Network Competition with Heterogeneous Customers and Calling Patterns *

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Abstract

The telecommunications industry is a fragmented market, characterized by a tremendous amount of customer heterogeneity. This paper shows how such customer heterogeneity dramatically affects nonlinear pricing strategies: (i) First, if there are unbalanced calling patterns between different customer types, networks make larger profits on the least attractive customers. In addition, the nature of the calling pattern substantially affects how networks discriminate implicitly between different customer types. (ii) Secondly, different customer types often perceive the substitutability of competing networks differently. The profit neutrality of the access charge, highlighted in the literature, is then shown to break down.

Keywords: *Telecommunications, Interconnection, Unbalanced Calling Patterns, Two-way Access, Competition Policy.*

JEL codes: D4, K21, L43, L51, L96.

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1 Introduction

The telecommunications industry is a fragmented market with a large number of customer segments, typically characterized by different volume demands for calls. Incumbent operators, for example, have in general at least three customer divisions, respectively focussing on the residential, the business and the corporate sector. Moreover, inside these customer categories, especially the residential segment, demand may differ tremendously. A standard assumption in the literature on competition between interconnected (telecommunications) networks and two-way access, started by Armstrong (1998) and Laffont, Rey and Tirole (1998a,b),[1] is that the calling pattern is *balanced*: for equal prices, flows in and out of a network are balanced — even if market shares are not. Heterogeneity in outgoing demand does not rule out balanced calling patterns: customers not only differ in outgoing calls, they also receive different volumes of incoming calls and if there is a perfect correlation, there will be no net flows between different segments for equal prices. This, however, is not observed in reality: although customers who call a lot (*'heavy users'*) effectively tend to receive more calls than people who call only moderately (*'light users'*), evidence indicates that call flows between different customer categories are often considerably unbalanced. In confidential data on aggregate calling patterns in a European country, business or corporate customers called during peak time 10% more to residential customers than the other way round. We denote the latter case, where light users tend to be called up more than they call, by *a light biased calling pattern.* Surprisingly, the opposite held for call flows between (small) business firms and (large) corporate firms. In our data, business firms called 20% more to corporate firms than vice versa. Similarly, off peak, residential customers have a net outflow of calls of the same order to the corporate and business segment. These are cases where heavy users tend to receive more calls than they originate, which we denote by *a heavy biased calling pattern.*

To incorporate these features of the industry, this paper generalizes the basic model of competition between interconnected networks, as developed in Laffont, Rey and Tirole (1998a,b) (LRT hereafter), and shows how operators optimally adjust their strategies and pricing schedules in response to such *heterogeneous calling patterns.* LRT present a duopoly model in which customers must decide which network to join and given this choice, how much to call. For each call a customer makes to someone subscribed to the rival network,

[1] Other papers in this literature include Carter and Wright (1999,2003), Gans and King (2001), DeGraba (2003), Jeon, Laffont and Tirole (2004), Peitz (2003) and Cambini and Valetti (2003). See Armstrong (2002) for an excellent survey.

an operator pays a — regulated or negotiated — access charge to his rival. It is assumed that reciprocal access pricing, that is the equality of the interconnect prices charged by the two networks, is mandated. Whereas in LRT, customers are identical, we explicitly model differences in outgoing calls by assuming that customers are either *heavy* (call a lot) or *light users.* Moreover, to capture the wide variety of calling patterns observed in reality, we allow for *heavy biased, light biased* and *balanced calling patterns.*

In the context of the above model, our companion paper, Dessein (2003), has shown that under certain conditions, profits are independent of the access charge, regardless of the nature of the calling pattern. This paper shows that unbalanced calling patterns nevertheless dramatically change the way networks compete for customers:

First, if networks can discriminate explicitly between customers, equilibrium tariffs reflect the *opportunity cost of not subscribing a particular customer.* Intuitively, customers which generate access revenues are not only profitable to have, they are also very costly *not to have,* since they then join the rival network and generate access deficits. As a result, we show that equilibrium tariffs for *access revenue generating customers* reflect a discount which is twice the value of the access revenues they generate. Similarly, equilibrium tariffs for *access deficit generating customers* reflect these access deficits *and* the lost access revenues these customers would have generated by joining the rival network. A corollary is that networks earn higher profits — including access revenues — on customers generating an access deficit than on those yielding access revenues.

Secondly, we show that calling patterns affect considerably *the way networks implicitly discriminate between customers of different types.* Similar to results in Armstrong and Vickers (2001) and Rochet and Stole (2002), we find that for an access charge equal to marginal cost, a simple two-part tariff offered to both heavy and light users allows for perfect discrimination in equilibrium. No incentive constraints are binding then. In contrast to Armstrong-Vickers and Rochet-Stole, however, we show that once the access charge is large (or small) enough, a menu of tariffs cannot mimic the explicit price discrimination outcome. Since the tariff which a customer pays depends to a large extent on the net outflow or inflow generated by him, calling patterns have an important impact on which incentive constraint exactly binds in equilibrium. For a given access charge, depending on the calling pattern, the incentive constraint of the light users will be binding, the incentive constraint of the heavy users will be binding or the equilibrium may be the same as with explicit price discrimination.

We conclude the paper by pointing to some *limits to the profit neutrality of the access* *charge*, highlighted in Dessein (2003).[2] Customer heterogeneity in outgoing volume demand is not only correlated with differences in incoming call volume, but typically as well with differences in how customers perceive competing networks. In particular, different customer types are likely to perceive the substitutability of the networks differently as they have different switching costs, different brand loyalty or a differentiated access to publicity and information about the networks. We show that when networks are seen as better substitutes by the heavy users than by the light users, networks obtain higher profits by agreeing on an access charge *below* marginal cost. In the opposite case, an access charge *above* marginal cost may boost profits.

This paper is organized as follows. Section 2 describes our model of heavy and light users. Section 3 investigates competition under explicit price discrimination. Section 4 considers price competition under implicit price discrimination. Section 5, finally, considers optimal pricing strategies if there is customer heterogeneity in perceived substitutability between networks. All proofs are provided in Appendix.

2 A model of heavy and light users

We consider the competition between two horizontally differentiated networks. The main elements are as follows:

Cost structure: The two networks have the same cost structure. Serving a customer involves a fixed cost f. Per call, a network also incurs a marginal cost c_o at the originating and terminating ends of the call and a marginal cost c_1 in between. The total marginal cost is thus

$$c = 2c_o + c_1 \tag{1}$$

Demand structure: The networks are differentiated à la Hotelling. Consumers are uniformly located on the segment $[0, 1]$ and networks are located at the two extremities, namely at $x_1 = 0$ and $x_2 = 1$. Given income y and telephone consumption q, a type $k-$consumer located at x joining network i has utility:

$$y + u_k(q) + v_o - \tau |x - x_i| \tag{2}$$

where v_o represents a fixed surplus from being connected[3], $\tau |x - x_i|$ denotes the cost of not being connected to its "most preferred" network, and the variable *gross surplus,* $u_k(q)$, is given

[2] Dessein (2003) shows, however, that this profit neutrality crucially depends on full customer participation. In particular, an access charge below marginal cost increases profits if there is limited customer participation.

[3] We will assume throughout the paper that v_o is "large enough", so that all consumers are connected in equilibrium.

by:

$$u_k(q) = \frac{k^{\frac{1}{\eta}} q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \tag{3}$$

Faced with a usage fee p, a customer consumes thus a quantity q_k given by

$$u_k'(q_k) = p \Leftrightarrow q_k = kp^{-\eta} \equiv kq(p) \tag{4}$$

Throughout the paper, we will say that the *usage fee is p if customers consume a quantity $q_k = kq(p)$.*

We consider two different customer types or customer segments:

- *light users*, fraction μ of the market, characterized by $k = k_L$.
- *heavy users*, fraction $1-\mu$ of the market, characterized by $k = k_H > k_L$.

The distribution of customers on the segment $[0,1]$ is assumed to be independent of their type k.

Calling patterns: We suppose that a fraction ℓ of calls terminates on the light user segment, where ℓ is independent of the type of customer who originates the call. As a benchmark, we are interested in the case where ℓ is such that the calling pattern is balanced:

Definition 1 *A calling pattern is balanced whenever for equal usage fees, each customer calls as much as he is being called.*

With homogeneous customers, this is realized very naturally by assuming that all customers receive the same amount of calls ($\ell = \mu$). With heterogeneous customers, a different assumption is needed:

Lemma 1 *A calling pattern is* balanced *if and only if* $\ell = \dfrac{\mu k_L}{\mu k_L + (1-\mu)k_H}$

Given that customers differ in their volume demand, the assumption of a balanced calling pattern is quite strong and often violated in reality. We therefore allow ℓ to be different from $\mu k_L / k$, which yields two types of unbalanced calling patterns:

Definition 2 *A calling pattern is:*

- Light biased *if* $\ell > \dfrac{\mu k_L}{\mu k_L + (1-\mu)k_H}$:
  Light users then receive more calls than they originate for equal prices.
- Heavy biased *if* $\ell < \dfrac{\mu k_L}{\mu k_L + (1-\mu)k_H}$
  Heavy users then receive more calls than they originate for equal prices.

3 Explicit Price Discrimination

We first consider competition when networks can discriminate explicitly between heavy and light users (third-degree price discrimination).[4] The next section then considers the more realistic case in which only implicit discrimination (second-degree price discrimination) is allowed.

Under explicit price discrimination, each network offers light users a quantity q_L for a tariff t_L and heavy users a quantity q_H for a tariff t_H. The variable net surplus of respectively a light user and a heavy user is thus

$$w_L \equiv u_{k_L}(q_L) - t_L \quad \textit{and} \quad w_H \equiv u_{k_H}(q_H) - t_H \tag{5}$$

For given net surpluses (w_L, w_H) and (w'_L, w'_H) offered by network 1 and 2, the market shares α_L and α_H of network 1 in respectively the light users' and the heavy users' segment are determined as in Hotelling's model. A consumer of type s ($s = L, H$) located at $x = \alpha_s$ is indifferent between the two networks if and only if

$$w_s - \tau\alpha_s = w'_s - \tau(1 - \alpha_s), \tag{6}$$

or

$$\alpha_s = \alpha(w_s, w'_s) \equiv \frac{1}{2} + \sigma\left[w_s - w'_s\right] \tag{7}$$

where

$$\sigma \equiv \frac{1}{2\tau} \tag{8}$$

is an index of substitutability between the two networks. Given our assumptions about the calling pattern, the *share in incoming calls* of network 1 is

$$\alpha^{IN} = \alpha_L\ell + \alpha_H(1 - \ell) \tag{9}$$

Let a denote the unit access charge to be paid for interconnection by a network to its competitor. Network 1's profits are

$$\begin{aligned} \pi \;=\; & \mu\alpha_L\left[t_L - \left(c + (1 - \alpha^{IN})(a - c_o)\right)q_L - f\right] + \\ & (1 - \mu)\alpha_H\left[t_H - \left(c + (1 - \alpha^{IN})(a - c_o)\right)q_H - f\right] + \\ & \alpha^{IN}\left[\mu(1 - \alpha_L)q'_L + (1 - \mu)(1 - \alpha_H)q'_H\right](a - c_o) \end{aligned} \tag{10}$$

These profits can be decomposed into a *retail profit*

$$\mu\alpha_L\left[t_L - cq_L - f\right] + (1 - \mu)\alpha_H\left[t_H - cq_H - f\right] \tag{11}$$

[4] For a related model of competition with explicit price discrimination, see De Bijl and Peitz (2003) Ch. 7.

which would be made if all calls terminated on net, plus an *access revenue*

$$A = \alpha^{IN}\left[\mu(1-\alpha_L)q'_L + (1-\mu)(1-\alpha_H)q'_H\right](a-c_o) - (1-\alpha^{IN})\left[\mu\alpha_L q_L + (1-\mu)\alpha_H q_H\right](a-c_o). \tag{12}$$

It will be useful to denote by A_L and A_H the access revenues respectively per light user and per heavy user:

$$A = \alpha_L A_L + \alpha_H A_H \tag{13}$$

The next proposition shows that whereas unbalanced calling patterns do not alter networks' aggregate profits (as discussed in Dessein (2003)), they do affect the way networks compete for customers. In particular, equilibrium tariffs reflect the *opportunity cost of not subscribing a particular customer*. With unbalanced calling patterns, this implies that profits made on heavy users differ from those on light users. Perhaps surprisingly, *higher* profits (including access revenues or deficits) are then made on customers who generate access deficits ($A_s < 0$).

Proposition 1 *i) In a symmetric equilibrium, profits are independent of both the access charge and the calling pattern, and are equal to* $1/4\sigma$.
ii) Equilibrium quantities reflect an implicit usage fee $p = c + \frac{a-c_o}{2}$, *whereas equilibrium tariffs are given by*

$$\hat{t}_L \equiv 1/2\sigma + f + c\hat{q}_L - 2A_L, \tag{14}$$

$$\hat{t}_H \equiv 1/2\sigma + f + c\hat{q}_H - 2A_H, \tag{15}$$

Per customer profits (including access revenues) are higher on consumers who generate an access deficit ($A_s < 0$)*:*

$$\pi_s \equiv \hat{t}_s + A_s - f - c\hat{q}_s = 1/2\sigma - A_s \qquad s = L, H \tag{16}$$

We first provide a sketch of the proof, followed by an intuitive explanation. Competition in tariffs is very similar to the one in a symmetric Hotelling model with unit demands, where the good offered to customers is a subscription. In the Hotelling model, the tariff must trade-off between maximizing (retail-)profits per customer and market share. In the case of competition between networks, firms face the same trade-off, but must also take into account the impact of the tariff on access revenues. Consider the benefits of a firm to increase its market share on the heavy user segment by ε. The cost of the tariff cut needed to achieve this equals $(1-\mu)\alpha_H\varepsilon/\sigma$. The benefits are the extra retail profits, $\varepsilon(1-\mu)R_H$, plus the change in access revenues $\varepsilon\partial A/\partial\alpha_H$. In equilibrium, these costs and benefits must exactly cancel out, hence retail profits per heavy user must satisfy

$$R_H = \frac{\alpha_H}{\sigma} - \frac{1}{1-\mu}\frac{\partial A}{\partial \alpha_H} \tag{17}$$

In a symmetric equilibrium $\alpha^{IN} = \alpha = 1/2$, from which

$$\frac{\partial A}{\partial \alpha_H} = \left[(1-\ell)\left(\mu q_L + (1-\mu)q_H\right) - (1-\mu)q_H\right](a - c_o) = 2(1-\mu)A_H \tag{18}$$

As a result, equilibrium profits *including* access revenues on a customer equal $1/2\sigma$ *minus* the access revenues made on this customer:

$$\pi_H \equiv R_H + A_H = \frac{1}{2\sigma} - A_H \tag{19}$$

Intuitively, equilibrium tariffs reflect the profits associated with a particular customer as well as the *opportunity cost of not subscribing that customer*. Suppose $A_H < 0$, then by subscribing a heavy user which generates a net access deficit, the network not only pays more access contributions to its rival (for an amount of A_H), he also foregoes the access contributions which he would have received if this heavy user had subscribed to his rival instead. The total economic cost in terms of access revenues is thus twice the access deficit generated by that heavy user. Similarly, if $A_H > 0$, a heavy user is worth twice the access revenues he procures. In a symmetric equilibrium, these 'access' costs (benefits) are completely passed on to the customer, who thus pays (is rewarded) twice for his contribution to the access deficit (revenues).

Because of the interconnection agreement between networks, there are no network externalities in our setting. Nevertheless, the above results have an interesting parallel with the literature on competition in industries with network externalities.[5] Indeed, it is a standard result that firms competing in a Hotelling setting earn less profit in equilibrium from customers who generate a positive network effect. Despite the absence of network externalities, a similar result thus arises in our model whenever interconnection is not priced at cost.

From proposition 1, calling patterns do not affect profits, that is the *average* tariff which customers pay is independent of the calling pattern. In contrast, calling patterns greatly affect what tariff a particular customer *type* pays. Obviously, if networks cannot explicitly

[5] I am grateful to a referee for pointing this out.

discriminate between different customer types, this will have an important impact on the incentive compatibility of the proposed tariffs. In particular, the more unbalanced a calling pattern, the more customers which generate access deficits will be tempted to choose calling plans designated for customers generating access revenues. The need to implicitly discriminate between customers of different types therefore puts a limit on how much customers can be charged or rewarded for generating an access deficit or revenue. The next section analyzes how implicit price discrimination affects our results.

4 Implicit price discrimination

If networks are not allowed to price discriminate explicitly according to whether a customer is a heavy or a light user (that is third-degree price discrimination is ruled out), the proposed menu of tariffs $\{q_L, t_L, q_H, t_H\}$ must be such that heavy users opt for (q_H, t_H) and light users choose (q_L, t_L). The incentive constraints (IC) are

$$w_H = u_{k_H}(q_H) - t_H \geq u_{k_H}(q_L) - t_L \quad (IC_H)$$

$$w_L = u_{k_L}(q_L) - t_L \geq u_{k_L}(q_H) - t_H \quad (IC_L)$$

We first consider pricing strategies when calling patterns are balanced. We subsequently discuss the impact of unbalanced calling patterns.

4.1 Balanced Calling patterns

If $a = c_o$, the quantities offered under explicit price discrimination are those generated by usage fees set at marginal cost, while tariffs are so that profits per customer equal $1/2\sigma$. It is easy to see that this tariff structure is incentive compatible; it can for example be implemented by a unique two-part tariff, $t(q) = pq + F$, in which $p = c$ and $F = 1/2\sigma + f$. This is in line with the results of Armstrong and Vickers (2001) and Rochet and Stole (2002), which show that simple two-part tariffs often arise in competitive environments where consumers have private information about their tastes.[6]

Since IC_H and IC_L are both satisfied with strict inequality whenever $k_L \neq k_H$ and thus $\hat{q}_L \neq \hat{q}_H$, the explicit discrimination equilibrium is still incentive compatible for a close to c_o. The equilibrium contract $\left\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\right\}$, however, then cannot be implemented anymore

[6] Rochet and Stole (2002) show, however, that this result is sensitive to the assumption that the customer's type is uncorrelated with the consumers location on the Hotelling line and that all consumer types are willing to participate with the candidate tariffs.

through two-part tariff(s). In order for customers to chose the optimal quantities $\hat{q}_L$ and $\hat{q}_H$, the usage fee for both types must be set at $p_L = p_H = c + \frac{a-c_o}{2}$; to implement $\hat{t}_L$ and $\hat{t}_H$, however, a different fixed fee is needed for heavy and light users, which, of course, is not incentive compatible.

In contrast, for a large or k_L close to k_H, $\left\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\right\}$ is no longer incentive compatible. Indeed, from Proposition 1, the explicit price discrimination equilibrium is given by $\hat{q}_s = k_s \hat{q}$ and $\hat{t}_s = \hat{t}(\hat{q}_s)$, $(s = L, H)$, where

$$\hat{q} = q(c + \tfrac{a-c_o}{2}) \tag{20}$$

and, since $A_L = A_H = 0$ with a balanced calling pattern and symmetric usage fees,

$$\hat{t}(q) = 1/2\sigma + f + cq. \tag{21}$$

Given (21), the net utility of a customer of type k is concave in q and reaches a maximum for $q = kq(c)$. From (20), an increase in the access charge lowers both $\hat{q}_L$ and $\hat{q}_H$ and, for large enough an access charge, $\hat{q}_L < \hat{q}_H \leq k_L q(c)$. It follows that given any k_L and k_H, for large enough an access charge, light users then strictly prefer $(\hat{q}_H, \hat{t}_H)$ to $(\hat{q}_L, \hat{t}_L)$ such that the IC of the *light users* is violated. Similarly, for a given access charge $a > c_o$, we have that $\hat{q}_L < k_L q(c)$ such that there exist always exists a value δ such that for $k_H - k_L < \delta$,

$$\hat{q}_L < \hat{q}_H = \frac{k_H}{k_L} \hat{q}_L \leq k_L q(c), \tag{22}$$

and again the IC of the light users is violated. The polar case is obtained for $a < c_o$: for a small enough or k_L close enough to k_H, $k_H q(c) \leq \hat{q}_L < \hat{q}_H$ and the IC of the *heavy users* is violated.

Intuitively, the IC which is violated under explicit discrimination, will be binding in the equilibrium under implicit discrimination. The next lemma guarantees this for $\delta = k_H - k_L$ small.

Lemma 2 *If the explicit price discrimination equilibrium,* $\left\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\right\}$ *violates the incentive constraint of the light (heavy) users, then for* $\delta = k_H - k_L$ *small, any symmetric equilibrium* $\{t_L^*, q_L^*, t_H^*, q_H^*\}$ *under implicit price discrimination is such that the incentive constraint of the light (heavy) users is binding.*

The next proposition characterizes equilibrium pricing when calling patterns are balanced. As shown by Laffont, Rey and Tirole (1998a), when the access charge is too large, no equilibrium exists in pure strategies. We therefore restricts ourselves to access charge for which an equilibrium in pure strategies does exist. Secondly, we assume that light users and heavy users are not too different such that it is optimal for both networks to serve both customer types in equilibrium.

Proposition 2 *i) In a symmetric equilibrium, profits are equal to* $1/4\sigma$*, irrespective of the access charge.*
ii) Fix the average customer type k *and let the difference* $\delta = k_H - k_L$ *vary. For any* δ_o*, there exists an access charge* $a_o > c_o$ *such that a symmetric equilibrium always exists for* $a \leq a_o$ *and* $\delta \leq \delta_o$*. Moreover:*
- For $a = c_o$*, the equilibrium menu of tariffs is equivalent to each network offering a simple two-part tariff. Whenever* a *differs from* c_o*, however, a simple two-part tariff is never an equilibrium outcome.*
- For any given $\delta \leq \delta_o$*, for* a *close to* c_o*, incentive constraints are nonbinding and the equilibrium is the same as if networks could explicitly discriminate between heavy and light users.*
- For any given $a \in \left]c_o, a_o\right]$*, for* δ *close to* 0*, the incentive constraint of the light users is binding. Compared to the equilibrium with explicit price discrimination: (1) the quantity (and tariff) offered to heavy users is higher, (2) a lower tariff for an unchanged quantity is charged to light users, (3) profits per light user are smaller than profits per heavy user.*

As mentioned previously, Armstrong and Vickers (2001) and Rochet and Stole (2002) have shown that in simple discrete choice frameworks, such as a Hotelling model with full participation, competitive price discrimination results in firms offering one simple two-part tariff. Proposition 2 shows that in the case of network competition, this result breaks down whenever the access charge differs from the marginal cost of access.

Intuitively, when firms compete in two-part tariffs, an access charge above marginal cost results in equilibrium usage prices above marginal cost, increasing the variable profits per customer. Competition for market share, however, induces networks to fully pass on these excess variable profits to customers in the form of a lower fixed fee. Indeed, profits equal $1/2\sigma$ per customer, independently of the access charge. Since variable profits are much higher on heavy users than on light users, this implies that the implicit 'discount' which heavy users receive on their fixed fee is much larger than the one received by light users. As a result, light users are then tempted to choose a tariff destined for heavy users.

For small access charge distortions, the explicit price discrimination outcome may still be implemented by a menu of tariffs (which is not a two-part tariff, however). If the difference between the access charge and the marginal cost of access is larger, however, the IC of light users will be binding in equilibrium. As we show in appendix, the symmetric equilibrium is then characterized by $\{q_L^*, t_L^*, q_H^*, t_H^*\}$, where

$$q_L^* = \hat{q}_L; \quad t_L^* < \hat{t}_L \tag{23}$$

$$q_H^* > \hat{q}_H; \quad t_H^* > \hat{t}_H + c(q_H^* - \hat{q}_H). \tag{24}$$

By increasing q_H and $(t_H - t_L)$, networks make $\{\hat{q}_H, \hat{t}_H\}$ less attractive to light users. Note that by providing a larger quantity to heavy users than is optimal given the perceived marginal cost, networks eliminate partly the distortion induced by this inflated perceived marginal cost. This is in contrast with standard results of nonlinear pricing where implicit price discrimination lowers the offered quantity and reduces efficiency. Interestingly, compared to the explicit price discrimination equilibrium, a higher surplus is left to the light users. While this lowers profits made on these customers, average profits are nevertheless unaffected, as the loss is exactly compensated by higher profits on the heavy users.

4.2 Unbalanced Calling patterns

While unbalanced calling patterns do not alter networks' aggregate profits (as discussed in Dessein (2003)), they do affect the way networks compete for customers. First, as shown in Proposition 1, calling patterns affect how much profits networks earn on a particular customer type. Second, as the next proposition shows, the calling pattern affects whether and which incentive constraints are binding in equilibrium.

Proposition 3 *i) In a symmetric equilibrium, profits are independent of both the access charge and the calling pattern, and are equal to* $1/4\sigma$.
ii) Fix the average customer type k *and let the difference* $\delta = k_H - k_L$ *vary. For any* δ_o, *there exists an access charge* $a_o > c_o$ *such that a symmetric equilibrium always exists for* $a \leq a_o$, $0 \leq \delta \leq \delta_o$ *and* $\ell \in [0, 1]$. *Moreover*
a) Given $\delta \in]0, \delta_o]$, *for* a *close to* c_o *and/or* ℓ *close to* $\frac{1}{2}\left(\frac{\mu k_L}{k} + \mu\right)$, *incentive constraints are nonbinding and the equilibrium is the same as if networks could explicitly discriminate between heavy and light users.*
b) Given $a \in]c_o, a_o]$, *for* δ *close* 0,
- the IC *of the light users is binding if* $\ell < \frac{1}{2}\left[\frac{\mu k_L}{k} + \mu\right]$, *that is when the calling pattern is heavy biased, balanced or slightly light biased.*

- *the IC of the heavy users is binding if* $\ell > \frac{1}{2}\left[\frac{\mu k_L}{k} + \mu\right]$, *that is when the calling pattern is substantially light biased calling pattern.*

We first provide a sketch of the proof, followed by an intuitive argument. Incentive conditions for $\left\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\right\}$ can be rewritten as

$$u_L(\hat{q}_L) - u_L(\hat{q}_H) \geq \hat{t}_L - \hat{t}_H \qquad (IC_L) \tag{25}$$

$$u_H(\hat{q}_L) - u_H(\hat{q}_H) \leq \hat{t}_L - \hat{t}_H \qquad (IC_H) \tag{26}$$

with IC_L and IC_H the incentive constraints of respectively the light and the heavy users. Compared to a balanced calling pattern, a *light biased calling pattern* decreases $\hat{t}_L - \hat{t}_H$ for $a > c_o$, as heavy (light) users pay (are rewarded) for their contribution to the access deficit (revenues). An access markup then has two opposite effects on incentive conditions. One is due to the inflated marginal cost $c + (a - c_o)/2$, which we analyzed in the previous section and makes $\left\{\hat{q}_L, \hat{t}_L\right\}$ relatively less attractive compared to $\left\{\hat{q}_H, \hat{t}_H\right\}$, and another, due to the access premium heavy users pay and light users receive, which decreases $\hat{t}_L - \hat{t}_H$. For $\ell = \frac{1}{2}\left[\mu k_L/k + \mu\right]$, the two effects exactly cancel out and IC_L and IC_H are strictly satisfied for any access charge. With a slightly light biased calling pattern, that is if $\ell < \frac{1}{2}\left[\mu k_L/k + \mu\right]$, IC_L will be violated for $\delta = k_H - k_L$ small. If, on the other hand, $\ell > \frac{1}{2}\left[\mu k_L/k + \mu\right]$, that is if the calling pattern is substantially light biased, IC_H will be violated for δ small. In each case, the IC violated by $\left\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\right\}$, is binding in the implicit price discrimination equilibrium. In contrast with this, a *heavy biased calling pattern* increases $\hat{t}_L - \hat{t}_H$ for $a > c_o$ so that the access markup has an unambiguous effect on incentive conditions: it makes $\left\{\hat{q}_L, \hat{t}_L\right\}$ relatively less attractive compared to $\left\{\hat{q}_H, \hat{t}_H\right\}$ so that the IC of the light users is always binding for δ small. Appendix 6.2 gives a characterization of the equilibrium when an incentive constraint is binding.

Intuitively, from proposition 1, calling patterns do not affect profits, that is the *average* tariff which customers pay is independent of the calling pattern. In contrast, we have shown that under explicit price discrimination, calling patterns greatly affect which tariff a particular customer type pays. Obviously, if network cannot explicitly discriminate between different customer types, this will have an important impact on the incentive compatibility of menus of tariffs. In particular, the more imbalanced a calling pattern, the more customers which generate access deficits will be tempted to choose calling plans destined for customers generating access revenues. Hence, if light users generate access revenues, this will make it more difficult to satisfy the incentive constraint of heavy users, as networks then would like to offer very favorable terms to light users. Note that this dramatically changes pricing strategies compared to a balanced calling pattern. Indeed, in the latter case, the main concern is that light users choose the tariff destined for heavy users.

Note that, as was the case under explicit price discrimination, networks tend to make less profits on customers generating access revenues than on customers generating access revenues (this is always ttue if the calling pattern is light-biased). The only difference is that the need to implicitly discriminate between customers of different types puts a limit on how much customers can be charged or rewarded for generating an access deficit or revenue.

5 Heterogeneity in network substitutability

In the previous analysis, the access charge substantially affected the pricing strategies of networks. As already demonstrated in Dessein (2003), however, the need to discriminate implicitly between different customer types does not affect average profits. In this section, we show that this result crucially relies on the assumption that heavy and light users perceive the substitutability of the competing networks in the same way.

Suppose therefore that customers types differ in the way they perceive the substitutability of the networks. Such different perceived substitutabilities can correspond to *different brand loyalties, different search costs, a differentiated access to product information or publicity, different switching costs.* We denote these perceived substitutabilities for light and heavy users respectively by σ_L and σ_H. To get some insight in how this affects pricing strategies, we consider the most simple scenario where calling patterns are balanced. If networks could discriminate explicitly, they would offer quantities that reflect the perceived marginal cost

$$\hat{q}_L = k_L q(c + \frac{a - c_o}{2}), \quad \hat{q}_H = k_H q(c + \frac{a - c_o}{2}) \tag{27}$$

and charge a tariff $t(q_s)$ such that profits per customer are $1/2\sigma_s$, $(s = L, H)$:

$$\hat{t}(\hat{q}_L) = 1/2\sigma_L + f + c\hat{q}_L \tag{28}$$

$$\hat{t}(\hat{q}_H) = 1/2\sigma_H + f + c\hat{q}_H \tag{29}$$

Total profits under explicit price discrimination, denoted by π_D^*, are thus independent of the access charge and are equal to

$$\pi_D^* = \frac{\mu}{4\sigma_L} + \frac{1-\mu}{4\sigma_H}. \tag{30}$$

If explicit price discrimination is not possible, networks still can discriminate implicitly through a menu of tariffs. A first consequence of $\sigma_L \neq \sigma_H$, is that incentive compatibility conditions may affect profits. Intuitively, if an IC is binding, networks must deviate from their best responses under explicit discrimination in order to meet incentive constraints. As only the difference between t_L and t_H matters, and profits are concave in t_L and t_H, networks deviate most on the segment where profits are least concave. Since profits are least concave on the segment with the smallest substitutability (market shares react much faster when the substitutability is stronger), networks deviate most on the segment where σ is smallest. Depending on whether the needed deviation on that particular segment is a tariff cut or a tariff raise, equilibrium profits are then either lower or higher than under explicit price discrimination. In case, for example, the IC of the light users is binding, networks must decrease $t_H - t_L$. If $\sigma_H > \sigma_L$, networks find it optimal to decrease more t_L than they increase t_H so that profits are lower than under explicit price discrimination. The opposite holds if $\sigma_H < \sigma_L$. In particular, denoting profits under implicit price discrimination by π^*, we show in appendix that

$$\pi^* < \pi_D^* \text{ when } \sigma_H > \sigma_L \quad \text{and} \quad \pi^* > \pi_D^* \text{ when } \sigma_H < \sigma_L \tag{31}$$

if the IC of the *light users* is binding, and

$$\pi^* > \pi_D^* \text{ when } \sigma_H > \sigma_L \quad \text{and} \quad \pi^* < \pi_D^* \text{ when } \sigma_H < \sigma_L \tag{32}$$

if the IC of the *heavy users* is binding.

As the *access charge affects incentive compatibility,* a second implication of $\sigma_L \neq \sigma_H$, is that the access charge may also affect profits. Suppose first that $a = c_o$. If Δk is relatively small compared to $|\sigma_H - \sigma_L|$, then the IC of the customers with the smallest perceived substitutability will be violated in the explicit price discrimination equilibrium: from (27),(28) and (29), the difference in equilibrium quantities is then small relative to the difference in equilibrium tariffs. From lemma 2, if Δk is small, the same IC is binding in the implicit price discrimination equilibrium. From (28) and (29), if $\sigma_H > \sigma_L$, the IC of the light users is thus binding for Δk small, from which $\pi^* < \pi_D^*$. Similarly, if $\sigma_H < \sigma_L$, the IC of the heavy users is binding for Δk small, from which also $\pi^* < \pi_D^*$. Of course, if Δk is relatively large, networks can perfectly discriminate and $\pi^* = \pi_D^*$.

Consider now $a \neq c_o$. In the explicit price discrimination equilibrium, customers of type k are offered a quantity $\hat{q}_k = kq(c + \frac{a-c_o}{2})$ for a tariff $\hat{t}_k = 1/2\sigma_k + f + c\hat{q}_k$. Defining

$$V_k(q) \equiv u_k(q) - cq, \quad \text{and} \quad F_k \equiv 1/2\sigma_k + f, \tag{33}$$

the IC of light and heavy users can be rewritten as

$$V_L(\hat{q}_L) - V_L(\hat{q}_H) > F_L - F_H \qquad (IC_L) \tag{34}$$

$$V_H(\hat{q}_L) - V_H(\hat{q}_H) < F_L - F_H \qquad (IC_H) \tag{35}$$

Note that $V_k(q)$ is strictly concave and reaches a maximum for $q = kq(c)$.

We distinguish the impact of a positive and a negative access markup:

(1) A negative access markup ($a < c_o$) has a clear impact on IC_L and IC_H. When $c + \frac{a-c_o}{2}$ goes to zero, $\hat{q}_H - \hat{q}_L$ tends to infinity such that $V_s(\hat{q}_L) - V_s(\hat{q}_H)$, $(s = L, H)$ increases without a bound when a gets smaller. For a small enough, IC_L is then always satisfied, while IC_H is violated. A negative access markup thus strengthens the IC_H and relaxes IC_L. Intuitively, the larger the quantities offered in equilibrium (due to the low access charge), the more everybody likes the smallest offered quantity: the incremental gross utility of consuming $\hat{q}_H$ instead of $\hat{q}_L$, given by $u(\hat{q}_H) - u(\hat{q}_L)$, decreases more and more compared to its incremental cost, $F_H - F_L + c(\hat{q}_H - \hat{q}_L)$. It follows that for $\sigma_H > \sigma_L$ and Δk small, a sufficiently negative access markup increases profits: IC_H is then binding so that $\pi^* > \pi_D^*$, while for $a = c_o$, the IC_L is binding and thus $\pi^* < \pi_D^*$.

(2) A positive access markup ($a > c_o$), on the other hand, always decreases $V_s(\hat{q}_L) - V_s(\hat{q}_H)$ $(s = L, H)$, though not without limits: $V_s(\hat{q}_L) - V_s(\hat{q}_H)$ $(s = L, H)$ reaches a negative underbound for some $a_H^* > c_o$ and $a_L^* > c_o$. For $|\sigma_H - \sigma_L|$, and thus also $|F_H - F_L|$ small enough, there exists then an $a > c_o$, such that IC_L is violated by the explicit price discrimination equilibrium. If follows that for $\sigma_H < \sigma_L$, IC_L is binding so that $\pi^* > \pi_D^*$.

Proposition 4 *i) Suppose $a = c_o$:*

- Given $|\sigma_H - \sigma_L|$, for Δk sufficiently small, the incentive constraint of customers with the smallest perceived substitutability is binding. Equilibrium profits π^* are then smaller than π_D^*.

- Given Δk, for $|\sigma_H - \sigma_L|$ sufficiently small, no incentive constraints are binding and $\pi^* = \pi_D^*$.

ii) The access charge may affect profits:

- For a sufficiently negative access markup ($a < c_o$) and Δk small, the incentive constraint of the heavy users is binding. If $\sigma_H > \sigma_L$, then $\pi^* > \pi_D^*$: a sufficiently negative access markup increases profits. If $\sigma_L < \sigma_H$, then $\pi^* < \pi_D^*$.

- For a given positive access markup ($a' > c_o$) and Δk small, if $|\sigma_H - \sigma_L|$ is sufficiently small, the incentive constraint of the light users is binding. If $\sigma_H < \sigma_L$, then $\pi^* > \pi_D^*$: $a' > c_o$ boosts profits relative to $a = c_o$. If $\sigma_H > \sigma_L$, $\pi^* < \pi_D^*$ for $a' > c_o$.

In Dessein (2003), we have shown under which conditions equilibrium profits are independent of the access charge, even when customers have private information about their preferences for volume demand and networks engage is second degree price discrimination. As argued in Dessein (2003), however, this profit neutrality is necessarily a knife-edge result. For example, in the presence of an elastic subscription demand, the access charge does affect profits. Similarly, Proposition 4 shows how the impact of the access charge on profits crucially depends on how customers differ in the way they perceive the substitutability of competing networks. This result further illustrates the rich array of outcomes that arise from competition between interconnected networks under customer heterogeneity.

6 Appendix

6.1 Explicit price discrimination

Proof of Proposition 1:

We only proof part (ii), the proof of (i) is provided in Dessein (2003). Since market shares only depend on the variable net surplus, it is convenient to view competition as one in which networks pick quantities (q_H, q_L) and net surpluses (w_H, w_L) rather than quantities and tariffs (t_H, t_L). Profits are then

$$\begin{aligned} \pi \;=\; & \mu\alpha_L \left[\tfrac{\eta}{\eta-1} k_L^{1/\eta} q_L^{1-1/\eta} - w_L - cq_L - f\right] \\ & +(1-\mu)\alpha_H \left[\tfrac{\eta}{\eta-1} k_H^{1/\eta} q_H^{1-1/\eta} - w_H - cq_H - f\right] \\ & - \left[\mu\alpha_L q_L + (1-\mu)\alpha_H q_H\right]\left[\ell(1-\alpha_L) + (1-\ell)(1-\alpha_H)\right](a - c_o) \\ & + \left[\mu(1-\alpha_L)q_L' + (1-\mu)(1-\alpha_L)q_H'\right]\left[\ell\alpha_L + (1-\ell)\alpha_H\right](a - c_o) \end{aligned} \tag{36}$$

We are looking for a symmetric equilibrium. For $a = c_o$, network i' s profits on the customer segment s are strictly concave in $\{q_s, w_s\}^7$. As a result, total profits given k_L, k_H are strictly concave in $\{q_L, w_L, q_H, w_H\}$ for $a = c_o$ and the Hessian matrix $D^2\pi(q_L, w_L, q_H, w_H)$ is negative semidefinite for $a = c_o$. Fix the average customer type k. As all terms of $D^2\pi(q_L, w_L, q_H, w_H)$ are continuous in k_L, k_H, ℓ and a, then for any $\delta_o = k_H - k_L$, one can find an access charge $a_o > c_o$ such that $D^2\pi(q_L, w_L, q_H, w_H)$ is still negative semidefinite and thus profits are strictly concave, for $c_o \leq a \leq a_o$, $0 \leq k_H - k_L \leq \delta_o$ and $\ell \in [0,1]$. A candidate equilibrium satisfying the FOC is then effectively an equilibrium. From the FOC with respect to q_L and q_H, equilibrium marginal fees are equal to perceived marginal costs, $\hat{p} = c + \frac{a-c_o}{2}$, leading to equilibrium quantities $\hat{q}_s = k_s q(c + \frac{a-c_o}{2})$, $(s = L, H)$. From the FOC with respect to w_L and w_H, equilibrium tariffs are given by

$$\hat{t}_L = 1/2\sigma + f + c\hat{q}_L + \frac{\mu\hat{q}_L - \ell\left[\mu\hat{q}_L + (1-\mu)\hat{q}_H\right]}{\mu}(a - c_o) \quad (37)$$

$$= 1/2\sigma + f + c\hat{q}_L - 2A_L \quad (38)$$

$$\hat{t}_H = 1/2\sigma + f + c\hat{q}_H - \frac{\mu\hat{q}_L - \ell\left[\mu\hat{q}_L + (1-\mu)\hat{q}_H\right]}{1-\mu}(a - c_o) \quad (39)$$

$$= 1/2\sigma + f + c\hat{q}_L - 2A_H \quad (40)$$

with A_L and A_H the access revenues respectively per light and per heavy user. It follows that profits per light user, respectively heavy user, are given by

$$\hat{\pi}_L = \hat{t}_L + A_L - f - c\hat{q}_L = 1/2\sigma - A_L \quad (41)$$

$$\hat{\pi}_H = \hat{t}_H + A_H - f - c\hat{q}_H = 1/2\sigma - A_H \quad (42)$$

6.2 Implicit price discrimination.

Proof of Lemma 2:

As we make also use of lemma 2 in the proof of proposition *4*, we provide a proof for the more general case in which σ_H may differ from σ_L, although we restrict ourselves to the case in which $a > c_o$, the case $a < c_o$ being similar. As symmetric equilibria in which networks serve only one segment can easily be ruled out for k_L close to k_H, the first order conditions must be satisfied. For $\lambda_L = \lambda_H = 0$, it follows from the FOC that the symmetric equilibrium is uniquely defined and given by $\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\}$, as characterized in the proof of proposition *1*. Consequently, if $\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\}$ is not incentive compatible and if a symmetric equilibrium exists, at least one incentive constraint is strictly binding: $\lambda_H > 0$ and/or $\lambda_L > 0$. We characterize the incentive compatible symmetric equilibrium contract, $\{t_L^*, q_L^*, t_H^*, q_H^*\}$. The first order condition with respect to q_L and q_H yield

$$\mu\alpha_L\left[k_L^{1/\eta}q_L^{*\,-1/\eta} - (c + \tfrac{a-c_o}{2})\right] - \lambda_H\left[k_H^{1/\eta} - k_L^{1/\eta}\right]q_L^{*\,-1/\eta} = 0 \quad (43)$$

$$(1-\mu)\alpha_H\left[k_H^{1/\eta}q_H^{*\,-1/\eta} - (c + \tfrac{a-c_o}{2})\right] + \lambda_L\left[k_H^{1/\eta} - k_L^{1/\eta}\right]q_H^{*\,-1/\eta} = 0 \quad (44)$$

with λ_H and/or λ_L strictly positive. Incentive constraints can never be binding at the same time: setting both IC's at equality and subtracting yields $q_H^* = q_L^* = q^*$; however, from (43) and (44), one must have then that

$$\left[k_H^{1/\eta} - k_L^{1/\eta}\right](q^*)^{-1/\eta} + \left(\frac{\lambda_L}{(1-\mu)\alpha_H} + \frac{\lambda_H}{\mu\alpha_L}\right)\left[k_H^{1/\eta} - k_L^{1/\eta}\right](q^*)^{-1/\eta} = 0 \quad (45)$$

which is impossible if both λ_L and λ_H are positive. Thus either $(\lambda_H > 0, \lambda_L = 0)$ or $(\lambda_H = 0, \lambda_L > 0)$. The proof can now be constructed by contradiction: $(\lambda_H = 0, \lambda_L > 0)$ holds if $(\lambda_H > 0, \lambda_L = 0)$ is impossible and the other way round. We distinguish two case:

a) The incentive constraint of the light users is violated in the explicit price discrimination equilibrium.

Suppose that $\{w_L^*, q_L^*, w_H^*, q_H^*\}$ is then such that the IC of the heavy users is binding, thus $\lambda_H > 0$ and $\lambda_L = 0$. We then have

$$w_H^* - w_L^* = \tfrac{\eta}{\eta-1}\left[k_H^{1/\eta} - k_L^{1/\eta}\right] q_L^{*\,1-1/\eta} \tag{46}$$

while from the FOC with respect to w_L and w_H,

$$\left[\tfrac{\eta}{\eta-1} k_L^{1/\eta} q_L^{*\,1-1/\eta} - w_L^* - cq_L^* - f\right] + \frac{(a-c_o)}{\mu} S = \lambda_H/\mu\sigma_L + 1/2\sigma_L \tag{47}$$

$$\left[\tfrac{\eta}{\eta-1} k_H^{1/\eta} q_H^{*\,1-1/\eta} - w_H^* - cq_H^* - f\right] - \frac{(a-c_o)}{1-\mu} S = -\lambda_H/(1-\mu)\sigma_H + 1/2\sigma_H \tag{48}$$

where S is the net inflow of calls in the light user's segment:

$$S = S(\lambda_L, \lambda_H) \equiv (\ell\,[\mu q_L^* + (1-\mu) q_H^*] - \mu q_L^*) \tag{49}$$

Subtracting (47) and (48) and substituting (46), we find

$$\tfrac{\eta}{n-1} k_H^{1/\eta}\left[q_H^{*\,1-1/\eta} - q_L^{*\,1-1/\eta}\right] - c(q_H^* - q_L^*) \tag{50}$$

$$= \frac{\sigma_L - \sigma_H}{2\sigma_H\sigma_L} + (a-c_o)\frac{S(\lambda_L,\lambda_H)}{\mu(1-\mu)} - \lambda_H/(1-\mu)\sigma_H - \lambda_H/\mu\sigma_L \tag{51}$$

For $\delta = k_H - k_L$ small, $S(\lambda_L, \lambda_H)$ can be approximated by a Taylor expansion. As for $\lambda_H > 0$ and $\lambda_L = 0$,

$$q_H^* = \hat{q}_H \quad \text{and} \quad q_L^* = \hat{q}_L\left(1 - \frac{2\lambda_H}{\mu}\frac{\left[k_H^{1/\eta} - k_L^{1/\eta}\right]}{k_L^{1/\eta}}\right)^{\eta}, \tag{52}$$

we find after some computations that

$$S(\lambda_L,\lambda_H) \cong \delta\left(\frac{\partial S(\lambda_L,\lambda_H)}{\partial\delta}_{|\delta=0,\lambda_H=\lambda_H^*}\right) = 2\delta\lambda_H(1-\mu)(c + \tfrac{a-c_o}{2})^{-\eta} \tag{53}$$

It follows that for δ or $a - c_o$ small, $\lambda_H/(1-\mu)\sigma_H + \lambda_H/\mu\sigma_L - (a-c_o)\frac{S(\lambda_L,\lambda_H)}{\mu(1-\mu)} > 0$, so that

$$\tfrac{\eta}{n-1} k_H^{1/\eta}\left[q_H^{*\,1-1/\eta} - q_L^{*\,1-1/\eta}\right] - c(q_H^* - q_L^*) < \frac{\sigma_L - \sigma_H}{2\sigma_H\sigma_L} \tag{54}$$

On the other hand, the explicit price discrimination outcome $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$ satisfies[8]

$$\left[\tfrac{\eta}{\eta-1} k_L^{1/\eta} \hat{q}_L^{1-1/\eta} - \hat{w}_L - c\hat{q}_L - f\right] = 1/2\sigma_L \tag{55}$$

$$\left[\tfrac{\eta}{\eta-1} k_H^{1/\eta} \hat{q}_H^{1-1/\eta} - \hat{w}_H - c\hat{q}_H - f\right] = 1/2\sigma_H, \tag{56}$$

and

$$\hat{w}_H - \hat{w}_L > \tfrac{\eta}{\eta-1}\left[k_H^{1/\eta} - k_L^{1/\eta}\right] q_L^{1-1/\eta} \tag{57}$$

Subtracting (55) and (56) and taking (57) and $q_H^* = \hat{q}_H$ into account, we find

$$\tfrac{\eta}{n-1} k_H^{1/\eta}\left[q_H^{*\,1-1/\eta} - \hat{q}_L^{1-1/\eta}\right] - c(q_H^* - \hat{q}_L) > \frac{\sigma_L - \sigma_H}{2\sigma_H\sigma_L} \tag{58}$$

But

$$\frac{\partial\left(\tfrac{\eta}{\eta-1} k_H^{1/\eta} q^{1-1/\eta} - cq_L\right)}{\partial q} = k_H^{1/\eta} q^{-1/\eta} - c > 0 \Leftrightarrow q < k_H q(c) \tag{59}$$

As for $\lambda_H > 0$, $q_L^* < \hat{q}_L = k_L q(c + \frac{a-c_o}{2}) < k_H q(c)$, from (58), also

$$\tfrac{\eta}{n-1} k_H^{1/\eta}\left[q_H^{*\,1-1/\eta} - q_L^{*\,1-1/\eta}\right] - c(q_H^* - q_L^*) > \frac{\sigma_L - \sigma_H}{2\sigma_H\sigma_L} \tag{60}$$

which is in contradiction with (50): $\lambda_H > 0$ and $\lambda_L = 0$ is impossible.

b) The incentive constraint of the heavy users is violated in the explicit price discrimination equilibrium.

Suppose that $\{w_L^*, q_L^*, w_H^*, q_H^*\}$ is then such that the IC of the light users is binding, thus $\lambda_L > 0$ and $\lambda_H = 0$. We then have

$$w_H^* - w_L^* = \tfrac{\eta}{\eta-1}\left[k_H^{1/\eta} - k_L^{1/\eta}\right] q_H^{*\,1-1/\eta} \tag{61}$$

$$\left[\tfrac{\eta}{\eta-1} k_L^{1/\eta} q_L^{*\,1-1/\eta} - w_L^* - cq_L^* - f\right] + \frac{(a-c_o)}{\mu} S = -\lambda_L/\mu\sigma_L + 1/2\sigma_L \tag{62}$$

$$\left[\tfrac{\eta}{\eta-1} k_H^{1/\eta} q_H^{*\,1-1/\eta} - w_H^* - cq_H^* - f\right] - \frac{(a-c_o)}{1-\mu} S = \lambda_L/(1-\mu)\sigma_H + 1/2\sigma_H, \tag{63}$$

from which

$$\tfrac{\eta}{\eta-1} k_L^{1/\eta}\left[q_H^{*\,1-1/\eta} - q_L^{*\,1-1/\eta}\right] - c(q_H^* - q_L^*) \tag{64}$$

$$= \frac{\sigma_L - \sigma_H}{2\sigma_H\sigma_L} + \frac{(a-c_o)}{\mu(1-\mu)} S + \lambda_L/(1-\mu)\sigma_H + \lambda_L/\mu\sigma_L \tag{65}$$

On the other hand, the explicit price discrimination outcome $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$ satisfies now

$$\tfrac{\eta}{\eta-1} k_L^{1/\eta}\left[\hat{q}_H^{1-1/\eta} - \hat{q}_L^{1-1/\eta}\right] - c(\hat{q}_H - \hat{q}_L) < \frac{\sigma_L - \sigma_H}{2\sigma_H\sigma_L} \tag{66}$$

Subtracting the previous equations, we find:

$$\tfrac{\eta}{\eta-1}k_L^{1/\eta}\left[q_H^{*\,1-1/\eta}-\hat{q}_H^{1-1/\eta}\right]-c(q_H^*-\hat{q}_H)>\tfrac{\lambda_L}{(1-\mu)\sigma_H}+\tfrac{\lambda_L}{\mu\sigma_L}+\tfrac{(a-c_o)}{\mu(1-\mu)}S \tag{67}$$

For $\delta=k_H-k_L$ small, the LHS of (67) can be approximated by a Taylor expansion. As for $\lambda_L>0$ and $\lambda_H=0$,

$$q_L^*=\hat{q}_L \quad \text{and} \quad q_H^*=\hat{q}_H\left(1+\frac{2\lambda_L}{(1-\mu)}\frac{\left[k_H^{1/\eta}-k_L^{1/\eta}\right]}{k_H^{1/\eta}}\right)^{\eta}, \tag{68}$$

we find

$$\tfrac{\eta}{\eta-1}k_L^{1/\eta}\left[q_H^{*\,1-1/\eta}-\hat{q}_H^{1-1/\eta}\right]-c(q_H^*-\hat{q}_H)\cong\delta\frac{\lambda_L}{(1-\mu)}(c+\tfrac{a-c_o}{2})^{-\eta}(a-c_o) \tag{69}$$

On the other hand, as $q_L^*=\hat{q}_L$ while $q_H^*>\hat{q}_H$, we have $S>0$, such that

$$\lambda_L/(1-\mu)\sigma_H+\lambda_L/\mu\sigma_L+\tfrac{(a-c_o)}{\mu(1-\mu)}S>\lambda_L/(1-\mu)\sigma_H+\lambda_L/\mu\sigma_L \tag{70}$$

It follows that for δ small, (67) is violated: $\lambda_L>0$ and $\lambda_H=0$ is impossible.

6.2.1 Balanced Calling Patterns

Proof of Proposition 2.

i) Equilibrium profits are independent of the access charge: See Dessein (2003).

ii) Equilibrium contract:

For $a=c_o$, network i' s profits on the customer segment s are strictly concave in $\{q_s,w_s\}$.[9] As a result, total profits given k_L,k_H are strictly concave in $\{q_L,w_L,q_H,w_H\}$ for $a=c_o$ and the Hessian matrix $D^2\pi(q_L,w_L,q_H,w_H)$ is negative semidefinite for $a=c_o$. Fix the average customer type k. As all terms of $D^2\pi(q_L,w_L,q_H,w_H)$ are continuous in k_L,k_H and a, then for any $\delta'=k_H-k_L$, one can find an access charge $a'>c_o$ such that $D^2\pi(q_L,w_L,q_H,w_H)$ is still negative semidefinite and thus profits are strictly concave, for $c_o\leq a\leq a'$ or $0\leq\delta\leq\delta'$. A candidate equilibrium satisfying the FOC's is then effectively an equilibrium. From these FOC's, if networks can discriminate explicitly, a unique symmetric equilibrium exists, given by $\{\hat{q}_L,\hat{w}_L,\hat{q}_H,\hat{w}_H\}$, where $\hat{w}_L$ and $\hat{w}_H$ denote the net utilities resulting from $\left\{\hat{q}_L,\hat{t}_L,\hat{q}_H,\hat{t}_H\right\}$. As shown in the text, given $k_H-k_L\in\left]0,\delta'\right]$, for a close enough to c_o, IC's are satisfied by $\{\hat{q}_L,\hat{w}_L,\hat{q}_H,\hat{w}_H\}$, which is thus also the equilibrium under implicit price discrimination. On the other hand, given $a\in\left]c_o,a'\right]$, for k_L close to k_H, the IC of the light users is violated by $\{\hat{q}_L,\hat{w}_L,\hat{q}_H,\hat{w}_H\}$. From lemma 2, then $\lambda_L>0$ and $\lambda_H=0$ and symmetric equilibrium quantities are characterized by

$$q_L^* = \hat{q}_L = k_L q(c + \tfrac{a-c_o}{2}) \quad \text{and} \quad q_H^* = \hat{q}_H \left(1 + \tfrac{2\lambda_L}{(1-\mu)} \tfrac{\left[k_H^{1/\eta} - k_L^{1/\eta}\right]}{k_H^{1/\eta}}\right)^{\eta} > \hat{q}_H \tag{71}$$

Similarly from (47) and (48), profits per heavy user, π_H^*, and light user, π_L^*, are

$$\pi_H^* = \frac{1}{2\sigma} + \frac{\lambda_L}{\sigma(1-\mu)} \quad \text{and} \quad \pi_L^* = \frac{1}{2\sigma} - \frac{\lambda_L}{\sigma(\mu)} \tag{72}$$

and thus

$$t_L^* = \hat{t}_L - \frac{\lambda_L}{\sigma\mu} < \hat{t}_L \quad \textit{and} \quad t_H^* - cq_H^* = \hat{t}_H - c\hat{q}_H + \frac{\lambda_L}{\sigma(1-\mu)} > \hat{t}_H - c\hat{q}_H. \tag{73}$$

6.2.2 Unbalanced Calling Patterns

Proof of Propostion 3:

Substituting the equilibrium under explicit price discrimination $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$ in the incentive constraints IC_H and IC_L, we find after some manipulations that, in order for the latter to be satisfied by $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$, one must have

$$\left[1 + 2\frac{\mu k_L - \ell k}{\mu(1-\mu)\Delta k}\right] \left[\frac{\frac{a-c_o}{2}}{c + \frac{a-c_o}{2}}\right] \geq 1 - \frac{\eta}{\eta - 1} k_H^{1/\eta} \frac{\left[k_H^{1-1/\eta} - k_L^{1-1/\eta}\right]}{\Delta k} \tag{74}$$

$$\left[1 + 2\frac{\mu k_L - \ell k}{\mu(1-\mu)\Delta k}\right] \left[\frac{\frac{a-c_o}{2}}{c + \frac{a-c_o}{2}}\right] \leq 1 - \frac{\eta}{\eta - 1} k_L^{1/\eta} \frac{\left[k_H^{1-1/\eta} - k_L^{1-1/\eta}\right]}{\Delta k} \tag{75}$$

One can verify that the RHS of (74) is strictly negative and the RHS of (75) is strictly positive as long as $k_L < k_H$. Denoting $\delta = k_H - k_L$, it follows that given any $\delta > 0$, for a close enough to c_o, both $IC's$ are satisfied and $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$ is also the equilibrium under implicit price discrimination.

Fix now k and $a > c_o$. As long as

$$1 + 2\frac{\mu k_L - \ell k}{\mu(1-\mu)\Delta k} > 0 \Leftrightarrow \ell < \frac{1}{2}\left[\frac{\mu k_L}{k} + \mu\right] \tag{76}$$

that is, with a heavy biased, balanced or slightly light biased calling pattern, (74), the incentive constraint of the heavy users will be satisfied for any $a > c_o$. On the other hand, given $a > c_o$, for $\delta = k_H - k_L$ small enough, the IC of the light users will be violated by $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$. Indeed, writing k_H and k_L respectively as $k_H = k + \mu\delta$ and $k_L = k - (1-\mu)\delta$ (and thus also seeing ℓ as a function of $\delta : \ell \equiv \ell(\delta, k, \mu)$ with $\ell(0, k, \mu) = \mu$), one can verify that the RHS of both (75) and (74) tend to zero when δ goes to zero, while the limit of the LHS stays then strictly positive. If on the other hand $\ell > \frac{1}{2}\left[\frac{\mu k_L}{k} + \mu\right]$, that is, if heavy users call considerably more than they are being called, given $a > c_o$, the IC of the heavy users will be violated when δ goes to zero. From the following lemma, the IC violated by $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$ is binding in a symmetric equilibrium under implicit price discrimination:

Lemma 3 *If the explicit price discrimination equilibrium, $\{\hat{q}_L, \hat{t}_L, \hat{q}_H, \hat{t}_H\}$ violates the incentive constraint of the light (heavy) users, then for $\delta = k_H - k_L$ small, a symmetric equilibrium $\{t_L^*, q_L^*, t_H^*, q_H^*\}$ under implicit price discrimination is such that the incentive constraint of the light (heavy) users is binding.*

Proof. The proof is a straightforward extension of the proof of lemma 2 ■

Finally, for $\ell = \frac{1}{2}\left[\frac{\mu k_L}{k} + \mu\right]$,the IC of both heavy and light users are always satisfied and for any access charge, if a symmetric equilibrium exists, it is given by $\{\hat{q}_L, \hat{w}_L, \hat{q}_H, \hat{w}_H\}$. We characterize now the equilibrium in case an incentive constraint is binding.
The FOC with respect to q_L and q_H yield

$$\begin{aligned}
\mu\alpha_L\left[k_L^{1/\eta}q_L^{-1/\eta} - (c + (a - c_o)/2)\right] - \lambda_H k_H^{1/\eta} - k_L^{1/\eta}q_L^{-1/\eta} &= 0 \qquad (77)\\
(1-\mu)\alpha_H\left[k_H^{1/\eta}q_H^{-1/\eta} - (c + (a - c_o)/2)\right] + \lambda_L\left[k_H^{1/\eta} - k_L^{1/\eta}\right]q_H^{-1/\eta} &= 0 \qquad (78)
\end{aligned}$$

If $\lambda_L > 0$, it follows that:

$$q_L^* = \hat{q}_L \quad \text{and} \quad q_H^* = k_H q\left(c + \tfrac{a-c_o}{2}\right)\left(1 + \frac{2\lambda_L}{(1-\mu)}\frac{\left[k_H^{1/\eta} - k_L^{1/\eta}\right]}{k_H^{1/\eta}}\right)^{\eta} > \hat{q}_H \qquad (79)$$

If on the other hand $\lambda_H > 0$, we find

$$q_H^* = \hat{q}_H \quad \text{and} \quad q_L^* = k_L q\left(c + \tfrac{a-c_o}{2}\right)\left(1 - \frac{2\lambda_H}{\mu}\frac{\left[k_H^{1/\eta} - k_L^{1/\eta}\right]}{k_L^{1/\eta}}\right)^{\eta} < \hat{q}_L \qquad (80)$$

From the FOC with respect to w_L and w_H, if $\lambda_L > 0$, profits per heavy user, π_H^*, and light user, π_L^*, are:

$$\pi_H^* = \hat{\pi}_H + \frac{\lambda_L}{\sigma(1-\mu)} \quad \text{and} \quad \pi_L^* = \hat{\pi}_L - \frac{\lambda_L}{\sigma(\mu)} \qquad (81)$$

while if $\lambda_H > 0$, we have

$$\pi_H^* = \hat{\pi}_H - \frac{\lambda_H}{\sigma(1-\mu)} \quad \text{and} \quad \pi_L^* = \hat{\pi}_L + \frac{\lambda_H}{\sigma(\mu)} \qquad (82)$$

6.2.3 Differences in Perceived Substitutability

Proof of proposition 4.

From the FOC with respect to (w_L, q_L) and (w_H, q_H), the equilibrium under explicit price discrimination is characterized by (27) and (28) and (29) and profits under explicit price discrimination are $\pi_D^* = \mu/4\sigma_L + (1-\mu)(4\sigma_H)$. From the first order condition with respect to w_L and w_H, profits under implicit price discrimination are given by $\pi^* = \pi_D^* + (\lambda_H - \lambda_L)(1/2\sigma_L - 1/2\sigma_H)$ with λ_H and λ_L the lagrange multipliers of the IC of respectively heavy users and light users. This proves (32) and (31). From (27) and (28) and (29), the explicit price discrimination equilibrium satisfies the IC of light users and heavy if and only if respectively (34) and (35) are satisfied.

We are now ready to prove the first part of the proposition. As for $a = c_o$, $V_H(\hat{q}_L) - V_H(\hat{q}_H)$ is negative and $V_L(\hat{q}_L) - V_L(\hat{q}_H)$ is positive, for $|\sigma_H - \sigma_L|$ sufficiently small $|F_H - F_L|$ tends to zero and both $IC's$ are satisfied and $\pi^* = \pi_D^*$. On the other hand, both $V_L(\hat{q}_L) - V_L(\hat{q}_H)$ and $V_H(\hat{q}_L) - V_H(\hat{q}_H)$ go to zero as Δk goes to zero, such that given $|\sigma_H - \sigma_L|$, the IC of customers with the smallest perceived substitutability are violated for Δk sufficiently small. From lemma 2, the same IC then is binding in the equilibrium under implicit discrimination and from (32) and (31), $\pi^* < \pi_D^*$.

The proof of the second part goes as follows. (1) When $c + \frac{a-c_o}{2}$ goes to zero, $\hat{q}_H - \hat{q}_L$ tends to infinity such that $V_s(\hat{q}_L) - V_s(\hat{q}_H)$, $(s = L, H)$ increases without a bound when a gets smaller. For a small enough IC_L is then always satisfied, while IC_H will be violated. From lemma 2, for Δk small, the IC of the heavy users is then binding under implicit discrimination such that from (32) and (31), $\pi^* > \pi_D^*$ if $\sigma_H > \sigma_L$ and $\pi^* < \pi_D^*$ if $\sigma_L < \sigma_H$. (2) A negative access markup $(a < c_o)$ always decreases $V_H(\hat{q}_L) - V_H(\hat{q}_H)$ and $V_L(\hat{q}_L) - V_L(\hat{q}_H)$, which reach a minimum respectively for $a_H^* > c_o$ and $a_L^* > c_o$ where

$$1 + (a_H^* - c_o)/2c = \Delta k / \left(k_H - k_H^{1/\eta} k_L^{1-1/\eta}\right) \quad (83)$$

$$1 + (a_H^* - c_o)/2c = \Delta k / \left(k_H^{1-1/\eta} k_L^{1/\eta} - k_L\right). \quad (84)$$

Denote this minimum respectively by $-\bar{V}_H$ and $-\bar{V}_L$. As for $a \geq c_o$, $V_H(\hat{q}_L) - V_H(\hat{q}_H)$ is always negative and for $\hat{q}_H \leq k_L q(c)$, $V_L(\hat{q}_L) - V_L(\hat{q}_H)$ is always negative, both $-\bar{V}_H$ and $-\bar{V}_L$ are negative. For $|\sigma_H - \sigma_L|$ - and thus also $|F_H - F_L|$ - small enough, there exists then an $a' > c_o$, such that the IC of the light users is violated by the explicit price discrimination equilibrium. If also Δk is small, the IC of the light users is then binding under implicit discrimination and $\pi^* > \pi_D^*$ if $\sigma_H < \sigma_L$, $\pi^* < \pi_D^*$ if $\sigma_L > \sigma_H$.

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