The Commitment Benefit of Consols in Government Debt Management*

Davide Debortoli† Ricardo Nunes‡ Pierre Yared§

October 6, 2021

Abstract

We consider optimal government debt maturity in a deterministic economy in which the government can issue any arbitrary debt maturity structure and in which bond prices are a function of the government's current and future primary surpluses. The government sequentially chooses policy, taking into account how current choices—which impact future policy—feed back into current bond prices. We show that issuing consols constitutes the unique stationary optimal debt portfolio, as it boosts government credibility to future policy and reduces the debt financing costs.

Keywords: Public debt, optimal taxation, fiscal policy

JEL Classification: H63, H21, E62

*For helpful comments, we are grateful to the editor and two anonymous referees; to Fernando Alvarez, Manuel Amador, Albert Marcet, Chris Moser, Rigas Oikonomou, Juan Pablo Nicolini, Facundo Piguillem, Jesse Schreger, Kjetil Storesletten, Harald Uhlig, and seminar participants at Columbia, Federal Reserve Bank of Boston, Humboldt, Sciences-Po, Surrey, University of Toronto, ASSA 2019 meetings, Royal Economic Society at Warwick, EEA-ESEM at Manchester, SED at St. Louis, and the Money Macro and Finance at LSE for comments. George Vojta provided excellent research assistance. Davide Debortoli acknowledges the financial support of the Spanish Agencia Estatal de Investigación through grants RyC-2016-20476 and PID2020-116268GB-I00 and through the Severo Ochoa Programme for Centres of Excellence in R&D (Barcelona School of Economics CEX2019-000915-S).

†Universitat Pompeu Fabra, CREi, Barcelona School of Economics and CEPR: davide.debortoli@upf.edu.

‡University of Surrey and CIMS: ricardo.nunes@surrey.ac.uk.

§Columbia University and NBER: pyared@columbia.edu.

1 Introduction

Numerous governments across the world have extended the maturity of their public debts. For example, since 2016, France, Indonesia, and Mexico have issued 50-year bonds, while Austria, Israel, and Peru have issued 100-year bonds.[1] The extension of the U.S. government debt maturity has been proposed by Cochrane (2015) and Orszag et al. (2021) and is under consideration by the U.S. Treasury.[2] Arguments in favor of maturity extension generally rest on the notion that long-term financing costs are low relative to future interest rate risks. Bhandari et al. (2021) verify these insights by introducing interest rate risk to a calibrated fiscal policy model and showing that the optimal maturity structure is very long and takes the (approximate) form of a consol.[3]

In this paper, we provide an alternative rationale for the optimal issuance of long maturities. We argue that consols are optimal because they boost government credibility to future fiscal policy and reduce the financing costs for the government. To illustrate this point, we consider a deterministic environment in which bond prices are a function of the government's current primary surplus and the path of future primary surpluses. The government sequentially chooses taxes and any arbitrary debt issuance strategy taking into account the impact of current policy on bond prices. Our main finding is that the unique stationary distribution of government debt is flat, with the government owing the same amount to the private sector at all future dates.

We establish this result in the dynamic fiscal policy model of Lucas and Stokey (1983). This is an economy with exogenous public spending and no capital in which the government chooses linear taxes on labor and issues public debt to finance government spending. In this environment, if the government could commit to policy at the beginning of time, then the choice of government debt maturity would be indeterminate. This is because any debt maturity structure would satisfy the present value constraints of the government at a given point in time.

We do not assume that the government commits ex-ante to policy, but we instead consider the sequentially optimal policy. More specifically, we characterize the Markov Perfect Competitive Equilibrium (MPCE). In our setup, the government without commitment chooses taxes and debt at every date, taking into account how current policy affects the price of bonds through expectations of future policy. Moreover, the government may

[1] See, e.g. Reuters (01-18-2021).

[2] See, e.g. Bloomberg (19-10-2021).

[3] Consols have been used in the past, for instance, consols were the largest component of the British government's debt during the Industrial Revolution (see Mokyr (2011)).

decide not to follow the optimal commitment policy. We characterize the entire set of MPCE's in a deterministic economy, including those with potentially discontinuous policy functions both on and off the equilibrium path.[4] Since we allow for any unconstrained structure of maturity issuance, the payoff relevant state—the government's portfolio of inherited maturities—is an infinite-dimensional and potentially complicated object. We focus our attention on the stationary maturity distribution that emerges when the inherited portfolio of maturities equals the issued portfolio.

Our main result is that any stationary maturity distribution under lack of commitment must be flat, with the government owing the same amount at all future dates. The fact that a flat maturity distribution is stationary is not surprising. Under a flat maturity distribution, the government lacking commitment can choose a tax rate to repay the debt immediately due without rebalancing its portfolio. The chosen tax rate coincides with the optimum under full commitment, and therefore maximizes government's welfare. What is less obvious is why no other maturity distribution is stationary. The reason is that a government that inherits a non-flat maturity distribution would always take advantage of the situation to front-load or back-load taxes in order to change interest rates. When the government does this, the issued maturity distribution does not coincide with the inherited one, and is therefore not stationary.

For example, suppose that the government inherits more long-term liabilities than short-term ones. Rather than issuing the same maturity distribution as the inherited one, the government can change taxes so as to increase short-term interest rates. This relaxes the government budget constraint by decreasing the market value of outstanding long-term liabilities, making the government strictly better off. The opposite is true if the government inherits more short-term liabilities than long-term ones. In this case, a policy that decreases short-term interest rates makes the government strictly better off by increasing the market value of newly issued liabilities.

We apply this simple logic to analyze the behavior of the government inheriting any infinite-dimensional maturity distribution. We show that only if the inherited maturity distribution is flat is the government unable to take advantage of imbalances in debt positions to relax its budget constraint. As such, any stationary maturity distribution must be flat.

Related Literature

The main contribution of this paper is to characterize optimal fiscal policy without

[4] In this regard, our approach is similar in spirit to that of Cao and Werning (2018) in their analysis of Markov equilibria in the hyperbolic consumption model.

commitment in the deterministic case of the Lucas and Stokey (1983) model. Lucas and Stokey (1983) argue that there is no distortion due to lack of commitment, since the government can structure its debt maturity to guarantee commitment to optimal fiscal policy by future governments.[5] However, Debortoli et al. (2021) show that this result does not generally hold in the same model as Lucas and Stokey (1983). As such, our analysis of optimal fiscal policy considers the entire set of MPCE's, not only the ones which coincide with the commitment policy (should they exist). Our paper is most related to the quantitative analysis of Debortoli et al. (2017), which considers lack of commitment under shocks and incomplete markets, either restricting the time-horizon (i.e., a three-period economy) or set of available maturities, and show that the optimal maturity converges to an approximately flat distribution.[6] In contrast to that work, we focus here on deterministic economies, where the time-horizon and the debt structure are unrestricted. We achieve general analytical results for the entire set of potential MPCE's, and show that issuing consols constitutes the *unique* stationary optimal debt portfolio.

This paper more broadly contributes to the literature on optimal government debt maturity in the absence of government commitment.[7] Our model is most applicable to economies where the risk of default and surprise inflation is not salient, but the government is still not committed to a path of taxes and debt maturity issuance, which affects the path of risk-free interest rates. For this reason, short-term debt does not dominate long-term debt in minimizing the government's lack of commitment problem in our framework. Even if the government were to issue only short-term debt, the government ex-post would deviate from the ex-ante optimal policy by pursuing policies which reduce short-term interest rates below the ex ante optimal level.[8]

The paper proceeds as follows. In Section 2, we describe the model. In Section 3, we formally define an MPCE. Section 4 establishes that any stationary maturity distribution under lack of commitment is flat. Section 5 concludes, and an online Appendix contains all the proofs, and additional results not included in the text.

[5]See also Alvarez et al. (2004) and Persson et al. (2006).

[6]Faraglia et al. (2010), Guibaud et al. (2013), and Lustig et al. (2008) also analyze optimal debt maturity in a stochastic setting, but they assume full commitment. Krusell et al. (2006) and Debortoli and Nunes (2013) consider instead a framework without commitment, but with only one-period bonds.

[7]Aguiar et al. (2019), Arellano and Ramanarayanan (2012), Dovis (2019), Fernandez and Martin (2015), Niepelt (2014), among others, analyze optimal government debt maturity and default risk. Bocola and Dovis (2019) additionally consider the effects of liquidity risk. Bigio et al. (2017) analyze debt maturity in a model with transactions costs. Missale and Blanchard (1994) and Arellano et al. (2013) consider lack of commitment when surprise inflation is possible.

[8]Note that in our closed-economy environment, the risk-free interest rate is endogenous and responds to fiscal policies, due to the concavity of the utility function. The same would happen in large open-economy environments or small-open economy environments with limited capital flows.

2 Model

2.1 Environment

We consider an economy identical to the deterministic case of Lucas-Stokey. There are discrete time periods $t = \{0, 1, ..., \infty\}$. The resource constraint of the economy is

$$c_t + g = n_t, \tag{1}$$

where c_t is consumption, n_t is labor, and $g > 0$ is government spending, which is exogenous and constant over time.

There is a continuum of mass 1 of identical households that derive the following utility:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right), \ \beta \in (0, 1). \tag{2}$$

$u(\cdot)$ is strictly increasing in consumption, strictly decreasing in labor, globally concave, and continuously differentiable. We also assume that $u_{cc}(c, c+g) + u_{cn}(c, c+g) < 0$ so that the marginal utility of consumption is decreasing in consumption in general equilibrium. As a benchmark, we define the first best consumption and labor $\left\{c^{fb}, n^{fb}\right\}$ as the values of consumption and labor that maximize $u(c_t, n_t)$ subject to the resource constraint (1).

Household wages equal the marginal product of labor (which is 1 unit of consumption), and are taxed at a linear tax rate τ_t. $b_{t,k} \gtreqless 0$ represents government debt purchased by a representative household at t, which is a promise to repay 1 unit of consumption at $t + k > t$. $q_{t,k}$ is the bond price at t. At every t, the household's allocation and portfolio $\left\{c_t, n_t, \{b_{t,k}\}_{k=1}^{\infty}\right\}$ must satisfy the household's dynamic budget constraint:

$$c_t + \sum_{k=1}^{\infty} q_{t,k}\left(b_{t,k} - b_{t-1,k+1}\right) = \left(1 - \tau_t\right) n_t + b_{t-1,1}. \tag{3}$$

Moreover, the household's transversality condition is

$$\lim_{T\to\infty} q_{0,T} \sum_{k=1}^{\infty} q_{T,k} b_{T,k} = 0$$

The variable $B_{t,k} \gtreqless 0$ represents debt issued by the government at t with a promise to repay 1 unit of consumption at $t+k > t$. At every t, government policies $\left\{\tau_t, g_t, \{B_{t,k}\}_{k=1}^{\infty}\right\}$

must satisfy the government's dynamic budget constraint:

$$g_t + B_{t-1,1} = \tau_t n_t + \sum_{k=1}^{\infty} q_{t,k} \left(B_{t,k} - B_{t-1,k+1} \right).^{9} \tag{4}$$

The economy is closed, which means that the bonds issued by the government equal the bonds purchased by households:

$$b_{t,k} = B_{t,k} \; \forall t, k. \tag{5}$$

Initial debt $\{B_{-1,k}\}_{k=1}^{\infty} = \{b_{-1,k}\}_{k=1}^{\infty}$ is exogenous. We assume that there exist debt limits:

$$b_{t,k} \in \left[\underline{b}, \overline{b} \right] \;\; \forall t, k. \tag{6}$$

In our recursive analysis, we will consider economies where these limits are not binding along the equilibrium path. The government is benevolent and shares the same preferences as the households in (2).

2.2 Primal Approach

We follow Lucas-Stokey by taking the primal approach to the characterization of competitive equilibria, since this allows us to abstract away from bond prices and taxes. Let

$$\{c_t, n_t\}_{t=0}^{\infty} \tag{7}$$

represent a sequence of consumption and labor allocations. We can establish necessary and sufficient conditions for (7) to constitute a competitive equilibrium. The household's optimization problem implies the following intratemporal and intertemporal conditions, respectively:

$$1 - \tau_t = -\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} \text{ and } q_{t,k} = \frac{\beta^k u_c(c_{t+k}, n_{t+k})}{u_c(c_t, n_t)}. \tag{8}$$

[9] We follow the same exposition as in Angeletos (2002) in which the government rebalances its debt in every period by buying back all outstanding debt and then issuing fresh debt at all maturities. This is without loss of generality. For example, if the government at $t-k$ issues debt due at date t of size $B_{t-k,k}$ which it then holds to maturity without issuing additional debt, then all future governments at date $t-k+l$ for $l = 1, ..., k-1$ will choose $B_{t-k+l,k-l} = B_{t-k,k}$, implying that $B_{t-1,1} = B_{t-k,k}$.

Substitution of these conditions into the household's dynamic budget constraint implies the following condition:

$$u_c(c_t, n_t)\, c_t + u_n(c_t, n_t)\, n_t + \sum_{k=1}^{\infty} \beta^k u_c(c_{t+k}, n_{t+k})\, b_{t,k} = \sum_{k=0}^{\infty} \beta^k u_c(c_{t+k}, n_{t+k})\, b_{t-1,k+1}. \quad (9)$$

Forward substitution into the above equation and taking into account the transversality condition implies the following implementability condition:

$$\sum_{k=0}^{\infty} \beta^k \left(u_c(c_{t+k}, n_{t+k})\, c_{t+k} + u_n(c_{t+k}, n_{t+k})\, n_{t+k} \right) = \sum_{k=0}^{\infty} \beta^k u_c(c_{t+k}, n_{t+k})\, b_{t-1,k+1}. \quad (10)$$

By this reasoning, if a sequence in (7) is generated by a competitive equilibrium, then it necessarily satisfies (1) and (10). We prove in the Appendix that the converse is also true, which leads to the below proposition that is useful for the rest of our analysis.

Lemma 1 ***(competitive equilibrium)*** *A sequence* (7) *is a competitive equilibrium if and only if it satisfies* (1) $\forall t$ *and* (10) *at* $t = 0$ *given* $\{b_{-1,k}\}_{k=1}^{\infty}$.

Note that to prove this result, we establish the fact that the satisfaction of (10) at $t = 0$ guarantees the satisfaction of (10) for all future dates, since bonds can be freely chosen so as to satisfy (10) at all future dates for any given sequence (7).

3 Markov Perfect Competitive Equilibrium

We characterize the MPCE in which the government without commitment chooses taxes and debt at every date, taking into account how current policy affects the price of bonds through expectations of future policy. In this section, we formally define our equilibrium, and then, using the primal approach, we provide a recursive representation of the equilibrium.

3.1 Equilibrium Definition

Formally, let $\mathbf{B}_t \equiv \{B_{t,k}\}_{k=1}^{\infty}$ and $\mathbf{q}_t \equiv \{q_{t,k}\}_{k=1}^{\infty}$. In every period t, the government chooses a policy $\{\tau_t, \mathbf{B}_t\}$ given $\mathbf{B}_{t-1}$. Households then choose an allocation and portfolio $\left\{c_t, n_t, \{b_{t,k}\}_{k=1}^{\infty}\right\}$. An MPCE consists of: a government strategy $\rho(\mathbf{B}_{t-1})$ which is a function of $\mathbf{B}_{t-1}$; a household allocation and portfolio strategy $\omega(\mathbf{B}_{t-1}, \rho_t, \mathbf{q}_t)$ which is a function of $\mathbf{B}_{t-1}$, the government policy $\rho_t = \{\tau_t, \mathbf{B}_t\}$, and bond prices $\mathbf{q}_t$; and a set of bond pricing functions $\left\{\varphi^k\left(\mathbf{B}_{t-1}, \rho_t\right)\right\}_{k=1}^{\infty}$ with $q_{t,k} = \varphi^k\left(\mathbf{B}_{t-1}, \rho_t\right)$ $\forall k \geq 1$ which depend on $\mathbf{B}_{t-1}$ and the government policy ρ_t. In an MPCE, these objects must satisfy the following conditions $\forall t$:

1. The government strategy $\rho\left(\cdot\right)$ maximizes (2) given $\omega\left(\cdot\right)$, $\varphi^k\left(\cdot\right)$ $\forall k \geq 1$, and the government budget constraint (4);

2. The household allocation and portfolio strategy $\omega\left(\cdot\right)$ maximizes (2) given $\rho\left(\cdot\right)$, $\varphi^k\left(\cdot\right)$ $\forall k \geq 1$, and the household budget constraint (3), and

3. The set of bond pricing functions $\varphi^k\left(\cdot\right)$ $\forall k \geq 1$ satisfy (5) given $\rho\left(\cdot\right)$ and $\omega\left(\cdot\right)$.

3.2 Recursive Representation

Given our definition, an MPCE is characterized by an equilibrium consumption and labor sequence (7) and an equilibrium debt sequence $\left\{\{b_{t,k}\}_{k=1}^{\infty}\right\}_{t=0}^{\infty}$, where each element at date t depends on history only through $\mathbf{B}_{t-1}$, the payoff relevant variables. Given this observation, in an MPCE, one can define a function $h^k\left(\cdot\right)$

$$h^k\left(\mathbf{B}_t\right) = \beta^k u_c\left(c_{t+k}, n_{t+k}\right) | \mathbf{B}_t \tag{11}$$

for $k \geq 1$, which equals the discounted marginal utility of consumption at $t+k$ given $\mathbf{B}_t$ at t. The function $h^k\left(\mathbf{B}_t\right)$ is determined by future government policies, which in turn are determined by future government strategies through the function $\rho\left(\cdot\right)$. As such, (11) is taken as given by a government at date t. This function is useful since, in choosing $\mathbf{B}_t$ at date t, the government must consider how it affects future expectations of policy, which in turn affect current bond prices through expected future marginal utility of consumption.

Note that choosing $\{\tau_t, \mathbf{B}_t\}$ at date t from the perspective of the government is equivalent to choosing $\{c_t, n_t, \mathbf{B}_t\}$ where one can write, with some abuse of notation, $\mathbf{B}_t = \{b_{t,k}\}_{k=1}^{\infty}$, and this follows from the primal approach delineated in Section 2.2. Removing the time subscript and defining $\mathbf{B} \equiv \mathbf{B}_{t-1} = \{b_k\}_{k=1}^{\infty}$ as the inherited portfolio of bonds, we can write the government's problem recursively as

$$V(\mathbf{B}) = \max_{c,n,\mathbf{B}'} u(c,n) + \beta V(\mathbf{B}') \tag{12}$$

s.t.

$$c + g = n, \text{ and} \tag{13}$$

$$u_c(c,n)\,c + u_n(c,n)\,n - u_c(c,n)\,b_1 + \sum_{k=1}^{\infty} h^k(\mathbf{B}')\,(b'_k - b_{k+1}) = 0, \tag{14}$$

where (14) is a recursive representation of (9). Let $f(\mathbf{B})$ correspond to the solution to (12) − (14) given $V(\cdot)$ and $h^k(\cdot)$ $\forall k \geq 1$. It therefore follows that the function $f(\cdot)$ necessarily implies functions $h^k(\cdot)$ $\forall k \geq 1$ which satisfy (11). An MPCE is therefore composed of functions $V(\cdot)$, $f(\cdot)$, and $h^k(\cdot)$ $\forall k \geq 1$ that are consistent with one another and satisfy (11) − (14).

Observe that any solution to (12) − (14) is an MPCE: Any choice of consumption and debt today implies a sequence of future consumption and debt policies in the future, where the level of consumption maps directly into a tax rate through the resource constraint and intratemporal condition.

4 Stationary Distribution of Debt Maturity

We focus on characterizing an economy in which the debt maturity distribution is stationary with $b_{t+1,k} = b_{t,k}$, $\forall t, k$, so that government debt maturity is time-invariant. Given the Markovian structure of the solution to the MPCE defined by (12) − (14), such a stationary maturity distribution is associated with tax rates, consumption, and interest rates that are constant over time. In this section, we show that any stationary maturity distribution must be flat, with the government owing the same amount of resources to the private sector at all future dates. To establish this result, we first impose a useful assumption in Section 4.1. In Section 4.2, we use this assumption show that a flat maturity distribution is stationary. Finally, in Section 4.3, we show that no other maturity distribution can be stationary.

4.1 Preliminaries

Before proceeding with our analysis, we impose a useful assumption. Using our recursive notation introduced in Section 3, define $W(\{b_k\}_{k=1}^{\infty})$ as the welfare of the government under full commitment given an initial starting debt position $\{b_k\}_{k=1}^{\infty}$:

$$W\left(\{b_k\}_{k=1}^{\infty}\right) = \max_{\{c_k,n_k\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} \beta^k u\left(c_k, n_k\right) \tag{15}$$

s.t.

$$c_k + g = n_k, \text{ and} \tag{16}$$

$$\sum_{k=0}^{\infty} \beta^k \left(u_c\left(c_k, n_k\right) c_k + u_n\left(c_k, n_k\right) n_k\right) = \sum_{k=0}^{\infty} \beta^k u_c\left(c_k, n_k\right) b_{k+1}. \tag{17}$$

Given Lemma 1, the program in (15) − (17) corresponds to that of a government under full commitment with $b_{-1,k} = b_{k+1}$.

Assumption 1. Consider the solution to (15) − (17) with $b_{k+1} = b \; \forall k \geq 0$. $\forall b \in \left[\underline{b}, \overline{b}\right]$, if the solution exists, then the solution is unique and admits $\{c_k, n_k\} = \{c^*\left(b\right), n^*\left(b\right)\}$ $\forall k \geq 0$, where

$$u_c\left(c^*\left(b\right), n^*\left(b\right)\right) c^*\left(b\right) + u_n\left(c^*\left(b\right), n^*\left(b\right)\right) n^*\left(b\right) = u_c\left(c^*\left(b\right), n^*\left(b\right)\right) b, \tag{18}$$

$$\text{and } c^*\left(b\right) + g = n^*\left(b\right). \tag{19}$$

This assumption states that if a government under full commitment is faced with a flat maturity distribution, then there is a unique optimum in which the government chooses a constant allocation of consumption and labor in the future.[10] This assumption is intuitive. Under a flat maturity distribution, every time period in the program in (15) − (17) is identical in the objective function and in the constraint set, which suggests that the optimal solution is a time-invariant allocation. A sufficient condition for Assumption 1 is that the government's objective is globally concave, which is guaranteed if the function $u_c\left(c, c+g\right)\left(c-b\right) + u_n\left(c, c+g\right)\left(c+g\right)$ is concave in c for all b. This is the case, for example, if the utility function is isoelastic with an elasticity of intertemporal substitution weakly below 1 (e.g., Werning, 2007) and if $\underline{b} = 0$ so that debt is non-negative.

4.2 Flat Maturity Distribution is Stationary

We begin by establishing that if the maturity distribution is flat, then it is stationary.

[10] Assumption 1 requires that the solution exists. If the upper bound on individual maturities $\overline{b}$ exceeds the highest primary surplus that can be raised at the peak of the Laffer curve, then there is no solution under a flat maturity for some high values of debt.

Lemma 2 *Suppose that there exists an MPCE starting from* $\mathbf{B}$ *which satisfies* $b_k = b\ \forall k$ *for some* $b \in \left[\underline{b}, \overline{b}\right]$. *Then,*

1. *In all solutions to* $(12) - (14)$, $c = c^*(b)$ *and* $n = n^*(b)$, *and*

2. *There exists a solution to* $(12) - (14)$ *that admits* $b'_k = b\ \forall k$.

The first part of the lemma states that in any MPCE, if the government inherits a flat maturity distribution with $b_k = b\ \forall k$, then the unique optimal response of the government is to choose consumption and labor that coincide with the commitment optimum. The second part of the lemma implies that one optimal—but not necessarily uniquely optimal—strategy for the government is to choose $b'_k = b\ \forall k$ so that debt is not rebalanced and the maturity distribution continues to be flat in the future. As such, there exists a stationary MPCE with a flat maturity distribution. Importantly, this lemma implies that in any MPCE for which $\mathbf{B}$ is a flat maturity distribution, it is necessary that

$$V(\mathbf{B}) = W(\mathbf{B}) \tag{20}$$

so that there is no welfare loss for the present government due to lack of commitment by future governments.

This result uses Assumption 1—which assumes a unique optimal allocation in the case of full commitment—to achieve an analogous statement of uniqueness for the case of lack of commitment. The logic behind this lemma is that a government inheriting a flat maturity distribution with $b_k = b\ \forall k$ can always decide to not rebalance its debt portfolio and to choose the tax rate associated with $\{c^*(b), n^*(b)\}$. Forward induction on this observation combined with Assumption 1 means that the government is able to achieve the commitment optimum with this strategy while inducing allocation $\{c^*(b), n^*(b)\}$ in all future periods. Note that the government can induce the commitment allocation in the future in *any* MPCE, including those where the government's continuation strategy off the equilibrium path given off equilibrium maturities does not coincide with the commitment solution.

4.3 No Other Maturity Distribution is Stationary

We now turn to the possibility that another maturity distribution is stationary. We show in this section that this is not possible by contradiction using an induction argument. The first step of the induction argument establishes that if a non-flat maturity distribution were stationary, then the debt immediately due, b_1, would be necessarily equal to the primary surplus. The second step of the induction argument establishes that if a non-flat maturity distribution were stationary with b_k equal to the primary surplus for all $k \leq m$, then b_{m+1} would necessarily be equal to the primary surplus. It follows then by induction that b_k equals the primary surplus for all maturities k and that the maturity distribution is flat, leading to a contradiction.

To pursue this induction argument, we establish a preliminary result that allows us to construct perturbations as part of the induction argument. To interpret this lemma, observe that since consumption is constant over time under a stationary maturity distribution, the price of a bond maturing in k periods is β^k.

Lemma 3 *Suppose that given* $\mathbf{B}$*, there exists a solution to* (12) $-$ (14) *with a stationary maturity distribution* $b'_k = b_k \; \forall k$ *and* $b'_l \neq b'_m$ *for some* l, m*. Then there exists another solution to* (12) $-$ (14) *with* $b'_k = \widehat{b} \; \forall k$ *where*

$$\widehat{b} = \sum_{k=1}^{\infty} \beta^{k-1} (1 - \beta) \, b_k. \tag{21}$$

This lemma states that under any MPCE with a stationary maturity distribution that is not flat, the government can choose the same tax rate but issue a flat maturity distribution with the same market value and achieve the same welfare. The proof of this lemma is facilitated by Lemma 2, which characterizes the continuation equilibrium following this choice of a flat maturity. Since current and future taxes and consumption remain unchanged from the issuance of a flat maturity, bond prices and welfare are also unchanged.

Lemma 3 implies that if there exists a stationary maturity distribution that is not flat, then the corresponding welfare is equal to that achieved under a flat maturity distribution with the same market value. Moreover, from (20), welfare under this MPCE equals that under commitment associated with a flat maturity distribution with the same market value:

$$V(\mathbf{B}) = W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k = \widehat{b} \; \forall k} = \frac{u(c(\widehat{b}), n(\widehat{b}))}{1 - \beta}. \tag{22}$$

Lemma 3 is useful since it characterizes welfare under a stationary maturity distribution that is not flat. Moreover, it allows us to consider off-equilibrium welfare following a deviation in maturity issuance strategy by the government, which is useful for establishing the first step of our induction argument in the next lemma.

Lemma 4 *Suppose that given* $\mathbf{B}$*, there exists a solution to* (12) $-$ (14) *with a stationary maturity distribution* $b'_k = b_k \; \forall k$ *and for which* $\{c, n\} \neq \{c^{fb}, n^{fb}\}$*. Then,* $\mathbf{B}$ *must satisfy*

$b_1 = \widehat{b}$ *for* $\widehat{b}$ *defined in* (21).

This lemma states that in any stationary maturity distribution in which the tax rate is not zero (so that consumption and labor do not equal the first best), short-term debt b_1 equals the annuitized value of total debt $\widehat{b}$. Therefore, the primary surplus equals the short-term debt b_1 and net debt issuance is zero.

The proof rests on showing that if the primary surplus is in excess of, or below, this short-term debt b_1, then the government can pursue a deviation from a smooth consumption path to boost welfare. For example, if the primary surplus is in excess of what the government immediately owes, then pursuit of a smooth consumption path would require the government to buy back some of its long-term debt. Rather than following a stationary debt issuance strategy, the government can back-load consumption to increase short-term interest rates and reduce the value of the long-term debt which it buys back. Since the deviation is beneficial, the maintenance of a stationary debt maturity distribution is not optimal if the primary surplus exceeds b_1.

If instead the primary surplus is below what the government immediately owes, then pursuit of a smooth consumption path would require the government to issue fresh debt in order to repay current short-term debt. Rather than following a stationary debt issuance strategy, the government can front-load consumption to decrease short-term interest rates and increase the value of newly issued debt. Since the deviation is beneficial, the maintenance of a stationary debt maturity distribution is not optimal if the primary surplus is below b_1.

Note that in constructing these deviations, we utilize Lemmas 2 and 3 which allow us to characterize the change in welfare if the government issues a flat government debt maturity today as part of its deviation. As such, we can explicitly show that these deviations increase welfare by relaxing the government's budget constraint.

Observe that the reason why our argument does not hold under a stationary distribution of debt maturities with zero taxes is that in this case, it is not possible to relax the government budget constraint further.

We now use analogous arguments to establish the second step of the induction argument.

Lemma 5 *Suppose that given* $\mathbf{B}$, *there exists a solution to* (12) $-$ (14) *with a stationary maturity distribution* $b'_k = b_k\ \forall k$ *and for which* $\{c, n\} \neq \left\{c^{fb}, n^{fb}\right\}$. *If* $b_l = \widehat{b}\ \forall l \leq m$, *then* $\mathbf{B}$ *must satisfy* $b_{m+1} = \widehat{b}$ *for* $\widehat{b}$ *defined in* (21).

This lemma considers the stationary maturity distribution when all bond maturities below m equal the primary surplus of the government (the annuitized value of government debt). When this is the case, then the bond of maturity $m+1$ must also equal the primary surplus of the government.

The argument, which relies on a proof by contradiction, starts from the fact that under a stationary maturity distribution, government's welfare satisfies (22), and thus equals welfare under commitment with a flat maturity distribution with the same market value. Now if the amount owed at date $m+1$ does not also equal the primary surplus, then there exists a feasible deviation from a stationary debt issuance strategy that can increase welfare above (22), leading to a contradiction.

More specifically, if $b_l = \widehat{b}\ \forall l \leq m$ but $b_{m+1} \neq \widehat{b}$, a feasible strategy for the government today is to continue to choose the same consumption and labor allocation today $\{c(\widehat{b}), n(\widehat{b})\}$ but to deviate by not retrading the inherited maturities (i.e., letting the bonds mature to next period). Such a deviation is feasible whatever the expectations of future policy and their impact on current bond prices since the government is not rebalancing its portfolio.

Without needing to specify the exact form of the continuation equilibrium, we can show that this deviation must necessarily increase welfare. The argument rests on putting a lower bound on the welfare of future governments following the deviation based on the feasible policies at their disposal. More specifically, note that after the initial deviation, future governments also have the opportunity to pursue the same strategy of choosing consumption and labor equal to $\{c(\widehat{b}), n(\widehat{b})\}$ and not rebalancing the portfolio of maturities. This is true up until some future date m periods in the future. Based on this logic, the welfare of the government today from pursuing the deviation must weakly exceed

$$\sum_{l=0}^{m-1} \beta^l u(c(\widehat{b}), n(\widehat{b})) + \beta^m V(\widehat{\mathbf{B}}\,(m)) \tag{23}$$

where $\widehat{\mathbf{B}}\,(m)$ satisfies $\widehat{b}\,(m)_k = b_{k+m}\ \forall k \geq 1$. Thus, for the initial deviation to be weakly dominated, this requires (23) to be weakly exceeded by (22), so that

$$V(\widehat{\mathbf{B}}\,(m)) \leq \frac{u(c(\widehat{b}), n(\widehat{b}))}{1-\beta}. \tag{24}$$

However, since $b_{m+1} \neq \widehat{b}$, the arguments of Lemma 4 imply that (24) cannot hold, leading to a contradiction. Intuitively, m periods into the future after following a strategy of no rebalancing, the primary surplus is above or below the debt immediately due. At this point in the future, pursuing a strategy that back-loads or front-loads consumption strictly increases welfare relative to a smooth consumption policy with a stationary debt issuance strategy. Therefore, the immediate deviation prior to reaching this m'th period is beneficial, and the maintenance of a stationary debt maturity distribution is not optimal.

Proposition 1 ***(flat maturity)*** *Suppose that conditional on* $\mathbf{B}$*, there exists a solution to* (12) − (14) *with a stationary maturity distribution* $b'_k = b_k \; \forall k$ *and for which* $\{c, n\} \neq \left\{c^{fb}, n^{fb}\right\}$*. Then it is necessary that* $b_k = \widehat{b} \; \forall k$ *so that the maturity distribution is flat.*

This proposition represents the main result of the paper. It states that if the maturity distribution is stationary and if the equilibrium does not entail first best consumption and labor, then the maturity distribution is flat. The reasoning for the proposition follows from induction arguments which appeal to Lemmas 4 and 5. Intuitively, if maturity distribution is not flat, then there are opportunities for the government take advantage of these imbalances to decrease the market value of its inherited portfolio or increase the market value of its newly-issued portfolio. Note that this result holds in any MPCE and does not appeal to any assumptions regarding the behavior of future governments.

Our result relies on the stationary maturity distribution not being associated with first best consumption and labor. Under such a stationary distribution, taxes would be zero, the market value of debt would be sufficiently negative to finance the stream of government spending forever, and the marginal benefit of resources for the government would be zero. For this reason, the stationary maturity distribution is not determined in this circumstance. We can trivially rule out this case if there are exogenous bounds on government debt which prevent such asset accumulation for the government.

Corollary 1 *Suppose that* $\underline{b} > -g$*. Then if conditional on* $\mathbf{B}$*, there exists a solution to* (12) − (14) *with a stationary maturity distribution* $b'_k = b_k \; \forall k$*, it is necessary that* $b_k = \widehat{b}$ $\forall k$ *so that the maturity distribution is flat.*

Finally, returning to Lemma 2, note that Proposition 1 also implies that starting from a flat maturity distribution, the unique continuation equilibrium requires the issuance of a flat maturity distribution. Therefore, in any MPCE, a flat government debt maturity is an absorbing state, and *all* flat maturity distributions are stationary.

Corollary 2 *Suppose that* $\mathbf{B}$ *satisfies* $b_k = b \; \forall k$ *for some* b *and that* $\{c, n\} \neq \left\{c^{fb}, n^{fb}\right\}$*. Then, in all solutions to* (12) − (14) $b'_k = b \; \forall k$.

Starting from a flat maturity distribution, the current government would like to guarantee a constant level of consumption and labor going forward. Choosing a maturity distribution that is not flat cannot guarantee such a continuation equilibrium, since future governments will deviate from a smooth policy in order to relax the government budget constraint. For this reason, the government chooses a flat maturity distribution, and a flat maturity distribution is an absorbing state.

4.4 Discussion

We have established that the unique stationary distribution of debt maturity is flat. A natural question concerns whether an MPCE converges to a stationary distribution over time. We can show that if the government program is concave at all dates and the MPCE coincides with the commitment solution—so that the analysis of Lucas and Stokey (1983) applies—then there exists an MPCE that converges to a stationary distribution if the inherited maturities at date 0 are flat beyond some horizon. Establishing an analogous result in the cases where the MPCE does not coincide with the commitment solution is challenging given the infinite choice of debt maturities. In light of this limitation, we can establish an analogous convergence result under a finite horizon using numerical methods in these cases.[11] These observations are consistent with the quantitative analysis of Debortoli et al. (2017) who consider a stochastic economy with either limited debt maturities or a finite time horizon, and who find that the maturity distribution transitions to an approximately flat one over time.

Another consideration is whether our results extend to an environment in which government spending follows a deterministic path but is not constant. In the special case of quasi-linear preferences that are linear in labor, we can construct examples where there exists an MPCE that transitions to a flat maturity, with total debt increasing or decreasing to accomodate evolving government spending needs, while preserving a smooth consumption path.[12] In this case the maturity is still flat but this policy differs from that in our current environment, since there is retrading; new debt is issued or old debt is bought back in different periods.[13] While this special case admits a flat maturity, it is not generally the case that an MPCE under deterministically evolving spending needs to admit a flat maturity structure, since counterexamples under alternate preferences can be constructed.

Another consideration is what happens in the presence of shocks and incomplete mar-

[11]See earlier version of our working paper Debortoli et al. (2018).

[12]Any deviation from a flat maturity would induce back-loading or front-loading consumption as described in the discussion of Lemma 4.

[13]While we can prove the existence of this MPCE, we cannot prove its uniqueness using our methods which rely on the absence of retrading.

kets. While exact analytical results are not feasible in such a setting, Debortoli et al. (2017) achieve numerical results while restricting the time horizon or the set of available government maturities. Consistent with our analysis, they find that the maturity distribution transitions to an approximately flat one that is actively managed by the government.

A final question concerns whether our results apply to the case of a growing economy with nominal debt. To address this, we can extend our framework to allow for a constant growth rate in labor productivity, a constant growth rate in spending, a constant inflation rate, and nominal—as opposed to real—government bonds.[14] Such an extension incorporates important features of the U.S. economy and it implies that nominal GDP grows at a constant long-run rate under a constant tax rate. In this environment, all of our results hold and the analog of a flat maturity structure is a consol with coupons that grow at the rate of nominal GDP.

5 Concluding Remarks

In this paper, we characterize optimal fiscal policy and debt management when the government reoptimizes sequentially. We consider an MPCE in which the government chooses policy as a function of the infinite-dimensional portfolio of government bonds that it inherits in every period. Our analysis applies to the entire set of MPCE's, including those that do not coincide with the commitment policy or those with potentially discontinuous policy functions both on and off the equilibrium path. We find that any stationary distribution of debt maturity must be flat, with the government owing the same amount at all future dates. Our analysis thus provides a theoretical argument for the use of consols in debt management based on the sequential optimization of fiscal policy by the government.

In our framework, we have considered a situation in which a government's objective in its debt issuance strategy is to minimize its financing costs. In practice, government debt management offices also pursue other objectives, such as supporting financial stability. For example, this can be achieved either by providing liquidity to segments of the market that lack it or through the bond auction process, which itself may serve a purpose of aggregating financial market information. How these factors matter for the optimal maturity management of government debt is an interesting question for future research.

[14] For this extension, we require preferences that are consistent with a balanced growth path.

References

Aguiar, M., M. Amador, H. Hopenhayn, and I. Werning (2019). Take the short route: Equilibrium default and debt maturity. *Econometrica 87*(2), 423–462.

Alvarez, F., P. J. Kehoe, and P. A. Neumeyer (2004). The Time Consistency of Optimal Monetary and Fiscal Policies. *Econometrica 72*(2), 541–567.

Angeletos, G.-M. (2002). Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure. *The Quarterly Journal of Economics 117*(3), 1105–1131.

Arellano, C., Y. Bai, P. Kehoe, and A. Ramanarayanan (2013). Credibility and the Maturity of Government Debt. Federal Reserve Bank of Minneapolis, Manuscript.

Arellano, C. and A. Ramanarayanan (2012). Default and the Maturity Structure in Sovereign Bonds. *Journal of Political Economy 120*(2), 187–232.

Bhandari, A., D. Evans, M. Golosov, and T. Sargent (2021). Managing public portfolios. Manuscript.

Bigio, S., G. Nuno, and J. Passadore (2017). A Framework for Debt-Maturity Management. Manuscript.

Bocola, L. and A. Dovis (2019). Self-fulfilling debt crises: A quantitative analysis. *American Economic Review 109*(12), 4343–77.

Cao, D. and I. Werning (2018). Saving and Dissaving with Hyperbolic Discounting. *Econometrica 86*(3), 805–857.

Cochrane, J. H. (2015). A New Structure for US Federal Debt. In D. Wessel (Ed.), *The 13 Trillion Question: Managing the U.S. Government's Debt*, pp. 91–146. Brookings Institution Press.

Debortoli, D. and R. Nunes (2013). Lack of Commitment and the Level of Debt. *Journal of the European Economic Association 11*(5), 1053–1078.

Debortoli, D., R. Nunes, and P. Yared (2017). Optimal Time-Consistent Government Debt Maturity. *The Quarterly Journal of Economics 132*(1), 55–102.

Debortoli, D., R. Nunes, and P. Yared (2018). Optimal Taxation and Debt Management without Commitment. Version 1 of Working Paper 24522, National Bureau of Economic Research, https://www.nber.org/system/files/working_papers/w24522/revisions/w24522.rev0.pdf.

Debortoli, D., R. Nunes, and P. Yared (2021). Optimal Fiscal Policy without Commitment: Revisiting Lucas-Stokey. *Journal of Political Economy 129*(5), 1640–1665.

Dovis, A. (2019). Efficient Sovereign Default. *The Review of Economic Studies 86*(1), 282–312.

Faraglia, E., A. Marcet, and A. Scott (2010). In Search of a Theory of Debt Management. *Journal of Monetary Economics 57*(7), 821–836.

Fernandez, R. and A. Martin (2015). The Long and the Short of It: Sovereign Debt Crises and Debt Maturity. Universitat Pompeu Fabra, Working Paper.

Guibaud, S., Y. Nosbusch, and D. Vayanos (2013). Bond Market Clienteles, the Yield Curve, and the Optimal Maturity Structure of Government Debt. *Review of Financial Studies 26*(8), 1914–1961.

Krusell, P., F. Martin, and J.-V. Ríos-Rull (2006). Time-Consistent Debt. Manuscript.

Lucas, R. E. and N. L. Stokey (1983). Optimal Fiscal and Monetary Policy in an Economy without Capital. *Journal of Monetary Economics 12*(1), 55–93.

Lustig, H., C. Sleet, and S. Yeltekin (2008). Fiscal Hedging with Nominal Assets. *Journal of Monetary Economics 55*(4), 710–727.

Missale, A. and O. J. Blanchard (1994). The Debt Burden and Debt Maturity. *The American Economic Review 84*(1), 309–319.

Mokyr, J. (2011). *The Enlightened Economy: Britain and the Industrial Revolution, 1700-1850*. Penguin UK.

Niepelt, D. (2014). Debt Maturity without Commitment. *Journal of Monetary Economics 68*(Supplement), S37–S54.

Orszag, P. R., R. E. Rubin, and J. E. Stiglitz (2021). Fiscal resiliency in a deeply uncertain world: The role of semiautonomous discretion. *Peterson Institute for International Economics Policy Brief* (21-2).

Persson, M., T. Persson, and L. E. O. Svensson (2006). Time Consistency of Fiscal and Monetary Policy: A Solution. *Econometrica 74*(1), 193–212.

Werning, I. (2007). Optimal Fiscal Policy with Redistribution. *The Quarterly Journal of Economics 122*(3), 925–967.