# A Dynamic Theory of War and Peace*

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August 6, 2009

**Abstract**

This paper develops a dynamic theory of war and peace. In our framework, an aggressive country can forcibly extract concessions from a non-aggressive country via war. Alternatively, it can avoid war and allow the non-aggressive country to make concessions on its own. Both countries suffer from limited commitment, and under peace, the non-aggressive country may receive a private shock which deems concessions too costly. We show that the realization of war sustains concessions along the equilibrium path. In the efficient sequential equilibrium, the aggressive country punishes failed concessions by requesting larger and larger concessions, and their failure eventually leads to a war which can be temporary. After a temporary war, the aggressive country forgives the non-aggressive country by re-engaging in peace because of the coarseness of public information. In the long run, temporary wars can be sustained only if countries are patient, if the cost of war is large, and if the cost of concessions is low. Otherwise, the aggressive country cannot continue to forgive the non-aggressive country, and countries converge to total war (permanent war).

**Keywords:** Asymmetric and Private Information, Contract Theory, International Political Economy, War

**JEL Classification:** D82, D86, F5, N4

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*I am especially grateful to Daron Acemoglu and to Mike Golosov for their support and encouragement. I would also like to thank Stefania Albanesi, Andrew Atkeson, V.V. Chari, Sylvain Chassang, Alexandre Debs, Oeindrila Dube, Roozbeh Hosseini, Larry Jones, Patrick Kehoe, Narayana Kocherlakota, David Laitin, Gerard Padro-i-Miquel, Santiago Oliveros, Christopher Phelan, Robert Powell, Debraj Ray, Aleh Tsyvinski, Ivan Werning, and three anonymous referees for comments. This paper replaces a previous version which circulated under the title "A Dynamic Theory of Concessions and War."

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# 1 Introduction

The subject of war, first formalized in an economic framework in the seminal work of Schelling (1966) and Aumann and Maschler (1995), is the original impetus for important advances in the field of game theory. While there is renewed theoretical interest in the subject of war in economics, no formal framework exists for investigating the transitional dynamics between war and peace.[1] In this paper, we apply the modern tools from the theory of repeated games developed by Abreu, Pearce, and Stacchetti (1986,1990) to the classical subject of war. Specifically, we present a dynamic theory of war in which countries suffer from limited commitment and asymmetric information, two frictions which hamper their ability to peacefully negotiate. Our main conceptual result is a dynamic theory of escalation, temporary wars, and total war. On the theoretical side, our framework additionally allows us to derive novel results on the role of information in repeated games and dynamic contracts.[2]

In our model, one country, which we refer to as the aggressive country, is dissatisfied with the status quo and seeks concessions from its rival, which we refer to as the non-aggressive country. In every period, the aggressive country can either forcibly extract these concessions via war, or it can let the non-aggressive country peacefully make the concessions on its own. While peaceful concession-making is clearly less destructive than war, there are two limitations on the extent to which peaceful bargaining is possible. First, there is limited commitment. Specifically, the non-aggressive country cannot commit to making a concession once it sees that the threat of war has subsided. Moreover, the aggressive country cannot commit to peace in the future in order to reward concession-making by the non-aggressive country today. Second, there is imperfect information. The aggressive country does not have any information regarding the non-aggressive country's ability to make a concession, and the non-aggressive country can use this to its advantage. Specifically, since there is always a positive probability that concessions are too costly to make, the non-aggressive country may wish to misrepresent itself as being unable to make a concession whenever it is actually able to do so.

There are many applications of our framework. As an example, consider the events which preceded the First Barbary War (1801-1805) which are described in detail in Lam-

[1] Examples of recent papers on war include but are by no means limited to Baliga and Sjostrom (2004), Chassang and Padro-i-Miquel, (2008), Dixit (1987), Esteban and Ray (1999,2008), Fearon (1995,2004), Hirschleifer (1995), Hirshleifer, Boldrin, and Levine (2008), Jackson and Morelli (2008), Leventoglu and Slantchev (2007), Powell (1999,2004), Schwarz and Sonin (2004), and Skarpedas (1992).

[2] Our theoretical framework is very close to applied models of dynamic optimal contracts with private information, for example Atkeson and Lucas (1992), Golosov, Kocherlakota, and Tsyvinski (2003), Hauser and Hopenhayn (2004), Phelan (1995), Spear and Srivastava (1987), and Thomas and Worrall (1990).

bert (2005). The Barbary States of North Africa (the aggressive country) requested tribute from the United States (the non-aggressive country) in exchange for the safe passage of American ships through the Mediterranean. The United States failed to make successful payments on multiple occasions, in part because of the small size of the government budget relative to the requested tribute and because of resistance from Congress. This resulted in the Pacha of Tripoli requesting ever-increasing concessions. Eventually, failure to make concessions resulted in the First Barbary War which culminated with the partial forgiveness of past American debts and a continuation of the peaceful relationship between the Barbary States and the United States. Eventually, however, the United States continued to miss payments, and this resulted in the Second Barbary War (1815) from which the United States emerged victorious and which ended the Mediterranean tribute system. This historical episode highlights how limited commitment to peace and to concessions together with imperfect information about the cost of concessions can lead to war.

We use our framework to consider the efficient sequential equilibria in which countries follow history-dependent strategies so as to characterize the rich dynamic path of war and peace. In our characterization, we distinguish between temporary war and total war, defining the latter as the permanent realization of war and the absence of negotiation. In our model, war is the unique static Nash equilibrium, so that total war is equivalent to the repeated static Nash equilibrium in which countries refrain from ever peacefully negotiating.

Our paper presents three main results. Our first result is that wars are necessary along the equilibrium path. This insight adds to the theory of war by showing how the realization of war serves as a *punishment* for the failure to engage in successful peaceful bargaining in the past. In our framework, both the aggressive and non-aggressive country recognize that war is ex-post inefficient, though it improves ex-ante efficiency by providing incentives for concession-making by the non-aggressive country. Our intuition for the realization of war is linked to the insights achieved by previous work on the theory of dynamic games which shows that the realization of inefficient outcomes (such as price wars) can sustain efficient outcomes along the equilibrium path (e.g., Green and Porter, 1984, Rotemberg and Saloner, 1986, and Abreu, Pearce, and Stacchetti, 1986,1990). An important technical distinction of our work from this theoretical work is that information in our environment is coarse. Specifically, though the aggressive country is always certain that the non-aggressive country is cooperating whenever concessions succeed, the aggressive country receives no information if concessions fail, and it cannot deduce the likelihood that the non-aggressive country is genuinely unable to make a concession. Therefore, there is a chance that the aggressive country is making a mistake by going to war. This technical distinction is important for our next results.

Our second result is that temporary war can occur along the equilibrium path. While the aggressive country must fight the non-aggressive country in order to sustain concessions, it need not engage in total war; it can forgive the non-aggressive country for the first few failed concessions by providing the non-aggressive country with another chance at peace after the first round of fighting. This insight emerges because of the coarseness of information in our environment. There is a large chance that the aggressive country is misinterpreting the failure to make a concession as being due to lack of cooperation. Consequently, even though it is efficient for the aggressive country to punish initial failed concessions with the most extreme punishment of total war, this is not necessary for efficiency since it may be making an error. More specifically, the equilibrium begins in the following fashion: Periods of peace are marked by escalating demands in which failure to make concessions by the non-aggressive country leads the aggressive country to request bigger and bigger concessions. Both countries strictly prefer this scenario to one in which initial failures to make a concession are punished by war since war is destructive and represents a welfare loss for both countries. With positive probability, the non-aggressive country is incapable of making concessions for several periods in sequence so that requested concessions become larger and larger, and the only way for the aggressive country to provide incentives for such large concessions to be made is to fight the non-aggressive country if these concessions fail. Consequently, some initial concessions fail, and war takes place, and this war may culminate with the aggressive country forgiving the non-aggressive country and giving peace another chance.

Our final result is that countries can engage in temporary wars in the long run only under special conditions, and countries necessarily converge to total war if these conditions are not satisfied. More specifically, temporary wars can be sustained in the long run equilibrium if countries are sufficiently patient, if the cost of war is sufficiently large, and if cost of concessions is sufficiently low. If countries are patient and if war is very costly relative to peace, then total war is an extremely costly punishment which need not be exercised to elicit peaceful concessions, particularly since these are not so costly for the non-aggressive country to make. In the long run, no matter how many concessions fail, the aggressive country can continue to forgive the non-aggressive country after a round of fighting and to provide the non-aggressive country with another chance at peace. In contrast, if countries are impatient, if the cost of war is low, or if the cost of making concessions is high, then countries must converge to total war. In this scenario, even the most extreme punishment of total war is not unpleasant enough for the non-aggressive country since it does not suffer so much under war and it does not place much value on the future. Moreover, the cost of making a peaceful concession for the non-aggressive country is so large that it eventually requires an extreme punishment for failure to meet its obligation. Consequently, even though temporary wars can occur along the equilibrium path through the process of escalating demands, eventually it becomes impossible for the aggressive country to continue to forgive the non-aggressive country and total war becomes a necessity.

Our paper makes two contributions. First, it is an application of a dynamic imperfect information game with history-dependent strategies to war. This is important since the study of war is a dynamic issue in which countries have long memories–particularly in long-lasting conflicts–and since the literature on war has recognized the importance of limited commitment and imperfect information. In contrast to the current work on war, we provide an explanation for war which combines these two frictions in a dynamic setting in which countries follow history-dependent strategies and in which neither peace nor war is an absorbing state. This allows the model to feature escalating demands, temporary wars, and total war.[3]

Second, our paper is an application of a dynamic imperfect information game in an environment with a coarse information structure. Much of the existing literature on dynamic games and dynamic contracts assumes a rich information structure, and this leads to the necessity of the *Bang-Bang* characterization of efficient equilibria.[4] In the context of war and diplomacy, this information structure and its equilibrium implications may not be appropriate. First, countries often have very little information about their enemy's behavior and intentions, particularly when their enemy is not cooperating. Second, even though temporary wars occur in actuality, in many environments in which total war represents the worst possible outcome, the *Bang-Bang* characterization of efficient equilibria implies that temporary wars do not occur.[5] In this paper, we show that under a coarse information structure, the *Bang-Bang* property is not necessary for efficiency since the prospect for error is large, and this allows us to generate temporary wars in equilibrium. Nevertheless, we show that the *Bang-Bang* property must hold in the long run under

[3]In addition to the work discussed in Footnote 1, our work is also related to the model of Acemoglu and Robinson (2005) on conflict and regime change, though they do not consider the role of asymmetric information.

[4]This literature is large and cannot be summarized here. See for example Abreu, Pearce, and Stacchetti (1986,1990), Fudenberg, Levine, and Maskin (1994), and Sannikov (2007a,2007b).

[5]That is if total war is the min-max. This characterization applies only to efficient equilibria. Efficient equilibria as opposed to other often-examined equilibria such as Markovian equilibria or trigger strategy equilibria are a useful selection device in our setting since rival countries have long memories of their past interactions which can lead to escalation.

some conditions in which countries converge to total war.[6]

The paper is organized as follows. Section 2 describes the model. Section 3 defines efficient sequential equilibria. Section 4 characterizes the equilibrium and provides our main results. Section 5 concludes. The Appendix contains all proofs and additional material not included in the text.

# 2 Model

We consider an environment in which an aggressive country seeks political or economic concessions from a non-aggressive country.[7] In every period, the aggressive country can enforce these concessions by war, or it can let the non-aggressive country make these concessions unilaterally under peace. With some positive probability, the non-aggressive country is incapable of making concessions because they are too costly. This may happen, for instance, because the non-aggressive country's government experiences severe domestic opposition to concession-making. Nevertheless, this cost of concession-making is not observed by the aggressive country, so that the non-aggressive country can always lie about the true reasons for the failure of concessions.

More formally, there are two countries $i = \{1, 2\}$ and time periods $t = \{0, ..., \infty\}$. Country 1 is the aggressive country and country 2 is the non-aggressive country. In every date $t$, country 1 publicly chooses $W_t = \{0, 1\}$. If $W_t = 1$, war takes place, each country $i$ receives $w_i$, and the period ends. Alternatively, if $W_t = 0$, peace occurs, and country 2 publicly makes a concession to country 1 of size $x_t \in [0, \overline{x}]$. Country 1 receives $x_t$ and country 2 receives $-x_t - c(x_t, s_t)$ for $c(x_t, s_t)$ which represents country 2's *private* additional cost of making a concession $x_t$ which is a function of the state $s_t = \{0, 1\}$. $s_t$ is observed by country 2 but not by country 1. Let $c(x_t, s_t) = \overline{c} > 0$ if $x_t > 0$ and $s_t = 0$ and let $c(x_t, s_t) = 0$ otherwise. $s_t$ is stochastic and determined as follows. If $W_t = 0$, then prior to the choice of $x_t$, nature chooses $s_t$ with $\Pr\{s_t = 1\} = \pi \in (0, 1)$.

Concessions by country 2 are more costly if $s_t = 0$, but this cannot be verified by country 1. For example, suppose that $\overline{c}$ is very high–as we will do–and suppose this implies that concessions cannot be positive if $s_t = 0$. Then, if $W_t = 0$ and if country 1 receives no concessions (i.e., $x_t = 0$), country 1 cannot tell if country 2 could not make concessions since the cost was too high (i.e., $s_t = 0$) or if country 2 could make concessions

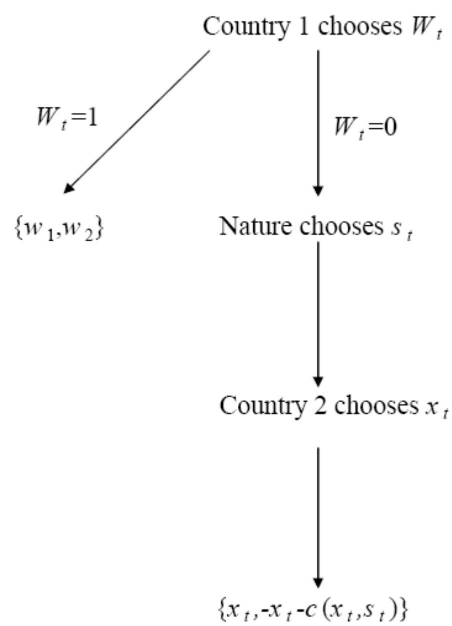
[6]See Fudenberg and Levine (2007) and Sannikov and Skrzypacz (2007) for an additional discussion of the characteristics of equilibria under different information structures.

[7]While we frame our discussion with respect to two countries, the insights from this model can apply to groups within countries.

but chose to not cooperate (i.e., $s_t = 1$).

We do not allow country 2 to choose to go to war or to receive concessions from country 1 only as a matter of parsimony. Under this additional refinement, the characterization of the equilibrium is identical to the one presented here, and all of our results are left unchanged.[8] Moreover, all of our results and intuitions generalize to an environment in which concessions are binary with $x_t \in \{0, \overline{x}\}$.[9] The game is displayed in Figure 1.

Figure 1: Game




Let $u_i(W_t, x_t, s_t)$ represent the payoff to $i$ at $t$.[10] Each country $i$ has a period zero welfare

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_i(W_t, x_t, s_t), \ \beta \in (0,1).$$

**Assumption 1 (inefficiency of war)** $\exists x \in [0, \overline{x}]$ s.t. $\pi x > w_1$ and $-\pi x > w_2$.

**Assumption 2 (military power of country 1)** $w_1 > 0$.

Assumption 1 captures the fact that war is destructive, since both countries can be made better off if war does not take place and country 2 makes a concession to country 1

[8] Such a model is isomorphic to the one here since country 1 always makes zero concessions. Details available upon request.

[9] This may be more appropriate for some applications. Details available upon request.

[10] Specifically, $u_1(W_t, x_t, s_t) = W_t w_1 + (1 - W_t) x_t$ and $u_2(W_t, x_t, s_t) = W_t w_2 - (1 - W_t)(x_t + c(x_t, s_t))$.

in state 1. Assumption 2 illustrates why country 1 is the aggressive country, since country 1's military power $w_1$ exceeds the economic resources under its control of size 0. The fact that $w_1$ exceeds $w_2$ (which is negative) is without loss of generality, and it is due to the normalization of both countries' resources to 0 which is purely for notational simplicity.[11]

Assumption 2 has an important implication. Specifically, in a one-shot equilibrium $W = 1$ is the unique static Nash equilibrium. This is because conditional on $W = 0$, country 2 chooses $x = 0$. Thus, by Assumption 2, country 1 chooses $W = 1$. Because the possibility of war precedes the possibility of peace, country 2 cannot commit to making concessions.[12] Consequently, in a static equilibrium, country 1, which is dissatisfied with the lack of concessions (by Assumption 2), will choose to enforce concessions via war rather than to provide country 2 with a chance at peace.

Since the static Nash equilibrium is inefficient (by Assumption 1), one can imagine that in a dynamic framework, country 1 may be able to enforce concessions from country 2 by rewarding successful concessions today by refraining from war in the future. Nevertheless, there are two obstacles to this arrangement which are important to consider. First, country 1 cannot commit to unconditionally refraining from war in the future, since it also suffers from limited commitment. Thus, whenever country 1 refrains from fighting at some date, it must be promised sufficient concessions in the future as a reward. Second, country 1 does not observe the state $s_t$ and the cost of concessions $c(\cdot, \cdot)$ which may be very large.

**Assumption 3 (high cost of concessions)** $\bar{c} > -\beta w_2 / (1-\beta)$.

In a dynamic environment, Assumption 3 implies that if $s_t = 0$, then country 2's concessions are so prohibitively costly that even the highest reward for a positive concession and the highest punishment for zero concessions together cannot induce a positive concession by country 2.[13] Therefore, concessions must be zero if $s_t = 0$. Consequently, if concessions fail (i.e., $x_t = 0$), country 1 cannot determine if this is unintentional because their cost is too high (i.e., $s_t = 0$) or if this is intentional because their cost is low (i.e., $s_t = 1$). This means that if country 1 goes to war in response to a failed concession, there is a chance that it is making a mistake since the concession's failure is unintentional.[14]

[11] Country 2 can be more powerful and control more resources than country 1 and vice versa as long as country 1's military power exceeds its economic power. More generally, $w_i$ can emerge from a possibly costly and unfair lottery over a set of resources.

[12] The fact that the opportunity to engage in war precedes the opportunity to engage in negotiations is important for the interpretation of dynamics since every period features either war or negotiations. This also captures the fact that negotiations are costly for country 1 since it forgoes the opportunity of war.

[13] This is because the discounted difference between the largest possible reward (permanent peace with continuation value of 0) and the largest possible punishment (permanent war with continuation value of $w_2 / (1-\beta)$) is not sufficiently large relative to $\bar{c}$.

[14] One can interpret our model as a reduced-form representation of an environment in which a stochastic

More formally, information in our environment is coarse. Though country 1 is always certain that country 2 is cooperating whenever concessions succeed, country 1 receives no information if concessions fail, and it cannot deduce the likelihood that country 2 is genuinely unable to make a concession. As we will discuss in Section 4.2, this detail is important as it will lead to temporary wars.

# 3 Efficient Sequential Equilibria

In this section, we formally define the equilibrium and we present our recursive method for the characterization of the efficient sequential equilibria between the two countries.

### 3.0.1 Equilibrium Definition

We begin by formally defining randomization since we allow countries to play correlated strategies. Let $z_t \in [0,1]$ represent an i.i.d. random variable independent of $s_t$ and all actions which is drawn from a continuous c.d.f. $G(\cdot)$ at the beginning of every period $t$. $z_t$ is observed by both countries and can be used as a randomization device which can improve efficiency by allowing country 1 to probabilistically go to war.

We consider equilibria in which each country conditions its strategy on past public information. Let $h_t = \{z^{t-1}, W^{t-1}, x^{t-1}\}$, the history of public information at $t$ prior to the realization of $z_t$.[15] Define a strategy $\sigma = \{\sigma_1, \sigma_2\} = \left\{\{W_t(h_t, z_t)\}_{t=0}^{\infty}, \left\{\{x_t(h_t, z_t, s_t)\}_{s_t=0,1}\right\}_{t=0}^{\infty}\right\}$. $\sigma$ is feasible if $\forall t \geq 0$ and $\forall (h_t, z_t)$,

$$\left\{W_t(h_t, z_t), \{x_t(h_t, z_t, s_t)\}_{s_t=0,1}\right\} \in \left\{\{0,1\}, [0, \overline{x}]^2\right\}.$$

Given $\sigma$, define the equilibrium continuation value for country $i$ at $(h_t, z_t)$ as

$$U_i(\sigma|_{h_t,z_t}) = \mathbf{E}\left\{\begin{array}{c} u_i(W_t(h_t, z_t), x_t(h_t, z_t, s_t), s_t) + \\ \beta \mathbf{E}\left\{U_i\left(\sigma|_{h_{t+1},z_{t+1}}\right)|h_t, z_t, W_t(h_t, z_t), x_t(h_t, z_t, s_t)\right\} \end{array} |h_t, z_t\right\} \tag{1}$$

for $\sigma|_{h_t,z_t}$ which is the continuation of a strategy after $(h_t, z_t)$ has been realized. Let $\Sigma_i|_{h_t,z_t}$ denote the entire set of feasible continuation strategies for $i$ after $(h_t, z_t)$ has been realized.

surplus only observable to and controlled by country 2 must be divided. This surplus is zero if $s_t = 0$ and positive if $s_t = 1$. The commitment problem arises because the surplus is divided after the possibility of war.

[15] Without loss of generality, we let $x_t = 0$ if $W_t = 1$.

**Definition 1** *$\sigma$ is a sequential equilibrium if it is feasible and if $\forall (h_t, z_t)$ and $i = 1, 2$*

$$U_i\left(\sigma|_{h_t,z_t}\right) \geq U_i\left(\sigma'_i|_{h_t,z_t}, \sigma_{-i}|_{h_t,z_t}\right)\ \forall \sigma'_i|_{h_t,z_t} \in \Sigma_i|_{h_t,z_t}.$$

In a sequential equilibrium, each country dynamically chooses its best response given the strategy of its rival. Because country 1's strategy is public by definition, any deviation by country 2 to a non-public strategy is irrelevant (see Fudenberg, Levine, and Maskin, 1994).

In order to build a sequential equilibrium allocation which is generated by a particular strategy, let $q_t = \{z^{t-1}, s^{t-1}\}$, the *exogenous* equilibrium history of public signals and states prior to the realization of $z_t$.[16] Define an equilibrium allocation as a function of the exogenous history:

$$\alpha = \left\{W_t\left(q_t, z_t\right), \left\{x_t\left(q_t, z_t, s_t\right)\right\}\right\}_{t=0}^{\infty}.$$

Let $\mathcal{F}$ denote the set of feasible allocations $\alpha$ with continuation allocations from $t$ onward which are measurable with respect to public information generated up to $t$. Let $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 1\right\}$ represent the expected continuation value to country 2 at $t+1$ conditional on $q_t$, $z_t$, and $s_t = 1$, and let $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 0\right\}$ be analogously defined for $s_t = 0$. Finally, define

$$\underline{U}_i = \frac{w_i}{1-\beta},$$

the payoff from the repeated static Nash equilibrium. Because the repeated static Nash equilibrium features the absence of negotiation, we refer to this event as *total war*.[17] We can thus provide necessary and sufficient conditions for $\alpha$ to be generated by sequential equilibrium strategies.

**Proposition 1** *$\alpha \in \mathcal{F}$ is a sequential equilibrium allocation if and only if $\forall (q_t, z_t)$,*

[16] Without loss of generality, let $s_t$ be revealed even if $W_t = 1$.

[17] One can interpret this event of total war by imagining that in every period of war, there is a probability that the conflict is resolved exogenously. One can formalize this idea easily in this setting by allowing for an exogenous probability that the game ends at $t$ conditional on war taking place at $t$. Alternatively, one can consider an extension of this model in which country 1 can arrive in a state where waging war is infinitely costly. None of our results or characterizations change, though total war now corresponds to a situation in which war only occur in periods in which country 1 is able to wage it, and there are zero concessions otherwise. Details available upon request.

$x_t(q_t, z_t, s_t = 0) = 0$ *if* $W_t(q_t, z_t) = 0,$

$$U_i\left(\alpha|_{q_t,z_t}\right) \geq \underline{U}_i \text{ for } i = 1, 2 \text{ and} \tag{2}$$

$$\begin{array}{c} -x_t(q_t, z_t, s_t = 1) + \\ \beta\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 1\right\} \end{array} \geq \beta\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 0\right\} \tag{3}$$
$$\text{if } W_t(q_t, z_t) = 0.$$

This proposition states that in a sequential equilibrium, both countries weakly prefer their equilibrium continuation values to total war, and country 2 weakly prefers to make a concession versus not making one. This proposition is a result from the original insight achieved by Abreu (1988) that sequential equilibria are sustained by the worst punishment. In this setting, all public deviations from equilibrium allocations lead countries to the worst punishment off the equilibrium path, which in our environment corresponds to total war.

We can now formally define efficient sequential equilibria since we focus on these. In contrast to trigger strategy equilibria, these equilibria can feature rich history-dependent dynamics such as escalating demands, and this is arguably a more accurate description of warring countries which are often motivated by long memories of their past interactions.[18] Let $U_i(\alpha)$ represent the period 0 continuation value to $i$ implied by $\alpha$ prior to the realization of $z_0$. Define $\Lambda$ as the set of sequential equilibrium allocations.

**Definition 2** *$\alpha \in \Lambda$ is an efficient sequential equilibrium allocation if $\nexists \alpha' \neq \alpha$ s.t. $\alpha' \in \Lambda$, $U_i(\alpha') > U_i(\alpha)$, and $U_{-i}(\alpha') \geq U_{-i}(\alpha)$ for $i = 1$ or $i = 2$.*

An efficient sequential equilibrium is therefore a solution to the following program, where $v_0$ is the minimum period 0 welfare promised to country 2:

$$\max_{\alpha} U_1(\alpha) \text{ s.t. } U_2(\alpha) \geq v_0 \text{ and } \alpha \in \Lambda. \tag{4}$$

#### 3.0.2 Recursive Representation

As is the case in many incentive problems, an efficient sequential equilibrium can be represented in a recursive fashion, and this is a useful simplification for characterizing equilibrium dynamics.[19] Specifically, at any date, the entire public history of the game is subsumed in the continuation value to each country, and associated with these two continuation values is a continuation sequence of actions and continuation values.

[18] This is also the approach pursued in the related work mentioned in Footnote 2.
[19] This is consequence of the insights from the work of Abreu, Pearce, and Stacchetti (1986,1990).

Formally, define $\Gamma = \{\{U_1(\alpha), U_2(\alpha)\} | \alpha \in \Lambda\}$ as the set of period 0 continuation values for both countries. By the stationarity of the game, $\{U_1(\alpha|_{q_t,z_t}), U_2(\alpha|_{q_t,z_t})\} \in \Gamma$ $\forall (q_t, z_t)$. Moreover, let $J(v)$ represent the value of $U_1(\alpha)$ at the solution to (4) subject to the additional restriction that $U_2(\alpha) = v$ for some $v \geq v_0$. Finally, define $\overline{U}_2 \geq \underline{U}_2$ as country 2's highest sequential equilibrium continuation value.

**Lemma 1** *(i) $\Gamma$ is convex and compact, (ii) $J(\underline{U}_2) = J(\overline{U}_2) = \underline{U}_1$, and (iii) $J(v)$ is weakly concave.*

The important features of the lemma are displayed in Figure 2 which depicts $J(v)$ as a function of $v$ for $v \in [\underline{U}_2, \overline{U}_2]$. The $y$-axis represents $J(v)$ and the $x$-axis represents $v$. All of the points underneath $J(v)$ and above the $x$-axis represent the set $\Gamma$ of sequential equilibrium continuation values.

There are three important features of Figure 2. First, $J(\underline{U}_2) = \underline{U}_1$. This is a consequence of (3), country 2's inability to commit to concessions. If country 2 could commit to concessions, then country 1 would choose no war and would request a level of concessions from country 2 sufficiently high so as to provide it with a continuation value of $\underline{U}_2$.[20] This would clearly be less destructive than total war by Assumption 1. However, under limited commitment, country 2 can always deviate from such an arrangement by making zero concessions today and guaranteeing itself a continuation value of at least $\underline{U}_2$ starting from tomorrow, so that its welfare today from the deviation is $\beta\underline{U}_2$ which exceeds its equilibrium welfare $\underline{U}_2$ (since $w_2$ is negative). Therefore, because country 2 cannot commit to concessions, country 1 must engage in total war in order to provide a continuation value of $\underline{U}_2$ to country 2.

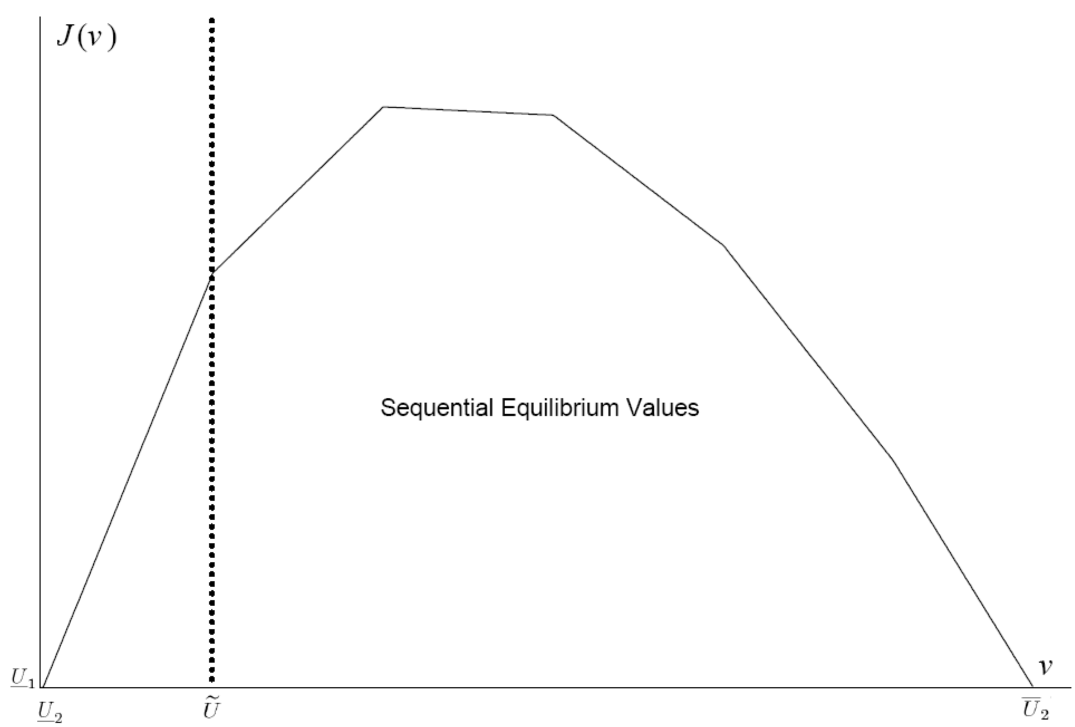
The second important feature of Figure 2 is that $J(\overline{U}_2) = \underline{U}_1$. This is a consequence of (2) for $i = 1$, country 1's inability to commit to peace. If country 1 could commit to peace, the highest continuation value to country 2 would be associated with permanent peace and zero concessions, yielding a continuation value of 0 to both countries. However, under limited commitment, country 1 can always deviate from such an arrangement by engaging in total war and guaranteeing itself a continuation value of $\underline{U}_1$ which exceeds 0 by Assumption 2. Therefore, because country 1 cannot commit to peace, the present discounted value of concessions must always be positive whenever country 1 is refraining from war, and this is embedded in the fact that $J(\overline{U}_2) = \underline{U}_1 > 0$.

The third important feature of Figure 2 is that $J(v)$ is inverse $U$-shaped. The increasing portion of $J(v)$ is a consequence of the fact that country 1 is made better off by the

[20] That is, assuming that $-\pi\overline{x} < w_2$ so that sufficiently large concessions are feasible.

increase in country 2's value since this implies a lower incidence of war and an increase in the size of the surplus to be shared by the two countries. The decreasing portion of $J(v)$ is a consequence of the fact that beyond a certain point, an increase in country 2's value entails a decrease in the size of the concessions made from country 2 to country 1, which means that country 1's value declines. Along this downward portion, it is not possible to make one country strictly better off without making the other country strictly worse off. As such, any efficient sequential equilibrium must begin on the downward-sloping potion of $J(v)$. Nevertheless, as we will see, the presence of imperfect information embedded in constraint (3) implies that it is not possible for the two countries to remain along the downward sloping portion of $J(v)$ forever.

Figure 2: $J(v)$




The important implication of Lemma 1 is that if $v$ represents the continuation value of country 2 at a given history in the efficient sequential equilibrium, then $J(v)$ represents the continuation value of country 1 at this given history.[21] We can therefore write (4)

[21] If country 1 were receiving any continuation value below $J(v)$ at a given public history, then the equilibrium would not be efficient.

recursively as:

$$J(v) = \max_{\{W_z, v_z^W, x_z, v_z^H, v_z^L\}_{z\in[0,1]}} \int_0^1 \left( \begin{array}{c} W_z \left[w_1 + \beta J\left(v_z^W\right)\right] + \\ (1-W_z)\left[\pi\left(x_z + \beta J\left(v_z^H\right)\right) + (1-\pi)\beta J\left(v_z^L\right)\right] \end{array} \right) dG_z \tag{5}$$

s.t.

$$v = \int_0^1 \left(W_z\left[w_2 + \beta v_z^W\right] + (1-W_z)\left[\pi\left(-x_z + \beta v_z^H\right) + (1-\pi)\beta v_z^L\right]\right) dG_z, \tag{6}$$

$$J\left(v_z^W\right), J\left(v_z^H\right), J\left(v_z^L\right) \geq \underline{U}_1 \; \forall z \in [0,1], \tag{7}$$

$$v_z^W, v_z^H, v_z^L \geq \underline{U}_2 \; \forall z \in [0,1], \tag{8}$$

$$-x_z + \beta v_z^H \geq \beta v_z^L \; \forall z \in [0,1], \tag{9}$$

$$v_z^H = v_z^L \text{ if } x_z = 0 \; \forall z \in [0,1], \tag{10}$$

$$W_z \in \{0,1\} \;\; \forall z \in [0,1], \text{ and } x_z \in [0, \overline{x}] \;\; \forall z \in [0,1]. \tag{11}$$

(2) represents the continuation value to country 1 written in a recursive fashion at a given history. With some abuse of notation, $W_z$ represents the realization of war today conditional on the random public signal $z$. $v_z^W$ represents the continuation value promised to country 2 for tomorrow conditional on war taking place today at $z$. If war does not take place at $z$, then concessions are zero if $s = 0$ (by Assumption 3), and concessions are equal to $x_z$ if $s = 1$. We refer to $x_z$ as the *requested* concession. Moreover, conditional on peace today, the continuation value promised to country 2 for tomorrow is $v_z^H$ if $s = 1$ and $v_z^L$ if $s = 0$.

Equation (6) represents the promise keeping constraint which ensures that country 2 is achieving a continuation value of $v$. Constraint (10) ensures that the continuation equilibrium is a function of public information. Constraints (11) ensure that the allocation is feasible.

Constraints (7) $-$ (9) represent the incentive compatibility constraints of this game. Without these constraints, the solution to the problem starting from an initial $v_0$ is simple: Countries refrain from war forever. Constraints (7) $-$ (9) captures the inefficiencies introduced by the presence of *limited commitment* and *imperfect information* which ultimately lead to the possibility of war. Constraint (7) captures the fact that at any history, country 1 cannot commit to refraining from total war which provides a continuation welfare of $\underline{U}_1$. Constraint (8) captures the fact that at any history, country 2 cannot commit to concession-making, as it can stop concessions forever and ensure itself a continuation value of at least $\underline{U}_2$. Therefore, country 2 cannot commit to making concessions and country 1 cannot commit to rewarding country 2 by refraining from war. Constraints (7) and (8) together capture the constraint of limited commitment. Under perfect information, they imply that if countries are sufficiently patient, permanent peace can be sustained by the off-equilibrium threat of total war. Constraint (9) captures the additional constraint of imperfect information: Country 1 does not observe the state $s$. If $s = 1$ and requested concessions $x > 0$ can be made, country 2 can always choose to pretend that $s = 0$ and make zero concessions without detection by country 1. Constraint (9) ensures that country 2's punishment from this deviation $(\beta v_z^L)$ is weakly exceeded by the equilibrium path reward from making the concession $(-x_z + \beta v_z^H)$.

# 4 Analysis

Let $\alpha^* (v)$ represent an argument which solves $(5) - (11)$, which consists of

$$\left\{W_z^* (v), v_z^{W*} (v), x_z^* (v), v_z^{H*} (v), v_z^{L*} (v)\right\}_{z\in[0,1]}.$$

Since $\alpha^* (v)$ may not be unique, we define the set of solutions for a particular $v$.

**Definition 3** $\Psi (v) = \{\alpha^* (v) | \alpha^* (v)$ *solves* $(5) - (11)\}$.

Let $W^* (v) = \int_0^1 W_z^* (v) dG_z$ and define $v^{W*} (v) = \int_0^1 W_z^* (v) v_z^{W*} (v) dG_z / W^* (v)$. Define $x^* (v) = \int_0^1 (1 - W_z^* (v)) x_z^* (v) dG_z / (1 - W^* (v))$ and $v^{H*} (v)$, and $v^{L*} (v)$ analogously. Note that given the concavity of (5) and the convexity of $(6) - (11)$, a solution always exists in which the only element of $\alpha^* (v)$ which depends on $z$ is $W_z^* (v)$ so that

$$\left\{v_z^{W*} (v), x_z^* (v), v_z^{H*} (v), v_z^{L*} (v)\right\} = \left\{v^{W*} (v), x^* (v), v^{H*} (v), v^{L*} (v)\right\} \ \forall z. \quad (12)$$

[22]

For the remainder of our discussion, we assume that countries are sufficiently patient that peace is incentive compatible for a positive mass of continuation values.[23]

**Assumption 4** $\beta > -w_1 / (\pi w_2)$.

In the following three sections, we characterize important features of the solution to the recursive program in $(5) - (11)$ (Section 4.1), and we use this characterization to

[22] Note that some of these variables are undefined whenever they become irrelevant if either $W^* (v)$ equals 0 or 1, but all of our results pertain to situations in which they are well defined.

[23] The precise implications of Assumption 4 are described in the Appendix.

describe the realization of war and peace along the equilibrium path (Section 4.2) and in the long run (Section 4.3).

## 4.1 Characterization

In this section, we characterize the solution to (5) – (11) in the below proposition. We provide a heuristic proof of this proposition in this section. Section 4.2 describes the economic intuition for this proposition together with its implications.

**Proposition 2** ***(characterization)***

1. $\exists \widetilde{U} \in \left(\underline{U}_2, \overline{U}_2\right)$ *s.t.* $\forall v \geq \widetilde{U}$ *and* $\forall \alpha^* (v) \in \Psi(v)$, $W^*_z(v) = 0 \ \forall z$,
2. $\forall \alpha^* (v) \in \Psi(v)$ *for* $v \geq \widetilde{U}$

$$x^*(v) = \min\left\{\beta \overline{U}_2 - v, \overline{x}\right\}, \ v^{H*}(v) = \min\left\{\frac{v+\overline{x}}{\beta}, \overline{U}_2\right\}, \text{ and } v^{L*}(v) = v/\beta, \text{ and} \tag{13}$$

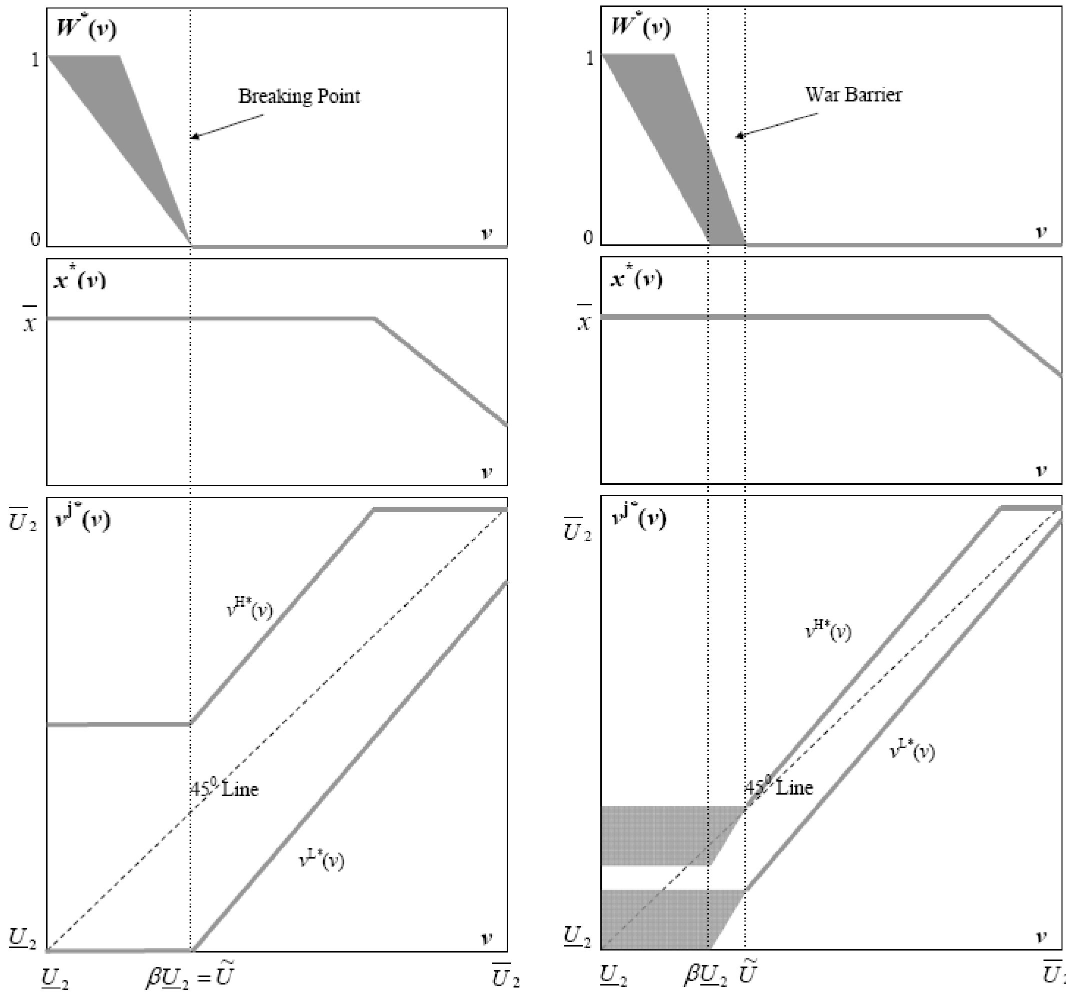
3. $\exists \alpha^* (v) \in \Psi(v)$ *for* $v \in \left(\underline{U}_2, \widetilde{U}\right)$ *s.t.* $W^*(v) > 0$ *and* $v^{W*}(v) > \underline{U}_2$.

The first part of this proposition states that above a continuation value $\widetilde{U}$, war ceases to occur. The second part of this proposition states that if $v \geq \widetilde{U}$, the average requested concession $x^*(v)$ weakly decreases in $v$, and the average reward $v^{H*}(v)$ and punishment $v^{L*}(v)$ weakly rise in $v$.[24] More specifically, the exact characterization in (13) captures the fact that (i) either the average requested concession is maximal or the average reward is maximal (i.e., $x^*(v) = \overline{x}$ or $v^{H*}(v) = \overline{U}_2$) and (ii) (9) must bind. The last part of this proposition states that for continuation values $v$ below $\widetilde{U}$ but above $\underline{U}_2$, there exist solutions for which war occurs and the average continuation value following the realization of war exceeds $\underline{U}_2$.

This proposition is displayed graphically in Figure 3. $v$ is on the $x$-axis. On the $y$-axis, $W^*(v)$ is in the in the top panel, $x^*(v)$ is in the middle panel, and $v^{H*}(v)$ and $v^{L*}(v)$ are in the bottom panel.[25] Note that in some cases–particularly in the top panel which features $W^*(v)$–these display correspondences. The left set of panels consider the case of a low cost of war to country 2 (i.e., $w_2 > -\overline{x}/\beta$) and the right set of panels consider the case of a high cost of war to country 2 (i.e., $w_2 < -\overline{x}/\beta$).[26] Proposition 2 applies to both cases and we distinguish between the two cases in Section 4.3.

[24] This is because $v < 0$ since $v = 0$ corresponds to permanent peace which is not incentive compatible.
[25] $v^{W*}(v)$ is excluded due to space constraints.
[26] We do not display the knife-edge case with $w_2 = -\overline{x}/\beta$ due to space constraints.

Figure 3: Recursive Solution




Case 1: $w_2 > -\overline{x}/\beta$ Case 2: $w_2 < -\overline{x}/\beta$

A heuristic proof of the first part of Proposition 2 is as follows. Countries can effectively randomize over the realization of war, and this ability to randomize implies a linearity in $J(v)$ for the continuation values which are generated by the positive probability of war. Since $W^*(\underline{U}_2) = 1$, it follows that there is an interval $\left[\underline{U}_2, \widetilde{U}\right)$ over which the probability of war is positive and over which $J(v)$ is linear. This is displayed in Figure 2. Thus, there exists a cutoff point $\widetilde{U}$ above which war ceases to occur since it reduces the welfare of both countries.[27]

[27] Note that by (8) and (9), $\widetilde{U} \geq \beta\underline{U}_2 > \underline{U}_2$. Whether $\widetilde{U}$ equals or exceeds $\beta\underline{U}_2$ is important for characterization of the long run in Section 4.3.

A heuristic proof of the second part of Proposition 2 is as follows. To understand why it is necessary that either $x^*(v) = \overline{x}$ or $v^{H*}(v) = \overline{U}_2$, consider a solution which satisfies (12) for $v \geq \widetilde{U}$ so that $W^*(v) = 0$. Fix $v^{L*}(v)$, and imagine if $x^*(v) < \overline{x}$ and $v^{H*}(v) < \overline{U}_2$. Then necessarily, a perturbation which increases $x^*(v)$ by $\epsilon$ and $v^{H*}(v)$ by $\epsilon/\beta$ for some $\epsilon > 0$ which is arbitrarily small continues to satisfy (6) $-$ (11). Moreover, the change in the welfare of country 1 is $\pi\left(\epsilon + \beta J\left(v^{H*}(v) + \epsilon/\beta\right) - J\left(v^{H*}(v)\right)\right)$ which is positive as long as the slope of $J(\cdot)$ is strictly above $-1$, which is the case in our framework. To see why, note that in an environment which ignores incentive compatibility constraints (7) $-$ (9), the slope of $J(\cdot)$ would be $-1$. This is because a transfer of 1 unit of welfare from country 1 to country 2 would occur through a reduction in concessions which the two countries equally value. In contrast, under (7) $-$ (9), the transfer of 1 unit of welfare from country 1 to country 2 occurs through a reduction in concessions *as well as a reduction in the probability of future war, which is beneficial to both countries.* Therefore, the implied reduction in size of the concession is not as large so that the slope of $J(\cdot)$ exceeds $-1$.

An additional rationale for this result can be generated by imagining an environment which ignores the upper bound on $x_z$ in (11) as well as the lower bound on $J\left(v_z^H\right)$ in (7). It can be shown that in such a setting, the efficient solution always requires country 1 to reward country 2's first successful concession with permanent peace. This is efficient since it allows country 1 to extract as much as possible in the current period while maximizing the duration of peace in the future, which is beneficial to both countries. This is insight is related to the "generalized no distortion at the top" result presented in Battaglini (2005). The result does not entirely hold in our environment (i.e. there may be a war even after a concession is made) because the upper bound on $x_z$ or the lower bound on $J\left(v_z^H\right)$ will bind.[28]

To understand why it is that (9) must bind–which is also embedded in the second part of Proposition 2–imagine if this were not the case, again considering a solution which satisfies (12). Then a perturbation which increases $v^{L*}(v)$ by $\epsilon/\beta$ either by increasing $x^*(v)$ by $\epsilon(1-\pi)/\pi$ or by reducing $v^{H*}(v)$ by $\epsilon(1-\pi)/(\pi\beta)$ strictly raises welfare. This is because an increase in $v^{L*}(v)$ raises the welfare of country 1 by reducing the incidence of war going forward. Technically, $v^{H*}(v)$ and $v^{L*}(v)$ are never inside the same line segment of $J(v)$, so that the strict concavity of $J(v)$ between these two points requires (9) to bind.

[28]More generally, in a situation in which country 1 can commit so that (7) can be ignored, $\overline{U}_2 = 0$ since country 1 can credibly promise to never attack country 2. Much of our characterization would be preserved in this case with the exception that one potential absorbing state in the long run is permanent peace following a sequence of successful concessions.

A heuristic proof of the third part of Proposition 2 is as follows. Consider a solution to the program for $v \in \left(\underline{U}_2, \widetilde{U}\right)$ for which $W^*(v) > 0$ and $v^{W*}(v) = \underline{U}_2$ associated with some $W^*(v) < 1$.[29] Consider a perturbation which increases the probability of war $W^*(v)$ by $\epsilon$ and increases the continuation value $v^{W*}(v)$ by an amount (which is a function of $\epsilon$) so as to leave (6) satisfied for some $\epsilon > 0$ arbitrarily small. This continues to satisfy $(6) - (11)$, and the linearity of $J(\cdot)$ in the interval $\left[\underline{U}_2, \widetilde{U}\right]$ implies that this perturbation yields the same welfare to country 1 as the original allocation. This idea is displayed in the top panel of Figure 3 which shows that in the interval $\left(\underline{U}_2, \widetilde{U}\right)$, $W^*(v)$ is a correspondence. It can take on low values if $v^{W*}(v)$ is chosen to be low, whereas it can take on high values if $v^{W*}(v)$ is chosen to be high. Intuitively, country 1 has flexibility in the intertemporal allocation of war. It can occur with high probability today, but with lower probability in the future, or alternatively, it can occur with low probability today, but with high probability in the future.

## 4.2 Equilibrium Path

In this section, we use the results of Proposition 2 to characterize the dynamics of war and peace along the equilibrium path. These are summarized in the below theorem where we let $\widehat{v} = \max_v \left\{v \in \left[\underline{U}_2, \overline{U}_2\right] \text{ s.t. } v = \arg\max_l J(l)\right\}$, the lowest continuation value on the downward sloping portion of $J(v)$, so that $U_2(\alpha) \geq v_0$ in (4) always binds if $v_0 \geq \widehat{v}$.[30]

**Theorem 1** ***(equilibrium path)***

1. *All solutions to* (4) *for all* $v_0$ *feature* $\Pr\{W_{t+k} = 1 | W_t = 0\} > 0$ $\forall t$ *for some* $k > 0$, *and*

2. *There exists a solution to* (4) *for almost all* $v_0 \geq \widehat{v}$ *which features*

$$\Pr\{W_t = 0 \textit{ and } W_{t+k} = 1 \textit{ and } W_{t+l} = 0\} > 0$$

*for some* $t \geq 0$ *and some* $l > k > 0$.

The first part of Theorem 1 states that in the solution to (4), if peace occurs at some date $t$, then war must be expected with positive probability at some date $t + k$. In other words, the realization of peace at any date must be followed by the positive probability of

[29] If $W^*(v) = 1$, then $v = \underline{U}_2$ by definition.

[30] We use $W_z^*(v)$ to refer to the probability of war in the recursive solution, whereas $W_t = \{0, 1\}$ corresponds to the stochastic realization of war in period $t$.

war in the future. The second part of Theorem 1 states that for almost all $v_0 \geq \widehat{v}$, there exists a solution to (4) which admits a sequence of allocations which feature temporary war with positive probability, where temporary war is defined formally as the realization of peace followed by probabilistic war followed by probabilistic peace.[31]

Both parts of this theorem are a direct implication of Proposition 2. To understand the proof of the first part of the theorem, note that if $v > \widetilde{U}$, then $W^*(v) = 0$ so that peace takes place today. Proposition 2 implies that for such $v < \overline{U}_2$,

$$v^{L*}(v) < v \text{ and } v^{H*}(v) > v.^{32} \tag{14}$$

Therefore, if a concession today is successful, the continuation value tomorrow increases, and by consequence, requested concessions tomorrow weakly decrease. Therefore, a reward for successful concessions is a reduction in future requested concessions. In contrast, if a concession today is unsuccessful, the continuation value tomorrow decreases, and by consequence, any concession requested tomorrow must weakly increase. This incentive scheme enforces concessions along the equilibrium path, since country 2 will always make a concession when it is able to, since failure to do so can result in an increase in future requested concessions. Therefore, along the equilibrium path, a long sequence of failed concessions by country 2 can cause requested concessions by country 1 to incrementally increase, and by Theorem 1, war eventually becomes necessary to enforce these ever-increasing concessions since continuation values decline beyond $\widetilde{U}$ with positive probability.

As an example, consider an environment in which $\overline{x}$ is arbitrarily large, so that the constraint that $x_z \leq \overline{x}$ never binds. In this scenario, Proposition 2 dictates that $v^{H*}(v) = \overline{U}_2$, $x^*\left(\overline{U}_2\right) = (\beta - 1)\overline{U}_2$, and $v^{L*}(v) \geq \widetilde{U}$ if $v \geq \widetilde{U}/\beta$. Now imagine that along the equilibrium path country 1 requests a concession $x_t$ from country 2, where associated with this concession is a continuation value $v_t \geq \widetilde{U}/\beta$ promised to country 2. It follows from Proposition 2 that $x_{t+1} = x^*\left(\overline{U}_2\right) + x_t/\beta$ if the concession at $t$ is unsuccessful and that $x_{t+1} = x^*\left(\overline{U}_2\right)$ if it is successful.[33] If $v_{t+1}$ in the former scenario exceeds $\widetilde{U}/\beta$, then it follows that a subsequent failed concession at $t+1$ causes country 1 to request a concession $x_{t+2} = x^*\left(\overline{U}_2\right) + x_{t+1}/\beta$ at $t+2$, and so on. Therefore, in this simple example, country 1 always requests a base concession of size $x^*\left(\overline{U}_2\right)$ plus accrued missed concessions from

[31] The theorem applies to almost all $v_0 \geq \widehat{v}$ because one can consider some environments where temporary wars do not occur starting from a countable finite set of $v_0$'s. See Appendix for details.

[32] The fact that $v^{H*}(v) > v$ is established in the proof of Proposition 2 since $v^{H*}\left(\widetilde{U}\right) \geq \frac{\widetilde{U}+\overline{x}}{\beta} \geq \widetilde{U}$.

[33] There is no randomization over the size of concessions in this example. The derivation of $x_{t+1}$ follows from the fact that $v_{t+1} = v_t/\beta$ if $s_t = 0$, $x_{t+1} = \beta\overline{U}_2 - v_{t+1}$, and $x_t = \beta\overline{U}_2 - v_t$.

the past, adjusting for discounting.

Why do failed concessions lead to escalating demands as opposed to immediate war? This is because war is costly to both countries, whereas larger concessions are only costly to country 2 and beneficial to country 1. Therefore, a sequence of initial missed concessions does not lead automatically to war, but to escalating demands. Country 1 forgives country 2 for the first missed concessions by requesting larger and larger concessions.[34] More specifically, it requests compensation for previously missed concessions, to the extent allowed by the upper bound $\overline{x}$. Country 1 would effectively like to postpone the realization of war as much as possible since it destroys surplus and harms both countries. An important way in which country 1 can postpone the realization of war is by requesting high enough concessions from country 2 today so as to reward their success with as high a reward as possible. This works since a higher reward is associated a longer duration of peace which benefits both countries going forward. Nevertheless, there is a limit to the feasibility of punishing country 2 with an increase in requested concessions, since beyond a certain point, requested concessions become so large that country 1 must punish their failure with the realization of war.

Efficient equilibria thus all begin on the downward sloping portion of $J(\cdot)$ in Figure 2. Continuation values decline with positive probability if concessions fail until the two countries inevitably transitions to the upward sloping portion of $J(\cdot)$. Once the two countries arrive at the upward sloping portion of $J(\cdot)$, they recognize that it is necessary for them to engage in an inefficient interaction in order to sustain the efficient interaction which has taken place in the past. Moreover, countries realize that *attempted* cooperation has in fact occurred in the past: War is by no means ex-post necessary, though it is ex-ante required for the enforcement of peace.[35]

To understand the proof of the second part of Theorem 1, note that once continuation values have declined beyond $\widetilde{U}$, Proposition 2 implies that war can take place with positive probability today and be followed by a continuation value which exceeds $\underline{U}_2$ starting from tomorrow. Such a continuation value must necessarily assign a positive probability to peace going forward since the payoff to country 2 exceeds that of total war. The intuition for this result is as follows. One obvious and efficient method of punishing country 2's failed concessions is for country 1 to engage in total war with low probability. What the theorem shows is that there exists an alternative method of efficiently punishing country

[34] An equivalent version of our model in which concessions are binary features escalation in the form of an increased probability of requested concessions.

[35] More generally, efficient sequential equilibria here are not renegotiation proof. According to the definition of Farrell and Maskin (1989), the only weakly renegotiation proof equilibrium in our setting is total war.

2 which is to engage in a temporary war with high probability. Both of these methods are equivalent from an efficiency perspective, and they deliver the same continuation value to country 2. Consequently, conditional on the two countries arriving to a history in the interval $\left(\underline{U}_2, \widetilde{U}\right)$–which is an event which occurs with positive probability starting from almost all $v_0 \geq \widehat{v}$–there is no need for country 1 to punish country 2's failed concessions with total war and temporary war can be generated along the equilibrium path.[36]

Theorem 1 delivers insights which build on the literature on dynamic games. In particular, the first part of the theorem is related to the work of Green and Porter (1984) who show that the realization of inefficient outcomes (such as price wars) can sustain efficient outcomes along the equilibrium path. In our setting, this insight implies that periods of peace are necessarily followed by periods of war. Without war, country 2 makes zero concessions, and by Assumption 2, country 1 cannot be satisfied by zero concessions. Note that as in Green and Porter (1984), this insight applies here to *all sequential equilibria*, and not just efficient sequential equilibria.

The second part of the theorem also relates to the work of Green and Porter (1984) since these authors present examples of sequential equilibria in which temporary price wars sustain cooperation. Nonetheless, the realization of temporary wars in their examples do not necessarily correspond to the efficient sequential equilibria as they do in our framework. For this reason, our second result is more closely related to the work of Abreu, Pearce, and Stacchetti (1986,1990) who analyze the efficient solution to a set of games related to that of Green and Porter (1984). These authors establish the necessity of the *Bang-Bang* property in the characterization of efficient equilibria. In this regard, our environment provides an example in which the satisfaction of the *Bang-Bang* property is not necessary for efficiency. As a consequence, in our setting temporary wars can be featured in an efficient equilibrium, and they are generated by a path of continuation values which fail to satisfy the *Bang-Bang* property.

More specifically, in our context, the *Bang-Bang* property implies that continuation values only travel to extreme points in the set of values $\Gamma$. Since all points on $J(\cdot)$ generated by probabilistic war are located on a line in the interval $\left[\underline{U}_2, \widetilde{U}\right)$ (see Figure 2), this implies that, if the *Bang-Bang* property were necessary in our context, then any realization of war would need to be associated with the absorbing state of total war. Thus, one would predict that in our framework, escalating demands and the failure to make concessions should necessarily lead directly to total war. What Theorem 1 shows is

[36] Hauser and Hopenhayn (2004) also present a game which features forgiveness, though their model considers the efficient transfer of favors whereas ours concerns the use of the inefficient action of war in eliciting concessions.

that even though such a dynamic path may be featured in an efficient equilibrium (i.e., satisfaction of the *Bang-Bang* property is sufficient for efficiency), an alternate dynamic path with temporary wars may also be featured in an efficient equilibrium (i.e., satisfaction of the *Bang-Bang* property is not necessary for efficiency).

The reason behind this is that information in our environment is coarse, and as a consequence, the *Bang-Bang* property–which is necessary in environments in which information is sufficiently rich–need not hold. More specifically, though country 1 is always certain that country 2 is cooperating whenever concessions succeed, country 1 receives no information if concessions fail, and it cannot deduce the likelihood that country 2 is genuinely unable to make a concession. Therefore, there is a chance that country 1 is making a mistake by going to war. Thus, total war does not dominate temporary wars as a punishment device since there is a limit on the information which is available to country 1 when it decides on the extent of war.

This situation would be significantly different, for instance, if country 1 could observe a sufficient amount of information in periods in which concessions fail. For example, suppose that a continuous public signal $y$ were revealed whenever concessions are zero, where this signal $y$ is informative about the state $s$ with higher values of $y$ being more likely if $s = 1$ (i.e., the cost of concessions is low). In this situation, escalating demands would always be followed by the stochastic realization of total war, with total war being more likely if concessions fail contemporaneously with the realization of a sufficiently high signal $y$. Country 1 would effectively use extreme rewards and punishments to provide incentives to country 2 while simultaneously utilizing the information in $y$ to optimally reduce the probability of error in going to war.[37] Our model highlights why this mechanism fails to work once information becomes coarse. Country 1 may make a mistake in going to total war, so that it does not strictly benefit from using such an extreme punishment.[38]

## 4.3 Long Run

Our model generates temporary wars along the equilibrium path, and a natural question concerns the extent to which such temporary wars can be sustained in the long run. This

[37] Specifically, $y$ has full support conditional on $s$ and it satisfies the monotone likelihood ratio property. In this circumstance, the failure of concessions cannot lead to continuation values in the interior of a line segment of $J(v)$ for any positive probability realizations of $y$ since this would imply that country 1 is not exploiting the full informational content of $y$. I thank Andrew Atkeson for pointing out this example.

[38] We conjecture that our result of temporary war holds for the intermediate case in which country 1 observes an signal $y$ which has $N$ realizations, with higher values being more likely if $s = 1$. As $N \to \infty$, we should converge to the continuous signal case in which temporary war cannot occur with positive probability.

is particularly relevant for understanding conflicts which have lasted a significant length of time but have not culminated in total war. We argue that even though temporary wars can occur along the equilibrium path, they can only be sustained in the long run if countries are sufficiently patient ($\beta$ is high), if the cost of war is sufficiently large ($w_2$ is low), and if the cost of concessions is sufficiently low ($\overline{x}$ is low).

**Theorem 2** ***(long run)***

1. *If* $w_2 \geq -\overline{x}/\beta$, $\nexists$ *a solution to* (4) *for any* $v_0$ *s.t.* $\lim_{t\to\infty} \Pr\{W_t = 0\} > 0$,
2. *If* $w_2 < -\overline{x}/\beta$, $\exists$ *a solution to* (4) *for all* $v_0$ *s.t.* $\lim_{t\to\infty} \Pr\{W_t = 0\} = 0$, *and*
3. *If* $w_2 < -\overline{x}/\beta$, $\exists$ *a solution to* (4) *for all* $v_0$ *s.t.* $\lim_{t\to\infty} \Pr\{W_t = 0\} > 0$.

The first part of Theorem 2 states that if $w_2 \geq -\overline{x}/\beta$, meaning if the cost of war $w_2$ is low relative to the discounted cost of the maximal concession $-\overline{x}/\beta$, then there is no efficient sequential equilibrium which features peace in the long run. Therefore, all allocations necessarily converge to total war. Together with Theorem 1, Theorem 2 effectively states that even though temporary wars can occur along the equilibrium path, eventual convergence to total war is necessary. The second and third parts of Theorem 2 state that if instead $w_2 < -\overline{x}/\beta$, then there exist efficient sequential equilibria which converge to total war as well as efficient sequential equilibria which feature temporary war in the long run. Thus, though convergence to total war constitutes an efficient equilibrium, convergence to total war is not necessary for efficiency. Note that if $w_2 \geq -\overline{x}/\beta$, then the *Bang-Bang* property described in Section 4.2 necessarily holds in the long run, whereas if $w_2 < -\overline{x}/\beta$, the *Bang-Bang* property is not necessary even in the long run.

A heuristic proof of the first part of Theorem 2 is as follows. One must first establish that in this case, $\widetilde{U} = \beta \underline{U}_2$, where $\widetilde{U}$ is the continuation value above which peace with probability 1 strictly dominates probabilistic war and $\beta \underline{U}_2$ is the continuation value below which peace with probability 1 ceases to be incentive compatible (since $v/\beta \geq v^{L*}(v) \geq \underline{U}_2$ by (9)). The reason why peace strictly dominates war whenever it is incentive compatible is because concessions can be rewarded with an *increase* in continuation value to country 2, and this reduces the incidence of war in the future which benefits country 1 and maximizes efficiency. Formally, $\overline{x}$ is large enough that one can choose $v^{H*}(v) \geq v$ for all $v$ which weakly exceed $\beta \underline{U}_2$. This means that any temporary wars which take place in the interval $\left(\underline{U}_2, \widetilde{U}\right)$ necessarily end at a *breaking point* in which country 2 receives promised utility $\widetilde{U} = \beta \underline{U}_2$ conditional on peace. At this point, country 1 gives country 2 a second chance at peace and requests a concession. If this concession fails, then total war ensues since

$v^{L*}\left(\widetilde{U}\right) = \underline{U}_2$. Alternatively, if this concession succeeds, country 2 is rewarded and peace ensues. Nonetheless, by Theorem 1 such a peace leads to a future war with positive probability which can involve either a total war or a temporary war which again ends at the breaking point. Because concessions can always fail with positive probability at the breaking point, and because the failure of concessions at the breaking point lead to total war, total war in this case is unavoidable in the long run.[39]

A heuristic proof of the second and third parts of Theorem 2 is as follows. Because the satisfaction of the *Bang-Bang* property is sufficient for efficiency, one can easily construct examples in which the first realization of war–which is necessary by Theorem 1–is associated with total war, which explains the second part of the theorem. To understand the third part of the theorem, note that in this case $\widetilde{U} > \beta\underline{U}_2$, so that even if peace with probability 1 is incentive compatible, it need not be strictly optimal. Specifically if $v$ is between $\beta\underline{U}_2$ and $\widetilde{U}$, then peace with probability 1 and probabilistic war are equally efficient means of providing continuation value $v$ to country 2. This is because in this region, $x^*(v) = \overline{x}$ and $v^{H*}(v) \leq \widetilde{U}$, so that even if peace takes place today and if concessions are successful today, war cannot be avoided in the future since otherwise country 2 would not be receiving the same continuation value. In other words, concessions today cannot be made large enough so as to allow country 1 to forgive country 2 for past failed concessions going forward. More generally, the failure of concessions along the equilibrium path leads continuation values to eventually decline beyond $\widetilde{U}$ and to effectively cross a *war barrier* from which there is no return to higher continuation value. Because peace with probability 1 in this region can be associated with a continuation value which exceeds $\beta\underline{U}_2$, the failure of concessions need not be punished with total war since one can always choose $v^{L*}(v) > \underline{U}_2$.[40] Thus, total war is not required for the enforcement of incentives and temporary wars can occur forever.

The intuition for the first part of the theorem is as follows. If $w_2$ is high and $\beta$ is low, then the cost of total war to country 2 is low relative to the cost of the maximal concession of size $\overline{x}$. As a consequence, it is necessary for country 1 to use the most extreme punishment to induce concessions from country 2, since the weaker punishment of temporary war cannot induce these large concessions. More specifically, since assured peace takes place whenever it is incentive compatible (i.e., $\widetilde{U} = \beta\underline{U}_2$), the duration of peace is prolonged as much as possible along the equilibrium path. Nonetheless, this

[39] In other words, a transition path which avoids total war occurs with probability zero since concessions would need to succeed every single time the breaking point is reached.

[40] These facts are displayed in Figure 3b which shows that the probability of war and continuation values as a function of $v$ are correspondences for continuation values below $\widetilde{U}$.

comes at a cost in the long run since eventually, a long stream of concessions fail, and this leads to inevitable total war.[41] Therefore, the two countries sacrifice their welfare in the long run in exchange for efficient incentive provision along the equilibrium path. Note that if $w_2 \geq -\overline{x}$, convergence to total war takes place even as $\beta$ approaches 1.[42]

The intuition for the second and third parts of the theorem are as follows. If $w_2 < -\overline{x}/\beta$, then the cost of total war to country 2 is extremely high relative to the cost of the maximal concession of size $\overline{x}$. As a consequence, though country 1 could use the most extreme punishment to induce concessions from country 2, this is not necessary for efficiency. This is because the weaker punishment of temporary wars is sufficiently painful. Specifically, peace with probability 1 does not strictly dominate probabilistic war in the range $\left(\underline{U}_2, \widetilde{U}\right)$ since successful concessions cannot be rewarded with the prolongation of the peace. Therefore, there is no sense in which the two countries maximize the duration of peace along the equilibrium path at the cost of total war in the long run. Specifically, even if a very long sequence of concessions fails in the long run, countries do not converge to total war because every missed concession can be punished with a war which stochastically ends with another chance at peace and this provides sufficient inducement for concessions.

Note that an implication of our model is that an increase in $\overline{x}$ from below $-\beta w_2$ to above $-\beta w_2$ leads countries away from the possibility of long run temporary wars to the necessity of long run total war. An important point to bear in mind is that this transformation *increases* period 0 welfare. The reason is that if $\overline{x}$ increases, it becomes easier for the two countries to postpone the realization of war, since escalating demands as opposed to war can be more easily used by country 1 to provide inducements to country 2. Rather than fight at a particular date, country 1 can request even larger concessions from country 2 and leave war for a later date. This raises welfare along the equilibrium path by prolonging the duration of peace. Nevertheless, this is made at the cost of total war in the long run which becomes necessary to induce the increase in concessions under peace.

To understand the role of the upper bound $\overline{x}$ in generating these distinct long run outcomes in our environment, it is useful to consider a more general environment in which the upper bound on concessions $\overline{x}$ is ignored but where country 2 experiences a cost of concessions equal to $-e(x_t)$ if $s_t = 1$ for some increasing function $e(\cdot)$. One can show

[41] Note that if in every period there is a positive probability that the government in country 1 is replaced by a government which does not remember the past, then the equilibrium restarts from some initial $v_0$ following such a shock. One can show that total war would be avoided in this case, though efficiency would be reduced since this undermines the discipline on country 2. Details available upon request.

[42] It is nevertheless the case that the probability of total war also approaches zero since $v^{L*}(v)$ approaches $v$ and the decrease in continuation value approaches zero (see Proposition 2).

that in this modified environment, wars continue to occur with positive probability only if $v \in \left[\underline{U}_2, \widetilde{U}\right)$. Moreover, by analogous reasoning to our previous arguments, convergence to total war is necessary in the efficient sequential equilibrium if $\widetilde{U} = \beta \underline{U}_2$, though it is not necessary if $\widetilde{U} > \beta \underline{U}_2$. Importantly, it can be shown that if $e(\cdot)$ is a smooth function, then necessarily $\widetilde{U} = \beta \underline{U}_2$ so that convergence to total war is necessary for efficiency. This is because starting from an equilibrium in which country 2 receives $\beta \underline{U}_2$ via peace with probability 1, country 1 optimally requests a high enough concession so as to to choose an incentive compatible reward $v^{H*}(\beta \underline{U}_2) \geq \beta \underline{U}_2$ since this benefits country 1 since it reduces the incidence of war going forward. In contrast, it may no longer be true that peace with probability 1 strictly dominates probabilistic war at $v = \beta \underline{U}_2$ if instead $e(\cdot)$ is a discontinuous function. This is because starting from an equilibrium in which country 2 receives $\beta \underline{U}_2$ via peace with probability 1, $v^{H*}(\beta \underline{U}_2) < \beta \underline{U}_2$ and a perturbation which increases concessions at $v = \beta \underline{U}_2$ requires an increase in $v^H$ which is either too high to be incentive compatible or too high to be efficient from the perspective of country 1 since it reduces concessions by too much going forward.[43] Therefore, if $e(\cdot)$ is discontinuous at particular values of $x_t$, it may be the case that $\widetilde{U} > \beta \underline{U}_2$ so that temporary wars can be featured in the efficient equilibrium. In sum, our environment considers a special case of this more general environment in which $e(x_t) = x_t$ for $x_t \in [0, \overline{x}]$ and $e(x_t)$ is arbitrarily large for $x_t > \overline{x}$. Thus, the exact value of $\overline{x}$–which corresponds to the maximal cost of concessions *in equilibrium*–determines whether $\widetilde{U}$ exceeds or is equal to $\beta \underline{U}_2$.

# 5 Conclusion

We have analyzed a dynamic model of war and peace to determine whether and how temporary wars between two countries can occur. In doing so, we have characterized the dynamics of escalation and highlighted how imperfect information generates temporary wars. Moreover, we present conditions which are necessary for the two countries to avoid total war and engage in temporary wars in the long run. Our analysis shows that countries which escalate to total war reach a breaking point at the culmination of every temporary war. In contrast, countries which are able to avoid total war find themselves having passed a war barrier beyond which forgiveness in the form of lower demands by the aggressive country become impossible. Our analysis provides us with a framework for understanding which conflicts can be sustained without convergence to total war.

[43] Such a perturbation unambiguously benefits country 1 if $v^H$ is rising on the upward sloping portion of $J(\cdot)$ but it does not unambiguously do so if $v^H$ is on the downward sloping portion. Details available upon request.

There are some important caveats in interpreting our results. First, we have ignored the fact that the cost of concession can be persistent by assuming that the state is i.i.d. This assumption is not made for realism but for convenience since it maintains the common knowledge of preferences over continuation contracts and simplifies the recursive structure of the efficient sequential equilibria. Future work should consider the effect of relaxing this assumption and potentially using some of the tools in Fernandes and Phelan (2000) in this regard. Second, we have ignored the possibility that the aggressive country may be able to exert some effort in more effectively monitoring its rival. In such an environment, both the precision as well as the coarseness of public signals becomes endogenous, and given our discussion in Section 4.2, this will affect the extent to which war is necessary as well as the extent to which temporary war can be sustained in an efficient equilibrium. Third, in choosing to focus on the role of diplomatic concessions, we have ignored the fact that military concessions such as disarmament could also serve to avert conflict by altering the payoff from war. Finally, we have implicitly assumed that there is a single good over which the two countries bargain and that in every period concessions are one sided. A realistic extension of this framework is one in which bilateral concessions are necessary to sustain peace. In such a setting, both countries could potentially have an incentive to engage in war. A thorough investigation of the implications of these issues for our results would be interesting for future research.

# 6 Appendix

## 6.1 Proofs for Section 3

### 6.1.1 Proof of Proposition 1

**Step 1.** If $\alpha$ is a sequential equilibrium allocation, then $x_t(q_t, z_t, s_t = 0) = 0 \ \forall (q_t, z_t)$ where $W_t(q_t, z_t) = 0$. If instead $x_t(q_t, z_t, s_t = 0) > 0$, consider a deviation by country 2 at $(q_t, z_t, s_t = 0)$ to $x'_k(q_k, z_k, s_k) = 0 \ \forall k \geq t$ and $\forall (q_k, z_k, s_k)$ which yields a minimum continuation value of $\beta \underline{U}_2$. Since $x_t(q_t, z_t, s_t = 0)$ is bounded from below by 0 so that $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1}, z_{t+1}}\right)|q_t, z_t, s_t = 1\right\}$ is bounded from above by 0, if this deviation is weakly dominated, then it must be that $-\bar{c} \geq \beta \underline{U}_2$, but this violates Assumption 3. **Step 2.** The necessity of (2) for $i = 1$ follows from the fact that country 1 can choose $W'_k(q_k, z_k) = 1$ $\forall k \geq t$ and $\forall (q_k, z_k)$ and this delivers continuation value $\underline{U}_1$. The necessity of (2) for $i = 2$ follows from the fact that country 2 can choose $x'_k(q_k, z_k, s_k) = 0 \ \forall k \geq t$ and $\forall (q_k, z_k, s_k)$, and this delivers a minimum continuation value $\underline{U}_2$. The necessity of (3) follows from the fact that conditional on $W_t(q_t, z_t) = 0$, country 2 can unobservably deviate to $x'_t(q_t, z_t, s_t = 1) = x_t(q_t, z_t, s_t = 0) = 0$ and follow the equilibrium strategy associated with $(q_t, z_t, s_t = 0)$ thereafter. **Step 3.** For sufficiency, consider an allocation in which $x_t(q_t, z_t, s_t = 0) = 0 \ \forall (q_t, z_t)$ which also satisfies (2) and (3), and construct the following off-equilibrium strategy. Any observable deviation results in a reversion to the repeated static Nash equilibrium. We only consider single period deviations since $\beta < 1$ and since continuation values are bounded. If $W_t(q_t, z_t) = 1$, a deviation to $W'_t(q_t, z_t) = 0$ is strictly dominated by (2) since $\beta \underline{U}_1 < \underline{U}_1$. If $W_t(q_t, z_t) = 0$, a deviation to $W'_t(q_t, z_t) = 1$ is weakly dominated by (2). If $W_t(q_t, z_t) = 0$, any deviation to $x'_t(q_t, z_t, s_t = 1) > 0$ is weakly dominated by a deviation to $x'_t(q_t, z_t, s_t = 1) = x_t(q_t, z_t, s_t = 0) = 0$ since $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1}, z_{t+1}}\right)|q_t, z_t, s_t = 0\right\} \geq \underline{U}_2$. A deviation to $x'_t(q_t, z_t, s_t = 1) = 0$ is weakly dominated by (3). Any deviation to $x'_t(q_t, z_t, s_t = 0) \neq x_t(q_t, z_t, s_t = 1)$ is strictly dominated since $\bar{c} > 0$. Since $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1}, z_{t+1}}\right)|q_t, z_t, s_t = 1\right\} \leq 0$, by Assumption 3 and (2), a deviation to $x'_t(q_t, z_t, s_t = 0) = x_t(q_t, z_t, s_t = 1)$ is strictly dominated.

**Q.E.D.**

### 6.1.2 Proof of Lemma 1

**Step 1.** Consider two continuation value pair $\{U'_1, U'_2\} \in \Gamma$ and $\{U''_1, U''_2\} \in \Gamma$ with corresponding allocations $\alpha'$ and $\alpha''$. It must be that

$$\{U_1^\kappa, U_2^\kappa\} = \{\kappa U'_1 + (1-\kappa) U''_1, \kappa U'_2 + (1-\kappa), U''_2\} \in \Gamma \ \forall \kappa \in (0,1).$$

Define $\alpha^{\kappa} = \{\alpha^{\kappa}|_{q_0,z_0}\}_{z_0\in[0,1]}$ as follows:

$$\alpha^{\kappa}|_{q_0,z_0} = \begin{cases} \alpha'|_{q_0,z_0} & \text{if } z_0 = 0 \\ \alpha'|_{q_0,\frac{z_0}{\kappa}} & \text{if } z_0 \in (0,\kappa) \\ \alpha''|_{q_0,\frac{z_0-\kappa}{1-\kappa}} & \text{if } z_0 \in [\kappa,1] \end{cases},$$

where $\alpha'|_{q_0,\frac{z_0}{\kappa}}$ for $z_0 \in (0,\kappa)$ is identical to $\alpha'|_{q_0,z_0}$ with the exception that $\frac{z_0}{\kappa}$ replaces $z_0$ in all information sets $q_t$, and $\alpha''|_{q_0,\frac{z_0-\kappa}{1-\kappa}}$ for $z_0 \in [\kappa,1]$ is analogously defined. $\alpha^{\kappa}$ achieves $\{U_1^{\kappa}, U_2^{\kappa}\}$, and since $\alpha', \alpha'' \in \Lambda$, then $\alpha^{\kappa} \in \Lambda$. **Step 2.** $\Gamma$ is bounded since $U_i(\alpha)$ is bounded for $i = 1,2$. **Step 3.** To show that $\Gamma$ is closed, consider a sequence $\{U'_{1j}, U'_{2j}\} \in \Gamma$ such that $\lim_{j\to\infty}\{U'_{1j}, U'_{2j}\} = \{U'_1, U'_2\}$. There exists one corresponding sequence of allocations $\alpha'_j$ which converges to $\alpha'_\infty$ since $U_i(\alpha'_j)$ is continuous in $\alpha'_j$. Since every element of $\alpha'_j$ at $(q_t, z_t)$ is contained in $\{0,1\} \times [0,\overline{x}]^2$, and since (2) and (3) are weak inequalities, then $\Lambda$ is closed and $\alpha'_\infty \in \Lambda$. Since $\beta \in (0,1)$, then by the Dominated Convergence Theorem, $U_i(\alpha'_\infty) = U'_i$ for $i = 1,2$. Therefore $\{U'_1, U'_2\} \in \Gamma$ so that $\Gamma$ is compact. **Step 4.** To show that $J(\underline{U}_2) = J(\overline{U}_2) = \underline{U}_1$, note that it is not possible that $J(\cdot) < \underline{U}_1$ since this violates (2) for $i = 1$. **Step 5.** Imagine if $J(\underline{U}_2) > \underline{U}_1$ and consider the associated $\alpha$. By Assumptions 1 and 2 and equation (2) for $i = 2$, equation (3) implies that $U_2(\alpha|_{q_0,z_0}) \geq \beta\underline{U}_2 > \underline{U}_2$ if $W_0(q_0,z_0) = 0$. Since $U_2(\alpha|_{q_0,z_0}) \geq \underline{U}_2$, then $U_2(\alpha|_{q_0,z_0}) = \underline{U}_2$ and $W_0(q_0,z_0) = 1$ $\forall(q_0,z_0)$. This requires $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\} = \underline{U}_2$ $\forall(q_0,z_0)$ and therefore $U_2(\alpha|_{q_1,z_1}) = \underline{U}_2$ $\forall(q_1,z_1)$. Forward induction on this argument implies that $W_t(q_t,z_t) = 1$ $\forall(q_t,z_t)$ so that $J(\underline{U}_2) = \underline{U}_1$. **Step 6.** Imagine if $J(\overline{U}_2) > \underline{U}_1$ and consider the associated $\alpha$. Since $U_2(\alpha|_{q_0,z_0}) \leq \overline{U}_2$, then $U_2(\alpha|_{q_0,z_0}) = \overline{U}_2$ $\forall(q_0,z_0)$ in order that $U_2(\alpha) = \overline{U}_2$. If $W_0(q_0,z_0) = 1$, then $\overline{U}_2 = w_2 + \beta\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\} \leq w_2 + \beta\overline{U}_2$, which means that $\underline{U}_2 = \overline{U}_2$ and by step 5, $J(\overline{U}_2) = \underline{U}_1$. Now consider $W_0(q_0,z_0) = 0$. It must be that $x_0(q_0,z_0) = 0$ and $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\} = \overline{U}_2$, otherwise it is possible to reduce $x_0(q_0,z_0)$ or increase $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\}$ while maintaining (2) and (3) and strictly increasing $U_2(\alpha)$. This means that $\overline{U}_2 = \beta\overline{U}_2$, but this violates the fact that $\overline{U}_2 < 0$, since $\overline{U}_2 = 0$ is not incentive compatible. This is because by Assumption 1, $J(\overline{U}_2) + \overline{U}_2 \leq 0$, so that if $\overline{U}_2 = 0$, then $J(\overline{U}_2) \leq 0$, which violates (2) for $i = 1$. **Step 7.** It follows that $J(v)$ is weakly concave since $J(\underline{U}_2) = J(\overline{U}_2) = \underline{U}_1$, $J(v) \geq \underline{U}_1$ $\forall v \in [\underline{U}_2, \overline{U}_2]$ by (2), and $\Gamma$ is a convex set.

**Q.E.D.**

### 6.1.3 Implications of Assumption 4

Assumption 4 implies that $J(v) > \underline{U}_1$ for some $v$. To see why, construct the following equilibrium. If $s_{t-1} = 1$, then $W_t = 0$ and $x_t = x = w_1/\pi + \epsilon$ if $s_t = 1$ and $x_t = 0$ otherwise for $\epsilon > 0$ arbitrarily small so as to continue to satisfy $-w_1 - \pi\epsilon > w_2$. If $s_{t-1} = 0$, both countries revert to the repeated static Nash equilibrium forever. Let $W_0 = 0$. By Assumption 1, country 1's welfare strictly exceeds $\underline{U}_1$ and country 2's welfare weakly exceeds $\underline{U}_2$ so that (2) is satisfied. To check (3), let $U_2|_{s=1}$ represent the continuation value to country 2 conditional on successful concessions yesterday. The stationarity of the equilibrium implies $U_2|_{s=1} = -\pi x + \beta\left(\pi U_2|_{s=1} + (1-\pi)\underline{U}_2\right)$, so that (3) which requires $-x + \beta U_2|_{s=1} \geq \beta \underline{U}_2$ becomes $-x \geq \beta w_2$ which is guaranteed by Assumption 4.

## 6.2 Proofs for Section 4

### 6.2.1 Proof of Proposition 2

We prove each part of this proposition separately as well as additional needed results. We use the following simplifying notation throughout:

$$\Upsilon^{+}(v,\epsilon) = \frac{J(v+\epsilon) - J(v)}{\epsilon} \text{ and } \Upsilon^{-}(v,\epsilon) = \frac{J(v) - J(v-\epsilon)}{\epsilon}.$$

**Proof of Part 1** **Step 1.** Consider two continuation values $U' < U''$ s.t. $W^*(U') > 0$ and $W^*(U'') > 0$. It follows that

$$J(v) = J(U') + \frac{J(U'') - J(U')}{U'' - U'}(v - U') \;\; \forall v \in [U', U''] \tag{15}$$
$$\text{where } m = \frac{J(U'') - J(U')}{U'' - U'}.$$

To see why, let

$$v^{F*}(v) = w_2 + \beta v^{W*}(v) \tag{16}$$
$$v^{P*}(v) = \pi\left(-x^*(v) + \beta v^{H*}(v)\right) + (1-\pi)\beta v^{L*}(v) \tag{17}$$

Since a perturbation which satisfies (12) satisfies (6) – (11), optimality and the concavity of $J(\cdot)$ require

$$J(v) = W^*(v) J\left(v^{F*}(v)\right) + (1 - W^*(v)) J\left(v^{P*}(v)\right). \tag{18}$$

By (18) and the concavity of $J(\cdot)$, it follows that $v^{F*}(v)$ and $v^{P*}(v)$ are on the same line segment in $J(\cdot)$ for a given $v$. Moreover, since $v^{W*}\left(v^{F*}(v)\right) = \frac{v^{F*}(v)-w_2}{\beta} > v$, it follows that

$$\frac{J\left(v^{F*}(U'')\right) - J\left(v^{F*}(U')\right)}{U''-U'} = \frac{\left(J\left(\frac{v^{F*}(U'')-w_2}{\beta}\right) - J\left(\frac{v^{F*}(U')-w_2}{\beta}\right)\right)}{\frac{U''-U'}{\beta}},$$

so that by the concavity of $J(\cdot)$, $v^{F*}(U')$ and $v^{F*}(U'')$ are on the same line segment. Therefore, (15) applies.

**Step 2.** Since $W^*(\underline{U}_2) = 1$ by step 5 of the proof of Lemma 1, it follows from step 1 that (15) applies for $U' = \underline{U}_2$ and some $U'' = \widetilde{U} \geq \underline{U}_2$. It follows that $W^*(v) = 0\ \forall v \geq \widetilde{U}$ if $\widetilde{U} > \underline{U}_2$ and $W^*(v) = 0\ \forall v > \widetilde{U}$ if $\widetilde{U} = \underline{U}_2$. **Step 3.** If $\widetilde{U} = \underline{U}_2$, then $W^*(v) = 0$ $\forall v > \underline{U}_2$, which by (9) implies that $v \geq \beta v^{L*}(v) \geq \beta \underline{U}_2 > \underline{U}_2\ \forall v > \underline{U}_2$, leading to a contradiction. Therefore by Lemma 1 and Assumption 4, $m > 0$ and $\widetilde{U} < \overline{U}_2$ so that

$$\Upsilon^+\left(\widetilde{U}, \epsilon\right) < m \text{ and } \Upsilon^-\left(\widetilde{U}, \epsilon\right) = m. \tag{19}$$

**Proof of Additional Lemmas**

**Lemma 2** *The following properties hold:*

1. *If $w_2 \geq -\overline{x}/\beta$, then $\widetilde{U} = \beta \underline{U}_2$, and*

2. *If $w_2 < -\overline{x}/\beta$ then $\widetilde{U} > \beta \underline{U}_2$.*

**Proof. Step 1.** Imagine if $\widetilde{U} < \beta \underline{U}_2$. This violates (8) and (9) which require $\widetilde{U} \geq \beta v^{L*}\left(\widetilde{U}\right) \geq \beta \underline{U}_2$. **Step 2.** $J(v+\epsilon) > J(v) - \epsilon$ for $\epsilon > 0$. If instead $\exists v$ s.t. $\Upsilon^-(v,\epsilon) \leq -1$, then by Lemma 1 and part 1, $\exists \widehat{U} \in \left[\widetilde{U}, \overline{U}_2\right)$ s.t. $\Upsilon^+\left(\widehat{U}, \epsilon\right) \leq -1$ and $\Upsilon^-\left(\widehat{U}, \epsilon\right) > -1$, so that $W^*\left(\widehat{U}\right) = 0$. Consider $\alpha^*\left(\widehat{U}\right) \in \Psi\left(\widehat{U}\right)$ which satisfies (12) and under which (9) binds, which is always weakly optimal by the weak concavity of the program and convexity of the constraint set. If $x^*\left(\widehat{U}\right) > 0$, then optimality requires that

$$J\left(\widehat{U}+\epsilon\right) \geq \pi\left(x^*\left(\widehat{U}\right) - \epsilon + \beta J\left(v^{H*}\left(\widehat{U}\right)\right)\right) + (1-\pi)\beta J\left(v^{L*}\left(\widehat{U}\right) + \frac{\epsilon}{\beta}\right) \tag{20}$$

since a perturbation to $x'\left(\widehat{U}+\epsilon\right) = x^*\left(\widehat{U}\right) - \epsilon$, $v^{H\prime}\left(\widehat{U}+\epsilon\right) = v^{H*}\left(\widehat{U}\right)$, and $v^{L\prime}\left(\widehat{U}+\epsilon\right) = v^{L*}\left(\widehat{U}\right) + \frac{\epsilon}{\beta}$ satisfies $(6)-(11)$ for $v = \widehat{U}+\epsilon$. Subtraction of $J\left(\widehat{U}\right)$ from both sides of (20) yields $\Upsilon^+\left(\widehat{U}, \epsilon\right) \geq -\pi + (1-\pi)\Upsilon^+\left(\frac{\widehat{U}}{\beta}, \frac{\epsilon}{\beta}\right) > -1$, which is a contradiction where we have used the fact that $\frac{v}{\beta} < v < 0$ from step 6 of the proof of Lemma 1. If instead

$x^*\left(\widehat{U}\right) = 0$ so that $v^{H*}\left(\widehat{U}\right) = v^{L*}\left(\widehat{U}\right) = \frac{\widehat{U}}{\beta}$, then analogous arguments can be made with a perturbation to $x'\left(\widehat{U}+\epsilon\right) = x^*\left(\widehat{U}\right)$ and $v^{H\prime}\left(\widehat{U}+\epsilon\right) = v^{L\prime}\left(\widehat{U}+\epsilon\right) = \frac{v}{\beta}+\frac{\epsilon}{\beta}$, so that $\Upsilon^+\left(\widehat{U},\epsilon\right) \geq \Upsilon^+\left(\frac{\widehat{U}}{\beta},\frac{\epsilon}{\beta}\right) > -1$ which is also a contradiction. **Step 3.** $\forall v \in \left[\underline{U}_2, \overline{U}_2\right]$ and $\forall \alpha^*(v) \in \Psi(v)$, if $W_z^*(v) = 0$ then $x_z^*(v) = \overline{x}$ or $v_z^{H*}(v) = \overline{U}_2$ $\forall z$. If $x^*(v) < \overline{x}$ and $v^{H*}(v) < \overline{U}_2$, then consider a perturbation to $x_z'(v) = x^*(v)+\epsilon$, $v_z^{H\prime}(v) = v^{H*}(v)+\epsilon/\beta$, and $v_z^{L\prime}(v) = v^{L*}(v)$ $\forall z$. Such a perturbation satisfies $(6)-(11)$ and strictly improves welfare by step 2. **Step 4.** Consider if $w_2 \geq -\overline{x}/\beta$ and imagine if $\widetilde{U} > \beta\underline{U}_2$. Let $J^P(v)$ denote the value of the constrained program $(5)-(11)$ s.t. $W^*(v) = 0$. By the proof of part 1, $J^P(v) \leq J(v)$ for $v < \widetilde{U}$ and $J^P(v) = J(v)$ for $v \geq \widetilde{U}$. Therefore,

$$m = \Upsilon^-\left(\widetilde{U},\epsilon\right) \leq \frac{J^P\left(\widetilde{U}\right) - J^P\left(\widetilde{U}-\epsilon\right)}{\epsilon} \tag{21}$$

for $\epsilon > 0$ arbitrarily small. By step 3, $x^*(v) = \overline{x}$ or $v^{H*}(v) = \overline{U}_2$, and by the same reasoning as step 2, (9) can bind. Therefore, if $\frac{\widetilde{U}+\overline{x}}{\beta} > \overline{U}_2$, then $\frac{J^P\left(\widetilde{U}\right)-J^P\left(\widetilde{U}-\epsilon\right)}{\epsilon} = -\pi + (1-\pi)\Upsilon^-\left(\frac{\widetilde{U}}{\beta},\frac{\epsilon}{\beta}\right) < m$, but this contradicts (21). If $\frac{\widetilde{U}+\overline{x}}{\beta} \leq \overline{U}_2$, then $\frac{J^P\left(\widetilde{U}\right)-J^P\left(\widetilde{U}-\epsilon\right)}{\epsilon} = \pi\Upsilon^-\left(\frac{\widetilde{U}+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) + (1-\pi)\Upsilon^-\left(\frac{\widetilde{U}}{\beta},\frac{\epsilon}{\beta}\right) < m$, but this also contradicts (21). This establishes the first part of the lemma. **Step 5.** Consider if $w_2 < -\overline{x}/\beta$ and imagine if $\widetilde{U} = \beta\underline{U}_2$. This implies that (19) holds for $\widetilde{U} = \beta\underline{U}_2$. By part 1, $W^*(\beta\underline{U}_2) = W^*(\beta\underline{U}_2+\epsilon) = 0$ for $\epsilon > 0$ arbitrarily small. By step 3, $x^*(v) = \overline{x}$ or $v^{H*}(v) = \overline{U}_2$ and (9) may bind for $v = \beta\underline{U}_2$ and $v = \beta\underline{U}_2+\epsilon$ by the same reasoning as step 2. Since $\frac{\widetilde{U}+\overline{x}}{\beta} < \beta\underline{U}_2 \leq \overline{U}_2$, then $\Upsilon^+\left(\widetilde{U},\epsilon\right) = \pi\Upsilon^+\left(\frac{\widetilde{U}+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) + (1-\pi)\Upsilon^+\left(\frac{\widetilde{U}}{\beta},\frac{\epsilon}{\beta}\right) = m$, but this contradicts (19). ■

**Lemma 3** *If* $\alpha^*(v) \in \Psi(v)$ *then it satisfies* $(6)-(11)$,

1. $W_z^*(v) = 0$ $\forall z$ *if* $v \geq \widetilde{U}$,
2. $v_z^{W*}(v) \leq \widetilde{U}$ $\forall z$,
3. $\pi\left(-x_z^*(v)+\beta v_z^{H*}(v)\right) + (1-\pi)\beta v_z^{L*}(v) \leq \widetilde{U}$ $\forall z$ *if* $v \leq \widetilde{U}$,
4. $x_z^*(v) = \overline{x}$ *or* $v_z^{H*}(v) = \overline{U}_2$ $\forall z$, *and*
5. (9) *binds* $\forall z$ *if* $v \geq \widetilde{U}$.

**Proof. Step 1.** The necessity of $(6)-(11)$ follows by definition. The necessity of $W_z^*(v) = 0$ $\forall z$ if $v \geq \widetilde{U}$ follows from part 1. The necessity of $x_z^*(v) = \overline{x}$ or $v_z^{H*}(v) = \overline{U}_2$ $\forall z$ follows from step 3 of the proof of Lemma 2. **Step 2.** Imagine if $v_z^{W*}(v) > \widetilde{U}$. Perturb the allocation as in step 1 of the proof of part 1. By (15), $\Upsilon^-\left(v^{W*}(v),\epsilon\right)=m$, which by the concavity of $J(\cdot)$ implies $v^{W*}(v)\leq\widetilde{U}$. Therefore, in order that this perturbation not strictly improve welfare, it is necessary that $\Upsilon^+\left(v_z^{W*}(v),\epsilon\right)=m\ \forall z$, which by (19) implies a contradiction. **Step 3.** Imagine if $\pi\left(-x_z^*(v)+\beta v_z^{H*}(v)\right)+(1-\pi)\beta v_z^{L*}(v)>\widetilde{U}$ for $v\leq\widetilde{U}$. Perturb the allocation as in step 1 of the proof of part 1. By step 1 of the proof of part 1, $\Upsilon^-\left(v^{P*}(v),\epsilon\right)=m$ and which by the concavity of $J(\cdot)$ implies $v^{P*}(v)\leq\widetilde{U}$. In order that this perturbation not strictly improve welfare, it is necessary that

$$\Upsilon^+\left(\pi\left(-x_z^*(v)+\beta v_z^{H*}(v)\right)+(1-\pi)\beta v_z^{L*}(v),\epsilon\right)=m\ \forall z$$

which by (19) implies a contradiction. **Step 4.** Imagine if (9) does not bind for some $z$ if $v\geq\widetilde{U}$. If $x^*(v)<\overline{x}$, consider a perturbation to $x_z'(v)=x^*(v)$, $v_z^{H\prime}(v)=\frac{v+x^*(v)}{\beta}<v^{H*}(v)$, and $v_z^{L\prime}(v)=\frac{v}{\beta}>v^{L*}(v)\ \forall z$. Such a perturbation satisfies (6) – (11) and weakly increases welfare by the concavity of $J(\cdot)$. However, the perturbed allocation is suboptimal by step 3 of the proof of Lemma 2 since $x_z'(v)<\overline{x}$ and $v_z^{H\prime}(v)<\overline{U}_2$. **Step 5.** If instead $x^*(v)=\overline{x}$, denote $v=U'$. If the perturbation of step 4 does not strictly improve welfare, then

$$\Upsilon^+(v,\epsilon)=\Upsilon^+\left(\frac{v+\overline{x}}{\beta},\epsilon\right)=\Upsilon^+\left(\frac{v}{\beta},\epsilon\right)\tag{22}$$

for $\epsilon>0$ arbitrarily low and $v=U'$. Note that since, $\Upsilon^+(v,\epsilon)=m$ for $v<\widetilde{U}$, it must be given (19) that $U'\geq\beta\widetilde{U}>\widetilde{U}$ so that $\Upsilon^+(U',\epsilon)<m$. By step 1 of the proof of part 1 and steps 2 and 3 of the proof of Lemma 2, $\forall v\in\left[\widetilde{U},U'\right]$, there exists a solution to (5) – (11) s.t. $x_z^*(v)=\overline{x}$ for which (9) binds so that $v_z^{H*}(v)=\frac{v+\overline{x}}{\beta}$ and $v_z^{L*}(v)=\frac{v}{\beta}\ \forall z$. Therefore, $\forall v\in\left[\widetilde{U},U'\right]$,

$$\Upsilon^+(v,\epsilon)=\pi\Upsilon^+\left(\frac{v+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right)+(1-\pi)\Upsilon^+\left(\frac{v}{\beta},\frac{\epsilon}{\beta}\right),\tag{23}$$

which by the concavity of $J(\cdot)$, the fact that $\Upsilon^+\left(\frac{\widetilde{U}}{\beta},\epsilon\right)=m$, and (22) implies that $\Upsilon^+(U',\epsilon)=m$, leading to a contradiction. ■

**Proof of Part 2** **Step 1.** Consider $\alpha^*(v)\in\Psi(v)$ for $v\geq\widetilde{U}$. Define

$$v_z=\pi\left(-x_z^*(v)+\beta v_z^{H*}(v)\right)+(1-\pi)\beta v_z^{L*}(v),$$

where by (6) and part 1, $v = \int_0^1 v_z dG_z$. **Step 2.** By parts 4 and 5 of Lemma 3, it is necessary that for a given $v \geq \widetilde{U}$

$$x_z^* = \min\left\{\beta\overline{U}_2 - v_z, \overline{x}\right\}, v_z^{H*} = \min\left\{\frac{v_z + \overline{x}}{\beta}, \overline{U}_2\right\}, \text{ and } v_z^{L*} = v_z/\beta. \tag{24}$$

**Step 3.** Integrating every term in (24) over $z$ yields (13).

**Proof of Additional Corollary**

**Corollary 1** *If $\alpha^*(v)$ satisfies Lemma 3's conditions and* (12)*, then $\alpha^*(v) \in \Psi(v)$.*

**Proof. Step 1.** If $v \geq \widetilde{U}$, then consider any solution which satisfies the conditions of Lemma 3. Since a perturbation as in step 1 of the proof of part 1 satisfies (12), yields a unique solution, and weakly improves welfare, then $\alpha^*(v) \in \Psi(v)$. **Step 2.** If $v < \widetilde{U}$, consider $w_2 \geq -\overline{x}/\beta$. Satisfaction of the conditions entails $v^{F*}(v) = w_2 + \beta v^{W*}(v) \in \left[\underline{U}_2, w_2 + \beta\widetilde{U}\right]$ and $v^{P*}(v) = \beta\underline{U}_2 = \widetilde{U}$ for $v^{F*}(v)$ and $v^{P*}(v)$ defined in (16) and (17). Suboptimality implies that

$$J(v) > W^*(v)\left(w_1 + \beta J\left(v^{W*}(v)\right)\right) + (1 - W^*(v))\, J\left(\widetilde{U}\right), \tag{25}$$

for $W^*(v) = \frac{\widetilde{U} - v}{\widetilde{U} - v^{F*}(v)}$, but (25) contradicts (15). **Step 3.** If $v < \widetilde{U}$, consider $w_2 < -\overline{x}/\beta$. Satisfaction of the conditions entails $v^{F*}(v) = w_2 + \beta v^{W*}(v) \in \left[\underline{U}_2, w_2 + \beta\widetilde{U}\right]$ and $v^{P*}(v) \in \left[\beta\underline{U}_2, \widetilde{U}\right]$. Suboptimality and (15) imply that

$$\begin{aligned} J(v) \;&>\; W^*(v)\left(w_1 + \beta J\left(v^{W*}(v)\right)\right) \\ &\quad + (1 - W^*(v))\left(\pi\overline{x} + \beta\left(\underline{U}_1 + m\left(\frac{v^{P*}(v) + \pi\overline{x}}{\beta} - \underline{U}_2\right)\right)\right), \end{aligned} \tag{26}$$

for $W^*(v) = \frac{v^{P*}(v) - v}{v^{P*}(v) - v^{F*}(v)}$ if $v^{F*}(v) \leq v \leq v^{P*}(v)$ and $W^*(v) = \frac{v^{F*}(v) - v}{v^{F*}(v) - v^{P*}(v)}$ if $v^{F*}(v) \geq v \geq v^{P*}(v)$. However, since (15) together with the fact that $W^*\left(\widetilde{U}\right) = 0$ implies that

$$\underline{U}_1 + m\left(\widetilde{U} - \underline{U}_2\right) = \pi\overline{x} + \beta\left(\underline{U}_1 + m\left(\frac{\widetilde{U} + \pi\overline{x}}{\beta} - \underline{U}_2\right)\right),$$

it follows that (26) contradicts (15). ■

**Proof of Part 3** By Lemma 3 and Corollary 1, $\exists \alpha^*(v) \in \Psi(v)$ s.t. $W^*(v) \geq 0$ and $v^{W*}(v) > \underline{U}_2\ \forall v \in \left(\underline{U}_2, \widetilde{U}\right)$.

**Q.E.D.**

#### 6.2.2 Proof of Theorem 1

**Proof of Part 1** Imagine if $\Pr\{W_{t+k}=1|W_t=0\}=0\ \forall t$ and $\forall k>0$. Constraint (9) implies that $\Pr\{v_{t+1}\leq v_t/\beta<v_t|W_t=0\}=1-\pi$, so that $\Pr\{v_{t+k}\in[\underline{U}_2,\beta\underline{U}_2)\,|W_t=0\}>0$ for some $k>0$. However, it cannot be the case that $\Pr\{W_{t+k}=1|v_{t+k}\in[\underline{U}_2,\beta\underline{U}_2)\}=0$ since from (8) and (9), this would require that $v_{t+k}\geq\beta\underline{U}_2$ which is a contradiction.

**Proof of Part 2** Since $\Pr\{W_{t+k}=1|W_t=0\}>0$ by part 1, and since $W_0=0$ since continuation values begin above $\widehat{v}$ which weakly exceeds $\widetilde{U}$, the absence of temporary wars implies that $\Pr\{W_{t+k}=0|W_t=1\}=0\ \forall t\geq 0$ and $\forall k>0$. Construct a solution using Corollary 1 starting from some $v_0\in\left[\widehat{v},\overline{U}_2\right]$ s.t. $v_0\notin\beta^T\underline{U}_2$ for some integer $T$. Since (9) binds for $v\geq\widetilde{U}$, then $\Pr\{v_{t+1}=v/\beta<v|v_t=v\}=1-\pi>0$. Therefore, $\exists t$ s.t. $\Pr\left\{v_t\in\left(\underline{U}_2,\widetilde{U}\right)\right\}>0$ and by Corollary 1, there exists a solution at $v_t\in\left(\underline{U}_2,\widetilde{U}\right)$ s.t. $\Pr\left\{W_t=1|v_t\in\left(\underline{U}_2,\widetilde{U}\right)\right\}>0$ and $\Pr\left\{W_{t+1}=0|W_t=1,v_t\in\left(\underline{U}_2,\widetilde{U}\right)\right\}>0$ since $\Pr\left\{v_{t+1}\in\left(\underline{U}_2,\widetilde{U}\right)|W_t=1,v_t\in\left(\underline{U}_2,\widetilde{U}\right)\right\}>0$.

**Q.E.D.**

#### 6.2.3 Proof of Theorem 2

**Proof of Part 1** **Step 1.** If $w_2\geq-\overline{x}/\beta$, imagine if $\exists$ a solution to (4) s.t. $\lim_{t\to\infty}\Pr\{W_t=0\}>0$, and consider a potential long run distribution of $v$. Since $\Pr\{v_{t+1}=\underline{U}_2|v_t=\underline{U}_2\}=1$, by step 5 of the proof of Lemma 1, then $\Pr\{v_t=\underline{U}_2\}=0$ under this long run distribution. **Step 2.** If $v\in(\underline{U}_2,\beta\underline{U}_2)$, then from Lemma 3, $v_z^{W*}(v)\leq\beta\underline{U}_2\ \forall z$ and $v_z^{L*}(v)=\underline{U}_2\ \forall z$. Therefore,

$$\Pr\{v_{t+1}=\underline{U}_2|W_t=0,v_t\in(\underline{U}_2,\beta\underline{U}_2)\} = 1-\pi,\text{ and} \tag{27}$$
$$\Pr\{v_{t+1}\leq\beta\underline{U}_2|W_t=1,v_t\in(\underline{U}_2,\beta\underline{U}_2)\} = 1 \tag{28}$$

under the long run distribution of $v$. **Step 3.** From (27),

$$\Pr\{v_{t+1}=\underline{U}_2\}\geq\Pr\{v_t\in(\underline{U}_2,\beta\underline{U}_2)\}\times\Pr\{W_t=0|v_t\in(\underline{U}_2,\beta\underline{U}_2)\}\times(1-\pi)$$

under the long run distribution. In order that $\Pr\{v_{t+1}=\underline{U}_2\}=0$, it is necessary that $\Pr\{W_t=0|v_t\in(\underline{U}_2,\beta\underline{U}_2)\}=0$. This is because $\Pr\{v_t\in(\underline{U}_2,\beta\underline{U}_2)\}>0$ since $\Pr\{v_t=\underline{U}_2\}=0$ and since part 1 of Proposition 2 and part 1 of Theorem 1 imply that

$\Pr\{v_t \in [\underline{U}_2, \beta\underline{U}_2)\} = \Pr\{v_t = \underline{U}_2\} + \Pr\{v_t \in (\underline{U}_2, \beta\underline{U}_2)\} > 0$. **Step 4.** The fact that $\Pr\{W_t = 1 | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} = 1$ combined with (28) implies $\Pr\{v_{t+1} \in (\underline{U}_2, \beta\underline{U}_2) | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} = 1$, and by forward induction

$$\Pr\{W_k = 1 \ \forall k \geq t+1 | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} = 1.$$

Since $\Pr\{v_t \in (\underline{U}_2, \beta\underline{U}_2)\} > 0$, then $\Pr\{W_k = 1 \ \forall k \geq t+1\} = \Pr\{v_{t+1} = \underline{U}_2\} > 0$ which is a contradiction.

**Proof of Part 2** **Step 1.** If $w_2 < -\overline{x}/\beta$, by Corollary 1, $\forall v \in \left(\underline{U}_2, \widetilde{U}\right)$, $\exists \alpha^*(v) \in \Psi(v)$ s.t. $W^*(v) = \frac{\widetilde{U}-v}{\widetilde{U}-\underline{U}_2}$, $v_z^{W*}(v) = \underline{U}_2$, $x_z^*(v) = \overline{x}$, $v_z^{H*}(v) = \frac{\widetilde{U}+\overline{x}}{\beta}$, and $v_z^{L*}(v) = \frac{\widetilde{U}}{\beta}$ $\forall z$. **Step 2.** Construct an equilibrium with the property of step 1, and imagine if $\lim_{t\to\infty}\Pr\{W_t = 0\} > 0$. Then it must be that $\Pr\{v_t = \underline{U}_2\} = 0$ under the long run distribution. However, in such an equilibrium, $\Pr\left\{v_{t+1} = \underline{U}_2 | v_t \in \left(\underline{U}_2, \widetilde{U}\right)\right\} = \frac{\widetilde{U}-v_t}{\widetilde{U}-\underline{U}_2} > 0$ under the long run distribution. By part 1 of Proposition 2 and part 1 of Theorem 1 , $\Pr\left\{v_t \in \left[\underline{U}_2, \widetilde{U}\right)\right\} > 0$. Consequently, $\Pr\{v_t = \underline{U}_2\} > 0$ under the long run distribution.

**Proof of Part 3** **Step 1.** If $w_2 < -\overline{x}/\beta$, by Corollary 1, $\forall v \in \left(\underline{U}_2, \widetilde{U}\right)$, $\exists \alpha^*(v) \in \Psi(v)$ s.t. $W^*(v) = \frac{\widetilde{U}-v}{\widetilde{U}-(\underline{U}_2+\epsilon(v))}$, $v_z^{W*}(v) = \underline{U}_2 + \frac{\epsilon(v)}{\beta}$, $x_z^*(v) = \overline{x}$, $v_z^{H*}(v) = \frac{\widetilde{U}+\overline{x}}{\beta}$, and $v_z^{L*}(v) = \frac{\widetilde{U}}{\beta}$ $\forall z$ for some $\epsilon(v) > 0$. **Step 2.** Construct an equilibrium with the property of step 1, and imagine if $\lim_{t\to\infty}\Pr\{W_t = 0\} = 0$. Then it must be that $\Pr\{v_t = \underline{U}_2\} > 0$ under the long run distribution. However, in such an equilibrium, $\Pr\left\{v_{t+1} = \underline{U}_2 | v_t \in \left(\underline{U}_2, \widetilde{U}\right)\right\} = 0$. Moreover, by Corollary 1, $\Pr\left\{v_{t+1} = \underline{U}_2 | v_t \in \left[\widetilde{U}, \overline{U}_2\right]\right\} = 0$. Since country 2 receives at least $\widetilde{U}$ starting from period 0, $\Pr\{v_t = \underline{U}_2\} = 0$ under the long run distribution.

**Q.E.D.**

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