SINGLE MACHINE SCHEDULING PROBLEMS WITH GENERAL BREAKDOWNS, EARLINESS AND TARDINESS COSTS

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In this paper we consider single machine scheduling problems with a common due-date for all jobs, *arbitrary* monotone earliness and tardiness costs and *arbitrary* breakdown and repair processes. We show that the problem is equivalent to a deterministic one *without* breakdowns and repairs and with an *equivalent cost function* of a job's completion time. A V-shaped schedule without idle times is shown to be optimal, if this equivalent cost function is *quasi-convex*.

Conversely, we show that a V-shaped schedule may fail to be optimal if the property does not apply. We derive general conditions for the earliness and tardiness cost structure and repair and breakdown processes under which the equivalent cost function is *quasi-convex*. When a V-shaped schedule is optimal, an efficient (though pseudo-polynomial) algorithm can be used to compute an optimal schedule.

Machine scheduling problems have traditionally been analyzed under fully deterministic assumptions. Over the last decade, significant attention has been given to the generally harder case where the jobs' *processing times* are random; see, e.g., Pinedo (1984). In practice, most of the uncertainty centers around the availability of the machine(s) which may well be subject to lengthy and unpredictable breakdowns. In many manufacturing lines, for example, there is little or no uncertainty with respect to the actual processing times of the different production lots or orders, but lengthy and poorly predictable breakdowns and repair times present major challenges to the production scheduler trying to meet deadlines with "minimal" inventories. Similarly, machine operations need to be interrupted when their performance violates quality control standards and the process of interruptions and the times required to restore the machine to acceptable performance are often of a highly stochastic nature.

To account for possible breakdowns and repair times, it is advisable to start a significant number of jobs well in advance of the times at which they would otherwise be started. To determine which jobs should be scheduled early on, in what sequence and at what specific times, one faces a fundamental tradeoff between so-called tardiness and earliness costs. An appropriate model should thus incorporate an accurate description of the breakdown and repair time processes, and schedules should be determined to minimize an appropriate, combined earliness and tardiness (E/T) cost objective.

In parallel to the above development of *stochastic* scheduling models (all with tardiness costs only, except for Forst (1993)), a significant literature on scheduling with E/T costs has arisen in the last decade, but it confines itself almost invariably to fully deterministic settings; see, e.g., Baker and Scudder (1990).

Under stochastic breakdowns, several assumptions may be made regarding the impact of a breakdown on the job in process. In the *preempt-resume* case, the breakdown merely acts as an interruption, i.e., the job in process can be resumed without loss of prior work as soon as the machine is back in operation. At the other extreme, the *preempt-repeat* case, all prior work on an interrupted job is lost. Another important distinction is whether the scheduler has *general* or *simple* recourse, i.e., whether the schedule can be dynamically adjusted in a general, nonanticipative way (see, e.g., Rockefellar and Wets (1976)) or a fixed permutation of jobs is to be chosen and only the jobs' starting times can be adjusted in response to breakdowns and repairs. General recourse models result in Markov Decision Processes which in general are too large to be solved to optimality. (See, however, Glazebrook (1984, 1987), Pinedo and Rammouz (1988) and Browne and Glazebrook (1992) for a number of models, with tardiness costs only.) In this paper we confine ourselves to preempt-resume and simple recourse settings.

Mittenthal and Raghavachari (1993) are the first to address a (preempt-resume, simple recourse) single machine model with E/T costs and breakdowns, building on Birge et al. (1991) who focused on tardiness costs only. They prove that a schedule of V-shape is optimal if the sum of squared deviations from a common due-date is to be minimized, the breakdown process is Poisson and the repair times are independent and identically distributed (i.i.d.). (A sequence is of V-shape if the corresponding sequence

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of processing times has a single local minimum, disregarding ties.)

In this paper we consider single machine scheduling problems with a common due-date for all jobs and (i) *arbitrary* monotone E/T cost functions and (ii) *arbitrary* breakdown and repair processes. We show that the problem is equivalent to a deterministic one without breakdowns and repairs and with an *equivalent* cost function of a job's completion time. A V-shaped schedule without idle times is shown to be optimal, if this equivalent cost function is *quasi-convex*. Conversely, we show that a V-shaped schedule may fail to be optimal if this property does not apply by giving examples of breakdown processes under which this occurs, even when the original E/T cost structure is quadratic (and in particular convex). As a special case, our results resolve the question raised in Mittenthal and Raghavachari whether a V-shaped schedule is optimal under nonhomogeneous Poisson breakdown processes. We derive general conditions for the E/T cost structure and repair and breakdown processes under which the equivalent cost function is quasi-convex. More generally, quasiconvexity can, however, be verified for any *specific* model by a one-time inspection of the equivalent cost function, either via standard calculus, or other numerical methods.

The significance of our results follows from the following observations: A large variety of earliness cost functions arise depending, e.g., on what types of inventory and job maintenance costs are incurred, or whether the jobs are perishable and if so, according to what pattern they decay. Many types of nonlinear tardiness costs arise depending, e.g., on the type of contractual penalties, expected goodwill or future revenue losses involved. We refer to Baker and Scudder and Federgruen and Mosheiov (1993a) for a discussion of many combined E/T cost structures. When a V-shaped schedule is optimal, a simple dynamic programming algorithm in Kahlbacher (1992) or Federgruen and Mosheiov (1993a) can be used to compute a schedule which is optimal among all schedules whose starting time is restricted to the points of any prespecified grid. Its running time is $O(NP_{\text{tot}}^2\Delta^{-1})$ where N denotes the number of jobs, P_{tot} the total processing time of the jobs and Δ the width of the chosen grid. Finally, the model and solution methods may be used to efficiently assess the impact of quality improvements.

1. THE GENERAL MODEL: OPTIMALITY OF A V-SHAPED SEQUENCE

Consider a (single machine) scheduling problem with N jobs, a common due date d and processing times $\{P_1, \ldots, P_N\}$. Let $P_{\text{tot}} = \sum_{J=1}^{N} P_j$. Breakdowns are generated by a point process $\{N(t)\colon t \geq 0\}$ with nondecreasing sample paths, where $N(t)$ = the number of breakdowns after t time units of work. Once the machine breaks down, it is repaired. Let Y_i = the duration of the ith repair time, $i \geq 1$.

The sequence of $\{Y_i\}$ is generated by an arbitrary (discrete-time) process. In particular we allow the sequence to be *nonstationary* (to reflect that repairs may be progressively easier or harder, or that they follow a more general learning curve), or *correlated* (so as to represent interdependencies between consecutive repairs). The repair and breakdown processes may even be *dependent* on each other.

For a given schedule of jobs π, with starting time s, let

$C_j(\pi)$ = the completion time of job j if *no* interruptions occur, and

$\bar{C}_j(\pi)$ = the *actual* completion time of job j under the schedule π.

In view of the preempt-resume assumption:

$$\bar{C}_j(\pi) = C_j(\pi) + \sum_{\ell=1}^{N(C_j(\pi)-s)} Y_\ell. \tag{1}$$

The earliness (tardiness) cost of job j ($j = 1, \ldots, N$) is given by a common function $F(\cdot)$ ($G(\cdot)$) of the amount of time the job is completed before (after) the due date, i.e., by $F(d - \bar{C}_j(\pi))$ and $G(\bar{C}_j(\pi) - d)$. We assume without loss of generality that $F(\cdot)$ and $G(\cdot)$ are nondecreasing and strictly increasing on some interval, with $F(x) = G(x) = 0$ for $x \leq 0$. Many scheduling models have focused on special choices, e.g., $F(x) = G(x) = x$ or $F(x) = G(x) = x^r$ for some $r > 1$, i.e., the objective is to minimize the sum of the rth power of the deviations of the jobs' completion times from the due-date, for some $r \geq 1$. As discussed, many other types of nonlinear cost functions may arise; in particular, $F(\cdot)$ and $G(\cdot)$ are often *different*, i.e., the cost structure is *asymmetric*. Our objective is to minimize the expected total cost:

$$= \min_{\pi} \sum_{j=1}^{N} E[F(d - \bar{C}_j(\pi)) + G(\bar{C}_j(\pi) - d)]. \tag{2}$$

The scheduling problem with stochastic breakdowns and repairs is thus equivalent to a deterministic problem *without* breakdowns, in which the cost of a job depends both on its completion time C and the starting time s of the entire schedule, and is given by:

$$\Phi_s(C) \stackrel{\text{def}}{=} EF\left(d - C - \sum_{\ell=1}^{N(C-s)} Y_\ell\right) + EG\left(C + \sum_{\ell=1}^{N(C-s)} Y_\ell - d\right). \tag{3}$$

We now obtain our main result:

Theorem 1. *Assume the cost function* $\Phi_s(\cdot)$ *is quasi-convex for all* $s > 0$, *i.e.,* $\max\{\Phi_s(C_1), \Phi_s(C_2)\} \geq \Phi_s(C)$ *for all* $C_1 < C < C_2$. *There exists an optimal schedule without idle times which is of V-shape.*

Proof. It suffices to prove that for *any* fixed starting time $s = s^*$, an optimal schedule exists which is of V-shape and inserts no idle times between consecutive jobs. Under the fixed starting time, the cost of a job is given by $\Phi^*(C) = \Phi_{s^*}(C)$, i.e., it is a function of its completion time only.

Since this function is quasi-convex, the result follows from Krieger and Raghavachari (1988) and Federgruen and Mosheiov (1993a, Theorem 1). □

Thus, if $\Phi_s(\cdot)$ is quasi-convex for all $s > 0$, the scheduling problem reduces to (i) finding an optimal starting time s and (ii) selecting one of the 2^{N-1} V-shaped job sequences. To determine an optimal schedule for a given starting time s, we need to distinguish between two cases:

Case 1. $\Phi_s(\cdot)$ is nondecreasing on the entire positive half line.

Case 2. $\Phi_s(\cdot)$ has a (possibly nonstrict) global minimum δ, i.e., $\Phi_s(C)$ is nonincreasing for $C < \delta$ and $\Phi_s(C)$ is nondecreasing for $C > \delta$. (We call the smallest such global minimum δ the due-date in the equivalent deterministic problem, since in the latter a job incurs minimal cost when completed at the due-date.)

The remaining case, where $\Phi_s(C)$ is nonincreasing for all $C > 0$, cannot occur as shown in the following lemma:

Lemma 2. *Fix $s > 0$. The function $\Phi_s(C)$ is nondecreasing for $C \geqslant d$.*

Proof. Note that for $C \geqslant d$, the first term in (3) vanishes. The family of random variables $X(C) \stackrel{\text{def}}{=} C + \sum_{\ell=1}^{N(C-s)} Y_\ell - d$ is stochastically increasing, since $N(C_2 - s) \geqslant N(C_1 - s)$ almost surely for all $C_1 < C_2$. This implies that $Eh(X(C))$ is nondecreasing in C for *any* nondecreasing function $h(\cdot)$, in particular for the function $h(x) = G(x)$, see, e.g., Whitt (1981). □

In Case 1 it is optimal to start the schedule at time s and schedule the jobs in nondecreasing order of their processing times. In Case 2, the existence of a V-shaped schedule implies that for any $k = 2, \ldots, N$ the subsequence of jobs $\{1, \ldots, k-1\}$ is *contiguous* and that job k is placed either immediately before or after the subsequence. Thus, for all $k = 1, \ldots, N$, let $S_k = \sum_{j=1}^{k} P_j$ and $V_s(k, t)$ = minimum cost incurred for jobs $1, \ldots, k$, given that the first of these jobs (in the equivalent deterministic model) is to start at time t while the entire sequence is to start at time s ($t = s, s+1, \ldots, s + S_N - S_k$). The function $V_s(k, t)$ clearly satisfies the recursion:

$$V_s(k, t) = \min\{\Phi_s(t + P_k) + V_s(k-1, t + P_k); \; \Phi_s(t + S_k) + V_s(k-1, t)\}, \quad k = 2, \ldots, N;$$

$$V_s(1, t) = \Phi_s(t + P_1).$$

The optimal schedule and its cost are obtained by computing $V_s(N, s)$.

This dynamic program, which is a variant of that in Kahlbacher, has complexity $O(NP_{\text{tot}})$. Given the nonlinearity of the functions $\Phi_s(\cdot)$ the exact optimal value of s can, in general, not be determined. Thus, restricting s to the points of an arbitrary discrete grid with width Δ, the dynamic program has to be repeated $\Delta^{-1} \min(d; P_{\text{tot}})$ times and the overall complexity is $O(NP_{\text{tot}} \min(d; P_{\text{tot}})\Delta^{-1})$. Alternatively, one can use the dynamic programming method of Federgruen and Mosheiov (1993a) which has significantly lower complexity when the cost function $\Phi(\cdot)$ is independent of the starting time s. Kahlbacher does not report any computational experience; Federgruen and Mosheiov show that problems with several hundreds of jobs, integer processing times between 1 and 100 and a grid of width $\Delta = 0.01$ can be solved in about one minute of CPU time on an IBM 4381 (VM/CMS).

Thus, to verify whether an optimal schedule exists which is of V-shape and avoids idle times, it suffices to determine whether for any $s > 0$, the function $\Phi_s(\cdot)$ has a strict (positive) local maximum or not. There are broad classes of functions F and G, combined with general types of breakdown and repair processes under which quasi-convexity of the expected tardiness costs, earliness costs or both can be proved; see, e.g., Theorem 4 below. In other settings, (e.g., Examples 1 and 2 below) one can obtain Φ_s as a closed form analytical and twice differentiable function, thus establishing quasi-convexity by inspecting the sign of its second derivative.

Example 1. Assume $F(x) = G(x) = x^r$, for some even integer r. Let breakdowns be generated by a nonhomogeneous Poisson process with rate $\lambda(t)$, $t > 0$. Assume repair times are i.i.d. with kth moment $\mu^{(k)}$, $k \leqslant r$. Conditioning on $N(C - s)$, a Poisson random variable with mean $\Lambda(C - s) \stackrel{\text{def}}{=} \int_0^{C-s} \lambda(t)\,dt$ one obtains for $r = 2$, $\Phi_s(C) = (C + \mu\Lambda(C - s) - d)^2 + \mu^{(2)}\Lambda(C - s)$ (see, e.g., Mittenthal and Raghavachari) and, in general,

$$\Phi_s(C) = \sum_{m=0}^{r} \binom{r}{m} (C - d)^{r-m} \sum_{\sum_{i=1}^{m} i a_i = m} \prod_{k=1}^{m} [(\mu^k)^{a_k}] \, (\Lambda(C - s))^{a_1 + \cdots + a_m}$$

(see, e.g., Proposition 2 in Federgruen and Katalan (1994)). Thus, whenever $\lambda(\cdot)$ is differentiable and integrable in closed form, $\Phi_s(C)$ is available as a closed form, twice differentiable function; e.g., when $\lambda(\cdot)$ is a polynomial, $\Phi_s(\cdot)$ is a polynomial as well where (local) minima can be found by computing the roots of the polynomial $\Phi'(\cdot)$ via a standard method.

Example 2. Consider example 1, however, with $\lambda(t) = \lambda$, i.e., breakdowns generated by an *ordinary* Poisson process. On the other hand, let $(Y_1, Y_2, \ldots)$ be a general multivariate Normal vector (possibly with nonidentical or correlated repair times). Let $\phi_n(\cdot)$ denote the (Normal) pdf of $S_n = \sum_{\ell=1}^{n} Y_\ell$, $n \geqslant 1$. Then

$$\Phi_s(C) = E\left[C + \sum_{\ell=1}^{N(C-s)} Y_\ell - d\right]^r = \sum_{n=0}^{\infty} \frac{e^{-\lambda(C-s)}[\lambda(C-s)]^n}{n!} \int_0^{\infty} [C + u - d]^r \phi_n(u)\,du$$

where each of the integrals can be obtained in closed form as a linear combination of the first r moments of the (normally distributed) variables (S_n). Thus $\Phi_s(C)$ is available in closed form as a so-called generalized exponential in C (and s).

Finally, if no closed form expression can be derived, one needs to search for local optima until more than one is found or it is verified that at most a *single* local minimum exists. This can be done by a variety of global optimization methods (see, e.g., Section 6 in Rinnooy Kan and Timmer (1989)) many of which operate without derivative information. In the worst case, quasi-convexity can be verified, under *integer* processing times and starting times restricted to the above mentioned grid, by (full) evaluation of the values $\{\Phi_s(C)\}$ on finitely many grid points.

To derive general conditions for quasi-convexity, we first need the following definitions: For a general point process $\{N(t)\colon t \geqslant 0\}$ one defines the *stochastic* intensity by $\zeta_t \overset{\text{def}}{=} \lim_{\downarrow 0} \epsilon^{-1}\text{Prob}[N(t+\epsilon) - N(t) = 1 | N(s), 0 \leqslant s \leqslant t]$ and the *conditional* intensity $\lambda(t, n)$ by $E[\zeta_t | N(t) = n]$, see, e.g., Brémaud (1981). (The stochastic intensity does not always exist; it exists, however, for most processes used in practical modelling, e.g. (nonhomogeneous) Poisson, renewal or Markov modulated Poisson processes, see Walrand (1988).) A family of random variables $\{X(C)\}$ is SICX (stochastically increasing convex), if $E\phi(X(C))$ is increasing (increasing convex) in C for every increasing (increasing, convex) function ϕ. The family is SIL (stochastically increasing linear) if it is SICX and in addition $E\phi(X(C))$ is increasing concave for every increasing, concave function ϕ.

Lemma 3. *Assume that the breakdown process $\{N(t)\}$ has a conditional intensity $\lambda(t, n)$ which is nondecreasing in t and in n, and that repair times are i.i.d. and independent of the breakdown process. Fix $s > 0$.*

$$X_s(C) \overset{\text{def}}{=} C + \sum_{\ell=1}^{N(C-s)} Y_\ell$$

$- d$ is SICX *in C, and* SIL *in C if $\{N(t)\}$ is Poisson.*

Proof. The family $\{N(C)\}$ is SICX (SIL in the Poisson case) since the conditional intensity λ is monotone (constant), see Theorem 5.11 in Shanthikumar and Yao (1992). Also $R(n) = \Sigma_{\ell=1}^{n} Y_\ell$ is SIL in n, see, e.g., Example 6.A.3 in Shaked and Shanthikumar (1994), so that $R(N(C-s))$ is SICX in C (and SIL in the Poisson case) by Theorem 6.A.13 there. □

Theorem 4. *Assume repair times are i.i.d. and independent of the breakdown process $\{N(t)\}$.*

(a) *Let $\{N(t)\}$ be Poisson. If F and G are convex, then so are the cost functions $\Phi_s(\cdot)$ for all $s > 0$. In particular, there exists an optimal schedule without idle times which is of V-shape.*

(b) *Assume the conditional intensity $\lambda(t, n)$ is nondecreasing in t and n. If $G(\cdot)$ is convex, then the expected tardiness cost is convex in C for every s.*

Proof. By Lemma 3 the expected tardiness cost is convex in C while $E\{-F(-X_s(C)\}$ is concave in C since $\phi(x) = -F(-x)$ is increasing concave in x; hence, the expected earliness cost is convex in C as well. Part (b) is again immediate from Lemma 3. □

Remark. Convexity of the expected earliness costs cannot be established for *general* breakdown processes with monotone conditional intensities; since for the latter $\{X_s(C)\}$ is only SICX, convex *decreasing* functions of $X_s(C)$ may fail to be convex in C.

We now discuss a number of important breakdown processes to which Theorem 4 is easily applied: (B1) The breakdown process is Poisson or a nonstationary Poisson process with nondecreasing rate $\lambda(b)$. (B2) Breakdowns are generated by a *mixed* Poisson process, i.e., $N(t) \overset{\text{def}}{=} P(W\Lambda(t))$ where $P(t)$, $t \geqslant 0$ is a standard Poisson process with *unit* rate, $\Lambda(\cdot)$ is a nonincreasing function with $\Lambda(0) = 0$, and W an independent positive-valued random variable with cdf $H(\cdot)$. (B3) Breakdowns arise according to a Markov modulated Poisson process, i.e., a Poisson process whose rate is an increasing function of the state of an underlying Markov chain, where the state is stochastically increasing in $N(t) = n$.

Regular nonstationary Poisson processes (B1) arise as a special case of (B2) with W a constant. (B2) is frequently used to model self-exciting failures or breakdowns, see, e.g., Browne and Glazebrook (1992). Special cases include the Polya processes where $\Lambda(t) \equiv t$ and W has a gamma (a, b) distribution, i.e., $H'(w) = w^{a-1}e^{-w/b}[b^a\Gamma(a)]^{-1}$ and $\lambda(t, n) = b(n + a)\,(1 + bt)^{-1}$. The *time homogeneous* Yule process has a conditional stochastic intensity which grows *linearly* with the number of failures experienced to date and independently of the time elapsed, i.e., $\lambda(t, n) = A(n + a)$ for some $a, A > 0$. This case arises when $\Lambda(t) = e^{At} - 1$ and W is a gamma $(a, 1)$. In all these cases, $\lambda(t, n)$ is linear in n, and the distribution of $N(t)$ is negative binomial $(t > 0)$. This fits many settings that cannot be modeled by regular nonstationary Poisson processes in which $N(t)$ is always Poisson $(t > 0)$.

The centrality of the mixed Poisson processes follows from its *characterization*, due to Feigin (1979), as *the* class of counting processes with the order statistic property, i.e., conditional upon $N(t) = n$, the first n breakdown times are distributed as the order statistics of n i.i.d. random variables with a general distribution on the interval $(0, t)$. If $\Lambda(t)$ is twice differentiable, Browne and Glazebrook show that $\lambda(t, n) = \Lambda''(t)\psi(n + 1, t)/\psi(n, t)$ where $\psi(n, t) = E[W^n e^{-W\Lambda(t)}]$, $(n = 1, 2, \ldots; t \geqslant 0)$, that $\lambda(t, n)$ is always increasing in n, and that it is increasing in t, if $\Lambda''(t) \geqslant \psi(n + 2, t)/\psi(n + 1, t) - \psi(n + 1, t)/\psi(n, t)$. Based on this simple characterization of the conditional intensity, there are established methods to fit an appropriate mixed Poisson process to empirically observed failure rates, see, e.g., Gerber (1981).

Theorem 4 generalizes the results in Mittenthal and Raghavachari who showed that a V-shaped schedule is optimal for the special case of $F(x) = G(x) = x^2$, $s = 0$ and breakdowns generated by a pure Poisson process. These authors also consider generalized or compound Poisson breakdown processes. However, with i.i.d. repair times that are independent of the breakdowns, this is equivalent to *pure* Poisson breakdowns and the total time required to resolve all breakdowns occurring simultaneously, as the repair time. Example 1 also provides a negative answer to the authors' open question whether a V-shaped schedule is necessarily optimal when the breakdowns are generated by a *nonhomogeneous* Poisson process. In this example, let $r = 2$, $d = 5$ and $\lambda(t) = e^{-2(t-5)}$, $t > 0$. With μ and σ the mean and standard deviation of the repair times, we have:

$$\Phi_0(C) = \left(C + \mu \int_0^C \lambda(t)\,dt - d\right)^2 + (\mu^2 + \sigma^2) \int_0^C \lambda(t)\,dt.$$

As μ tends to zero, we have $\Phi_0'(C) = 2(C - 5) + 0.01e^{-2(C-5)}$, $C > 0$. Φ_0 is increasing for $C < 1.76$, has a local maximum in $\bar{\delta} \simeq 1.76$, a local minimum in $\underline{\delta} \simeq 4.975$ and it is increasing for $C > \underline{\delta}$. In other words, the function fails to be quasi-convex. Consider now a problem instance with $N = 3$ and $P_1 = 0.5$, $P_2 = 2.5$ and $P_3 = 6$. Let $s = 0$ be the starting time. The schedule with *non-V-shaped* sequence $q = (1, 3, 2)$ and no idle times is cheaper than any V-shaped sequence: see Federgruen and Mosheiov (1993b) for details.

The following provides an intuitive explanation for the peculiar shape of $\Phi_0(\cdot)$. Clearly for small values of C, $\Phi_0(C)$ increases since the increase in expected tardiness costs exceeds the decrease in expected earliness costs and this in spite of $C < d$. However, because the failure rate *decreases* with time there exists a local maximum $\bar{\delta} < d$ such that for $C > \bar{\delta}$ sufficiently small, i.e., $\bar{\delta} < C < \underline{\delta}$, $\Phi_0(\cdot)$ decreases; the increase in expected tardiness costs over this interval is smaller than the decrease in expected earliness costs.

Federgruen and Mosheiov (1993b) show that the optimal cost and optimal schedule can be significantly different from those arising (i) in the absence of breakdowns, and (ii) when each processing time is replaced by the expected total time in process including all repair times, a common practice in scheduling systems.

Finally, quality control focuses on improving the reliability of manufacturing processes, e.g., by improved training, by expediting repairs, or by more reliable technologies. The model and solution methods above are well suited to quantify the impact of such quality improvements. We illustrate this with the special case of Example 1 with $r = 2$, and Poisson breakdowns with rate λ. The analysis of the example shows that $\Phi_s(C)$ and hence the cost of any schedule is a simple quadratic function of d and the *three* dimensions of reliability λ, μ, and σ. It follows that the minimum cost z^* is increasing in λ, μ and σ, decreasing in d, and piecewise quadratic in all four parameters, see Federgruen and Mosheiov (1993b, Proposition 2.1). Figures 1–4 there, exhibit, for a specific 100 job problem, how z^* varies with each of the parameters. The ability to generate such optimal cost curves efficiently is important in design and quality studies.

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