Relational Contracts With and Between Agents

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Abstract. Firms often use both objective/verifiable and subjective/non-verifiable performance measures to provide employees with effort incentives. We study a principal/two-agent model in which an objective team-based performance measure and subjective individual performance measures are available for contracting. A problem with tying rewards to subjective measures is that the principal may have incentives to understate the realization of those measures in order to reduce compensation. We compare two mechanisms for overcoming this credibility problem: bonus pools and reputation. While reputation is fostered by repeated interactions (a low discount rate), repeated interactions create opportunities for agent-agent collusion under bonus pools. These opportunities for collusion can be exacerbated by the team performance measure, to the point that it can be optimal to make the size of the bonus pool independent of the realization of the team measure. In general, strong task interdependencies—a strategic complementarity or a strategic substitutability of the objective team measure in the agents' actions—improve the effectiveness of reputation-based contracting and reduce the effectiveness of bonus pool arrangements.

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1. Introduction

Firms often use both objective/verifiable and subjective/non-verifiable performance measures to provide employees with effort incentives (Gibbs, 1995; Gibbs et al., 2004). While objective measures usually relate to the output of agents' productive actions—and thus reflect agents' effort only with noise—subjective measures often capture agents' effort input more precisely. At the same time, tying compensation to subjective measures creates credibility problems as the principal has incentives to claim the (non-verifiable) measure is low even though it is high. The key mechanisms suggested by the prior literature to mitigate this problem are *reputation* (in repeated relationships) and *bonus pools* (in multi-agent settings).[1] In this paper we study both these mechanisms in a multi-period/multi-agent setting and show that the presence of multiple agents working in a team makes reputation *more* effective, whereas repeated interactions make bonus pools *less* effective as a means to sustain implicit incentives.

Consider a team of agents collaborating to produce a joint output over an infinite horizon. In each period, the agents' actions can exhibit either strategic complementarity or strategic substitutability in that the marginal productivity of an agent's effort (i.e., the change in the probability that the team output is a success) is higher—or lower, respectively—if the other agent also chooses high effort. [2] Aside from verifiable team output, compensation can also be based on subjective performance measures that are agent-specific.

Under a bonus pool, the principal commits to an amount to be paid out to a group of agents and uses her subjective assessment of the agents' performance only in deciding how to divide the bonus pool among the agents. While, by construction, such an arrangement lends

[1] See, among others, Baker, Gibbons, and Murphy (1994), Levin (2002, 2003), MacLeod (2003), Rayo (2007) on the role of reputation; and Baiman and Rajan (1995), Rajan and Reichelstein (2006, 2009), and Ederhof (2007) on the role of bonus pools.

[2] An example of a team setting in which the agents' actions are strategic complements is a cross-functional team in which effort from each team member is necessary to pull off a success, since each team member provides a

credibility to implicit contracts, it has features of relative performance evaluation and therefore is susceptible to *collusion*. Such agent-agent relational contracting becomes a concern in particular for repeated interactions: the agents then can credibly commit to implicit side contracts to undermine the effort choices desired by the principal. As far as we are aware, ours if the first study to analyze bonus pools under repeated interactions.

In the most commonly encountered form of bonus pools, the total amount to be distributed among the agents is contingent on the objective performance measure.[3] Moreover, the payout tends to be symmetric in that agents who have been observed subjectively to have chosen the same effort level will receive the same bonus portion.[4] We show that the threat of collusion under such a bonus pool arrangement takes one of two forms. If their actions are strategic complements, the agents will side-contract on always choosing low effort in each period ("*Shirk*"); if their actions are strategic substitutes, the agents will conspire to alternate between one agent choosing high effort and the other agent choosing low effort and vice versa ("*Cycle*"). We characterize the contracting cost to the principal associated with a collusion-proof contract and show that this cost is particularly high if the discount rate is small and if the tasks are either strong complements or strong substitutes.

To illustrate the role of task interdependency, consider the case of strongly complementary efforts. In that case, the principal's "bribe" to an agent to get him to (unilaterally) deviate from *Shirk* is very costly because the associated increase in the

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unique input. An example of a team setting in which the agents' actions are strategic substitutes is one in which agent effort is interchangeable and there are decreasing returns to total effort.

[3]An example is described in Reuters, February 21, 2007: " A maximum bonus pool of $165 million has been established for a group of five senior executives that includes Bear Stearns Chief Executive James Cayne, the company said. Payout will be pegged to the company's return on equity. No executive can get more than 30 percent of the total pool, which can be as little as zero. Bear Stearns' compensation committee also approved the performance goals for a second bonus pool for seven other top executives. The maximum amount will be $140 million, with awards based on pretax return on equity, departmental income and expense controls. Bear Stearns did not disclose the exact performance targets."

[4] We also show that such a symmetric payout policy can be improved upon by favoring one agent off the equilibrium path.

probability of the bonus pool being paid out is small (the marginal productivity of unilateral effort is small for strong complements). Similar arguments demonstrate that the cost to the principal of breaking a collusive agreement of the *Cycle* variation is particularly high if the agents' tasks are strong substitutes. In summary, the cost of relational side contracting under bonus pools is particular high if (i) the technological interdependency between the agents is high and (ii) the discount rate is small (or, equivalently, their expected collaborative time horizon is long).

As an alternative to bonus pools, the principal could write individual (relational) contracts, one with each agent.[6] To make the principal's implicit promises credible, the agents threaten to insist on purely objectively determined rewards in all future periods (or to quit), should the principal ever renege on her promises. It is well known that the smaller the discount rate, the more credible the principal's promises as future punishments by the agents then would be more costly. A more subtle result that emerges from Baker, Gibbons, and Murphy (1994) is that a poor quality (less informative in the sense of Holmstrom, 1979) objective performance measure may be desirable. The threat of having to rely on such a poor quality performance measure in future periods makes it more credible for the principal to honor his promises in the current period. In our team setting, the production technology (whether the agents' efforts are substitutes or complements with regard to team output) plays a key role in determining the quality of the fallback contract and, thereby, the principal's ability to commit to implicit contracts.

If the agents' actions are strategic complements, they can punish the principal for reneging on the implicit contract by insisting that in all future periods the contract be based only on the (objective) team performance measure and that the contract ensure that working hard is the only equilibrium. A naïve fallback incentive contract with complementary efforts however would produce both a working and a shirking equilibrium. To eliminate the shirking equilibrium, higher-powered incentives are needed. The associated cost of deterring off-equilibrium "bad" play helps the principal commit to honoring implicit contracts. If instead the agents' actions are strategic substitutes, then high effort will be the unique equilibrium under the naïve fallback contract. As a result, the optimal contract turns out to be a stark one. It uses either only the subjective individual performance measures (for low discount rates) or only the objective team performance measure (for high discount rates). Compared with strategic complements, the principal's ability to use implicit incentives is more limited under strategic substitutes. Moreover, with strategic complements, relational contracts are feasible even in finite horizon settings, not so for actions that are substitutes.

The preceding arguments imply that individual contracts (principal-agent relational contracts) are the preferred solution for low discount rates (expected long-term relationships) and strong interdependencies among tasks. Bonus pools on the other hand perform well for high discount rates (expected short-term relationships) and tasks that are technologically largely independent. So, while relational contracts between the principal and agents can enhance efficiency, and should hence be bolstered by cultivating long-term relationships, relational contracts among agents tend to undermine bonus pool arrangements. Job rotation might mitigate such problems as it shortens the agent-agent relationship horizon without affecting the horizon of the principal-agent relationship.[7]

We also consider alternative payout policies for bonus pools. In particular, the principal may be better off committing to pay out a fixed bonus pool, independent of the realization of the objective performance measure ("pay without performance"). This seems to contradict conventional wisdom that the size of bonus pools should be varied with some

[6] The relational contracting literature has focused for the most part on principal-single-agent models. Two notable exceptions are Rayo (2007), who studies relational contracts in a team setting *without* a principal (i.e., imposing a budget balance constraint), and Levin (2002), who compares bilateral with multilateral contracts.
[7] A related argument is made by Tirole (1986) in a hierarchical principal-supervisor-agent model.

objective measure of performance. A fixed bonus pool is preferred when the agents' actions are strong strategic complements or strong strategic substitutes in the objective team-based measure. Lastly, we show that the threat of collusion can be reduced (although not eliminated) by adopting asymmetric payout policies whereby the principal favors one of the agents in the (off-equilibrium) event that both choose low effort.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Sections 3 and 4, respectively, address individual contracts and bonus pools in an infinitely repeated relationship. Section 5 presents results for a finite-horizon (two-period) version of the model, and Section 6 concludes.

2. Model

A principal contracts with two agents, $i = A, B.$ Each agent i provides personally costly effort $a_t^i \in \{L, H\}$ in period t, where $L = 0 < H$. In a joint and stochastic fashion, these efforts result in concurrent team output $x_t \in \{0,1\}$. In particular, let

$$\begin{aligned} p_H &= Pr(x_t = 1 \mid a_t^A = a_t^B = H), \\ &> p = Pr(x_t = 1 \mid a_t^i = H, a_t^j = L), \quad i \neq j, \\ &> p_L = Pr(x_t = 1 \mid a_t^A = a_t^B = L). \end{aligned}$$

The team output x_t is commonly observable and contractible. Aside from output, compensation contracts can also depend on signals, $y_t^i \in \{0,1\}$, about agent i's effort in period t. While these signals are more informative, we assume they are *non-verifiable, subjective* metrics. Any contractual obligations based on them therefore need to be self-enforcing. We consider short-term contracts, only. At the beginning of period t, the principal offers agent i the compensation contract $w_t^i(x_t, (y_t^i, y_t^j))$. Agent i then either accepts the contract or leaves the employment relationship and receives a reservation utility of 0 in perpetuity. If an agent quits, the firm will shut down or, equivalently, the principal would have to incur prohibitively high search costs in order to find a replacement for the agent.

For simplicity, we assume the principal observes a_t^i perfectly, i.e., $y_t^i = 1$ if and only if $a_t^i = H$. Because the subjective measures are perfect, the optimal contract can then be expressed as an additive contract of the following form:

$$w_t^i = \alpha \hat{y}_t^i H + \beta x_t$$

where $\hat{y}_t^i \in \{0,1\}$ is the principal's report of the subjective metric regarding agent i's effort, a_t^i. We adopt this additive representation because of its ease in conveying the intuition.

Both agents and the principal are risk-neutral and share a common discount rate of r; r can be interpreted as capturing the time value of money or as the probability the relationship will end at the conclusion of the current period (or a combination of the two). The agents are protected by limited liability in that $w_t^i \geq 0$ for all i, t. We assume that agent effort is sufficiently important that the principal always finds it worthwhile to elicit high effort from each agent in each period. Agent i's period-t payoff is normalized to $w_t^i - a_t^i$, and the principal's period-t payoff is $x_t - \sum_i w_t^i$.

We evaluate the efficiency of any contractual arrangement by the expected periodic cost, C_t, to the principal of eliciting (H,H) efforts from the agents. As a benchmark, in the first-best solution agent efforts are contractible, so that the principal would simply direct the agents to take high efforts in each period and reimburse them for their disutility. The resulting first-best expected periodic cost to the principal would equal $C_t^{FB} = 2H$. Another useful benchmark is the case of contracts based on only the objective measure (i.e., $\alpha_t^i = 0$), so that $w_t^i(x_t,(y_t^i, y_t^j)) \equiv w_t^i(x_t)$. It is straightforward to see that playing (H,H) will then be a Nash equilibrium for the two agents provided the principal sets $\beta \geq \beta^{HH} \equiv \frac{H}{p_H - p}$. Since making this inequality strict would result in excessive rents earned by the agents, the expected periodic cost to the principal equals $\bar{C}_t \equiv 2 p_H \beta^{HH} = \frac{p_H}{p_H - p} 2H$. Note also that playing (L,L) is *not* a Nash equilibrium whenever $\beta \geq \beta^{LL} \equiv \frac{H}{p - p_L}$.

Throughout the paper, we assume the agents perfectly observe each other's efforts and they will play as the principal intends as long as doing so constitutes a subgame perfect Nash equilibrium in the overall game, which is not Pareto-dominated by any other subgame perfect Nash equilibrium.

We will distinguish between two cases: efforts are either *strategic complements* in that $p_H - p > p - p_L$ (equivalently, $Z \equiv p_H - 2p + p_L > 0$), or *strategic substitutes* in that

$p - p_L \geq p_H - p$ (i.e., $Z < 0$). To illustrate these two cases, consider again the benchmark case where contracts are based on only the objective measure, i.e., $\alpha_t^i = 0$. If efforts are strategic complements, then $\beta^{HH} < \beta^{LL}$ and, therefore, for any $\beta \in \left[\beta^{HH}, \beta^{LL}\right]$ there exist two pure-strategy Nash equilibria, (H,H) and (L,L). For (H,H) to be the unique (in fact, a dominant-strategy) equilibrium, $\beta > \beta^{LL}$ has to hold. If efforts are strategic substitutes, on the other hand, then $\beta^{HH} \geq \beta^{LL}$ and (absent implicit contracts) the effort profile (H,H) constitutes the unique pure-strategy equilibrium for any $\beta > \beta^{HH}$.[8]

We consider an infinitely repeated contractual relationship, first assuming the principal in each period offers the agents individual contracts and then allowing for bonus pool arrangements.

3. Individual Rewards: Principal-Agent Relational Contracting

We first derive the optimal contract offered to the agents individually when both objective and subjective measures are available for contracting. Assuming the principal honors the implicit contract, playing (H,H) constitutes a Nash equilibrium for the agents if and only if

$$\alpha H + (p_H - p)\beta \geq H. \tag{1}$$

The agent's periodic expected rent is $U_t^i = \alpha H + p_H \beta - H$. There exists a one-dimensional set of incentive coefficients $\left\{(\alpha, \beta^*(\alpha)) \mid \beta^*(\alpha) \equiv (1-\alpha)\beta^{HH}\right\}$ satisfying (1). That is, the explicit and implicit performance measures are substitutes in providing effort incentives. Plugging $\beta^*(\alpha)$ into U_t^i yields

$$U_t^i(\alpha) \equiv U_t^i(\alpha, \beta^*(\alpha)) = (1-\alpha)H \frac{p}{p_H - p},$$

which is decreasing in α. Because the implicit measure y_t^i is assumed noiseless, the principal wants to set α as high as possible so as to reduce limited-liability related rents. However, since y_t^i is a non-verifiable measure, there are limits to the principal's power to commit to this measure.

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[8] With efforts being strategic substitutes, there exist two asymmetric Nash equilibria (L,H) and (H,L) for $\beta^{LL} < \beta < \beta^{HH}$, and a unique (dominant-strategy) equilibrium (L,L) for $\beta < \beta^{LL}$.

We consider the following trigger strategy to support the implicit contract. As long as the principal honors the implicit contract, the agents are willing to play the desired (H,H) strategy (provided it constitutes a subgame perfect equilibrium that is not Pareto-dominated by any other subgame perfect equilibrium). If however the principal reneges on the implicit contract by claiming that $\hat{y}_t^i = 0$ for some i and t, whereas in fact $a_t^i = H$, then both agents will not respond to implicit incentives any longer. Instead they will punish the principal by playing (L,L) indefinitely, unless (H,H) is a unique equilibrium under the continuation contract which is solely dependent on the objective measure.

Strategic Complements

Recall that with efforts being strategic complements ($Z > 0$), by setting $\beta = \beta^{HH}$ the principal makes (H,H) a Nash equilibrium, but not a unique one. As a result, the contract offered by the principal has to satisfy the following *reneging constraint*:

$$\alpha H \leq \frac{1}{r}\left[p_H \beta^{LL} - (\alpha H + p_H \beta^*(\alpha))\right].$$

The left-hand side of this constraint gives the principal's benefit of reneging on the implicit contract, in which case she would save αH for each agent. The right-hand side states the principal's cost of reneging: to prevent the agents from playing (L,L) in all future periods, the principal will have to raise the explicit bonus from $\beta^{HH} = \frac{H}{p_H - p}$ to $\beta^{LL} = \frac{H}{p - p_L}$ in order to ensure the (H,H) equilibrium is unique in the stage game, while at the same time avoiding the expected "status quo" compensation of $\alpha H + p_H \beta^*(\alpha)$. By revealed preference, the term in square brackets on the right-hand side is strictly positive for any $\alpha > 0$ (otherwise, the principal would not have set a positive α to begin with). Plugging in $\beta^*(\alpha)$ and rearranging yields:

$$\alpha\left(r - \frac{p}{p_H - p}\right) \leq p_H \left(\frac{1}{p - p_L} - \frac{1}{p_H - p}\right). \tag{2}$$

The right-hand side of (2) is always positive for strategic complements, whereas the left-hand sideis negative for $r < \frac{p}{p_H - p}$. In the latter case the optimal incentive weights are $\alpha^* = 1$ and $\beta^* = 0$. For $r > \frac{p}{p_H - p}$ the left-hand side is also positive and we can rewrite the reneging constraint as follows:

$$\alpha \le \bar{\alpha} \equiv \frac{p_H\left(\frac{1}{p-p_L}-\frac{1}{p_H-p}\right)}{r-\frac{p}{p_H-p}}.$$

The optimal α^* in the case of strategic complements therefore case is given by $\alpha^* = \min\{1, \bar{\alpha}\}$ and $\beta^* = (1-\alpha^*)\frac{H}{p_H-p}$. It is easy to show that $\bar{\alpha} \ge 1$ whenever $r \le \frac{Z+p}{p-p_L}$. As one would expect, the reneging constraint is easier to satisfy if: (i) the discount rate r is small and (ii) the agents' actions are strong strategic complements in that the difference $\left(\frac{1}{p-p_L}-\frac{1}{p_H-p}\right)$ is large. If the agents' actions are strategic complements, the expected cost to the principal equals

$$C_t^{comp} = 2[\alpha^* H + p_H B^*(\alpha^*)] = \bar{C}_t - 2H\frac{p}{p_H-p}\alpha^*.$$

Note that as α^* increases, such an arrangement becomes increasingly efficient and reaches first-best benchmark performance for α^* = 1.

Suppose we increase the degree of complementarity by decreasing p while holding p_L and p_H constant. By examining C_t^{comp} when both the objective and the subjective measures are used, one can see that the expected compensation cost is decreasing in the degree of the complementarity. The reason is that as p becomes smaller, the fallback contract, which has to ensure the (H, H) equilibrium is unique while relying on only the objective measure, becomes costlier. Proposition 1 and its corollary summarize the strategic complements case.

Proposition 1. If the agents' actions are strategic complements (i.e., $Z > 0$), then the expected per-period compensation cost is $C_t^{comp} = \frac{(p_H - \alpha^* p)}{p_H - p}2H$. The weight placed on the subjective measure equals:

$$\alpha^* = \begin{cases} \bar{\alpha} = \dfrac{p_H\left(\dfrac{1}{p-p_L}-\dfrac{1}{p_H-p}\right)}{r-\dfrac{p}{p_H-p}} \in (0,1), & \text{if } r > \dfrac{Z+p}{p-p_L}, \\ 1, & \text{otherwise.} \end{cases}$$

Corollary 1. If the agents' actions are strategic complements, the use of implicit incentives is:

(i) decreasing in the discount rate r and

(ii) increasing in the degree of strategic complementarity (captured by decreasing p while holding p_H and p_L constant).

Counting on the agents punishing the principal in a way that also punishes themselves is nothing new. In a repeated Prisoners' Dilema, one subgame perfect equilibrium is "tit-for-tat." The agents threaten to revert to the stage game equilibrium if the other ever defects from cooperating. (A difference between our setting and the repeated Prisoners' Dilema is that the stage game equilibrium is unique in the repeated Prisoners' Dilema.) In contrast, Bernheim and Whinston (1998) assume that only Pareto-unranked equilibria can be used as punishments. Under that alternative approach, the following analysis for the strategic subsitutes case also applies to strategic complements.

Strategic Substitutes

If instead the agents' actions are strategic substitutes ($Z < 0$), then the same (IC) constraint (1) applies in that (H,H) will be a Nash equilibrium for any $\beta = \beta^*(\alpha) = (1-\alpha)\beta^{HH}$. In the strategic substitutes case, this equilibrium can be made unique by increasing the bonus payment by any arbitrarily small positive amount. For ease of exposition, we ignore this small additional cost throughout the paper. The principal's reneging constraint for efforts that are strategic substitutes is

$$\alpha H \le \frac{1}{r}\left[p_H \beta^{HH} - (\alpha H + p_H \beta^*(\alpha)) \right],$$

which simplifies to

$$r \le \frac{p}{p_H - p}. \tag{3}$$

This condition is independent of α. If (3) is not satisfied, then implicit incentives will not be sustainable, i.e., $\alpha^* = 0$ and $\beta^* = \beta^{HH}$. If (3) is satisfied, then $\alpha^* = 1$ and $\beta^*(1) = 0$, i.e., the first-best solution obtains. As a result, there is a discontinuity at $r = \frac{p}{p_H - p}$, in that the first-best solution is realized if and only if implicit contracts are feasible: $C_t^{sub} = C_t^{FB}$ if (3) holds, and $C_t^{sub} = \bar{C}_t$ otherwise. In comparison with the reneging constraint (2) for the case of strategic complements, (3) is a stronger condition. Whenever (3) is satisfied, then so is (2), and the first-best solution will also be attainable under strategic complements. If (3) is not satisfied but (2) is, then no implicit incentives are feasible under strategic substitutes whereas some positive α^* can be applied under strategic complements, which will make the principal better off.

Suppose we again increase the degree of complementarity by decreasing p while holding p_L and p_H constant. By examining C_t^{sub}, one can see the expected compensation cost is discontinuously increasing in the degree of the complementarity. When $r - \frac{p}{p_H - p} < 0$, the first-best can be achieved; otherwise only the objective measure can be used. The reason for both the discontinuity and the non-monotonicity (when we put the strategic complements and strategic substitutes cases together) is that, under strategic complements, the off-equilibrium fallback contract uses the objective measure differently ($p - p_L$ comes into play) than under the on-equilibrium contract ($p_H - p$ comes into play). Under strategic substitutes, both the on- and off-equilibrium contracts use the objective measure in the same way (only $p_H - p$ comes into play). Proposition 2 and its corollary summarize the strategic substitutes case.

Proposition 2. Suppose the agents' actions are strategic substitutes.

(i) If $r > \frac{p}{p_H - p}$, then $\alpha^* = 0$ and $C_t^{sub} = \bar{C}_t = \frac{p_H}{p_H - p} 2H$.

(ii) If $r \le \frac{p}{p_H - p}$, then $\alpha^* = 1$ and $C_t^{sub} = C_t^{FB} = 2H$.

Corollary 2. If the agents' actions are strategic substitutes, the use of implicit incentives is:

(i) decreasing in the discount rate r and

(ii) discontinuously decreasing in the degree of strategic complementarity.

4. Bonus Pools: Agent-Agent Relational Contracting

As the preceding analysis has shown, a key impediment to relational contracting is the principal's limited ability to commit to honoring implicit contracts. Bonus pools avoid any such commitment issues. A bonus pool is a contractual agreement by which the principal commits to split some total bonus amount among a set of agents, where the total does not depend on any subjective signals observed. That way, the principal is indifferent as to how to split the total bonus as it is a sunk cost anyway. Earlier literature (e.g., Baiman and Rajan

1995, Rajan and Reichelstein 2006, 2009) has shown that bonus pools can be powerful contracting tools in static settings, but has remained silent on how these arrangements perform in dynamic settings. In this section, we show that the repeated nature of transactions can impede the effectiveness of bonus pools as a result of *agent-agent relational* contracting.

Within the class of bonus pool arrangements various payout policies are conceivable. We refer to a payout policy as *symmetric* if, given a total bonus pool amount B to be distributed in period *t*, each agent will receive $B/2$, if $y_t^i = y_t^j$; whereas the entire amount B goes to that agent with the higher y_t^i measure, if $y_t^i \neq y_t^j$. The payout policy is labeled *conditional* if is it contingent on the realization of the objective signal x_t. We will begin our analysis with the most commonly-encountered form of bonus pools in which the payout is *symmetric* and *conditional* in that neither agent receives any bonus based on y_t^i if $x_t = 0$, whereas the full amount *B* is paid out (to one agent or split evenly, depending on the subjective measures) if $x_t = 1$. Below we will consider alternative payout policies.[9]

4.1 Bonus Pools with Conditional, Symmetric Payout

We denote by $U_{kl}^{C,S}$ an agent's expected period-*t* utility from choosing effort $a_t^i = k$ under a bonus pool arrangement with *conditional, symmetric* payout when the other agent chooses effort $a_t^j = l$:

		Agent B: L	Agent B: H
Agent A	L	$U_{LL}^{C,S} = p_L \dfrac{B}{2}$, $U_{LL}^{C,S} = p_L \dfrac{B}{2}$	$U_{LH}^{C,S} = 0$, $U_{HL}^{C,S} = pB - H$
	H	$U_{HL}^{C,S} = pB - H$, $U_{LH}^{C,S} = 0$	$U_{HH}^{C,S} = p_H \dfrac{B}{2} - H$, $U_{HH}^{C,S} = p_H \dfrac{B}{2} - H$

Table 1: Payoffs under Bonus Pool with Conditional, Symmetric Payout

Under such an arrangement playing (H,H) will constitute a Nash equilibrium for the agents' stage subgame, if and only if the following incentive compatibility constraint is satisfied:

[9] In particular, we show below that asymmetric payout policies can help reduce collusion costs. While we confine attention to these discrete payout policies, it would be desirable for future research to characterize *optimal* payout policies in a more general contracting framework.

$$U_{HH}^{C,S}(B) \geq U_{LH}^{C,S}(B) \tag{4}$$

Denote by B^{HH} the (unique) bonus pool amount that satisfies this incentive constraint (4) as an equality. Using the above expressions for the agents' payoffs yields $B^{HH} = 2H / p_H$. Yet, other equilibria then may exist and, worse, they may Pareto-dominate (H,H) for the two agents. We therefore now turn to the issue of collusion — i.e., agent-agent relationship contracting — so as to derive the optimal collusion-proof contract. To that end, we will abstract from monetary transfers between agents, as those will be difficult to enforce.

Given the infinitely repeated nature of the game, even without monetary transfers among the agents, the side contracting space for the two agents is very rich. In general, the agents can agree to play any strategy profile $\mathbf{a}_0 = \left\{a_t^A, a_t^B\right\}_{t=0}^{\infty}$, $a_t^A \in \{L, H\}$, provided $\mathbf{a}_0$ forms a subgame perfect equilibrium. To support such a side contract, the agents would adopt a trigger strategy whereby each agent sticks to the side contract until some agent *j* defects, in which case agent i will retaliate by choosing $a_\tau^i = H,\ \tau > t$, indefinitely thereafter. That is, the agents would return to the (H,H) effort profile (as desired by the principal) in each subsequent period. It can be shown however that attention can be confined to two specific and intuitive collusive strategies:

Lemma 1. In designing a renegotiation-proof bonus pool with symmetric, conditional payout it is without loss of generality to consider only the following two collusive strategies:

(i) "*Shirk*": $\mathbf{a}_0^{SHK} = \left\{a_t^A = L, a_t^B = L\right\}_{t=0}^{\infty}$, i.e., each agent chooses low effort in each period.

(ii) "*Cycle*": $\mathbf{a}_0^{CYC} = \left\{\left\{a_t^A = H, a_t^B = L\right\}_{t=0,2,4,\ldots} \cup \left\{a_t^A = L, a_t^B = H\right\}_{t=1,3,5,\ldots}\right\}$, i.e., the agents alternate choosing high and low effort.[10]

Why would the *Cycle* strategy ever be the "binding" collusion constraint? After all, it involves agents incurring disutility of high effort every other period, whereas under *Shirk* effort cost is avoided altogether. Note however that the relevant constraint regarding collusion-proofness depends not on which collusive strategy yields the highest payoff to the agents, but on which collusive arrangement is most costly for the principal to *break up*. If the agents' actions are substitutes, then the *B* amount required to prevent side contracting of the

[10] We arbitrarily label Agent A the one to choose high effort in the first period. This is without loss of generality; the agents could toss a coin to determine who chooses high effort first.

Cycle type will be high because the probability of realizing a high outcome increases only marginally (from p to p_H) when the agents' action profile changes to (H,H) from (L,H). With complementary actions, on the other hand, it is relatively cheap for the principal to prevent collusion via *Cycle* because an agent who defects from colluding by choosing high effort benefits from a steep increase in the probability that $x_t = 1$ given that the other agent also chooses high effort. In that case, the binding collusion constraint will be to prevent *Shirk*.

In light of Lemma 1, we need only consider the following two collusion-proofness constraints: to prevent *Shirk*, *B* needs to be set high enough to ensure

$$U_{HL}^{C,S}(B) + \frac{U_{HH}^{C,S}(B)}{r} \geq \frac{1+r}{r} U_{LL}^{C,S}(B). \tag{5}$$

At the same time, preventing *Cycle* requires the following inequality to hold:

$$\frac{1+r}{r}\frac{U_{HH}^{C,S}(B)}{r} \geq \bar{U}_{0,i}^{cyc}(B), \quad i = A, B, \tag{6}$$

where $\bar{U}_{0,A}^{cyc}(B) = \sum_{t=0,2,4,...} \frac{pB-H}{(1+r)^t}$ and $\bar{U}_{0,B}^{cyc}(B) = \sum_{t=1,3,5,...} \frac{pB-H}{(1+r)^t}$ are the respective agents' expected utilities in present value terms when abiding to the *Cycle* side contract.

In summary, to derive the cost-minimizing collusion-proof bonus pool arrangement, the principal needs to ensure that:

- (H,H) is a Nash equilibrium in the stage game (requires $B \geq B^{HH}$);
- (L,L) is not an equilibrium in the stage game (requires $B \geq B^{LL}$); or if it is, it must be Pareto-dominated by the (H,H) equilibrium from the point of view of the agents (requires $B \geq B^{Pareto}$ — both B^{Pareto} and B^{LL} are derived in the Appendix);
- The bonus pool must be collusion-proof. Let B^{SHK} and B^{CYC}, respectively, denote the lower bounds for *B* defined by (5) and (6). By the above logic, it is easy to show that $B \geq B^{SHK} > B^{CYC}$ holds for complementary actions, whereas $B \geq B^{CYC} > B^{SHK}$ for substitute actions.

Our next result summarizes the contracting cost under this arrangement:

Proposition 3. The expected per-period compensation cost using a bonus pool with conditional, symmetric payout equals $C^{C,S} = p_H B^{C,S}$, where:

$$B^{C,S} = \begin{cases} B^{Pareto} = \dfrac{2H}{p_H - p_L}, & p < \dfrac{p_H}{2} \\ B^{SHK} = \dfrac{2(1+r)H}{p_H + 2rp - (1+r)p_L}, & p \in \left[\dfrac{p_H}{2}, \dfrac{p_H + p_L}{2}\right] \\ B^{CYC} = \dfrac{2(1+r)H}{(2+r)p_H - 2p}, & p > \dfrac{p_H + p_L}{2} \end{cases}$$

Proof: See Appendix.

For strongly complementary actions ($p < p_H / 2$) the threat of the agents colluding on *Shirk* is very costly to the principal. In response, the principal has to raise *B* to the point where (H, H) becomes the Pareto-dominant equilibrium. There is no need to increase *B* beyond that level, and hence B^{Pareto} constitutes an upper bound on the contracting cost for complementary actions. If efforts are mild complements ($p \in [p_H / 2, (p_H + p_L) / 2]$), the binding constraint is to prevent the agents from colluding on *Shirk*. For substitute efforts ($p > (p_H + p_L) / 2$), on the other hand, the relevant collusive strategy for the principal to combat is *Cycle*. Note that B^{SHK} is decreasing in *p*, whereas B^{CYC} is increasing in *p*.[11] Therefore:

Corollary 3.

(i) The expected per-period compensation cost is non-monotonic in *p*: (weakly) decreasing in *p* for complementary efforts, and strictly increasing in *p* for substitute efforts.

(ii) Bonus pools with conditional, symmetric payout never achieve first-best performance.

(iii) $C^{C,S}$ is weakly decreasing in *r*.

---

[11] With symmetric payout and substitute tasks ($Z < 0$), the payoffs for the agents of the stage game when colluding via *Cycle* cannot be Pareto-ranked with their respective payoffs from obeying the principal's intention by playing (H, H). The agent whose turn under *Cycle* it is to exert high effort always receives a higher payoff than under (H, H), but the reverse holds for the agent who is supposed to lie low. Hence, for substitute tasks there does not exist an upper bound on *B* akin to the upper bound B^{Pareto} in the case of complements.

Proof: Parts (i) and (ii) are trivial. As for part (iii), B^{Pareto} is independent of *r*, whereas differentiating B^{SHK} and B^{CYC} shows each term to be decreasing in *r*, if and only if $p \geq p_H / 2$. Given the cutoffs for *p* in Proposition 3, part (iii) of the Corollary follows. □

For a bonus pool to achieve first-best performance, the binding constraint has to be that (H,H) be a Nash equilibrium (because $p_H B^{HH} = C_t^{FB}$). Proposition 3 and its corollary however show that the specter of side contracting always prevents the principal from attaining this benchmark. Depending on the production technology, the principal always has to worry about the agents colluding via one of the two strategies defined in Lemma 1. From the viewpoint of the agents, the effectiveness of colluding via *Shirk* decreases as task complementarity becomes smaller (i.e., as p approaches $\frac{p_H + p_L}{2}$ from below). Similarly, collusion via *Cycle* becomes less effective as the degree of task substitutability decreases (i.e., as p approaches $\frac{p_H + p_L}{2}$ from above). As a result, the principal's contracting cost will reach its lowest level when tasks are "technologically independent," i.e., when $Z = 0$. See Figure 1 for illustration (the effective bonus pool amount $B^{C,S}$ is depicted in boldface).[12]

Lastly, recall that the preceding section has confirmed the intuition that when the principal contracts individually with each agent, efficiency will improve as the discount rate goes down, because the principal will then find it easier to commit to honor his promises. In the case of bonus pools, by the same logic, implicit contracts again perform better as *r* decreases. Now, however, the implications for the principal's welfare are reversed: the more patient all players, the more effectively the agents can side-contract and thereby obstruct the principal's intentions — this gives rise to part (iii) of Corollary 3.

[12] It is easy to show that preventing (L,L) is never a binding constraint. For $p < p_H / 2$, $B^{LL} > B^{SHK}$, but at the same time, $B^{LL} > B^{Pareto}$.

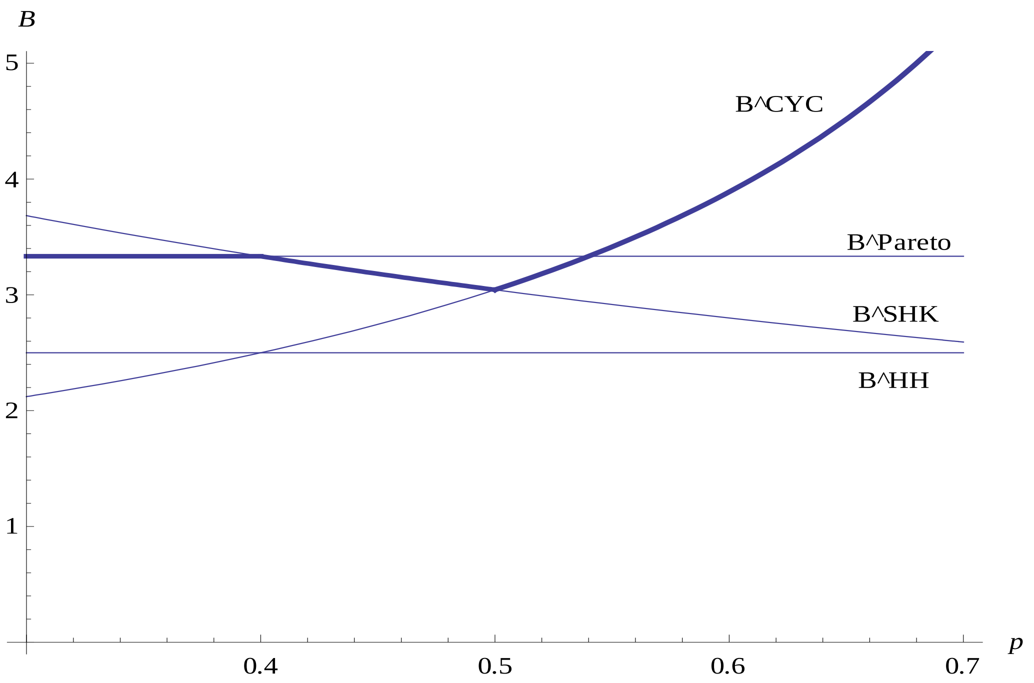

Figure 1: Bonus Pool with Conditional, Symmetric Payout for
($p_L = .2,\ p_H = .8,\ H = 1,\ r = .4$)

We are now in a position to compare the performance of individual contracting and bonus pools — the latter in their most common form with conditional and symmetric payout. Individual contracts perform particularly well for low discount rates and for settings that exhibit either strong complementarity or strong substitutability among the agents' actions. The reverse holds for bonus pools where high discount rates and technologically "independent" actions (Z close to zero) make it harder for the agents to collude.

Corollary 4. Let $C_t^{indiv} \in \left\{C_t^{sub}, C_t^{comp}\right\}$ denote the principal's per-period cost under individual contracting for efforts that are substitutes or complements, respectively (and recall that $C_t^{C,S}$ is the per-period cost under conditional-symmetric- bonus pools).

(i) If efforts are substitutes ($p > \frac{p_H + p_L}{2}$), then $C_t^{indiv} < C^{C,S}$ if and only if

$$r < \frac{p}{p_H - p} \equiv r_{(i)}.$$

(ii) If efforts are strong complements ($p < \frac{p_H}{2}$), then $C_t^{indiv} < C^{C,S}$ if and only if

$$r < \frac{p(p_H - p)}{(p - p_L)^2} \equiv r_{(ii)} .$$

(iii) If efforts are weak complements ($p \in \left[\frac{p_H}{2}, \frac{p_H + p_L}{2}\right]$), then $C_t^{indiv} < C^{C,S}$ if and only if

$$r < \frac{2pp_L - p_L^2 + \sqrt{16p^3(p_H + p_L) - 12p^4 + p_L^4 - 4p^2 p_H(p_H + 5p_L) + 4pp_L(p_H^2 + p_H p_L - p_L^2)}}{2(p - p_L)(3p - p_H - p_L)} \equiv r_{(iii)}$$

.

(iv) Furthermore, if the technological interdependence between the agents' tasks becomes stronger, the relative performance of individual contracting improves—i.e., $r_{(i)}$ is increasing in p, whereas $r_{(ii)}$ and $r_{(iii)}$ are decreasing in p.

Proof: Parts (i)-(iii) follows from straightforward comparison of the per-period costs given in Propositions 1-3. Equating C_t^{sub} with $p_H B^{CYC}$ yields $r_{(i)}$ for substitutes; equating C_t^{comp} with $p_H B^{Pareto}$ yields $r_{(ii)}$ for strong complements; and equating C_t^{comp} with $p_H B^{SHK}$ yields $r_{(iii)}$ for weak complements. Part (iv) is established by taking derivatives of $r_{(i)}$ and $r_{(ii)}$ with respect to p; the result that $r_{(iii)}$ is decreasing in p follows from the facts that C_t^{comp} is increasing p (Corollary 1) while B^{SHK} is decreasing in p. □

Since bonus pools perform poorly if agents are sufficiently patient and anticipate repeated team transactions with each other, this suggests job rotation as a way to reduce the cost of collusion to the organization.

4.2 Alternative Payout Policies

Unconditional Payout Policy

While in most firms the total financial reward to be distributed among employees by means of a bonus pool is contingent on some verifiable outcome such as income, EPS, or sales, we now allow for the possibility that the principal commits to paying out B *irrespective of the realization of* x_t. *A priori*, from an agency perspective, this is a plausible construction as we assume here that the principal can observe the agents' efforts perfectly, while the objective output measure x_t is observed with noise. Under such a bonus pool arrangement with *unconditional, symmetric payout ("U,S")*, the agents' payoffs are independent of the success probabilities for high output; instead they read:

		Agent B: L	Agent B: H
Agent A	L	$U_{LL}^{U,S}=\frac{B}{2}$, $U_{LL}^{U,S}=\frac{B}{2}$	$U_{LH}^{U,S}=0$, $U_{HL}^{U,S}=B-H$
	H	$U_{HL}^{U,S}=B-H$, $U_{LH}^{U,S}=0$	$U_{HH}^{U,S}=\frac{B}{2}-H$, $U_{HH}^{U,S}=\frac{B}{2}-H$

Table 2: Payoffs under Bonus Pool with Unconditional, Symmetric Payout

The requirement that (H,H) be a Nash equilibrium now reduces to $B\geq 2H$, which also rules out (L,L) as an equilibrium. But the principal again faces a collusion problem. Since now the payout is independent of the project success probabilities, the only relevant collusive strategy is *Shirk* (note that *Cycle* now is dominated from the agents' point of view by *Shirk* as the former involves positive effort costs without altering the total monetary rewards). To prevent the agents from colluding on *Shirk*, the principal needs to set *B* high enough so that the collusion-proofness condition (5) holds (with superscript "U,S" substituted for "C,S"). It is easy to show that this requirements boils down to $B\geq \frac{1+r}{r}2H$. Thus, with unconditional payout, the collusion-proofness constraint is always the binding one. Since the bonus is now paid out with probability one, the resulting cost to the principal of securing high effort from the agents is $C_t^{U,S}=\frac{1+r}{r}2H$. A comparison with the contracting cost under conditional bonus pool payout $C_t^{C,S}$ — characterized in Proposition 3 — yields our next result (the proof follows from straightforward algebra and is hence omitted).

Proposition 4. If efforts are strategic complements, an unconditional-symmetric payout policy dominates conditional-symmetric payout if and only if *r* is sufficiently high. For substitute efforts, conditional-symmetric payout dominates unconditional-symmetric payout for any *r*.

To illustrate, write out the collusion-proofness conditions in (5) for unconditional and conditional payout, respectively:

$$U_{HL}^{U,S}+\frac{U_{HH}^{U,S}}{r}=B-H+\frac{\frac{B}{2}-H}{r} \quad > \quad \frac{1+r}{r}U_{LL}^{U,S}=\frac{1+r}{r}\frac{B}{2},$$

$$U_{HL}^{C,S}+\frac{U_{HH}^{C,S}}{r}=pB-H+\frac{p_H\frac{B}{2}-H}{r} \quad > \quad \frac{1+r}{r}U_{LL}^{C,S}=\frac{1+r}{r}p_L\frac{B}{2}.$$

Rearranging terms shows how breaking away from *Shirk* affects the deviating agent's payoff in the *current* and in all *future* periods:

	T	t+1	…	t+n
Unconditional Payout ("*U*,*S*")	$\frac{B}{2}-H$	$-H$	…	$-H$
Conditional Payout ("*C*,*S*")	$pB-p_L\frac{B}{2}-H$	$-H+(p_H-p_L)\frac{B}{2}$	…	$-H+(p_H-p_L)\frac{B}{2}$

Table 3: Effect on the Payoff of an Agent who Deviates from *Shirk*

With efforts being strategic complements, the contemporaneous reward from defecting is smaller with conditional than with unconditional payout because there is a positive probability that the bonus pool will not be paid out—namely in case $x_t = 0$ (note that $2p - p_L < 1$ if $Z > 0$). At the same time, the future punishment from triggering a reversal back to (H,H) indefinitely is reduced under conditional payout by the fact that the probability of the bonus pool being paid out increases by $(p_H - p_L)$ in each period. Thus, if agents are sufficiently impatient, then it will be cheaper for the principal to entice them to deviate from *Shirk* by using an unconditional payout policy. This is the intuition behind the first part of Proposition 4.

For the second part of the proposition, when efforts are substitutes, the relevant collusion-proofness constraint under the conditional payout policy is to prevent *Cycle*, while it remains *Shirk* for the unconditional policy. Since *Cycle* forces each agent to incur the disutility of high effort every other period, it is less costly for the principal to induce them to break away from this side contract.[13] Figure 2 illustrates Proposition 4, with part a) depicting

[13] An interesting open question is what the optimal payout policy would look like. Proposition 4 indicates that it will not always pay out zero if $x_t = 0$ (as does our "conditional" payout policy), but also not always the same amount regardless of the realization of x_t (as does our "unconditional" payout policy). In a more general formulation, there are two bonus pool amounts, $B_k, k = 0,1,$ to be distributed between the agents conditional on

the case of *r* sufficiently high so that for strong complements (*p* small) the contracting cost under unconditional payout (dashed horizontal line) is less than with conditional payout. Part b) depicts the case of low *r* in which case conditional payout is always the dominant policy.

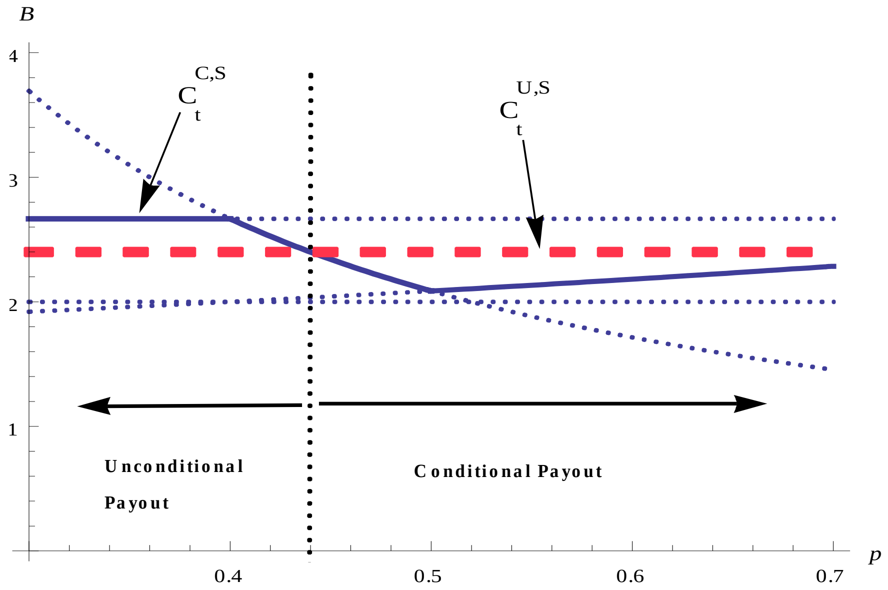

Figure 2a: High *r*

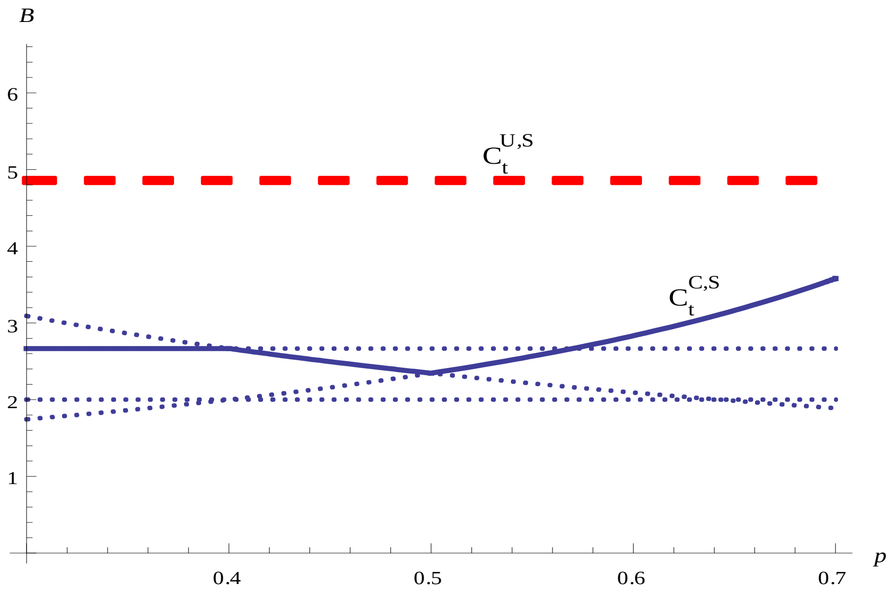

Figure 2b: Low *r*

<u>Figure 2</u>: Compare Conditional and Unconditional Payout Policies (Proposition 4)

$x_t = k$. Such a formulation method would nest the two symmetric payout regimes considered here. A conceptual challenge to the modeler, however, is that the agents' collusive strategies depend endogenously on the payout policy.

Asymmetric Payout Policy

Bonus pools with conditional payout suffer from two qualitatively different threats of collusion, *Shirk* and *Cycle*. The *Shirk* strategy in particular capitalizes on the fact that with complementary effort there may be multiple equilibria in the stage game, specifically (H,H) and (L,L). In a multi-agent adverse selection setting Demski and Sappington (1984) have shown that (some) undesirable equilibria can be eliminated by treating agents asymmetrically. That way, dominant-strategy incentives can be created for the disadvantaged player to comply with the principal's preferred action. The other player then chooses his best response.

Similar logic can be applied to our setting. Suppose without loss of generality that the principal discriminates against agent B by allocating the entire bonus pool amount to agent A if he observes (subjectively, but perfectly) that both agents have chosen low effort. The agents' expected payoffs under such a conditional, asymmetric payout policy then are as in Table 1 except for the (L,L) cell:

		Agent B L	Agent B H
Agent A	L	$U^{C,A}_{LL,A} = p_L B$, $U^{C,A}_{LL,B} = 0$	$U^{C,A}_{LH} = 0$, $U^{C,A}_{HL} = pB - H$
	H	$U^{C,A}_{HL} = pB - H$, $U^{C,A}_{LH} = 0$	$U^{C,A}_{HH} = p_H \dfrac{B}{2} - H$, $U^{C,A}_{HH} = p_H \dfrac{B}{2} - H$

<u>Table 4</u>: Payoffs under Bonus Pool with Conditional, Asymmetric Payout

It is easy to see that agent A now has dominant-strategy incentives to choose high effort provided $B \geq B^{HH}$. Agent B will respond by choosing high effort also, because (H,H) is a Nash equilibrium. It is easy to see that such an asymmetric payout policy deters collusion by the agents using the *Shirk* strategy. At the same time, the threat of agent side contracting via *Cycle* remains unmitigated, as this strategy is unaffected by the manipulation of the agents' payoffs in the (L,L) cell. Therefore, the bonus pool amount to be paid out in case $x_t = 1$ equals $B^{C,A} = \max\{B^{HH}, B^{CYC}\}$. Solving for the cutoff *p*-value that equates B^{HH} and B^{CYC} — and noting that the outcome of an analogous unconditional-asymmetric is identical to the unconditional-symmetric outcome described above[14] — yields:

Corollary 5. Bonus pools with conditional, asymmetric payout policy result in periodic contracting cost of $C^{C,A} = p_H B^{C,A}$, where:

$$B^{C,A} = \begin{cases} B^{HH}, & p < \dfrac{p_H}{2} \\ B^{CYC}, & p > \dfrac{p_H}{2} \end{cases},$$

and therefore $C^{C,A} \leq \min\left\{C^{C,S}, C^{U,A} = C^{U,S}\right\}$, i.e., this payout policy dominates all other bonus pool arrangements considered.

In summary, an asymmetric payout policy dominates symmetric ones as it costlessly eliminates the undesired (L,L) equilibrium. It also removes the indeterminacy regarding the ranking of conditional and unconditional payout. It is beyond the scope of this paper to address the question why asymmetric contracts favoring some agents over others are rarely seen in practice. Aside from obvious fairness problem, such contracts could suffer from excessive risk premiums in case agents are risk averse and subjective measures are observed with noise.

5. Finite Horizon Model

To illustrate the effects of a finite horizon, we now consider a two-period model. For simplicity, we set the discount rate, r, equal to zero as it plays less of a role now. This is clearly an extreme situation because in practice, while planning horizons are finite, they are usually subject to an uncertain ending date, which is conceptually equivalent to the above infinite horizon specification.

The scope for relational contracts is severely impaired in finite horizon settings due to the well-known unraveling problem. As a result, bonus pools become very effective because the agents will find it hard or impossible to collude. In fact, it is easy to show that bonus pools can then achieve first-best performance even their simplest and most common form (with conditional and symmetric payout). The reason is that, by backward induction, the two-period contracting problem collapses to a twofold repetition of a static bonus pool, in

[14] As for conditional payout, introducing asymmetric payouts in case the principal observes (L,L) eliminates the *Shirk* strategy for agent side contracting. Yet the agents again can collude via *Cycle*. Now, for unconditional

which the principal only needs to ensure (H,H) is a Nash equilibrium and, as such, is not Pareto-dominated from the agents' point of view by (L,L). Thus, conditional-symmetric bonus pools achieve first-best if and only if the agents' efforts are strategic substitutes. To close, we reconsider the alternative payout policies analyzed in Section 4.2. It is easy to see that in the two-period model both the unconditional (symmetric) payout policy, as well as the asymmetric, conditional payout policy will achieve first-best performance for any *p*. Under either of these alternative policies, by setting $B \geq B^{HH}$ the principal not just makes (H,H) a Nash equilibrium in each period, but also precludes (L,L) as an equilibrium.[15]

Using similar logic one would also expect that relational principal-agent contracting will become infeasible, again by backward induction. Surprisingly, however, this logic turns out to be incomplete. Suppose that principal contracts with each agents individually. Then in the second period implicit contracting is not credible since there are no future cooperation benefits to the relationship. So, the principal pays each agent a bonus of $\beta_2^{HH} = \dfrac{H}{p_H - p}$ if and only if $x_2 = 1$. Under strategic substitutes, the (H,H) equilibrium is unique in Period 2, so the agents have no threat to use against any promise by the principal in Period 1. Hence, under strategic substitutes, the principal uses only the objective performance measure in both periods, with a total (over two periods and two agents) expected compensation cost of $2\bar{C}_t = \dfrac{p_H}{p_H - p}4H$. That is, our earlier intuition holds for efforts are strategic substitutes: introducing a definitive end to the relationship at some future point rules our implicit incentives altogether.

Under strategic complements and the above contract, (L,L) is also an equilibrium in Period 2. Hence, the agents can threaten to play this equilibrium if the principal does not honor the implicit promise he made for Period 1. In response to the agents' threat, the principal would increase the bonus to ensure that the (H,H) equilibrium is unique. This enables the principal to make a promise to pay a subjective performance measure-based reward in Period 1that is bounded by the following reneging constraint:

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payout it is easy to show that the bonus pool amount required to prevent *Cycle* again just equals $[(1+r)/r]2H$.

[15] Bonus pools couples with either of the alternative payout policies analyzed in Section 4.2 achieve first-best performance for any *p*. Under either of these alternative policies, by setting $B \geq B^{HH}$ the principal not just makes (H,H) a Nash equilibrium in each period, but also precludes (L,L) as an equilibrium.

$$\alpha H \le p_H\left(\frac{H}{p - p_L} - \frac{H}{p_H - p}\right),$$

in analogy with the infinite horizon model. Denote by $\bar{\alpha}_1$ that value of α at which this period-1 reneging constraint is binding. Then the optimal period-1 weight on the subjective measure equals $\alpha_1^* = \min\{\bar{\alpha}_1, 1\}$ and the total expected compensation cost (over two periods and two agents) equals $\left(\frac{p_H - \alpha_1^* p}{p_H - p} + \frac{p_H}{p_H - p}\right)2H < 2\bar{C}_t$, as α_1^* will be strictly positive. Moreover, it is easy to see that as p decreases, α_1^* will increase and eventually reaches one, in which case period-1 contracting cost achieve the first-best level and total contracting cost (across two agents and two periods) equals $C_t^{FB} + \bar{C}_t$. Hence, once again, if the agents' actions are strategic complements, increasing the complementarity facilitates implicit contracting.

To summarize, under strategic substitutes, the problem behaves largely as a single-agent one because of the uniqueness of the equilibrium. With a finite horizon, relational contracts between the principal and a single agent unravel by backwards induction. Under strategic complements, the multiple equilibria that emerge in the last period facilitate implicit contracting in the first period.[16] Introducing a definitive end to the contracting relationship thus sharpens our predictions of Section 3 that effort complementarity is a key factor in facilitating implicit contracts. These findings are summarized in our next result.

Proposition 4. In the two-period setting:

(i) If the agents' actions are strategic substitutes, the subjective measure will not be used in a finite horizon setting.

(ii) If the agents' actions are strategic complements, the subjective measure will be used, and the use of implicit incentives is increasing in the degree of the complementarity.

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[16] Arya, Fellingham, and Glover (1997) study implicit side contracting and mutual monitoring among agents in a two-period model. The principal intentionally creates multiple equilibria in the agents' second-period subgame to facilitate the implicit side contracting, whether the actions are strategic complements or strategic substitutes.

6. Conclusion

Contracting relationships within firms are dynamic in nature, and they often involve technological interdependencies in that the firm (or division) output depends on a vector of actions taken by a number of productive agents. The expectation that the employment relationship endures creates scope for implicit incentives. While the prior literature has focused on the bright side of relational contracts (between principal and agents), this paper argues that there is a dark side too: agent-agent side contracting. As we have shown, the effectiveness of bonus pools is severely adversely affected by such collusion due to the tournament-like structure of such arrangements. If all parties are sufficiently patient and, thus, expect significant future gains to be had from current cooperation, then bonus pools will be dominated by individual contracting arrangements whereby the principal contracts with each agent unilaterally and replies on her reputation to pay out rewards tied to nonverifiable performance measures.

We also show that the greater the technological interdependencies among agents' productive efforts, the stronger the principal's preference for individual contracts, all else equal. Such individual contracts are particularly effective if the agents' actions are strategic complements because then the agents can credibly threaten to play the shirking equilibrium in case the principal reneges on her promises. The cost this punishment would impose on the principal serves as a useful commitment device for the latter to remain truthful.

A simplifying assumption throughout this paper is that all players observe the nonverifiable performance measure equally and without noise. Arguably, in many settings the principal will observe the agents' effort only with noise, even though the agents can observe each other's effort perfectly. While analyzing this extension is beyond the scope of this paper, it turns out that such observation noise reduces the relative performance of bonus pools even further. The reason is that the additional noise will strengthen the agents' incentive compatibility constraints without at the same time relaxing their collusion-proofness constraint (because they observe each other's efforts perfectly). A formal analysis of imperfect measurement, both for bonus pools and individual contracts, seems an interesting avenue for future work.

Appendix

Proof of Lemma 1: Consider any generic collusive strategy $\mathbf{a}_0$. Denote by $\mathbf{a}_t = \left\{a_\tau^A, a_\tau^B\right\}_{\tau=t}^{\infty}$ the continuation strategy at date *t*, and by $U_t^{cont}\left(B\middle|\mathbf{a}_t\right)$ the attendant continuation payoff for a representative agent in present value terms at date *t*. Under the collusive strategy $\mathbf{a}_0$, one of two cases can arise in any period *t:* (i) $a_t^A = a_t^B = L$ or (ii) $a_t^i = L$ and $a_t^j = H, \quad j \neq i$. In case (i), collusion will be prevented, if and only if

$$U_{HL}^{C,S}(B) + \frac{U_{HH}^{C,S}(B)}{r} \geq U_{LL}^{C,S}(B) + U_{t+1}^{cont}\left(B\middle|\mathbf{a}_{t+1}\right). \tag{7}$$

Denote by $B_{(i)}$ the value of *B* at which this requirement holds with equality.

In case (ii), collusion-proofness requires *B* to be high enough such that

$$U_{HH}^{C,S}(B) + \frac{U_{HH}^{C,S}(B)}{r} \geq U_{LH}^{C,S}(B) + U_{t+1}^{cont}\left(B\middle|\mathbf{a}_{t+1}\right). \tag{8}$$

Evaluating this condition at $B = B_{(i)}$, we find that (8) will be satisfied at $B_{(i)}$, if and only if

$$U_{HH}^{C,S}(B_{(i)}) - U_{LH}^{C,S}(B_{(i)}) \geq U_{HL}^{C,S}(B_{(i)}) - U_{LL}^{C,S}(B_{(i)}).$$

Simple algebra shows that this condition is equivalent to $Z \geq 0$. That is, if $Z \geq 0$, then the bonus pool amount required to prevent collusion in period *t* is higher if the agents collude on (L,L); whereas for $Z < 0$ it is higher if the agents in that period collude asymmetrically on (L,H) (or (H,L)).

In a last step, proceed recursively by applying similar arguments to period *t*-1 with $\mathbf{a}_t = \left\{\left(a_t^A, a_t^B\right) \cup \mathbf{a}_{t+1}\right\}$ where, by the preceding arguments, $\left(a_t^A, a_t^B\right) = \left(L, L\right)$ if $Z \geq 0$, and $\left(a_t^A, a_t^B\right) = \left(L, H\right)$ (or $\left(H, L\right)$) if $Z < 0$. Thus, if $Z \geq 0$, the binding collusion-proofness constraint is (7) (i.e., the agents adopt the *Shirk* strategy $\mathbf{a}_0^{SHK}$ as defined in the Lemma). For $Z < 0$, on the other hand, the binding collusion-proofness constraint is (8). The last step required to establish that the agents will adopt the *Cycle* strategy $\mathbf{a}_0^{CYC}$ for $Z < 0$ is to note that all (infinitely many) collusive strategies in which $a_t^i = L$ and $a_t^j = H, \quad j \neq i$, yield the same aggregate payoff to the agents in present value terms. To break this collusive arrangement, the principal needs to set *B* high enough to induce that agent to break away who has the lowest payoff in present value terms, the agents will settle on that collusive strategy which yields the most symmetrical payoffs in present value terms, among those described by $a_t^i = L$ and $a_t^j = H, \quad j \neq i$, i.e., they choose *Cycle*. $\square$

Proof of Proposition 3: As shown in the main text, for (H,H) to be an equilibrium *B* has to exceed $B^{HH} = \frac{2H}{p_H}$. At the same time (L,L) will *not* be an equilibrium, if and only if $U_{HL}^{C,S} \geq U_{LL}^{C,S}$, which is equivalent to $B \geq B^{LL} = \frac{2H}{2p - p_L}$. If $B < B^{LL}$, so that (L,L) is an equilibrium, then this equilibrium has to be Pareto-dominated by (H,H). Given the inherent symmetry among the agents, this amounts to:

$$U_{HH}^{C,S} \geq U_{LL}^{C,S} \quad \Longleftrightarrow \quad B \geq \frac{2H}{p_H - p_L} \equiv B^{Pareto}.$$

Collusion-proofness with regard to the *Shirk* strategy in Lemma 1 requires that

$$U_{HL}^{C,S}(B) + \frac{U_{HH}^{C,S}(B)}{r} \geq \frac{1+r}{r} U_{LL}^{C,S}(B) \quad \Longleftrightarrow \quad B \geq \frac{2(1+r)H}{p_H + 2rp - (1+r)p_L} \equiv B^{SHK}.$$

Lastly, consider agent-agent side contracting via the *Cycle* strategy in Lemma 1. Taking Agent A to be the one to choose high effort in period 0 (without loss of generality), the respective agents' expected utilities in present value terms read:

$$\bar{U}_{0,A}^{cyc}(B) = \sum_{t=0,2,4,\ldots} \frac{pB-H}{(1+r)^t} = \frac{(1+r)^2}{r(2+r)}(pB-H),$$
$$\bar{U}_{0,B}^{cyc}(B) = \sum_{t=1,3,5,\ldots} \frac{pB-H}{(1+r)^t} = \frac{\bar{U}_{0,A}^{cyc}(B)}{1+r}.$$

Agent B realizes a lower payoff than Agent A due to time value of money-reasons. To prevent such *Cycle* collusion, the principal has to entice the "weak link" — i.e., Agent B — to break away from the side contract:

$$\frac{1+r}{r} U_{HH}(B) \geq \bar{U}_{0,B}^{cyc}(B) \quad \Longleftrightarrow \quad B \geq \frac{2(1+r)H}{(2+r)p_H - 2p} \equiv B^{cyc}.$$

In the last step, it is a matter of straightforward algebra, holding constant (p_H, p_L), to derive cutoffs for *p* that permit a ranking of the relevant *B*-values; the three cases in Proposition 3 then follow. □

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