Welfare Consequences of Sustainable Finance

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Abstract

Sustainable finance pledges to invest in firms that qualify by meeting net-zero carbon emissions targets. We model how these mandates incentivize companies to address the global warming externality by contributing to the accumulation of decarbonization capital. The lower cost-of-capital for qualified compared to unqualified firms equals a sustainable firm’s decarbonization contributions divided by its Tobin’s q. The ratio of decarbonization to productive capital mitigates economic damages from global warming. Due to adjustment costs, this ratio rises gradually—as do the costs to investors—until reaching the steady-state. Our model matches macro-finance moments and mitigation pathways compatible with reforestation. Mandates attain welfare outcomes close to that of the planner’s first-best solution.

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1 Introduction

In light of the failure of legislative bodies to implement carbon emissions taxes globally, there is growing pressure on the financial sector and regulators to address the global warming externality. The most widely-discussed alternative is sustainable investment mandates which only invest in firms that meet net-zero emissions targets by either 2030 or 2050, as part of the 2015 Paris Climate Agreement. For instance, nearly 40 trillion dollars of assets under management (predominantly from sovereign wealth funds and pension plans) signed up for the Net-Zero-Managers Initiative. More generally, according to US SIF Foundation in January 2019, around 38% of assets under management already undergo some type of sustainability screening and over 80% of these screens as implemented as passive portfolios. Banks and central banks have indicated interest in similar mandates with regards to their lending policies, as reflected in the Network for Greening the Financial System (NGFS).

To meet net-zero targets and to qualify to be included in investors’ sustainable portfolios, firms have to spend enough on mitigation measures. According to a recent Intergovernmental Panel on Climate Change (IPCC) special report (Rogelj et.al. (2018)), most mitigation pathways to net-zero emissions require a portfolio of decarbonization measures, including renewables and negative emission technologies (NETs) such as afforestation and reforestation, soil carbon sequestration, bioenergy with carbon capture and storage (BECCs), and direct air capture (DAC).\(^1\) While a number of these measures exist, the stock of decarbonization capital (e.g. plants to do air capture, forests, etc...) are low relative to what is needed to stabilize our climate and need to be accumulated.

We model how these mandates incentivize firms to address the global warming externality via contributions to the accumulation of decarbonization capital stock. Productive capital is the only input of production and generates carbon emissions. Emissions lead to extreme global temperatures, which damage economic growth (Dell, Jones, and Olken (2012), Burke, Hsiang, and Miguel (2015)).\(^2\) But decarbonization capital can offset carbon emissions

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\(^1\)One reason is that for heavy industrial sectors like cement and steel, which generate nearly 20% of global CO2 emissions, switching fuel sources is not a viable option for achieving net-zero (de Pee (2018)).

\(^2\)According to the National Academy of Sciences (2016)), extreme temperatures lead to increased fre-
and hence mitigate damages. The ratio of decarbonization capital to productive capital reduces the frequency of extreme temperatures and hence expected losses from these disasters. Contributions to decarbonization capital, which comes at the expense of firm productivity, increase this ratio. Extreme temperatures damage decarbonization and productive capital equally, and do not influence this ratio.

As in integrated assessment models (Nordhaus (2017)), there is a high willingness-to-pay for mitigation among our households with non-expected utility ((Epstein and Zin (1989) and Weil (1990)) since disasters cause significant welfare losses (Barro (2006), Weitzman (2009), and Pindyck and Wang (2013)). But there is an externality when it comes to mitigating the damages of emissions. Since the benefits of this mitigation only affect the aggregate risk and the market price of risk, which firms take as given, firms do not contribute to decarbonization capital in competitive markets (Hong, Wang and Yang (2020)) — i.e., there is over-accumulation of productive capital and under-accumulation of decarbonization capital. A capital tax is needed in our model to fund mitigation to address this market failure.\(^3\)

We introduce restrictions on the representative investor’s portfolio, i.e., sustainable finance mandates that can vary with the scaled decarbonization stock, into competitive markets to examine the extent to which they can address emissions externalities. The representative investor, who has access to a complete set of financial securities (e.g., all contingencies including idiosyncratic shocks are dynamically spanned), is restricted to passively index a fixed fraction of total wealth to firms that meet sustainability guidelines. To be included in the representative investor’s sustainable portfolio, otherwise ex-ante identical firms have to spend a minimally required amount on decarbonization which they otherwise would not due to externalities.

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\(^3\)This is analogous to a tax on emissions (Golosov, Hassler, Krusell and Tsyviski (2014)) in traditional integrated assessment models featuring an emissions sector.
The value of productive capital, i.e., Tobin’s $q$ for sustainable and unsustainable firms, are endogenously determined so as to leave value-maximizing firms indifferent between being sustainable and not — the Tobin’s $q$ or stock price is the same for all firms in equilibrium. The decarbonization capital, which is unproductive and does not contribute to output, sits in the firm’s assets but is not priced by markets other than through the mandate qualification mechanism. The risk-free rate, stock-market risk premium, Tobin’s $q$ for aggregate productive capital, and growth rates are jointly determined in equilibrium. They in turn depend on the ratio of decarbonization to productive capital, which governs transition dynamics and the steady state. Despite being a dynamic stochastic general equilibrium model, the solution is intuitive and has a number of implications.

The cost-of-capital wedge between qualified and unqualified firms equals firm decarbonization contributions divided by its Tobin’s $q$. Since firms have the same Tobin’s $q$ in equilibrium, the investment and growth paths of both sustainable and unsustainable firms are identical (path by path) over time. Sustainable firms have lower cashflows to pay out due to mitigation spending but have lower cost of capital (the expected return required by the representative investor). The cash-flow effect and the discount-rate effect have to offset each other so as to leave all firms indifferent between being a sustainable and an unsustainable firm. The lower cost of capital for sustainable firms subsidizes their decarbonization, which they would have otherwise invested or distributed to shareholders. The benefits of this mitigation accrue to the entire economy.

The rise of sustainable finance raises difficult questions, which we can use our model to address. To what extent can these mandates achieve first-best outcomes, i.e., how close can welfare-maximizing mandates in a competitive market setting get to the planner’s solution? How large would these mandates have to be? How many years does it take for the economy to transition to the steady state? What are the consequences for extreme temperatures? And what is the impact on economic growth, particularly during the transition? And what about the consequences for investors’ portfolio performance?

To answer these questions, we use our model to conduct quantitative analysis. We cali-
brate the parameters of our model to match not only key macro-finance moments (Bansal and Yaron (2004)) but also climate mitigation pathways involving reforestation—one of the most cost-effective forms of carbon capture (Rogelj et al. (2018), Bastin et al. (2019), Griscom et al. (2017)). First, we use estimates of the damage to GDP growth from abnormal annual country temperatures of 1.5°Celsius (relative to pre-industrial era). Such events are still uncommon, occurring in in a few percent of the country-year observations but damage conditional on such an event is around minus four percentage points of GDP growth (Dell, Jones, and Olken (2012)). Second, we calibrate parameters governing the adjustment cost and efficiency of decarbonization capital to reforestation. Since the pre-industrial age, the planet has gone from 5.9 billion hectares of forests to currently 4 billion hectares. Scientists estimate that reforestation of 2 billion hectares (i.e., of new forecast capital) will keep global temperatures from exceeding the 1.5°Celsius barrier. This can be feasibly done in several decades.

Assuming that the current level of mitigation is negligible, we can pin down both the economic damage absent mitigation and the cost of mitigation from the first-order conditions of the planner’s problem.\footnote{For instance, there are some attempts already at reforestation around the world such as in US, China, Turkey, Canada and a number of countries in Europe but these are relatively small efforts.} We then consider a comparative static where we increase the frequency of annual extreme temperatures. In a 1.5°Celsius world, the frequency of extreme temperature events will rise (from a few percent of country-year observations to a much larger fraction). Zero mitigation will no longer be optimal. In our baseline quantitative exercise, we report the solutions (for both the planner’s problem and market economy with mandates) as we increase frequencies of extreme annual temperatures to one (or 100% of country-year observations).

There are several key messages from our quantitative analysis. First, large enough sustainable finance mandates or portfolio restrictions (on the order of real world estimates of 20 to 40 percent of wealth pledged to sustainable firms) can attain outcomes close to the first-best. The decarbonization to productive capital ratio in the market economy with mandates is around 4.6% in steady state. Given that the world capital stock is around 600 trillion
dollars,\textsuperscript{6} this would imply around 27.6 trillion dollars of decarbonization capital in the steady state. Incidentally, the current book value of forests is around 24 trillion dollars (Kappen et al. (2020)). That is, this calculation corresponds to roughly increasing forest size by double its current size, consistent with reasonable reforestation pathways proposed by scientists (Bastin et al. (2019)). The steady-state ratio of decarbonization to productive capital in the planner’s solution is close to 5.7%. This short-fall of about 1.1% decarbonization capital stock relative to productive capital stock reflects that there is still too high an investment rate in productive capital compared to the planner’s first-best level.

Despite this short-fall, the welfare gains from mandates are substantial, almost 25% higher measured in the certainty equivalent wealth than in an unmitigated competitive market setting and comparable to the welfare gain obtained the planner’s solution. The reason is that the rise of decarbonization capital gradually brings down the jump arrival rate of extreme temperature country-year events from one to around 0.6.

Aggregate contributions to decarbonization capital stock each year under mandates is around 0.23% of physical capital stock in the steady state, which means spending of around 1.4 trillion dollars per year towards decarbonization under mandates. As we discuss below, these numbers are in the ballpark of what is often discussed as the decarbonization spending needed for firms to reach net-zero. Decarbonization contributions can peak before the steady state is reached in a market economy with mandates. The transition time to the steady state is about 23 years (a number that we target in our calibration to be consistent with the time it takes to reforest). To the extent one interprets 2030 or 2050 targets as the steady states, our analysis suggests that 2030 is unlikely to be achieved. Our model’s output can also be related to the notion of a net-zero economy promoted by practitioners and regulators.

Second, there is mixed news when it comes to economic growth. European Union’s 2050 climate plan sees benefits of up to 2% of GDP from the push to decarbonize even excluding the benefits of mitigation in reducing disaster losses (Euractiv.com June 3, 2021). The EU is assuming that decarbonization capital is essentially the same as physical capital

\textsuperscript{6}Gadzinki, Schuller and Vacchino (2018) estimate global capital stock (including both traded and non-traded assets) in 2016 to be between 500 and 600 trillion dollars.
in producing output—i.e., that renewables are just as productive as fossil fuels which is debatable. But decarbonization capital such as forests or direct-air capture plants are much less productive since their sole role is to offset carbon emissions. Indeed, if it were the case that decarbonization capital were highly productive, then there is no need for sustainable finance mandates to push firms to do decarbonization. We make the polar assumption that decarbonization capital is non-productive so there is a role for mandates. In other words, the representative investor pays for decarbonization capital to manage aggregate risks. The key benefits to growth come from the mitigation benefits of unproductive capital since decarbonization does not contribute to output.

Third, the news is again mixed when it comes to investors’ portfolio performance. The sustainable investment thesis is often presented as rational anticipation of incipient carbon taxes. In reality and in our model, sustainable finance mandates emanate from portfolio restrictions imposed by activist investors who want to incentivize firms to reform as we have emphasized. The initial cost-of-capital wedge is low since there is little absolute level decarbonization contributions in the beginning. When the fraction of wealth that is indexed to sustainable finance mandates is smaller, each sustainable firm needs to do more contributions (i.e. qualifying standards are higher for being labeled sustainable) and gets compensated with a larger cost of capital wedge relative to unsustainable firms. At 20%, the wedge rises to 0.70% per annum in steady state. The cost of capital wedge tracks annual firm decarbonization contributions, and hence also peaks before steady-state. Assuming that 50% of wealth is restricted to invest in sustainable firms, the cost-of-capital wedge will rise 0.25% per annum.

In our model, the size of these mandates is essentially a substitute for a carbon tax. While the carbon tax is highly regressive (affecting poor households’ consumption much more), a sustainable finance mandate essentially taxes rich stock owners. Perhaps this is one reason why such mandates seem to be arising as an alternative solution to global warming in the face of political economy considerations. Our analysis is focused purely on the welfare consequences for a given mandate size. We leave the endogenous determination of the size of these mandates or constraints for future research.
2 Related Literatures

When it comes to calculating the optimal carbon tax, our approach differs from the integrated assessment models (IAM) literature. In that literature, emissions or temperature is the state variable and there is no decarbonization (mitigation) capital stock (see, e.g., Jensen and Traeger (2014), Barnett, Brock, and Hansen (2020), Cai and Lontzek (2019), Daniel, Litterman, and Wagner (2019)). In order to tractably accommodate a different approach to decarbonization, where mitigation capital stock is the key state variable, we have chosen to model extreme temperature events (i.e. 1.5°C Celsius above a country’s historical norms) via jump shocks. The frequency of these events, which have disastrous consequences for economic growth, drives mitigation decisions.

Our two capital stock approach builds on Eberly and Wang (2009), who consider a general equilibrium model with two sectors of different productivity. Despite using a different approach from the IAM literature, our model can speak to the impact of mitigation on the frequency of these events as well as economic growth. In our quantitative analysis, we show how sustainable finance can incentivize firms to invest in decarbonization capital while still being able to match key macro-finance moments and generate realistic climate mitigation pathways.

Another way to rationalize these investment mandates is used by an earlier generation of papers where investors have ethical and socially responsible investing preferences for non-pecuniary rationales. Notably, Heinkel, Kraus, and Zechner (2001) use a static constant absolute risk aversion (CARA) utility framework with no capital accumulation, an exogenous interest rate, and a given fixed cost that brown firms can pay to become green. Hong and Kacperczyk (2009) find that ethical investing emanating from institutions such as religious organizations indeed generate cost-of-capital wedges. But to conduct welfare calculations when it comes to firm mitigation of global warming, it is desirable to have a dynamic stochastic general equilibrium model where the mandate qualification criterion is based on decarbonization spending, interest rates, and capital accumulation are endogenous. Our paper combines integrated assessment models of climate change, typically done in a social
planner setting, with richer competitive financial markets to evaluate the viability of sustainable finance mandates to avoid climate disasters.

Our model makes a sharp prediction linking the cost-of-capital wedge between sustainable and unsustainable firms to the amount of mitigation. Recent evidence based on return differences for high versus low carbon emissions companies in the last few years (Bolton and Kacperczyk (2020)) point a cost-of-capital wedge of around 1%. Our assumption of a restriction on the representative investor’s portfolio to passively index to sustainable firms generates realistic downward sloping demand curves even with complete financial spanning.

But a cost-of-capital wedge also arises whenever there is demand for firms with sustainability attributes by a subset of investors who cannot perfectly hedge the idiosyncratic firm risks due to lack of full financial spanning. In this vein, Pastor, Stambaugh, and Taylor (2020) model how some investors’ non-pecuniary taste for green stocks generates three-fund separation in a static CAPM setting and induces an expected-return wedge between green and brown stocks. The cost-of-capital wedge in their model also depends on investor preferences and incomplete financial spanning. Pedersen, Fitzgibbons, and Pomorski (2020) also incorporate potential mispricing of fundamental information captured by sustainability measures. To the extent that demand curves are flat, it can be more efficient to engage activist strategies relying on voting as opposed to divestment to effect change (Gollier and Pouget (2014), Broccardo, Hart, and Zingales (2020), and Oehmke and Opp (2020)).

We proceed as follows. We describe our model in Section 3 and then solve our sustainable finance mandate model in Section 4. Section 5 summarizes the planner’s first-best solution. We provide a quantitative analysis in Section 6 and conclude in Section 7.

3 Model

While mitigating climate disaster risk benefits the society, doing so is privately costly for the firm. We model sustainable finance mandates as portfolio restrictions on the representative agent’s portfolio and examine the extent to which it encourages firms to provide risk

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7See e.g., Shleifer (1986), Chang, Hong and Liskovich (2015), Kashyap, Kovrijnykh, Pavlova (2018), and Kojien and Yogo (2019).
mitigation and quantify its implications for social welfare. We use a representative-agent framework for expositional simplicity, where this agent can be interpreted as representing both public (e.g., sovereign wealth funds) and private investors.

On the demand side for financial assets, the representative agent holds and invests the entire wealth of the economy between sustainable ($S$) firms, unsustainable ($U$) firms, and the risk-free bonds. The agent has to invest an $\alpha$ fraction of the entire aggregate wealth in a sustainable type-$S$ firm. The risk-averse representative agent is required to meet the sustainable investment mandate at all times when allocating assets.

On the supply side, a portfolio of $S$ firms and a portfolio of $U$ firms will arise endogenously in equilibrium, which we refer to as $S$-portfolio and $U$-portfolio, respectively. For a firm to qualify to be type-$S$, it has to spend at least a fraction $m$ of its capital on mitigation via a portfolio of decarbonization technologies so as to reduce disaster risk. Otherwise, it is labeled a type-$U$ for unsustainable.

### 3.1 Firm Production and $K$ Capital Accumulation

The firm’s output at $t$, $Y_t$, is proportional to its capital stock, $K_t$, which we refer to as productive capital and is the only factor of production:

$$Y_t = AK_t,$$

where $A > 0$ is a constant that defines productivity for all firms. This is a version of widely-used $AK$ models in macroeconomics and finance. All firms start with the same level of initial capital stock $K_0$ and have the same production and capital accumulation technology. Additionally, they are subject to the same shocks (path by path).

That is, there is no idiosyncratic shock in our model. This simplifying assumption makes our model tractable and allows us to focus on the impact of the investment mandate on equilibrium asset pricing and resource allocation. Despite being identical in all aspects, some firms choose to be sustainable while others remain unsustainable in equilibrium.

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8There are pros and cons of using an $AK$ model for our climate-change analysis. For analyzing weather disasters such as hurricanes which have been shown to have permanent effects on capital and output, an $AK$ model setup is natural. But an $AK$ setup might miss important features of growth rate dynamics in other settings (Jones (1995)).
**Investment.** Let $I_t$ denote the firm’s investment. As in Pindyck and Wang (2013), the firm’s productive capital stock, $K_t$, evolves as:

$$dK_t = \Phi(I_{t-}, K_{t-})dt + \sigma K_{t-}dB_t - (1-Z)K_{t-}dJ_t .$$

(2)

As in Lucas and Prescott (1971) and Jerrmann (1998), we assume that $\Phi(I, K)$, the first term in (2), is homogeneous of degree one in $I$ and $K$, and thus can be written as

$$\Phi(I, K) = \phi(i)K ,$$

(3)

where $i = I/K$ is the firm’s investment-capital ratio and $\phi(\cdot)$ is increasing and concave. This specification captures the idea that changing capital stock rapidly is more costly than changing it slowly. As a result, installed capital earns rents in equilibrium so that Tobin’s $q$, the ratio between the value and the replacement cost of capital exceeds one.

The second term captures continuous shocks to capital, where $B_t$ is a standard Brownian motion and the parameter $\sigma$ is the diffusion volatility (for the capital stock growth). This $B_t$ is the source of shocks for the standard $AK$ models in macroeconomics. This diffusion shock is common to all firms. Had we introduced an additional shock that is idiosyncratic across firms, our solution would remain unchanged as firms can perfectly hedge idiosyncratic shocks at no cost and our aggregation results remain valid.

**Jump shocks.** The firm’s $K$ capital stock is also subject to an aggregate jump shock. We capture this jump effect via the third term, where $J_t$ is a (pure) jump process with an endogenously determined arrival rate, which we denote by $\lambda_t > 0$, which we discuss in detail later. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at $t$. Examples of jumps include hurricanes or wildfires that destroy physical and housing capital stock.

When a jump arrives ($dJ_t = 1$), it permanently destroys a stochastic fraction $(1-Z)$ of the firm’s capital stock $K_{t-}$, as $Z$ is the recovery fraction where $Z \in (0, 1)$. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = .85$.) There is no limit
to the number of these jump shocks.\(^9\) If a jump does not arrive at \(t\), i.e., \(dJ_t = 0\), the third term disappears. We assume that the cumulative distribution function (cdf) and probability density function (pdf) for the recovery fraction, \(Z\), conditional on a jump arrival at any time \(t\), are time invariant. Let \(\Xi(Z)\) and \(\xi(Z)\) denote the cdf and pdf of \(Z\), respectively.

We use **boldfaced** notations for aggregate variables. Before discussing the endogenous jump arrival rate \(\lambda_t\), we first introduce emissions, emission removals, and the dynamics of decarbonization capital stock \(N\).

### 3.2 Aggregate Emissions, Emission Removals, and Decarbonization Capital Stock \(N\)

We assume that the aggregate emissions \(E\) is proportional to \(K\):

\[
E_{t-} = eK_{t-},
\]

where \(e > 0\) is a constant. That is, aggregate emissions increases linearly with the size of the production sector of the economy, which is measured by the aggregate capital stock \(K\) or equivalently GDP (\(AK\)).

Similarly, we assume that the aggregate emission removals \(R\) is proportional to the decarbonization capital stock \(N\):

\[
R_{t-} = \tau N_{t-},
\]

where \(\tau > 0\) is a constant. Equations (4) and (5) state that both aggregate emissions and carbon removals are given by an “\(AK\)”-type of technology.

Let \(X_t\) denote the aggregate mitigation spending. The aggregate decarbonization capital stock \(N\) evolves as follows:

\[
\frac{dN_t}{N_{t-}} = \omega(X_{t-}/N_{t-})dt + \sigma dB_t - (1 - Z)dJ_t.
\]

In (6), \(\omega(X_{t-}/N_{t-})\) is the rate at which aggregate mitigation spending \(X_{t-}\) increases \(dN_t/N_{t-}\). We assume that \(\omega(\cdot)\) is increasing and concave as we do for \(\phi(i)\). This specification captures

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\(^9\)Stochastic fluctuations in the capital stock have been widely used in the growth literature with an \(AK\) technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.
the idea that changing the decarbonization capital stock rapidly is more costly than changing it slowly.

We further assume that the growth rate $dN_t/N_t$ for decarbonization capital stock $N_t$ is subject to the same diffusion and jump shocks as the growth rate of capital stock $K$, $dK_t/K_t$, path by path (e.g., for each realized jump and recovery fraction $Z$). This explains why the last two terms in (6) take the same form as those in (2).

Let $n_t$ denote the decarbonization stock $N_t$ scaled by $K_t$:

$$n_t = \frac{N_t}{K_t}.$$  \hspace{1cm} (7)

Using Ito’s lemma, we obtain the following dynamics for $n_t$:

$$\frac{dn_t}{n_t} = [\omega(x_t/n_t) - \phi(i_t)] \, dt.$$  \hspace{1cm} (8)

Since the two types of capital stock are subject to the same jump-diffusion shocks, there is no uncertainty for the dynamics of $n_t$. Next, we describe the distribution for the recovery fraction $Z$.

### 3.3 Mitigation and Externality

Since global warming is expected to increase the frequency of disasters, we assume that the jump arrival rate $\lambda_t$ increases with the aggregate emissions $E_t$ and decreases with the aggregate emissions removals $R_t$. As $E_t = eK_t$ and $R_t = \tau N_t$ (see equations (4) and (5)), we may write $\lambda_t$ as a function that is increasing in $K_t$ and decreasing in $N_t$.

The pre-jump expected damage over a small $dt$ period is $\lambda_t \mathbb{E}(1 - Z)K_t \, dt$, where $\mathbb{E}(\cdot)$ is the expectation operator. We further make the following homogeneity assumption: the expected damage doubles if we simultaneously double both the size of the productive sector ($K_t$) and the size of the protective sector ($N_t$). This boils down to assuming that $\lambda_t$ is homogeneous of degree zero in $K_t$ and $N_t$, which means $\lambda_t$ is simply a function of the pre-jump scaled aggregate decarbonization stock $n_t = N_t/K_t$. It is useful to make the dependence of $\lambda_t$ on $n_t$ explicit: $\lambda_t = \lambda(n_t)$. Intuitively, increasing $n$ lowers the jump arrival rate, $\lambda'(n) < 0$. Additionally, the marginal impact of $N$ on the change of $\lambda$ decreases as $N$ increases, i.e., $\lambda''(n) > 0$. 

As disaster shocks are aggregate and disaster damages are only curtailed by aggregate decarbonization stock $N$, absent mandates or other incentive programs, firms have no incentives to mitigate on their own as the economy is competitive and their own mitigation spending have no impact on the aggregate mitigation spending (Hong, Wang, and Yang (2020)).

### 3.4 Sustainable Investment Mandates

Let $1^S_t$ be an indicator function describing the status of a firm at $t$. To qualify as a sustainable ($S$) firm at $t$, the firm has to spend at least $M_t$ at $t$ on disaster risk mitigation, which contributes to the reduction of aggregate risk. That is, $1^S_t = 1$ if and only if the firm’s mitigation spending $X_t$ satisfies:

$$X_t \geq M_t.$$  \hfill (9)

Otherwise, $1^S_t = 0$ and the firm is unsustainable ($U$).

To preserve our model’s homogeneity property, we assume that the mandated mitigation spending is proportional to firm size $K_t$ for given $n_t$:

$$M_t = m_t(n_t)K_t,$$  \hfill (10)

where $m_t$ is the minimal level of mitigation per unit of the firm’s capital stock to qualify a firm to be sustainable. That is, it is cheaper for a firm (with smaller $K_t$) to qualify as a sustainable firm. Later, we endogenize the $S$-firm qualification threshold, $m(n_t)$, to maximize the representative agent’s utility.

The investment mandate $\alpha$ creates the inelastic demand for $S$ firms. In equilibrium, the remaining $1 - \alpha$ fraction is invested in the $U$-portfolio so that the agent has no investment in the risk-free bonds in equilibrium.

### 3.5 Optimal Firm Mitigation

Each firm can choose to be either a sustainable ($S$) or a unsustainable firm ($U$). We assume that a firm’s mitigation is observable and contractible. While spending on aggregate risk
mitigation yields no monetary payoff for the firm, doing so allows it to be included in the S-portfolio.

A value-maximizing firm chooses whether to be sustainable or unsustainable depending on which strategy yields a higher value. Let $Q^j_t$ denote the the market value of a type-$j$ firm at $t$, where $j = \{S, U\}$. By exploiting our model’s homogeneity property, we conjecture and verify that the equilibrium value of a type-$j$ firm at time $t$ must satisfy:

$$Q^j_t = q^j(n_t)K^j_t,$$

where $q^j$ is Tobin’s average $q$ for a type $j$-firm for given $n_t$.

In equilibrium, as mitigation spending has no direct benefit for the firm, if the firm chooses to be $U$, i.e., $I^S_t = 0$, it will set $X_t = 0$. Moreover, even if a firm chooses to be a $S$ firm, it has no incentive to spend more than $M_t$, i.e., $(9)$ always binds for a type-$S$ firm.

As we later verify, the equilibrium expected rate of return for a type-$j$ firm, which we denote by $r^j(n_t)$, is function of $n_t$. A type-$j$ firm maximizes its present value:

$$\max_{I_t, X_t} \mathbb{E} \left( \int_0^\infty e^{-r^j(n_t)t} CF^j(n_t) dt \right)$$

subject to the standard transversality condition specified in the Appendix A. In equation (12), $CF^j(n_t)$ is the firm’s cash flow at $t$, which is given by

$$CF^S(n_t) = AK^S_t - I^S_t(n_t) - X^S_t(n_t) \quad \text{and} \quad CF^U(n_t) = AK^U_t - I^U_t(n_t),$$

as an unsustainable firm spends nothing on mitigation.

Since $I_t$ and $X_t$ are both proportional to $K_t$, spending on $X_t$ effectively reduces the productivity of firms. Hence, $X_t$ can be broadly interpreted as various mitigatory activities that reduce firm productivity including limiting carbon emissions or spending on other forms of mitigation.

### 3.6 Dynamic Consumption and Asset Allocation

The representative agent makes all the consumption and asset allocation decisions. We thus use individual and aggregate variables for the agent interchangeably. For example,
the aggregate wealth, $W_t$, is equal to the representative agent’s wealth, $W_t$. Similarly, the aggregate consumption, $C_t$, is equal to the representative agent’s consumption, $C_t$.

The representative agent has the following investment opportunities: (a) the $S$ portfolio which includes all the sustainable firms; (b) the $U$ portfolio which includes all other firms that are unsustainable; (c) the risk-free asset that pays interest at a constant risk-free interest rate $r$ determined in equilibrium; and (d) actuarially fair insurance claims for disasters with every possible recovery fraction $Z$ (and also for diffusion shocks.)

**Type-$S$ and type-$U$ portfolios.** The $S$ and $U$ portfolios include all the $S$ and $U$ firms, respectively. Let $Q^S_t$ and $Q^U_t$ denote the aggregate market value of the $S$ portfolio firm and the $U$ portfolio at $t$, respectively. Similarly, Let $D^S_t$ and $D^U_t$ denote the aggregate dividend of the $S$ portfolio firm and the $U$ portfolio at $t$, respectively.

We conjecture and then verify that the cum-dividend return for the type-$n$ portfolio is given by

$$\frac{dQ^j_t + D^j_t dt}{Q^j_t} = r^j(n_{t-})dt + \sigma dB_t - (1 - Z)(d\mathcal{J}_t - \lambda(n_{t-})dt),$$

where $r^j(n_{t-})$ is the endogenous expected cum-dividend return for a type-$j$ firm in equilibrium for given $n$. In equation (14), the diffusion volatility is equal to $\sigma$ as in equation (2). The third term on the right side of equation (14) is a jump term capturing the effect of disasters on return dynamics. Both the diffusion volatility and jump terms are martingales (and this is why $r^j(n_{t-})$ is the expected return.) Note that the only difference between the $S$- and $U$-portfolio is the expected return. The diffusion and jump terms are the same as those in the capital evolution dynamics given in equation (2).

**Disaster risk insurance (DIS).** We define DIS as follows: a DIS for the survival fraction in the interval $(Z, Z + dZ)$ is a swap contract in which the buyer makes insurance payments $p(Z)dZ$, where $p(Z)$ is the equilibrium insurance premium payment, to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z + dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined disaster event occurs and then collects one unit of the consumption good as a payoff from the seller. The
DIS contract is priced at actuarially fairly terms so that investors earn zero profits.

**Preferences.** We use the Duffie and Epstein (1992) continuous-time version of the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that the representative agent has homothetic recursive preferences given by:

\[ V_t = \mathbb{E}_t \left[ \int_t^\infty f(C_s, V_s) ds \right], \tag{15} \]

where \( f(C, V) \) is known as the normalized aggregator given by

\[ f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1 - \gamma)V)^\chi}{((1 - \gamma)V)^{\chi-1}}. \tag{16} \]

Here \( \rho \) is the rate of time preference, \( \psi \) the elasticity of intertemporal substitution (EIS), \( \gamma \) the coefficient of relative risk aversion, and we let \( \chi = (1 - \psi^{-1})/(1 - \gamma) \). Unlike expected utility, recursive preferences as defined by (15) and (16) disentangle risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is \( f_C = \rho C^{-\psi^{-1}}/[(1 - \gamma)V]^{\omega-1} \), which depends not only on current consumption but also (through \( V \)) on the expected trajectory of future consumption.

If \( \gamma = \psi^{-1} \) so that \( \chi = 1 \), we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

\[ f(C, V) = \frac{\rho C^{1-\gamma}}{1 - \gamma} - \rho V. \tag{17} \]

This more flexible utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature.

**Wealth dynamics.** Let \( W_t \) denote the representative agent’s wealth. Let \( H_t^S \) and \( H_t^U \) denote the dollar amount invested in the \( S \) and \( U \) portfolio, respectively. Let \( H_t \) denote the agent’s wealth allocated to the market portfolio at \( t \). That is, \( H_t = H_t^S + H_t^U \). The dollar
amount, \((W_t - H_t)\) is the dollar amount invested in the risk-free asset. For disasters with recovery fraction in \((Z, Z + dZ)\), \(\delta(Z, n_{t_{-}})W_{t}dt\) gives the total demand for the DIS over time period \((t, t + dt)\).

The agent accumulates wealth as:

\[
dW_t = \left[ r(n_{t_{-}})(W_{t_{-}} - H_{t_{-}}) - C_{t_{-}} \right] dt + \left( r_s(n_{t_{-}})H_{t_{-}}^S + r_u(n_{t_{-}})H_{t_{-}}^U \right) dt + \sigma H_{t_{-}}dB_t
\]

\[
- (1 - Z) H_{t_{-}} (dJ_t - \lambda(n_{t_{-}})dt) - \left( \int_0^1 \delta(Z, n_{t_{-}})p(Z, n_{t_{-}})dZ \right) W_{t_{-}}dt + \delta(Z, n_{t_{-}})W_{t_{-}}dJ_t .
\]

The first term in (18) is the interest income from savings in the risk-free asset minus consumption. The second term is the expected return from investing in the \(S\) and \(U\) portfolios. Note that the expected returns are different: \(r^S(n)\) and \(r^U(n)\) for the \(S\) and \(U\) portfolios, respectively. The third and fourth terms are the diffusion and jump martingale terms for the stock market portfolio. Note that the stochastic (shock) components of the returns (diffusion and jumps) for the two portfolios are identical path by path. The fifth term is the total DIS premium paid by the consumer before the arrival of jumps and captures the financial hedging cost. The last term describes the DIS payments by the DIS seller to the household when a jump occurs.

The total market capitalization of the economy, \(Q_t\), is given by

\[
Q_t = q^S(n_t)K_t^S + q^U(n_t)K_t^U .
\]

Let \(\pi_t^S\) and \(\pi_t^U\) denote the fraction of total wealth \(W_t\) allocated to the \(S\) and \(U\) portfolio at time \(t\), respectively. That is, \(H_t^S = \pi_t^S H_t\), \(H_t^U = \pi_t^U H_t\), and the remaining fraction \(1 - (\pi_t^S + \pi_t^U)\) of \(W_t\) is allocated to the risk-free asset.

In equilibrium, the investment mandate requires that the total capital investment in the \(S\) portfolio has to be at least an \(\alpha\) fraction of the total stock market capitalization \(Q_t\):

\[
H_t^S \geq \alpha Q_t .
\]

In equilibrium, the total stock market capitalization \(Q_t\) depends on the mandate. We later derive a closed-form expression for the relation between \(Q_t\) and \(\alpha\).
Let $Y_t$, $C_t$, $I_t$, and $X_t$ denote the aggregate output, consumption, investment, and mitigation spending, respectively. Adding across all type-$S$ and $U$ firms, we obtain the aggregate resource constraint:

$$Y_t = C_t + I_t + X_t.$$  

(21)

3.7 Competitive Equilibrium with Mandates

We define the competitive equilibrium subject to the investment mandate as follows: (1) the representative agent dynamically chooses consumption and asset allocation among the $S$ portfolio, the $U$ portfolio, and the risk-free asset subject to the investment mandate given in (20); (2) each firm chooses its status ($S$ or $U$), and investment policy $I$ to maximizes its market value; (3) all firms that choose sustainable investment policies are included in the $S$ portfolio and all remaining firms are included in the $U$ portfolio; and (4) all markets clear.

The market-clearing conditions include (i) the net supply of the risk-free asset is zero; (ii) the representative agent’s demand for the $S$ portfolio is equal to the total supply by firms choosing to be sustainable; (iii) the representative agent’s demand for the $U$ portfolio is equal to the total supply by firms choosing to be brown; (iv) the net demand for the DIS of each possible recovery fraction $Z$ is zero; and (v) the goods market clears, i.e., the resource constraint given in (21) holds.

Because the risk-free asset and all DIS contracts are in zero net supply, the agent’s entire wealth $W_t$ is invested in the $S$ and $U$ portfolios.

3.8 Welfare-Maximizing Mandate

For a given level of $\alpha$, we endogenize the criterion at the firm level characterized by the scaled mitigation threshold $M_t = m(n_t)K_t$, for a firm to qualify as a sustainable firm. Specifically, at time 0, the planner announces $\{M_t; t \geq 0\}$ and commits to the announcement with the goal of maximizing the representative agent’s utility given in equation (15) taking into account that the representative agent and firms take the mandate as given and optimize in
competitive equilibrium.\textsuperscript{10} Since no firm spends more than $M_t$ to qualify as an $S$ firm, the equilibrium aggregate mitigation spending satisfies:

$$X_t = \alpha M_t.$$  \hfill (22)

**Comment.** In our model, the representative agent represents investors in the whole economy including both the private and public sectors. We may also interpret our representative-agent model as one with heterogeneous agents where an $\alpha$ fraction of them are sustainable investors, who have investment mandates (e.g., large asset managers and sovereign wealth funds), and the remaining $1 - \alpha$ fraction do not. The sustainable investors group has inelastic demand for sustainable firms and moreover they do not lend their shares out for other investors to short sustainable firms.

### 4 Equilibrium Solution

In this section, we solve for the equilibrium solution with the sustainable finance mandate. First, we introduce the investment mandate at the firm level.

#### 4.1 Sustainability Investment Mandate

For a firm to be sustainable at $t$, it is required to spend the minimal required $m_t$ fraction of its productive capital stock $K_t$. We assume that $m_t$ is a function of $n_t$ to preserve our model’s homogeneity property:

$$x_t^S = \frac{X_t^S}{K_t^S} = m(n_t).$$  \hfill (23)

Any additional spending on mitigation is suboptimal as it yields no further benefit to the firm. All other firms spend nothing on mitigation and hence are unsustainable, i.e., $x_t^{U} = 0$. The fraction of total wealth allocated to meet the sustainability investment mandate is $\alpha$.

Next, we consider the firm’s decision problem when it takes the sustainability mandate \{m_t : t \geq 0\} as given.

\textsuperscript{10}Broadly speaking, our mandate choice is related to the optimal fiscal and monetary policy literature (e.g., Lucas and Stokey, 1983) in macroeconomics. See Ljungqvist and Sargent (2018) for a textbook treatment.
4.2 Firm Optimization

Next, we solve for optimal investment policies for both types of firms. The firm’s objective (12) implies that \( \int_0^s e^{-\int_0^s r^j(n_t) dt} CF^j(n_t) dt + e^{-\int_0^s r^j(n_t) dt} Q^j_s \) is a martingale under the physical measure. We obtain the following Hamilton-Jacobi-Bellman (HJB) equation:

\[
r^j(n) Q^j(K^j, n) = \max_{I^j} CF^j(n) + \left( \Phi(I^j, K^j) Q^j_K(K^j, n) + \frac{1}{2}(\sigma K^j)^2 Q^j_{KK}(K^j, n) \right) + \left[ \omega(x/n) - \phi(i) \right] n Q^j_n(K^j, n) + \lambda(n) \mathbb{E} \left[ Q^j(ZK^j, n) - Q^j(K^j, n) \right],
\]

where \( r^j(n) \) is the cost of capital and \( CF^j(n) \) is the cash flow for a type-\( j \) firm given by (13). The preceding equation takes the aggregate decarbonization stock \( n \), aggregate mitigation spending \( x \), and aggregate investment \( i \) as given. In (24), \( \mathbb{E} [\cdot] \) is the conditional expectation operator with respect to the distribution of recovery fraction \( Z \). The last term depends on the scaled aggregate decarbonization capital stock \( n \) and has the same effect on all firms.

By using our model’s homogeneity property, \( Q^j_t = q^j(n_t)K_t \), we obtain the following

\[
r^j(n) q^j(n) = \max_{i^j} cf^j(n) + g(i^j) q^j(n) + [\omega(x/n) - \phi(i)] n q''(n),
\]

where \( g(i(n)) \) is the expected firm growth rate:

\[
g(i(n)) = \phi(i(n)) - \lambda(n)(1 - \mathbb{E}(Z)),
\]

and \( cf^j(n) = CF^j(n)/K^j \) is the scaled cash flow for a type-\( j \) firm. As \( x^S(n) = m(n) \) and \( x^U(n) = 0 \), we have \( cf^S(n) = A - i^S(n) - m(n) \) for a type-S firm and \( cf^U(n) = A - i^U(n) \) for a type-U firm.

The investment FOC for both types of firms implied by (25) is the following well known condition in the \( q \)-theory literature:

\[
q^j(n) = \frac{1}{\phi'(i^j(n))}.
\]

A type-\( j \) firm’s marginal benefit of investing is equal to its marginal \( q \), \( q^j(n) \), multiplied by \( \phi'(i^j(n)) \). Equation (27) states that this marginal benefit, \( q^j(n)\phi'(i^j(n)) \), is equal to one, the marginal cost of investing at optimality. The homogeneity property implies that a firm’s marginal \( q \) is equal to its average \( q \) (Hayashi, 1982).
4.3 Representative Agent’s Optimization

In the appendix, we show that both the optimal risk-free asset holding and the jump hedging demand $\delta(Z, n)$ are zero for all $Z$ and $n$ in equilibrium. Additionally, the fraction of total wealth allocated to the S-portfolio, which we denote by $\pi^S = H^S / (H^S + H^U) = H^S / W$, is equal to the fraction of wealth mandated to invest in the S portfolio: $\pi^S = \alpha$. The remaining $1 - \pi^S$ fraction of total wealth is allocated to the U-portfolio. hat is, $H^S_t = \alpha W_t = Q^S_t = \alpha Q_t$, $H^U_t = (1 - \alpha) W_t = Q^U_t = (1 - \alpha) Q_t$, and $W_t = Q_t = Q^S_t + Q^U_t$.

To ease exposition, here we only highlight the FOC with respect to consumption and the following associated simplified HJB equation for the agent’s value function, $V(W, n)$:

$$
0 = \max_C f(C, V) + \left[ (r^S(n) \alpha + r^U(n)(1 - \alpha)) W - C \right] V_W(W, n) + \left[ \omega(x/n) - \phi(i) \right] n V_n(W, n) + \frac{\sigma^2 W^2 V_{WW}(W, n)}{2} + \lambda(n) \int_0^1 [V(ZW, n) - V(W, n)] \xi(Z) dZ. \tag{28}
$$

The FOC for consumption $C$ is the standard condition:

$$
f_C(C, V) = V_W(W, n). \tag{29}
$$

4.4 Market Equilibrium

The equilibrium risk-free rate $r(n)$, the expected returns ($r^S(n)$ and $r^U(n)$) for the $S$ and $U$ portfolios, Tobin’s average $q$ for all firms are all functions of $n$.

As a firm can choose being either sustainable or not, it must be indifferent between the two options at all time. That is, in equilibrium, all firms have the same Tobin’s $q$, which in equilibrium is also Tobin’s $q$ for the aggregate economy:

$$
q^S(n) = q^U(n) = q(n). \tag{30}
$$

Equations (27) and (30) imply that all firms also have the same equilibrium investment-capital ratio, which is also the aggregate $i(n)$ for given $n$:

$$
i^S(n) = i^U(n) = i(n) \tag{31}
$$
As a result, the cash flows difference between a $U$ and an $S$ firm is exactly the mitigation spending:

$$cf^U(n) - cf^S(n) = m(n),$$

where $cf^U(n) = A - i(n)$.

Since each $S$ firm spends $m(n)K^S_t$ units on mitigation and all firms are of the same size, we have the following relation between the scaled mitigation $m(n)$ at the firm level and scaled mitigation at the aggregate level $x(n) = X(n)/K$:

$$m(n) = \frac{x(n)}{\alpha} \geq x(n).$$

The mitigation spending mandate for a firm, $m(n)$, is larger than the aggregate scaled mitigation, $x(n)$, as only an $\alpha$ fraction of firms are sustainable.

In equilibrium, the aggregate consumption is equal to the aggregate dividend:

$$c(n) = cf(n) = A - i(n) - x(n).$$

**Equilibrium risk-free rate $r(n)$ and expected market return $r^M(n)$ for a given $n$.**

Building on Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we calculate the aggregate stock-market risk premium, $r^M(n) - r(n)$, by using

$$r^M(n) - r(n) = \gamma \sigma^2 + \lambda(n) \mathbb{E}[(1 - Z)(Z^{-\gamma} - 1)].$$

The risk-free rate is

$$r(n) = \frac{c(n)}{q(n)} + \phi(i(n)) + [\omega(x/n) - \phi(i(n))] \frac{\eta q'(n)}{q(n)} - \gamma \sigma^2 - \lambda(n) \mathbb{E}[(1 - Z)Z^{-\gamma}].$$

**Aggregate $i(n)$, $q(n)$, and $c(n)$ for a given $x(n)$ process.** For a given $x(n)$ process, we obtain the aggregate scaled investment $i(n)$ by solving

$$0 = \frac{(A - i(n) - x(n)) \phi'(i(n))}{1 - \psi^{-1}} + \phi(i(n)) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(n)}{1 - \gamma} \mathbb{E}(Z^{1-\gamma} - 1)$$

$$+ [\omega(x/n) - \phi(i)] \left( \frac{\psi}{1 - \psi} \frac{q'(n)}{q(n)} - \frac{1}{1 - \psi} \frac{ni'(n) + nx'(n)}{A - i(n) - x(n)} \right),$$

where $q(n)$ is given by

$$q(n) = \frac{1}{\phi'(i(n))}.$$
Welfare, optimal mitigation, and equilibrium investment. In Appendix C, we show that the welfare measure per unit of capital, \( b(n) = u(n) \times q(n) \), satisfies the following ODE:

\[
0 = \frac{\rho}{1 - \psi^{-1}} \left[ \left( \frac{A - i - x}{b(n)} \right)^{1-\psi^{-1}} - 1 \right] + \phi(i) + \left( \omega(x/n) - \phi(i) \right) \frac{nb'(n)}{b(n)} \\
- \frac{\gamma \sigma^2}{2} + \frac{\lambda(n)}{1 - \gamma} \left[ \mathbb{E}(Z^{1-\gamma}) - 1 \right].
\]  

(38)

The FOC for the welfare-maximizing level of \( x \) is given by

\[
b(n)^{-\psi^{-1}}b'(n) = (A - i - x)^{-\psi^{-1}} \frac{\rho}{\omega'(x/n)}.
\]  

(39)

The FOC for the optimal investment is

\[
b(n)^{1-\psi^{-1}} = (A - i - x)^{-\psi^{-1}} \frac{\rho}{\phi'(i)}.
\]  

(40)

At the steady state, the drift of \( n \) is zero. Let \( i^* \) and \( x^* \) denote the corresponding steady-state investment-capital ratio and mitigation spending, respectively. We have

\[
\omega(x^*/n^*) - \phi(i^*) = 0
\]  

(41)

and

\[
0 = \frac{\rho}{1 - \psi^{-1}} \left[ \left( \frac{A - i^* - x^*}{b(n^*)} \right)^{1-\psi^{-1}} - 1 \right] + \phi(i^*) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(n^*)}{1 - \gamma} \left[ \mathbb{E}(Z^{1-\gamma}) - 1 \right].
\]  

(42)

Summary. The steady state is an endogenously determined boundary that satisfies (39), (40), (41), and (42). For the transition path, we solve the ODE (38) together with the FOCs (39) and (40) subject to the boundary conditions for the steady state given above.

We next calculate the costs of capital for \( S \) and \( U \) firms.

Cost-of-capital wedge. It is helpful to use \( \theta^j(n) \) to denote the wedge between the expected return for a type-\( j \) firm, \( r^j(n) \), and the aggregate stock-market return, \( r^M(n) \), and write for \( j = \{S,U\} \),

\[
r^j(n) = r^M(n) + \theta^j(n).
\]  

(43)
As an $\alpha$ fraction of the total stock market is the $S$ portfolio and the remaining $1-\alpha$ fraction is the $U$ portfolio, we have

$$r^M(n) = \alpha \cdot r^S(n) + (1-\alpha) \cdot r^U(n). \quad (44)$$

Using (25) for both $S$- and $U$-portfolios, we obtain

$$\theta^U(n) = \frac{x(n)}{q(n)} = \frac{\alpha m(n)}{q(n)} > 0. \quad (45)$$

Equation (45) states that investors demand a higher rate of return to invest in $U$ firms than in the aggregate stock market. The expected return wedge between the $U$-portfolio and the market portfolio is equal to $\theta^U(n)$, which is equal to the aggregate mitigation spending $X(n)$ divided by aggregate stock market value $Q(n)$. This ratio $x(n)/q(n)$ can be viewed as a "tax" on the unsustainable firms by investors in equilibrium.

Substituting (43) into (44) and using (45), we obtain:

$$\theta^S(n) = -\frac{1-\alpha}{\alpha} \theta^U(n) = -\frac{1-\alpha x(n)}{q(n)} = -(1-\alpha) \frac{m(n)}{q(n)} < 0. \quad (46)$$

The cost-of-capital difference between $U$ and $S$ firms is given by

$$r^U(n) - r^S(n) = \theta^U(n) - \theta^S(n) = \frac{1}{\alpha} \frac{x(n)}{q(n)} = \frac{m(n)}{q(n)}. \quad (47)$$

By being sustainable, a firm lowers its cost of capital from $r^U(n)$ to $r^S(n)$ by $r^U(n) - r^S(n)$. To enjoy this benefit, the firm spends $m(n)$ on mitigation. To make it indifferent between being sustainable and not, the cost-of-capital wedge is given by $r^U(n) - r^S(n) = m(n)/q(n)$, the ratio between the firm’s mitigation spending, $m(n)K$, and its market value, $q(n)K$.

5 First Best: Planner’s Solution

Before quantifying our model for the sustainable finance mandate economy, we first report the first-best solution where the planner chooses aggregate $C$, $I$, and $X$ to maximize the representative agent’s utility defined in (15)-(16).

As our model features the homogeneity property, it is convenient to work with scaled variables at both aggregate and individual levels. We use lower-case variables to denote the
corresponding upper-case variables divided by contemporaneous capital stock. For example, at the firm level, \( i_t = I_t/K_t, \phi_t = \Phi_t/K_t, \) and \( x_t = X_t/K_t \). Similarly, at the aggregate level, \( i_t = I_t/K_t, x_t = X_t/K_t \). For consumers, \( c_t = c_t = C_t/K_t \). 

Let \( V(K, N) \) denote the representative agent’s value function. As in Hong, Wang, and Yang (2020), the following Hamilton-Jacobi-Bellman (HJB) equation characterizes the planner’s optimization problem:

\[
0 = \max_{C, i, x} f(C, V) + \phi(i)KV + \omega(x/n)NV_N + \frac{K^2V_{KK} + 2NKV_{NK} + N^2V_{NN}}{2} + \lambda(n) \int_0^1 [V(ZK, ZN) - V(K, N)] \xi(Z)dZ ,
\]

subject to the following aggregate resource constraint at all \( t \):

\[
AK_t = C_t + i_tK_t + x_tK_t .
\]

The first-order condition (FOC) for the scaled investment \( i \) is

\[
f_C(C, V) = \phi'(i)V_K(K, N) .
\]

The first-order condition (FOC) for the scaled aggregate mitigation spending \( x \) is

\[
f_C(C, V) = \omega'(x/n)V_N(K, N) ,
\]

if the solution is strictly positive, \( x > 0 \). Otherwise, \( x = 0 \) as mitigation cannot be negative.

The representative agent’s value function takes the following homothetic form:

\[
V(K, N) = \frac{1}{1-\gamma} (b(n)K)^{1-\gamma} ,
\]

where \( b(n) \) is a function measuring the agent’s certainty-equivalent wealth and is endogenously determined.

Substituting (52) into the FOCs (50)-(51) and the HJB equation (48) and simplifying, we obtain the following two equations for optimal policies

\[
b(n)^{1-\psi^{-1}} = (A - i - x)^{-\psi^{-1}} \rho \left[ \frac{n}{\omega'(x/n)} + \frac{1}{\phi'(i)} \right] ,
\]

\[
\frac{b(n) - nb'(n)}{b'(n)} = \frac{\omega'(x/n)}{\phi'(i)} ,
\]
and the following ODE:

\[ 0 = \frac{\rho}{1 - \psi^{-1}} \left[ \frac{A - i - x}{\sigma^2} + 1 \right] + \phi(i) - \frac{\gamma \sigma^2}{2} + \frac{\omega(x/n) - \phi(i)}{1 + \frac{\omega'(x/n)}{n \phi'(i)}} + \frac{\lambda(n)}{1 - \gamma} \left[ \int_0^1 \xi(Z)Z^{1-\gamma} dZ - 1 \right]. \tag{55} \]

At the first-best steady state \( n^{FB} \), we have

\[ \omega(x^{FB}/n^{FB}) - \phi(i^{FB}) = 0, \tag{56} \]

and

\[ 0 = \frac{\rho}{1 - \psi^{-1}} \left[ \frac{A - i^{FB} - x^{FB}}{\sigma^2} + 1 \right] + \phi(i^{FB}) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(n^{FB})}{1 - \gamma} \left[ \int_0^1 \xi(Z)Z^{1-\gamma} dZ - 1 \right]. \tag{57} \]

**Summary.** The first-best steady state is an endogenously determined boundary that satisfies (53), (54), (56), and (57). For the transition path, we solve the ODE (55) together with the FOCs (53) and (54) subject to the boundary conditions for the steady state given above.

6 Quantitative Analysis

In this section, we operationalize our model. First, we specify various functional forms in our model. Second, we calibrate our model and choose parameter values based on business-as-usual projections of the damage of global warming to economic growth. Finally, we describe our quantitative results and findings.

6.1 Functional Form Specifications for the Model

Capital accumulation processes for \( K \) and \( N \). As in Pindyck and Wang (2013), we specify the investment-efficiency function \( \phi(i) \) as

\[ \phi(i) = i - \frac{\eta_K i^2}{2}, \tag{58} \]

where \( \eta_K \) measures the degree of adjustment costs.
We assume that the controlled drift function for decarbonization stock $N$ takes the same form as that for capital stock $K$:

$$
\omega(x/n) = (x/n) - \frac{\eta_N (x/n)^2}{2},
$$

where $\eta_N$ measures the degree of adjustment costs for decarbonization capital. Note that $x/n = X/N$ is the firm mitigation spending $X$ scaled by $N$, which is analogous to the firm’s investment scaled by capital stock: $i = I/K$.

**Conditional damage and disaster arrival rate.** We define disasters as events that cause the temperature to be 1.5° Celsius higher than the historical normal level. As in Barro (2006) and Pindyck and Wang (2013), we model the stochastic damage upon the arrival of a disaster by assuming that the stochastic recovery fraction, $Z \in (0, 1)$, of capital stock is governed by the following cdf:

$$
\Xi(Z) = Z^\beta,
$$

where $\beta > 0$ is a constant. To ensure that our model is well defined (and economically relevant moments are finite), we require $\beta > \max\{\gamma - 1, 0\}$. That is, the damage caused by a disaster follows a fat-tailed power-law function (Gabaix, 2009).

Decarbonization capital can ameliorate the damage of extreme weather to economic growth by reducing the frequencies of these extreme events. Specifically, we use the following specification for the disaster arrival rate $\lambda(n)$:

$$
\lambda(n) = \lambda_0(1 - n^{\lambda_1}),
$$

where $\lambda_0 > 0$, and $0 < \lambda_1 < 1$.

For a given $n$, the expected aggregate growth rate, $g$, is

$$
g = \phi(i) - \lambda(n)\mathbb{E}(1 - Z) = \phi(i) - \frac{\lambda(n)}{\beta + 1} = \phi(i) - \lambda(n)\ell.
$$

### 6.2 Baseline Calibration

Our calibration exercise is intended to highlight the importance of mitigation for welfare analysis. Our model has ten parameters in total. We summarize the values of these ten
parameters for our baseline analysis in Table 1.

Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>elasticity of intertemporal substitution</td>
<td>( \psi )</td>
<td>1.5</td>
</tr>
<tr>
<td>time rate of preference</td>
<td>( \rho )</td>
<td>5%</td>
</tr>
<tr>
<td>coefficient of relative risk aversion</td>
<td>( \gamma )</td>
<td>3</td>
</tr>
<tr>
<td>productivity for ( K )</td>
<td>( A )</td>
<td>14%</td>
</tr>
<tr>
<td>adjustment parameter for ( K )</td>
<td>( \eta_K )</td>
<td>9</td>
</tr>
<tr>
<td>diffusion volatility for ( N ) and ( K )</td>
<td>( \sigma )</td>
<td>14%</td>
</tr>
<tr>
<td>power-law exponent</td>
<td>( \beta )</td>
<td>24</td>
</tr>
<tr>
<td>jump arrival rate with no mitigation</td>
<td>( \lambda_0 )</td>
<td>1</td>
</tr>
<tr>
<td>adjustment cost parameter for ( N )</td>
<td>( \eta_N )</td>
<td>12</td>
</tr>
<tr>
<td>mitigation technology parameter</td>
<td>( \lambda_1 )</td>
<td>0.3</td>
</tr>
</tbody>
</table>

All parameter values, whenever applicable, are continuously compounded and annualized.

Preferences parameters. We choose consensus values for the coefficient of relative risk aversion, \( \gamma = 3 \), and the time rate of preferences, \( \rho = 5\% \) per annum. Estimates of the EIS \( \psi \) in the literature vary considerably, ranging from a low value near zero to values as high as two.\(^\text{11}\) We choose \( \psi = 1.5 \) which is larger than one, as in Bansal and Yaron (2004) and the long-run risk literature for asset-pricing purposes.

Parameters for productive capital accumulation process. We set the productivity parameter to \( A = 14\% \) per annum and the capital adjustment parameter \( \eta_K = 9 \) to primarily target an average \( q \) of 1.87 and an average growth rate of \( g = 3.8\% \) per annum in the pre-climate-change sample when the disaster arrival rate is low. The estimates of \( A \) and \( \eta_K \) are in the range of estimates reported in Eberly, Rebelo, and Vincent (2012). We set the annual

\(^{11}\)Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. Guvenen (2006) reconciles the conflicting evidence on the elasticity of intertemporal substitution from a macro perspective.
diffusion volatility at $\sigma = 14\%$ primarily to target a historical stock market risk premium of 6% per annum (Mehra and Prescott, 1985).

**Parameters for disaster arrival and conditional damage function.** We calibrate the parameter ($\lambda_0$) describing the arrival rate of extreme temperature disasters (i.e., abnormal temperatures above 1.5° Celsius) and damages conditioned on arrival ($\beta$) using a set of panel regressions documenting the adverse effects of exogenous annual changes in temperature (i.e., weather shocks) for economic growth (Dell, Jones, and Olken (2012)).

First, we calibrate $\beta$ as follows. For the median country, a 1.5° Celsius abnormal temperature over one year results in a 4% lower GDP growth rate. Conditional on a jump arrival, the expected fractional capital loss, $\ell$, is given by

$$\ell = \mathbb{E}(1 - Z) = \frac{1}{\beta + 1}.$$  

To match this moment, we set the power-law parameter to $\beta = 24$ as the implied expected fractional capital loss is $\ell = 1/(\beta + 1) = 1/25 = 4\%$. Second, using Dell, Jones, and Olken (2012), we infer that the jump arrival rate is $\lambda_0 = 0.05$ per annum in the pre-climate-change sample.

**Parameters for decarbonization capital adjustment and its benefits to mitigation.**

As discussed in the Introduction, reforestation has the potential to keep global temperatures from breaching the 1.5° Celsius barrier assuming that we can roughly double the size of forests. This corresponds to roughly adding 24 trillion dollars of forest capital stock. As the adjustment is likely to take two to three decades, we set $\eta_N = 12$ for our baseline analysis. We then calibrate the efficiency parameter of decarbonization capital stock ($\lambda_1$) in reducing the disaster arrival rate. Since there has only been negligible attempts at reforestation, we determine the value of $\lambda_1$ by using the planner’s FOCs for mitigation and investment by

---

12This panel regression approach initially focused on how weather affects crop yields (Schenkler and Roberts (2009)) by using location and time fixed effects. But it is now applied to many other contexts including economic growth and productivity. The main idea is that extreme annual temperature fluctuations are plausibly exogenous shocks that causally trace out the impact of higher temperatures on output. Burke, Hsiang, and Miguel (2015) find that the effects of temperature on growth is nonlinear. But we stay with the linear specification from Dell, Jones and Olken (2012) in this paper.
targeting a small amount of mitigation ($x = 0.003\%$) and a low level of scaled decarbonization stock ($n = 0.05\%$) in the data. Doing so yields a value of $\lambda_1 = 0.3$.

<table>
<thead>
<tr>
<th>variable</th>
<th>notation</th>
<th>$\lambda_0 = 0.05$</th>
<th>$\lambda_0 = 0.5$</th>
<th>$\lambda_0 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>scaled mitigation spending</td>
<td>$x$</td>
<td>0.003%</td>
<td>0.09%</td>
<td>0.23%</td>
</tr>
<tr>
<td>scaled decarbonization stock</td>
<td>$n$</td>
<td>0.05%</td>
<td>1.57%</td>
<td>4.59%</td>
</tr>
<tr>
<td>scaled aggregate investment</td>
<td>$i$</td>
<td>5.16%</td>
<td>4.73%</td>
<td>4.35%</td>
</tr>
<tr>
<td>Tobin’s $q$</td>
<td>$q$</td>
<td>1.869</td>
<td>1.740</td>
<td>1.644</td>
</tr>
<tr>
<td>scaled aggregate consumption</td>
<td>$c$</td>
<td>8.83%</td>
<td>9.19%</td>
<td>9.42%</td>
</tr>
<tr>
<td>expected GDP growth rate</td>
<td>$g$</td>
<td>3.78%</td>
<td>2.30%</td>
<td>1.09%</td>
</tr>
<tr>
<td>(real) risk-free rate</td>
<td>$r$</td>
<td>2.58%</td>
<td>1.27%</td>
<td>0.21%</td>
</tr>
<tr>
<td>stock market risk premium</td>
<td>$r^M - r$</td>
<td>5.93%</td>
<td>6.31%</td>
<td>6.60%</td>
</tr>
<tr>
<td>transition time from $n = 0$ to $n_{0.99}$</td>
<td>$t_{0.99}$</td>
<td>3.15</td>
<td>11.35</td>
<td>22.64</td>
</tr>
</tbody>
</table>

$n_{0.99}$ is the 99% level of that in the steady state: $n_{0.99} = 0.99 \times n^*$ and $t_{0.99}$ is the transition time from 0 to $n_{0.99}$.

The first column ($\lambda_0 = 0.05$) in Table 2 reports the pre-climate-change steady-state equilibrium when extreme temperature events are uncommon for the economy to which we calibrate our parameters. Targeting Tobin’s average $q$ at 1.87, the expected annual growth rate at $g = 3.8\%$, and the annual stock market risk premium at 5.93%, we obtain an annual risk-free rate of 2.6%, an investment-capital ratio of $i = 5.2\%$ per annum, and the aggregate consumption-capital ratio of $c = 8.8\%$ per annum.

### 6.3 Steady States Under (Welfare-Maximizing) Mandates

To analyze the effects of various key parameter values, we conduct comparative statics for the steady-state solution.

**Varying disaster arrival rate $\lambda_0$.** Now consider how the steady-state equilibrium outcomes change as we increase $\lambda_0$. We focus our discussion on the effect of increasing $\lambda_0$ from 5% to 1, which is the value of $\lambda_0$ for our baseline. Mitigation rises from $x = 0.003\%$ to 0.23% per annum. Since the physical capital stock is 600 trillion dollars, the aggregate

---

13The results for $\lambda_0 = 0.5$ are between those for the $\lambda = 5\%$ and $\lambda = 1$ scenarios.
contribution to decarbonization stock is about 1.4 trillion dollars per year. The ratio of decarbonization to physical capital stock $n$ is 4.6%, which means the aggregate decarbonization capital stock $N$ is about 28 trillion dollars.

As we mentioned in the Introduction, our model generates mitigation spending that is in line with projections for decarbonization using reforestation. Of course, our model can be applied to other forms of decarbonization capital accumulation such as direct carbon capture. We interpret our model through reforestation simply because it is currently the most feasible and well understood technology. Other forms of decarbonization methods are less mature and the costs of these methods are similar to if not more expensive than reforestation.\footnote{Gates (2021) also proposes that spending on net emission targets (NETs) needs to be around 1% of capital stock annually to address global warming. According to estimates from a McKinsey Sustainability report (de Pee (2018)), decarbonization of just the heavy industries that account for around 20% of the global carbon emissions will cost around 20 trillion dollars up to 2050 (or around 1 trillion dollars per year just for heavy sectors) to be net-zero.}

As a result of mitigation, aggregate investment is modestly lower at 4.35% as is Tobin’s $q$ at 1.644. The expected growth rate is still positive at 1.09% per annum as a result of mitigation, down from 3.78% absent global warming. The market risk premium increases from 5.93% to 6.60%, and the real risk-free rate falls to 0.21% from 2.6% per annum. From the time that mitigation starts, it takes 23 years to transition to the 99% of the steady state.

**Varying decarbonization capital adjustment parameter $\eta_N$.** Next, we examine in Table 3 how the equilibrium outcomes change as we change the decarbonization capital adjustment parameter $\eta_N$. The middle column ($\eta_N = 12$) summarizes our baseline case results. As we increase $\eta_N$ to 14 from 12, the annual mitigation spending $x$ falls to 0.10% and the steady-state decarbonization to physical capital ratio $n$ falls to 1.62% corresponding to a drop of $N$ to 9 trillion dollars. Aggregate investment, Tobin’s $q$, consumption $c$ are hardly changed from our baseline case. However, the expected growth rate $g$ decreases significantly to 0.62%. The reason is that the higher adjustment cost leads to lower levels of optimal mitigation and steady-state decarbonization capital. The risk-free rate now approaches zero and the stock market risk premium increased slightly. The transition time to the 99% of the steady state is now 38.07 years due to the higher adjustment cost.
Table 3: The effect of $\eta_N$ in the steady state under mandates.

<table>
<thead>
<tr>
<th>notation</th>
<th>$\eta_N = 9$</th>
<th>$\eta_N = 12$</th>
<th>$\eta_N = 14$</th>
</tr>
</thead>
<tbody>
<tr>
<td>scaled mitigation spending</td>
<td>x</td>
<td>0.33%</td>
<td>0.23%</td>
</tr>
<tr>
<td>scaled decarbonization stock</td>
<td>n</td>
<td>7.60%</td>
<td>4.59%</td>
</tr>
<tr>
<td>scaled aggregate investment</td>
<td>i</td>
<td>4.38%</td>
<td>4.35%</td>
</tr>
<tr>
<td>Tobin’s $q$</td>
<td>q</td>
<td>1.651</td>
<td>1.644</td>
</tr>
<tr>
<td>scaled aggregate consumption</td>
<td>c</td>
<td>9.28%</td>
<td>9.42%</td>
</tr>
<tr>
<td>expected GDP growth rate</td>
<td>g</td>
<td>1.36%</td>
<td>1.09%</td>
</tr>
<tr>
<td>(real) risk-free rate</td>
<td>$r$</td>
<td>0.46%</td>
<td>0.21%</td>
</tr>
<tr>
<td>stock market risk premium</td>
<td>$r^M - r$</td>
<td>6.52%</td>
<td>6.60%</td>
</tr>
<tr>
<td>transition time from $n = 0$ to $n_{0.99}$</td>
<td>$t_{0.99}$</td>
<td>16.73</td>
<td>22.64</td>
</tr>
</tbody>
</table>

$n_{0.99}$ is the 99% level of that in the steady state: $n_{0.99} = 0.99 \times n^*$ and $t_{0.99}$ is the transition time from 0 to $n_{0.99}$.

When we decrease $\eta_N$ to 9, the scaled mitigation spending increases to 0.33% and the steady state $n$ is higher at 7.60% since adjustment costs are lower. Again, investment, Tobin’s $q$, and consumption $c$ are hardly changed compared to our baseline case. The growth rate $g$ is higher as is the risk-free rate. Moreover, the transition time to the steady state falls only slightly to 16.73 years since the targeted steady-state decarbonization level $n$ is also higher. We view the $\eta_N$ results as being quite pertinent since adjustment costs of decarbonization capital has significant effects on the steady-state accumulation level and hence welfare outcomes.

### Varying damages from extreme temperatures $\ell = 1/(\beta+1)$ conditional on arrival.

Next, we consider in Table 4 how varying the conditional damage $\ell = 1/(\beta+1)$ changes equilibrium outcomes. Recall that $\beta$ measures the damage to the economy from extreme weather in the absence of mitigation. Our baseline of the conditional damage of $\ell = 4\%$ (as $\beta = 24$) is informed by existing empirical studies based on historical data on extreme temperature events. We are extrapolating from these estimates. Needless to say, there is bound to be uncertainty regarding these estimates. In this vein, it is instructive to consider values for $\beta$ that are higher and lower than our baseline. A higher $\beta = 49$ corresponds to less damage absent mitigation (about $\ell = 2\%$ conditioned on an arrival of an extreme $1.5^\circ$
Celsius event). A lower $\beta = 11.5$ corresponds to a higher conditional damage of $\ell = 8\%$.

Table 4: The effect of conditional damage $\ell = 1/(1 + \beta)$ in the steady state under mandates.

<table>
<thead>
<tr>
<th>notation</th>
<th>$\ell = 2%$</th>
<th>$\ell = 4%$</th>
<th>$\ell = 8%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>scaled mitigation spending</td>
<td>$x$</td>
<td>0.08%</td>
<td>0.23%</td>
</tr>
<tr>
<td>scaled decarbonization stock</td>
<td>$n$</td>
<td>1.41%</td>
<td>4.59%</td>
</tr>
<tr>
<td>scaled aggregate investment</td>
<td>$i$</td>
<td>4.75%</td>
<td>4.35%</td>
</tr>
<tr>
<td>Tobin’s $q$</td>
<td>$q$</td>
<td>1.747</td>
<td>1.644</td>
</tr>
<tr>
<td>aggregate consumption/dividends</td>
<td>$c$</td>
<td>9.17%</td>
<td>9.42%</td>
</tr>
<tr>
<td>expected GDP growth rate</td>
<td>$g$</td>
<td>2.29%</td>
<td>1.09%</td>
</tr>
<tr>
<td>(real) risk-free rate</td>
<td>$r$</td>
<td>1.47%</td>
<td>0.21%</td>
</tr>
<tr>
<td>stock market risk premium</td>
<td>$r^{M} - r$</td>
<td>6.07%</td>
<td>6.60%</td>
</tr>
<tr>
<td>transition time from $n = 0$</td>
<td>$t_{0.99}$</td>
<td>9.58</td>
<td>22.64</td>
</tr>
</tbody>
</table>

$n_{0.99}$ is the 99% level of that in the steady state: $n_{0.99} = 0.99 \times n^*$ and $t_{0.99}$ is the transition time from 0 to $n_{0.99}$.

When $\ell = 2\%$, the optimal annual mitigation spending $x$ is lower at 0.08% and so is the steady-state $n$ at 1.41%, though these are still quite significant amounts. Again, the effects on investment, Tobin’s $q$, and consumption are limited. The expected growth rate $g$ is now surprisingly higher at 2.29% and so is the risk-free rate (at 1.47%) even though mitigation $x$ and decarbonization capital $n$ are much lower in equilibrium. The transition to the 99% of the steady state is now much faster (it takes about 9.58 years.)

When $\ell = 9\%$ (as $\beta = 11.5$), optimal mitigation $x$ increases significantly to 0.67% per annum and the steady-state $n$ is now 16.32%. The growth rate $g$ is negative at -0.27% and the risk-free rate is significantly more negative at -2.35%. It also takes much longer to reach the 99% of the steady state at 74.30 years.

Overall, our quantitative analysis suggests that our main conclusions are fairly robust across a number of perturbations. The one outcome that is very sensitive is transition time to the steady state, which depends on $\eta_N$ and the severity of the conditional damage $\ell$. A conservative estimate of a higher adjustment cost for decarbonization capital suggests that it may take up to 38 years to reach steady state should the economy start today to accumulate decarbonization capital stock. In this vein, 2030 net-zero targets widely thrown around by
policy markers seem unrealistic since this decarbonization drive has not even begun.

### 6.4 Transition Dynamics and Comparison to Planner’s Outcomes

In this subsection, we discuss the transition dynamics. We also highlight the extent to which welfare-maximizing mandates (Section 3.8) can attain the planner’s first best outcomes (Section 5).

![Graphs showing transition dynamics](image)

**Figure 1:** This figure plots the aggregate mitigation spending $x$, aggregate investment $i$, aggregate consumption $c$ and the aggregate welfare measure $b$ as functions of the scaled decarbonization capital stock $n$. The parameters values are reported in Table 1.

**Mitigation, consumption, investment and welfare gains under mandates versus planner’s solution.** In Figure 1, we examine the transition dynamics for the optimal mitigation $x$, investment $i$, consumption $c$ and certainty-equivalent welfare measure $b$. All these aggregates are dependent on the underlying state variable $n$ — the ratio of decarbonization
capital to physical capital. For all four panels, the blue lines indicate the optimal mandate solution and the red lines describe the planner’s solution.

Panel A shows that the two solutions track each other closely up to the level of $n = 4.59\%$. The planner’s solution peaks at a higher value and is also higher in the steady state. That is, the mitigation spending under optimal mandate is only materially below the first-best when $n$ is sufficiently high. This is intuitive as the marginal return of mitigation is quite high when $n$ is not too high.

Panel B shows that investment $i$ is higher under the mandate than the planner’s solution, whereas Panel C shows that consumption $c$ is lower under the mandate than the planner’s solution. Note that this is to a large extent expected as the sum of $i$, $c$, and $x$ is the constant productivity $A$. As risk mitigation is a public good, $n = 0$ is the competitive markets outcome with no mandate. Therefore, mandates move all three of these policies from the market solution towards the planner’s solution. However, the mandate solution does not track well the planner’s solution.

Nonetheless, as Panel D shows, the welfare measure $b(n)$ for the mandate solution is quite close to that for the planner’s solution. This is good news for the usefulness of mandates in incentivizing firms to reform and contribute to decarbonization. In fact, the welfare gains are massive. Under the unmitigated competitive market solution, the certainty equivalent wealth measure $b(0)$ is 0.0498. The certainty equivalent at the steady state is 0.0627 under the mandate’s solution and 0.0638 under the planner’s solution. We thus obtain a 26% gain in welfare with an optimal mandate than without. The magnitudes are large as the world (based on the estimates we use from the literature) at 1.5$^\circ$ Celsius with no mitigation is dismal.

**Extreme temperature event arrivals $\lambda(n)$ and the aggregate growth rate $g(n)$**. In Figure 2, we examine how the extreme temperature event arrival rate $\lambda(\cdot)$ and the expected aggregate growth rate $g(\cdot)$ vary with $n$. Panel A shows that the disaster arrival rate $\lambda(\cdot)$ falls with $n$, as we expect. In competitive markets (and hence $n = 0$), the arrival rate is $\lambda(0) = 1$ at 1.5$^\circ$ Celsius with no mitigation. As the society builds up the decarbonization
capital, the disaster arrival rate falls and approaches around 0.6 per annum at the steady state: \( \lambda(n^*) = 0.6 \) where \( n^* = 4.59\% \).

The high jump arrival rate in competitive markets implies that growth \( g(0) \) is low at around -0.42\%. Note that this result depends on the assumptions of our production economy absent mitigation in the pre-climate-change period. (We have chosen a modest productivity scenario where the expected growth rate is about 4\%.) Hence, a 1.5\(^\circ\) Celsius world if unmitigated will lead to low growth. As the society accumulates the decarbonization capital, the growth rate increases. For the mandate solution, the expected growth rate at the steady-state is positive at 1.09\% per annum as we have discussed above. The planner’s steady-state growth rate is lower.

The difference of growth rates between the two solutions is because the planner solution emphasizes de-risking in building up more decarbonization capital at the expense of investing in productive capital. Despite a lower growth rate, the welfare is higher in the planner’s solution because the planner fully fixes the under-provision of risk mitigation (a public good).

Next, in Figure 3, we plot the transition path of \( n_t \) over time \( t \). We see that the society reaches a higher steady state under the planner’s solution than under the mandate.
6.5 Asset Prices

In Figure 4, we report how key asset pricing variables, the risk-free rate $r$, the stock market risk premium $r_p$, and Tobin’s average $q$, vary with $n$. At $n = 0$, the unmitigated competitive market equilibrium has a negative interest rate of around -1.1%, a market risk premium of around 7% per annum, and Tobin’s $q$ of 1.58. As $n$ increases, the risk-free rate increases, the risk premium falls, and Tobin’s $q$ rises.

In Figure 5, we analyze the costs of accumulating decarbonization capital to firms and investors. We consider three investment mandate levels: $\alpha = 0.2, 0.5, 0.8$. For these three levels of $\alpha$, our mandate solution can all be implemented. Naturally when $\alpha$ is lower, firms need to spend more to qualify for the sustainable portfolio but they get compensated with a larger cost-of-capital wedge in equilibrium.

The blue line depicts the solution when 20% of wealth is indexed to sustainable mandates ($\alpha = 0.2$). The qualifying standard $m$ increases with $n$, peaking at 1.15% per annum at the
Figure 4: This figure plots the equilibrium interest rate $r$, stock market risk premium $rp$, and Tobin’s $q$ as functions of the scaled decarbonization capital stock $n$. The parameters values are reported in Table 1.

Figure 5: This figure plots the mitigation spending mandate $m(n)$ and the cost-of-capital wedge $r^n_U(n) - r^n_S(n)$ as functions of scaled decarbonization capital stock $n$. The parameters values are reported in Table 1.

steady state. That is, a firm would need to spend 1.15% of its capital on decarbonization to qualify for the sustainable portfolio at the steady state. The sustainable firms get compensated for their contributions with a significant cost-of-capital wedge $r^n_U(n) - r^n_S(n)$ of over 0.7% per annum in at the steady state.
Interestingly, the optimal ramp-up schedules of both \( m(n) \) and cost-of-capital wedge \( r^U(n) - r^S(n) \) are non-linear. As we increase \( \alpha \) from 0.2 to 0.5 (the grey dotted line) and 0.8 (the red dotted line), the qualification standard falls and so do the cost-of-capital wedges. The non-linearity discussed above remains however. Current estimates have sustainable finance mandates \( \alpha \) in the range of 20\% to 50\%. Moreover, our model clearly predicts that as decarbonization \( n \) ramps up, qualification standards start rising and the cost to investors also rise.

### 6.6 Connecting to Net-zero Emissions Targets

Finally, we show that our calibration implies that a sustainable finance solution based on reforestation can meet a significant fraction of net-zero emissions targets. First, \( e = E/K \) in equation (4) is approximately 0.067 since net emissions is about 40 billion metric tons and there is 600 trillion dollars of aggregate productive capital stock \( K \). Second, \( \tau = N/R \) in equation (5) is 0.42=10/24. We obtain this value by dividing the forests’ aggregate net absorption of 10 billion metric tons of carbon per annum by the book value of forests in aggregate (24 trillion dollars). To achieve net-zero emissions, we need to target \( n = 0.12 = 0.067/0.42 - 0.04 \).\(^ {15} \) The steady-state \( n \) from reforestation alone in our baseline is 4.59\%. Hence, sustainable finance mandates can get us about 40\% of the way towards net-zero through the funding of reforestation pathways. Of course, society also needs to pursue other strategies to get the rest of the way there.

### 7 Conclusion

Sustainable finance mandates have grown significantly in the last decade in lieu of government failures to address climate disaster externalities. Firms that spend enough resources on mitigation of these externalities qualify for sustainable finance mandates. These mandates incentivize otherwise ex-ante identical unsustainable firms to become sustainable for a lower cost of capital. We present and solve a dynamic stochastic general equilibrium model fea-

\(^ {15} \)We subtract 0.04, which is the current value of \( n_0 = 24/600 = 0.04 \), the ratio of current stock of forests (24 trillion dollars) divided by the current stock of capital stock (600 trillion dollars.)
turing the gradual accumulation of nonproductive but protective decarbonization capital to study the welfare consequences. The model is highly tractable, including a simple formula that characterizes the cost-of-capital wedge between sustainable and unsustainable firms as the tax rate on firm value to subsidize mitigation. There are a number of testable implications that can be taken to the data. The model is also useful for quantitative analysis of both transition dynamics and the steady state.
References


Gates, B., 2021. *How to Avoid a Climate Disaster: The Solutions We Have and the Breakthroughs We Need*. Knopf.


Appendices

A Firm Value Maximization

Using the standard dynamic programming, we obtain the following HJB equation for $q^j$ given aggregate decarbonization stock $n$, aggregate mitigation spending $x$, and aggregate investment $i$:

\[
r^j(n)Q^j(K^j, n) = \max_{i', x'} AK^j - I^j - X^j + \left( \Phi(I^j, K^j)Q^j_K(K^j, n) + \frac{1}{2}(\sigma K^j)^2 Q^j_K(K^j, n) \right) + [\omega(x/n) - \phi(i)] nQ^n_j(K^j, n) + \lambda(n)E^n \left[ Q^j(ZK^j, n) - Q^j(K^j, n) \right]. \tag{A.64}
\]

And then substituting $Q^j(K^j, n) = q^j(n)K^j$ into (A.64), we obtain

\[
r^j(n)q^j(n) = \max_{i', x'} A - i^j - x^j + \phi(i^j)q^j(n) + \lambda [E^n(Z) - 1] q^j(n) + [\omega(x/n) - \phi(i)] nq'^j(n). \tag{A.65}
\]

The FOC for investment implied by (A.65) is

\[
q^j(n) = \frac{1}{\phi'(i^j)}, \tag{A.66}
\]

which is the standard Tobin’s $q$ formula (e.g., Lucas and Prescott, 1971; Hayashi, 1982). As $x^U \geq 0$ and $x^S \geq m$, the optimal mitigation spending is $x^U = 0$ for a type-$U$ firm and $x^S = m$ for a type-$S$ firm as no firm wants to spend more than it has to on mitigation.

As all firms have the same Tobin’s $q$ in equilibrium, we have $i^S(n) = i^U(n) = i(n)$ and

\[
q(n) = \frac{A - i - m + [\omega(x/n) - \phi(i)] nq'(n)}{r^S - g(i)} = \frac{A - i + [\omega(x/n) - \phi(i)] nq'(n)}{r^U - g(i)}. \tag{A.67}
\]

As in the steady state, $\omega(x^*/n^*) - \phi(i^*) = 0$, we have

\[
q(n^*) = \frac{A - i^* - m}{r^S - g(i^*)} = \frac{A - i^*}{r^U - g(i^*)}. \tag{A.68}
\]

B Household’s Optimization Problem

Using the same procedure as in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we can show that both the optimal risk-free asset holding and the jump hedging demand for all levels of $Z$ are zero in equilibrium. Therefore, we may rewrite the household’s wealth dynamics given by (18) as follows

\[
dW_t = W_{t-} \left[ \left( r(n_{t-}) + r^S(n_{t-}) - r(n_{t-}) \right) \pi^S_{t-} + \left( r^U(n_{t-}) - r(n_{t-}) \right) (1 - \pi^S_{t-}) \right] dt + \sigma dB_t
\]

\[-W_{t-} [(1 - Z)(dJ_t - \lambda(n_{t-})dt)] - C_{t-} dt, \tag{B.69}
\]

where $\pi^S = H^S/(H^S + H^U) = H^S/W$. 

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The post-jump wealth is \( W^J = W - (1 - Z)W = ZW \). And by using the standard dynamic programming method, we may use the following HJB equation to characterize \( J(W, n) \):

\[
0 = \max_{C, \pi^S} \left[ r(n)W + \left( (r^S(n) - r(n))\pi^S + (r^U(n) - r(n))(1 - \pi^S) + \lambda(n) \right) (1 - \mathbb{E}(Z)) \right] W - C \ 
+ \ 
+ f(C, V) + [\omega(x/n) - \phi(i)] nV_n(W, n) + \frac{\sigma^2 W^2 V_{WW}(W, n)}{2} + \lambda(n) \int_0^1 [V (ZW, n) - V(W, n)] \xi(Z) dZ,
\]

subject to \( \pi^S \geq \alpha \). And the FOC for consumption \( C \) is the standard condition given by (29). Because the \( S \)- and the \( U \)-portfolio have exactly the same (diffusion and jump) risk exposures with probability one, the optimality for \( \pi^S \) is positive infinity if \( r^S > r^U \) as we can see from (28). This is not an equilibrium. In equilibrium, \( r^S \leq r^U \) and \( \pi^S = \alpha \) holds, which implies the agent’s value function satisfies the HJB equation (28). We later pin down the equilibrium relation between \( r^S \) and \( r^U \).

Let \( V_t = V(W_t, n_t) \) denote the household’s value function. We show that

\[
V(W, n) = \frac{1}{1 - \gamma} (u(n)W)^{1-\gamma}, \tag{B.71}
\]

where \( u(n) \) is determined endogenously. Substituting (B.71) into the FOC (29) yields the following linear consumption rule:

\[
C(W, n) = \rho^\psi u(n)^{1-\psi} W. \tag{B.72}
\]

## C Market Equilibrium

First, a sustainable firm spends minimally on mitigation: \( x^S = \frac{x^S}{K^S} \). Second, in equilibrium, the household invests all wealth in the stock market and holds no risk-free asset, \( H = W \) and \( W = Q^S + Q^U \), and has zero disaster hedging position, \( \delta(Z, n) = 0 \) for all \( Z \). Third, the representative agent’s (dollar amount) investment in the \( S \) portfolio is equal to the total market value of sustainable firms, \( \pi^S = \alpha \) and (dollar amount) investment for the \( U \) portfolio is equal to the total market value of unsustainable firms, \( \pi^U = 1 - \alpha \). Finally, goods market clears.

By using the preceding equilibrium conditions together with \( H = W = Q^S + Q^U = q^S(n)K^S + q^U(n)K^U = q(n)(K^S + K^U) = q(n)K, \) \( W^J = ZW \) and \( \pi^S = \alpha \), we obtain

\[
\alpha r^S(n) + (1 - \alpha)r^U(n) = r^M(n) = r^M(n) + \gamma \rho^2 \sigma^2 + \lambda(n) \mathbb{E} [(1 - Z)(Z^{-\gamma} - 1)] = r^M(n). \tag{C.73}
\]

Using \( \alpha r^S(n) + (1 - \alpha)r^U(n) = r^M(n), \) \( x = \alpha m(n), \) and (A.67), we obtain

\[
A - i - x + [\omega(x/n) - \phi(i)] nq'(n) \] 
\[
\frac{r^M(n) - g(i)}{r^M(n) - g(i) + (1 - \alpha)(r^U(n) - g(i))} = q(n). \tag{C.74}
\]
we obtain the risk-free rate is which implies (36).

And then substituting into (C.77) and combining we obtain (A – i + [ω(x/n) – φ(i)] nq'(n))θU(n) = x(rU(n) – g(i)) and θU(n) = x/q(n) = αm(n)/q(n) as shown in (45).

In addition, the optimal consumption rule given in (B.72) implies

\[ c(n) = \frac{C}{K} = \frac{C}{W} q(n) = \rho^\psi u(n)^{1-\psi} q(n). \] (C.76)

And then substituting c given by (C.76) and the value function given in (B.71) into the HJB equation (28), we obtain

\[ 0 = \frac{1}{1-\psi^{-1}} \left( \frac{c(n)}{q(n)} - \rho \right) + \left( \alpha r^S(n) + (1 - \alpha) r^U(n) - \frac{c(n)}{q(n)} + \lambda(n)(1 - \mathbb{E}(Z)) \right) \]

\[ + [\omega(x/n) - \phi(i)] \frac{nu'(n)}{u(n)} - \frac{\gamma \sigma^2}{2} + \frac{\lambda(n)}{1-\gamma} \left[ \mathbb{E}(Z^{1-\gamma}) - 1 \right] \]

\[ = \frac{1}{1-\psi^{-1}} \left( \frac{c(n)}{q(n)} - \rho \right) + \left( r^M(n) - \frac{c(n)}{q(n)} + \lambda(n)(1 - \mathbb{E}(Z)) \right) \]

\[ + [\omega(x/n) - \phi(i)] \frac{nu'(n)}{u(n)} - \frac{\gamma \sigma^2}{2} + \frac{\lambda(n)}{1-\gamma} \left[ \mathbb{E}(Z^{1-\gamma}) - 1 \right]. \] (C.77)

By using (C.75) and the goods market clear condition, we obtain

\[ \frac{c(n)}{q(n)} = r^M(n) - g(i) - [\omega(x/n) - \phi(i)] \frac{nq'(n)}{q(n)}. \] (C.78)

And then by substituting it into (C.77) and combining c(n) = A – i(n) – x(n), we have

\[ 0 = \frac{1}{1-\psi^{-1}} \left( \frac{A - i(n) - x(n)}{q(n)} - \rho \right) + \phi(i(n)) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(n)}{1-\gamma} \left[ \mathbb{E}(Z^{1-\gamma}) - 1 \right] \]

\[ + [\omega(x/n) - \phi(i)] \left( \frac{nq'(n)}{q(n)} + \frac{nu'(n)}{u(n)} \right) \]

\[ 0 = \frac{1}{1-\psi^{-1}} \left( \frac{A - i(n) - x(n)}{q(n)} - \rho \right) + \phi(i(n)) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(n)}{1-\gamma} \left[ \mathbb{E}(Z^{1-\gamma}) - 1 \right] \]

\[ + [\omega(x/n) - \phi(i)] \left( \frac{\psi}{1-\psi} \frac{nq'(n)}{q(n)} - \frac{1}{1-\psi} \frac{ni'(n) + nx'(n)}{\psi A - i(n) - x(n)} \right), \] (C.79)

which implies (36).

And then by substituting it into (C.77) and combining r^M(n) = r(n) + γσ^2 + λ(n)E[(1 - Z)(Z^{-γ} - 1)], we obtain the risk-free rate is

\[ r(n) = \frac{c(n)}{q(n)} + \phi(i(n)) + [\omega(x/n) - \phi(i(n))] \frac{nq'(n)}{q(n)} - \frac{\gamma \sigma^2}{2} - \lambda(n)E \left[ (1 - Z)Z^{-\gamma} \right]. \] (C.80)

In addition, under the steady state the risk-free rate satisfies

\[ r(n^*) = \rho + \psi^{-1} \phi(i(n^*)) - \frac{\gamma(\psi^{-1} + 1)\sigma^2}{2} - \lambda(n^*)E \left[ (Z^{-\gamma} - 1) + (\psi^{-1} - \gamma) \left( \frac{1 - Z^{1-\gamma}}{1-\gamma} \right) \right]. \] (C.81)
**Welfare-maximizing mandate.** As the household’s value function is given by (B.71) and $W = q(n)K$ in equilibrium, we may write

$$V = \frac{1}{1-\gamma} (u(n)W)^{1-\gamma} = \frac{1}{1-\gamma} (u(n) \times q(n)K)^{1-\gamma} = \frac{1}{1-\gamma} (b(n)K)^{1-\gamma}, \quad (C.82)$$

where $b(n) = q(n) \times u(n)$ is proportional to the certainty equivalent wealth (welfare) per unit capital. And then substituting $b(n) = q(n) \times u(n)$ into (C.79) and using $c(n) = \frac{C}{K} = \frac{C}{W}q(n) = \rho^\psi u(n)^{1-\psi}q(n)$, we obtain the ODE given in (38) for $b(n)$. Immediately, by following ODE (38) we have the FOC for $x$ to maximize welfare $b(n)$ is given by (39). And then substituting the good market clearing conditions $c(n) = A - i(n) - x(n)$ under equilibrium into the optimal consumption rule (C.76), and recalling $b(n) = q(n) \times u(n)$ we obtain the optimal investment satisfying (40). Finally, the welfare-maximizing mandate is given by (33) for given $\alpha$ by following the welfare-maximizing mitigation obtained above.