innovations to stimulate economic exchanges among users. Recently, blockchain technology offers alternative solutions. A new generation of digital platforms introduce crypto-tokens as local currencies and allow the use of smart contracts to facilitate transactions among users as well as the financing of ongoing platform development.

We develop a dynamic model of a digital platform to analyze the equilibrium dynamics of token-based communities. In our model, transactions on the platform are settled with native tokens. Users demand tokens as a means of payment, and their holdings are exposed to the fluctuation of token price. The entrepreneur (representing platform owners) manages token issuance by solving a dynamic problem of token-based payout and token-financed investment in platform productivity. The token market-clearing condition determines the evolution of token price. The ratio of token supply to platform productivity (“normalized token supply”) is the key state variable. When it is high (i.e., the system is “inflated”), the platform cuts back investment and refrains from payout. To reduce token supply and boost token price, the platform may find it optimal to buy back tokens, but doing so requires costly external funds.¹ The financing cost of token buyback is the key friction that causes underinvestment in productivity. Our analysis applies to both traditional and blockchain-based platforms.

Our model delivers several unique insights on the economics of tokens and platforms. First, tokens are akin to durable goods but defy Coase’s conjecture (Coase, 1972). While the marginal cost of producing digital tokens is zero, the entrepreneur refrains from over-supply, and the equilibrium token price is positive. In contrast to the stationary demand in durable goods models (Stokey, 1981; Bulow, 1982), token demand grows endogenously as the platform invests in its productivity.² Under a growing token demand, the entrepreneur optimally spreads out her token payouts over time, trading off between milking the system now or in the future. The entrepreneur maintains balanced growth of token supply and productivity. Specifically, the productivity-normalized token supply is endogenously bounded.

Second, underinvestment arises from the conflict of interest between the entrepreneur and platform users. When tokens are issued to finance investment, token supply increases

¹Buying back and burning tokens means sending them to a public “eater address” from which they can never be retrieved because the address key is unobtainable. Practitioners often burn tokens to boost token price and reward token holders (e.g., Binance and Ripple).

²The platform network effect—the positive externality of one user’s adoption on other users—amplifies the demand growth in a mechanism that reminisces the knowledge spillover effect in Romer (1986).

but the investment outcome is random. If productivity improves, both the entrepreneur and users benefit; otherwise, the users are free to reduce token holdings or even abandon the platform, while the entrepreneur, now facing an inflated system, may have to raise costly external funds to buy back tokens. Such asymmetry dampens the entrepreneur’s incentive to invest. The underinvestment in turn reduces user welfare, the equilibrium token price, and eventually, the entrepreneur’s value from token payouts.

The root of the underinvestment problem is the entrepreneur’s time inconsistency. If the entrepreneur is able to commit against underinvestment, the users would demand more tokens, which then increases the token price and the value of entrepreneur’s token payouts. However, time inconsistency arises as the predetermined level of investment (optimal *ex ante*) can be deemed suboptimal *ex post* as the conflict of interest arises between the entrepreneur and platform users. Blockchain technology enables commitment to predetermined rules of investment and can thus add value by addressing time inconsistency. Our paper is among the first to show the value of commitment brought by blockchains.³ That said, we recognize that in practice, blockchain commitment is far from perfect, and that is why it is important to consider both predetermined and discretionary token supply (our baseline case).⁴

We focus on analyzing tokens as monetary assets that facilitate transactions in a fully dynamic setting rather than tokens as dividend-paying assets and their difference from traditional securities (e.g., Gryglewicz, Mayer, and Morellec, 2020). Our paper builds upon Cong, Li, and Wang (2020) (henceforth CLW). While CLW assume a fixed token supply (standard in the literature), in this paper, we analyze the optimal token supply and explore new questions on the dynamics of platform investment and financing, the conflict of interest between the entrepreneur and users, and the role of blockchain technology in platform economics.

Next, we further elaborate on our model setup and mechanisms. Users hold tokens as a means of payment on the platform, enjoying convenience yield that increases in platform productivity. Intuitively, the more productive a platform is, the more activities (and transactions) it supports. To capture the network effect, a distinguishing feature of platform

³In a dynamic system where a large player interacts with a group of small players, the small players respond to the large player’s discretionary action. As a result, the large player’s decision-making environment changes in response to her own action. The large player may obtain a higher value by forgoing discretion and committing to a plan. This mechanism lies behind studies on the commitment of monetary and fiscal policies (Fischer, 1980; Kydland and Prescott, 1980; Lucas and Stokey, 1983; Barro and Gordon, 1983; Ljungqvist and Sargent, 2004) and, more recently, optimal corporate capital structure (DeMarzo and He, 2020).

⁴While some platforms restrict the maximum of token supply, such caps are typically large, leaving the gradual release of token reserves under the discretion of platform designers. Moreover, platform designers are often entitled to a significant fraction of total allocation, through which they can influence token supply despite various vesting schemes. Appendix A illustrates the discretionary allocation with examples.

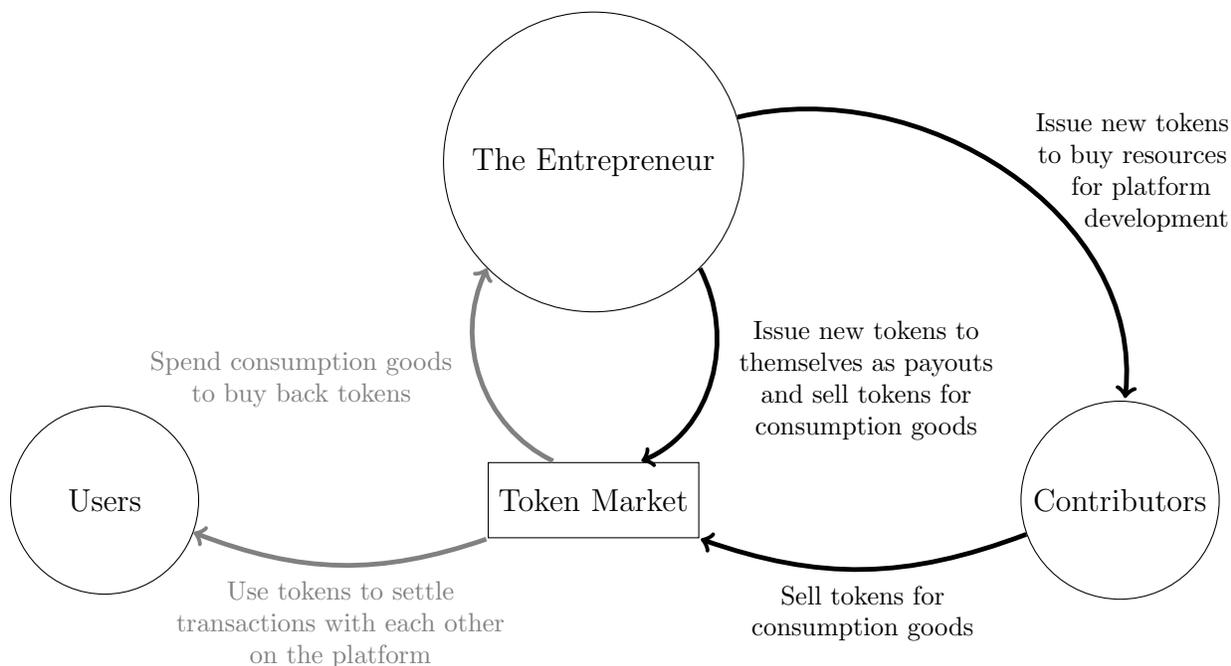


Figure 1: **Token Ecosystem.** The black and gray arrows represent token supply and demand, respectively.

businesses, we allow the convenience yield to depend on the number of users. The user base evolves endogenously for two reasons. First, the stochastic growth of productivity directly affects adoption. Second, users’ expectation of future token price varies over time. An intertemporal complementarity amplifies the effects of productivity growth on user-base dynamics — when potential users expect productivity growth and more users to join in future, they expect token price to appreciate and thus have a stronger incentive to adopt now.

The platform’s investment in productivity is financed by token issuances. Therefore, tokens not only enable users to transact with one another, but also serves as financing instruments. The platform can increase token supply and pay tokens to a pool of contributors for their efforts and resources that improve productivity.⁵ Because the contributors sell tokens to users who value the convenience yield and thus are the natural buyers, the amount of resources the platform can raise by issuing tokens depends on the token price. Token price is determined endogenously by the users’ token demand and the platform’s supply.

The entrepreneur’s value is the present value of tokens paid to herself net the costs of token buybacks. In the Markov equilibrium, the entrepreneur’s value is a function of the current

⁵Our focus on decentralized contributions is consistent with the vision of major token-based platforms that once the platform launches, the founding entrepreneurs’ contributions tend to be limited relative to the decentralized contributors’, such as KIN, a blockchain-based social network, and TON, a payment network.

platform productivity and token supply, which are the two state variables. The marginal value of productivity is positive, capturing the equilibrium dynamics of users' adoption and valuation of tokens. The marginal value of token supply is negative due to the downward pressure of supply on token price and the entrepreneur's token payout. In order to protect the continuation value (i.e., the present value of future token payout), the entrepreneur may even find it optimal to buy back tokens (through external financing) and burn them out of circulation.⁶ In equilibrium, the entrepreneur receives token payout when token supply is low relative to platform productivity and buys back tokens when token supply is relatively high. Figure 1 summarizes the circulation of tokens in our dynamic platform economy.⁷

A key friction in our model is that when buying back tokens, the entrepreneur has to raise costly external funds. While token buybacks occur occasionally, the associated financing cost propagates into a *dynamic token issuance cost* in every state of the world because every time more tokens are issued, the entrepreneur's expectation of costly future buyback changes accordingly. Specifically, the entrepreneur's cost of issuing one more token (i.e., the marginal decline of continuation value) is larger than the market price of tokens (i.e., the users' valuation of tokens). This wedge causes the platform to under-invest in productivity.

In our model, tokens are perfectly liquid. For example, newly issued tokens are not subject to discounts due to informational asymmetry. They are simply valued by the marginal user's indifference condition. Despite the perfect liquidity, tokens are not immune to financial frictions. The token issuance cost emerges because the entrepreneur's optimal buyback relies on costly external funds. In other words, the optimal strategy of token management transmits the traditional costs of external financing into an endogenous cost of token issuance.

The dynamic token issuance cost implies a conflict of interest between the entrepreneur and users. Productivity enhancement paid with tokens benefits users via a higher convenience yield. But more tokens in circulation implies a higher likelihood of costly token buyback in the future. Therefore, while the entrepreneur bears the costs of future token buyback, the benefits of token-financed investment are shared with users. Admittedly, part of such benefits flow to the entrepreneur through a higher token price (and higher value of token payout), but the entrepreneur cannot seize all surplus from users. Users are heterogeneous in deriving

⁶Burning tokens means sending them to a public "eater address" from which they can never be retrieved because the address key is unobtainable. Practitioners often burn tokens to boost token price and reward token holders (e.g., Binance and Ripple). Some also use Proof-of-Burn as an environmentally friendly alternative to Proof-of-Work to generate consensus (e.g., Counterparty (XCP) blockchain), or destroy unsold tokens or coins after an ICO or seasoned token issuances for fair play (e.g., Neblio's burning of NEBL tokens).

⁷We thank our discussant Sebastian Gryglewicz for sharing this figure with us in his discussion slides.

convenience yield from tokens, so only the marginal user breaks even while those who derive more convenience yield enjoy a positive surplus.⁸ *Token overhang*, which is underinvestment due to the surplus leakage to users, is a fundamental feature of token-based financing.

After characterizing the optimal token-management strategy (i.e., investment, payout, and buyback), we analyze the value of introducing blockchain technology in our setting. Blockchains distinguish themselves from traditional technologies in several aspects: immutable record keeping due to time-stamping and linked-list data structure, smart contracting for automating and ensuring execution, and distributed design for easier monitoring and decentralized governance. These features enable the commitment of predetermined token-supply rules that, we show, are valuable in addressing the underinvestment problem.

Specifically, motivated by Ethereum, we consider a constant rate of token issuances that finance investment. We find that commitment mitigates the underinvestment problem by severing the state-by-state linkage between investment and the token issuance cost. While the increased amount of tokens issued for investment results in more frequent costly token buyback, the entrepreneur’s value is higher than the case with discretionary token supply, because the token price is higher under faster trajectories of productivity and user-base growth. Previous studies of tokens assume predetermined rules of supply. In contrast, our analysis starts from the fully discretionary supply of tokens. By comparing the discretionary case with the predetermined case, we are able to identify the value-added of commitment and to partly explain the popularity of blockchain technology among the platform businesses.

Finally, our model also has implications for the design of stablecoins. Different from the approaches based on collateralization, the entrepreneur in our setting supports the franchise value by occasionally buying back tokens out of circulation. When the token supply is high relative to the platform productivity — precisely at the moment that token price is low but the marginal value of reducing token supply is high — the buyback happens. The resulting token supply dynamics moderate the token price fluctuations. Therefore, for platforms with endogenous productivity growth, their tokens are inherently stable.

Overall, our model sheds light on the equilibrium dynamics of token-based communities and also provides a guiding framework for practitioners. The various token offering schemes observed in practice can be viewed as special (suboptimal) cases. Appendix A discusses the institutional background and offers a rich set of real-life examples.

⁸The intuition is related to the surplus that a monopolistic producer forgoes to consumers when price discrimination is impossible. Here tokens are traded at a prevailing price among competitive users, so the entrepreneur cannot extract more value from users who derive a higher convenience yield from tokens.

Literature. The paper that is most related to ours is Gryglewicz, Mayer, and Morellec (2020) who also study endogenous platform productivity but focus on the founders’ efforts, rather than decentralized contributors’ efforts and resources. Mayer (2020) further introduces speculators and studies the conflict of interest among various token holders. Our paper differs in our focus on tokens as a means of payment and on a different stage of platform life-cycle. Gryglewicz, Mayer, and Morellec (2020) model uncertainty in the exogenous arrival of platform launching and a constant token price post-launch. We study a post-launch platform with uncertainty from Brownian productivity shocks, and model the endogenous fluctuation of token price. Finally, while they consider a fixed token supply, we characterize the optimal state-contingent supply and highlight the value of commitment brought by blockchains.

Our paper connects the literature on platform economics to dynamic corporate finance, especially the studies emphasizing the role of financial slack and issuance costs (e.g., Bolton, Chen, and Wang, 2011; Hugonnier, Malamud, and Morellec, 2015; Décamps, Gryglewicz, Morellec, and Villeneuve, 2016). Instead of cash management, we analyze platforms’ token-supply management when investment induces user network effects and, importantly, the token price varies endogenously as users respond to supply variation.⁹ From a methodological perspective, our paper is related to Brunnermeier and Sannikov (2016) and Li (2017), who both study the endogenous price determination of inside money (deposits) issued by banks. The key distinction is that tokens are outside money instead of liabilities of the platforms.

Our paper contributes to the broad literature on digital platforms. Studies on traditional platforms (e.g., Rochet and Tirole, 2003) do not consider the use of tokens as platforms’ native currencies (local means of payment).¹⁰ We share the view on platform tokens with Brunnermeier, James, and Landau (2019): a platform is a currency area where a unique set of economic activities take place and its tokens derive value by facilitating the associated transactions.¹¹ Beyond this, we emphasize that a platform can invest in its quality, for example, payment efficiency (Duffie, 2019), thereby raising token value. We are the first to formally analyze how platforms manage their investment and payout through token supply, and provide insights into the incentives and strategies of platform businesses.

Our paper adds to emerging studies on blockchains and cryptocurrencies. We innovate upon CLW by endogenizing token supply and incorporating the entrepreneur’s long-term interests (franchise value), which allows us to explore new issues concerning the dynamics of

⁹Related to Cagan (1956), the entrepreneur essentially maximizes the present value of seigniorage flows.

¹⁰Stulz (2019) reviews the recent financial innovations by major digital platforms.

¹¹Our model differs from the majority of monetary-policy models because token issuance finances investment, as in Bolton and Huang (2017), and payout rather than to stimulate nominal aggregate demand.

optimal platform investment and financing, the conflict of interest between the entrepreneur and users, and the role of blockchain technology in platform economics.¹² By doing so, we are able to provide the first unified theory of dynamic corporate finance of post-launch platforms: optimal monetary, investment, and payout policies with both token price and user base being endogenously determined. We also demonstrate the commitment value of blockchains.¹³ Hinzen, John, and Saleh (2019) show that limited adoption is an equilibrium outcome in Proof-of-Work (PoW) blockchains and Irresberger, John, and Saleh (2019) empirically document that Proof-of-Stake (PoS) blockchains dominate on adoption scale. Our focus is on the use of tokens for platform finance and endogenous adoption, regardless of the consensus protocol and level of decentralization issues we explore in CLW.

Furthermore, our paper adds to the discussion on token price volatility and stablecoins. On the demand side, high token price volatility could be an inherent feature of platform tokens due to technology uncertainty and endogenous user adoption (see CLW).¹⁴ Saleh (2018) emphasizes that token supply under proof-of-burn (PoB) protocols can reduce price volatility. We endogenize both the demand for tokens driven by users’ transaction needs and dynamic adoption, and the supply of tokens for platform development and the founders’ rent extraction. We show that the optimal token supply strategy stabilizes token price.

Finally, our paper is broadly related to the literature on crowdsourcing and the gig economy. Blockchain-based consensus provisions in the form of cryptocurrency mining and resources (capital) raised via initial coin offerings (ICOs) are salient examples of decentralized on-demand contributions. Existing studies on ICOs and crowdfunding focus on one-time issuance of tokens before the platform launches (e.g., Canidio, 2018; Garratt and Van Oordt, 2019; Chod and Lyandres, 2018), yet platforms increase token supply on an ongoing basis. Existing studies also center around the founders’ hidden efforts or asymmetric information pre-launch, whereas we emphasize decentralized contributors’ effort post-launch that is highly relevant for digital platforms and the gig economy.¹⁵ This distinction is a key

¹²The studies on the design issues of tokens (e.g., proof-of-work protocols) typically assume a fixed user base (e.g., Chiu and Wong, 2015; Chiu and Koepl, 2017). A fixed token supply is a common feature among the models that examine the roles of tokens among users and contributors (e.g., miners in Sockin and Xiong, 2018; Pagnotta, 2018) and the existing models of token valuation (e.g., Fanti, Kogan, and Viswanath, 2019).

¹³Even though commitments through tokens can be valuable in various settings (e.g., Goldstein, Gupta, and Sverchkov, 2019), in practice, the reliability of blockchain and the associated commitment value are not free of costs (Abadi and Brunnermeier, 2018). As analyzed by Biais, Bisiere, Bouvard, and Casamatta (2019), proof-of-work protocols can lead to competing records of transactions (“forks”). Commitment is often implemented via smart contracts, for which Cong and He (2019) provide some examples.

¹⁴Hu, Parlour, and Rajan (2018) and Liu and Tsyvinski (2018) document token price dynamics empirically.

¹⁵The entrepreneurs in the ICO models do not engage in dynamic token management for long-term platform

consideration in determining whether tokens are securities or not based on the Howey test.¹⁶

2 Model

Three types of agents interact in a continuous-time economy: an entrepreneur (used interchangeably with “platform owners”), a pool of contributors, and a unit measure of users. The entrepreneur, representing the group of platform founders, key personnel, and venture investors, designs the platform’s protocol. Contributors, who represent individual miners (transaction ledger keepers), third-party app developers, and other providers of on-demand labor in practice, devote efforts and resources required for the operation and continuing development of the platform. Users conduct peer-to-peer transactions and realize trade surpluses on the platform. A generic consumption good serves as the numeraire.

2.1 Platform productivity and contributors

We study a dynamically evolving platform whose productivity (synonymous with quality), A_t , evolves as follows:

$$\frac{dA_t}{A_t} = L_t dH_t, \quad (1)$$

where L_t is the decentralized contribution (contributors’ resources and labor as described in Appendix A) the entrepreneur gathers through token payments to grow A_t . dH_t is an investment efficiency shock,

$$dH_t = \mu^H dt + \sigma^H dZ_t. \quad (2)$$

Here Z_t is a standard Brownian motion that generates the information filtration.¹⁷ A_t broadly captures marketplace efficiencies, network security, processing capacity, regulatory conditions, users’ interests, the variety of activities feasible on the platform, etc. It therefore affects directly users’ utility on the platform, which shall be made clear below.

Our focus is on the dynamic interaction between the entrepreneur and users, so we do

development, and thus, do not concern the franchise value. Despite newly issued tokens from platforms, decentralized contributors such as miners can also receive transaction fees (Basu, Easley, O’Hara, and Sirer, 2019; Easley, O’Hara, and Basu, 2019; Huberman, Leshno, and Moallemi, 2019; Lehar and Parlour, 2020).

¹⁶For example, the SEC sued Telegram/TON that raised US\$1.7 billion through a private placement for not complying with securities laws (Michaels, 2019). The issue boils down to whether token investors post-launch expect to profit from the entrepreneurs’ effort or decentralized contributions.

¹⁷The process H_t may result from the entrepreneur’s efforts prior to platform launch, which we take as exogenous to differentiate our model from models on founders’ efforts as discussed in the literature review.

not explicitly model contributors’ decision-making but instead specify directly the required numeraire value of compensation for L_t to be $F(L_t, A_t)$, which is increasing and convex in L_t and may also depend on A_t . Let P_t denote the unit price of the token in terms of the numeraire goods. Given A_t , to gather L_t , the platform needs to issue $F(L_t, A_t)/P_t$ units of new tokens to workers, which adds to the total amount of circulating tokens, M_t .

A distinguishing feature of the labor supply in a “platform economy” or “gig economy” is that contributions are on-demand and contributors such as miners in Proof-of-Work-based public blockchains or ride-share drivers receive on-the-spot payments instead of long-term employment contracts. Tokens facilitate the acquisition of on-demand labor by avoiding the limited commitment on the part of platform that arises in the implementation of deferred compensation, especially when workers and the platform belong to different judicial areas. Moreover, since digital tokens are often programmable (via smart contracting), escrow accounts can be set up and enforced automatically so that tokens are released to workers only if their inputs (e.g., programming codes or solutions to cryptography puzzles) are received. Therefore, tokens also reduce the platform’s exposure to workers’ limited commitment.¹⁸ Finally, L_t can also include the capital received from crowd-based investors. Investors receive tokens immediately, instead of receiving contracts of future payments.

When the platform is token-based, the concern of dilution naturally arises – workers and investors’ tokens may depreciate if the platform issues more tokens in the future. In other words, while tokens avoid limited commitments by facilitating spot payments, the platform’s lack of commitment against increasing the token supply is still a concern. To see how our analysis of optimal token supply addresses this question, we first introduce platform users.

2.2 Platform users

As in CLW, users can conduct transactions by holding tokens. We use $x_{i,t}$ to denote the value (real balance) of agent i ’s holdings in unit of numeraires. By facilitating transactions, these holdings generate a flow of utility (or convenience yield) over dt given by:

$$x_{i,t}^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha dt, \tag{3}$$

¹⁸Another reason to introduce tokens as a means of payment for L_t is the heterogeneity in labor quality. Consider a subset of workers who supply high-quality efforts because they better understand the technologies behind the platform. Naturally, these capable workers assign a higher value to tokens because they are not concerned about the adverse selection problem that low-quality workers face due to their lack of technological knowledge. In other words, in contrast to cash-based compensation, token-based compensation screens out high-type workers and thereby improves the match between employer (the platform) and employees (workers).

where N_t is the platform user base, u_i captures agent i 's needs for platform transactions, and $\alpha, \gamma \in (0, 1)$ are constants. Similar to CLW, we provide a theoretical foundation in Appendix B. A crucial difference from CLW is that we endogenize A_t and the token supply M_t .

The flow utility of token holdings depends on N_t , the total measure of users on the platform with $x_{i,t} > 0$.¹⁹ This specification captures the network externality among users, such as the greater ease of finding trading or contracting counterparties in a larger community.

We allow users' transaction needs, u_i , to be heterogeneous. Let $G_t(u)$ and $g_t(u)$ denote the cross-sectional cumulative distribution and density function respectively that are continuously differentiable over a positive support $[\underline{U}_t, \bar{U}_t]$. u_i can be broadly interpreted: For payment blockchains (e.g., Ripple and Bitcoin), a high value of u_i reflects user i 's needs for international remittance. For smart-contracting platforms (e.g., Ethereum), u_i captures user i 's project productivity, and token holdings facilitate contracting.²⁰ In decentralized computation (e.g., Dfinity) and data storage (e.g., Filecoin) applications, u_i corresponds to the need for secure and fast access to computing power and data.

Recall that P_t denotes the unit price of a token in terms of the numeraire. Let $k_{i,t}$ denote the number of tokens that user i holds, then the *real balance* is:

$$x_{i,t} = P_t k_{i,t}. \tag{4}$$

To join the platform (i.e., $k_{i,t} > 0$), a user incurs a flow cost ϕdt . For example, transacting on the platform requires attention; account maintenance and data migration also take effort. Therefore, only agents with sufficiently high u_i choose to join the platform.

Let $y_{i,t}$ denote user i 's *cumulative* utility from platform activities. We follow CLW and assume that the users are well-diversified so that their transaction surpluses and financial gains on the platform are priced by an exogenous stochastic discount factor. Thus, we can interpret the equilibrium dynamics as dynamics under the risk-neutral measure. When users are risk neutral, the risk-neutral measure coincides with the data-generating probability

¹⁹One example involves a producer who accepts tokens as a means of payment and earns net profits equal to the full transaction surplus. The profits depend on the scale of operation, i.e., the sales $x_{i,t}$, and variables that determine the profit margin, which include the total customer outreach, N_t , the platform efficiency A_t , and the producer's idiosyncratic productivity u_i .

²⁰For example, in a debt contract, the borrower's Ethereum can be held in an escrow or "margin" account, which is automatically transferred to the lender in case of default. Posting more Ethereum as margin allows for larger debt contracts, which in turn lead to projects of larger scale and profits.

(physical measure). The user i 's objective is given by:

$$\mathbb{E} \left[\int_0^\infty e^{-rt} dy_{i,t} \right], \quad (5)$$

where the incremental utility $dy_{i,t}$ is:

$$dy_{i,t} = \max \left\{ 0, \max_{k_{i,t} > 0} \left[(P_t k_{i,t})^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha dt + k_{i,t} \mathbb{E}_t [dP_t] - \phi dt - P_t k_{i,t} r dt \right] \right\}. \quad (6)$$

The outer “max” operator in (6) reflects user i 's option to leave and obtain zero surplus from platform activities, and the inner “max” operator reflects user i 's optimal choice of $k_{i,t}$. Inside the inner max operator are four terms that give the incremental transaction surpluses from platform activities. The first corresponds to the payment convenience yield given in (3). The second is the expected capital gains from holding $k_{i,t}$ units of tokens. The third is the participation cost and the last term is the financing) cost of holding $k_{i,t}$ units of tokens.

It is worth emphasizing that platform users must hold tokens for at least an instant, dt , to complete transactions and derive utility flows, and are therefore exposed to token price change over dt . Appendix A contains motivating examples and institutional details. We implicitly assume a liquid secondary market for tokens. Hence, after receiving tokens, decentralized contributors can immediately sell tokens to users. Contributors can also be users themselves, and the model is not changed at all as long as the utility from token usage and the disutility from contributing L_t (which gives rise to $F(\cdot)$) are additively separable.

2.3 The entrepreneur

We refer to the founding entrepreneurs, the key developers, and initial investors who own the platform collectively as the entrepreneur. Importantly, the entrepreneur designs the platform protocols and determines the investment strategies $\{L_t, t \geq 0\}$. Over time, the entrepreneur receives a *cumulative* number of tokens D_t as dividends and, similar to users, evaluates the tokens with a risk-neutral objective function and discount rate r :

$$\max_{\{L_t, D_t\}_{t \geq 0}} \int_{t=0}^{+\infty} \mathbb{E} \left[e^{-rt} P_t dD_t \left[\mathbb{I}_{\{dD_t \geq 0\}} + (1 + \chi) \mathbb{I}_{\{dD_t < 0\}} \right] \right]. \quad (7)$$

When $dD_t > 0$, the entrepreneur receives token dividends that have a market value P_t per unit, as a form of compensation for his essential human capital.²¹ Note that token dividends could be either continuous (i.e., of dt order) or lumpy, and that in equilibrium, the entrepreneur immediately sells her tokens to users who are the natural buyers of tokens because they derive an extra convenience yield from token holdings.

We allow the entrepreneur to buy back and burn tokens to reduce the token supply (i.e., $dD_t < 0$). When $dD_t < 0$, the entrepreneur raises external financing (numeraire goods) at a proportional cost χ for token buyback.²² By reducing token supply, the entrepreneur can boost token price, and consequently increase the value of future token dividends. A higher token price also allows the platform to gather more resources for productivity growth. We allow the amount of token buyback to be continuous (i.e., of dt order) or lumpy.

The key accounting identity that describes the evolution of token supply entails both the tokens issued for financing platform investment and the entrepreneur's dividend/buyback:

$$dM_t = \frac{F(L_t, A_t)}{P_t} dt + dD_t. \quad (8)$$

When the platform invests (the first term on the right side) or distributes token dividends ($dD_t > 0$), the total amount of tokens in circulation increases; the token supply decreases when the entrepreneur burns tokens out of circulation ($dD_t < 0$).²³ In the next section, we show that the entrepreneur's financial slack decreases in M_t . An increase in M_t depresses token price P_t , so when M_t rises to a sufficiently high level, the entrepreneur, who is concerned over the value of future token payouts (i.e., the continuation value), pays the financing cost to raise funds for token buyback that token price. Therefore, under the financing cost χ , managing the token stock is akin to managing cash inventory in Bolton, Chen, and Wang (2011) and Hugonnier, Malamud, and Morellec (2015). A firm's financial slack increases in its cash holdings, because when its cash dries up, the firm has to resort to costly external

²¹For example, blockchain behemoth Bitmain Technologies Ltd and Founders Fund (known for early bets on SpaceX and Airbnb) invest in EOS and hold ownership stakes that entitle them to future token rewards. The gradual distribution of token dividends can be viewed as contingent vesting in reality – a certain amount of total tokens D_t have been allocated by time t but are distributed over time (via dD_t) depending on the stages of platform development and the tokens outstanding (i.e., different values of A_t and M_t).

²²The external financing cost assumption (via parameter χ) is in line with those in the corporate finance literature, e.g., Bolton, Chen, and Wang (2011) and Hugonnier, Malamud, and Morellec (2015), who model in reduced form information, incentive, and transactions costs of raising external funds.

²³It is suboptimal for the platform to pay contributors with costly external financing. Instead, it is generally optimal to use tokens (internal financing) to compensate contributors as doing so delays incurring the costs of external financing. However, using tokens as internal funds incurs a shadow cost because an increase in token supply depresses token price, which reduces the entrepreneur's token payout value.

funds. In our model, the financial slack decreases in token supply, because when the token supply rises too high, the entrepreneur has to buy back tokens with costly external funds.

In what follows, we characterize a Markov equilibrium. The two state variables are the productivity A_t , which measures the technological aspect of the platform, and the token supply M_t , which inversely measures the financial slack.

Definition 1. *A Markov equilibrium with state variable A_t and M_t is comprised of agents' decisions and token price dynamics, such that the token market-clearing condition holds, users optimally decide to participate (or not) and choose token holdings, contributors supply resources for the compensation of $F(L_t, A_t)$ in numeraire value, and the platform strategies, i.e., L_t and D_t , are optimally designed to maximize the entrepreneur's value.*

3 Dynamic equilibrium

We first derive the entrepreneur's optimal investment and token payout and buyback, which in turn pin down the token supply. We then derive platform users' optimal decisions on adoption and token holding in order to aggregate token demand. Finally, token market clearing yields the equilibrium dynamics of token price.

3.1 Optimal token supply

At time t , the entrepreneur's continuation or franchise value V_t (i.e., the time- t value function) satisfies the following Hamilton–Jacobi–Bellman (HJB) equation:

$$rV(M_t, A_t) dt = \max_{L_t, dD_t} P_t dD_t [\mathbb{I}_{\{dD_t \geq 0\}} + (1 + \chi) \mathbb{I}_{\{dD_t < 0\}}] + V_{M_t} \left[\frac{F(L_t, A_t)}{P_t} dt + dD_t \right] + V_{A_t} A_t L_t \mu^H dt + \frac{1}{2} V_{A_t A_t} A_t^2 L_t^2 (\sigma^H)^2 dt . \quad (9)$$

The first term in this HJB equation reflects the dividend payout ($dD_t > 0$) and buyback ($dD_t < 0$). When there are more tokens in circulation, the token price is depressed and the entrepreneur's continuation value is reduced. Therefore, we expect $V_{M_t} < 0$, which we later confirm in the numerical solution. Payout occurs only if $-V_{M_t} \leq P_t$, i.e., the market value of token weakly exceeds the marginal cost of increasing token supply. Token buyback happens when $-V_{M_t} \geq P_t(1 + \chi)$, i.e., the marginal benefit of decreasing token supply is not lower than the cost of burning tokens. The second term is the product of the marginal

value of token supply, V_{M_t} , and the drift of token supply, which consists of tokens paid to contributors and tokens distributed to or burned by the entrepreneur. The third term is the marginal benefit of an increase in A_t . The productivity increases in L_t , but obtaining L_t with token payments increases the token supply M_t , which has a marginal cost of $V_{M_t} \frac{F_{L_t}}{P_t} dt$. Moreover, investment outcome is uncertain, so the fourth term captures how such risk enters into the choice of L_t . The next proposition summarizes the optimal policies.

Proposition 1 (Optimal Token Supply). *The optimal L_t is solved implicitly as a function of state variables, A_t and M_t , by*

$$V_{A_t} A_t \mu^H + V_{A_t A_t} A_t^2 L_t^* (\sigma^H)^2 = F_L(L_t^*, A_t) \left(\frac{-V_{M_t}}{P_t} \right). \quad (10)$$

The optimal dD_t is characterized as follows: the entrepreneur receives token payouts ($dD_t^ > 0$) when $P_t \geq -V_{M_t}$, and buys back and burns tokens ($dD_t^* < 0$) when $-V_{M_t} \geq P_t(1 + \chi)$.*

Equation (10) equates the marginal benefit of investment to the marginal cost. The left side is the marginal impact on the drift of A_t , evaluated by the entrepreneur's marginal value of A_t growth and adjusted for the risk of productivity shock via the second term. The right side is the marginal cost of investment. Since the entrepreneur's marginal cost of token supply can be larger than the market value of tokens, the physical marginal cost F_L is multiplied by $-V_{M_t}/P_t$. This multiplier reflects a *token issuance cost*. Here the platform pays for investment with "undervalued" tokens. The payout/buyback policy in Proposition 1 implies that $-V_{M_t}/P_t \in [1, 1 + \chi]$. Because the entrepreneur incurs a financing cost $\chi > 0$ when burning tokens, there exists a region of (M_t, A_t) such that $V_{M_t}/P_t > 1$, which reflects the cost of issuing tokens. A corollary from Proposition 1 highlights the link between off-platform capital-market frictions and the platform's token issuance cost:

Corollary 1 (Token Issuance Cost). *The financing cost $\chi > 0$ leads to a token issuance cost for the entrepreneur (i.e., $-V_{M_t}/P_t > 1$ for a positive measure of (M_t, A_t)). The issuance cost distorts the investment policy by amplifying the marginal cost of investment in (10).*

Token issuance cost arises even though the token market is perfectly liquid. The financing cost creates a conflict of interest between insider (the entrepreneur) and outsiders (users). A productivity enhancement paid with new tokens benefits users via a higher convenience yield. But more tokens in circulation implies a higher likelihood of token buyback and incidence of financing cost in the future for the entrepreneur. While the entrepreneur bears the financing

cost, the benefits are shared with users. Admittedly, as the token demand strengthens following a productivity increase, the entrepreneur benefits from a higher token price (and higher value of token payout), but the entrepreneur cannot capture the full surplus.

Users are heterogeneous in deriving convenience yield from tokens, so only the marginal user breaks even after token price increases, while those who derive more convenience yield capture a positive surplus. The intuition is similar to that in a monopolistic producer’s problem when full price discrimination is impossible. Here tokens are traded at a prevailing price among competitive users, so the entrepreneur cannot extract more value from users who derive a higher convenience yield than the marginal token holder.

As such, token-based financing naturally exhibits *token overhang*, which is underinvestment due to the leakage of surplus to users. Uncertainty also plays a critical role here. Without dZ_t , the productivity shock, L_t , always increases A_t . Then, with a sufficiently efficient investment technology $F(\cdot)$ (so that relatively few new tokens are needed to pay for L_t), we arrive at a situation where, following investment, A_t always grows faster than M_t . As we will show below, the entrepreneur conducts costly token buyback when M_t is too high relative to A_t . Thus, with A_t always growing faster than M_t , the entrepreneur always moves away from costly token buyback after making investment. As a result, the financing cost is never a concern given this sufficiently efficient $F(\cdot)$. However, in the presence of uncertainty in investment outcome, there always exists a probability that M_t increases faster than A_t after investment, moving the platform closer to costly buyback.

In sum, the mechanism of token overhang relies on three ingredients in the model. First, when the entrepreneur raises consumption goods to buy tokens out of circulation, the entrepreneur faces a financing cost. Second, users are heterogeneous in deriving convenience yield from token holdings, so under a single token price that clears the competitive market, only the marginal user breaks even. Third, the outcome of platform investment is uncertain. The first ingredient creates a private cost of investment for the entrepreneur, and the second implies a surplus leakage to users. Together, they generate a conflict of interest between the entrepreneur and users. Finally, the third ingredient, uncertainty, is needed so that despite the specification of $F(\cdot)$, token overhang always exists.

Our characterization of the optimal investment and payout/buyback policies in some sense allays the concern over fraudulent designs or manipulations by the founding developers, for example, through building “back doors” in the protocol to steal tokens and depress the token price when selling the stolen tokens in secondary markets. As shown in Proposition 1, our setup allows the entrepreneur to extract tokens as dividends, and the optimal payout

policy already maximizes the entrepreneur's value. In other words, the policy is incentive-compatible in this subgame perfect equilibrium between a large player (the entrepreneur) and a continuum of small players (users). From a regulatory perspective, a proposal of blockchain or platform design should disclose the policy of token payout to the platform owners, and it should be broadly in line with the above characterization.

3.2 Aggregate token demand

We conjecture and later verify that in equilibrium, the token price, P_t , evolves as

$$dP_t = P_t \mu_t^P dt + P_t \sigma_t^P dZ_t, \quad (11)$$

where μ_t^P and σ_t^P are endogenously determined. Agents take the price process as given under rational expectation. Conditioning on joining the platform, user i chooses the optimal token holdings, $k_{i,t}^*$, by using the following first-order condition,

$$(1 - \alpha) \left(\frac{N_t^\gamma A_t u_i}{P_t k_{i,t}^*} \right)^\alpha + \mu_t^P = r, \quad (12)$$

which states that the sum of *marginal* transaction surplus on the platform and the expected token price change is equal to the required rate of return, r .

Rearranging this equation, we obtain the following expression for optimal token holdings:

$$k_{i,t}^* = \frac{N_t^\gamma A_t u_i}{P_t} \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \quad (13)$$

$k_{i,t}^*$ has several properties. First, users hold more tokens when the common productivity, A_t , or user-specific transaction need, u_i , is high, and also when the user base, N_t , is larger due to network effects. Equation (13) reflects an investment motive to hold tokens, that is $k_{i,t}^*$ increases in the expected token appreciation, μ_t^P .

Using $k_{i,t}^*$, we obtain the following expression for the user's maximized profits conditional on participating on the platform:

$$N_t^\gamma A_t u_i \alpha \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1-\alpha}{\alpha}} - \phi. \quad (14)$$

User i only participates when the preceding expression is non-negative. That is, only those

users with sufficiently large u_i participate. Let \underline{u}_t denote the type of the marginal user, then

$$\underline{u}_t = \underline{u}(N_t; A_t, \mu_t^P) = \frac{\phi}{N_t^\gamma A_t \alpha} \left(\frac{r - \mu_t^P}{1 - \alpha} \right)^{\frac{1-\alpha}{\alpha}}. \quad (15)$$

The adoption threshold \underline{u}_t decreases in A_t because a more productive platform attracts more users. The threshold also decreases when users expect a higher token price appreciation (i.e., higher μ_t^P). Because only agents with $u_i \geq \underline{u}_t$ participate, the user base is then:

$$N_t = 1 - G_t(\underline{u}_t). \quad (16)$$

Equations (15) and (16) jointly determine the user base N_t given A_t and μ_t^P .²⁴

Proposition 2 (Token Demand and User Base). *Given A_t and μ_t^P , the platform has a positive user base when Equations (15) and (16) have solutions for \underline{u}_t and N_t . Conditional on participating, user i 's optimal token holding, $k_{i,t}^*$, is given by Equation (13). The token holding, $k_{i,t}^*$, decreases in P_t and increases in A_t , μ_t^P , u_i , and N_t .*

3.3 Token market clearing

Clearing the token market pins down the token price. We define the *participants' aggregate transaction need* by aggregating u_i of participating users:

$$U_t := \int_{u \geq \underline{u}_t} u g_t(u) du. \quad (17)$$

The market-clearing condition is:

$$M_t = \int_{i \in [0,1]} k_{i,t}^* di. \quad (18)$$

Substituting optimal holdings in Equation (13) into the market-clearing condition in Equation (18), we arrive at the *token pricing formula* in Proposition 3.

Proposition 3 (Token Pricing). *The equilibrium token price is given by*

$$P_t = \frac{N_t^\gamma U_t A_t}{M_t} \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \quad (19)$$

²⁴We do not consider the trivial solution of zero adoption, which always leads to a zero token price.

Token price increases in N_t . The larger the user base is, the higher the trade surplus individual participants can realize by holding tokens, and the stronger the token demand. The price-to-user base ratio increases in the productivity, the expected price appreciation, and the network participants' aggregate transaction need, while it decreases in the token supply M_t .²⁵ Equation (19) implies a differential equation for P_t in the state space of (M_t, A_t) . This can be clearly seen once we apply the infinitesimal generator to $P_t = P(M_t, A_t)$, expressing μ_t^P into a collection of first and second derivatives of P_t by Itô's lemma. Note that the equilibrium user base, N_t , is already a function of A_t and μ_t^P as shown in Proposition (2). Therefore, the collection of token market-clearing conditions at every t essentially characterize the full dynamics of token price. This method of solving token price follows CLW.

Equations (8) and (18) describe the primary and secondary token markets. The change of M_t is a flow variable, given by Equation (8), that includes the new issuances from platform investment and payout and the repurchases by the entrepreneur. The token supply M_t is a stock variable, and through Equation (18), it equals the token demand of users.

Discussion: Durable-good monopoly. The problem faced by a token-based platform reminisces a durable-good monopoly problem (e.g., Coase, 1972; Stokey, 1981; Bulow, 1982). First, token issuance permanently increases the supply. When issuing tokens to finance investment or payout, the entrepreneur is competing with future selves. Second, given a zero physical cost of creating tokens, the Coase intuition seems applicable: The entrepreneur can be tempted to satisfy the residual demand by ever lowering token price as long as the price is positive (i.e., above the marginal cost of production). Thus, users wait for lower prices, driving token price to zero. Our model differs from the Coasian setting in two aspects. First, even though the physical cost of producing tokens is zero, the dynamic token issuance cost increases in the token supply as we show in the next section. This reminisces the result in Kahn (1986) that the Coase intuition does not hold in the presence of increasing marginal cost of production. Second, in contrast to theories of durable-good monopoly, token demand in our model is not stationary; in fact, it increases geometrically with the endogenously growing A_t , so users cannot expect lower token price in the future. Therefore, we can solve an equilibrium with a positive token price in the next section.

²⁵The formula reflects certain observations by practitioners, such as incorporating DAA (daily active addresses) and NVT Ratio (market cap to daily transaction volume) in token valuation framework, but instead of heuristically aggregating such inputs into a pricing formula, we solve *both* token pricing and user adoption as an equilibrium outcome. See, for example, *Today's Crypto Asset Valuation Frameworks* by Ashley Lannquist at Blockchain at Berkeley and Haas FinTech.

4 Equilibrium characterization

We further characterize the equilibrium by analytically deriving and numerically solving the system of differential equations concerning token price and the entrepreneur's value function. To streamline exposition and focus on core economic insights, we make some intuitive parametric assumptions.

4.1 User distribution and investment cost function

We assume that u_i follows the commonly used Pareto distribution on $[\underline{U}_t, +\infty)$ with cumulative probability function (c.d.f.) given by the Pareto distribution:

$$G_t(u) = 1 - \left(\frac{\underline{U}_t}{u}\right)^\xi, \quad (20)$$

where $\xi > 1$ and $\underline{U}_t = 1/(\omega A_t^\kappa)$, $\omega > 0$, $\kappa \in [0, 1]$. The cross-section mean of u_i is $\frac{\xi \underline{U}_t}{\xi - 1}$. It is assumed that \underline{U}_t decreases in A_t , which reflects the competition from follower platforms inspired by the success (high A_t) of the platform in question. For example, after the success of Bitcoin, alternative blockchains emerge as competitors in the area of payments. Similarly, there are alternative platforms to Ethereum for smart contracting. The overall effects of competition depend on the parameters ω and κ , while the parameter ξ governs how heterogeneous users transaction needs (u_i) are (i.e., the dispersion of u_i distribution).

Lemma 1 (Parameterized User Base). *Given A_t and μ_t^P , from Proposition 2, we have a unique non-degenerate solution, N_t , from Equations (15) and (16), given by:*

$$N_t = A_t' \left(\frac{\alpha}{\omega \phi}\right)^{\frac{\xi}{1-\xi\gamma}} \left(\frac{1-\alpha}{r-\mu_t^P}\right)^{\left(\frac{\xi}{1-\xi\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)}, \quad \text{where } A_t' \equiv A_t^{(1-\kappa)\left(\frac{\xi}{1-\xi\gamma}\right)}, \quad (21)$$

if $\underline{u}_t \geq \frac{1}{\omega A_t^\kappa}$, i.e., $A_t^{1-\kappa}\left(\frac{1-\alpha}{r-\mu_t^P}\right)^{\frac{1-\alpha}{\alpha}} \leq \frac{\omega \phi}{\alpha}$; otherwise, $N_t = 1$.

A_t' is a transformed version of A_t . It is the effective productivity that captures user homogeneity, platform competition, and user network effects. Intuitively, the way productivity matters is amplified by the network-effect parameter γ (introduced in (3)) but dampened by the competition parameter κ . When there is no network effect ($\gamma = 0$) or competition ($\kappa = 0$), the exponent is simply ξ , which measures user heterogeneity. The effect of user heterogeneity has two components. One is the interaction component with γ , as seen in the

denominator of $\frac{\xi}{1-\xi\gamma}$. When agents are more homogeneous (larger ξ), small changes in A_t brings big changes in adoption, which is amplified by network effects. The second component is in the numerator of $\frac{\xi}{1-\xi\gamma}$. Even without the network effect, greater homogeneity still means that there is a bigger adoption sensitivity with respect to platform productivity.

Our later discussion focuses on $\xi\gamma < 1$, so that the user base, N_t , increases in the platform productivity despite platform competition. This is realistic because a technology leader usually benefits from its innovation despite the presence of competing followers. Moreover, we focus on low values of A_t , such that $N_t < 1$ in the Markov equilibrium so as to examine how token allocation interacts with user base dynamics. Under the Pareto distribution, the aggregate transaction need is given by:

$$U_t = N_t \left(\frac{\xi u_t}{\xi - 1} \right). \quad (22)$$

Lemma 2 (Parameterized Token Price). *The equilibrium token price in Proposition 3 when $N_t < 1$ is given by:*

$$P_t = \frac{A'_t}{M_t} \frac{\xi}{(\xi - 1)\omega^{\frac{\xi}{1-\xi\gamma}}} \left(\frac{\alpha}{\phi} \right)^{\frac{\xi}{1-\xi\gamma} - 1} \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{1 + \left(\frac{\xi}{1-\xi\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)}. \quad (23)$$

Next, we follow the literature on investment in finance and macroeconomics (e.g., Hayashi, 1982) to specify a convex (quadratic) investment cost function:

$$F(L_t, A_t) = \left(L_t + \frac{\theta}{2} L_t^2 \right) A'_t, \quad (24)$$

and $\theta \geq 0$ can depend on the elasticity of contributors' resource supply. The particular functional form ensures analytical tractability because $F(L_t, A_t)$ being linear in A' not only captures the reality that contribution compensation depends on the *effective productivity* of the platform, but also allows us to reduce the dimensionality of the state variables when solving the model. The specification also allows us to characterize optimality via the first-order condition for investment, similar to the F.O.Cs in Hayashi-style q-theoretic models with convex adjustment costs. In general, the investment cost is higher when the platform is more productive because incremental improvements in the productivity become harder (note that L_t enters into the growth *rate* of A_t). It is quadratic in the decentralized contribution

the entrepreneur gathers to reflect the increasing marginal cost of adding L_t .²⁶ For example, to induce more miners to mine Bitcoin, more rewards must be given as miners' competition drives up their cost of mining through higher electricity prices.

We characterize a Markov equilibrium in a transformed state space. The equilibrium variables depend on (m_t, A_t) , where the productivity-normalized token supply is given by:

$$m_t = \frac{M_t}{A_t}. \quad (25)$$

By inspecting Equation (23), we can see that m_t is the only state variable driving the token price. By Itô's lemma, μ_t^P is a function of m_t if and only if P_t is a function of m_t .

Moreover, the entrepreneur's value function exhibits a homogeneity property, $V(M_t, A_t) = v(m_t)A_t'$. These properties significantly simplify our analysis. In the interior region where $dD_t = 0$, denoting $(1 - \kappa)(\frac{\xi}{1 - \xi\gamma})$ by δ , we simplify the HJB equation (9) under $V_t = v(m_t)A_t'$ to an ordinary differential equation for $v(m_t)$, as follows:

$$rv(m_t) = \max_{L_t} v'(m_t) \frac{(L_t + \frac{\theta}{2}L_t^2)}{P_t} + [v(m_t) - v'(m_t)m_t] \left[\delta\mu^H L_t + \frac{1}{2}\delta(\delta - 1)(\sigma^H)^2 L_t^2 \right] + \frac{1}{2}v''(m_t)m_t^2\delta^2(\sigma^H)^2 L_t^2. \quad (26)$$

Under the specification of $F(\cdot)$ in (24), the HJB equation implies the following optimal investment via the standard first-order condition.

Lemma 3 (Parameterized Optimal Investment). *Under the specification of investment cost function, $F(\cdot)$, in Equation (24), the optimal investment is given by:*

$$L_t^* = \frac{[v(m_t) - v'(m_t)m_t] \delta\mu^H + \frac{v'(m_t)}{P_t}}{-\frac{v'(m_t)}{P_t}\theta - v''(m_t)m_t^2\delta^2(\sigma^H)^2 - \delta(\delta - 1)(\sigma^H)^2[v(m_t) - v'(m_t)m_t]}. \quad (27)$$

The optimality conditions for dD_t give us the boundary conditions for solving $v(m_t)$ and

²⁶While we view this as a reasonable starting point, we are fully aware that the implicit adjustment costs for labor or capital may not be convex. Investment may be lumpy and fixed costs can be important in reality. In corporate finance, Hugonnier, Malamud, and Morellec (2015) show that lumpy investment for a financially constrained firm facing costly external equity issuance generates different investment and financing dynamics from those in a model based on smooth investment adjustment costs, e.g., Bolton, Chen, and Wang (2011). For example, Hugonnier, Malamud, and Morellec (2015) show that value function may not be globally concave and smooth pasting conditions may not guarantee optimality.

$P(m_t)$. The marginal value of retained token must be equal to the market value,

$$-v'(\underline{m}) = P(\underline{m}), \quad (28)$$

at the optimal payout boundary, \underline{m} , and we have the standard “super contact” condition:

$$-v''(\underline{m}) = P'(\underline{m}). \quad (29)$$

As the payout boundary is a reflecting boundary, to rule out arbitrage in the token market, we have:

$$P'(\underline{m}) = 0. \quad (30)$$

Intuitively, the distribution of token dividends happens when the token supply is sufficiently small relative to the platform productivity, i.e., low m_t . Similarly, at the buyback boundary, denoted by \bar{m} , we have the following conditions:

$$-v'(\bar{m}) = P(\bar{m})(1 + \chi); \quad (31)$$

$$-v''(\bar{m}) = P'(\bar{m})(1 + \chi); \quad (32)$$

$$P'(\bar{m}) = 0. \quad (33)$$

The optimal amounts of token payout or buyback are determined as follows. The payout boundary, \underline{m} , is a reflecting boundary of $m_t = M_t/A'_t$. When $m_t = \underline{m}$, any decrease of m_t (for example, due to a positive shock to A'_t) leads to payout, and the payout amount is precisely equal to the increase in M_t (the numerator) that is required to bring m_t back up to \underline{m} . Similarly, at the buyback boundary, \bar{m} , is also a reflecting boundary of m_t . At \bar{m} , any increase of m_t (for example, due to a negative shock to A'_t) leads to token buyback, and the buyback amount is equal to the decrease in M_t that is required to bring m_t back down to \bar{m} .

Proposition 4 (Solving the Markov Equilibrium). *With Lemmas 1, 2, and 3, there exists a Markov equilibrium with A_t (equivalently A'_t) and $m_t = M_t/A'_t$ as the state variables, and the equilibrium has the following properties:*

(i) $P(m_t)$ and $v(m_t)$ uniquely solve the system of ordinary differential equations given by Equation (23) and (26) subject to boundary conditions given by Equations (28) to (33).

(ii) The entrepreneur’s optimal investment L_t and decisions on whether to payout ($dD_t > 0$) or buy back tokens ($D_t < 0$) all depend on m_t only.

(iii) Users’ optimal token holdings and participation decisions, together with the user base

depend on both m_t and A_t according to Proposition 2.

For the parameters that affect user activities, we follow CLW to set $\alpha = 0.3$, $\phi = 1$, $r = 0.05$, and the volatility parameter, $\sigma^H = 2$. For the mean productivity growth, we set $\mu^H = 0.5$, which generates a μ_t^P in line with the values in CLW. We set $\chi = 7\%$ for the financing cost following the empirical literature on equity issuance (Eckbo, Masulis, and Norli, 2007). We set $\theta = 10000$ so the growth rate of productivity is in line with that in CLW. The rest of parameters are to illustrate the qualitative implications of the model: $\gamma = 1/8$ for the network effect and $\xi = 2$, $\kappa = 5/8$, and $\omega = 100$ for the distribution parameters of u_i . The model’s qualitative implications are robust to the choice of these parameters.

Discussion: token supply limit. Blockchain platforms often feature a maximum total token supply. One way to incorporate this is to have an absorbing upper bound of m_t , say \tilde{m} . In such case, once reaching a multiple of the platform productivity, i.e., $\tilde{m}A'_t$, the supply would grow proportionally with A'_t forever, and according to Lemma 2, token price will then be a constant. As for newly issued tokens, they are divided between the entrepreneur and contributors, and here the entrepreneur faces a standard consumption-savings trade-off: If she takes a larger share of the new tokens, the productivity grows slower.

4.2 Endogenous platform development

Panel A of Figure 2 plots the A' -scaled value function $v(m_t)$. The curve starts at the payout boundary where the entrepreneur receives payouts in the form of newly issued tokens, and it ends at the buyback boundary where the entrepreneur raises funds to buy back and burn tokens out of circulation in order to support token price and the continuation value.

The entrepreneur’s value declines in the normalized token supply (a notion of “inflation” practitioners casually refer to). Intuitively, when more tokens are circulating relative to productivity, it is more likely for the entrepreneur to reach the buyback (upper) boundary and pay the financing cost, and in the less likely event of token payout, the entrepreneur receives a lower value due to the depressed token price. The value function is always positive in Panel A, suggesting that the entrepreneur never abandons the platform.

Panel B of Figure 2 plots the optimal platform investment (given by (27)) against the normalized token supply. In (27), the optimal L_t^* decreases in $-V_{M_t}/P_t$, the ratio of the marginal cost of token issuance, $-V_{M_t} = -v'(m_t)$, to the token market price, P_t . This ratio measures the valuation gap that exists between the entrepreneur and the platform users

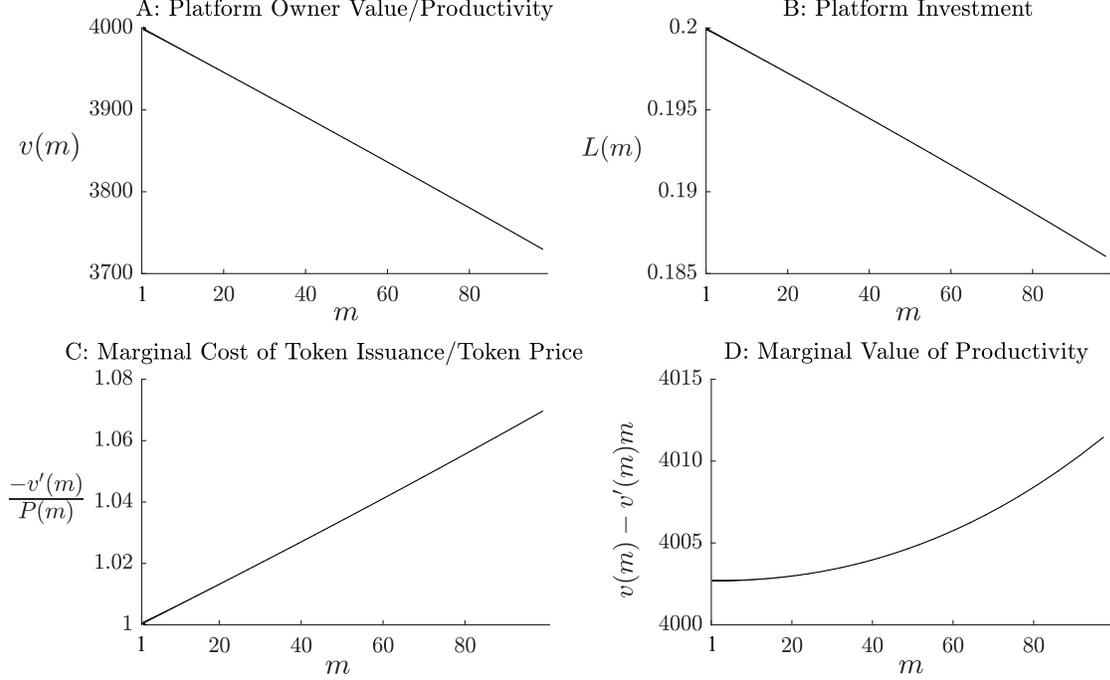


Figure 2: **Platform Value and Investment.** Panel A plots the A'_t -scaled value function. Panel B plots the optimal investment as a function of productivity-normalized token supply, m_t . Panel C shows the ratio of the entrepreneur's marginal value of tokens to the market price of tokens, and the wedge between this ratio and one represents the token issuance cost. Panel D shows the entrepreneur's marginal value of A'_t .

(i.e., the token issuance cost). When the gap is high, it is costly from the entrepreneur's perspective to finance investment with tokens. The ratio starts at one, as implied by the value-matching condition of the payout boundary. This is when the entrepreneur's private valuation of tokens, which incorporates the expected cost of token buyback, coincides with the market or users' valuation. The gap widens as the token supply outpaces the growth of the effective productivity, i.e., as m_t increases, and eventually, when the gap reaches $(1 + \chi)$, the entrepreneur optimally buys back tokens. The increasing token issuance cost in Panel C (i.e., $-V_{M_t}/P_t$ increasing in m_t) largely contributes to the decreasing pattern of L_t^* .

The optimal L_t^* given by (27) increases in the marginal value of effective productivity, $\frac{\partial V_t}{\partial A'_t} = v(m_t) - v'(m_t)m_t$, because on average, investment has a positive outcome, i.e., $\mu^H > 0$, so more resources gathered by token payments, L_t , leads to a higher expected growth of A_t and an expected increase in the entrepreneur's value. Moreover, the marginal value of effective productivity also has a positive impact on L_t^* via the denominator of L_t^* in (27). As shown in the HJB equation (26), the marginal value of A'_t is multiplied by the drift of A' ($= A_t^\delta$), which is equal to $\delta\mu^H L_t + \frac{1}{2}\delta(\delta - 1)(\sigma^H)^2 L_t^2$ given dA_t in (2). The denominator effect follows the quadratic term in the drift of A'_t . Near the buyback (upper) boundary, $\frac{\partial V_t}{\partial A'_t}$

is particularly high because an increase in A'_t pulls down m_t and thus reduces the likelihood of costly buyback. Overall, even though the marginal value of productivity is increasing in m_t in Panel D, the economic force of token overhang (Panel C) dominates, resulting in an optimal investment that declines in the normalized token supply m_t .

Finally, according to (27), the second-order derivative of the A' -scaled value function, $v''(m_t)$, also affects the optimal investment L_t^* via the denominator. Its impact is small under the current parameterization, so the plot is omitted from Figure 2. However, the intuition of the potential precautionary motive ($v''(m_t) < 0$) is still interesting. Token payout is largely a real option decision. While it is not completely irreversible, reversing it (i.e., buying back tokens) incurs the financing cost.²⁷ The probability of incurring such cost increases as m_t approaches the buyback boundary, so the entrepreneur becomes increasingly cautious on making a risky investment given the shock in (2). Therefore, the negative impact of precaution on investment is more prominent near the buyback (upper) boundary of m_t .

Overall, our model reveals a rich set of trade-offs in the choice of token-financed investment. The model has the potential to explain various features of token distribution to open-source engineers, miners (ledger maintainers), and crowd-sourced financiers in practice.

4.3 Token price and user adoption

The dynamics of token price are directly linked to that of productivity-normalized token supply. As shown in Panel A of Figure 3, P_t declines in m_t . In Appendix C, we derive the negative shock loading (diffusion) of m_t . Thus, a positive shock in productivity decreases m_t by increasing A'_t , thus moving the economy closer to the payout (lower) boundary of m_t . Token price increases in response and is therefore procyclical with respect to productivity shock. In stark contrast to the 200% per annum volatility of productivity shock that we input, i.e., the fundamental volatility, σ_t^P is surprisingly small (below 0.15% in Panel B of Figure 3) because the entrepreneur receives newly issued tokens as payout when m_t is low and raise funds to buy back and burn tokens when m_t is high, actively moderating the variation of token price by controlling the supply. The entrepreneur's incentive to regulate token supply is governed by her marginal cost of raising token supply, $-\frac{\partial V_t}{\partial M_t} = v'(m_t) < 0$. The next corollary directly follows Corollary 1.

Corollary 2. *From Corollary 1, the token price is bounded in $\left[-\left(\frac{1}{1+\chi}\right)v'(m_t), -v'(m_t)\right]$.*

²⁷For example, the Synereo team has to hold multiple meetings and incur effort cost to explain to users when the team burned 33% of its cryptocurrency reserves.

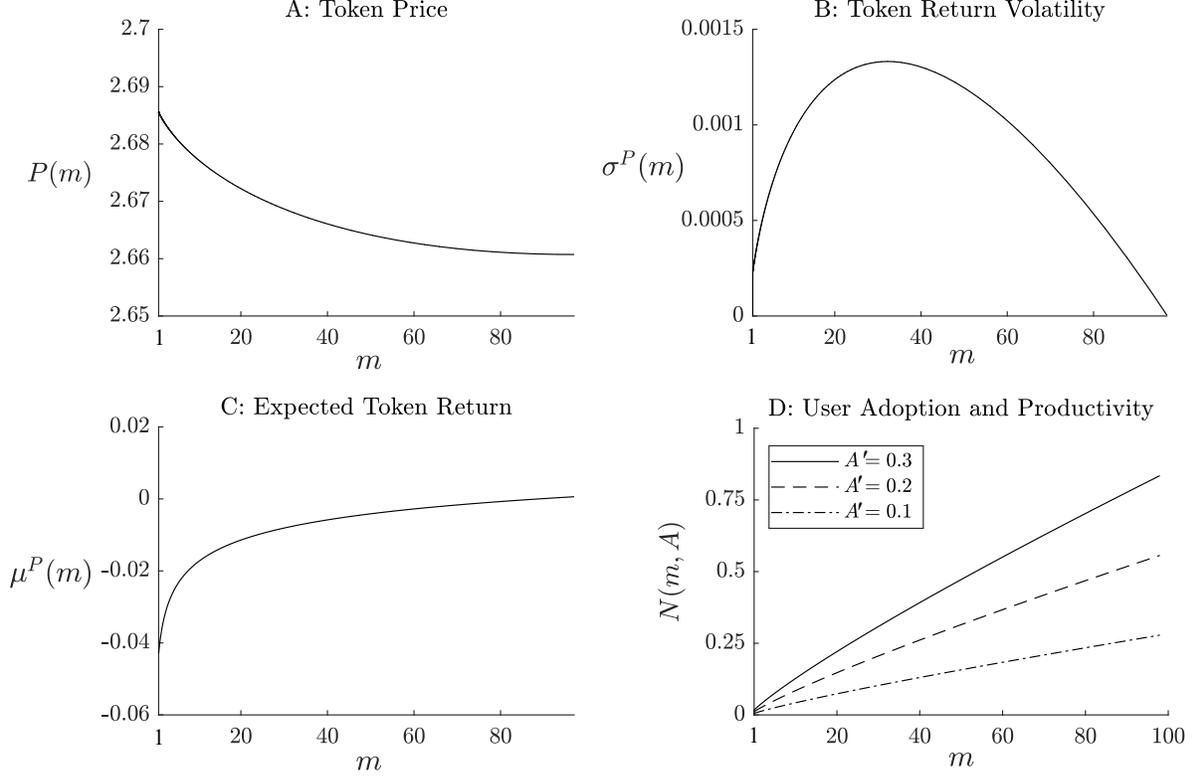


Figure 3: **Token Price Dynamics and User Adoption.** Panel A plots token price against productivity-normalized token supply, m_t . Panel B plots σ_t^P , the P_t -scaled diffusion term of token price. Panel C shows the P_t -scaled drift of token price. Panel D plots the user base, which depends on both m_t and A'_t .

The optimality condition on payout imposes an upper bound on token price. At any m_t , $P(m_t) \leq -v'(m_t)$ because otherwise the entrepreneur prefers obtaining token payout (worth $P(m_t)$ per unit of token) over preserving the continuation value (worth $-v'(m_t)$). The optimality condition on token buyback imposes a lower bound on token price. At any m_t , $P(m_t) \geq -\left(\frac{1}{1+\chi}\right)v'(m_t)$ because otherwise the entrepreneur finds tokens too cheap in the secondary market and prefers raising costly funds to buy back tokens. In our model, the token value has two anchors. First, users need tokens for transactions. Second, to preserve the continuation value, the entrepreneur is willing to pay the financing cost to raise funds and use such real resources to buy back and burn tokens out of circulation.

Panel C of Figure 3 shows the expected token price change. When m_t is low, the expectation is negative, reflecting the likely token-supply increase due to token payout to the entrepreneur and increasing investment needs (Panel A of Figure 2). As m_t increases, the expected change in token price gradually increases and eventually becomes positive because, first, the investment needs decline, and second, the likelihood of token buyback increases.

Finally, we report the results on user-base dynamics. As shown in Proposition 2, unlike other endogenous variables that only depends on m_t , the user base N_t depends on both m_t (through the expected token price change $\mu^P(m_t)$) and A'_t . Panel D of Figure 3 plots the user base against m_t under different values of A'_t . Given A'_t , Panel D shows that as m_t increases (and μ_t^P increases), the user base increases because agents expect an improving capital gain from token holdings. Given any value m_t , a higher value of productivity A_t leads to a larger user base because A_t directly enters users' convenience yield from token holdings in (3).

Discussion: Stablecoins. Our model features mild volatility of token price. The entrepreneur's optimal payout and token buyback decisions impose two reflecting boundaries on the state variable m_t . At both boundaries, the first derivative of token price with respect to m_t must be zero (e.g., see (30)) because otherwise arbitrage opportunities emerge: For example, at \underline{m} , m_t will be reflected upward for sure, so $P'(m_t) > 0$ (< 0) implies guaranteed instantaneous profits from a long (short) position. By Itô's lemma, $P'(m_t) = 0$ at the boundaries implies $\sigma^P(m_t) = 0$. Therefore, even in the interior region, token volatility $\sigma^P(m_t)$ can exceed zero; however, it cannot go far beyond zero as it is tied to zero at both boundaries.

Therefore, in our model, the stability of token price relies on the dynamic payout and token buyback decisions of the entrepreneur. This mechanism differs significantly from the stablecoin designs proposed by practitioners. A popular approach is to mimic open market operations by central banks. When token price is low, the platform issues token bonds to buy back tokens. Token bonds promise to pay the principal with interest in the future, but all payments are in tokens. The problem with this design is that an inter-temporal substitution between current and future tokens does not introduce any real resources to support token price, nor does it provide any incentive to economic agents to devote such resources. A champion of this design, the Basis stablecoin project, which attracted \$133 million of venture capital in April 2017, has closed down all operations, citing US securities regulations as the reason for its decision.²⁸ An alternative design is collateralization, which backs token value with real resources, such as the U.S. dollar (e.g., Tether, Circle, Gemini, JPM coin, or Paxos), oil reserves (e.g., Venezuela's El Petro, OilCoin, or PetroDollars).²⁹ A derivative of such design is to further tranche the claims on real resources, so tokens are the most senior tranche, which is less information-sensitive and thus has a stable secondary-

²⁸<https://icoexaminer.com/ico-news/133-million-basis-stablecoin-project-ceases-and-desists-citing-regulatory-concerns/>

²⁹Such designs are often subject to manipulations (e.g., Griffin and Shams, 2018).

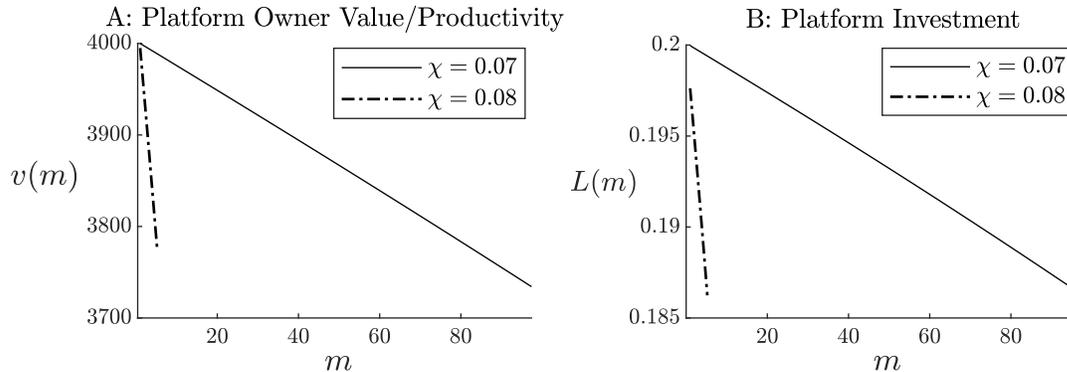


Figure 4: **Comparative Statics: Financing Cost.** This figure compares the A' -scaled value function (Panel A) and optimal investment (Panel B) under $\chi = 0.08$ and the baseline value $\chi = 0.07$.

market value. Li and Mayer (2020) provide a model on collateralized stablecoins.

4.4 Comparative statics

We further explore the mechanism of underinvestment through the analysis of comparative statics. Specifically, we focus on the root of the problem, which is the cost of external financing for token buyback, and by comparing platforms with different degrees of network effects, we highlight the key role of risk in generating underinvestment.

4.4.1 External financing cost

The entrepreneur's external financing cost drives the divergence of interest between the entrepreneur and users. If the cost of buyback, χ , is zero, the dynamic token issuance cost and thus underinvestment disappear as shown in Equation (10) because $-V_{M_t} = P_t$.³⁰ We now compare the model's performances under $\chi = 7\%$ (the baseline value) and $\chi = 8\%$.

In Panel A of Figure 4, we see that the entrepreneur's (A' -scaled) value function curve ends at a lower level of m_t when χ is higher. While the buyback (right) boundary is lower, the payout (left) boundary remain roughly unchanged. Therefore, the overall level of m_t is lower when χ increases. When the financing cost is higher, the entrepreneur faces a higher cost of issuing tokens. Therefore, the entrepreneur optimally maintains a low level of normalized token supply. Panel A of Figure 4 also shows that, intuitively, a higher external financing cost causes the value function to shift downward across different values of m_t .

³⁰In this case, the HJB equation of the owner's valuation function degenerates. Once the token price is solved via the ODE implied by the token market-clearing condition, the owner's value function can be calculated as an integral of token price, because $-V_{M_t} = -v'(m_t) = P(m_t)$.

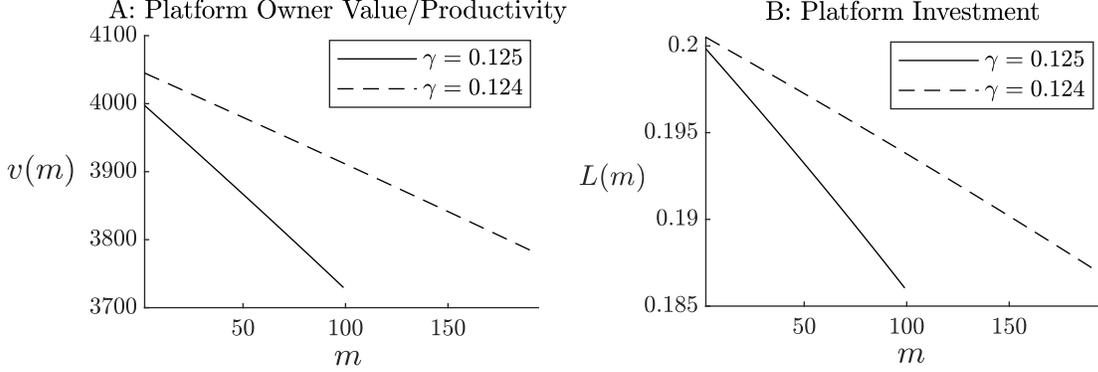


Figure 5: **Comparative Statics: Network Effect.** This figure compares the A' -scaled value function (Panel A) and optimal investment (Panel B) under $\gamma = 0.124$ and the baseline value $\gamma = 0.125$.

We compare platform investment under different values of χ in Panel B of Figure 4. A higher financing cost leads to lower investment as the conflict of interest between the entrepreneur and users is exacerbated. Following a positive shock, the investment successfully enhances productivity, benefiting both the entrepreneur and users; following a negative shock, only the entrepreneur bears the downside of a higher likelihood of paying the financing cost for token buyback because users are free to reduce token holdings and abandon the platform. A higher financing cost implies a greater downside for the entrepreneur.

4.4.2 Network effect

As specified in (3), the parameter γ governs the strength of the network effect. When γ is higher, an increase in the total number of users, N_t , causes each user to demand more tokens. In Figure 5, we compare the entrepreneur's value function (Panel A) and optimal investment (Panel B) under $\gamma = 0.125$ (the baseline value) and $\gamma = 0.124$.

Interestingly, a weaker network effect induces the entrepreneur to be more aggressive in token issuance. Moreover, the entrepreneur's value function shifts upward (Panel A), as a weaker network effect induces more investment (Panel B). These observations are counterintuitive because when the network effect is weaker, the positive feedback effect of increasing productivity to attract more and more users is dampened, which discourages investment in productivity. However, our model features a counteracting force.

As previously discussed, the conflict of interest between the entrepreneur and users and the resultant underinvestment problem depend on three ingredients: (1) the external financing cost, (2) user heterogeneity, and (3) the uncertainty in investment outcome. When investment outcome is uncertain, the entrepreneur has to consider the potential downside, i.e.,

the increase in the likelihood of costly token buyback follows a negative shock that increases m_t (by decreasing productivity in the denominator). A weaker network effect dampens uncertainty. Following a positive shock, investment increases productivity, but under a weaker network effect, the resultant increase in token demand and price is smaller. Likewise, following a negative shock, a weaker network effect implies a smaller decline in token demand and price. Therefore, even though a weaker network effect reduces the average positive impact of investment on token price and the entrepreneur’s payout (the mean effect), it also dampens the risk of investment, which is a key ingredient of the underinvestment problem.

5 Blockchain and investment efficiency

The entrepreneur faces a time inconsistency problem that features prominently in studies on macroeconomics (Kydlund and Prescott, 1980; Barro and Gordon, 1983; Lucas and Stokey, 1983) and corporate capital structure (DeMarzo and He, 2020). If the entrepreneur is able to commit against underinvestment, the users would have demanded more tokens, which then increases token price, as well as the value of token payouts to the entrepreneur. However, a predetermined level of investment can be deemed suboptimal ex post as the conflict of interest arises between the entrepreneur and users, reflected in the gap between the entrepreneur’s private valuation of tokens, $-V_{M_t}$, and users’ valuation, P_t .

So far, we have focused on the discretionary token-supply policies of the platform. Next, we study how commitment to predetermined token-supply rules adds value. Our analysis sheds light on why tokens become a viable payment solution after the blockchain technology matures. The rise of tokens as a means of payment on digital platforms is a recent phenomenon with many applications inspired by the success of Bitcoin, Ethereum, and other blockchain-based startups. In Appendix A, we summarize the three aspects of blockchain technology that are critical in enabling commitment, including data structure, smart contracting, decentralized governance, and we highlight both the advantages of blockchain-based commitment over traditional approaches (e.g., collateral) and its limitations.

5.1 Constant token growth as commitment to investment

To illustrate the impact of commitment brought forth by the blockchain technology, we consider a specific case where:

$$dM_t = F(L_t, A_t) / P_t dt = \mu^M M_t dt, \quad (34)$$

i.e., a constant growth rate of token supply in the interior region ($dD_t = 0$) to finance the enhancement of platform productivity. This commitment is popular among blockchain applications primarily as a way to address users' concern over token-holding dilution via inflation.³¹ We show that the fundamental role of such commitment actually lies in the mitigation of underinvestment. This rule of token supply implies that the resources a platform gathers, L_t , become a predetermined function of the state variables.

We still allow the entrepreneur to receive token dividends and buy back tokens, but with L_t following a predetermined rule, the entrepreneur's only control variable is dD_t . The following HJB equation characterizes the value function:

$$\begin{aligned} rV(M_t, A_t) dt = \max_{dD_t} & P_t dD_t [\mathbb{I}_{\{dD_t \geq 0\}} + (1 + \chi) \mathbb{I}_{\{dD_t < 0\}}] + V_{M_t} (\mu^M M_t dt + dD_t) \\ & + V_{A_t} A_t L_t \mu^H dt + \frac{1}{2} V_{A_t A_t} A_t^2 L_t^2 (\sigma^H)^2 dt. \end{aligned} \quad (35)$$

Comparing it with Equation (9), the tokens used to pay for L_t is replaced by $\mu^M M_t dt$. Under the same change of variable as in Section 4.1, we have, in the interior region ($dD_t = 0$)

$$\begin{aligned} rv(m_t) = v'(m_t) & \frac{(L_t + \frac{\theta}{2} L_t^2)}{P(m_t)} + [v(m_t) - v'(m_t) m_t] \left[\delta \mu^H L_t + \frac{1}{2} \delta (\delta - 1) (\sigma^H)^2 L_t^2 \right] \\ & + \frac{1}{2} v''(m_t) m_t^2 \delta^2 (\sigma^H)^2 L_t^2, \end{aligned} \quad (36)$$

where L_t , as a function of m_t , is implicitly defined by:

$$L_t + \frac{\theta}{2} L_t^2 = \mu^M P(m_t) m_t. \quad (37)$$

As the left side of (37) is an increasing and convex function of L_t , investment increases in the A' -scaled token market capitalization, i.e., $P(m)m = P_t M_t / A'_t$. Intuitively, when tokens are

³¹Blockchain applications emphasize predetermined rules of token-supply growth. Ethereum has roughly fixed increments while Bitcoin's minting rate is a constant that halves as the system matures via a longer transaction chain (specifically, every 210,000 more blocks).

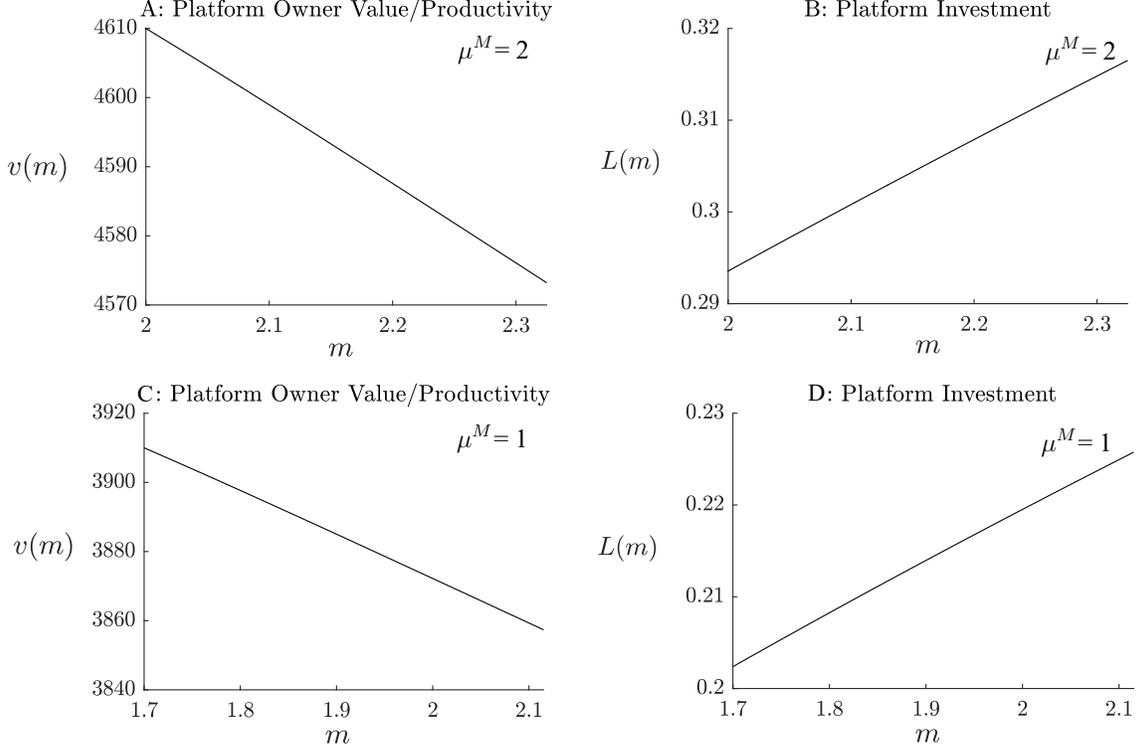


Figure 6: **Platform Value and Investment under Predetermined Token-Supply Growth.** Panels A and C plot, respectively, the A'_t -scaled value functions under $\mu^M = 2$ and $\mu^M = 1$. Panel B and D plot, respectively, the optimal investment under $\mu^M = 2$ and $\mu^M = 1$.

more valuable, the constant growth of token supply gathers more resources for development. The boundary conditions are the same as those of the baseline model.

Proposition 5 (Solution under Predetermined Token Growth). *Under the commitment to a constant growth rate of token supply for investment in productivity, the investment given by Equation (37) increases in A' -scaled token market capitalization, i.e., $P_t m_t$. The entrepreneur receives token dividends ($dD_t^* > 0$) when $P_t \geq -V_{M_t}$, and buys back and burns tokens out of circulation ($dD_t^* < 0$) when $-V_{M_t} \geq P_t(1 + \chi)$. Token price is determined by Equation (23) as in the baseline model.*

In our numerical solution, we start with $\mu^M = 2$ for illustrative purpose. Comparing Panel A of Figure 6 with Panel A of Figure 2, we can see that the commitment increases the entrepreneur's value by around 15% near the payout (lower) boundary and around 20% near the buyback (upper) boundary. By comparing Panel B in the two figures, we see that such an increase mainly comes from a higher level of investment as the commitment mitigates the problem of underinvestment under token overhang.

As previously discussed, to the entrepreneur, the value of tokens is $-V_{M_t}$, while to the users, it is P_t . The wedge between $-V_{M_t}$ and P_t widens as the normalized token supply, m_t , increases, and the ratio, $-V_{M_t}/P_t$, reaches $1 + \chi$, the financing cost of token buyback, at the buyback (upper) boundary of m_t . Therefore, as shown in (10), the closer to the buyback boundary, the more concerned the entrepreneur is over the token issuance cost, $-V_{M_t}/P_t > 1$. This implies that investment chosen by the entrepreneur declines in m_t as shown in Panel B of Figure 2. Therefore, commitment to the predetermined investment rule creates more value when m_t is higher and closer to its upper boundary. This explains why the commitment-induced improvement in the entrepreneur's value is greater near the buyback boundary (around 20% increase) than near the payout boundary (around 15% increase).

Another difference between the commitment case and the baseline case of discretionary investment is that under commitment, investment increases in m_t (Panel B of Figure 6), while in the baseline case, investment decreases in m_t (Panel B of Figure 2). As previously discussed, the declining pattern in the baseline case is due to the increasing cost of issuing tokens, i.e., the widening wedge between $-V_{M_t}$ and P_t . The increasing pattern in the commitment case is a numeric result. As m_t increases, token price, $P(m_t)$, decreases, but the A'_t -scaled market capitalization, i.e., $P(m_t)m_t$, may increase or decrease. Under the current parameterization, the $P(m_t)m_t$ increases in m_t , so, according to (37), L_t increases in m_t .

To further demonstrate the economic mechanism, we also consider the solution when the committed growth of token supply is half as large, i.e., $\mu^M = 1$. The impact of reducing token-supply growth rate on investment and the entrepreneur's value is unclear a priori. By reducing inflation, a lower growth rate of token supply tends to increase the token price that users are willing to pay and thus increase L_t , the resources gathered via token issuance. However, as shown in (37), L_t depends on the token market capitalization, i.e., both the unit price and the quantity of tokens, so reducing the token-supply growth can also negatively impact L_t . Under the current parameterization, the latter force dominates, which leads to a lower level of investment under $\mu^M = 1$ (Panel D of Figure 6) than the solution under $\mu^M = 2$ (Panel B of Figure 6). Comparing Panels A and C of Figure 6, we can see that the entrepreneur's value also declines when μ^M declines from 2 to 1.

What is more interesting is that the entrepreneur's value near the payout (lower) boundary of m_t is not only below the value under $\mu^M = 2$ but also around 2% below the value in the baseline case of discretionary investment (Panel A of Figure 2). As previously discussed, the wedge between the entrepreneur's private cost of token issuance, $-V_{M_t}$, and users' valuation, P_t , widens as m_t increases, so commitment adds more value when m_t is higher. Indeed,

the entrepreneur’s value is around 3% higher under $\mu^M = 1$ than in the discretionary case near the buyback (upper) boundary. In contrast, commitment adds less value in the low- m_t region, i.e., near the payout (lower) boundary, because the conflict of interest between the entrepreneur and users is less severe. Therefore, near \underline{m} , the value added from commitment is small while the drawback of commitment—the entrepreneur cannot coordinate investment (L_t) and payout/buyback (dD_t) decisions—dominates. As a result, the entrepreneur’s value actually decreases under commitment relative to the discretionary case.

Regarding the coordination between investment and payout/buyback decisions, its importance can be seen from the difference in the range of m_t on the horizontal axis between commitment cases (Figure 6) and the baseline case (Figures 2 and 3). Under commitment, the range is much smaller. The entrepreneur pays the financing cost to buy back and burn tokens at a much lower level of m_t (i.e., chooses a lower buyback boundary). Moreover, the payout boundaries under commitment (both $\mu^M = 1$ and $\mu^M = 2$) are higher than that of the discretionary case. Therefore, when the entrepreneur loses control of the amount of tokens issued for investment, she turns more active in payout and buyback, effectively narrowing the equilibrium range of m_t . This results in more frequency payments of the financing cost.

To sum up, commitment to predetermined investment rules adds value by addressing the token overhang problem, but it also forces the entrepreneur to control the token supply more actively via the remaining margins (i.e., payout and buyback), and to pay the financing cost more frequently. Overall, when the former force dominates, the entrepreneur obtains a higher value via commitment: A higher level of investment translates into a higher token price through users’ expectation of faster productivity growth, and a higher token price in turn implies more valuable token payout for the entrepreneur.

5.2 Mitigating underinvestment through fees

As demonstrated, commitment enabled by blockchains alleviates the problem of token-overhang and underinvestment, but commitment to predetermined investment rules forces the entrepreneur to manage the token supply through more active payout and buyback, which results in paying the financing cost more frequently. An alternative solution is to finance investment with fees collected from users.

To analyze the impact of fees, let $f_{0,t}$ and $f_{1,t}$ denote respectively the fixed and propor-

tional fees users pay at t . The users' objective (6) is modified to:

$$\max \left\{ 0, \max_{k_{i,t} > 0} \left[(P_t k_{i,t})^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha dt + k_{i,t} \mathbb{E}_t [dP_t] - (\phi + f_{t,0}) dt - P_t k_{i,t} (r + f_{1,t}) dt \right] \right\}. \quad (38)$$

Under the parameterized distribution of u_i , we follow the same procedure to solve the participation threshold and then obtain a new measure of users:

$$N_t = A_t' \left(\frac{\alpha}{\omega(\phi + f_{0,t})} \right)^{\frac{\xi}{1-\xi\gamma}} \left(\frac{1-\alpha}{r + f_{1,t} - \mu^P(m_t)} \right)^{\left(\frac{\xi}{1-\xi\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)} \equiv A_t' n(f_{0,t}, f_{1,t}, m_t). \quad (39)$$

It is clear that $\frac{\partial n(f_{0,t}, f_{1,t}, m_t)}{\partial f_{0,t}} < 0$ and $\frac{\partial n(f_{0,t}, f_{1,t}, m_t)}{\partial f_{1,t}} < 0$ (as $\alpha \in (0, 1)$ and $\xi \in (1, 1/\gamma]$). The platform faces a trade-off. Higher fees, either via $f_{0,t}$ or $f_{1,t}$, lead to lower user participation, which directly reduces the revenue from fixed fees, i.e., $f_{0,t} N_t$.³²

Meanwhile, higher fees also reduce users' token demand, which leads to a lower token price and proportional fees. We follow the same procedure of solving users' token demand, $k_{i,t}$, and then from the token market-clearing condition, we obtain:

$$P(f_{0,t}, f_{1,t}, m_t) = \frac{\xi}{m_t(\xi - 1)\omega^{\frac{\xi}{1-\xi\gamma}}} \left(\frac{\alpha}{\phi + f_{0,t}} \right)^{\frac{\xi}{1-\xi\gamma}-1} \left(\frac{1-\alpha}{r + f_{1,t} - \mu^P(m_t)} \right)^{1+\left(\frac{\xi}{1-\xi\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)}. \quad (40)$$

We have $\frac{\partial P(f_{0,t}, f_{1,t}, m_t)}{\partial f_{0,t}} < 0$ and $\frac{\partial P(f_{0,t}, f_{1,t}, m_t)}{\partial f_{1,t}} < 0$ (as $\frac{\xi}{1-\xi\gamma} - 1 > 0$ under $\xi\gamma < 1$), so token price is negatively affected by the platform imposing fees on users.

Overall, when choosing fees, the entrepreneur solves a problem akin to a monopolistic producer, trading off unit prices and quantities. The total fee revenues are equal to $f_{0,t} N_t + f_{1,t} P_t M_t$, where we apply the token market-clearing condition, $\int_{i \in [0,1]} k_{i,t} di = M_t$ to substitute out $k_{i,t}$. Such revenues can be used to finance investment in productivity so that the need to issue tokens is reduced. The new law of motion of token supply is thus given by:

$$dM_t = \frac{F(L_t, A_t) - (f_{0,t} N_t + f_{1,t} P_t M_t)}{P_t} dt + dD_t, \quad (41)$$

³²Note that the users' expectation of the rate of token price change, $\mu^P(m_t)$, is a function of the key state variable m_t , which the platform takes as given when choosing its fees.

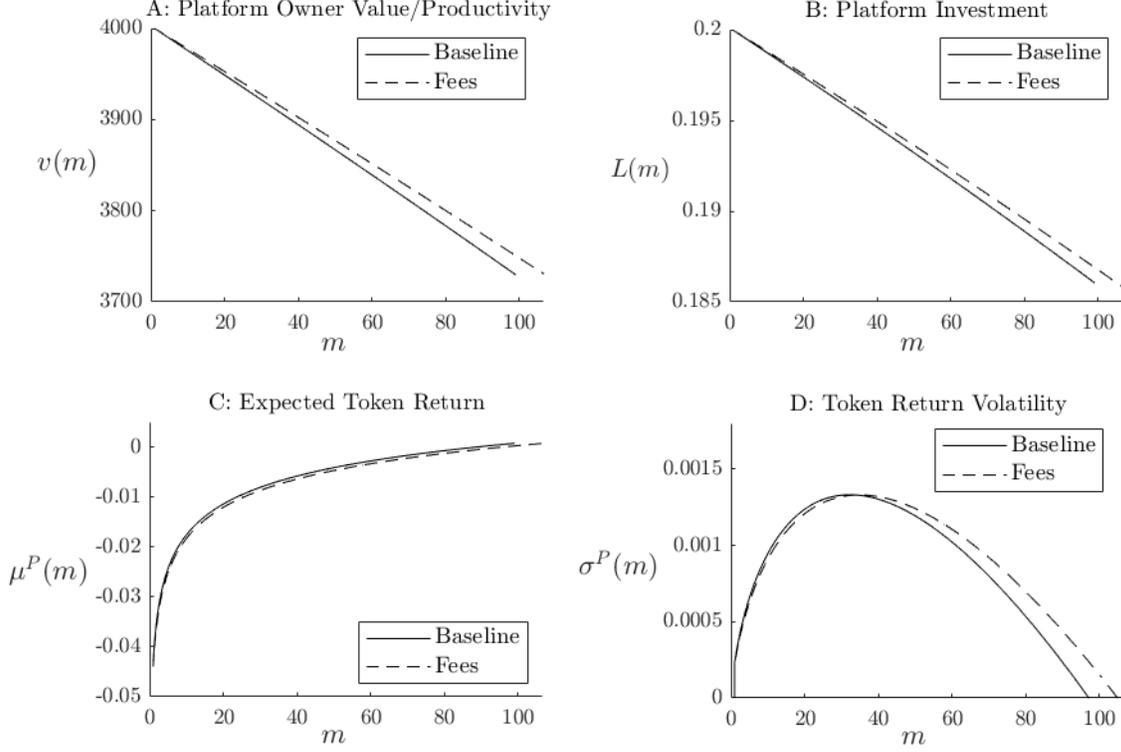


Figure 7: **Platform Value, Investment, and Token Price Dynamics under Fees.** Panel A, B, C, and D compare, respectively, the entrepreneur’s A_t -scaled value function, optimal investment, the P_t -scaled token-price drift, and the P_t -scaled token-price diffusion under fees and in the baseline case without fees.

and in the interior region where $dD_t = 0$, the platform’s HJB equation (9) becomes:

$$rV(M_t, A_t) dt = \max_{\{f_{0,t}, f_{1,t}, L_t, dD_t\}} V_{M_t} \left[\frac{F(L_t, A_t) - (f_{0,t}N_t + f_{1,t}P_tM_t)}{P_t} dt \right] + V_{A_t} A_t L_t \mu^H dt + \frac{1}{2} V_{A_t A_t} A_t^2 L_t^2 (\sigma^H)^2 dt. \quad (42)$$

Under a negative marginal value of outstanding token supply, i.e., $V_{M_t} < 0$, the entrepreneur chooses fees to minimize the expression in the square bracket on the right side of (42). Allowing the platform to charge fees provides an additional source of revenues that likely increases investment and the entrepreneur’s value, especially when the likelihood of costly token buyback is high near the upper (buyback) boundary of m_t .

Introducing fees entails costs and benefits that are associated with traditional platform businesses. There are many studies on the complex economic forces that drive the determination of fees on platforms (Hagiu, 2006; Rochet and Tirole, 2006; Rysman, 2009). Given our focus on tokens, our setup does not capture all those forces. Therefore, we do not address

the question of optimal fee setting. Instead, we illustrate the impact of fees in Figure 7 using a particular fee structure $f_{0,t} = 0.1\%$ and $f_{1,t} = 0.1\%$.³³ The fee revenue, $f_{0,t}N_t + f_{1,t}P_tM_t$, varies endogenously in the model, along with the user base, N_t , the token price, P_t , and the outstanding token amount, M_t . The constant fee parameters allow us to remain as close as possible to the baseline model and its solution method.

The dashed line in Panel A of Figure 7 shows that fees increase the platform owner’s value especially near the buyback (upper) boundary of m_t . As previously discussed, near the upper boundary, the token issuance cost is high, so the platform refrains from token-financed investment. This underinvestment problem is now mitigated by fee revenues (Panel B of Figure 7). Introducing fees does not significantly affect the dynamics of token price (Panels C and D of Figure 7) in terms of the drift and diffusion. A notable difference between our main model and the model with fees is that under fees, the entrepreneur delays costly token buyback, which is reflected in a higher upper boundary of m_t (Panel D of Figure 7). This additional financial slack also helps to boost the entrepreneur’s value.

Overall, fees increase the entrepreneur’s value by both alleviating the underinvestment problem and allowing the platform to reduce the impact of financing costs by postponing token buybacks. Our analysis thus points towards an interesting direction for future research: Analogous to macroeconomic management through monetary and fiscal policies, a platform may dynamically manage its state-contingent token-supply policy and fee structure.

6 Conclusion

We develop a dynamic model of a platform economy, where tokens are used as a means of payment among users and issued to finance platform operation and growth. Tokens facilitate user transactions and compensate distributed ledger-keepers, open-source developers, and crowdfunders for their contributions to platform development. The platform owners maximize their seigniorage by managing token supply, subject to the conditions that users optimally decide on token demand and rationally form expectation of token price dynamics.

We characterize the optimal token-supply strategy and its implications for user-base dynamics, endogenous platform growth, and token price dynamics. A key mechanism is

³³When fees are set by centralized platform owners, they are relatively stable, as we see in IBM Blockchain’s flat fee for IBM Cloud, for example (<https://cloud.ibm.com/docs/blockchain?topic=blockchain-ibp-saas-pricing>). Fee setting on permissionless, decentralized platforms has traditionally been linked to network congestion and service capacity, which are outside our model but discussed in Cong, Li, and Wang (2018) and Basu, Easley, O’Hara, and Sirer (2019).

the wedge between insiders' (the platform owners') token valuation and that of outsiders (users). When the valuation wedge falls to zero, the platform owners optimally receive token dividends; when it rises to an endogenously determined threshold, the platform optimally burns tokens out of circulation to stabilize token value. The wedge creates underinvestment in platform productivity under the financing cost of token buyback.

By enabling commitment, blockchains enable rule-based token supply, thereby mitigating underinvestment by overcoming the platform owners' time inconsistency. Financing investment with fees charged on users reduces the investment inefficiency at the expense of user participation and token demand. Beyond the main focus on token-financed platform development, our paper provides broad implications of dynamic token allocation for token price, user adoption, stablecoins, among other issues.

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