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Network competition in nonlinear pricing

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Previous research, assuming linear pricing, has argued that telecommunications networks may use a high access charge as an instrument of collusion. I show that this conclusion is difficult to maintain when operators compete in nonlinear pricing: (i) As long as subscription demand is inelastic, profits can remain independent of the access charge, even when customers are heterogeneous and networks engage in second-degree price discrimination. (ii) When demand for subscriptions is elastic, networks may increase profits by agreeing on an access charge below marginal cost (relative to cost-based access pricing). Welfare is typically increased by setting the access charge above marginal cost.

1. Introduction

■ Most industrialized countries are engaged in a project to create competition in one of the largest noncompetitive industries of modern economies: local telecommunications. Because of technological advances, the local network has ceased to be a natural monopoly, and competing operators are starting to develop their own local and interurban networks. This is likely to affect considerably the way the industry operates. As customers of different operators want to get in touch with each other, competing networks must be interconnected. This requires a mutual provision of access: each competitor must terminate calls from his rival's customers in exchange for a fee or access charge. Such a two-way access problem differs considerably in its nature from the more familiar one-way access situation in which an (integrated) monopolist controls the local network and is required to interconnect with entrants competing on complementary segments such as long-distance or value-added services. Whereas in the latter case, the economic literature and practice has made clear that regulation is necessary, this is less obvious for the two-way bottleneck situation (see, e.g., Laffont and Tirole, 2000). Should access charges be freely negotiated between operators, with a possible recourse to private arbitration in case of conflict, or should there be a (strong) regulatory involvement? Should private agreements between competitors be trusted to bring about effective competition? No consensus has yet been reached on this point, and the telecommunications industry and regulators are still defining principles for two-way interconnection fees.

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Up to now, the economic literature on two-way access charges has advocated the regulatory approach or at least taken a very ambivalent position. In two seminal articles, Armstrong (1998) and Laffont, Rey, and Tirole (1998a) independently warn that competing networks may use a high access charge as an instrument of collusion due to a "raise-each-other's-cost effect": for given market shares, a higher access charge increases each network's average marginal cost and thus induces networks to raise their retail price. The symmetric equilibrium price is therefore increasing in the access charge.[1]

To derive the latter result, however, both articles assume that networks compete in linear pricing. This assumption is not very harmless, as noted by Laffont, Rey, and Tirole (1998a): the collusive power of the access charge disappears completely when networks compete in two-part tariffs and, hence, customers pay a monthly fixed fee in addition to a usage price per call. A higher access charge then still boosts usage prices, but the positive effect from this on retail profits is totally neutralized by a lower fixed fee. Intuitively, nonlinear pricing erodes the fat profits generated by a high access charge, as networks then have an instrument to build market share without inflating their outflow. It seems nevertheless extreme that there is no net effect on profits, in particular since Laffont, Rey, and Tirole obtain their result in a simple model with *homogeneous* customers. Indeed, the literature on nonlinear pricing has shown that once customers are *heterogeneous* in volume demand, results under nonlinear pricing typically resemble again those obtained under linear pricing, as firms then try to discriminate implicitly between different types (for example, usage fees are set above marginal cost, some surplus is left to customers). Laffont, Rey, and Tirole (1998a), p. 22, conjecture therefore that the collusive effect of high access charges is likely to be partially restored if one would generalize the model "so as to allow consumers to differ ... in their taste for variable consumption" (see also (Laffont, Rey, and Tirole (1998b), p. 53, and Armstrong (1998), p. 557, for similar statements). Given that nonlinear pricing is prevailing in the industry, the main question the literature tries to answer, whether effective competition between telecommunications networks is possible in a deregulated environment, thus remains open.

The main aim of this article is to provide a more realistic analysis of competition in nonlinear pricing. For this purpose, I shall introduce heterogeneity in volume demand and heterogeneity in subscription demand. This allows me to analyze network competition in the presence of second-degree price discrimination and elastic subscription demand.

□ **Second-degree price discrimination.** Volume demand for calls differs tremendously among customers in the telecommunications industry. I also observe that operators make abundant use of calling plan menus in order to discriminate implicitly between different types of customers. I model this heterogeneity in a straightforward way by assuming that customers are either high-demand or low-demand users, and I allow for a wide range of calling patterns, where high-demand users tend to call more than they are being called and vice versa. I consider both optimal nonlinear pricing and the case where networks are restricted to a menu of two-part tariffs.

□ **Elastic subscription demand.** Customer participation is a significant issue in the telecommunications industry. In the mobile sector, for example, actual penetration rates are currently closer to 50% than 100% in most countries. I allow for an elastic subscription demand by assuming that consumers are heterogeneous in their reservation value for telecommunications services. In equilibrium, some customers then drop out of the market.

I derive two main insights:

(i) *Introducing heterogeneity in volume demand is not sufficient to restore the collusive effect of high access charges*. Extending the model of Laffont, Rey, and Tirole (1998a) with high- and low-demand customers, I obtain again the result that profits are independent of the access charge, even when differences in demand cannot be perfectly overcome through a menu of calling plans.[2] Intuitively, the main effect of an access markup is that it makes it optimal

[1] No equilibrium exists, however, when access charges and/or the substitutability between networks are too large.

[2] Independently, Hahn (2000) has shown that this result also holds for a continuum of consumer types.

for both networks to offer lower quantities (i.e., the raise-each-other's-cost effect). With linear pricing, a low quantity of calls is equivalent to a high price and high profits. Once networks compete in nonlinear pricing, however, there is by definition no direct link between the quantities of calls offered to customers and the tariffs that are charged by the operators. If customers are heterogeneous in volume demand, then quantities and tariffs must play an additional role in helping discriminate between customers of different types. The need to meet certain incentive constraints, however, does not reduce a network's incentive or ability to reduce its *average* tariff in order to build market share. In particular, I show that the benefits of reducing tariffs remain independent of the equilibrium quantities of calls offered to customers. Therefore, unlike under linear pricing, agreeing to offer low quantities to customers—by means of agreeing on a high access charge—does not soften competition for market share.

(ii) *If subscription demand is elastic, firms prefer an access charge below marginal cost.* As I shall argue, the exact "profit-neutrality" of the access charge relies on some assumptions that are unlikely to be satisfied in more complex settings. In these cases, however, networks may easily prefer an access charge below marginal cost. An important example is elastic subscription demand. If some customers choose *not* to subscribe in equilibrium, as customers are heterogeneous in their reservation value for a subscription,[3] networks can increase profits by agreeing on an access charge below marginal cost (relative to a cost-based access charge). Intuitively, one consequence of an elastic subscription demand is that the industry exhibits positive network externalities: as customers derive utility from calling other customers, the value of a subscription to one customer is increasing in the total number of subscriptions. Furthermore, these externalities are larger for lower usage fees, as a customer then makes more calls to each additional subscriber. Finally, the larger these externalities, the more each duopolist acts like a monopolist, since a decrease in its tariffs then also benefits his rival's customers. It follows that by agreeing on a low access charge and thus, implicitly, on a low usage fee, networks can increase their equilibrium profits. For example, if for a cost-based access charge two duopolists offer customers a larger net surplus than a monopolist would offer, then a decrease in the access charge results in a decrease in this net customer surplus, a decrease in overall welfare (as market participation decreases), and an increase in profits. Conversely, an increase in the access charge then decreases profits but increases welfare and customer surplus.

From an economic theory perspective, the above results show that the collusion concern should not necessarily be associated with a high access charge (as argued by Laffont, Rey, and Tirole (1998a) and Armstrong (1998)). From a policy perspective, they suggest that an optimal regulation of the access charge is likely to be tricky. Ensuring that the access charge is close to cost may therefore be a good second-best policy.

This article is organized as follows. Section 2 describes the model of high- and low-demand users. In Section 3 I review the collusive effect of a high access charge under linear pricing, highlighted in the previous literature, and show that it should be qualified by a study of the calling pattern. Indeed, unbalanced calling patterns may increase or decrease this effect depending on the direction of the call bias. Section 4 analyzes competition in nonlinear pricing when networks engage in second-degree price discrimination. It derives the profit-neutrality result and discusses its limits. Section 5 looks at the impact of an elastic subscription demand. Section 6 concludes.

2. A model with heterogeneous customers

■ I consider the competition between two horizontally differentiated networks. The main elements are as follows:

□ **Cost structure.** The two networks have the same cost structure. Serving a customer involves a fixed cost f. Per call, a network also incurs a marginal cost c_0 at the originating and terminating

[3] For tractibility, customers are assumed to be homogeneous in volume demand.

ends of the call and a marginal cost c_1 in between. The total marginal cost is thus

$$c = 2c_0 + c_1.$$

□ **Demand structure.** The networks are differentiated à la Hotelling. Consumers are uniformly located on segment [0, 1] and networks are located at the two extremities, namely at $x_1 = 0$ and $x_2 = 1$. Given income y and telephone consumption q, a type-k consumer located at x joining network i has utility

$$y + k^{1/\eta}u(q) + v_0 - \tau \left| x - x_i \right|,$$

where v_0 represents a fixed surplus from being connected, $\tau|x - x_i|$ denotes the cost of not being connected to its "most-preferred" network, and $k^{1/\eta}u(q)$ is the variable gross surplus, with $u'(\cdot) > 0$ and $u''(\cdot) < 0$.[4] Faced with a usage fee p, a customer thus consumes a quantity $q_k(p)$, where $k^{1/\eta}u'(q_k) = p$. I consider two different customer types or customer segments:

(i) Light users: fraction μ of the market, characterized by $k = k_L$.
(ii) Heavy users: fraction $1 - \mu$ of the market, characterized by $k = k_H > k_L$.

I denote $q_L \equiv q_{k_L}$ and $q_H \equiv q_{k_H}$. By definition, $q_H(p) > q_L(p)$. The distribution of customers on segment [0, 1] is assumed to be independent of their type k_i $(i = L, H)$.

□ **Calling patterns.** Customers may differ in their likelihood of receiving a call, where this likelihood may be a function of the originator of the call. Light users, for example, will typically also be "light receivers." Customers may further have a tendency to mainly call customers of their own type. The only restriction I put on calling patterns is that the probability that a subscriber receives a call is independent of both his address on segment [0, 1] as well as the address of the originator of the call. Under these assumptions, a calling pattern is fully characterized by the pair $(\ell_H, \ell_L) \in [0, 1]^2$, where ℓ_H denotes the fraction of calls that heavy users make to light users, and ℓ_L is the fraction of calls that light users make to other light users.

□ **Reciprocal access pricing.** Each network charges a per-unit access charge a for terminating its rival's off-net calls. As a result, a network faces a marginal cost for off-net calls equal to $c + a - c_0$. The access charge or termination charge is assumed to be the same for both networks. This assumption is standard in the literature on two-way access charges, and regulators tend to insist on the reciprocity of access charges.[5]

□ **Retail pricing.** I consider competition both in linear and nonlinear pricing. I do not, however, allow networks to charge different prices for on-net and off-net calls. This type of price discrimination has been extensively discussed in Laffont, Rey, and Tirole (1998b). I allow networks neither to charge nor subsidize customers for receiving calls.[6]

3. Competition in linear pricing

■ I first review the argument for collusion developed by Armstrong (1998) and Laffont, Rey, and Tirole (1998a). They show, with homogeneous customers, that an increase in the access charge results in a higher equilibrium price due to a "raise-each-other's-cost effect": for given market

[4] With the exception of Section 5, I will assume that v_0 is "large enough" so that all consumers are connected in equilibrium.

[5] The U.S. Telecommunications Act of 1996 mandates reciprocal interconnection pricing unless there is a rebuttal of symmetry. Symmetry may be violated if traffic is unbalanced or if access costs are very unequal. Although I investigate unbalanced calling patterns, traffic flows between networks are balanced in equilibrium. Hence there is no rebuttal of symmetry.

[6] As argued by Laffont and Tirole (2000), this extension, as well as the previous one, is likely to reduce the (under linear pricing) collusion effect of a high access charge.

shares, the average marginal cost of a call increases with the access charge so that a higher access charge induces the networks to set a higher retail price. In this section I show that similar results hold when customers are heterogeneous, but the calling pattern may then considerably affect the impact of the access charge.

Without loss of generality, I assume in this section that $\ell_L = \ell_H \equiv \ell$.[7] I further assume that $u(q) = \eta - 1/\eta q^{1-1/\eta}$, which results in an iso-elastic demand function,

$$q_L(p) = k_L q(p) \quad \text{and} \quad q_H(p) = k_H q(p),$$

with $q(p) \equiv p^{-\eta}$. Denoting network 1's market shares in the heavy users' and the light users' segment by α_H and α_L respectively, it follows that network 1's and network 2's shares in the volume of incoming calls are given by

$$\hat{\alpha}_1 = \alpha_L \ell + \alpha_H (1 - \ell) \quad \text{and} \quad \hat{\alpha}_2 = 1 - \hat{\alpha}_1, \tag{1}$$

where α_H and α_L are a function of both p_1 and p_2. As a result, network 1's profits are given by

$$\begin{aligned}\pi_1(p_1, p_2) = {}& \mu\alpha_L \left([p_1 - c - \hat{\alpha}_2(a - c_0)]\, k_L q(p_1) - f\right) \\ & + (1-\mu)\alpha_H \left([p_1 - c - \hat{\alpha}_2(a - c_0)]\, k_H q(p_1) - f\right) \\ & + \hat{\alpha}_1 \left[(1 - \alpha_L)\mu k_L + (1 - \alpha_H)(1 - \mu) k_H\right] q(p_2)(a - c_0).\end{aligned} \tag{2}$$

The first and second lines in the above expression denote the retail profits of network 1 in the light users' and the heavy users' market segment respectively. Indeed, given that $\hat{\alpha}_2$ is the fraction of calls that terminate on network 2, the average marginal cost of a call (including any access charges being paid) equals $c + \hat{\alpha}_2(a - c_0)$. The last line then denotes the access revenues of network 1.

Based on the parameters (ℓ, μ, k_L, k_H), we can distinguish three types of calling patterns. I denote by *light biased* those calling patterns where for equal usage fees ($p_1 = p_2$), light users receive in aggregate more calls than they originate. This will be the case if and only if $\ell > \mu k_L / k$, where k denotes the average customer type:

$$k \equiv \mu k_L + (1 - \mu) k_H.$$

A *heavy biased* calling pattern is then a calling pattern where for equal usage fees, heavy users receive in aggregate more calls than they originate and thus $\ell < \mu k_L / k$. If light (heavy) users send in aggregate as many calls as they receive for equal usage fees, that is, $\ell = \mu k_L / k$, I will speak of a *balanced* calling pattern.

Let us first assess the impact of the access charge, assuming a balanced calling pattern, a case that is most closely related to the one studied by Armstrong (1998) and Laffont, Rey, and Tirole (1998a). I then discuss the impact of unbalanced calling patterns.

Proposition 1. With a balanced calling pattern,

(i) For a close to c_0, there exists a unique, symmetric equilibrium, characterized by $p_1 = p_2 = p^*$, given by

$$\frac{p^* - (c + a - c_0/2)}{p^*} = \frac{1}{\eta}\left(1 - 2\sigma\left[(1 + h)k(p^* - c)q(p^*) - f\right]\right), \tag{3}$$

[7] In a previous draft, I consider the case where customers are allowed to call more their own type, that is, $\ell_L = \ell_H + \Delta$. This case is equivalent to one in which a fraction Δ of consumers only calls its own type, whereas for all other consumers $\ell \equiv \ell_H/(1 - \Delta)$. I show that Proposition 1 is not affected, whereas in Propostion 2, one should replace ψ by $(1 - \Delta)\psi$.

where $\sigma \equiv 1/2\tau$ is an index of substitutability between the two networks and h is a measure of heterogeneity in demand:

$$h \equiv \frac{\operatorname{var} k_i}{k^2} = \frac{\mu(k_L)^2 + (1-\mu)(k_H)^2}{k^2} - 1. \tag{4}$$

(ii) The access charge is an instrument of collusion: as long as the equilibrium price $p_1 = p_2 = p^*(a)$ exists, p^* increases with a.

(iii) Keeping average demand constant, an increase in the heterogeneity of volume demand (h) lowers the equilibrium price. Similarly, keeping heterogeneity in demand constant, an increase in average demand (k) results in tougher competition.

Proof. See the Appendix.

Condition (3) tells us that the equilibrium price p^* reflects a markup over a network's average marginal cost, $c + (a - c_0)/2$, where this markup is decreasing in the elasticity of demand η, the substitutability of the two networks σ, the equilibrium profits per customer $k(p^* - c)q^* - f$, and the heterogeneity of demand h. A high access charge is thus still an instrument of collusion: as long as the equilibrium $p_1 = p_2 = p^*$ exists, p^* increases with a. Laffont and Tirole (2000) call this the "raise-each-other's-cost" effect. Each network's average marginal cost, $c + (a - c_0)/2$, increases with the access charge, leading to a higher equilibrium price.

The impact of heterogeneity in volume demand ($h > 0$) on the equilibrium price stems from the fact that heavy users are more price-sensitive in the choice of their network: since they benefit more from a low price, they more easily accept not being connected to their preferred network for a given price differential.[8] A first consequence is that competition is tougher with only heavy users than with only light users: an increase in the average type of customers (k) leads to a lower equilibrium price. Secondly, a low price-network will have a relatively high fraction of heavy users among its clientele: there is an endogenous selection of customers. As a consequence, competition is tougher than what the "average" size of users may suggest: all other things equal, the equilibrium price is lower when customers are more heterogeneous in demand.

I now show that if calling patterns are unbalanced, this endogenous selection changes the impact of the access charge on the equilibrium price.

Proposition 2. For any calling pattern,

(i) For a close to c_0, there exists a unique, symmetric equilibrium characterized by $p_1 = p_2 = p^*$, given by

$$\frac{p^* - \left(c + \dfrac{a - c_0}{2}\right)}{p^*} = \frac{1}{\eta}\left(1 - 2\sigma\left[(1+h)k(p^* - c)q^* - f + (a - c_0)\psi k q^*\right]\right), \tag{5}$$

where

$$\psi \equiv \frac{k_H - k_L}{k}\left(\frac{\mu k_L}{k} - \ell\right).$$

We have that $\psi = 0$ for a balanced calling pattern, $\psi < 0$ for a light-biased calling pattern, and $\psi > 0$ for a heavy-biased calling pattern.

(ii) Compared with the case of a balanced calling pattern, for a close to c_0, the impact of an increase in the access charge on the equilibrium price is

(a) even more positive when the calling pattern is biased toward light users;

[8] The fact that high-demand users are more price sensitive depends on the lump-sum nature of the transport cost. If the transport cost were per unit, things would be more ambiguous. It is, however, a widely observed fact that high-volume customers are quickest to switch to lower-price alternatives, which is what drives my results.

(b) still positive but smaller when the calling pattern is biased toward heavy users and

$$4\sigma \cdot \psi \cdot k\tilde{p}q(\tilde{p}) < \eta, \tag{6}$$

where $\tilde{p} = p^*|_{a=c_0}$; and

(c) reversed (negative) when the calling pattern is biased toward heavy users and

$$4\sigma \cdot \psi \cdot k\tilde{p}q(\tilde{p}) > \eta. \tag{7}$$

Proof. See the Appendix.

The nature of the calling pattern affects price competition as soon as the access charge differs from the termination cost ($a \neq c_0$), because the composition of a network's customer portfolio then also affects access revenues. To see this, suppose first that $a > c_0$ and the calling pattern is light biased ($\psi < 0$). Heavy users then have a negative impact on the access revenues because they tend to call more than they are called. A cut in the usage price is thus also less profitable, as it attracts especially these heavy users. Compared to the balanced calling pattern benchmark, an access markup thus has a bigger collusive effect: it further reduces incentives to set low prices. A polar picture is obtained if heavy users receive more calls than they make, that is, if there is a heavy-biased calling pattern ($\psi > 0$). The effect of the access markup on the equilibrium price is then ambiguous: on the one hand, the access markup still reduces incentives to lower prices through the raise-each-other's-cost effect; on the other hand, lowering prices attracts mainly heavy users, which are now a source of access revenues. Whenever (7) holds, an increase in the access charge reduces p^*.

Conditions (6) and (7) suggest that the raise-each-other's-cost effect becomes relatively less important as demand gets more inelastic. Intuitively, in the limit where $\eta = 0$ (demand per customer is constant), the marginal cost of a good does not affect the equilibrium price in an oligopoly. The collusive (or procompetitive) effect of the access charge then stems only from unbalanced calling patterns.[9] The impact of unbalanced calling patterns, for its part, will be stronger the larger the substitutability. Indeed, for a given price cut, the effect on the composition of the customer portfolio is larger if the substitutability is high, as customers then switch faster: for σ very large, a price cut by an ε may attract almost all heavy users and still leave a lot of light users to the rival network.

4. Competition in nonlinear pricing: the profit-neutrality result

■ Perhaps the most striking result in Laffont, Rey, and Tirole (1998a) is the dichotomy between competition in linear and nonlinear pricing. In their model with homogeneous customers and a balanced calling pattern, the collusive power of the access charge disappears completely when networks compete in (then optimal) two-part tariffs ($t(q) = F + pq$). An access charge above marginal cost then still boosts final usage prices, since in equilibrium, networks set usage fees equal to the average marginal cost of a call:

$$p_1 = p_2 = p^* = c + (a - c_0)/2.$$

The positive effect from this on retail profits, however, is totally neutralized by a lower fixed fee. As a result, total profits are independent of the access charge and firms may as well set the access charge equal to marginal cost.

Intuitively, if customers are homogeneous, price schedules must achieve two goals: creating surplus by offering a certain volume of calls and sharing this surplus with customers. By agreeing

[9] Suppose that consumers have a constant demand $q(p) = k$ ($k = k_L, k_H$) and assume for simplicity that $f = 0$; then the equilibrium price is given by $p^* - c = [1/(1+h)][(1/2\sigma k) - \psi(a - c_0)]$.

on a high access charge, networks basically agree on offering a low quantity of calls to customers. Since under linear pricing the quantity offered to customers is linked to the way the resulting surplus is shared with customers (there is only one pricing instrument), an increase in the access charge then results in higher profits. Under nonlinear pricing, however, there is no such link. Networks then have two instruments (quantities and tariffs) that allow them to separate the two roles of pricing.

If customers are heterogeneous, on the other hand, and networks are not allowed to discriminate explicitly, a new, third, role of pricing is to discriminate implicitly between customers of different types. The literature on nonlinear pricing has then shown that results in general resemble those obtained with linear prices (for example, usage fees are set above marginal cost, some surplus is left to customers).[10] The central question is thus whether the presence of heterogeneity in volume demand and the resulting difference between the number of goals and instruments allows networks again to use a high access charge as an instrument of collusion, as under linear pricing.

I consider competition in optimal nonlinear pricing both under the assumption that networks can discriminate explicitly between heavy and light users (third-degree price discrimination) and in the more realistic case in which only implicit discrimination (second-degree price discrimination) is allowed. From the revelation principle, networks cannot do better than offering customers a quantity q_L for a tariff t_L and a quantity q_H for a tariff t_H where, under implicit discrimination, $\{q_L, t_L, q_H, t_H\}$ must be such that light users opt for (q_L, t_L) and heavy users choose (q_H, t_H). The variable net surplus of respectively a light and a heavy user is then

$$w_L \equiv k_L^{1/\eta} u(q_L) - t_L \quad \text{and} \quad w_H \equiv k_H^{1/\eta} u(q_H) - t_H.$$

For given net surpluses (w_L, w_H) and (w'_L, w'_H) offered by networks 1 and 2, the market share α_s of network 1 in segment s is then

$$\alpha_s = \alpha(w_s, w'_s) \equiv \frac{1}{2} + \sigma\left[w_s - w'_s\right] \qquad (s = L, H), \tag{8}$$

where $\sigma \equiv 1/2\tau$ is an index of substitutability between the two networks. If networks are not allowed or not able to price discriminate explicitly, the following incentive constraints must be satisfied:

$$w_H = k_H^{1/\eta} u(q_H) - t_H \geq k_H^{1/\eta} u(q_L) - t_L \tag{IC$_\mathrm{H}$}$$

$$w_L = k_L^{1/\eta} u(q_L) - t_L \geq k_L^{1/\eta} u(q_H) - t_H. \tag{IC$_\mathrm{L}$}$$

I now show that whether networks engage in second- or third-degree price discrimination, profits are independent of the access charge.[11] I use the following calling pattern property:

Lemma 1 (calling pattern property). If both firms offer the same pair of quantities (q_L, q_H) and network 1's market share in the heavy- and light-user's segment satisfies $\alpha_L = \alpha_H \equiv \alpha$, then there is no net outflow of calls from a network.

Proof. If both firms offer (q_L, q_H), the net outflow of network 1 is given by

$$\alpha_L \mu q_L + \alpha_H(1-\mu)q_H - \mu q_L\left[\ell_L\alpha_L + (1-\ell_L)\alpha_H\right] - (1-\mu)q_H\left[(1-\ell_H)\alpha_L + \ell_H\alpha_H\right],$$

which equals zero if $\alpha_L = \alpha_H$. *Q.E.D.*

Let $\{q_L, t_L, q_H, t_H\}$ be the symmetric equilibrium, with profits per type-s subscriber equal to π_s $(s = L, H)$. From Lemma 1, since $\alpha_L = \alpha_H = 1/2$ at a symmetric equilibrium, firms incur

[10] See, for example, Varian (1989) and Wilson (1993).

[11] I am very grateful to one of the referees for suggesting this proof, which is at once more general and more straightforward than the original proof.

no access deficit and average profits per subscriber are equal to $\pi = \mu\pi_L + (1 - \mu)\pi_H$. What happens if one firm reduces both its tariffs t_L and t_H by the same amount, say ε? This means that no incentive constraints are affected, and from (8), the market shares of the deviating firm are increased by the same amount $\sigma\varepsilon$ for the two types of subscriber. From Lemma 1, the deviating firm then still does not incur an access deficit (or surplus), and its total profits are

$$\left(\frac{1}{2} + \sigma\varepsilon\right)(\pi - \varepsilon).$$

Therefore, for the original contract to be an equilibrium (i.e., for $\varepsilon = 0$ to maximize the above), I require that $\pi = 1/2\sigma$ and so profits per firm equal $1/4\sigma$:

Proposition 3. In any symmetric equilibrium, profits are independent of the access charge and given by $1/4\sigma$.

Intuitively, the need to meet certain incentive constraints (i.e., discriminate implicitly between customers) means that the role of the offered quantity is not just to maximize surplus, and the role of tariffs is not just to share this surplus optimally with customers, but also to help discriminate between customers of different types. Concerns for incentive compatibility, however, never lead networks to use the quantity instrument to share surplus with customers. In this sense, agreeing to offer low quantities to customers (by way of agreeing on a high access charge) does not soften competition for market share. This leads to the profit-neutrality result in my basic model.

Note that the same argument and result would hold if networks were constrained to compete with (a menu of) two-part tariffs. Also, the argument has nothing whatsoever to do with the "two-type" model of private information and would apply equally to a continuum model, provided all types were served.[12]

□ **Equilibrium pricing.** While the access charge has no impact on profits, it considerably affects the way networks compete for customers. I briefly provide the main insights but refer to Dessein (1999a, 1999b) for a detailed discussion.

For an access charge equal to the marginal cost of terminating a call, $a = c_0$, my model is identical to those of Armstrong and Vickers (2001) and Rochet and Stole (2002). Hence, I obtain their "no screening" result:[13] despite the fact that customers are heterogeneous, for $a = c_0$ a single two-part tariff $t^*(q)$ is optimal, where

$$t^*(q) = cq + f + 1/2\sigma.$$

Whenever $a \neq 0$, however, this "no screening" result disappears. Assuming a balanced calling pattern, Dessein (1999a) shows how an increase in the access charge makes the contract offered to heavy users more and more attractive to light users. Under explicit price discrimination, for a high-enough access charge, light users then prefer the tariff and quantity designated to heavy users. To discriminate implicitly, networks must therefore increase the tariff for heavy users and lower the tariff for light users. The incentive constraint of the light users is then binding, and heavy users face an implicit marginal price that is smaller than the perceived marginal cost. Obviously, if there were a continuum of customer types, the incentive constraint would be binding whenever $a \neq 0$.

The main insight of Dessein (1999b) is that the nature of the calling pattern considerably affects the attractiveness of certain customer types, and hence the way firms compete for—and discriminate between—customers. Dessein (1999b) shows how for $a \neq c_0$, the tariff that a

[12] See Hahn (2000) for a formal proof.

[13] Rochet and Stole show, however, that this result is highly sensitive to the assumption that the customer's type is uncorrelated with the consumers location on the Hotelling line and that all consumer types are willing to participate with the candidate tariffs.

customer pays depends on the net outflow or inflow of calls he generates. Therefore, depending on the calling pattern, for a given access charge, the incentive constraint of the light users may be binding or the incentive constraint of the heavy users may be binding, and hence marginal prices may be smaller, larger, or equal to the perceived marginal cost.[14]

□ **Limits to the profit-neutrality result.** I do not make any claim of robustness of the above profit-neutrality result. In particular, as is clear from the proof of Proposition 3, what is needed is (i) symmetry in demand, (ii) distribution of volume types (high, low) to be independent of the location, and (iii) transport costs τ to be independent of the variable consumption q_k or volume type. If (iii) is not verified, for example, then starting from a symmetric equilibrium where $\alpha_L = \alpha_H = 1/2$, an increase in t_L and t_H by ε will result in $\alpha_L \neq \alpha_H$, such that Lemma 1 cannot be applied. Dessein (1999a), assuming a balanced calling pattern, then shows that when operators are seen as better substitutes by heavy users than by light users, networks obtain higher profits by agreeing on an access charge below marginal cost. In the opposite case, an access charge above marginal cost may boost profits.[15] If (i) is not verified and, as a result, networks differ in size, Carter and Wright (2003) show that the larger network prefers a reciprocal access charge to be set at cost. For sufficiently large asymmetries, the smaller network will have the same preference.

In my view, Proposition 3 should therefore be seen as a benchmark; in more complex settings, high access charges may as easily increase as reduce operators' profits, depending on the specifics of the model. What is important is that, *a priori,* the access charge has no clear-cut impact on profits. The next section, which considers a model in which demand for subscription is elastic, provides a detailed example of the breakdown of the access charge's profit neutrality.

5. Elastic subscription demand and network externalities

■ In the previous analysis I have assumed that in equilibrium, all customers subscribe to one of the two networks. Because market participation is such an important issue, especially in the mobile sector, I now use a more general discrete-choice framework where demand for subscription is elastic and, in equilibrium, some customers choose not to subscribe.[16] I show that networks then prefer an access charge below marginal cost, whereas overall welfare typically increases by setting the access charge above marginal cost.

Let firms 1 and 2 be two symmetrically placed operators facing differentiated customers, who are further homogeneous in their volume demand.[17] Consumers subscribe to one or the other firm, or opt for an outside option. Suppose that the subscription offered by firm i gives each consumer a net surplus w_i, and that the outside option generates a net surplus w_0. I use a discrete-choice framework (see Anderson, de Palma, and Thisse (1992) or, more recently, Armstrong and Vickers (2001)) and assume that a consumer of type $\varepsilon = (\varepsilon_0, \varepsilon_1, \varepsilon_2)$ obtains a utility $w_i + \varepsilon_i$ if he buys from firm i, and a utility $w_0 + \varepsilon_0$ if he does not subscribe. The triplet $\varepsilon = (\varepsilon_0, \varepsilon_1, \varepsilon_2)$ is distributed across the population according to a known continuous distribution. There is a continuum of potential consumers, with measure normalized to one.

In the above framework, the number of subscribers to firm i, α_i, is given by the probability that $w_i + \varepsilon_i = \max\{w_1 + \varepsilon_1, w_2 + \varepsilon_2, w_0 + \varepsilon_0\}$. I denote $\alpha_1 \equiv s(w_1, w_2)$ and, by symmetry,

[14] Similarly, assuming a uniform calling pattern, where light users receive many more calls than they originate, Hahn (2000) finds that for $a > c_0$, the marginal price is larger than the perceived marginal cost. From Dessein (1999b), the opposite result is likely to hold for more balanced calling patterns.

[15] Since the mechanism at work is quite technical, I refer to Dessein (1999a) for more details.

[16] The Hotelling model is ill-suited to analyze limited participation. Only customers in the middle of the Hotelling line drop out then, such that each network has a local monopoly. The Hotelling model can nevertheless be considered as a special case of the more general discrete-choice framework that I consider.

[17] As noted by a referee, if subscribers also differ in volume demand, then the marginal subscriber is likely to make and receive far fewer calls than the average subscriber. While this reduces network externalities, it is unlikely to affect my results qualitatively, since they hold for any level of externalities.

$\alpha_2 \equiv s(w_2, w_1)$. Total subscription demand or market participation is denoted by

$$\phi = \phi(w_1, w_2) \equiv s(w_1, w_2) + s(w_2, w_1).$$

Assumption 1.

$$\left.\frac{\partial \phi(w_1, w_2)}{\partial w_1}\right|_{w_1=w_2=w^*} \text{ and } \left.\frac{\partial s(w_1, w_2)}{\partial w_1}\right|_{w_1=w_2=w^*} \text{ are nonincreasing in } w^*.$$

Assumption 1 implies that for larger values of $\phi(w, w)$ and $s(w, w)$, it should not become easier to further increase, respectively, market penetration and market share. These conditions are verified for the logit model[18] and the Hotelling model.[19]

Since volume demand is known and the same for all customers, network i cannot do better than offer a two-part tariff $t_i(q) = F_i + p_i q$. Given a market participation $\phi \equiv \phi(w_1, w_2)$, demand for calls by a given customer and the variable gross surplus derived from making these calls is given respectively by $\phi q(p_i)$ and $\phi u(q_i)$. Thus, a lower market participation lowers the per-customer demand for calls, as there are fewer customers to whom a call can be made. As a result, net surpluses w_1 and w_2 are the solution to

$$w_i = \phi(w_1, w_2)v(p_i) - F_i \qquad i = 1, 2, \tag{9}$$

where $v(p_i) = \max_q\{u(q) - p_i q\}$.

I assume that w_1 and w_2 are continuous in, and uniquely defined by, the variables (p_1, p_2, F_1, F_2) in the relevant parameter range.

From (9), the industry exhibits positive network externalities: customers value the fact that more customers are connected to one of the two operators. Each connected customer thus has an externality on all other customers, where this externality is increasing in $v(p_i)$ and, hence, decreasing in p_i. Intuitively, if usage fees are smaller, customers value more an additional connected customer, as they will make more calls to this customer. Since a lower access charge results in lower equilibrium usage fees p_i and p_j, it also results in larger network externalities. This will drive my main result.

I look for a candidate symmetric equilibrium that satisfies the first-order conditions. Let F^* and p^* denote respectively the symmetric equilibrium fixed fee and usage fee, and w^* the resulting net surplus to customers. As usual, I obtain marginal-cost pricing, $p^* = c + (a - c_0)/2$. From (9), symmetric equilibrium profits can therefore be written as

$$\pi_i(w^*, w^*) \equiv s(w^*, w^*)\left[\phi(w^*, w^*)V(a - c_0) - w^* - f\right], \tag{10}$$

with

$$V(a - c_0) \equiv v\left(c + \frac{a - c_0}{2}\right) + \left(\frac{a - c_0}{2}\right) q\left(c + \frac{a - c_0}{2}\right).$$

I am interested in the sign of $d\pi_i(w^*, w^*)/da$ for a close to c_0.

From (10), a change in the access charge may affect symmetric equilibrium profits in two ways:

(i) The access charge affects $\pi_i(w^*, w^*)$ through its effect on $V(a - c_0)$. It is easy to verify that $V(a - c_0)$ is maximized for $a = c_0$ and is decreasing in $|a - c_0|$. Intuitively, setting $a \neq c_0$

[18] The most commonly used discrete-choice model is the logit model where $\varepsilon_i - \varepsilon_j$ ($i \neq j$) is distributed according to the logistic function, yielding $s(w_i, w_j) = e^{w_i/\rho}/(e^{w_i/\rho} + e^{w_j/\rho} + 1)$, where ρ is a measure of differentiation between the two firms. Both derivatives in Assumption 1 are then strictly decreasing in w.

[19] In a Hotelling model with limited participation, both derivatives of Assumption 1 equal 2σ and are thus constant in w.

results in a distorted usage price $p^* \neq c$, which, all other things equal, reduces total surplus and profits per customer. This force thus tends to push the profit-maximizing access charge close to c_0. For $a = c_0$, however, this effect is only of second order and can be neglected. Indeed, we have that

$$\left.\frac{\partial V(a - c_0)}{\partial a}\right|_{a=c_0} = 0.$$

(ii) The access charge affects $\pi_i(w^*, w^*)$ through its impact on w^*. As hinted above, an increase in the access charge reduces the amount of network externalities, which in turn affects w^*. Because for a close to c_0 the impact of a on $V(a - c_0)$ is of second order, the following result holds:

Lemma 2. For $a = c_0$, $d\pi_i(w^*, w^*)/da$ has the same sign as

$$\frac{dw^*}{da}\left[w^M - w^*\right],$$

where w^M is the net surplus offered by a monopolist.

Proof. See the Appendix.

Intuitively, for $a = c_0$, industry profits are concave in w and maximized for $w = w^M$. Hence if $w^* < w^M$, profits will increase with a if and only if $dw^*/da > 0$. Similarly, if $w^* > w^M$, profits will increase with a if and only if $dw^*/da < 0$.

Before we discuss the sign of dw^*/da, note first that both $w^* < w^M$ and $w^* > w^M$ are possible. On the one hand, a duopolist i does not take into account that by decreasing F_i he may reduce the market share of his competitor. This is the standard business-stealing effect of competition, which, in the absence of network externalities, would always imply that $w^* > w^M$. On the other hand, when setting F_i, neither does a duopolist internalize the impact of an increase in the overall market participation $\phi(w_i, w_j)$ on his competitor's profits per customer. As a result, if network externalities are large, it may be possible that in equilibrium two competitors offer less surplus than would a monopolist, that is, $w^* < w^M$. Intuitively, we may expect $w^* > w^M$ unless $-\partial s(w_1, w_2)/\partial w_2$ is relatively small (networks are very differentiated) and $\partial\phi(w_1, w_2)/\partial w_1$ is relatively large (the market expansion effect is very important).[20]

Let us now investigate the sign of dw^*/da. Due to network externalities, when network 1 reduces its fixed fee F_1, it increases both w_1 and w_2. From (9), in a symmetric equilibrium, we have that

$$\chi^* \equiv \left.\frac{dw_2/dF_1}{dw_1/dF_1}\right|_{p_1=p_2=p^*, F_1=F_2=F^*} = \frac{v(p^*)\phi_1(w^*, w^*)}{1 - v(p^*)\phi_1(w^*, w^*)}, \tag{11}$$

where $p^* = c + (a - c_0)/2$, and $\phi_1(w_1, w_2)$ is the derivative of ϕ with respect to w_1.[21] χ^* can be considered a measure of the importance of network externalities in equilibrium. As hinted before, since $v(p^*)$ is decreasing in a, (11) suggests that network externalities are more important when the access charge a is smaller.

What is the impact of χ^* on w^*? Intuitively, the more important network externalities are, and thus the larger χ^* is, the more each competitor will act as a monopolist, as a decrease in his own tariff then also benefits his rival's customers. One might thus conjecture that an increase in χ^*—through a decrease in the access charge—will bring w^* closer to w^M. In the Appendix, I prove that this intuition is correct:

Lemma 3. For $a = c_0$,

$$\frac{dw^*}{da^*}\left[w^* - w^M\right] > 0.$$

[20] See the Appendix, proof of Lemma A1, *Remark*, for a formal argument.

[21] See the Appendix for a formal derivation.

Proof. See the Appendix.

From Lemma 3, a lower access charge decreases w^* only when a decrease in consumer surplus w^* increases networks' profits, despite a decrease in overall consumer participation. This yields the following proposition:

Proposition 4. For a close to c_0, in any symmetric equilibrium:

(i) Profits are decreasing in a.

(ii) Total welfare is increasing in a if and only if $w^* > w^M$ (that is, if and only if two duopolists offer a larger net surplus to customers than would a monopolist).

Proof. Part (i) stems directly from Lemmas 2 and 3. I now discuss the impact of the access charge on total welfare, given by

$$S = S(w^*(a), a) \equiv \phi(w^*, w^*)\left[\phi(w^*, w^*)V(a - c_0) - w^* - f\right] + \Phi(w^*, w^*),$$

where $\Phi(w_1, w_2) \equiv E\left[\max\left\{w_1 + \varepsilon_1, w_2 + \varepsilon_2, w_0 + \varepsilon_0\right\}\right]$ is total consumer surplus. For $a = c_0$, $\partial V(a - c_0)/\partial a = 0$, from which

$$\left.\frac{dS(w^*, a)}{da}\right|_{a=c_0} = \frac{\partial S(w^*, a)}{\partial w}\frac{dw^*}{da}. \tag{12}$$

By the envelope theorem, $\partial\Phi(w_1, w_2)/\partial w_1 = s(w_1, w_2)$. (See, for example, Armstrong and Vickers (2001).) Hence,

$$\frac{\partial S}{\partial w} = 2\phi_1(w^*, w^*)\left[\phi(w^*, w^*)V(a - c_0) - w^* - f\right] + 2\phi_1(w^*, w^*)\phi(w^*, w^*)V(a - c_0),$$

which is strictly positive because firms must obtain nonnegative profits in equilibrium. From Lemma 3, then $dS/da > 0$ for $a = c_0$ if and only if $w^* > w^M$. *Q.E.D.*

Proposition 4 removes the idea that the collusion concern should be associated with high access charges. Indeed, if demand for subscription is elastic and, hence, the industry exhibits network externalities, firms may increase their profits by setting the access charge below marginal cost (relative to a case where $a = c_0$). Whenever $w^* > w^M$, that is, whenever competition between the two networks results in a larger consumer surplus relative to a case in which both colluded explicitly, such a decrease in the access charge reduces consumer surplus and, hence, market participation. Intuitively, a social planner then prefers an access charge above marginal cost. In contrast, if $w^* < w^M$, an access charge below marginal cost increases consumer surplus, and both a social planner and the networks prefer an access charge below marginal cost. As argued above, $w^* > w^M$ is likely unless networks are very differentiated and/or network externalities are very important.[22]

While Proposition 4 only shows how profits move with the access charge for a close to c_0, in simulations with a constant elasticity demand and logit preferences,[23] I always found an access charge $a^* < c_0$, such that symmetric equilibrium profits are maximized for $a = a^*$. In contrast, in these simulations, total welfare was maximized for $a = a^R > c_0$.[24]

[22] In my model, reciprocal access charges are optimal. In the context of mobile telecommunications, however, it may be optimal to set the fixed-to-mobile access charge above the mobile-to-fixed access charge (and above cost). This subsidizes the mobile segment and boosts consumer participation. See Armstrong (2002) for a formal argument.

[23] See footnote 18 for a description of the logit model. The parameter values used were the following: constant elasticity $\eta = 1.5$, $c_1 = c_0 = .05$ (so $c = .15$), $f = 2$, and ρ respectively equal to 10, 50, and 100.

[24] I am grateful to Jullian Wright, Steve Poletti, and Aaron Schiff for providing me with these simulations. Independently, Schiff (2002) has developed an alternative model of partial consumer participation. Although Schiff cannot solve his model analytically, simulation results are exactly as suggested by Proposition 4.

In my model, network externalities are industrywide: the value of a subscription is increasing in the total number of subscribers to any of the two networks. One of my assumptions, however, is that networks are not allowed to charge a different price for intra- and internetwork calls.[25] If one allows for such termination-based price discrimination, firm-specific network externalities may arise even with full market participation. Indeed, whenever $a > c_0$ ($a < c_0$), firms then charge a higher (lower) price for internetwork calls than for intranetwork calls. All else equal, the value of a subscription is then increasing (decreasing) in the market share of the network one subscribes to. Gans and King (2001) show that, as with industrywide network externalities, the presence of such firm-specific network externalities induces firms to agree on an access charge below marginal cost ($a < c_0$). Intuitively, if off-net calls are cheaper than on-net calls, customers prefer to subscribe to a smaller network, reducing the incentives of firms to compete for market share.[26]

6. Concluding remarks

■ Previous research on network interconnection and two-way access has mainly focused on linear pricing, thereby emphasizing the collusive power of a high access charge. Given that competition between telecommunications operators is de facto competition in nonlinear pricing, this can only be justified by an implicit assumption that linear prices are a good shortcut to nonlinear prices. Though Laffont, Rey, and Tirole (1998a) show in a simple model with homogeneous customers that this collusive effect then completely disappears, both Laffont, Rey, and Tirole (1998a, 1998b) and Armstrong (1998) argue that once customers are heterogeneous in demand and marginal prices differ from marginal cost, results are likely to more closely resemble those under linear pricing. Subsequent research and policy-oriented publications have continued to report results obtained under the assumption of linear pricing, often without mentioning the result on two-part tariffs in Laffont, Rey, and Tirole (1998a) (see, for example, Carter and Wright (1999) and Doganoglu and Tauman (1996)). As a result, the idea that high access charges are an instrument of collusion has become widespread among policy makers.

This article shows that introducing customer heterogeneity does not restore the collusive effect of a high access charge. First, the access charge may have no effect on profits even when customers are heterogeneous, whether networks engage in second- or third-degree price discrimination. Second, if subscription demand is elastic and, hence, the industry exhibits network externalities, operators may increase their profits by agreeing on an access charge below the marginal cost of access, an act that typically decreases both overall welfare and consumer surplus relative to cost-based access pricing. In fact, total welfare and consumer surplus can typically be increased by setting the access charge above marginal cost.

From an economic theory perspective, the above results remove the idea that the collusion concern should be associated with high access charges. From an economic policy perspective, my results warn that defining the best regulation may be tricky. On the one hand, welfare maximization may require an access charge above marginal cost. On the other hand, in complex environments, the impact of the access charge on profits and welfare may *a priori* go either way, depending on often unobservable variables. Indeed, while I have shown that limited participation results in a collusive access charge below marginal cost, I have also argued that other forces may push the collusive access charge in a different direction. To the extent that these forces are difficult to evaluate by a regulator, ensuring that the access charge is close to cost is likely to be a good second-best policy.

To conclude, it should be noted that in this article I have analyzed competition between two symmetric operators, a situation that mirrors a well-developed industry. As networks then have similar preferences with respect to the level of the access charge, my focus has been on the

[25] This may be due to regulatory restrictions or may reflect that consumers are not aware of the network they are calling (for example, given the existence of local number portability).

[26] In contrast to my model, however, the welfare-maximizing access charge equals c_0.

potentially collusive nature of access agreements. My results, however, do not purport to speak about the potential need for regulation in the entry phase of competition, where entrants and incumbents may prefer a different (reciprocal) access charge.

Appendix

■ Proofs of Propositions 1 and 2 and Lemmas 2 and 3 follow.

□ **Competition in linear pricing.**

Proof of Proposition 1. (i) For given prices p_1 and p_2, α_L and α_H are given by

$$\alpha_s = \alpha_s(p_1, p_2) \equiv \frac{1}{2} + k_s\sigma\left[v(p_1) - v(p_2)\right] \qquad s = L, H,$$

where $k_s v(p)$ is the variable net surplus from a subscription,

$$k_s v(p) \equiv \max_q \left\{k_s^{1/\eta}u(q) - pq\right\} = k_s\left(\frac{p^{-(\eta-1)}}{\eta - 1}\right),$$

and $\sigma \equiv 1/2\tau$ is an index of substitutability between the two networks. Let us maximize network 1's profit, given by (2), over p_1. At a symmetric equilibrium, $p_1 = p_2 = p$ and $\alpha_i = \hat{\alpha}_i = 1/2$. Denoting $q^* \equiv q(p^*)$, the first-order condition yields

$$\frac{p^* - \left(c + \dfrac{a - c_0}{2}\right)}{p^*} = \frac{1}{\eta}\left(1 - 2\sigma\left[(1 + h)k(p^* - c)q^* - f + (a - c_0)\psi kq^*\right]\right), \tag{A1}$$

where

$$h \equiv \frac{\operatorname{var} k_i}{k^2} = \frac{\mu(k_L)^2 + (1 - \mu)(k_H)^2}{k^2} - 1$$

and

$$\psi \equiv \frac{\Delta k}{k}\left(\frac{\mu k_L}{k} - \ell\right).$$

We have that $\psi = 0$ for a balanced calling pattern, hence (A1) simplifies to (3). The proof of existence and uniqueness of the symmetric candidate equilibrium is an extension of Proposition 1 in Laffont, Rey, and Tirole (1998a).

(ii) and (iii) The symmetric equilibrium price p^* must satisfy the first-order condition

$$\begin{aligned}\left.\frac{\partial\pi_i}{\partial p_i}\right|_{p_1=p_2=p^*} &= \frac{k}{2}\left[R'(p^*) - a - c_0/2q'(p^*)\right] - \sigma kq(p^*)\left[k(1 + h)R(p^*) - f\right] \\ &= \frac{k}{2}\frac{q^*}{p^*}\left[(\eta - 1)(p^M - p^*) + \eta a - c_0/2 - 2\sigma p^*\hat{\pi}(p^*)\right] = 0,\end{aligned}$$

with $\hat{\pi}(p) = k(1 + h)R(p) - f$ and $R(p) = (p - c)q(p)$. Using the implicit function theorem, I obtain

$$\frac{\partial p^*}{\partial a} = \frac{\dfrac{\eta}{2}}{\eta - 1 + 2\sigma\left[\hat{\pi}(p^*) + p^*\hat{\pi}'(p^*)\right]}$$

and

$$\frac{\partial p^*}{\partial k} = \frac{-2\sigma(1 + h)p^*R(p^*)}{\eta - 1 + 2\sigma\left[\hat{\pi}(p^*) + p^*\hat{\pi}'(p^*)\right]}.$$

Using exactly the same argument as Laffont, Rey, and Tirole,[27] it is shown that the denominator of both terms is positive. It follows that $\partial p^*/a$ is positive and $\partial p^*/\partial k$ is negative. By the same argument, $\partial p^*/\partial h$ is negative also. *Q.E.D.*

[27] Laffont, Rey, and Tirole (1998a) explain the uniqueness and monotonicity of p^* with respect to a; just substitute for $\pi(p)$ with $\hat{\pi}(p)$ and use $\pi(p) > 0 \Rightarrow \hat{\pi}(p) > 0$.

Proof of Proposition 2.

(i) See proof of Proposition 1.

(ii) The symmetric equilibrium price p^* must satisfy the first-order condition,

$$\begin{aligned}
&\frac{k}{2}\left[R'(p^*) - a - c_0/2q'(p^*)\right] - \sigma k^2 q(p^*)\left[(1+h)R(p^*) - f/k\right] - \psi(a-c_0)\sigma k q(p^*)^2 \\
&= \frac{k}{2}\frac{q^*}{p^*}\left[(\eta - 1)(p^M - p^*) + (\eta - 4\sigma k p^* q^* \psi)\frac{a-c_0}{2} - 2\sigma p^* \hat{\pi}(p^*)\right] \\
&= 0,
\end{aligned}$$

and thus

$$\frac{\partial p^*}{\partial a} = \frac{\frac{\eta}{2} - 2\psi k p^* q(p^*)\sigma}{\eta - 1 + 2\sigma\left[\hat{\pi}(p^*) + p^*\hat{\pi}'(p^*) - \psi q^*(\eta-1)(a-c_0)\right]}.$$

As for $a = c_0$, p^* is independent of ψ and the denominator is positive, so it follows that $\partial^2 p^*/\partial a \partial\psi < 0$. By continuity, this also holds for a close to c_0. To conclude the proof, I thus only have to show that if $\psi > 0$ (heavy-biased calling pattern), p^* decreases with a if and only if $4\sigma\psi k\tilde{p}q(\tilde{p}) > \eta$, where $\tilde{p}$ denotes the equilibrium price in case $a = c_0$. For a close to c_0, the denominator of $\partial p^*/\partial a$ is positive and I denote by A the largest access charge below which this denominator is still strictly positive. I show that if for $a = c_0$, $4\psi k\tilde{p}q(\tilde{p})\sigma > \eta$ and thus $\partial p^*/\partial a < 0$, then $\partial p^*/\partial a < 0$ for any access charge smaller than A. Indeed, suppose that there exists an access charge $a' < A$ for which $\partial p^*/\partial a > 0$ and thus $4\psi k p^* q(p^*)\sigma < \eta$; then, by continuity, there exists an access charge $a'' < a'$ for which $4\psi k p^* q(p^*)\sigma = \eta$. Consequently, at a'', all the derivatives of p^* with respect to a are zero, so that $\partial p^*/\partial a = \partial p^*/\partial a|_{a=a''} = 0$ for any access charge smaller than A. As a result, $4\psi k\tilde{p}q(\tilde{p})\sigma = \eta$ also, a contradiction. By the same argument, if $4\psi k\tilde{p}q(\tilde{p})\sigma < \eta$, then for any access charge smaller than A, $4\psi k p^* q(p^*)\sigma < \eta$ and p^* increases with a. *Q.E.D.*

☐ **Elastic subscription demand.**

Remark on notation. In what follows, $s_i(w_1, w_2)$ and $\phi_i(w_1, w_2)$, $i \in \{1, 2\}$, denote the derivative with respect to w_i of $s(w_1, w_2)$ and $\phi(w_1, w_2)$.

First-order conditions. To prove Lemmas 2 and 3, I first derive the first-order conditions and show that in a symmetric equilibrium, usage prices $p_1 = p_2 = p^*$ are equal to the average marginal cost of a call. The profits of firm 1 are given by

$$\pi_1 = s(w_1, w_2)\left[F_1 + \left[p_1 - c - s(w_2, w_1)/\phi(w_1, w_2)(a - c_0)\right]\phi(w_1, w_2)q(p_1) - f\right] \tag{A2}$$

$$+ s(w_2, w_1)\phi(w_1, w_2)q(p_2)\frac{s(w_1, w_2)}{\phi(w_1, w_2)}(a - c_0), \tag{A3}$$

which, from (9), can be rewritten as

$$\begin{aligned}
\pi_1 &= s(w_1, w_2)\left[\phi(w_1, w_2)\left(v(p_1) + (p_1 - c)q(p_1)\right) - w_1 - f\right] \\
&\quad + s(w_2, w_1)s(w_1, w_2)\left[q(p_2) - q(p_1)\right](a - c_0).
\end{aligned} \tag{A4}$$

I look for the candidate symmetric equilibrium that satisfies the first-order conditions with respect to F_1 and w_1. Let F^* and p^* be the symmetric equilibrium fixed fee and usage fee, and $w^* = w(p^*, F^*)$ the resulting net surplus to customers.

From (9), we have that

$$\frac{dw_2}{dF_1} = \frac{\partial\phi(w_1, w_2)}{\partial w_1}v(p_2)\frac{dw_1}{dF_1} + \frac{\partial\phi(w_1, w_2)}{\partial w_2}v(p_2)\frac{dw_2}{dF_1}.$$

Let us denote

$$\chi^* \equiv \left.\frac{dw_2/dF_1}{dw_1/dF_1}\right|_{p_1=p_2=p^*, F_1=F_2=F^*}.$$

Since $\phi_2(w, w) = \phi_1(w, w)$, then

$$\chi^* = \phi_1(w^*, w^*)v(p) + \phi_1(w^*, w^*)v(p)\chi^*$$

and thus

$$\chi^* = \frac{\phi_1(w^*, w^*)v(p^*)}{1 - \phi_1(w^*, w^*)v(p^*)}. \tag{A5}$$

For w_1 and w_2 to be continuous in F_1 and F_2, one must have that $\chi^* < 1$, or $\phi_1(w^*, w^*)v(p) < 1/2$. Similarly, we have that

$$\frac{dw_2}{dp_1} = \frac{\partial\phi(w_1, w_2)}{\partial w_1} v(p_2) \frac{dw_1}{dp_1} + \frac{\partial\phi(w_1, w_2)}{\partial w_2} v(p) \frac{dw_2}{dp_1}$$

and thus also

$$\left.\frac{dw_2/dp_1}{dw_1/dp_1}\right|_{p_1=p_2=p^*, F_1=F_2=F^*} = \chi^*.$$

Assuming a symmetric equilibrium, from (9), the first-order condition with respect to F_1 yields

$$Z(w^*, \chi^*, a)\frac{dw_1}{dF_1} = 0, \tag{A6}$$

where

$$\begin{aligned} Z(w^*, \chi^*, a) \equiv & \left[s_1(w^*, w^*) + \chi^* s_2(w^*, w^*)\right] \pi^* \phi_1(w^*, w^*) \left[1 + \chi^*\right] s(w^*, w^*) \left[v(p^*) + (p^* - c)q(p^*)\right] \\ & - s(w^*, w^*), \end{aligned}$$

with π^* denoting equilibrium profits per customer:

$$\pi^* \equiv \phi(w^*, w^*) \left(v(p^*) + (p^* - c)q(p^*)\right) - w^* - f.$$

Similarly, assuming a symmetric equilibrium, the first-order condition with respect to p_1 yields

$$Z(w^*, \chi^*, a)\frac{dw_1}{dp_1} + s(w^*, w^*)\phi(w^*, w^*) \left[p^* - c - \frac{a - c_0}{2}\right] \frac{\partial q(p^*)}{\partial p} = 0. \tag{A7}$$

Together with (A6), the latter implies that $p^* = c + (a - c_0)/2$.

Proof of Lemma 2. Given $p^* = c + (a - c_0)/2$, symmetric equilibrium profits can be written as

$$\pi_i(w^*, w^*) \equiv s(w^*, w^*) \left[\phi(w^*, w^*)V(a - c_0) - w^* - f\right], \tag{A8}$$

where

$$V(a - c_0) \equiv v\left(c + \frac{a - c_0}{c}\right) + \frac{a - c_0}{c} q\left(c + \frac{a - c_0}{c}\right).$$

For $a = c_0$, $V(a - c_0) = v(c)$ and $\partial V(a - c_0)/\partial a = 0$.

Hence, at $a = c_0$, $d\pi_i^*(w^*, w^*)/da$ equals

$$\frac{1}{2}\left[\begin{array}{c} 2\phi_1(w^*, w^*)\left(\phi(w^*, w^*)v(c) - w^* - f\right) - \phi(w^*, w^*) \\ +2\phi_1(w^*, w^*)\phi(w^*, w^*)v(c) \end{array}\right]\frac{dw^*}{da}, \tag{A9}$$

where I have used the fact that $s(w^*, w^*) = \phi(w^*, w^*)/2$.

What is the net surplus w^M that customers would receive if a monopolist owned the two firms? A monopolist would set $p^M = c$ and charge a fixed fee F^M such that resulting net surplus w^M maximized total industry profits

$$\phi(w, w)\left[\phi(w, w)v(c) - w - f\right]. \tag{A10}$$

Note that the derivative of (10) is given by the term in brackets of (A9). Since total industry profits are concave in w, it thus follows that at $a = c_0$,

$$\frac{d\pi_i^*(w^*, w^*)/da}{dw^*/da}$$

equals zero for $w^* = w^M$, is positive for $w^* < w^M$, and is negative for $w^* > w^M$. *Q.E.D.*

To prove Lemma 3, I first establish the following result:

Lemma A1. In a symmetric equilibrium, for $a = c_0$:

$$\left(\frac{dw^*}{da}\right) / \left(\frac{d\chi^*}{da}\right) < 0 \Leftrightarrow w^* > w^M, \tag{A11}$$

where

$$\frac{d\chi^*}{da} = \frac{\partial\chi^*}{\partial a} + \frac{\partial\chi^*}{\partial w^*}\frac{dw^*}{da}. \tag{A12}$$

Proof. From (A6), one must have that

$$\frac{dZ}{da} \equiv \frac{\partial Z}{\partial w}\frac{dw^*}{da} + \frac{\partial Z}{\partial\chi^*}\frac{d\chi^*}{da} + \frac{\partial Z}{\partial a} = 0.$$

For $a = c_0$, we have that $\partial V(a - c_0)/\partial a = 0$, and (A6) implies

$$\left(\frac{dw^*}{da}\right) / \left(\frac{d\chi^*}{da}\right) = -\left(\frac{\partial Z}{\partial\chi^*}\right) / \left(\frac{\partial Z}{\partial w}\right). \tag{A13}$$

I now show that for $a = c_0$, $\partial Z/\partial w^* < 0$, while $\partial Z/\partial\chi^* < 0 \Longleftrightarrow w^* > w^M$.

(i) For $a = c_0$, $\partial Z/\partial\chi^* < 0 \Longleftrightarrow w^* > w^M$. For $a = c_0$, Z can be rewritten as

$$\begin{aligned} Z = {} & \left[s_1(w^*, w^*) + s_2(w^*, w^*\right]\pi^* + 2\phi_1(w^*, w^*)s(w^*, w^*)v(c) - s(w^*, w^*) \\ & - \left(1 - \chi^*\right)\left[s_2(w^*, w^*)\pi^* + (1 - \chi^*)\phi_1(w^*, w^*)s(w^*, w^*)v(c)\right]. \end{aligned}$$

The expression in the first line is identical to the derivative of (industry) divided by 2 and, hence, equals zero for $w^* = w^M$, is positive for $w^* < w^M$, and is negative for $w^* > w^M$. Denoting

$$M \equiv s_2(w^*, w^*)\pi^* + (1 - \chi^*)\phi_1(w^*, w^*)s(w^*, w^*)v(c), \tag{A14}$$

we have that $\partial Z/\partial\chi^* = M$. Since $Z = 0$ and $(1 - \chi^*) > 0$, we must have that $M > 0$ if and only if $w^* < w^M$.

(ii) For $a = c_0$, $\partial Z/\partial w^* < 0$. For $a = c_0$, Z can be rewritten as

$$Z = s(w^*, w^*)\left[\frac{[s_1(w^*, w^*) + \chi^* s_2(w^*, w^*)]}{s(w^*, w^*)}\pi^* + \phi_1(w^*, w^*)\left[1 + \chi^*\right]v(c) - 1\right].$$

Since Z and thus also the expression in brackets equals zero, I need only show that the expression in brackets is decreasing in w^*.

From Assumption 1, $\phi_1(w^*, w^*)$ is decreasing in w^*. Furthermore, as $\chi^* < 1$ and $s_2(w, w) = \phi_1(w^*, w^*) - s_1(w^*, w^*)$, from Assumption 1, also

$$\frac{\partial}{\partial w^*}\left(\frac{s_1(w^*, w^*) + \chi^* s_2(w^*, w^*)}{s(w^*, w^*)}\right) < 0.$$

Finally,

$$\frac{\partial\pi^*}{\partial w^*} = 2\phi_1(w^*, w^*)v(c) - 1,$$

which must be negative because $\chi^* < 1$ implies $\phi_1(w^*, w^*)v(c) < 1/2$.

As all terms in the expression in brackets are positive and their derivatives negative, it follows that $\partial Z/\partial w^* < 0$.

Remark. Since for $a = c_0$, $w^* < w^M \Leftrightarrow M > 0$, from (A14), $w^* < w^M$ only if (i) $-s_2(w_1, w_2)$ is very small, (ii) $\phi_1(w^*, w^*)$ is very large, or (iii) fixed costs f are very large such that π^* is small. *Q.E.D.*

Proof of Lemma 3. Lemma A1 implies Lemma 3. Indeed, let us denote

$$\mathsf{F} \equiv \left(\frac{dw^*}{da}\right) / \left(\frac{d\chi^*}{da}\right)\Bigg|_{a=c_0}.$$

From Lemma A1, $\mathsf{F} > 0 \Leftrightarrow w^* < w^M$. From (A12), we have that

$$\mathsf{F}\frac{\partial \chi^*}{\partial a} = \left(1 - \frac{\partial \chi^*}{\partial w^*}\mathsf{F}\right)\frac{dw^*}{da}. \tag{A15}$$

From (A5), as $v(p^*)$ is decreasing in a and $\phi_1(w^*, w^*)$ is decreasing in w, we have that $\partial \chi^*/\partial a < 0$ and $\partial \chi^*/\partial w^* < 0$.

I consider three cases:

(i) First, suppose that $\mathsf{F} > 0$, then from (A15), $dw^*/da < 0$.

(ii) Second, if $\mathsf{F} = 0$, then $dw^*/da = 0$.

(iii) Third, if $\mathsf{F} < 0$, then as long as $(\partial \chi^*/\partial w^*)\mathsf{F} < 1$, from (A15), $dw^*/da > 0$. I argue now that $(\partial \chi^*/\partial w^*)\mathsf{F} \geq 1$ is impossible. Note first that given $\partial \chi^*/\partial w^* < 0$ and (A13), F is continuous in the level of fixed costs f. Assume that for some value of f, we have that $(\partial \chi^*/\partial w^*)\,\mathsf{F} \geq 1$. In the proof of Lemma A1(i), I have shown that $w^* < w^M$, and hence $\mathsf{F} > 0$, if and only if $M > 0$, where I have defined M by (A14) with $\pi^* = \phi(w^*, w^*)v(c) - w^* - f$. Since π^* must go to zero for f sufficiently large, from (A14), I can always find an f' sufficiently large such that $M > 0$ and, hence, $(\partial \chi^*/\partial w^*)\mathsf{F} < 0$. Since F is continuous in f, there exists then a value f'' for f such that $(\partial \chi^*/\partial w^*)\mathsf{F} = 1$, which is impossible given (A15). Hence, whenever $\mathsf{F} < 0$, it must be that $(\partial \chi^*/\partial w^*)\mathsf{F} < 1$ and thus $dw^*/da > 0$.

Lemma A1 together with observations (i)–(iii) yields Lemma 3. *Q.E.D.*

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