Future Rent-Seeking and Current Public Savings*

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Abstract

The conventional wisdom is that politicians' rent-seeking motives increase public debt and deficits. This is because myopic politicians face political risk and prefer to extract political rents as early as possible. In this paper we study the determination of government debt and deficits in a dynamic political economy model. We show that this conventional wisdom relies on economic volatility being low relative to political uncertainty. If economic volatility is high relative to political uncertainty, then a rent-seeking government actually *over-saves* and *over-taxes* along the equilibrium path relative to a benevolent government. This result emerges because of the *option value of rent-seeking*: A rent-seeking government over-values future funds because of the possibility of using them for future rents instead of cutting taxes in the event of a future boom (when marginal utility of private consumption is low). This over-saving bias is temporary since, in the long run, the rent-seeking government over-borrows relative to the benevolent government as it eventually squanders the funds it has accumulated. We find that both the under-saving and over-saving bias of the government can be solved by a rule of capping deficits.

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1 Introduction

The conventional wisdom is that the rent-seeking motives of politicians increase public debt and deficits. This is because myopic politicians face political risk and prefer to extract rents as early as possible. An implication of this argument is that governments will under-save during a boom, leaving the economy unprotected in the event of a downturn.[1] This view is not only of theoretical interest, but it motivates a number of fiscal rules in the world which are aimed at cutting deficits and constraining borrowing so as to limit the size of this political distortion.[2]

In this paper we study the determination of government debt and deficits in a dynamic political economy model.[3] We show that the conventional wisdom that rent-seeking governments under-save holds if economic volatility is low and if political uncertainty is high. Nonetheless, if economic volatility is high and political uncertainty is low, then a rent-seeking government actually *over-saves* and *over-taxes* along the equilibrium path relative to a benevolent government. This result emerges because of the *option value of rent-seeking*: A rent-seeking government over-values future funds because of the possibility of using them for future rents instead of cutting taxes in the event of a future boom (when marginal utility of private consumption is low). This over-saving bias is temporary since, in the long run, the rent-seeking government over-borrows relative to the benevolent government as the government eventually squanders the funds it has accumulated. We find that both the under-saving and over-saving bias of the government can be solved by a rule of capping deficits.

More specifically, we study an economy managed by a sequence of politicians who face political risk and who care about household welfare and rents conditional on remaining in power. In contrast to the previous work on the political economy of debt, we consider the interrelated implications of three important features: economic uncertainty, incomplete markets, and transitional dynamics. The economy begins in a boom, and this boom can come to a permanent end at any date. Throughout the length of the boom, the benevolent government gradually reduces its debt in order to prepare for the potential downturn. We compare this optimal behavior to that of a rent-seeking government managed by

[1] See the survey article of Alesina and Perotti (1994) for a discussion of this view.

[2] Chile provides a recent example which has become a reference for fiscal reforms in Latin America and commodity producing economies more broadly. The fiscal rule establishes a structural (i.e., at "normal" terms of trades) surplus of 0.5 percent of GDP. Thus, when terms of trade rise as a result of a commodity boom, the state runs very large fiscal surpluses (the sum of the structural surplus target plus the excess fiscal income due to high commodity prices).

[3] Acemoglu, Golosov, and Tsyvinski (2008a, 2008b) also study the effect of political economy distortions on taxes, though they do not consider the effect on government debt.

politicians.

Our first result is that while a rent-seeking government reduces its debt at the beginning of the boom, it stops reducing its debt if the boom is sufficiently prolonged. This is because beyond a certain date, government resources become so abundant that rent-seeking considerations come to dominate intertemporal smoothing considerations. A rent-seeking government realizes that if it were to save more, then a future replacement government would use the additional funds for rent-seeking (which only benefits incumbent politicians) as opposed to tax-cutting (which benefits households), and the government therefore restrains its savings in order to starve the future government of funds. Therefore, in the long run, a prolonged boom always leads a benevolent government to hold more assets and to tax less than a rent-seeking government. This result is consistent with that emphasized by Battaglini and Coate (2008a,2008b). Our main contribution is to show that while this characterization applies to the long run fairly generally, whether or not it applies to the transitional dynamics of the economy depends on the level of economic volatility.

Our second result is that if economic volatility is sufficiently low relative to political uncertainty, then the rent-seeking government *over-borrows* and *under-taxes* along the equilibrium path relative to a benevolent government. This insight–which is consistent with the conventional wisdom–emerges because low economic volatility implies that politicians are biased toward extracting rents today versus in the future since political risk is high and the cost of leaving the economy exposed in the downturn is low. This causes governments to over-borrow and under-tax at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate this behavior of politicians in the future, and for this reason, they choose to over-borrow and to under-tax themselves. Thus the prospect of future rent-seeking reinforces over-borrowing and under-taxation in the present.

Our third and most important result–which stands in contrast to the conventional wisdom–is that if economic volatility is sufficiently high relative to political uncertainty, then the rent-seeking government *over-saves* and *over-taxes* along the equilibrium path relative to a benevolent government. Whenever economic volatility is high, politicians are less likely to consume rents today and more likely to consume them tomorrow since this simultaneously protects the economy while providing them with potential rents in the event of a boom during which they are not replaced. In anticipation of these rents in the future, the rent-seeking government actually over-saves relative to a benevolent government since the marginal value of additional funds in the future boom due to rent-seeking exceeds the marginal value of additional funds for a benevolent government who would instead use the additional savings to cut taxes. This causes governments to over-save and over-tax at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate this behavior of politicians in the future, and for this reason, they choose to over-save and to over-tax themselves. The prospect of future rent-seeking therefore reinforces over-saving and over-taxing in the present. Importantly, in light of our first result, this over-saving bias is temporary since the rent-seeking government eventually squanders the funds it has accumulated on rents and holds more debt than the benevolent government.

Our last result is that the popular fiscal rule of capping deficits brings deficits and surpluses closer to those of the benevolent government, although the mechanism is different in the under-saving and over-saving cases. In the under-saving region, the government would like to save less in order to starve the future government of resources which it would otherwise squander on rents. However, the rule does not permit the government to do this, so that it must necessarily bind and it forces the rent-seeking government to save more and to behave more like a benevolent government. In the over-saving region, the rule works through expectations by reducing the value of *future* public funds. More specifically, unconstrained governments over-save because they look forward to squandering public funds in the future if the boom persists for sufficiently long. The fiscal rule however makes it impossible to squander these public funds in the future since it forces a future government to save more. Therefore, the rule reduces the value of future funds from today's perspective, and this induces today's government to save less. Part of this reduction in savings comes not from deep tax cuts but from earlier and higher levels of rent extraction relative to the economy in the absence of fiscal rules. More generally, on its own, the fiscal rule cannot force governments to cut taxes when resources become sufficiently abundant, and in the long run, additional increases in savings are used purely for rent-seeking.

This paper builds on the literature on optimal fiscal policy and debt management dating back to the classical work of Barro (1979) and Lucas and Stokey (1983).[4] We depart from this work by relaxing the assumption of a benevolent government and by assuming that the economy is managed by politicians who derive partial utility from rents and who face potential replacement. In this regard, this paper is most closely related to the literature on the political economy of debt. More specifically, our work complements that of Battaglini and Coate (2008a,2008b). As in our work, they consider a setting in which current governments face economic risk and political risk. They show that the presence of

[4] See also Aiyagari, Marcet, Sargent, and Seppala (2002), Bohn (1990), and Chari and Kehoe (1993a, 1993b).

political risk implies that in the long run, a rent-seeking government holds a level of debt which exceeds that of the benevolent government. We depart from their work by focusing on the implications of political economy *along the equilibrium path* and away from steady state. In the process, we describe a novel over-saving mechanism. Our work is also related to that of Song, Storesletten, and Zilibotti (2009) who show that intergenerational conflict in a dynamic model can cause a government to under-save or over-save relative to the social optimum. We depart from their work by abstracting from intergenerational conflict and considering instead the impact of political and economic risk.[5] Finally, our over-saving result is related to the work of Yared (2010) who argues that prescribing high levels of savings in the presence of rent-seeking politicians is distortionary since it is associated with the anticipation of future rents. In contrast, in the current paper we explain these high savings as an endogenous mechanism to extract future rents when effective economic uncertainty is high.

This introduction is followed by six sections and an appendix. Section 2 describes the environment. Section 3 describes our main over-saving result using a simple two period example. Section 4 describes the infinite horizon equilibrium under a benevolent government. Section 5 describes the infinite horizon equilibrium under a rent-seeking government and compares it to that of a benevolent government. Section 6 describes a simulation of our economy and discusses policy implications. Section 7 concludes. The Appendix contains the proofs and additional material.

2 Model

2.1 Economic Environment

There are discrete time periods $t = \{0, ..., \infty\}$ and a continuum of mass 1 of identical households with the following period 0 welfare:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t) \right), \ \beta \in (0,1), \tag{1}$$

for $c_t \geq 0$ which represents consumption and for $u(\cdot)$ which satisfies $u'(\cdot), -u''(\cdot) > 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Households hold a constant endowment $e > 0$, they pay lump

[5] For additional work on the political economy of debt, see for example Aghion and Bolton (1990), Aguiar, Amador, and Gopinath (2009), Alesina and Perotti (1994), Alesina and Tabellini (1990), Amador (2003), Lizzeri (1999), and Persson and Svensson (1989). Our work is also related to the large literature on dynamic voting games and taxation, for example Krusell and Rios-Rull (1999).

sum taxes to the government $\tau_t \leq e$, and they balance their budget so that $c_t = e - \tau_t$. Since τ_t can be negative, it can also be interpreted as the negative of public spending.

There is a large number of potential and identical politicians who derive the flow utility $u(c_t)$ when out of power and who derive the flow utility $u(c_t) + \theta x_t$ when in power for $x_t \geq 0$ which represents socially wasteful rents.[6] $\theta \geq 0$ and we refer to the special case of $\theta = 0$ as a benevolent government since it corresponds to the case in which incumbent politicians have the same preferences as households. Levels of θ which exceed 0 capture the inverse cost of rent-seeking for the politician so that higher levels of θ are associated with less costly rent-seeking.

A politician in power in period t is permanently removed from office and replaced with an identical politician from $t+1$ onward with exogenous probability $1-q \in (0,1)$, so that q represents the survival rate of a politician. Therefore, the welfare of the incumbent at $t = 0$ can be written as

$$E_0\left(\sum_{t=0}^{\infty} \beta^t \left(u(c_t) + q^t \theta x_t\right)\right), \tag{2}$$

where we have taken into account that a politician in period zero survives to period t with probability q^t.[7]

In every period, the government finances rents $x_t \geq 0$ and debt $b_t \gtreqless 0$ by raising revenue $\tau_t \leq e$ and borrowing $b_{t+1} \gtreqless 0$ from international markets at a price $\beta \in (0,1)$. In addition, the government experiences an exogenous endowment shock y_t.[8] The government's dynamic budget constraint is

$$\beta b_{t+1} = b_t + x_t - (\tau_t + y_t) \tag{3}$$

for a given b_0 subject to $\lim_{t \to \infty} \beta^t b_{t+1} \leq 0$.

The endowment y_t is stochastic and depends on the state $s_t \in \{L, H\}$ with $y(H) = -y(L) = \sigma > 0$. The government therefore exists to smooth household's consumption. s_t follows a first order Markov process and is independent of the political replacement

[6] While the linearity of rents in the utility function is important for the full characterization of the model, the over-saving mechanism we describe depends on the existence of a region in which rents are zero. In a two-period economy, for example, if $v(x)$ represents the flow utility of rents, we require $v'(0) < \infty$. Details available upon request.

[7] The politician in power can be an individual from the population if one interprets x_t as per capita public spending which only provides utility to the individual in power. Because the probability of entering politics for any given individual is zero, it does not enter the welfare criterion of the benevolent planner. Note that what is critical for our results is not that $\theta = 0$ for the benevolent planner, but that the benevolent planner values rents by less than the politician in power.

[8] There is no difference between letting the government or the households experience this endowment shock.

shock. We simplify our discussion by assuming that $\Pr\{s_t = L | s_{t-1} = L\} = 1$ and that $\Pr\{s_t = H | s_{t-1} = H\} = \alpha \in (0, 1)$. We refer to state H as the boom and state L as the downturn. We will focus on the path of the economy with $s_0 = H$. Therefore, the economy is experiencing a temporary boom which may permanently end at any date with probability $1 - \alpha$.[9]

2.2 Political Environment

The order of events at every period t is as follows:

1. Nature determines y_t and potentially replaces the period $t-1$ incumbent.

2. The period t politician chooses policies $\{\tau_t, x_t, b_{t+1}\}$.

3. Households receive consumption and the politician receives rents.

Given that there are many potential equilibria which can emerge in this setting, we consider the symmetric Markov Perfect Equilibrium which coincides with the limit of our economy with T periods as $T \to \infty$.[10] In this equilibrium, the incumbent politician–independently of identity and of past political shocks–chooses policies as a function of the state s_t and the level of debt b_t. Note that in choosing τ_t, the incumbent effectively chooses c_t, so that without loss of generality, we will refer to $c(b, s)$, $x(b, s)$ and $b'(b, s)$ as the politician's choices of c_t, x_t, and b_{t+1}, respectively, conditional on $b_t = b$ and $s_t = s$. Define $V^N(b, s)$ and $V^P(b, s)$ as the continuation value of being out of office and in office, respectively, with debt b in state s. The set of policies $\{c(b, s), x(b, s), b'(b, s)\}_{s=L,H}$ constitutes an Markov Perfect Equilibrium if $\{c(b, s), x(b, s), b'(b, s)\}$ maximizes $V^P(b, s)$ given b and s and subject to the government's dynamic budget constraint.

3 Two Period Example

Before proceeding to analyze the fully dynamic economy, it is useful to characterize a two period version of our economy with $t = 0, 1$ in order to present the main novel insight

[9]This formulation allows for tractability. If the economy instead experiences a temporary downturn followed by a permanent boom, then debt expands and there are no deviations from the benevolent benchmark starting from sufficiently high levels of debt. We have also numerically simulated economies in which neither state is absorbing and achieved similar characterization to our analytical results here. Details available upon request.

[10]That is, subject to the constraint that $\beta^T b_{T+1} \leq 0$. This is a refinement of Markov Perfect Equilibria since others could also exist in the infinite horizon game.

of our model regarding the potential over-saving bias of the government. To do this we consider the extreme case for which $q \to 1$ so that political uncertainty is low relative to economic uncertainty and the example starkly illustrates the *option value of rent-seeking*.

In a two period economy, (3) implies that

$$\begin{aligned} c_0 &= e + y_0(s_0) - b_0 + \beta b_1 - x_0 \\ c_1 &= e + y_1(s_1) - b_1 - x_1. \end{aligned}$$

A benevolent government clearly chooses $x_0 = x_1 = 0$. Moreover, it chooses the level of b_1 so as to equalize the expected marginal utility of households across dates so that the optimal level of debt satisfies the following standard Euler equation:

$$u_c\left(e + \sigma - b_0 + \beta b_1^B(b_0, H)\right) = \alpha u_c\left(e + \sigma - b_1^B(b_0, H)\right) + (1-\alpha) u_c\left(e - \sigma - b_1^B(b_0, H)\right). \tag{4}$$

$b_1^B(b_0, H)$ corresponds to this optimal choice of debt which depends on initial debt b_0 and the initial state s_0 which is H. Note that $b_1^B(b_0, H)$ is a strictly increasing function of initial debt b_0.

Now consider the level of debt chosen by a rent-seeking government. At date 1, a politician maximizing $u(c_1) + \theta x_1$ chooses the following level of consumption and rents, $c_1^P(b_1, s_1)$ and $x_1^P(b_1, s_1)$, respectively as a function of outstanding debt b_1 and the state s_1:

$$c_1^P(b_1, s_1) = \min\left\{e + y_1(s_1) - b_1, u_c^{-1}(\theta)\right\} \tag{5}$$

$$x_1^P(b_1, s_1) = \max\left\{0, e + y_1(s_1) - b_1 - u_c^{-1}(\theta)\right\} \tag{6}$$

Consider the problem of the rent-seeking government from the perspective of date 0. Note that if $b_0 \geq b_0^* = b_1^{B^{-1}}(e + \sigma - u_c^{-1}(\theta))$, then the level of debt is sufficiently high that there are not enough resources for the rent-seeking government to extract rents at any date. More specifically, if the rent-seeking government chooses a level of debt equal to $b_1^B(b_0, H)$, then the marginal utility of consumption at all dates exceeds θ, meaning it is inefficient to extract rents at any date. Therefore, the equilibrium level of debt chosen by the politician $b_1^P(b_0, H)$ equals that of the benevolent government $b_1^B(b_0, H)$ and the presence of a rent-seeking government has no impact on policies.

In contrast, suppose $b_0 < b_0^* = b_1^{B^{-1}}(e + \sigma - u_c^{-1}(\theta))$ so that a positive level of rents would be extracted at date 1 if the politician replicated the policy of the rent-seeking government. In this situation, the politician has enough funds to finance rent-extraction at date 1 conditional on the realization of the boom. This is because the marginal value of public funds is the lowest in this state. Now consider $b_0 \in (b_0^{**}, b_0^{*})$ for $b_0^{**} = (e - u_c^{-1}((\theta)))(1+\beta) + \sigma(1-\beta)$. In this situation, the level of initial debt is sufficiently low to allow for rent-extraction during a boom at date 1, but not sufficiently low to allow for rent-extraction at date 0. In this scenario, the first order condition for the politician extracting only rents in the date 1 boom can be written as:

$$u_c\left(e + \sigma - b_0 + \beta b_1^P(b_0, H)\right) = \alpha\theta + (1-\alpha)\, u_c\left(e - \sigma - b_1^P(b_0, H)\right), \tag{7}$$

where $b_1^P(b_0, H)$ is analogously defined as $b_1^B(b_0, H)$ but for the rent-seeking government. Note that (7) takes into account that an additional unit of savings is used for rents in the date 1 boom so that its marginal value is θ.[11]

Equation (7) captures the option value of rent-seeking and it explains why $b_1^P(b_0, H) < b_1^B(b_0, H)$ in this region so that the rent-seeking government *saves more* than a benevolent government. To see why, compare (4) and (7). Clearly, the marginal value of public funds at date 0 and at date 1 during the downturn is the same for the benevolent government and the rent-seeking government conditional on the same hypothetical level of debt b_1. This is because the rent-seeking government does not engage in rent-extraction at that date. Nevertheless, the marginal value of public funds for the rent-seeking government in the event of a boom at date 1 is θ, and this value exceeds the marginal value of public funds for the benevolent government. This causes the politician to over-save relative to the benevolent benchmark. Intuitively, the politician has an option to extract rents in the boom and the presence of this option increases the marginal value of his savings which would otherwise be used by a benevolent government for cutting taxes. Note that though we focus on the special case for which $q \to 1$, the key assumption driving this result is that political risk is sufficiently low (q is sufficiently high) relative to economic uncertainty so that the politician can exercise this option with high probability, and this motivates his desire to over-save.

Finally, we can consider the remaining case with low initial levels of debt with $b_0 < b_0^{**}$, where the government extracts rents at date 1 during the boom as well as at date 0. In this situation, the rent-seeking government chooses a level of debt $b_1^P(b_0, H) = e - \sigma - u_c^{-1}(\theta)$ which is independent of initial debt, and taxes are independent of initial debt implying a consumption equal $u_c^{-1}(\theta)$. Importantly, the government extracts enough rents at date 0 so as to not leave enough savings to allow for rent-seeking at date 1 during the downturn. Intuitively, given the presence of political risk, the government at date 0 prefers to consume

[11] More precisely, the marginal value of these savings is $q\theta$ which is arbitrarily close to θ since $q \to 1$.

rents today versus leaving additional rents for the government date 1, since it knows that any alternative date 1 government will use additional savings not to cut taxes but to seek rents. An implication of our analysis of the region for $b_0 < b_0^{**}$, is that there is an additional cutoff point $b_0^{***} < b_0^{**}$, where if $b_0 < b_0^{***}$, the rent-seeking government *saves less* than the benevolent government (since $b_1^B(b_0, H)$ is a strictly increasing function of b_0). In other words, even though a benevolent government utilizes its initial wealth to cut taxes at all dates, a rent-seeking government keeps taxes high and it squanders any initial increases in initial wealth on initial rents.

In sum, our analysis of a two period economy shows the following three patterns: (i) for high initial debt, the rent-seeking government behaves exactly like a benevolent government, (ii) for intermediate initial debt, the rent-seeking government over-saves relative to the benevolent government, and (iii) for low initial debt, the rent-seeking government under-saves relative to the benevolent government.

These results serve as a useful guide for interpreting patterns in the the infinite horizon economy. More specifically, our infinite horizon analysis allows us to characterize transitional dynamics for debt and also to more explicitly determine the parameter regions for which the over-saving bias for the rent-seeking government exists. It also allows us to show how expectations of future government behavior can reinforce current behavior by the rent-seeking government, and it allows us to consider the role of fiscal rules.

4 Benevolent Government Benchmark

We begin by considering the policies of the benevolent government which corresponds to a special case of our economy with $\theta = 0$. In this circumstance, $V^P(b, s)$ equals $V^N(b, s)$, and to facilitate future discussion, we let the superscript B denote the continuation value and the policies of the benevolent government. The problem of the government in the *downturn* can be written as

$$V^B(b, L) = \max_{c,x,b'} u(c) + \beta V^B(b', L) \tag{8}$$

$$\text{s.t. } x \geq 0 \text{ and}$$

$$\beta b' = b + x + c - (e - \sigma), \tag{9}$$

Since households are always better off consuming more, the solution to this problem assigns $x^B(b, L) = 0$. Conditional on b', the politician is always better off taxing less versus extracting more rents. Therefore, the problem is mathematically equivalent to a personal consumption problem in which smoothing consumption is optimal. Thus, $c^B(b,L) = e - \sigma - b(1-\beta)$, $b'^B(b,L) = b$, and $V^B(b,L) = u(e - \sigma - b(1-\beta))/(1-\beta)$.

Using this characterization, we can now consider the government's problem during the preceding *boom*:

$$V^B(b,H) = \max_{c,x,b'} u(c) + \beta E_s V^B(b', s) \tag{10}$$

$$\text{s.t. } x \geq 0 \text{ and}$$

$$\beta b' = b + x + c - (e + \sigma). \tag{11}$$

As in the downturn, the solution to this problem yields $x^B(b,H) = 0$, and optimality requires $c^B(b,H)$ to be defined by the following Euler equation:

$$u_c\left(c^B(b,H)\right) = \alpha u_c\left(c^B\left(b^{B\prime}(b,H), H\right)\right) + (1-\alpha) u_c\left(c^B\left(b^{B\prime}(b,H), L\right)\right). \tag{12}$$

Lemma 1 *$c^B(b,H)$ is strictly decreasing in b, $b'^B(b,H)$ is strictly increasing in b, and $b'^B(b,H) < b$.*

The government taxes more and carries more debt into the future when government debt is high since the economy is relatively poor. The government always raises its savings in the boom in preparation for the downturn and it continues to drive down its debt until the boom ends. Note that as the boom persists, the size of the government asset position approaches infinity since the government always benefits from saving more in preparation for the downturn.

5 Rent-Seeking Government

We now consider the behavior of a government more generally for all $\theta > 0$. Here we write the problem of the government recursively (Section 5.1), characterize the dynamics of consumption and debt (Section 5.2), and compare these policies to those of a benevolent government (Section 5.3).

5.1 Recursive Program

Conditional on entering a downturn, the incumbent politician solves the following problem:

$$V^P(b,L) = \max_{c,x,b'} \{u(c) + \theta x + \beta W(b',L)\} \tag{13}$$

$$\text{s.t. } x \geq 0 \text{ and}$$

$$\beta b' = b + x + c - (e - \sigma). \tag{14}$$

for $W(b',s) = qV^P(b',s) + (1-q)V^N(b',s)$. $W(b',s)$ represents the ex-ante continuation value to the incumbent politician facing the possibility of removal conditional on the state s.

The government clearly wishes to smooth consumption, though it is also interested in rent-seeking which provides a marginal utility of θ and sets a lower bound for the marginal utility of consumption. This means that during the downturn, politicians choose the following policies, where the superscript P denotes the policies of a rent-seeking government:

$$\begin{aligned}
c^P(b,L) &= \min\left\{e - \sigma - (1-\beta)b, u_c^{-1}(\theta)\right\} \\
x^P(b,L) &= \max\left\{0, \frac{e - \sigma - u_c^{-1}(\theta)}{1-\beta} - b\right\} \\
b'^P(b,L) &= \max\left\{b, \frac{e - \sigma - u_c^{-1}(\theta)}{1-\beta}\right\}
\end{aligned}$$

The rent-seeking government follows the same smooth policies with zero rent-seeking as those of a benevolent government as long as its initial stock of debt b is above a threshold $(e - \sigma - u_c^{-1}(\theta))/(1-\beta)$. In this case, the government is relatively poor and any additional reductions in b are used for reducing taxes on households as opposed to raising rents (since the marginal benefit of cutting those taxes exceeds θ.).

If b is below this threshold, then the government is rich. Politicians extract positive rents, they tax households more than the benevolent government, and they borrow more than the benevolent government. More specifically, consumption is held at $u_c^{-1}(\theta)$, so that the marginal benefit of rent-seeking equals the marginal benefit of consumption. Moreover, debt is held at $(e - \sigma - u_c^{-1}(\theta))/(1-\beta)$. Therefore, any additional reductions in b are used only for rent-seeking as opposed to tax or debt reduction. By following this strategy, the incumbent politician who may be replaced in the future chooses to frontload all rent-extraction and leaves all future politicians with zero rents. Note that the threshold which separates the zero rent region from the positive rent region rises with the rent-seeking bias θ.

Given these policies, we can characterize $V^P(b,L)$ and $W(b,L)$.

Lemma 2 *The following conditions hold:*

1. *$V^P(b,L)$ and $W(b,L)$ are strictly decreasing in b, strictly concave in b, and continuously differentiable in b for $b > (e-\sigma-u_c^{-1}(\theta))/(1-\beta)$ with $V^P(b,L) = W(b,L)$,*

2. *$V^P(b,L)$ is linear in b and continuously differentiable in b for $b \leq (e-\sigma-u_c^{-1}(\theta))/(1-\beta)$ with $V_b^P(b,L) = -\theta$,*

3. *$W(b,L)$ is linear in b and continuously differentiable in b for $b < (e-\sigma-u_c^{-1}(\theta))/(1-\beta)$ with $W_b(b,L) = -q\theta$.*

The important feature of Lemma 2 is that $W(b,L)$ is not differentiable at the cutoff point $(e-\sigma-u_c^{-1}(\theta))/(1-\beta)$ where rent-seeking begins. This is because additional resources are no longer used for cutting taxes and are instead used for raising rents which is only beneficial to the politician conditional on being in power. We will see that an analogous result to Lemma 2 holds in the boom.

Given the behavior of the economy in the downturn, we characterize the policy of the rent-seeking government in the boom. The incumbent politician solves the following problem:

$$V^P(b,H) = \max_{c,x,b'} \{u(c) + \theta x + \beta E_s(W(b',s))\} \tag{15}$$

$$\text{s.t. } x \geq 0 \text{ and}$$

$$\beta b' = b + x + c - (e+\sigma). \tag{16}$$

To facilitate discussion, we define the following cut-off point:

$$\underline{b} = \left(e+\sigma-\max\left\{u_c^{-1}(\theta), 2\sigma + u_c^{-1}(\theta(1-\alpha q)/(1-\alpha))\right\}\right)/(1-\beta). \tag{17}$$

We will show that $\underline{b}$ represents the steady state level of debt to which the economy converges during a sustained boom. Note that the exact characterization of $\underline{b}$ depends on the level of volatility σ, and this is important since there are two cases two consider. Specifically, define σ^* as

$$\sigma^* = \frac{1}{2}\left(u_c^{-1}(\theta) - u_c^{-1}\left(\theta\frac{1-\alpha q}{1-\alpha}\right)\right).$$

Note that since $q < 1$, $\sigma^* > 0$. The cutoff value σ^* decreases in q, so that as political survival q goes to 1, σ^* goes to 0. Moreover, as the persistence of the boom α increases, σ^* increases. Finally, it can be shown by implicit differentiation given that if $u'''(\cdot) > 0$ then σ^* is decreasing in the rent-seeking bias θ.[12] Therefore, σ is more likely to exceed σ^* if political risk is low, the boom is temporary, and the rent-seeking bias θ is high.

As we will show, rent-seeking begins at levels of debt below $e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$. Thus, an analogous result to Lemma 2 holds and we can characterize $V^P(b, H)$ and $W(b, H)$.

Lemma 3 *The following conditions hold:*

1. *$V^P(b, H)$ and $W(b, H)$ are strictly decreasing in b, strictly concave in b, and continuously differentiable in b for $b > e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ with $V^P(b, H) = W(b, H)$,*

2. *$V^P(b, H)$ is linear in b and continuously differentiable in b for $b \leq e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ with $V_b^P(b, H) = -\theta$,*

3. *$W(b, H)$ is linear in b and continuously differentiable in b for $b < e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ with $W_b(b, H) = -q\theta$.*

The first order conditions and the envelope condition imply that if $b'^P(b, H) > e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$, then

$$u_c\left(c^P(b, H)\right) = \alpha u_c\left(c^P\left(b'^P(b, H), H\right)\right) + (1 - \alpha) u_c\left(c^P\left(b'^P(b, H), L\right)\right), \tag{18}$$

so that the Euler equation holds as under a benevolent government. Moreover, if $b'^P(b, H) < e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$, then

$$u_c\left(c^P(b, H)\right) = \alpha q \theta + (1 - \alpha) u_c\left(c^P\left(b'^P(b, H), L\right)\right). \tag{19}$$

These two equations relate the marginal cost of public funds today to the expected marginal cost of public funds tomorrow. They show that the marginal cost of public funds tomorrow depends on whether or not rent-seeking takes place during the boom.[13] If no rent-seeking takes place, the marginal cost of public funds equals the marginal utility of consumption since additional resources are used to boost consumption (equation (18)). In contrast, if rent-seeking takes place, the marginal cost of public funds is $q\theta$ since

[12] Formally, $\frac{d\sigma^*}{d\theta} < \frac{1}{2}\left(\left(u_{cc}\left(u_c^{-1}(\theta)\right)\right)^{-1} - \left(u_{cc}\left(u_c^{-1}\left(\theta \frac{1 - \alpha q}{1 - \alpha}\right)\right)\right)^{-1}\right) < 0.$

[13] Savings are never high enough for rent-seeking to occur both in the boom and in the downturn since this is suboptimal for today's government.

today's politician maintains power with probability q and extracts rents in the future which provide marginal benefit θ (equation (19)).[14]

5.2 Transitional Dynamics

We begin by describing the transitional dynamics of policies under a rent-seeking government.

Proposition 1 ***(dynamics)*** *Policies satisfy the following properties for some $\overline{b} > \underline{b}$:*

1. *$b'^P(b,H) = \underline{b}$ if $b \leq \overline{b}$, $b'^P(b,H) < b$ if $b > \overline{b}$, and $b'^P(b,H)$ weakly increases in b,*

2. *If $\sigma \leq \sigma^*$, then $c^P(b,H) < (=)\, u_c^{-1}(\theta)$ and $x^P(b,H) = (>)\, 0$ if $b > (<)\, \underline{b}$, and*

3. *If $\sigma > \sigma^*$, then $c^P(b,H) < (=)\, u_c^{-1}(\theta)$ and $x^P(b,H) = (>)\, 0$ if $b > (<)\, \overline{b}$.*

Figures 1 and 2 display this proposition graphically. Specifically, they depict $b'^P(b,H)$ as a function of b for $\sigma \leq \sigma^*$ and $\sigma > \sigma^*$, respectively. Much like the benevolent government, the rent-seeking government lets debt decline monotonically throughout the boom, but *unlike* the benevolent government, government assets do not rise forever. Beyond $\overline{b}$, a prolonged boom causes the government to stabilize tomorrow's debt at a minimum point $\underline{b}$. These figures also depict the rent-seeking regions for different levels of σ. If $\sigma \leq \sigma^*$, then rent-seeking begins when debt goes below $\underline{b}$. In contrast, if $\sigma > \sigma^*$, then rent-seeking begins when debt drops below $\overline{b} > \underline{b}$.

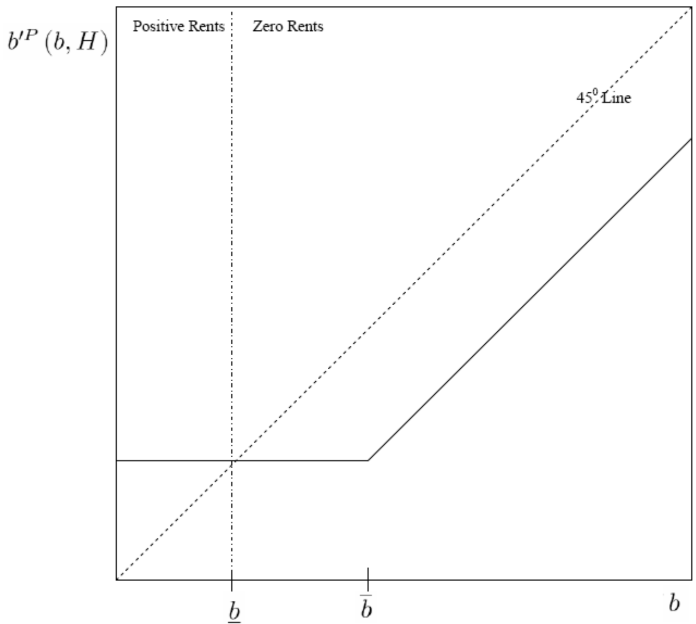
The implied dynamics of consumption and rents depend crucially on the degree of economic uncertainty σ. If $\sigma \leq \sigma^*$, then starting from $b_0 > \underline{b}$, the governments saves and it never extracts rents along the path. Once debt b first reaches $\overline{b}$, the government chooses $b'^P(b,H) = \underline{b}$ so that the economy reaches the steady state with zero rents. The government never saves beyond $\underline{b}$ since politicians know that rents would be extracted by a likely replacement government, and the additional benefit of making these savings available for a downturn do not outweigh the cost of leaving additional rents for a replacement government in a boom. For the same reason, if the economy starts from $b_0 < \underline{b}$, the government chooses $c^P(b_0,H) = u_c^{-1}(\theta)$, $x^P(b_0,H) = \underline{b} - b_0$, and $b'^P(b_0,H) = \underline{b}$, in

[14] Note that if $b'^P(b,H) = e + \sigma - u_c^{-1}(\theta) + \beta\underline{b}$, then $W(b,H)$ is not differentiable, though $u_c\left(c^P(b,H)\right)$ must be in the range between the right hand side of (19) and the right hand side of (18). Specifically,

$$u_c\left(c^P(b,H)\right) \in \left[\alpha q\theta + (1-\alpha)\, u_c\left(c^P\left(b'^P(b,H),L\right)\right), \alpha\theta + (1-\alpha)\, u_c\left(c^P\left(b'^P(b,H),L\right)\right)\right].$$

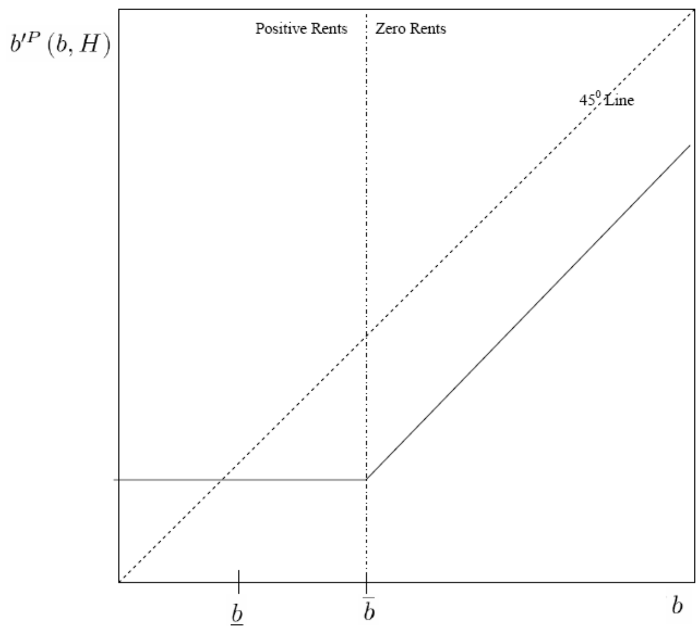
order to starve the future government of resources. In summary, a prolonged boom in this environment leads debt to $\underline{b}$ and to zero rent-seeking.

Figure 1: $b'^P(b,H)$ vs. b for $\sigma \leq \sigma^*$



These dynamics are different if $\sigma > \sigma^*$. Starting from $b_0 > \bar{b}$, the government chooses zero initial rents, and it gradually saves during the boom until debt eventually reaches $\bar{b}$. Once debt b drops below $\bar{b}$, the government chooses positive rents so that $c^P(b,H) = u_c^{-1}(\theta)$, $x^P(b,H) = \bar{b} - b$, and $b'^P(b,H) = \underline{b}$. Therefore, the government reaches a steady state with positive rents, which is in contrast to the $\sigma \leq \sigma^*$ case. Thus, even if the economy starts from zero rents, there is a possibility that rents may be positive in the future if the boom persists for sufficiently long. The current politician does not want to fully starve the future government of rents since he knows that it would expose the economy to too much volatility, and he may as well postpone rent-seeking given that he has a sufficiently high survival probability and is likely to consume these rents himself.

Figure 2: $b'^{P}(b,H)$ vs. b for $\sigma > \sigma^{*}$



5.3 Comparison to Benevolent Government

In this section, we compare the path of debt and consumption under a rent-seeking government to that under a benevolent government. We begin by considering the implications of the equilibrium if the boom is prolonged. Let $\left\{c_t^B\right\}_{t=0}^{\infty}$ and $\left\{b_{t+1}^B\right\}_{t=0}^{\infty}$ correspond to the equilibrium sequence of consumption and debt, respectively, conditional on a boom persisting forever under a benevolent government starting from some initial debt b_0. Define $\left\{c_t^P\right\}_{t=0}^{\infty}$, and $\left\{b_{t+1}^P\right\}_{t=0}^{\infty}$ analogously for a rent-seeking government.

Proposition 2 ***(long run)***

$$\begin{aligned} \lim_{t\to\infty} b_{t+1}^B &= -\infty < \lim_{t\to\infty} b_{t+1}^P = \underline{b} \textit{ and} \\ \lim_{t\to\infty} c_t^B &= \infty > \lim_{t\to\infty} c_t^P = u_c^{-1}(\theta). \end{aligned}$$

Proposition 2 implies that a prolonged boom leads a rent-seeking government to hold more debt than a benevolent government and to consume less (tax more) than a benevolent government. Though a rent-seeking government reduces its debt at the beginning of the boom, it stops reducing its debt if the boom is sufficiently prolonged. This is because beyond a certain date, government resources become so abundant that rent-seeking considerations come to dominate intertemporal smoothing considerations. A rent-seeking government realizes that if it were to save more, then a future replacement government would use the additional funds for rent-seeking (which only benefits incumbent politicians) as opposed to tax-cutting (which benefits households), and the government therefore restrains its savings in order to starve the future government of funds. Therefore, in the long run, a prolonged boom always leads a benevolent government to hold more assets and to tax less than a rent-seeking government. This result is consistent with that emphasized by Battaglini and Coate (2008a,2008b). Our main contribution is to show that while this characterization applies to the long run fairly generally, whether or not it applies to the transitional dynamics of the economy depends on the level of economic volatility.

Next we consider the dynamics of public debt and taxes along the equilibrium path. With some abuse of notation, let $u_c\left(c^B\left(b,H;\sigma\right)\right)$ represent the value of $u_c\left(c^B\left(b,H\right)\right)$ for a benevolent government facing uncertainty σ. Define $\underline{\sigma}$ and $\overline{\sigma}$ as the unique solutions to the following two equations:

$$\begin{aligned}\underline{\sigma} &: u_c\left(c^B\left(\frac{e-\underline{\sigma}-u_c^{-1}\left(\theta\left(1-\alpha q\right)/\left(1-\alpha\right)\right)}{1-\beta},H;\underline{\sigma}\right)\right)=q\theta \\ \overline{\sigma} &: u_c\left(c^B\left(e+\overline{\sigma}-u_c^{-1}\left(\theta\right)+\beta\frac{e-\overline{\sigma}-u_c^{-1}\left(\theta\left(1-\alpha q\right)/\left(1-\alpha\right)\right)}{1-\beta},H;\overline{\sigma}\right)\right)=q\theta\end{aligned}$$

Lemma 4 *(i)* $0<\sigma^*<\underline{\sigma}<\overline{\sigma}$, *(ii)* $\underline{\sigma}$ *and* $\overline{\sigma}$ *are decreasing in* q *and increasing in* α, *(iii)* $\underline{\sigma}$ *and* $\overline{\sigma}$ *approach* 0 *as* q *approaches* 1, *(iv)* $u_c\left(c^B\left(\underline{b},H\right)\right)<q\theta$ *iff* $\sigma>\underline{\sigma}$, *and (v)* $u_c\left(c^B\left(\overline{b},H\right)\right)<q\theta$ *iff* $\sigma>\overline{\sigma}$.

The lemma states that like σ^*, the cutoff points $\underline{\sigma}$ and $\overline{\sigma}$ decrease in survival rate q and increase in the persistence parameter α.[15] Moreover, like σ^*, these converge to zero as q approaches 1, so that any positive value of σ must necessarily exceed $\overline{\sigma}$ as q approaches 1. The parameter $\underline{\sigma}$ is the level of volatility for which $\sigma>\underline{\sigma}$ implies $u_c\left(c^B\left(\underline{b},H\right);\sigma\right)<q\theta$. $u_c\left(c^B\left(\underline{b},H\right);\sigma\right)$ decreases in σ since as economic volatility σ increases, the steady state level of debt $\underline{b}$ decreases, and it decreases by an amount large enough to cause the benevolent government's consumption at $\underline{b}$ to rise. Eventually, the marginal utility of this consumption goes below $q\theta$. Analogous arguments hold for the level of debt $\overline{b}$, where $\overline{\sigma}$ is the level of volatility such that $\sigma>\overline{\sigma}$ implies $u_c\left(c^B\left(\overline{b},H\right)\right)<q\theta$.

The interpretation of these cutoff points for economies with $\sigma>\sigma^*$ is as follows: If $\sigma<\underline{\sigma}$, then $u_c\left(c^B\left(b,H\right)\right)>q\theta$ for $b\in\left[\underline{b},\overline{b}\right]$, which is the region in which debt exceeds steady state debt and in which rent-seeking is positive. Therefore, the marginal

[15]Comparative statics with respect to θ are ambiguous.

value of public funds for a benevolent government in the boom exceeds the (expected) marginal value of public funds for a rent-seeking government in the boom who survives with probability q and who values marginal rents with weight θ. In contrast, if $\sigma > \overline{\sigma}$, then $u_c\left(c^B\left(b, H\right)\right) < q\theta$ for $b \in \left[\underline{b}, \overline{b}\right]$. In this case, the marginal value of public funds for a benevolent government in the boom is below the (expected) marginal value of public funds for a rent-seeking government in the boom.

As we will show, whether the marginal value of public funds for a benevolent government exceeds or is below $q\theta$ in the rent-seeking region affects whether or not the rent-seeking government saves less or more than a benevolent government. We show that economies with $\sigma < \underline{\sigma}$ feature over-borrowing along the equilibrium path (Section 5.3.1), and we show that economies with $\sigma > \overline{\sigma}$ feature over-saving along the equilibrium path (Section 5.3.2). In the Appendix, we consider economies with $\sigma \in (\underline{\sigma}, \overline{\sigma})$, and we show that both over-borrowing or over-saving can occur along the equilibrium path, and this depends on initial condition b_0.

5.3.1 Low Economic Volatility

We begin by showing that the rent-seeking government over-borrows if economic volatility is low.

Proposition 3 ***(starve the beast)*** *If $\sigma < \underline{\sigma}$, then $b'^P\left(b, H\right) > b'^B\left(b, H\right)\ \forall b$ and $c^P\left(b, H\right) > c^B\left(b, H\right)\ \forall b \geq \overline{b}$.*

This proposition states that if economic volatility is low, then the rent-seeking government always borrows more than the benevolent government, and it consumes more than the benevolent government for levels of debt which exceed $\overline{b}$.[16] Therefore, the transition path starting from $b_0 > \overline{b}$ features over-spending and over-borrowing, which is in line with the conventional wisdom in the political economy literature.

The intuition for this result is that low economic volatility implies that politicians are biased towards extracting rents today versus in the future, since political risk is high and the cost of leaving the economy exposed in the downturn is low. This causes governments to over-borrow and over-consume at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate this behavior of politicians in the future, and for this reason, they choose to over-borrow and to over-consume. The prospect of future rent-seeking therefore reinforces over-borrowing and over-consumption in the present.

[16] Whenever $\sigma < \underline{\sigma}$, there is some cutoff level of debt below which $c^P\left(b, H\right) < c^B\left(b, H\right)$. For the $\sigma < \sigma^*$ case, this cutoff point is below $\underline{b}$.

More formally, suppose volatility is so low that rents are never extracted under levels of debt which exceed $\underline{b}$ (i.e., $\sigma < \sigma^*$). Since $x^P(b, H) = 0\ \forall b \geq \underline{b}$, then $c^P(b, H) > c^B(b, H)$ if and only if $b'^P(b, H) > b'^B(b, H)$ from the dynamic budget constraint of the economy. Since $b'^P(\underline{b}, H) > b'^B(\underline{b}, H)$, the rent-seeking government must be choosing $c^P(\underline{b}, H) > c^B(\underline{b}, H)$. Therefore, in steady state, the government over-borrows and over-consumes, and the marginal cost of public funds at $\underline{b}$ under a benevolent government which equals $u_c\left(c^B(\underline{b}, H)\right)$ exceeds the marginal cost of public funds under a rent-seeking government which equals $u_c\left(c^P(\underline{b}, H)\right) = \theta$. This affects savings decisions for all levels of debt above $\underline{b}$. Consider the Euler conditions of the benevolent and rent-seeking government, (12) and (18), respectively, for $b \in \left[\underline{b}, \overline{b}\right]$. Since $b \geq \underline{b}$, $c^P(b, L) = c^B(b, L)$ because debt is never sufficiently low in the downturn to induce rent-seeking. Therefore, satisfaction of (12) and (18) implies that $b'^B(b, H) < b'^P(b, H) = \underline{b}$, since the benevolent government perceives a higher marginal cost of public funds in the future than the rent-seeking government. Thus, $u_c\left(c^P(b, H)\right) < u_c\left(c^B(b, H)\right)$ so that the marginal cost of public funds is higher at b under a benevolent government. Forward iteration of this argument implies that all rent-seeking governments perceive a lower marginal cost of public funds in the future than the benevolent government, and they consequently save less than the benevolent government.

An analogous argument holds if instead volatility is low, though rents are extracted under levels of debt that exceed $\underline{b}$ and are below $\overline{b}$ (i.e., $\sigma^* < \sigma < \underline{\sigma}$). In this case, $x^P(b, H) > 0$ for some b and it is no longer the case that $c^P(b, H) > c^B(b, H)$ if and only if $b'^P(b, H) > b'^B(b, H)$. Nonetheless, note that the marginal cost of public funds in the boom for the rent-seeking government for $b \in \left[\underline{b}, \overline{b}\right]$ equals $q\theta$ since the government expects to survive with probability q and to extract rents which provide marginal utility θ. However, given the definition of $\underline{\sigma}$, $u_c\left(c^B(b, H)\right) > q\theta$ in this region so that the benevolent government values public funds more on the margin than the rent-seeking government. Therefore, analogous arguments to the previous case comparing (12) and (19) imply that for $b > \overline{b}$ for which $b'^P(b, H) \in \left[\underline{b}, \overline{b}\right]$, it is the case that $b'^P(b, H) > b'^B(b, H)$ and $c^P(b, H) > c^B(b, H)$ (since $x^P(b, H) = 0$) so that the rent-seeking government over-borrows and over-consumes. Since $u_c\left(c^P(b, H)\right) < u_c\left(c^B(b, H)\right)$, the rent-seeking government under-values public funds at b and forward iteration on this argument implies that over-saving occurs for all b.

5.3.2 High Economic Volatility

The previous picture changes dramatically for high levels of economic volatility.

Proposition 4 ***(feed the beast)*** *If $\sigma > \overline{\sigma}$, then $b'^{P}(b,H) < b'^{B}(b,H)$ $\forall b \geq \overline{b}$ and $c^{P}(b,H) < c^{B}(b,H)$ $\forall b$.*

This proposition states that if economic volatility is high, then the rent-seeking government saves *more* than the benevolent government for levels of debt which exceed $\overline{b}$, and it consumes less (taxes more) than the benevolent government.

The intuition for this result is as follows. Whenever economic volatility is high, the politician is less likely to consume rents today and more likely to consume them tomorrow since this simultaneously protects the economy while providing him with potential rents in the event of a boom during which he is not replaced. In anticipation of these rents in the future, the rent-seeking government may actually over-save relative to a benevolent government since the marginal value of additional funds in the future boom due to rent-seeking exceeds the marginal value of additional funds for a benevolent government who would instead use the additional savings to increase consumption. This causes governments to over-save and under-consume at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate this behavior of politicians in the future, and for this reason, they choose to over-save and to under-consume themselves. The prospect of future rent-seeking therefore reinforces over-saving and under-consumption in the present. Future governments are not cutting taxes during the boom in response to additional savings–the natural response of a benevolent government–and this provides additional incentives for savings today.

More formally, consider the government at values of debt $b \in \left[\underline{b}, \overline{b}\right]$. In this region, the government chooses positive rents, and the marginal value of public funds for a rent-seeking government who may be potentially replaced prior to entering the boom is $q\theta$. Moreover, by the definition of $\overline{\sigma}$, the benevolent government is so wealthy in this region that its marginal value of public funds $u_c\left(c^{B}(b,H)\right)$ is below $q\theta$. The rent-seeking government is extracting rents and also over-taxing in order to do so. Now consider values of $b > \overline{b}$ for which $b'^{P}(b,H) \in \left[\underline{b}, \overline{b}\right]$. In this region, $x^{P}(b,H) = 0$ so that $c^{P}(b,H) < c^{B}(b,H)$ if and only if $b'^{P}(b,H) < b'^{B}(b,H)$. Given (12) and (19), it must be the case that $b'^{P}(b,H) < b'^{B}(b,H)$ and $c^{P}(b,H) < c^{B}(b,H)$ so that the rent-seeking government over-saves and under-consumes. Since $u_c\left(c^{P}(b,H)\right) > u_c\left(c^{B}(b,H)\right)$, the rent-seeking government over-values public funds at b and forward iteration on this argument implies that over-borrowing occurs for all b.

Note that even though the rent-seeking government over-saves along the equilibrium path, in steady state it over-borrows relative to a benevolent government who instead drives its asset position to infinity.[17] In a sense then, it is the prospect of rent-seeking and over-borrowing in the future which induces politicians to over-save in the present. This induces the rent-seeking government to over-tax both when it is anticipating future rent-seeking and also in steady state when rent-seeking takes place.

6 Policy Implications and Discussion

A central implication of our model is that rent-extraction does *not* actually have to take place for distortions to emerge. The main mechanism in our framework operates through *expectations*. For example, when debt is sufficiently high, there are no rents independently of the regime. However, there are important distortions in both the low and high volatility scenarios.

In the low volatility scenario there is a wedge pushing the government to tax and save too little, since the government is worried that its potential replacement will squander everything. That is, the current government is too expansionary and borrows too much. In contrast, in the high volatility scenario, there is a wedge pushing the government *to tax and save too much.* Here, fiscal policy is actually too contractionary, and society would benefit from cutting taxes and saving less. In what follows, we illustrate these scenarios and conclude by analyzing the impact of standard fiscal rules.

6.1 The Two Scenarios

Consider an economy with $u(c) = \log(c)$ and $\{\beta, e, \sigma, \alpha, \theta\} = \{.95, 100, 1.5, .95, .001\}$, where we have chosen θ such that the long run level of debt in a boom in the $\sigma < \sigma^*$ case is equal to 10. Consider two economies: $q = .2$ and $q = .99$, so that in one economy, the current incumbent has an 80% chance of being replaced and in the other economy the incumbent has virtually no chance of being replaced. Under this parameterization, the low q case corresponds to an economy with $\sigma < \underline{\sigma}$, so that the government under-taxes and over-borrows, and the high q case corresponds to an economy with $\sigma > \overline{\sigma}$ so that the government over-taxes and over-saves.

Figures 3 and 4 illustrate the path of debt and consumption in the $q = .2$ economy during a prolonged boom starting from a level of debt $b_0 = 30$ for a rent-seeking and a benevolent government. The rent-seeking government over-borrows relative to the benevolent government. This difference can be substantial. For example, at $t = 40$, the rent-

[17] Formally, there exists a cutoff point in the range $[\underline{b}, \overline{b}]$ below which the rent-seeking government over-borrows.

seeking government holds a level of debt equal to 10 whereas the benevolent government holds a level of debt equal to -33, a difference equal to over 40% of the endowment of the economy. The counterpart of the path of debt is not rent extraction (since $\sigma < \sigma^* < \underline{\sigma}$) but excessive consumption (low taxes) during the transition (Figure 4), and economic fragility during the downturn (not shown).

Figure 3: Path of Debt ($\sigma < \underline{\sigma}$)

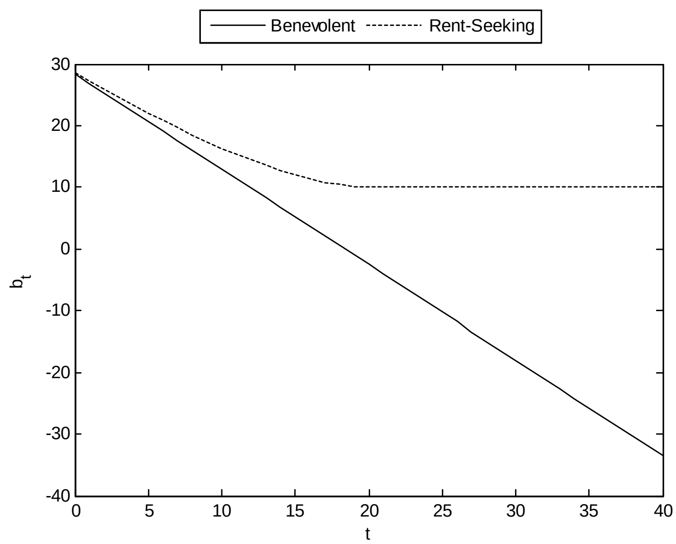
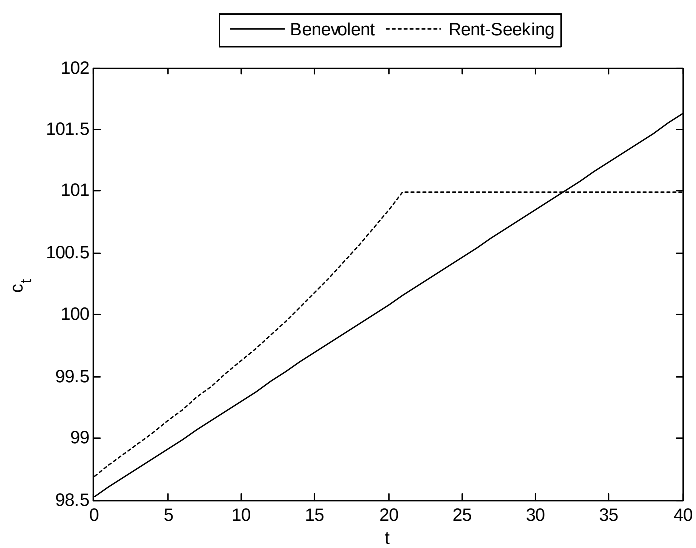


Figure 4: Path of Consumption ($\sigma < \underline{\sigma}$)



In contrast, Figures 5 and 6 consider the $q = .99$ economy during a prolonged boom also starting from a level of debt $b_0 = 30$. In this situation, the rent-seeking government over-saves early on relative to the benevolent government (Figure 5). The difference between the two governments can be substantial. For example, at $t = 40$ the rent-seeking government holds level of debt equal to -46 whereas the benevolent government holds a level of debt equal to -33, a difference equal to over 10% of the endowment of the economy. Early on, the high taxes are used to reduce debt but later on they finance government rents. As a result, consumption is lower than under the benevolent government throughout the boom (Figure 6). Early on, when no rents are extracted, the economy gains in terms of extra protection against the contraction. Later on, consumption is lower both during the boom and the contraction.

Figure 5: Path of Debt ($\sigma > \overline{\sigma}$)

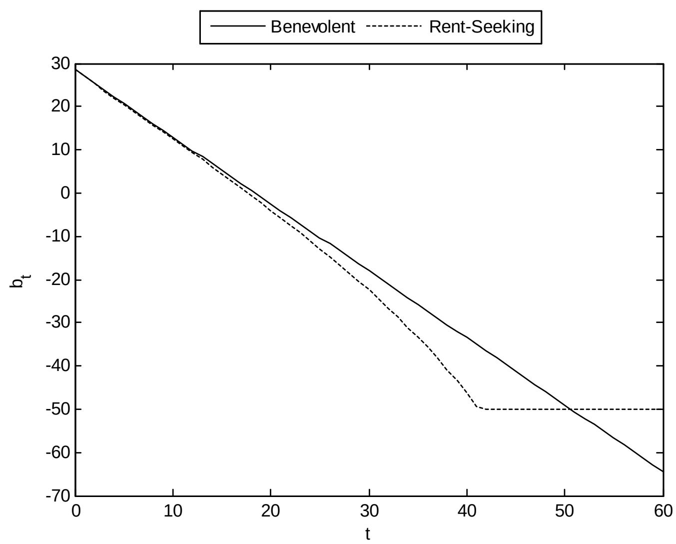
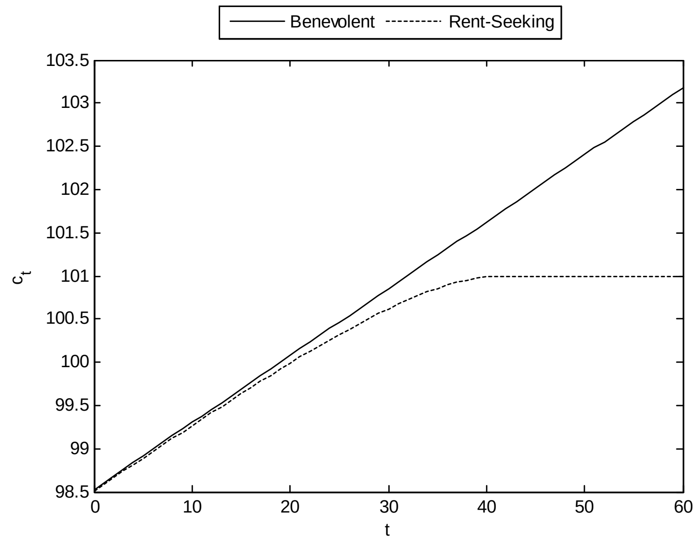


Figure 6: Path of Consumption ($\sigma > \overline{\sigma}$)



6.2 Fiscal Rules

The conventional view, captured in Figures 3 and 4, has given support to the increasingly popular policy option of adopting fiscal rules that essentially cap deficits (or require surpluses) during booms (the budget, surplus or deficit rules). A natural question concerns the degree to which such fiscal rules are useful in economies in which over-saving occurs along the equilibrium path as in Figures 5 and 6. This question is particularly relevant for commodity-economies which experience high economic volatility.

More specifically, consider an economy starting from b_0 in which a benevolent government would choose a sequence of consumption $\left\{c_t^B\right\}_{t=0}^{\infty}$ in the boom. Imagine a fiscal rule whereby the rent-seeking government in period t is allowed to choose any policy subject to the constraint that such a policy must satisfy

$$c_t + x_t \leq c_t^B, \tag{20}$$

so that the government effectively cannot run a primary deficit above that of the benevolent government at any given date. The political environment is as described in Section 2.2 with the exception that (20) must be satisfied by every government in every period. Since rents are zero under a benevolent government, (20) implies that the rent-seeking government must save at least as much as the benevolent government at every date. The next proposition characterizes the behavior of the economy under the fiscal rule where $\left\{\widetilde{c}_t^P\right\}_{t=0}^{\infty}$ and $\left\{\widetilde{x}_t^P\right\}_{t=0}^{\infty}$ correspond to the path of consumption and rents, respectively, during the boom under a rent-seeking government subject to the fiscal rule.

Proposition 5 ***(fiscal rules)*** *$\widetilde{c}_t^P + \widetilde{x}_t^P = c_t^B$ at every t in the economy under the fiscal rule and*

$$\begin{aligned} \widetilde{c}_t^P &= \min\left\{c_t^B, u_c^{-1}(\theta)\right\} \text{ and} \\ \widetilde{x}_t^P &= \max\left\{0, c_t^B - u_c^{-1}(\theta)\right\}. \end{aligned}$$

Proposition 5 states that the fiscal rule (20) binds, and $\widetilde{c}_t^P$ and $\widetilde{x}_t^P$ are chosen as in Section 5 so that rents are only positive if the marginal value of consumption equals θ. The rule binds in economies in which $\sigma < \underline{\sigma}$ since the unconstrained rent-seeking government has higher equilibrium path deficits than the benevolent government. Thus the fiscal rule reduces the government deficit along the equilibrium path and increases public saving.

More surprisingly, the rule binds in economies in which $\sigma > \overline{\sigma}$ so that the unconstrained rent-seeking government has a *lower* equilibrium path deficit than the benevolent government in the early phase of the boom. Therefore, even though the fiscal rule imposes a cap on deficits, it actually induces the rent-seeking government to borrow *more* than it would if it were unconstrained. The reason for this is that the rule works through expectations by reducing the value of *future* public funds. More specifically, in this region unconstrained governments over-save because they look forward to squandering public funds in the future if the boom persists for sufficiently long. The fiscal rule however makes it impossible to squander these public funds in the future since it forces a future government to save more. Therefore, the rule reduces the value of future funds from today's perspective, and this induces today's government to save less.

Note that the rule induces the government to consume more (tax less) and to extract more rents than it would if it were unconstrained along the equilibrium path.[18] This is because since the marginal value of funds in the future is lower, the current government decides to use funds for itself today, and it does so in the form of higher consumption and higher rent-seeking. This means that the government will begin to extract rents at an earlier date than it would in the absence of rules, since rent-seeking begins at higher levels of debt in comparison to an economy in the absence of rules.

Finally, note that while a fiscal deficit rule can force a rent-seeking government to save in the same fashion as the benevolent government, it cannot control the composition of public spending. Specifically, the government continues to squander resources on rents as

[18] More specifically, the fiscal rules induce more consumption at high levels of debt and more rent-seeking at intermediate levels of debt.

opposed to cutting taxes if the boom is sufficiently prolonged or if initial resources are very abundant. This suggests that a deficit rule must be combined with a cap on taxes, so as to achieve the social optimum.

7 Final Remarks

We developed a dynamic political economy model of debt that characterizes public debt and deficits along the transitional path and in the long run. This allowed us to re-examine the conventional wisdom regarding the nature of political distortions. Our main result is that in the short run phase of a boom–when the level of public debt is still high–it matters whether the government faces high or low economic volatility. While the conventional wisdom of under-saving holds in the latter case, it does not in the former. If economic volatility is high, politicians *over-save* in the short run by *keeping taxes too high.*

In future work we intend to extend our analysis of fiscal policy in high economic volatility environments. The natural next steps are to study the qualitative and welfare properties of a broad class of fiscal rules found in practice,[19] and to pursue empirical work aimed at aligning these different rules with the characteristics of different countries and regions.

[19] See for example Azzimonti, Battaglini, and Coate (2008) for an analysis of a balanced budget ammendment to the US constitution.

8 Appendix

8.1 Proofs

8.1.1 Proof of Lemma 1

Step 1. $V^B(b,H)$ is strictly decreasing and concave in b by standard arguments, and differentiability follows from Benveniste and Sheinkman (1979).

Step 2. First order conditions and the envelope condition imply that $V_b^B(b,H) = -u_c\left(c^B(b,H)\right)$, which by step 1 implies that $c^B(b,H)$ is decreasing in b. These also imply that $V_b^B(b,H) = \alpha V_b^B\left(b'^B(b,H),H\right) + (1-\alpha)V_b^B\left(b'^B(b,H),L\right)$ so that $b'^B(b,H)$ is strictly increasing in b.

Step 3. If $b'^B(b,H) \geq b$, then from steps 1 and 2, $u_c\left(c^B\left(b'^B(b,H),H\right)\right) \geq u_c\left(c^B(b,H)\right)$, which from (12) implies $c^B\left(b'^B(b,H),L\right) \geq c^B((b,H))$. However, given the budget constraint (3), this contradicts $b'^B(b,H) \geq b$. **Q.E.D.**

8.1.2 Proof of Lemma 2

Step 1. Given the characterization of policies in the text and the dynamic budget constraint, we can write

$$\begin{aligned} V^P(b,L) &= \frac{u\left(\min\left\{e-\sigma-b(1-\beta),u_c^{-1}(\theta)\right\}\right)}{1-\beta} + \theta\max\left\{0,\frac{e-\sigma-u_c^{-1}(\theta)}{1-\beta}-b\right\} \text{ and} \\ V^N(b,L) &= \frac{u\left(\min\left\{e-\sigma-b(1-\beta),u_c^{-1}(\theta)\right\}\right)}{1-\beta}. \end{aligned}$$

Step 2. All of the properties follow from this characterization and the definition of $W(b,L)$. **Q.E.D.**

8.1.3 Proof of Lemma 3

Step 1. This is proved by induction. In a T period economy, define

$$\begin{aligned} \underline{b}_t &= \left(e+\sigma-\max\left\{u_c^{-1}(\theta),2\sigma+u_c^{-1}(\theta(1-\alpha q)/(1-\alpha))\right\}\right)\left(\sum_{k=0}^{T-t}\beta^k\right) \quad \forall t\leq T \text{ and} \\ \underline{b}_{T+1} &= 0. \end{aligned}$$

Define $\widehat{b}_t = e+\sigma-u_c^{-1}(\theta)+\beta\underline{b}_{t+1}$. Let $V_t^P(b_t,s)$ and $W_t(b_t,s)$ correspond to the values of $V^P(\cdot)$ and $W(\cdot)$, respectively in a T period economy in period $t\leq T$.

Step 2. By analogous arguments to those of Lemma 2, we can write

$$\begin{aligned} W_t(b_t, L) &= u\left(\min\left\{e - \sigma - \frac{b_t}{\left(\sum_{k=0}^{T-t}\beta^k\right)}, u_c^{-1}(\theta)\right\}\right)\left(\sum_{k=0}^{T-t}\beta^k\right) + \\ &\quad q\theta \max\left\{0, \left(e - \sigma - u_c^{-1}(\theta)\right)\left(\sum_{k=0}^{T-t}\beta^k\right) - b_t\right\}. \end{aligned}$$

Step 3. Consider an economy with $T = 0$. It follows that the solution to the government's program sets

$$\begin{aligned} c_T^P(b_T, H) &= \min\left\{e + \sigma - b_T, u_c^{-1}(\theta)\right\} \text{ and} \\ x_T^P(b_T, H) &= \max\left\{0, e + \sigma - b_T - u_c^{-1}(\theta)\right\}, \end{aligned}$$

and this implies all of the properties of the lemma for $T = 0$.

Step 4. Consider the economy with $T = 1$. Step 3 implies the properties of the lemma for $V_t^P(b_t, H)$ and $W_t(b_t, H)$ at $t = 1$. Now consider $t = 0$. Let us assume and later verify that if $b_t \geq \widehat{b}_t$, then $x_t^P(b_t, H) = 0$ and if $b_t < \widehat{b}_t$, then $c_t^P(b_t, H) = u_c^{-1}(\theta)$, $b_t^{P\prime}(b_t, H) = \underline{b}_{t+1}$, and $x_t^P(b_t, H) = \widehat{b}_t - b_t$. This means that $V_t^P(b_t, H) = W_t(b_t, H)$ if $b_t \geq \widehat{b}_t$. That they are both decreasing and concave follows by standard arguments, and differentiability follows from Benveniste and Sheinkman (1979). The linearity of both functions for $b_t < \widehat{b}_t$ together with their derivative follows from the characterization of the equilibrium for this range. We now verify our assumption by first proving that $x_t^P(b_t, H) > 0$ implies that $b_t^{P\prime}(b_t, H) = \underline{b}_{t+1}$. Note that intratemporal optimality implies that $c_t^P(b_t, H) = u_c^{-1}(\theta)$. If $b_t^{P\prime}(b_t, H) < \underline{b}_{t+1} \leq \widehat{b}_{t+1}$, then intertemporal optimality taking into account that $x_{t+1}^P\left(b_t^{P\prime}(b_t, H), H\right) > 0$ requires

$$\theta = \alpha q\theta + (1 - \alpha)\, u_c\left(c_{t+1}^P\left(b_t^{P\prime}(b_t, H), L\right)\right). \tag{21}$$

However, (21) is violated since $b_t^{P\prime}(b_t, H) < \underline{b}_{t+1}$. If instead $b_t^{P\prime}(b_t, H) > \underline{b}_{t+1}$, then there are two cases to consider. If $\sigma \leq \sigma^*$, then $\underline{b}_{t+1} = \widehat{b}_{t+1}$ so that $x_{t+1}^P\left(b_t^{P\prime}(b_t, H), H\right) = 0$ and intertemporal optimality requires

$$\theta = \alpha u_c\left(c_{t+1}^P\left(b_t^{P\prime}(b_t, H), H\right)\right) + (1 - \alpha)\, u_c\left(c_{t+1}^P\left(b_t^{P\prime}(b_t, H), L\right)\right), \tag{22}$$

which is violated since the right hand side exceeds θ by step 2. If instead $\sigma > \sigma^*$, then

$\underline{b}_{t+1} < \widehat{b}_{t+1}$ so that $x^P_{t+1}\left(b^{P\prime}_t(b_t,H),H\right) \geq 0$ and intertemporal optimality requires

$$\theta > \alpha q\theta + (1-\alpha)\, u_c\left(c^P_{t+1}\left(b^{P\prime}_t(b_t,H),L\right)\right), \tag{23}$$

which is violated since $b^{P\prime}_t(b_t,H) > \underline{b}_{t+1}$. Therefore, if $x^P_t(b_t,H) > 0$ then $c^P_t(b_t,H) = u_c^{-1}(\theta)$ and $b^{P\prime}_t(b_t,H) = \underline{b}_{t+1}$. Now suppose that $b_t \geq \widehat{b}_t$. If it were that $x^P_t(b_t,H) > 0$, then this would violate the budget constraint since $c^P_t(b_t,H) = u_c^{-1}(\theta)$ and $b^{P\prime}_t(b_t,H) = \underline{b}_{t+1}$ cannot hold. Therefore, $x^P_t(b_t,H) = 0$. Suppose that $b_t < \widehat{b}_t$. If it were the case that $x^P_t(b_t,H) = 0$, then the fact that intratemporal optimality requires $c^P_t(b_t,H) \leq u_c^{-1}(\theta)$ implies that $b^{P\prime}_t(b_t,H) < \underline{b}_{t+1} \leq \widehat{b}_{t+1}$ which violates intertemporal optimality since it implies

$$u_c\left(c^P_t(b_t,H)\right) \geq \theta > \alpha q\theta + (1-\alpha)\, u_c\left(c^P_{t+1}\left(b^{P\prime}_t(b_t,H),L\right)\right)$$

Step 5. Successive application of 4 taking T to ∞ yields the result. **Q.E.D.**

8.1.4 Proof of Proposition 1

Step 1. Define $\overline{b}$ as

$$\overline{b} = \begin{cases} e - u_c^{-1}\left(\alpha\theta + (1-\alpha)\, u_c\left(u_c^{-1}(\theta) + 2\sigma\right)\right) + \sigma + \beta\underline{b} & \text{if } \sigma \leq \sigma^* \\ e - u_c^{-1}(\theta) + \sigma + \beta\underline{b} & \text{if } \sigma > \sigma^* \end{cases}$$

Step 2. The fact that $b'^P(b,H) = \underline{b}$ if $b \leq \overline{b}$ and property (iii) for $\sigma > \sigma^*$ follows from step 4 of the proof of Lemma 3. The fact that $b'^P(b,H) = \underline{b}$ if $b \leq \overline{b}$ and property (ii) for $\sigma \leq \sigma^*$ follows from step 4 of the proof of Lemma 3 which states that $x^P(b,H) = 0$ for $b > \underline{b}$ and from (18) and (19).

Step 3. To prove that $b'^P(b,H) < b$, note that if $b'^P(b,H) \geq b$, then necessarily $u_c\left(c^P\left(b'^P(b,H),H\right)\right) \geq u_c\left(c^P(b,H)\right)$ from the envelope condition since $x^P(b,H) = 0$ for $b > \overline{b}$. Satisfaction of (18) then implies $c^P\left(b'^P(b,H),L\right) \geq c^P(b,H)$, but this is a contradiction given the dynamic budget constraints. Therefore, $b'^P(b,H) < b$ if $b > \overline{b}$.

Step 4. To prove that $b'^P(b,H)$ weakly increases in b, substitute the envelope condition into (18) and (19) to achieve:

$$V^P_b(b,H) = E_s\left\{W_b\left(b'^P(b,H),H\right)\right\}. \tag{24}$$

If $b \leq \widetilde{b}$ for $\widetilde{b}$ which satisfies

$$V^P_b\left(\widetilde{b},H\right) = -\alpha q\theta + (1-\alpha)\, W_b\left(e + \sigma - u_c^{-1}(\theta) + \beta\underline{b}, L\right),$$

then (24) implies that $b'^P(b,H)$ is strictly increasing in b with $b'^P\left(\widetilde{b},H\right)=e+\sigma-u_c^{-1}(\theta)+\beta\underline{b}$. If instead $b\geq\widetilde{\widetilde{b}}>\widetilde{b}$ for $\widetilde{\widetilde{b}}$ which satisfies

$$V_b^P\left(\widetilde{\widetilde{b}},H\right)=-\alpha\theta+(1-\alpha)W_b\left(e+\sigma-u_c^{-1}(\theta)+\beta\underline{b},L\right),$$

then (24) implies that $b'^P(b,H)$ is strictly increasing in b with $b'^P\left(\widetilde{\widetilde{b}},H\right)=e+\sigma-u_c^{-1}(\theta)+\beta\underline{b}$. If $\widetilde{b}\leq b\leq\widetilde{\widetilde{b}}$, then (24) implies $b'^P(b,H)=e+\sigma-u_c^{-1}(\theta)+\beta\underline{b}$ which completes the proof. **Q.E.D.**

8.1.5 Proof of Proposition 2

Step 1. Proposition 1 implies that $b_{t+1}^P<b_t$ for $b_t\geq\overline{b}$ and that $b_{t+1}^P=\underline{b}$ if $b_t\leq\overline{b}$ for some $\overline{b}>\underline{b}$. Since $b'^P(b,H)$ is continuous, this implies that $\lim_{t\to\infty}b_{t+1}^P=\underline{b}$. Therefore, $\lim_{t\to\infty}c_t^P=u_c^{-1}(\theta)$.

Step 2. Lemma 1 implies that $b_{t+1}^B\in\left(-\infty,b_t^B\right)$. It cannot be that $\lim_{t\to\infty}b_{t+1}^B=b_\infty^B>-\infty$ since $b'^P(b,H)<b$ for all b and since $b'^P(b,H)$ is a continuous function. Given (3), this implies that $\lim_{t\to\infty}c_t^B=\infty$.

8.1.6 Proof of Lemma 4

Step 1. We first show that $\underline{\sigma}$ and $\overline{\sigma}$ exist and are uniquely defined. To do this we present the difference equations which characterize the equilibrium value of consumption. Let c_t^j for $j=H,L$ correspond to the equilibrium value of consumption at date t as a function of the shock j for an economy beginning with debt b_0 and state s_0. We can manipulate (3) to write

$$-\left(b_t-\frac{b_{t-1}}{\beta}\right)=-\frac{1}{\beta}c_{t-1}^H+\frac{1}{\beta}(e+\sigma).$$

Note that $c_0^L=e-\sigma-b_0(1-\beta)$, and more generally $c_t^L=e-\sigma-b_t(1-\beta)$. Substitution into the above equation then yields a difference equation for consumption in the downturn

$$c_t^L=\frac{1}{\beta}c_{t-1}^L-\frac{1}{\beta}(1-\beta)c_{t-1}^H+\frac{1}{\beta}2\sigma(1-\beta). \tag{25}$$

Therefore c_t^L is increasing in c_{t-1}^L and σ and decreasing in c_{t-1}^H. Substitution of this equation into the Euler equation yields

$$u'\left(c_t^H\right) = \frac{u'\left(c_{t-1}^H\right) - (1-\alpha)\, u'\left(\frac{1}{\beta} c_{t-1}^L - \frac{1}{\beta}(1-\beta)\, c_{t-1}^H + \frac{1}{\beta} 2\sigma (1-\beta)\right)}{\alpha}. \tag{26}$$

Therefore c_t^H is increasing in c_{t-1}^H and decreasing in c_{t-1}^L and σ. The path of consumption follows (25) and (26) subject to $c_0^L = e - \sigma - b_0 (1-\beta)$ and c_0^H chosen to satisfy the present value budget constraint of the government

$$\sum_{t=0}^{\infty} \beta^t c_t^H = \sum_{t=0}^{\infty} \beta^t (e+\sigma) - b_0. \tag{27}$$

Step 2. Define $\underline{b}$ and $\overline{b}$ as under the case for $\sigma > \sigma^*$. We can now show that $\underline{\sigma}$ and $\overline{\sigma}$ exist and are uniquely defined. First consider the value of $\underline{\sigma}$ by letting $b_0 = \underline{b}$. An increase in σ leaves c_0^L unchanged and raises the right hand side of (27). If c_0^H weakly declines then forward iteration on (25) and (26) implies that c_t^H declines for all t, violating (27). Therefore c_0^H strictly increases in σ. If $\sigma = 0$, then $u_c\left(c_0^H\right) = \theta (1-\alpha q) / (1-\alpha) > \theta$. As σ approaches ∞, $\underline{b}$ approaches $-\infty$ so that c_0^L approaches ∞, and $u_c\left(c_0^H\right)$ approaches $0 < q\theta$. Therefore $\underline{\sigma} > 0$ exists and is uniquely defined. Now consider the value of $\overline{\sigma}$, letting $b_0 = \overline{b}$. An increase in σ by reduces c_0^L and raises the right hand side of (27). If c_0^H weakly declines then forward iteration on (25) and (26) implies that c_t^L weakly increases so that c_t^H weakly decreases for all $t \geq 1$, violating (27). Therefore c_0^H strictly increases in σ. Analogous reasoning as in the previous case implies that $\overline{\sigma} > 0$ exists and is uniquely defined.

Step 3. Properties (iv) and (v) follow from steps 1 and 2.

Step 4. An increase in q reduces $\underline{b}$ and $\overline{b}$ which increases c_0^L and increases the right hand side of (27). Analogous arguments to those of step 2 imply that c_0^H must increase in response so that $u_c\left(c_0^H\right)$ decreases whereas $q\theta$ increases. This implies that both $\underline{\sigma}$ and $\overline{\sigma}$ must decrease to compensate. Analogous arguments imply that $\underline{\sigma}$ and $\overline{\sigma}$ increase if α rises. This establishes property (ii).

Step 5. As q approaches 1, $\underline{b}$ approaches $\left(e - \sigma - u_c^{-1}(\theta)\right) / (1-\beta)$ and $\overline{b}$ approaches $2\sigma + \left(e - \sigma - u_c^{-1}(\theta)\right) / (1-\beta)$. This means that if $\sigma = 0$, $u_c\left(c_0^H\right) = \theta$, which implies that $u_c\left(c_0^H\right) < \theta$ for any $\sigma > 0$ since $u_c\left(c_0^H\right)$ is declining in σ. Therefore $\underline{\sigma}$ and $\overline{\sigma}$ must approach 0, establishing property (iii).

Step 6. To establish property (i), suppose $\sigma \geq \overline{\sigma}$ so that $u_c\left(c^B\left(\overline{b}, H\right)\right) \leq q\theta$. By Lemma 1, $u_c\left(c^B\left(\underline{b}, H\right)\right) < u_c\left(c^B\left(\overline{b}, H\right)\right) \leq q\theta$, which from step 3 implies that

$\sigma > \underline{\sigma}$. Therefore, $\overline{\sigma} > \underline{\sigma}$. Suppose $\sigma = \sigma^*$. Then $\underline{b} = \left(e + \sigma - u_c^{-1}(\theta)\right)/(1-\beta)$ and $u_c\left(c^B\left(\underline{b}, H\right)\right) > \theta > q\theta$ since $b'^B(b,H) < \underline{b}$ by Lemma 1. Therefore, $\sigma^* < \underline{\sigma}$. **Q.E.D.**

8.1.7 Proof of Proposition 3

Step 1. Since $b'^P(b,H) = \underline{b}\ \forall b \leq \underline{b}$ from Proposition 1, then from Lemma 1, $b'^P(b,H) > b'^B(b,H)\ \forall b \leq \underline{b}$.

Step 2. Suppose $\sigma \leq \sigma^*$. Then $c^P\left(\underline{b}, H\right) = u_c^{-1}(\theta) > c^B\left(\underline{b}, H\right)$ since $x^P\left(\underline{b}, H\right) = 0$ from Proposition 1. If $b \in \left[\underline{b}, \overline{b}\right]$, then from Proposition 1, $b'^P(b,H) = \underline{b}$, and since $x^P\left(\underline{b}, H\right) = 0$, the Euler equation implies that

$$u_c\left(c^P(b,H)\right) \leq \alpha u_c\left(c^P\left(\underline{b},H\right)\right) + (1-\alpha)\, u_c\left(c^P\left(\underline{b},L\right)\right). \tag{28}$$

Since $c^P\left(\underline{b}, L\right) = c^B\left(\underline{b}, L\right)$ but $c^P\left(\underline{b}, H\right) > c^B\left(\underline{b}, H\right)$, then in order that (12) hold given (28), it must be that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$ in this region.

Step 3. If $b \in \left[\overline{b}, b'^{P^{-1}}\left(\overline{b}, H\right)\right]$, then from Proposition 1 $b'^P(b,H) \in \left[\underline{b}, \overline{b}\right]$, and from step 2, $c^P\left(b'^P(b,H), L\right) = c^B\left(b'^P(b,H), L\right)$ but $c^P\left(b'^P(b,H), H\right) > c^B\left(b'^P(b,H), H\right)$. Therefore, in order that (12) hold, it must be that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$ in this region. Successive applications of this argument until the natural debt limit implies that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)\ \forall b \geq \overline{b}$.

Step 4. Suppose $\sigma \geq \sigma^*$. For any $b \in \left[\overline{b}, b'^{P^{-1}}\left(\overline{b}, H\right)\right)$, $b'^P(b,H) \in \left[\underline{b}, \overline{b}\right)$, and (19) holds since $x^P\left(b'^P(b,H), H\right) > 0$ from Proposition 1. Since $c^P\left(b'^P(b,H), L\right) = c^B\left(b'^P(b,H), L\right)$ but $u_c\left(c^B\left(b'^P(b,H), H\right)\right) > q\theta$, then in order that (12) hold given (19) it must be that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$ in this region. Note that because $b'^P\left(\overline{b}, H\right) = b'^P(b,H) = \underline{b} > b'^B\left(\overline{b}, H\right)$ for $b \in \left[\underline{b}, \overline{b}\right]$, this furthermore implies that $b'^P(b,H) > b'^B(b,H)$ for $b \in \left[\underline{b}, \overline{b}\right]$.

Step 5. If $b'^P(b,H) = \overline{b}$, then since $c^B\left(b'^P(b,H), H\right) < c^P\left(b'^P(b,H), H\right)$, then given (18) and (19), in order that (12) hold it must be that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$.

Step 6. For all b s.t. $b'^P(b,H) > \overline{b}$ successive applications of the analogue to step 3 implies that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)\ \forall b \geq \overline{b}$. **Q.E.D.**

8.1.8 Proof of Proposition 4

Step 1. Given the definition of $\overline{\sigma}$, $c^P(b,H) = u_c^{-1}(\theta) < u_c^{-1}(q\theta) < c^B(b,H)$ for $b \leq \overline{b}$.

Step 2. For any $b \in \left[\overline{b}, b'^{P^{-1}}\left(\overline{b}, H\right)\right)$, $b'^P(b,H) \in \left[\underline{b}, \overline{b}\right)$, and (19) holds since $x^P\left(b'^P(b,H), H\right) > 0$ from Proposition 1. Since $c^P\left(b'^P(b,H), L\right) = c^B\left(b'^P(b,H), L\right)$

but $u_c\left(c^B\left(b'^P(b,H),H\right)\right) < q\theta$, then in order that (12) hold it must be that $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$ in this region.

Step 3. If $b'^P(b,H) = \bar{b}$, then since $u_c^B\left(b'^P(b,H),H\right) < \theta$, then given (18) and (19), in order that (12) hold it must be that $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$ $\forall b \geq \bar{b}$.

Step 4. For any $b \in \left(b'^{P^{-1}}\left(\bar{b},H\right), b'^{P^{-1}}\left(b'^{P^{-1}}\left(\bar{b},H\right)\right),H\right)$, $c^P\left(b'^P(b,H),L\right) = c^B\left(b'^P(b,H),L\right)$ but $c^B\left(b'^P(b,H),H\right) > c^P\left(b'^P(b,H),H\right)$ by steps 2 and 3. In order that (12) hold it must be that $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$ in this region. Successive applications of this step then imply that $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$ $\forall b > \bar{b}$. **Q.E.D.**

8.1.9 Proof of Proposition 5

Step 1. Given (3), (20) implies that $\widetilde{b}_{t+1}^P \leq b_{t+1}^B$ along the equilibrium path, where $\widetilde{b}_{t+1}^P$ corresponds to the equilibrium level of debt under a politician constrained by the deficit rule and b_{t+1}^B corresponds to the equilibrium level of debt under a benevolent government.

Step 2. Consider an economy in final period T in which $\widetilde{b}_T^P \leq b_T^B$. If (20) does not bind, then this implies that $\widetilde{b}_{T+1}^P < b_{T+1}^B = 0$, implying that the rent-seeking government can strictly raise welfare by raising $\widetilde{c}_T^P$ or $\widetilde{x}_T^P$ and increasing $\widetilde{b}_{T+1}^P$. Therefore, (20) binds at T, and intratemporal optimal requires $\widetilde{c}_T^P = \min\left\{c_T^B, u_c^{-1}(\theta)\right\}$ and $\widetilde{x}_T^P = \max\left\{0, c_T^B - u_c^{-1}(\theta)\right\}$.

Step 3. Consider an economy in period $t < T$ in which t $\widetilde{b}_t^P \leq b_t^B$ and (20) binds for all $k > t$ if $\widetilde{b}_k^P \leq b_k^B$ with $\widetilde{c}_k^P = \min\left\{c_k^B, u_c^{-1}(\theta)\right\}$ and $\widetilde{x}_k^P = \max\left\{0, c_k^B - u_c^{-1}(\theta)\right\}$. If (20) does not bind at t, then this implies that $\widetilde{b}_{t+1}^P < b_{t+1}^B$. Given that (20) binds for all $k > t$, this implies that $\widetilde{b}_{T+1}^P < b_{T+1}^B = 0$. This implies that the rent-seeking government strictly raise welfare by raising $\widetilde{c}_t^P$ or $\widetilde{x}_t^P$ and increasing $\widetilde{b}_{t+1}^P$, leaving $\widetilde{c}_k^P$ and $\widetilde{x}_k^P$ unchanged for all $k > t$ since this increases $\widetilde{b}_{T+1}^P$. Therefore, (20) binds at $t < T$ and $\widetilde{c}_t^P = \min\left\{c_t^B, u_c^{-1}(\theta)\right\}$ and $\widetilde{x}_t^P = \max\left\{0, c_t^B - u_c^{-1}(\theta)\right\}$.

Step 4. By forward induction, (20) binds for all t and as $T \to \infty$. **Q.E.D.**

8.2 Intermediate Volatility: $\sigma \in (\underline{\sigma}, \overline{\sigma})$

In this section, we briefly describe the region of intermediate volatility which we do not consider in the text. We show that the path taken by the economy depends on the region in which b_0 is located.

Given the definitions of $\underline{\sigma}$ and $\overline{\sigma}$, there exists a cutoff point $\widetilde{b} \in \left[\underline{b}, \overline{b}\right]$ s.t. $u_c\left(c^B\left(\widetilde{b},H\right)\right) =$

$q\theta$ so that $u_c\left(c^B(b,H)\right) < (>)\, q\theta$ if $b < (>)\, \widetilde{b}$. Consider $b \in \left[\bar{b}, b'^{P^{-1}}\left(\widetilde{b}, H\right)\right)$. The application of step 2 in the proof of Proposition 4 implies that $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$ in this region. Moreover, for $b \in \left(b'^{P^{-1}}\left(\widetilde{b}, H\right), b'^{P^{-1}}\left(\bar{b}, H\right)\right)$, then the application of step 3 in the proof of Proposition 3 implies that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$ in this region.

Now consider the region for which $b'^P(b,H) = \bar{b}$. Equation (19) holds with equality at a minimum value of b in this region, which means that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$ at this point. Equation (18) holds with equality at the maximum point in this region, which means that $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$ at this point. Since $b'^P(b,H) = \bar{b}$ in this region and since $b'^B(b,H)$ is monotonically increasing, there exists a cutoff point $\widetilde{\widetilde{b}}$ which splits the region such that if $b < \widetilde{\widetilde{b}}$ and b is in this region then $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$, and if $b > \widetilde{\widetilde{b}}$ and b is in this region then $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$.

Therefore, we can apply step 3 in the proof of Proposition 3 to the set of b's for which $b'^P(b,H) \in \left(b'^{P^{-1}}\left(\widetilde{b}, H\right), \widetilde{\widetilde{b}}\right)$ and show that $b'^B(b,H) < b'^P(b,H)$ and $c^B(b,H) < c^P(b,H)$. Analogously, we can find a cutoff $\widetilde{\widetilde{\widetilde{b}}}$ such that we can apply step 2 in the proof of Proposition 4 to the set of b's for which $b'^P(b,H) \in \left(\widetilde{\widetilde{b}}, \widetilde{\widetilde{\widetilde{b}}}\right)$ and show that $b'^B(b,H) > b'^P(b,H)$ and $c^B(b,H) > c^P(b,H)$. Forward iteration on this argument implies that there is a sequence of regions between $\bar{b}$ and the natural debt limit in which there is either over-borrowing and over-spending or over-saving and under-spending.

Thus, the path taken by the economy depends on the region in which b_0 is located. If b_0 is in the over-borrowing region, then over-borrowing occurs along the equilibrium path until $\bar{b}$ is passed and if b_0 is in the over-saving region, then over-saving occurs along the equilibrium path until $\bar{b}$ is passed.

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