The Commitment Benefit of Consols in Government Debt Management*

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Abstract

We consider optimal government debt maturity in a deterministic economy in which the government can issue any arbitrary debt maturity structure and in which bond prices are a function of the government's current and future primary surpluses. The government sequentially chooses policy, taking into account how current choices—which impacts future policy—feed back into current bond prices. We show that issuing consols constitutes the unique stationary optimal debt portfolio, as it boosts government credibility to future policy and reduces the debt financing costs.

Keywords: Public debt, optimal taxation, fiscal policy

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1 Introduction

Numerous governments across the world have extended the maturity of their public debts. For example, since 2016, France, Indonesia, and Mexico have issued 50-year bonds, while Austria, Israel, and Peru have issued 100-year bonds.[1] The extension of the U.S. government debt maturity has been proposed by Cochrane (2015) and Orszag et al. (2021) and is under consideration by the U.S. Treasury.[2] Arguments in favor of maturity extension generally rest on the notion that long-term financing costs are low relative to future interest rate risks. Bhandari et al. (2021) verify these insights by introducing interest rate risk to a calibrated fiscal policy model and showing that the optimal maturity structure is very long and takes the (approximate) form of a consol.[3]

In this paper, we provide an alternative rationale for the optimal issuance of long maturities. We argue that consols are optimal because they boost government credibility to future fiscal policy and reduce the financing costs for the government. To illustrate this point, we consider a deterministic environment in which bond prices are a function of the government's current primary surplus and the path of future primary surpluses. The government sequentially chooses taxes and any arbitrary debt issuance strategy taking into account the impact of current policy on bond prices. Our main finding is that the unique stationary distribution of government debt is flat, with the government owing the same amount to the private sector at all future dates.

We establish this result in the dynamic fiscal policy model of Lucas and Stokey (1983). This is an economy with exogenous public spending and no capital in which the government chooses linear taxes on labor and issues public debt to finance government spending. In this environment, if the government could commit to policy at the beginning of time, then the choice of government debt maturity would be indeterminate. This is because any debt maturity structure would satisfy the present value constraints of the government at a given point in time.

We do not assume that the government commits ex-ante to policy, but we instead consider the sequentially optimal policy. More specifically, we characterize the Markov Perfect Competitive Equilibrium (MPCE). In our setup, the government without commitment chooses taxes and debt at every date, taking into account how current policy affects the price of bonds through expectations of future policy. Moreover, the government may

[1] See, e.g. Reuters (01-18-2021).

[2] See, e.g. Bloomberg (19-10-2021).

[3] The use of consols has been pursued historically, most notably by the British government during the Industrial Revolution, when consols were the largest component of the British government's debt (see Mokyr (2011)).

decide not to follow the optimal commitment policy. We characterize the entire set of MPCE's in a deterministic economy, including those with potentially discontinuous policy functions both on and off the equilibrium path.[4] Since we allow for any unconstrained structure of maturity issuance, the payoff relevant state—the government's portfolio of inherited maturities—is an infinite-dimensional and potentially complicated object. We focus our attention on the stationary maturity distribution that emerges when the inherited portfolio of maturities equals the issued portfolio.

Our main result is that any stationary maturity distribution under lack of commitment must be flat, with the government owing the same amount at all future dates. The fact that a flat maturity distribution is stationary is not surprising. Under a flat maturity distribution, the government lacking commitment can choose a tax rate to repay the debt immediately due without rebalancing its portfolio. The chosen tax rate coincides with the optimum under full commitment, and therefore maximizes government's welfare. What is less obvious is why no other maturity distribution is stationary. The reason is that a government that inherits a non-flat maturity distribution would always take advantage of the situation to front-load or back-load taxes in order to change interest rates. When the government does this, the issued maturity distribution does not coincide with the inherited one, and is therefore not stationary.

For example, suppose that the government inherits more long-term liabilities than short-term ones. Rather than issuing the same maturity distribution as the inherited one, the government can change taxes so as to increase short-term interest rates. This relaxes the government budget constraint by decreasing the market value of outstanding long-term liabilities, making the government strictly better off. The opposite is true if the government inherits more short-term liabilities than long-term ones. In this case, a policy that decreases short-term interest rates makes the government strictly better off by increasing the market value of newly issued liabilities.

We apply this simple logic to analyze the behavior of the government inheriting any infinite-dimensional maturity distribution. We show that only if the inherited maturity distribution is flat is the government unable to take advantage of imbalances in debt positions to relax its budget constraint. As such, any stationary maturity distribution must be flat.

Related Literature

The main contribution of this paper is to characterize optimal fiscal policy without

[4] In this regard, our approach is similar in spirit to that of Cao and Werning (2018) in their analysis of Markov equilibria in the hyperbolic consumption model.

commitment in the deterministic case of the Lucas and Stokey (1983) model. Lucas and Stokey (1983) argue that there is no distortion due to lack of commitment, since the government can structure its debt maturity to guarantee commitment to optimal fiscal policy by future governments.[5] However, Debortoli et al. (2021) show that this result does not generally hold in the same model as Lucas and Stokey (1983). As such, our analysis of optimal fiscal policy considers the entire set of MPCE's, not only the ones which coincide with the commitment policy (should they exist).

Our work also contributes to a literature on optimal government debt maturity in the absence of government commitment. We depart from this literature in two ways. First, we consider the optimal maturity without imposing arbitrary constraints on the bonds available to the government.[6] Second, our model is most applicable to economies where the risk of default and surprise in inflation are not salient, but the government is still not committed to a path of taxes and debt maturity issuance.[7] In this regard, our paper is related to the quantitative analysis of Debortoli et al. (2017). We differ from this work in two respects. First, we do not arbitrarily confine the set of bonds available to the government, as they do. Second, we consider a deterministic economy and ignore the presence of shocks.[8] These two departures allow us to achieve exact theoretical characterization of the stationary maturity distribution. Our finding that the maturity distribution is exactly flat is consistent with their quantitative result that the maturity distribution is approximately flat.

Our paper proceeds as follows. In Section 2, we describe the model. In Section 3, we formally define an MPCE. Section 4 establishes that any stationary maturity distribution under lack of commitment is flat. Section 5 concludes, and the Appendix provides all of the proofs and additional results not included in the text.

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[5] See also Alvarez et al. (2004) and Persson et al. (2006).

[6] Krusell et al. (2006) and Debortoli and Nunes (2013) consider a similar environment to ours in the absence of commitment, but with only one-period bonds, for example.

[7] Other work considers optimal government debt maturity in the presence of default risk, for example, Aguiar et al. (2017), Arellano and Ramanarayanan (2012), Dovis (2019), Fernandez and Martin (2015), and Niepelt (2014), among others. Bocola and Dovis (2016) additionally consider the presence of liquidity risk. Bigio et al. (2017) consider debt maturity in the presence of transactions costs. Arellano et al. (2013) consider lack of commitment when surprise inflation is possible.

[8] Angeletos (2002), Bhandari et al. (2017), Buera and Nicolini (2004), Faraglia et al. (2010), Guibaud et al. (2013), and Lustig et al. (2008) also consider optimal government debt maturity in the presence of shocks, but they assume full commitment.

2 Model

2.1 Environment

We consider an economy identical to the deterministic case of Lucas-Stokey. There are discrete time periods $t = \{0, 1, ..., \infty\}$. The resource constraint of the economy is

$$c_t + g = n_t, \tag{1}$$

where c_t is consumption, n_t is labor, and $g > 0$ is government spending, which is exogenous and constant over time.

There is a continuum of mass 1 of identical households that derive the following utility:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right), \; \beta \in (0,1). \tag{2}$$

$u(\cdot)$ is strictly increasing in consumption, strictly decreasing in labor, globally concave, and continuously differentiable. We also assume that $u_{cc}(c, c+g) + u_{cn}(c, c+g) < 0$ so that the marginal utility of consumption is decreasing in consumption in general equilibrium. As a benchmark, we define the first best consumption and labor $\left\{c^{fb}, n^{fb}\right\}$ as the values of consumption and labor that maximize $u\left(c_t, n_t\right)$ subject to the resource constraint (1).

Household wages equal the marginal product of labor (which is 1 unit of consumption), and are taxed at a linear tax rate τ_t. $b_{t,k} \gtreqless 0$ represents government debt purchased by a representative household at t, which is a promise to repay 1 unit of consumption at $t + k > t$. $q_{t,k}$ is the bond price at t. At every t, the household's allocation and portfolio $\left\{c_t, n_t, \{b_{t,k}\}_{k=1}^{\infty}\right\}$ must satisfy the household's dynamic budget constraint:

$$c_t + \sum_{k=1}^{\infty} q_{t,k}\left(b_{t,k} - b_{t-1,k+1}\right) = \left(1 - \tau_t\right) n_t + b_{t-1,1}. \tag{3}$$

$B_{t,k} \gtreqless 0$ represents debt issued by the government at t with a promise to repay 1 unit of consumption at $t + k > t$. At every t, government policies $\left\{\tau_t, g_t, \{B_{t,k}\}_{k=1}^{\infty}\right\}$ must satisfy the government's dynamic budget constraint:

$$g_t + B_{t-1,1} = \tau_t n_t + \sum_{k=1}^{\infty} q_{t,k}\left(B_{t,k} - B_{t-1,k+1}\right).[9] \tag{4}$$

[9] We follow the same exposition as in Angeletos (2002) in which the government rebalances its debt in every period by buying back all outstanding debt and then issuing fresh debt at all maturities. This

The economy is closed, which means that the bonds issued by the government equal the bonds purchased by households:

$$b_{t,k} = B_{t,k} \ \forall t, k. \tag{5}$$

Initial debt $\{B_{-1,k}\}_{k=1}^{\infty} = \{b_{-1,k}\}_{k=1}^{\infty}$ is exogenous. We assume that there exist debt limits to prevent Ponzi schemes:

$$b_{t,k} \in \left[\underline{b}, \overline{b}\right] \ \ \forall t, k. \tag{6}$$

In our recursive analysis, we will consider economies where these limits are not binding along the equilibrium path. The government is benevolent and shares the same preferences as the households in (2).

2.2 Primal Approach

We follow Lucas-Stokey by taking the primal approach to the characterization of competitive equilibria, since this allows us to abstract away from bond prices and taxes. Let

$$\{c_t, n_t\}_{t=0}^{\infty} \tag{7}$$

represent a sequence of consumption and labor allocations. We can establish necessary and sufficient conditions for (7) to constitute a competitive equilibrium. The household's optimization problem implies the following intratemporal and intertemporal conditions, respectively:

$$1 - \tau_t = -\frac{u_n\left(c_t, n_t\right)}{u_c\left(c_t, n_t\right)} \text{ and } q_{t,k} = \frac{\beta^k u_c\left(c_{t+k}, n_{t+k}\right)}{u_c\left(c_t, n_t\right)}. \tag{8}$$

Substitution of these conditions into the household's dynamic budget constraint implies the following condition:

$$u_c\left(c_t, n_t\right) c_t + u_n\left(c_t, n_t\right) n_t + \sum_{k=1}^{\infty} \beta^k u_c\left(c_{t+k}, n_{t+k}\right) b_{t,k} = \sum_{k=0}^{\infty} \beta^k u_c\left(c_{t+k}, n_{t+k}\right) b_{t-1,k+1}. \tag{9}$$

is without loss of generality. For example, if the government at $t-k$ issues debt due at date t of size $B_{t-k,k}$ which it then holds to maturity without issuing additional debt, then all future governments at date $t-k+l$ for $l = 1, ..., k-1$ will choose $B_{t-k+l,k-l} = B_{t-k,k}$, implying that $B_{t-1,1} = B_{t-k,k}$.

Forward substitution into the above equation and taking into account the absence of Ponzi schemes implies the following implementability condition:

$$\sum_{k=0}^{\infty} \beta^k \left(u_c\left(c_{t+k}, n_{t+k}\right) c_{t+k} + u_n\left(c_{t+k}, n_{t+k}\right) n_{t+k}\right) = \sum_{k=0}^{\infty} \beta^k u_c\left(c_{t+k}, n_{t+k}\right) b_{t-1,k+1}. \quad (10)$$

By this reasoning, if a sequence in (7) is generated by a competitive equilibrium, then it necessarily satisfies (1) and (10). We prove in the Appendix that the converse is also true, which leads to the below proposition that is useful for the rest of our analysis.

Lemma 1 ***(competitive equilibrium)*** *A sequence (7) is a competitive equilibrium if and only if it satisfies (1) $\forall t$ and (10) at $t = 0$ given $\{b_{-1,k}\}_{k=1}^{\infty}$.*

Note that this result rests on the fact that the satisfaction of (10) at $t = 0$ guarantees the satisfaction of (10) for all future dates, since bonds can be freely chosen so as to satisfy (10) at all future dates for any given sequence (7).

3 Markov Perfect Competitive Equilibrium

We characterize the MPCE in which the government without commitment chooses taxes and debt at every date, taking into account how current policy affects the price of bonds through expectations of future policy. In this section, we formally define our equilibrium, and then, using the primal approach, we provide a recursive representation of the equilibrium.

3.1 Equilibrium Definition

Formally, let $\mathbf{B}_t \equiv \{B_{t,k}\}_{k=1}^{\infty}$ and $\mathbf{q}_t \equiv \{q_{t,k}\}_{k=1}^{\infty}$. In every period t, the government chooses a policy $\{\tau_t, \mathbf{B}_t\}$ given $\mathbf{B}_{t-1}$. Households then choose an allocation and portfolio $\left\{c_t, n_t, \{b_{t,k}\}_{k=1}^{\infty}\right\}$. An MPCE consists of: a government strategy $\rho\left(\mathbf{B}_{t-1}\right)$ which is a function of $\mathbf{B}_{t-1}$; a household allocation and portfolio strategy $\omega\left(\mathbf{B}_{t-1}, \rho_t, \mathbf{q}_t\right)$ which is a function of $\mathbf{B}_{t-1}$, the government policy $\rho_t = \rho\left(\mathbf{B}_{t-1}\right)$, and bond prices $\mathbf{q}_t$; and a set of bond pricing functions $\left\{\varphi^k\left(\mathbf{B}_{t-1}, \rho_t\right)\right\}_{k=1}^{\infty}$ with $q_{t,k} = \varphi^k\left(\mathbf{B}_{t-1}, \rho_t\right)$ $\forall k \geq 1$ which depend on $\mathbf{B}_{t-1}$ and the government policy $\rho_t = \rho\left(\mathbf{B}_{t-1}\right)$. In an MPCE, these objects must satisfy the following conditions $\forall t$:

1. The government strategy $\rho\left(\cdot\right)$ maximizes (2) given $\omega\left(\cdot\right)$, $\varphi^k\left(\cdot\right)$ $\forall k \geq 1$, and the government budget constraint (4);

2. The household allocation and portfolio strategy $\omega(\cdot)$ maximizes (2) given $\rho(\cdot)$, $\varphi^k(\cdot)$ $\forall k \geq 1$, and the household budget constraint (3), and

3. The set of bond pricing functions $\varphi^k(\cdot)$ $\forall k \geq 1$ satisfy (5) given $\rho(\cdot)$ and $\omega(\cdot)$.

3.2 Recursive Representation

Given our definition, an MPCE is characterized by an equilibrium consumption and labor sequence (7) and an equilibrium debt sequence $\{\{b_{t,k}\}_{k=1}^{\infty}\}_{t=0}^{\infty}$, where each element at date t depends on history only through $\mathbf{B}_{t-1}$, the payoff relevant variables. Given this observation, in an MPCE, one can define a function $h^k(\cdot)$

$$h^k(\mathbf{B}_t) = \beta^k u_c(c_{t+k}, n_{t+k})\,|\mathbf{B}_t \tag{11}$$

for $k \geq 1$, which equals the discounted marginal utility of consumption at $t+k$ given $\mathbf{B}_t$ at t. This function is useful since, in choosing $\mathbf{B}_t$ at date t, the government must take into account how it affects future expectations of policy, which in turn affect current bond prices through expected future marginal utility of consumption.

Note that choosing $\{\tau_t, \mathbf{B}_t\}$ at date t from the perspective of the government is equivalent to choosing $\{c_t, n_t, \mathbf{B}_t\}$ where one can write, with some abuse of notation, $\mathbf{B}_t = \{b_{t,k}\}_{k=1}^{\infty}$, and this follows from the primal approach delineated in Section 2.2. Removing the time subscript and defining $\mathbf{B} \equiv \mathbf{B}_{t-1} = \{b_k\}_{k=1}^{\infty}$ as the inherited portfolio of bonds, we can write the government's problem recursively as

$$V(\mathbf{B}) = \max_{c,n,\mathbf{B}'} u(c,n) + \beta V(\mathbf{B}') \tag{12}$$

s.t.

$$c + g = n, \text{ and} \tag{13}$$

$$u_c(c,n)\,c + u_n(c,n)\,n - u_c(c,n)\,b_1 + \sum_{k=1}^{\infty} h^k(\mathbf{B}')(b'_k - b_{k+1}) = 0, \tag{14}$$

where (14) is a recursive representation of (9). Let $f(\mathbf{B})$ correspond to the solution to (12) − (14) given $V(\cdot)$ and $h^k(\cdot)$ $\forall k \geq 1$. It therefore follows that the function $f(\cdot)$ necessarily implies functions $h^k(\cdot)$ $\forall k \geq 1$ which satisfy (11). An MPCE is therefore composed of functions $V(\cdot)$, $f(\cdot)$, and $h^k(\cdot)$ $\forall k \geq 1$ that are consistent with one another and satisfy (11) − (14).

4 Stationary Distribution of Debt Maturity

We focus on characterizing an economy in which the debt maturity distribution is stationary with $b_{t+1,k} = b_{t,k}$, $\forall t, k$, so that government debt maturity is time-invariant. Given the Markovian structure of the solution to the MPCE defined by (12) − (14), such a stationary maturity distribution is associated with tax rates, consumption, and interest rates that are constant over time. In this section, we show that any stationary maturity distribution must be flat, with the government owing the same amount of resources to the private sector at all future dates. To establish this result, we first impose a useful assumption in Section 4.1. In Section 4.2, we use this assumption show that a flat maturity distribution is stationary. Finally, in Section 4.3, we show that no other maturity distribution can be stationary.

4.1 Preliminaries

Before proceeding with our analysis, we impose a useful assumption. Using our recursive notation introduced in Section 3, define $W\left(\{b_k\}_{k=1}^{\infty}\right)$ as the welfare of the government under full commitment given an initial starting debt position $\{b_k\}_{k=1}^{\infty}$:

$$W\left(\{b_k\}_{k=1}^{\infty}\right) = \max_{\{c_k,n_k\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} \beta^k u\left(c_k, n_k\right) \tag{15}$$

s.t.

$$c_k + g = n_k, \text{ and} \tag{16}$$

$$\sum_{k=0}^{\infty} \beta^k \left(u_c\left(c_k, n_k\right) c_k + u_n\left(c_k, n_k\right) n_k\right) = \sum_{k=0}^{\infty} \beta^k u_c\left(c_k, n_k\right) b_{k+1}. \tag{17}$$

Given Lemma 1, the program in (15) − (17) corresponds to that of a government under full commitment with $b_{-1,k} = b_{k+1}$.

Assumption 1. Consider the solution to (15) − (17) with $b_{k+1} = b\ \forall k \geq 0$. $\forall b \in \left[\underline{b}, \overline{b}\right]$, if the solution exists, then the solution is unique and admits $\{c_k, n_k\} = \{c^*(b), n^*(b)\}$ $\forall k \geq 0$, where

$$u_c\left(c^*(b), n^*(b)\right) c^*(b) + u_n\left(c^*(b), n^*(b)\right) n^*(b) = u_c\left(c^*(b), n^*(b)\right) b, \tag{18}$$

$$\text{and } c^*(b) + g = n^*(b). \tag{19}$$

This assumption states that if a government under full commitment is faced with a flat maturity distribution, then there is a unique optimum in which the government chooses a constant allocation of consumption and labor in the future.[10] This assumption is intuitive. Under a flat maturity distribution, every time period in the program in (15) − (17) is identical in the objective function and in the constraint set, which suggests that the optimal solution is a time-invariant allocation. A sufficient condition for Assumption 1 is that the function $u_c(c, c+g)(c-b) + u_n(c, c+g)(c+g)$ is concave in c for all b, which is the case, for example, if the utility function satisfies is isoelastic (e.g., Werning, 2007) and if $\underline{b} = 0$ so that debt is non-negative.

4.2 Flat Maturity Distribution is Stationary

We begin by establishing that if the maturity distribution is flat, then it is stationary.

Lemma 2 *Suppose that there exists an MPCE starting from* $\mathbf{B}$ *which satisfies* $b_k = b\ \forall k$ *for some* $b \in \left[\underline{b}, \overline{b}\right]$*. Then,*

1. *In all solutions to* (12) − (14), $c = c^*(b)$ *and* $n = n^*(b)$, *and*
2. *There exists a solution to* (12) − (14) *that admits* $b'_k = b\ \forall k$.

The first part of the lemma states that in any MPCE, if the government inherits a flat maturity distribution with $b_k = b\ \forall k$, then the unique optimal response of the government is to choose consumption and labor that coincide with the commitment optimum. The second part of the lemma implies that one optimal—but not necessarily uniquely optimal—strategy for the government is to choose $b'_k = b\ \forall k$ so that debt is not rebalanced and the maturity distribution continues to be flat in the future. As such, there exists a stationary MPCE with a flat maturity distribution. Importantly, this lemma implies that in any MPCE for which $\mathbf{B}$ is a flat maturity distribution, it is necessary that

$$V(\mathbf{B}) = W(\mathbf{B}) \tag{20}$$

so that there is no welfare loss for the present government due to lack of commitment by future governments.

The logic behind this lemma is that a government inheriting a flat maturity distribution with $b_k = b\ \forall k$ can always decide to not rebalance its debt portfolio and to choose

[10] Assumption 1 requires that the solution exists. If the upper bound on individual maturities $\overline{b}$ exceeds the highest primary surplus that can be raised at the peak of the Laffer curve, then there is no solution under a flat maturity for some high values of debt.

the tax rate associated with $\{c^* (b), n^* (b)\}$. Forward induction on this observation combined with Assumption 1 means that the government is able to achieve the commitment optimum with this strategy while inducing allocation $\{c^* (b), n^* (b)\}$ in all future periods. Note that the government can induce the commitment allocation in the future in *any* MPCE, including those where the government's continuation strategy off the equilibrium path given off equilibrium maturities does not coincide with the commitment solution.

4.3 No Other Maturity Distribution is Stationary

We now turn to the possibility that another maturity distribution is stationary. We show in this section that this is not possible by contradiction using an induction argument. The first step of the induction argument establishes that if a non-flat maturity distribution were stationary, then the debt immediately due, b_1, would be necessarily equal to the primary surplus. The second step of the induction argument establishes that if a non-flat maturity distribution were stationary with b_k equal to the primary surplus for all $k \leq m$, then b_{m+1} would necessarily be equal to the primary surplus. It follows then by induction that b_k equals the primary surplus for all maturities k and that the maturity distribution is flat, leading to a contradiction.

To pursue this induction argument, we establish a preliminary result that allows us to construct perturbations as part of the induction argument. To interpret this lemma, observe that since consumption is constant over time under a stationary maturity distribution, the price of a bond maturing in k periods is β^k.

Lemma 3 *Suppose that given* $\mathbf{B}$*, there exists a solution to* (12) $-$ (14) *with a stationary maturity distribution* $b'_k = b_k \; \forall k$ *and* $b'_l \neq b'_m$ *for some* l, m*. Then there exists another solution to* (12) $-$ (14) *with* $b'_k = \widehat{b} \; \forall k$ *where*

$$\widehat{b} = \sum_{k=1}^{\infty} \beta^{k-1} (1 - \beta) \, b_k. \tag{21}$$

This lemma states that under any MPCE with a stationary maturity distribution that is not flat, the government can choose the same tax rate but issue a flat maturity distribution with the same market value and achieve the same welfare. The proof of this lemma is facilitated by Lemma 2, which characterizes the continuation equilibrium following this choice of a flat maturity. Since current and future taxes and consumption remain unchanged from the issuance of a flat maturity, bond prices and welfare are also unchanged.

Lemma 3 implies that if there exists a stationary maturity distribution that is not flat, then the corresponding welfare is equal to that achieved under a flat maturity distribution with the same market value. Moreover, from (20), welfare under this MPCE equals that under commitment associated with a flat maturity distribution with the same market value:

$$V(\mathbf{B}) = W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k} = \frac{u(c(\widehat{b}), n(\widehat{b}))}{1-\beta}. \tag{22}$$

Lemma 3 is useful since it characterizes welfare under a stationary maturity distribution that is not flat. Moreover, it allows us to consider off-equilibrium welfare following a deviation in maturity issuance strategy by the government, which is useful for establishing the first step of our induction argument in the next lemma.

Lemma 4 *Suppose that given* $\mathbf{B}$*, there exists a solution to* (12) − (14) *with a stationary maturity distribution* $b'_k = b_k\ \forall k$ *and for which* $\{c, n\} \neq \{c^{fb}, n^{fb}\}$*. Then,* $\mathbf{B}$ *must satisfy* $b_1 = \widehat{b}$ *for* $\widehat{b}$ *defined in* (21)*.*

This lemma states that in any stationary maturity distribution in which the tax rate is not zero (so that consumption and labor do not equal the first best), short-term debt b_1 equals the annuitized value of total debt $\widehat{b}$. Therefore, the primary surplus equals the short-term debt b_1 and net debt issuance is zero.

The proof rests on showing that if the primary surplus is in excess of, or below, this short-term debt b_1, then the government can pursue a deviation from a smooth consumption path to boost welfare. For example, if the primary surplus is in excess of what the government immediately owes, then pursuit of a smooth consumption path would require the government to buy back some of its long-term debt. Rather than following a stationary debt issuance strategy, the government can back-load consumption to increase short-term interest rates and reduce the value of the long-term debt which it buys back. Since the deviation is beneficial, the maintenance of a stationary debt maturity distribution is not optimal if the primary surplus exceeds b_1.

If instead the primary surplus is below what the government immediately owes, then pursuit of a smooth consumption path would require the government to issue fresh debt in order to repay current short-term debt. Rather than following a stationary debt issuance strategy, the government can front-load consumption to decrease short-term interest rates and increase the value of newly issued debt. Since the deviation is beneficial, the maintenance of a stationary debt maturity distribution is not optimal if the primary surplus is below b_1.

Note that in constructing these deviations, we utilize Lemmas 2 and 3 which allow us to characterize the change in welfare if the government issues a flat government debt maturity today as part of its deviation. As such, we can explicitly show that these deviations increase welfare by relaxing the government's budget constraint.

Note that the reason why our argument does not hold under a stationary distribution of debt maturities with zero taxes is that in this case, it is not possible to relax the government budget constraint further.

We now use analogous arguments to establish the second step of the induction argument.

Lemma 5 *Suppose that given* $\mathbf{B}$, *there exists a solution to* (12) $-$ (14) *with a stationary maturity distribution* $b'_k = b_k \; \forall k$ *and for which* $\{c, n\} \neq \{c^{fb}, n^{fb}\}$. *If* $b_l = \widehat{b} \; \forall l \leq m$, *then* $\mathbf{B}$ *must satisfy* $b_{m+1} = \widehat{b}$ *for* $\widehat{b}$ *defined in* (21).

This lemma considers the stationary maturity distribution when all bond maturities below m equal the primary surplus of the government (the annuitized value of government debt). When this is the case, then the bond of maturity $m+1$ must also equal the primary surplus of the government.

The argument, which relies on a proof by contradiction, starts from the fact that under a stationary maturity distribution, government's welfare satisfies (22), and thus equals welfare under commitment with a flat maturity distribution with the same market value. Now if the amount owed at date $m+1$ does not also equal the primary surplus, then there exists a feasible deviation from a stationary debt issuance strategy that can increase welfare above (22), leading to a contradiction.

More specifically, if $b_l = \widehat{b} \; \forall l \leq m$ but $b_{m+1} \neq \widehat{b}$, a feasible strategy for the government today is to continue to choose the same consumption and labor allocation today $\{c(\widehat{b}), n(\widehat{b})\}$ but to deviate by not retrading the inherited maturities (i.e., letting the bonds mature to next period). Such a deviation is feasible whatever the expectations of future policy and their impact on current bond prices since the government is not rebalancing its portfolio.

Without needing to specify the exact form of the continuation equilibrium, we can show that this deviation must necessarily increase welfare. The argument rests on putting a lower bound on the welfare of future governments following the deviation based on the feasible policies at their disposal. More specifically, note that after the initial deviation, future governments also have the opportunity to pursue the same strategy of choosing consumption and labor equal to $\{c(\widehat{b}), n(\widehat{b})\}$ and not rebalancing the portfolio of maturities. This is true up until some future date m periods in the future. Based on this logic, the welfare of the government today from pursuing the deviation must weakly exceed

$$\sum_{l=0}^{m-1} \beta^l u(c(\widehat{b}), n(\widehat{b})) + \beta^m V(\widehat{\mathbf{B}}\,(m)) \tag{23}$$

where $\widehat{\mathbf{B}}\,(m)$ satisfies $\widehat{b}\,(m)_k = b_{k+m}\ \forall k \geq 1$. Thus, for the initial deviation to be weakly dominated, this requires (23) to be weakly exceeded by (22), so that

$$V(\widehat{\mathbf{B}}\,(m)) \leq \frac{u(c(\widehat{b}), n(\widehat{b}))}{1-\beta}. \tag{24}$$

However, since $b_{m+1} \neq \widehat{b}$, the arguments of Lemma 4 imply that (24) cannot hold, leading to a contradiction. Intuitively, m periods into the future after following a strategy of no rebalancing, the primary surplus is above or below the debt immediately due. At this point in the future, pursuing a strategy that back-loads or front-loads consumption strictly increases welfare relative to a smooth consumption policy with a stationary debt issuance strategy. Therefore, the immediate deviation prior to reaching this m'th period is beneficial, and the maintenance of a stationary debt maturity distribution is not optimal.

Proposition 1 ***(flat maturity)*** *Suppose that conditional on* $\mathbf{B}$*, there exists a solution to* (12) $-$ (14) *with a stationary maturity distribution* $b'_k = b_k\ \forall k$ *and for which* $\{c, n\} \neq \{c^{fb}, n^{fb}\}$*. Then it is necessary that* $b_k = \widehat{b}\ \forall k$ *so that the maturity distribution is flat.*

This proposition represents the main result of the paper. It states that if the maturity distribution is stationary and if the equilibrium does not entail first best consumption and labor, then the maturity distribution is flat. The reasoning for the proposition follows from induction arguments which appeal to Lemmas 4 and 5. Intuitively, if maturity distribution is not flat, then there are opportunities for the government take advantage of these imbalances to decrease the market value of its inherited portfolio or increase the market value of its newly-issued portfolio. Note that this result holds in any MPCE and does not appeal to any assumptions regarding the behavior of future governments.

Our result relies on the stationary maturity distribution not being associated with first best consumption and labor. Under such a stationary distribution, taxes would be zero, the market value of debt would be sufficiently negative to finance the stream of government spending forever, and the marginal benefit of resources for the government would be zero. For this reason, the stationary maturity distribution is not determined in this circumstance. We can trivially rule out this case if there are exogenous bounds on government debt which prevent such asset accumulation for the government.

Corollary 1 *Suppose that $\underline{b} > -g$. Then if conditional on* $\mathbf{B}$*, there exists a solution to* (12) $-$ (14) *with a stationary maturity distribution $b'_k = b_k \; \forall k$, it is necessary that $b_k = \widehat{b}$ $\forall k$ so that the maturity distribution is flat.*

Finally, returning to Lemma 2, note that Proposition 1 also implies that starting from a flat maturity distribution, the unique continuation equilibrium requires the issuance of a flat maturity distribution. Therefore, in any MPCE, a flat government debt maturity is an absorbing state, and *all* flat maturity distributions are stationary.

Corollary 2 *Suppose that* $\mathbf{B}$ *satisfies $b_k = b \; \forall k$ for some b and that $\{c, n\} \neq \{c^{fb}, n^{fb}\}$. Then, in all solutions to* (12) $-$ (14) $b'_k = b \; \forall k$.

Starting from a flat maturity distribution, the current government would like to guarantee a constant level of consumption and labor going forward. Choosing a maturity distribution that is not flat cannot guarantee such a continuation equilibrium, since future governments will deviate from a smooth policy in order to relax the government budget constraint. For this reason, the government chooses a flat maturity distribution, and a flat maturity distribution is an absorbing state.[11]

5 Concluding Remarks

In this paper, we characterize optimal fiscal policy and debt managment when the government reoptimizes sequentially. We consider an MPCE in which the government chooses policy as a function of the infinite-dimensional portfolio of government bonds that it inherits in every period. Our analysis applies to the entire set of MPCE's, including those that do not coincide with the commitment policy or those with potentially discontinuous policy functions both on and off the equilibrium path. We find that any stationary distribution of debt maturity must be flat, with the government owing the same amount at all future dates. Our analysis thus provides a theoretical argument for the use of consols in debt management based on the sequential optimization of fiscal policy by the government.

In our framework, we have considered a situation in which a government's objective in its debt issuance strategy is to minimize its financing costs. In practice, government debt management offices also pursue other objectives, such as supporting financial stability. For example, this can be achieved either by providing liquidity to segments of the

[11] These results are consistent with the quantitative analysis of Debortoli et al. (2017) who consider a stochastic infinite horizon with limited debt maturities, and who find that the maturity distribution transitions to an approximately flat one.

market that lack it or through the bond auction process, which itself may serve a purpose of aggregating financial market information. How these factors matter for the optimal maturity management of government debt is an interesting question for future research.

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Appendix

Appendix A. Proofs of Section 2

Proof of Lemma 1

The necessity of these conditions is proved in the text. To prove sufficiency, let the government choose the associated level of debt $\left\{\{b_{t,k}\}_{k=1}^{\infty}\right\}_{t=0}^{\infty}$ which satisfies (9) and a tax sequence $\{\tau_t\}_{t=0}^{\infty}$ which satisfies (8). Let bond prices satisfy (8). (9) given (1) implies that (3) and (4) are satisfied. Therefore household optimality holds and all dynamic budget constraints are satisfied along with the market clearing, so the equilibrium is competitive.■

Appendix B. Proofs of Section 4

Proof of Lemma 2

Note that if $b_k = b\ \forall k$, then from Assumption 1, the solution under commitment admits $\{c_t, n_t\} = \{c^*(b), n^*(b)\}\ \forall t$, and this solution can be implemented with $b'_k = b$ given (18)–(19). Since the MPCE satisfies the same constraints of the problem under commitment plus additional constraints regarding sequential optimality, it follows that

$$W(\mathbf{B}) = \frac{u(c^*(b), n^*(b))}{1-\beta} \geq V(\mathbf{B}) \tag{B.1}$$

if $b_k = b\ \forall k$. Now consider optimal policy under the MPCE in (12) − (14) given $b_k = b\ \forall k$. A government has the option of choosing $c = c^*(b)$ and $n = n^*(b)$ together with $b'_k = b$ $\forall k$. This satisfies the resource constraint (13) and the implementability constraint (14). Therefore, it follows that

$$V(\mathbf{B}) \geq u(c^*(b), n^*(b)) + \beta V(\mathbf{B}). \tag{B.2}$$

Equations ($B.1$) and ($B.2$) imply that

$$V(\mathbf{B}) = W(\mathbf{B}). \tag{B.3}$$

By Assumption 1, $W(\mathbf{B})$ is uniquely characterized by $\{c_k, n_k\} = \{c^*(b), n^*(b)\}$ $\forall k$. Therefore, it follows that any solution to (12) − (14) given $b_k = b$ $\forall k$ admits $c = c^*(b)$ and $n = n^*(b)$.■

Proof of Lemma 3

Conditional on $\mathbf{B}$, if a solution admits $b'_k = b_k$, then this means that $\mathbf{B}$ is an absorbing state with $\mathbf{B} = \mathbf{B}'$ and consumption and labor are constant and equal to some $\{c, n\}$ from that period onward. Therefore, $h^k(\mathbf{B}') = \beta^k u_c(c,n)$ $\forall k \geq 1$ for $h^k(\mathbf{B}')$ defined in (11). As such, (14) can be rewritten as

$$u_c(c,n)\,c + u_n(c,n)\,n - u_c(c,n)\,b_1 + u_c(c,n)\sum_{k=1}^{\infty}\beta^k\left(b'_k - b_{k+1}\right) = 0 \tag{B.4}$$

which combined with (21) and the fact that $b'_k = b_k$ implies that

$$u_c(c,n)\,c + u_n(c,n)\,n = u_c(c,n)\,\widehat{b}. \tag{B.5}$$

Now consider the solution to the following problem given $\widehat{b}$:

$$\max_{c,n} \frac{u(c,n)}{1-\beta} \text{ s.t. } c + g = n \text{ and } (B.5)\,. \tag{B.6}$$

It is necessary that $V(\mathbf{B})$ be weakly below the value of $(B.6)$. This is because the solution to $V(\mathbf{B})$ also admits a constant consumption and labor (as in the program in $(B.6)$) and since the constraint set in $(B.6)$ is slacker, since the program ignores the role of government debt in changing future policies. Note furthermore that the value of $(B.6)$ equals $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k}$, where this follows from Assumption 1. Therefore,

$$V(\mathbf{B}) \leq W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k}. \tag{B.7}$$

Now consider the welfare of the government in the MPCE if, instead of choosing $b'_k = b_k$ $\forall k$, it instead chooses $b'_k = \widehat{b}$ $\forall k$ with $c = c^*(\widehat{b})$ and $n = n^*(\widehat{b})$. It follows from Lemma 2 that under this perturbation, $h^k(\mathbf{B}') = \beta^k u_c(c^*(\widehat{b}), n^*(\widehat{b}))$ $\forall k \geq 1$, which implies that the resource constraint (13) and implementability constraint (14) are satisfied under this deviation. Because the continuation value associated with this deviation is $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k}$, it follows that for this deviation to be weakly dominated:

$$W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k} \leq V(\mathbf{B})\,. \tag{B.8}$$

Given $(B.7)$ and $(B.8)$, it follows that $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k} = V(\mathbf{B})$. Therefore, given $\mathbf{B}$, there exists another solution to (12) − (14) with $b'_k = \widehat{b}\ \forall k$ which achieves the same welfare.■

Proof of Lemma 4

Before proving this lemma, define c^{laffer} as:

$$c^{laffer} = \arg\max_{c}\left\{c + \frac{u_n(c, c+g)}{u_c(c, c+g)}(c+g)\right\}, \tag{B.9}$$

and let b^{laffer} correspond to the value of the maximized objective in $(B.9)$. It follows that a solution to (15) − (17) exists if $b_k = b\ \forall k \geq 1$ if $b \leq b^{laffer}$.

Given this definition, we can proceed to prove this lemma by contradiction. By Lemma 3,

$$V(\mathbf{B}) = W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k} = \frac{u(c^*(\widehat{b}), n^*(\widehat{b}))}{1-\beta} \tag{B.10}$$

for $\widehat{b}$ defined in (21). Now suppose that $b_1 \neq \widehat{b}$. Given the definition of $\widehat{b}$, this means that $\widehat{b} \in (\underline{b}, \overline{b})$ and that $\widehat{b} \leq b^{laffer}$. We consider two cases separately.

Case 1. Suppose that $\widehat{b} < b^{laffer}$, and suppose that the government locally deviates to $b'_k = \widetilde{b} \neq \widehat{b}\ \forall k$ so that from tomorrow onward, consumption is $c^*(\widetilde{b})$ and labor is $n^*(\widetilde{b})$, where this follows from Lemma 2. This means that $h^k(\widetilde{\mathbf{B}}) = \beta^k u_c(c^*(\widetilde{b}), n^*(\widetilde{b}))$ under the deviation. In order to satisfy the resource constraint and implementability condition, let the government deviate today to a consumption and labor allocation $\{\widetilde{c}, \widetilde{n}\}$ which satisfies

$$\widetilde{c} + g = \widetilde{n} \tag{B.11}$$

and

$$\begin{aligned} u_c(\widetilde{c}, \widetilde{n})\widetilde{c} + u_n(\widetilde{c}, \widetilde{n})\widetilde{n} - (u_c(\widetilde{c}, \widetilde{n}) - u_c(c^*(\widetilde{b}), n^*(\widetilde{b})))b_1 = \\ u_c(c^*(\widetilde{b}), n^*(\widetilde{b}))\left(\widehat{b} + \frac{\beta}{1-\beta}(\widehat{b} - \widetilde{b})\right) \end{aligned} \tag{B.12}$$

where we have appealed to the definition of $\widehat{b}$ in (21). For such a deviation to be weakly dominated, it must be that

$$V(\mathbf{B}) \geq u(\widetilde{c}, \widetilde{n}) + \beta W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widetilde{b}\ \forall k}. \tag{B.13}$$

Clearly, the value of the right hand side of ($B.13$) equals $V(\mathbf{B})$ if $\widetilde{b} = \widehat{b}$. Therefore, it must be that $\widetilde{b} = \widehat{b}$ with $\{\widetilde{c}, \widetilde{n}\} = \{c^*(\widehat{b}), n^*(\widehat{b})\}$ maximizes the right hand side of ($B.13$) subject to ($B.11$), and ($B.12$). More specifically, we can consider the solution to the following program

$$\max_{\widetilde{c},\widetilde{n},\widetilde{b}} u(\widetilde{c},\widetilde{n}) + \beta W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widetilde{b}\ \forall k} \text{ s.t. } (B.11) \text{ and } (B.12). \tag{B.14}$$

For the deviation to not strictly increase welfare, $\widetilde{b} = \widehat{b}$ must be a solution to ($B.14$). By Assumption 1, $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widetilde{b}\ \forall k} = u(c^*, n^*)/(1-\beta)$ where $\{c^*, n^*\} = \{c^*(\widetilde{b}), n^*(\widetilde{b})\}$ are the unique levels of consumption and labor which maximize welfare given $\widetilde{b}$ and are defined in (18) and (19). Letting μ_1 represent the Lagrange multiplier on the implementability condition for the program defining $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widetilde{b}\ \forall k}$ in (15) $-$ (17), it follows from first order conditions that

$$\begin{gathered} u_c(c^*, n^*) + u_n(c^*, n^*) + \\ \mu_1 \begin{pmatrix} u_c(c^*, n^*) + u_n(c^*, n^*) \\ +u_{cc}(c^*, b^*)(c^* - \widetilde{b}) + u_{cn}(c^*, n^*)(c^* - \widetilde{b} + n^*) + u_{nn}(c^*, n^*)n^* \end{pmatrix} = 0. \end{gathered} \tag{B.15}$$

Since $\{c^*, n^*\} \neq \{c^{fb}, n^{fb}\}$ by the statement of the lemma, ($B.15$) implies that $\mu_1 \neq 0$. Using this observation, implicit differentiation of (18) and (19) taking ($B.15$) into account implies

$$c^{*\prime}(\widetilde{b}) = n^{*\prime}(\widetilde{b}) = -\mu_1 \frac{u_c(c^*, n^*)}{u_c(c^*, n^*) + u_n(c^*, n^*)}. \tag{B.16}$$

Finally, by the Envelope condition,

$$\frac{dW\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widetilde{b}\ \forall k}}{d\widetilde{b}} = -\mu_1 \frac{u_c(c^*, n^*)}{1-\beta}. \tag{B.17}$$

Now consider the solution to ($B.14$). Let μ_0 correspond to the Lagrange multiplier on ($B.12$). First order conditions with respect to $\widetilde{c}$ and $\widetilde{n}$ imply

$$\begin{gathered} u_c(\widetilde{c}, \widetilde{n}) + u_n(\widetilde{c}, \widetilde{n}) + \\ \mu_0 \begin{pmatrix} u_c(\widetilde{c}, \widetilde{n}) + u_n(\widetilde{c}, \widetilde{n}) \\ +u_{cc}(\widetilde{c}, \widetilde{n})(\widetilde{c} - b_1) + u_{cn}(\widetilde{c}, \widetilde{n})(\widetilde{c} - b_1 + \widetilde{n}) + u_{nn}(\widetilde{c}, \widetilde{n})\widetilde{n} \end{pmatrix} = 0. \end{gathered} \tag{B.18}$$

Since $\{\widetilde{c}, \widetilde{n}\} \neq \{c^{fb}, n^{fb}\}$ by the statement of the lemma, ($B.18$) implies that $\mu_0 \neq 0$. Since the solution admits $\widetilde{b} = \widehat{b} \in (\underline{b}, \overline{b})$, then we can ignore the bounds on government debt, and first order conditions with respect to $\widetilde{b}$ taking into account ($B.16$) and ($B.17$)

yields

$$\mu_0\mu_1\frac{u_{cc}(c^*,n^*)+u_{cn}(c^*,n^*)}{u_c(c^*,n^*)+u_n(c^*,n^*)}\left(\widehat{b}-b_1+\frac{\beta}{1-\beta}(\widehat{b}-\widetilde{b})\right)+\frac{\beta}{1-\beta}(\mu_0-\mu_1)=0. \tag{B.19}$$

Note that ($B.15$) and ($B.18$) imply that

$$\frac{\beta}{1-\beta}(\mu_0-\mu_1)=\frac{\beta}{1-\beta}\mu_0\mu_1\frac{u_{cc}(c^*,n^*)+u_{cn}(c^*,n^*)}{u_c(c^*,n^*)+u_n(c^*,n^*)}(\widetilde{b}-b_1) \tag{B.20}$$

Now consider if $\widetilde{b}=\widehat{b}$ so that $\{\widetilde{c},\widetilde{n}\}=\{c^*,n^*\}$. In that case, use ($B.20$) to substitute into ($B.19$) to achieve:

$$\mu_0\mu_1\frac{u_{cc}(c^*,n^*)+u_{cn}(c^*,n^*)}{u_c(c^*,n^*)+u_n(c^*,n^*)}(\widehat{b}-b_1)=0. \tag{B.21}$$

If it were the case that $\widehat{b}\neq b_1$, then ($B.21$) would require that $u_{cc}(c^*,n^*)+u_{cn}(c^*,n^*)=0$, which contradicts the fact that $u_{cc}(c^*,n^*)+u_{cn}(c^*,n^*)<0$. Therefore, $\widehat{b}=b_1$.

Case 2. Suppose that $\widehat{b}=b^{laffer}$. In this case, consider an analogous perturbation as in case 1 which reduces $\widehat{b}$ locally. For such a perturbation to be weakly dominated, the analog of ($B.21$) requires

$$\mu_0\mu_1\frac{u_{cc}(c^*,n^*)+u_{cn}(c^*,n^*)}{u_c(c^*,n^*)+u_n(c^*,n^*)}(\widehat{b}-b_1)\geq 0 \tag{B.22}$$

It follows from ($B.17$) that $\mu_1>0$ since any reduction in inherited debt can facilitate higher consumption and higher welfare. Since $u_{cc}(c^*,n^*)+u_{cn}(c^*,n^*)<0$, satisfaction of ($B.22$) requires

$$\mu_0(\widehat{b}-b_1)\leq 0. \tag{B.23}$$

Given that $\{\widetilde{c},\widetilde{n}\}=\{c^*,n^*\}=\{c^{laffer},c^{laffer}+g\}$, it can be verified from ($B.9$) that if $b_1<(>)\ \widehat{b}=b^{laffer}$, then ($B.18$) implies that $\mu_0>(<)\ 0$. This follows from the fact that $c^{laffer}<c^{fb}$ and the term in parentheses multiplying μ_0 in equation ($B.18$) is equal to 0 if $b_1=b^{laffer}$ and is increasing in b_1. Therefore, ($B.23$) cannot hold unless $\widehat{b}=b_1$.■

Proof of Lemma 5

Suppose that $b_l=\widehat{b}\ \forall l\leq m$. Given $\mathbf{B}$, let $\widehat{\mathbf{B}}(1)$ represent the portfolio which sets $\widehat{b}_k=b_{k+1}$ so that no retrading takes place. Note that in such a portfolio, $\widehat{b}_1=b_2$. Define $\widehat{\mathbf{B}}(2)$ analogously as the portfolio involving no retrading at the next date, so that

$\widehat{b}_k = b_{k+2}$ under $\widehat{\mathbf{B}}(2)$, and define $\widehat{\mathbf{B}}(l)$ $\forall l \leq m$ analogously. In any MPCE for which $b_1 = \widehat{b}$, a possible deviation sets $\{c, n\} = \{c^*(\widehat{b}), n^*(\widehat{b})\}$ and $b'_k = b_{k+1}$ so that no retrading takes place, where this deviation satisfies the resource constraint and implementability condition given (18) − (19). For such a deviation to be weakly dominated, it is necessary that:

$$V(\mathbf{B}) \geq u(c^*(\widehat{b}), n^*(\widehat{b})) + \beta V(\widehat{\mathbf{B}}(1)). \tag{B.24}$$

Forward induction on this argument implies that

$$V(\mathbf{B}) \geq \sum_{l=0}^{m-1} \beta^l u(c^*(\widehat{b}), n^*(\widehat{b})) + \beta^m V(\widehat{\mathbf{B}}(m)). \tag{B.25}$$

Combining ($B.10$) with ($B.25$), we achieve

$$V(\mathbf{B}) \geq V(\widehat{\mathbf{B}}(m)). \tag{B.26}$$

Now consider optimal policy starting from $\widehat{\mathbf{B}}(m)$. Note that since $b_l = \widehat{b}$ $\forall l \leq m$, then following the same arguments as in the proof of Lemma 3, a feasible strategy starting from $\widehat{\mathbf{B}}(m)$ is to issue a flat debt maturity with all bonds equal to $\widehat{b}$. Such a strategy ensures a constant consumption and labor allocation forever equal to $\{c^*(\widehat{b}), n^*(\widehat{b})\}$. As such, it follows that ($B.26$) holds with equality and that choosing a flat maturity distribution going forward is optimal.

Now we prove by contradiction that $b_{m+1} = \widehat{b}$. Suppose it were the case that $b_{m+1} \neq \widehat{b}$. This means that starting from $\widehat{\mathbf{B}}(m)$, the immediate debt which is owed by the government does not equal $\widehat{b}$. If this is the case, then the same arguments as those in the proof of Lemma 4 imply that there exists a deviation from the government's equilibrium strategy at $\widehat{\mathbf{B}}(m)$ which can strictly increase the government's welfare. However, if this is the case, ($B.26$) which holds with equality is violated. Therefore, it must be that $b_{m+1} = \widehat{b}$.■

Proof of Proposition 1 and Corollaries 1 and 2

The proof of Proposition 1 follows directly by induction after appealing to Lemmas 4 and 5.

To prove the first corollary, note that for the statement of Proposition 1 to be false, it is necessary that $\{c, n\} = \{c^{fb}, n^{fb}\}$. However, if this is the case, then ($B.4$) implies that

$$c^{fb} + \frac{u_n(c^{fb}, n^{fb})}{u_c(c^{fb}, n^{fb})} n^{fb} = -g = \sum_{k=1}^{\infty} \beta^{k-1} (1 - \beta) b_k \geq \underline{b} \tag{B.27}$$

which contradicts $b_k > -g$.

To prove the second corollary, note that from Lemma 2, it is necessary that the continuation equilibrium starting from a flat government debt maturity entail consumption and labor equal to $\{c^*(b), n^*(b)\}$ forever. The arguments in the proof of Lemmas 4 and 5 imply that if the government were to choose a non-flat maturity distribution going forward, future governments would not choose $\{c^*(b), n^*(b)\}$ forever. Therefore, all solutions to (12) $-$ (14) admit $b'_k = b \; \forall k$.■